MOHAMED SATHAK
A. J. COLLEGE OF ENGINEERING
SIRUSERI IT PARK,OMR, CHENNAI 603 103

DEPARTMENT OF CIVIL ENGINEERING

CE 8491 - SOIL MECHANICS

Course Objectives
The Student should be able

S. No.	Course Objective
1	To impart knowledge to classify the soil based on index properties and to assess their engineering properties based on the classification. To familiarize the students about the fundamental concepts of compaction, flow through soil, stress transformation, stress distribution, consolidation and shear strength of soils. To impart knowledge of design of both finite and infinite slopes.

Course Outcomes:
On Completion of the course the students will be able to

CO No.	Course Outcome
1	classify the soil and assess the engineering properties, based on index properties.
2	Understand the stress concepts in soils
3	Understand and identify the settlement in soils.
4	Determine the shear strength of soil
5	Analyze both finite and infinite slopes.

Prepared by,
Mrs.V.JANAKI,B.E,M.E,.
Assistant professor
Department of Civil Engineering

SYLLABUS

UNIT I SOIL CLASSIFICATION AND COMPACTION 9

History – formation and types of soil – composition - Index properties – clay mineralogy structural arrangement of grains – description – Classification – BIS – US – phase relationship – Compaction – theory – laboratory and field technology – field Compaction method – factors influencing compaction.

UNIT II EFFECTIVE STRESS AND PERMEABILITY 9

Soil - water – Static pressure in water - Effective stress concepts in soils – Capillary phenomena– Permeability – Darcy's law – Determination of Permeability – Laboratory Determination (Constant head and falling head methods) and field measurement pumping out in unconfined and confined aquifer – Factors influencing permeability of soils – Seepage - Two-dimensional flow – Laplace's equation – Introduction to flow nets – Simple problems Sheet pile and wier.

UNIT III STRESS DISTRIBUTION AND SETTLEMENT 9

Stress distribution in homogeneous and isotropic medium – Boussines of theory – (Point load, Line load and udl) Use of Newmarks influence chart –Components of settlement – Immediate and consolidation settlement – Factors influencing settlement – Terzaghi's one dimensional consolidation theory – Computation of rate of settlement. – √t and log t methods. e-log p relationship consolidation settlement N-C clays – O.C clays – Computation.

UNIT IV SHEAR STRENGTH 9

Shear strength of cohesive and cohesion less soils – Mohr-Coulomb failure theory – shear strength - Direct shear, Triaxial compression, UCC and Vane shear tests – Pore pressure parameters – Factors influences shear strength of soil.

UNIT V SLOPE STABILITY 9

Infinite slopes and finite slopes — Friction circle method – Use of stability number –Guidelines for location of critical slope surface in cohesive and c - soil – Slope protection measures.

UNIT-1

SOIL CLASSIFICATION AND COMPACTION

History - formation and types of soil - composition - Index properties - clay mineralogy structural arrangement of grains - description - classification - BIS - US - phase relationship - compaction - theory - laboratory and field technology - field compaction method - factors influencing compaction.

Introduction

Soil is an unconsolidated material. It comprises of solids, air, water or both or all. It is derived from disintegration of rock.

The process of soil formation is called pedogenesis. It is a cyclic process called geological cycle.

Soil Mechanics

Soil mechanics is the study of engineering behaviour of soil when it is used either as a construction material or as a foundation material.

Classification of Soil / Types of Soil

Almost all the soils are derived from weathering of rock material is called sediment. Weathering may be due to physical or chemical agencies. If weathered material is transported and deposited at other place then it is called transported soil. If weathered material remains over parent rocks then soil is called residual soil.

1) Alluvial Soils.

Found in Indo - Ganga - Brahmaputra plains and river plains. These are deposited by river. These may contain gravel, sand, silt and clay.

2) Talus

These are found in mountain valley. These are formed by gravity forces. Due to temperature and moisture variation, creeping of soil and land sliding, such sediments get deposited in lower part of mountain valleys.

3) Lacustrine soil

These are deposited by still water such as lakes. These are uniformly graded.

4) Glacial Soil

It is formed by glaciers and ice berg. These contain mixture of gravel sands and clays. These are well graded.

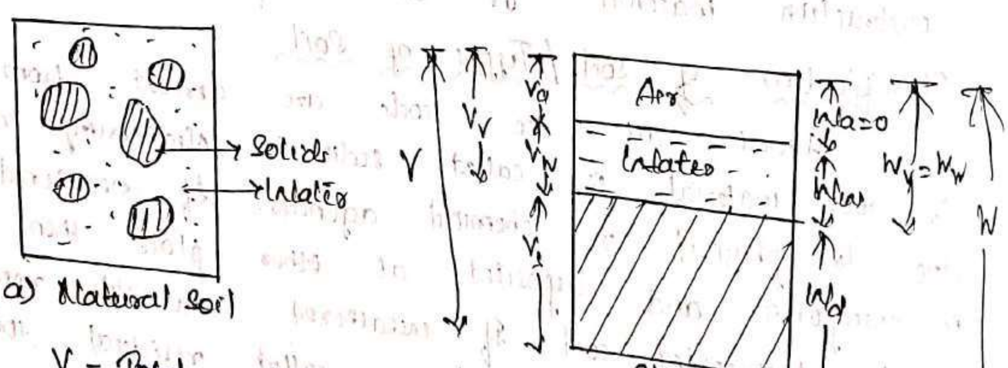
Nature of Soil

A soil mass is a three phase system consisting of solid particles, water and air. The void space between the soil grains is filled partly with water and partly with air.

Soil → 3 phase < Solid [soil grains], Water, Air } Voids.

Dry soil mass → soil grains + air

Perfectly saturated soil → soil grains + water.



a) Natural soil

Element separated into 3 phases.

V - Total volume of the soil mass

V_v - Volume of voids.

V_s - Volume of solids

V_w - Volume of water

V_a - Volume of air

$V = V_v + V_s$

$V_v = V_w + V_a$

$V = V_w + V_a + V_s$.

W - Total weight of sample $\qquad W = W_d + W_v$

W_d - Weight of solids

W_w - Weight of water $\qquad W_v = W_a + W_w$

W_a - Weight of air (always zero) $\qquad W_a = 0$

W_v - Weight of voids. $\qquad W_w = W_v$

$\therefore W = W_d + W_v$

$= W_d + W_w$

a) Water Content (w)

Water content is also called as moisture content is defined as ratio of weight of water (W_w) to weight of solids (W_d)

$$w = \frac{W_w}{W_d} \times 100 \qquad [W_w = W - W_d]$$

w is always expressed as percentage.

$$w = \left[\frac{W - W_d}{W_d}\right] \times 100$$

$$= \left[\frac{W}{W_d} - 1\right] \times 100.$$

b) Density of soil

It is defined as mass of the soil per unit volume.

i) Bulk density (ρ) $\rho = \frac{M}{V}$ [g/cm^3 or kg/m^3]

Bulk density is also known as moist density. It is the total mass of soil per unit of its total volume.

ii) Dry density (ρ_d).

It is the mass of solids per unit of total volume of soil mass.

$$\rho_d = \frac{M_d}{V}.$$

iii) Density of solids

It is the mass of soil solids (M_d) per unit volume of solids (V_s)

$$\rho_s = \frac{M_d}{V_s}$$

iv) Saturated density (ρ_{sat})

When soil mass is saturated, its bulk density is called saturated density. It is the ratio of total soil mass to its total volume.

$$\rho_{sat} = \frac{M}{V}$$

v) Submerged density (ρ') / Buoyant density

It is the submerged mass of soil solids $(M_d)_{sub}$ per unit volume V of soil mass.

$$\rho' = \frac{(M_d)_{sub}}{V}$$

$$= \rho_{sat} - \rho_w$$

c) Unit Weight of soil mass

It is defined as weight of soil per unit volume

i) Bulk unit weight (γ) / Moist unit weight

It is the total weight (W) of a soil mass per unit of its total volume (V)

$$\gamma = \frac{W}{V}$$

ii) Dry unit weight (γ_d)

It is the ~~to~~ weight of solids per unit of its total volume of soil mass.

$$\gamma_d = \frac{W_d}{V}$$

iii) Unit weight of solids (γ_s)

It is the weight of soil solids (W_d) per unit of its volume of solids (V_s)

$$\gamma_s = \frac{W_d}{V_s}$$

iv) Saturated Unit Weight (γ_{sat})

When soil mass is saturated, its bulk unit weight is called saturated unit weight. It is the ratio of total weight of saturated soil sample to its total volume.

$$\gamma_{sat} = \frac{W}{V}$$

Inter convention between density and unit weight.

To convert density into unit weight, multiply density by 9.81

$$1\ g/cm^3 = 9.81\ kN/m^3 \quad ; \quad \gamma = 9.81 \times \rho$$

Specific Gravity

It is defined as the ratio of weight of soil solids at a given temperature to the weight of an equal volume of distilled water at that temperature, both weights taken in air.

It is the ratio of unit weight of soil solids to that of water.

$$G = \frac{\gamma_s}{\gamma_w}$$

Voids are excluded for determining true volume of soils, then it is absolute/true specific gravity

The Apparent / mass / bulk specific gravity (G_m) denotes specific gravity of soil mass.

$$G_m = \frac{\gamma}{\gamma_m}.$$

~~Void Ratio~~

~~It is the ratio of volume of voids to total volume of given mass.~~

Void Ratio

It is the ratio of volume of voids to the volume of soil solids in given soil mass.

$$e = \frac{V_v}{V_s}$$

Porosity

It is the ratio of volume of voids to the total volume of given mass.

$$n = \frac{V_v}{V}.$$

(5)

Relation between void ratio and porosity.

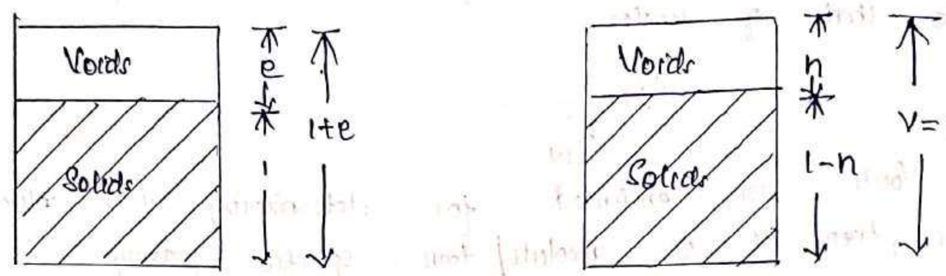
$$e = \frac{V_v}{V_s} \quad ; \quad n = \frac{V_v}{V}$$

If $V_s = 1$; $e = V_v$, then $V = V_v + V_s$

$$= e + 1$$
$$= 1 + e$$

$$n = \frac{V_v}{V} = \frac{e}{1+e}$$

$$\boxed{n = \frac{e}{1+e}}$$



a) In terms of e b) In terms of n

If $V = 1$, $n = V_v \rightarrow n = \frac{V_v}{V}$ V→which is 1

Then $V_s = V - V_v$

$$= 1 - n$$

$$e = \frac{V_v}{V_s} = \frac{n}{1-n}$$

$$\therefore e = \frac{n}{1-n}$$

$$n = \frac{e}{1+e} = e\left[\frac{1}{1+e}\right]$$

$$\cancel{n} = \frac{\cancel{n}}{1-n}\left[\frac{1}{1+e}\right]$$

$$1 - n = \frac{1}{1+e}$$

Degree of Saturation (S)

It is defined as ratio of volume of water present in a given soil mass to the total volume of voids in it.

$$S = \frac{V_w}{V_v} \rightarrow \text{always expressed as percentage.}$$

Fully saturated soil ($S=1$)

$$V_w = V_v$$

$$S = \frac{V_w}{V_w} = 1$$

Perfectly dry soil

$$V_w = 0 \; ; \; S = 0$$

Percentage air voids (n_a)

It is defined as the ratio of volume of air voids to total volume of soil mass.

$$n_a = \frac{V_a}{V} \times 100 \rightarrow \text{Expressed as percentage.}$$

Air content (a_c)

It is defined as the ratio of volume of air voids to volume of voids.

$$a_c = \frac{V_a}{V_v}$$

$$V_a = V_v - V_w$$

$$a_c = \frac{V_v - V_w}{V_v} = 1 - \frac{V_w}{V_v}$$

$$= 1 - S$$

Density Index (I_D)

Density Index (or) relative density (or) density of density is defined as the ratio of difference between void ratio of soil in its loosest state (e_{max}) and its natural void ratio (e) to the difference between void ratios in the loosest and densest states.

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

e_{max} = void ratio in loosest state

e_{min} = void ratio in densest state

e = natural voids ratio of the deposit

It is only used for cohesionless soil.

When cohesionless soil is in its loosest form

$e = e_{max}$; $I_D = 0$

When cohesionless soil is in its densest form

$e = e_{min}$; $I_D = 1$

Any intermediate state ; $I_D = 0$ to 1

I_D in terms of density

$$e = \frac{G\gamma_w}{\gamma_d} - 1 \quad ; \quad e_{min} = \frac{G\gamma_w}{\gamma_{dmax}} - 1$$

$$\therefore I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

$$= \frac{\left[\frac{G\gamma_w}{\gamma_{dmin}} - 1\right] - \left[\frac{G\gamma_w}{\gamma_d} - 1\right]}{\left[\frac{G\gamma_w}{\gamma_{dmin}} - 1\right] - \left[\frac{G\gamma_w}{\gamma_{dmax}} - 1\right]}$$

$$= \left[\frac{\frac{G\gamma_w}{\gamma_{dmin}} - \frac{G\gamma_w}{\gamma_d}}{\frac{G\gamma_w}{\gamma_{dmin}} - \frac{G\gamma_w}{\gamma_{dmax}}}\right]$$

$$= \frac{G\gamma_w}{G\gamma_w}\left[\frac{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_d}}{\frac{1}{\gamma_{dmin}} - \frac{1}{\gamma_{dmax}}}\right]$$

$$= \frac{\frac{\gamma_d - \gamma_{dmin}}{\gamma_d \cdot \gamma_{dmin}}}{\frac{\gamma_{dmax} - \gamma_{dmin}}{\gamma_{dmax} \cdot \gamma_{dmin}}}$$

$$\boxed{I_D = \frac{\gamma_d - \gamma_{dmin}}{\gamma_{dmax} - \gamma_{dmin}} \times \frac{\gamma_{dmax}}{\gamma_d}} \quad (8)$$

I_D in terms of porosity

$$I_D = \frac{(n_{max} - n)(1 - n_{min})}{(n_{max} - n_{min})(1 - n)}$$

γ_d - in situ dry density

γ_{dmax} - maximum dry density

γ_{dmin} - minimum dry density

n - insitu porosity

n_{max} - maximum porosity at loosest state.

n_{min} - minimum porosity at densest state.

Relative Density (I_D) %	Description
0-15	Very loose
15-35	Loose
35-65	Medium
65-85	Dense
85-100	Very dense

Relative Compaction (R_c)

Degree of compaction expressed in terms of index is called relative compaction.

$$R_c = \frac{\gamma_d}{\gamma_{dmax}}$$

γ_{dmax} - maximum dry density from compaction test

$\because \gamma_d = \gamma_s (1+e)$

$$R_c = \frac{1 + e_{min}}{1+e}$$

In terms of relative density,

$$R_c = \frac{R_0}{1 - I_D(1 - R_0)} \quad ; \quad R_0 = \frac{\gamma_{dmin}}{\gamma_{dmax}}$$

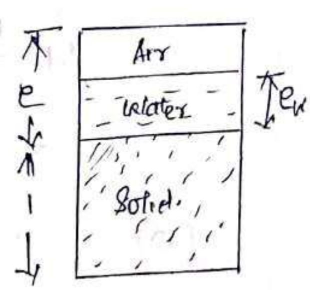
Phase Relationships

1) Relation between e, G, w and S

Let volume of water (V_w) = e_w

Volume of voids (V_v) = e

Volume of solids (V_s) = 1



(9)

$$S = \frac{V_{w}}{V_{v}} = \frac{e_{w}}{e}$$

$$e_{w} = e \cdot S \quad \text{——— (a)}$$

e_w - water voids ratio ; for fully saturated sample, $e_w = e$

$$w = \frac{W_{w}}{W_{d}} = \frac{e_{w} \cdot \gamma_{w}}{\gamma_{s} \times 1}$$

$\gamma_s = \frac{W_d}{V_s}$

$V_s = 1$; $W_d = \gamma_s \times 1$

$\gamma_w = \frac{W_w}{V_w}$

$\therefore G = \frac{\gamma_s}{\gamma_w}$; $w = \frac{e_w}{G}$

$\therefore V_w = e_w$; $W_w = \gamma_w \cdot e_w$

$$e_{w} = w \cdot G \quad \text{——— (b)}$$

Equating (a) & (b)

$$eS = w \cdot G$$

$$e = \frac{w \cdot G}{S}$$

Fully saturated soil,

$S = 1$

$w = w_{sat}$

$e = w_{sat} \cdot G$

ii) Relation between e, S and n_a.

$$n_a = \frac{V_a}{V} = \frac{e - e_w}{1+e} = \frac{e - e \times S}{1+e} = \frac{e[1-S]}{1+e}$$

$V_a = V_v - V_w = e - e_w$

$\therefore e_w = eS$

$V = V_s + V_v = 1 + e$

iii) Relation between n_a, a_c and n

$$a_c = \frac{V_a}{V_v} \; ; \; n = \frac{V_v}{V} \; ; \; n_a = \frac{V_a}{V}$$

$$n_a = \frac{V_a}{V} \times \frac{V_v}{V_v} = a_c \cdot n$$

$$n_a = n \cdot a_c$$

iv) Relation between γ_d, G and e (or) n

$$\gamma_d = \frac{W_d}{V} = \frac{\gamma_s}{1+e}$$

(or)

$$= \frac{G\gamma_w}{1+e}$$

$\therefore W_d = \gamma_s \cdot 1$

$V = 1+e$

$\therefore G = \frac{\gamma_s}{\gamma_w}$

$\gamma_s = G \cdot \gamma_w$.

For calculating void ratio.

$$1+e = \frac{G \cdot \gamma_w}{\gamma_d} \Rightarrow e = \frac{G\gamma_w}{\gamma_d} - 1$$

w.k.t, $\quad 1-n = \frac{1}{1+e}$

$$\therefore \gamma_d = G\gamma_w (1-n)$$

$$= (1-n)\, G \cdot \gamma_w.$$

v) Relation between γ_{sat}, G and e (or) n

$$\gamma_{sat} = \frac{W_{sat}}{V} = \frac{W_d + W_w}{V}$$

$$= \frac{\gamma_s \cdot V_s + \gamma_w \cdot V_w}{V}$$

For fully saturated sample, $e_w = e$ $\quad \therefore V_w = e$

$$\gamma_{sat} = \frac{\gamma_s \cdot 1 + \gamma_w \cdot e}{1+e} \qquad \because \gamma_s = G \cdot \gamma_w$$

$$= \frac{G \cdot \gamma_w + e \cdot \gamma_w}{1+e}$$

$$= \frac{(G+e)\gamma_w}{1+e}$$

w.k.t, $\quad (1-n) = \frac{1}{1+e}$ & $n = \frac{e}{1+e}$

$$\gamma_{sat} = G \cdot \gamma_w \left[\frac{1}{1+e}\right] + \gamma_w \left[\frac{e}{1+e}\right]$$

$$= (1-n)\, G \cdot \gamma_w + n\gamma_w.$$

vi) Relation between γ, G, e and S

$$\gamma = \frac{W}{V} = \frac{W_d + W_w + W_a}{V} \qquad \because W_a = 0$$

$$\gamma = \frac{\gamma_s V_s + \gamma_w \cdot V_w}{V}$$

$V_s = 1$; $V_w = e_w$ & $V = 1+e$

$$\gamma = \frac{\gamma_s \cdot 1 + \gamma_w \cdot e_w}{1+e}$$

$$= \frac{G \cdot \gamma_w + e_w \cdot \gamma_w}{1+e}$$

$$= \frac{G \cdot \gamma_w + e \cdot S \cdot \gamma_w}{1+e}$$

$$\gamma = \frac{(G+es)\gamma_w}{1+e}$$

vii) Relation between γ_d, γ and w

$$w = \frac{W_w}{W_d}$$

$$1+w = 1+\frac{W_w}{W_d}$$

$$= \frac{W_d + W_w}{W_d} = \frac{W}{W_d}$$

$$W_d = \frac{W}{1+w}$$

$$\therefore \gamma_d = \frac{W_d}{V} = \frac{W}{(1+w)V}$$

$$\gamma_d = \frac{\gamma}{1+w}$$

viii) Relation between γ_{sat}, γ, γ_d and S

$$\gamma = \frac{(G+eS)\gamma_w}{1+e}$$

$$= \frac{G\gamma_w}{1+e} + \frac{eS\gamma_w}{1+e}$$

$$= \gamma_d + S\left[\frac{(G+e)\gamma_w}{1+e} - \frac{G\gamma_w}{1+e}\right]$$

$$= \gamma_d + S\left[\gamma_{sat} - \gamma_d\right]$$

ix) Relation between γ_d, G, w and S

$$\gamma_d = \frac{G\gamma_w}{1+e}$$

$$= \frac{G\gamma_w}{1+\frac{wG}{S}} \qquad e = \frac{wG}{S}$$

For fully saturated soil, $S = 1$

$$\gamma_d = \frac{G\gamma_w}{1+w_{sat}\cdot G}$$

Problems

1) A soil sample has a porosity of 40%. The specific gravity of soils is 2.7. Calculate a) Void ratio.
b) Dry density
c) Unit weight of the soil is 50% saturated
d) Unit weight of the soil is completely saturated.

Soln:

$n = 40\% = 0.4 \quad ; \quad G = 2.7$

a) $e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$

b) $\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.667}$ $\qquad \gamma_w = 9.81\ kN/m^3$

$= 15.89\ kN/m^3$.

c) $\gamma_d = \frac{\gamma}{1+w}$ $\qquad Gw = eS$.

$\gamma = \gamma_d (1+w)$ $\qquad S = 0.5$

$w = \frac{eS}{G} = \frac{0.667 \times 0.5}{2.7} = 0.124$

$\gamma = 15.89\ (1+0.124)$

$= 17.85\ kN/m^3$.

d) $\gamma = \gamma_d\ (1+w)$ $\qquad w = \frac{eS}{G} = \frac{0.667 \times 1}{2.7}$

$= 15.89\ (1+0.247)$ $\qquad = 0.247$

$= 19.81\ kN/m^3$.

2) An undisturbed sample of soil has a volume of $100\ cm^3$ and mass of 190 g. On oven drying for 24 hours, the mass is reduced to 160g. If the specific gravity of grains is 2.68, determine the water content, void ratio & degree of saturation of soil.

Soln:

$M = 190\ g$; After drying ; $M_d = 160\ g$

$M_w = 190 - 160 = 30g$

$M_d = 160g$

$$w = \frac{M_{(w)}}{M_d} = \frac{30}{100} = 0.188$$

$$= 18.8\%.$$

$$e = \frac{G\gamma_{(w)}}{\gamma_d} - 1 \qquad \because \gamma_d = \frac{G\gamma_{(w)}}{1+e}$$

$$\gamma_d = \frac{\gamma}{1+w} \quad ; \gamma = 9.81 \times \rho$$

$$\rho = \frac{M}{V} = \frac{190}{100} = 1.9 \ g/cm^3.$$

$$\gamma = 9.81 \times 1.9 = 18.64 \ kN/m^3.$$

$$\gamma_d = \frac{18.64}{1+0.188} = 15.69 \ kN/m^3$$

$$e = \frac{2.68 \times 9.81}{15.69} - 1$$

$$= 0.67$$

$$eS = wG \Rightarrow S = \frac{wG}{e} = \frac{0.188 \times 2.68}{0.67}$$

$$= 0.744 = 74.4\%.$$

3) The insitu percentage voids of a sand deposit is 34%. For determining the density index, dried sand from the stratum was first filled loosely in a 1000 cm^3 mould and was then vibrated to give a maximum density. The loose dry mass in a mould was 1610 g and the dense dry mass of maximum compaction was found to be 1980 g. Determine the density index if the specific gravity of the sand particles is 2.67

<u>Soln:</u>

Porosity is referred as percentage voids.

$$n = 34\% = 0.34$$

$$V = 1000 \ cm^3$$

Loose dry mass $= 1610 g.$; $G = 2.67$

Dense dry mass $= 1980 g.$; $\gamma_w = 9.81 \ kN/m^3$

Density Index $(I_D) = \frac{e_{max} - e}{e_{max} - e_{min}}.$

$$e = \frac{n}{1-n} = \frac{0.34}{1-0.34} = 0.515$$

$$e_{max} = \frac{G\gamma_w}{(\gamma_d)_{min}} - 1 \quad ; \quad e_{min} = \frac{G\gamma_w}{(\gamma_d)_{max}} - 1$$

$$(\gamma_d)_{min} = \frac{1610}{1000} \times 9.81 = 15.79\ kN/m^3 \qquad (\gamma_d)_{max} = \frac{1980}{1000} \times 9.81 = 19.42\ kN/m^3$$

$$e_{max} = \frac{2.67 \times 9.81}{15.79} - 1 = 0.659 \qquad e_{min} = \frac{2.67 \times 9.81}{19.42} - 1 = 0.349.$$

$$= 0.465$$

$$\therefore I_D = 46.5\ \%$$

4) It is required to prepare a compacted cylindrical specimen of 40mm dia and 80mm length from oven dry soil. The specimen is required to have water content of 16% and percent air voids of 18%. Taking G = 2.7, determine the mass of soil and mass of water, required for preparation of above specimen.

Soln:

Diameter of specimen = 40 mm

Length of specimen = 80 mm

$\omega = 16\%$

$n_a = 18\%$; $G = 2.7$

$$\text{Volume of sample} = \frac{\pi}{4} d^2 h = \frac{\pi}{4}\left[(0.04)^2 \times 0.08\right]$$

$$= 1.0053 \times 10^{-4}\ m^3 \quad \text{—— (1)}$$

$$\omega = \frac{M_w}{M_d}$$

$$M_w = 0.16 M_d.$$

$$\gamma_s = G\gamma_w$$

$$\frac{W_d}{V_s} = G\gamma_w$$

$$\frac{1}{V_s} = \frac{G\gamma_w}{W_d} \quad ; \quad V_s = \frac{W_d}{G\gamma_w}$$

$$V_s = \frac{M_d}{G \rho_w} \qquad \rho_w = 1000\ kg/m^3$$

$$= \frac{M_d}{2.7 \times 1000} = \frac{M_d}{2700}\ m^3 \quad \text{——— (2)}$$

$$V_w = \frac{M_w}{\rho_w} = \frac{0.16 M_d}{1000} = 1.6 \times 10^{-4} M_d\ m^3 \quad \text{——— (3)}$$

$$V_a = n_a \cdot V = 0.18 \times 1.0053 \times 10^{-4}$$
$$= 1.8095 \times 10^{-5}\ m^3$$

Total volume of $V = V_s + V_w + V_a$.

$$1.0053 \times 10^{-4} = \frac{M_d}{2700} + 1.6 \times 10^{-4} M_d + 1.8095 \times 10^{-5}$$

$$M_d = 0.1554\ kg$$
$$= 155.4\ g$$

$$M_w = 0.16\ M_d$$
$$= 0.16 \times 155.4$$
$$= 24.9\ g.$$

Index Properties

* Water Content
* Specific Gravity
* Particle size distribution
* Consistency limits
* Insitu density
* Density Index.

i) Water Content

Water content of soil sample can be determined by following methods.

a) Oven drying method
b) Sand Bath method
c) Alcohol method
d) Calcium Carbide method
e) Pycnometer method
f) Radiation method
g) Torsion Balance method.

(16)

a) Oven drying Method (Accurate method)

A specimen of soil sample is kept in a clean container and put in a thermostatically controlled oven with interior of non corroding material to maintain temperature between 105°C to 110°C. Usually sample is kept for about 24 hours in oven so that complete drying is assured.

Temperature > 110°C - Break crystalline structure

For highly organic soils, lower temperature of 60°C is preferred to prevent oxidation of organic matter.

If gypsum present in soil, sample is dried more than 80°C but for a long time.

$$w = \frac{M_2 - M_3}{M_3 - M_1} \times 100.$$

M_1 - Mass of container with lid

M_2 - Mass of container with lid and wet soil.

M_3 - Mass of container with lid and dry soil.

b) Sand bath method (Field method)

The container with soil is placed on a sand bath. The sand bath is heated over a kerosene stove. The soils becomes dry within 1/2 to 1 hour.

$$w = \frac{M_2 - M_1}{M_3 - M_1} \times 100.$$

c) Alcohol method (Crude Field Method)

The wet soil sample is kept in a evaporating dish and mixed with sufficient quantity of methylated spirit. The dish is then properly covered and the mixture is ignited. It is kept stirred by a wire during ignition. Since there is no control over temperature, it should not be used for soils containing large % of organic matter or gypsum.

d) Calcium Carbide Method (Quick Method)

6g of wet soil sample is placed in an air tight container and is mixed with sufficient quantity of fresh calcium carbide powder. The mixture is shaken vigorously. The acetylene gas produced, exerts pressure on a sensitive diaphragm placed at end of container.

The dial gauge located at diaphragm reads the water content directly.

$$w = \frac{w'}{1-w'}$$

w' = water content based on wet weight
w - water content based on dry weight.

b) Pycnometer Method (Quick Method)

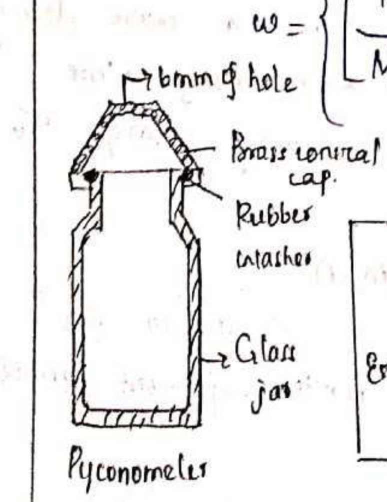
It is used for soils whose specific gravity is accurately known.

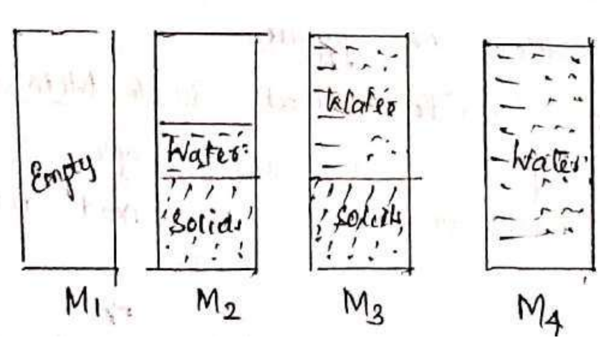
Pyconometer- large size density bottle of 900ml capacity
- Conical brass cap, 6mm dia hole at its top
- Rubber washer placed between conical cap and rim of bottle so that there is no leakage of water.

Procedure

* Take clean, dry pyconometer and find its mass with cap and washer (M_1)
* Put about 200g to 400g of wet soil in pyconometer and find mass (M_2)
* Add water and stir it. Fill the pyconometer with water and find its mass (M_3)
* Empty the pyconometer, clean it and fill with clean water to hole of conical cap and find its mass (M_4)

$$w = \left\{ \left[\frac{M_2-M_1}{M_3-M_4}\right] \cdot \left[\frac{G-1}{G}\right] - 1 \right\} \times 100.$$



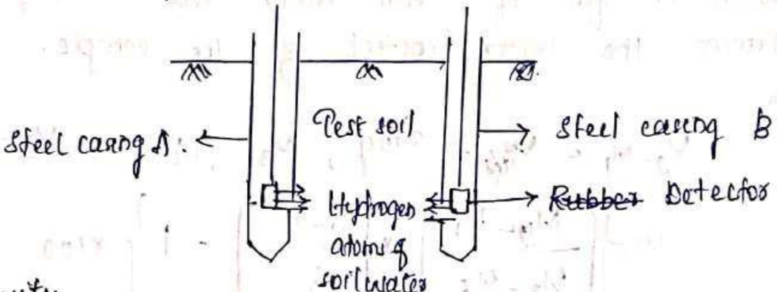


C(8)

Radiation Method

It is useful for determining water content of soil deposits in insitu condition. It uses two steel casings - casing A and casing B which are placed in two bore holes at some distance apart in soil deposit.

* Device containing some radio active isotope material is placed in a capsule which is lowered into casing A.
* Detector unit is lowered in steel casing B.
* Small openings are made in A and B facing each other.
* When radio active device is activated, it emit neutrons.
* Neutrons strike through hydrogen atoms of water, they loose energy.
* The loss of energy is equal to water content of soil.
* The detector device is calibrated to given directly the water content of subsoil at level of emission.



Specific Gravity

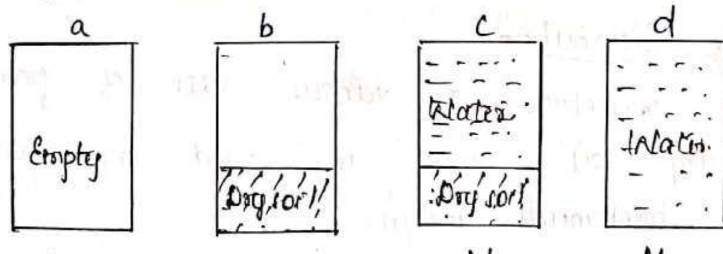
It is determined by

i) 50 ml density bottle → most accurate and suitable for all types of soils

ii) 500 ml flask.

iii) Pycnometer → coarse grained soils.

Density bottle method is the standard method used in lab.



Dry mass of soil, $M_d = M_2 - M_1$

Mass of water in c $= M_3 - M_2$

Mass of water in d $= M_4 - M_1$

Mass of water having the same volume as that of soil solids $= (M_4 - M_1) - (M_3 - M_2)$

$$G = \frac{\text{Dry mass of soil}}{\text{Mass of water at equal volume}}$$

$$= \frac{M_2 - M_1}{(M_4 - M_1) - (M_3 - M_2)} = \frac{M_2 - M_1}{M_4 - M_1 - M_3 + M_2}$$

$$= \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)} = \frac{M_d}{M_d - (M_3 - M_4)}$$

Distilled water is used in flask or pycnometer but kerosene is used in density bottle method.

Problem

1) In order to determine the water content, 370 g of a wet sandy sample was placed in a pycnometer. The mass of the pycnometer, sand and water full to the top of the conical cap was found to be 2148 g. The mass of pycnometer full of clean water was 1932 g. Taking $G = 2.65$, determine the water content of the sample.

Soln:

$$M_2 - M_1 = M_{wet} = 370g \; ; \; M_3 = 2148g \; ; \; M_4 = 1932g \; ; \; G = 2.65$$

$$w = \left\{ \left[\frac{M_2 - M_1}{M_3 - M_4} \right] \times \left[\frac{G-1}{G} \right] - 1 \right\} \times 100$$

$$= \left\{ \left[\frac{370}{2148 - 1932} \times \frac{2.65 - 1}{2.65} \right] - 1 \right\} \times 100$$

$$= 6.5\%.$$

(ii) Particle Size Distribution

The percentage of various sizes of particles in a given dry soil sample is found by particle size analysis (or) mechanical analysis.

It is meant for separation of soil into its different size fractions. It is performed in two stages

i) Sieve Analysis — Coarse grained soils

ii) Sedimentation Analysis / Wet Mechanical analysis.
→ Fine grained soils.

Sieve Analysis

In BIS and ASTM standards, sieve sizes are given in terms of number of openings per inch. The number of openings per square inch is equal to square of number of sieve.

Code - IS : 460 : 1962

Sieve Analysis — Coarse Analysis / Fine Analysis.

An oven dried sample of soil is seperated into two fraction by sieving it through 4.75 mm IS sieve.

Portions retained on 4.75mm sieve - Gravel / Coarse Analysis

Portions passing through 4.75mm sieve - Fine Analysis.

Set of Sieves

Coarse sieve analysis	Fine sieve analysis
IS : 100 mm	IS : 2mm
63 mm	1 mm
20 mm	600 μ
10 mm	425 μ
4.75 mm	300 μ
	212 μ
	150 μ
	75 μ

* It is advisable to wash the soil portion passing through 4.75mm sieve over 75μ sieve so that clay and silt particles sticking to sand particles may be dislodged.
* 2 g of sodium hexametaphosphate is added per litre of water used.
* Washing should be continued until the water passing through 75μ sieve is substantially clean.
* The fraction retained on 75μ sieve is dried in oven. The dried portion is resieved through fine analysis.

Sedimentation Analysis

In sedimentation analysis, the soil fraction finer than 75μ sieve is kept in suspension in a liquid medium.

The analysis is based on Stoke's law, according to which velocity at which grains, settle out of suspension all other factors being equal, is dependent on shape, weight and size of grain.

It is assumed that soil particle are spherical and have same specific gravity. With this assumption, coarser particles settle more quickly than finer ones.

γ - terminal velocity of sinking of spherical particle.

$$\gamma = \frac{2}{9} r^2 \frac{\gamma_s - \gamma_w}{\eta} \qquad r = D/2 \; ; \; r^2 = \frac{D^2}{4}$$

$$= \frac{1}{18} D^2 \frac{\gamma_s - \gamma_w}{\eta}.$$

r - radius of spherical particle (m)
D - Diameter of spherical particle (m)
γ - Terminal velocity (m/s)
γ_s - unit weight of particle (kN/m^3)
γ_w - unit weight of water / liquid (kN/m^3).
η - viscosity of water / liquid (kNs/m^2) $= \frac{\mu}{g}$
μ - viscosity of absolute units of poise
g - acceleration due to gravity.

If water is used as medium for suspension,

$$\gamma_w = 9.81 \; kN/m^3 \; , \quad \gamma_s = G\gamma_w$$

$$\gamma = \frac{1}{18} D^2 \frac{(G-1)\gamma_w}{\eta}$$

If D is in mm, then

$$\gamma = \frac{1}{18}\left(\frac{D}{1000}\right)^2 \frac{(G-1)\gamma_w}{\eta} = \frac{D^2 \gamma_w (G-1)}{18 \times 10^6 \eta}.$$

$$\gamma_w = 9.81 \; kN/m^3 ; \quad \gamma = \frac{D^2 \times 9.81 (G-1)}{18 \times 10^6 \eta}$$

$$= \frac{D^2 (G-1)}{1.835 \times 10^6 \eta}$$

$$D^2 = \frac{18 \times 10^6 \eta \cdot \gamma}{9.81 (G-1)}$$

(22)

$$D = \sqrt{\frac{18 \times 10^6 \eta \cdot \gamma}{\gamma_w (G-1)}}.$$

Hydrometer Analysis

* The mass M_D per ml of suspension is computed indirectly by reading the density of soil suspension at depth H_e at various time intervals.
* The sampling depth H_e goes on increasing as the particles settle with increase in time intervals.
* It is necessary to calibrate the hydrometer and sedimentation jar before of start of sedimentation test, to find relation between H_e and density readings of hydrometer.
* The readings on hydrometer stem gives the density of soil suspension situated at centre of bulb at any time.
* For convenience, hydrometer readings are recorded after subtracting 1 and multiplying the remaining digits by 1000.
* Such a reduced reading is R_R

If reading is 1.010, $R_R = (1.010 - 1) \times 1000 = 10$
If reading is 0.995, $R_R = (0.995 - 5) \times 1000 = -5$

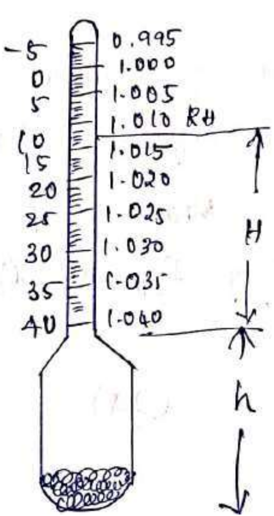
R_R increase in downward direction towards hydrometer bulb.

$$H_e = \left[H + \frac{h}{2} + \frac{V_R}{2A} \right] - \frac{V_R}{A}$$

$$= H + \frac{h}{2} + \frac{2V_R - V_R}{2A}$$

$$= H + \frac{h}{2} + \frac{V_R}{2A}$$

$$= H + \frac{1}{2}\left[h + \frac{V_R}{A} \right].$$



(23)

Particle Size Distribution Curve

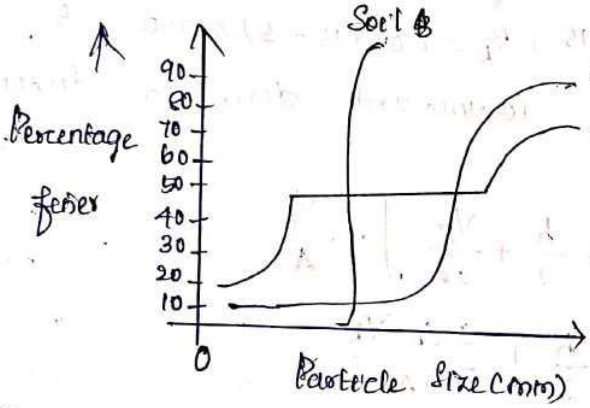
The results of the mechanical analysis are plotted to get a particle size distribution curve with the percentage finer N as ordinate and particle diameter as abscissa, diameter being plotted on logarithmic scale.

It gives an idea about type and gradation of soil.

A curve situated higher up or to the left represents a relatively fine grained soil while a curve situated to the right represents coarse grained soil.

Soil:
- Well graded → Particles of all sizes
- Poorly graded → Excess of certain particles and deficiency of others.
- Uniformly graded → Particles of about same size.

A curve with flat portion represent a soil in which some intermediate size particles are missing. Such a soil is known as gap graded or skip graded.



A – well graded soil
B – Uniform soil
C – gap graded soil

For coarse grained soil, certain particle sizes such as D_{10}, D_{30}, D_{60} are important.

D_{10} – Size in mm such that 10% of particles are finer than this size.

D_{60} – 60% of total mass of sample are finer.

D_{10} – Effective size or effective diameter.

Uniformity Coefficient (C_u) – Measure of particle size range and is given by ratio of D_{60} and D_{10} size

$$C_u = \frac{D_{60}}{D_{10}}$$

Coefficient of curvature (C_c)

$$C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}}$$

Gravels, $C_u > 4$

Sand $C_u > 6$

Uniformly graded soil, $C_u = 1$

Well graded soil, $C_c = 1$ to 3.

Problem

1) A soil sample, consisting of particles of size ranging from 0.5mm to 0.01mm is put on the surface of still water tank 5m deep. Calculate the time of settlement of the coarsest and finest particles of the sample to the bottom of the tank. Assume average specific gravity of soil particles as 2.66 and viscosity of water as 0.01 poise.

Soln:

$G = 2.66$; $\eta = 0.01 \times 10^{-4}$ kN s/m^2 ; $D = 0.5$mm to 0.01mm

$h = 5$m.

$$v = \frac{D^2 \gamma_w (G-1)}{18 \times 10^6 \, \eta}$$

$$= \frac{D^2 \times 9.81 \, (2.66-1)}{18 \times 10^6 \times 0.01 \times 10^{-4}}$$

$$= 0.905 D^2.$$

If $D = 0.5$mm ; $v = 0.905 \, (0.5)^2$

$= 0.2263$ m/s.

$$t = \frac{h}{v}$$

$$= \frac{5}{0.2263} = 22.1 s.$$

If $D = 0.01$mm , $v = 0.905 (0.01)^2$

$= 9.05 \times 10^{-5}$ m/s.

$$t = \frac{h}{v} = \frac{5}{9.05 \times 10^{-5}}$$

$= 55249$ sec

(25) $t = 15$ hrs 20 min 49 sec.

Consistency of soils

Consistency denotes degree of firmness of soil which may be termed as soft, firm, stiff or hard.

Atterberg divided the entire range from liquid to solid state into four stages.

Solid	Semi solid	Plastic	Liquid
		Working	$\tau = 0$
w_s	w_p test	w_L	Flow test

τ - shear strength $= 0$

w_L - Liquid limit $\quad w > w_L$ - Liquid state

w_p - Plastic limit $\quad w < w_s$ - Solid state

w_s - Shrinkage limit

Liquid Limit (w_L)

It is defined as the minimum water content at which the soil is still in the liquid state, but has a small shearing strength against flowing.

It is the minimum water content at which a part of soil cut by a groove of standard dimensions will flow together for a distance of 12 mm ($\frac{1}{2}$ inch) under an impact of 25 blows in the device.

Plastic Limit (w_p)

It is defined as the minimum water content at which a soil will just begin to crumble when rolled into a thread approximately 3mm in diameter.

Shrinkage Limit (w_s)

It is defined as the maximum water content at which a reduction in water content will not cause a decrease in volume of soil mass.

Plasticity Index (I_p)

The range of consistency within which soil exhibits plastic properties is called plastic range and is indicated by plasticity index.

It is defined as the difference between liquid limit and plastic limit of soil

$$I_p = W_L - W_p$$

If $W_p \geq W_L$; $I_p = 0$

Plasticity

It is defined as the property of soil which allows it to be deformed rapidly without rupture, without elastic rebound and without volume change.

Consistency Index (I_c) / Relative Consistency.

It is defined as the ratio of liquid limit minus natural water content to the plasticity index of soil

$$I_c = \frac{W_L - W_c}{I_p}$$

W - Natural water content

$I_c = 1 \rightarrow$ Plastic limit

$I_c = 0 \rightarrow$ Liquid limit

$I_c > 1 \Rightarrow$ Semi solid state

$I_c < 1 \Rightarrow W > W_L$, Behaves like liquid

Liquidity Index (I_L) / Water plasticity ratio.

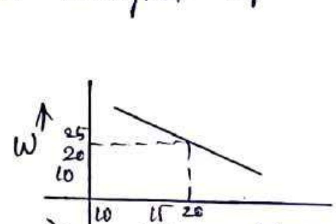
It is the ratio expressed as percentage, of the natural water content of a soil minus its plastic limit to its plasticity index.

$$I_L = \frac{W - W_p}{I_p}$$

Determination of liquid limit and plastic limit

The liquid limit is determined in lab with the help of standard liquid limit apparatus designed by Casagrande,

$$W_1 - W_2 = I_f \log_{10}\left(\frac{n_2}{n_1}\right)$$



Flow curve.

Flow Index

Flow index / slope of curve is determined by

$$I_f = \frac{W_1 - W_2}{\log_{10}\left(\frac{n_2}{n_1}\right)}$$

Selecting the values of n_2 & n_1 corresponding to number of blows over one log cycle difference $\log_{10}\left(\frac{n_2}{n_1}\right)$

becomes equal to unity. If becomes equal to difference between corresponding water contents.

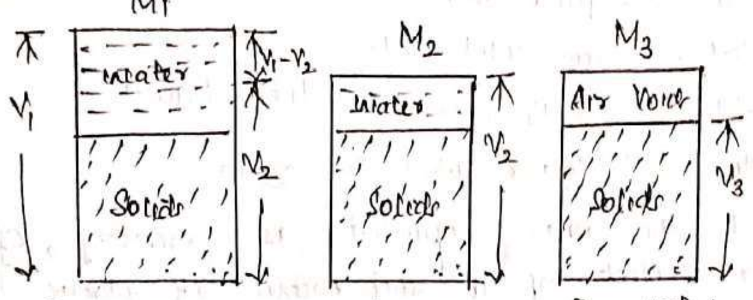
Toughness Index

It is defined as the ratio of plasticity index to flow index.

$$I_T = \frac{I_P}{I_f}$$

Shrinkage Limit

If a saturated soil sample is taken and allowed to dry up gradually, its volume will go on reducing till a stage will come after which the reduction in soil water will not result in further reduction in total volume of soil sample. The water content at this stage is shrinkage limit.



Original soil (a) Soil @ shrinkage limit (b) Dry soil (c)

Mass of water in (a) $= M_1 - M_d$

Loss of water from (a) to (b) $= (V_1 - V_2)\rho_w$

Mass of water in (b) $= (M_1 - M_d) - (V_1 - V_2)\rho_w$

$$w_s = \frac{(M_1 - M_d) - (V_1 - V_2)\rho_w}{M_d} \times 100$$

$$= \left[w_1 - \frac{(V_1 - V_d)\rho_w}{M_d} \right] \times 100$$

$$= \left[w_1 - \frac{(V_1 - V_d)\gamma_w}{W_d} \right] \times 100.$$

w_1 – water content of original saturated sample of volume V_1

V_d – Dry volume of soil sample.

W_d – dry weight of soil sample

M_d – dry mass of soil sample.

Value of G from shrinkage limit test

$$\gamma_s = G\gamma_w = \frac{M_d \times 9.81}{V_s}$$

$$G = \frac{M_d}{V_s}$$

$$V_s = V_1 - \frac{M_1 - M_d}{\rho_w}$$

$$G = \frac{M_d}{V_1 - \frac{M_1 - M_d}{\rho_w}}$$

$$= \frac{M_d \cdot \rho_w}{V_1 \rho_w - (M_1 - M_d)}$$

$\rho_w = 1\ g/cc.$

$$= \frac{M_d}{V_1 - (M_1 - M_d)}$$

$$G = \frac{1}{\frac{\gamma_w}{\gamma_d} - \frac{w_s}{100}}$$

Shrinkage Ratio (SR)

It is defined as the ratio of given volume change expressed as a percentage of dry volume, to the corresponding change in water content above the shrinkage limit expressed as a percentage of weight of oven dried soil

$$S.R = \frac{\frac{V_1 - V_2}{V_d} \times 100}{w_1 - w_2}$$

V_1 - volume of soil mass at water content w_1

V_2 - volume of soil mass at water content w_2

V_d - volume of dry soil mass.

At shrinkage limit

$$SR = \frac{\left[\frac{V_1 - V_d}{V_d}\right] \times 100}{w_1 - w_s}$$

Shrinkage ratio of soil is equal to mass specific gravity of soil in dry state.

Volumetric shrinkage (VS)

It is defined as the decrease in volume of soil mass, expressed as percentage of dry volume of soil mass, when the water content is reduced from a given percentage to shrinkage limit.

$$VS = \frac{V_1 - V_d}{V_d} \times 100 \qquad \therefore SR = \left[\frac{\frac{V_1 - V_d}{V_d}}{w_1 - w_s}\right] \times 100$$

But $\frac{V_1 - V_d}{V_d} \times 100 = (w_1 - w_s)\, SR$

$$VS = (w_1 - w_s)\, SR$$

Linear shrinkage (L_s)

It is defined as the decrease in one dimension of soil mass expressed as percentage of original dimension when water content is reduced from a given value to shrinkage limit.

$$L_s = 100\left[1 - \left(\frac{100}{VS + 100}\right)^{1/3}\right]$$

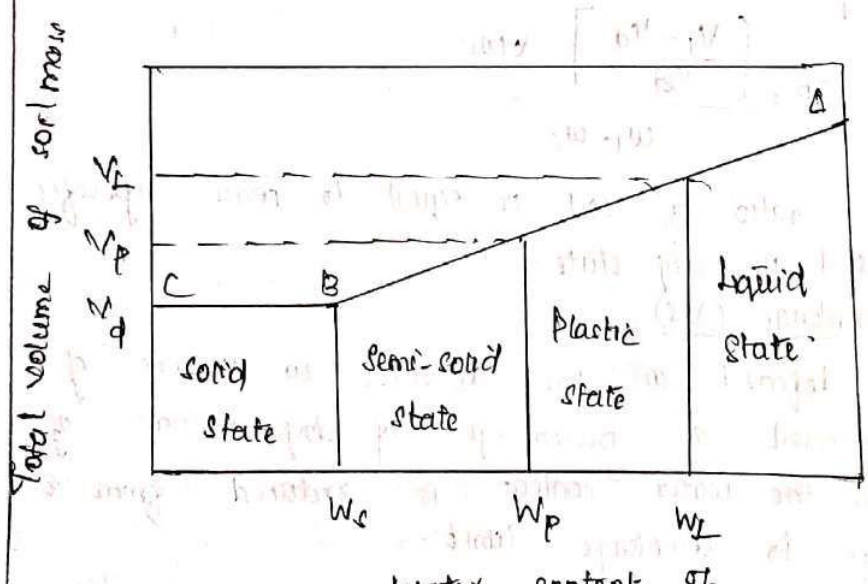
Activity of clays

It is defined as the ratio of plasticity index to the percent by weight of soil particles of diameter smaller than two microns present in the soil.

$$A_c = \frac{I_P}{C_w}$$

C_w – Percentage, by weight of clay sizes (i.e., particles of size less than 2μ).

Activity	Classification
< 0.75	Inactive
$0.75 - 1.4$	Normal
> 1.4	Active



Consistency Limits

Sensitivity of clays

The degree of disturbance of undisturbed clay sample due to remoulding is expressed by sensitivity (S_t)

Sensitivity (S_t)

It is defined as the ratio of its unconfined compression strength in the natural or undistributed state without change in water content.

$$S_t = \frac{q_u \text{ (undistributed)}}{q_u \text{ (remoulded)}} \qquad S_t = 1 \text{ to } 8.$$

Sensitivity	Classification	Structure
1	Insensitive	
2 to 4	Normal / less sensitive	Honey comb structure
4 to 8	Sensitive	Honey comb or flocculent structure
8 to 16	Extra sensitive	Flocculent structure
> 16	Quick	Unstable.

Thixotropy of clays

The phenomenon of 'strength loss – strength gain' with no change in volume or water content is called thixotropy. It is defined as an isothermal, reversible time dependent process which occurs under constant composition and volume, thereby a material softens as a result of remoulding and then gradually returns to its original strength when allowed to rest.

Classification of soils.

Soils may be classified by following system

1) Particle size classification
2) Textural classification.
3) Highway Research Board (HRB) classification
4) Unified soil classification and IS classification system

Particle Size Classification

In this system, soils are arranged according to grain size. Terms such as gravel, sand, silt and clay are used to indicate grain size.

Textural Classification

Soil classification of composite soils exclusively of the based on the particle size distribution is known as textural classification

Highway Research Board (HRB) Classification

HRB classification system also known as Public Road Administration classification system, is based on both particle size composition as well as plasticity characteristics.

* Used for pavement construction

Unified Soil Classification System (USCS)

Soils are classified into four major groups
i) Coarse grained ii) Fine grained iii) Organic soils iv) Peat.

Indian Standard Classification System (ISCS)

Code IS : 1498 - 1970

Soil is divided into 3 groups

1) Coarse grained
2) Fine grained
3) Highly organic.

Compaction

Compaction is a process by which the soil particles are artificially rearranged and packed together into a closer state of contact by mechanical means inorder to decrease the porosity of soil and thus increase its dry density.

It may be accomplished by rolling, tamping or vibration.

At maximum dry density, optimum water content is reached.

Laboratory Compaction Methods

The tests are based on any one of the types of compaction : dynamic or impact, static and vibration.

Usual compaction methods used in laboratory to determine water density relationships

* Standard Proctor Test
* Modified Proctor Test
* Harvard Miniature Compaction Test
* Abbot Compaction Test
* Jodhpur mini compactor Test

Standard Proctor Test

It was developed by R.R. Proctor (1933) for construction of Earth fill dams in state of California. The test equipment consists of

i) Cylindrical metal mould, having an internal diameter of 4 inches, an effective height of 4.6 in (11.7 cm) and a capacity of 1/30 cu.ft.

ii) detachable base plate

iii) Collar

iv) Rammer 5.5 lb (2.5 kg) in mass falling through a height of 12 inch (30.5 cm)

* The test consists in compacting soil at various water contents in the mould, in three equal layers, each layer being given 25 blows of the 5.5 lb rammer dropped from a height of 12 inch.

* Dry density obtained in each test is determined by knowing the mass of compacted soil and its water content.

* The compactive energy used for this test is 6065 kgcm per 1000 ml of soil.

IS: 2720 (Part VII) 1980

Mould - 1000 ml capacity

Internal dia - 100 mm

Internal effective height - 127.5 mm

Rammer Mass - 2.6 kg

Drop - 310 mm

* About 3 kg of air dried soil passing 4.75 mm sieve is mixed thoroughly with water

* The quantity of water to be added may be taken 4% for coarse grained and 10% for fine grained soils.

* The empty mould attached to base plate is weighed without collar.

* The collar is then attached to mould.

* The mixed and mature soil is placed on the mould and compacted by giving 25 blows of rammer uniformly distributed over surface, such that compacted height of soil is about 1/3 of height of mould.

* The second and third layers are similarly compacted each layer being 25 blows.

* The last compacted layer should project not more than 6mm into collar.

* The collar is removed and excess soil is trimmed off to make it level with top of mould.

* Weight of mould, base plate and compacted soil is taken

* A representative sample is taken from centre of compacted specimen and kept for water content determination

* Bulk density (ρ) and dry density (ρ_d) for compacted soil is calculated from

$$\rho = \frac{M}{V} \ (g/cm^3) \qquad \rho_d = \frac{\rho}{1+\omega} \ (g/cm^3)$$

M – mass of wet compacted specimen
V – Volume of mould.

* The compacted soil is taken out of the mould broken with hand and remixed with raised water content (By 2 to 4%)

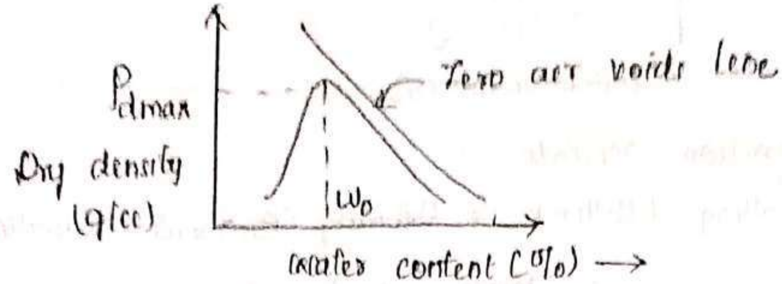
* The test is repeated on soil samples with increasing water contents and corresponding dry density ρ_d obtained is determined.

* A compaction curve is plotted between water content as abscissa and dry densities as ordinate

ρ increases, w increases till maximum dry density.

Optimum water content

The water content corresponding to maximum density is optimum water content w_0.



Zero air voids line

A line shows the water content dry density relation for compacted soil containing a constant percentage air voids is known as zero air voids line.

$$\rho_d = \frac{(1-n_a) G \rho_w}{1+wG}$$

n_a - percentage air voids

G - Specific Gravity.

The line showing dry density as a function of water content for soil containing no air voids is called zero air voids line or saturation line

$$\rho_d = \frac{G \rho_w}{1+wG}$$

Modified Proctor Test / AASHO Test

* It was developed to give higher standard of compaction.

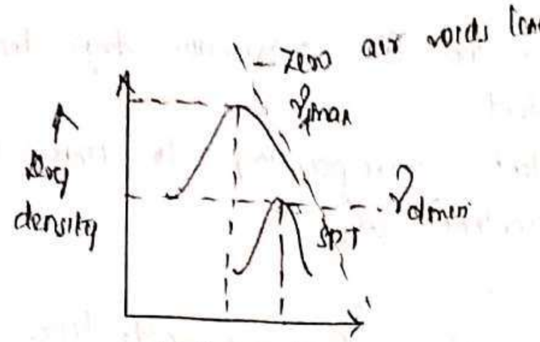
* In this test, soil is compacted in standard proctor mould but in 5 layers, each layer being given 25 blows of 10 lb (4.9 kg) rammer dropped through height of 18 inches.

* Compactive energy - 27260 kg cm / 1000 cm^2 of soil which is 4½ times that of SPT

IS 2720 (Part VII) - 1980 / 87

- Rammer - 4.9 kg

Drop - 45 cm



Field Compaction Methods

* Rolling (Rollers) * Ramming (Rammers) * Vibration (vibrat

Rolling equipments —
- Smooth wheel rollers
- Pneumatic type rollers
- Sheep foot rollers
- Loris & Pneumatic tyred construction plant
- Track laying vehicle

Ramming equipments —
- Dropping weight type
- Internal combustion type
- Pneumatic type

Vibrating equipment —
- Dropping weight type
- Pulsating hydraulicity.

Factors affecting compaction

(i) Water Content

When water content increased, compacted density goes on increasing till a maximum dry density is achieved after which further addition of water decreases density

Upto γ_{dmax}, $w \uparrow$, $\gamma_d \uparrow$

After γ_{dmax}, $W \uparrow$, $\gamma_d \downarrow$

(ii) Amount of compaction

It affects γ_{dmax} and OMC

The effect of increasing compactive energy results in increasing maximum dry density and decreasing OMC

Compacting energy $\uparrow$, $\gamma_{dmax} \uparrow$, OMC $\downarrow$

iii) Method of compaction

Density obtained depends on type of compaction or manner in which compactive effort is applied. Variables in this aspect are

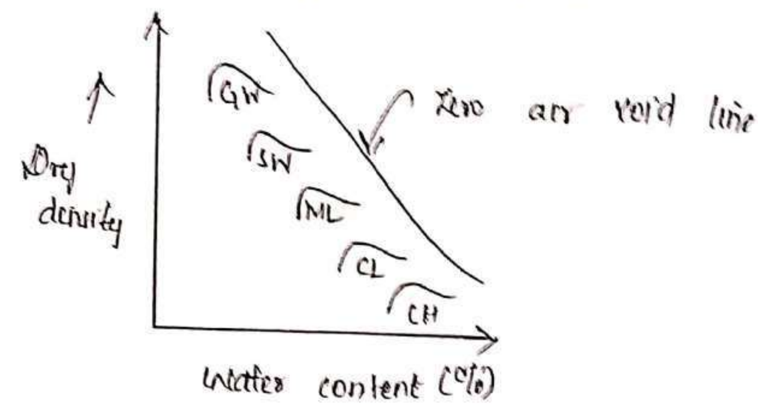
i) Weight of compacting equipment

ii) Manner of operation - dynamic or impact, static,

iii) Time and area of contact between corresponding compacting element and soil.

iv) Type of soil

$\gamma_{d\,max}$ achieved depends on type of soil. Well graded soils - higher density and lower optimum moisture content than fine grained soil which require more water for lubrication because of greater specific surface.



v) Addition of admixtures

Compaction properties of soil can be modified by number of admixtures other than soil material. Certain chemicals are added to soil cement to reduce cement consumption.

Clays and silts → Lime and calcium chloride
→ Sodium carbonate & sulphate.

Fly ash - additive - Stabilisation of sand.

Fly ash - Filler - increasing density.

— x —

Unit - II

Effective Stress and Permeability

Soil - water - Static pressure in water - Effective stress concepts in soils - Capillary phenomena - Permeability - Darcy's law - Determination of permeability - Laboratory Determination (Constant head and falling head methods) and field measurement pumping out in unconfined and confined aquifer - Factors influencing permeability of soils - Seepage - Two dimensional flow - Laplace's equation - Introduction to flow nets - Simple problems sheet pile and weir.

Soil Water

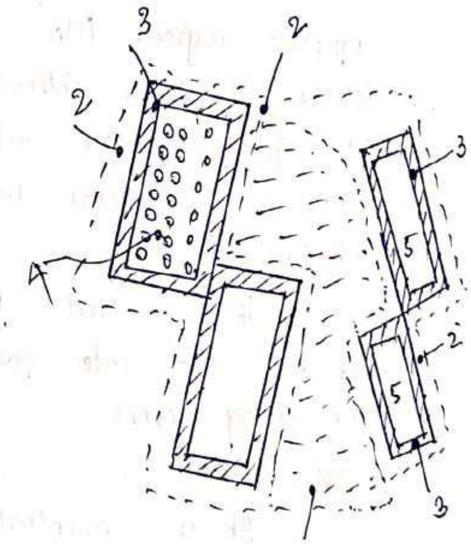
Water present in the voids of soil mass is called soil water.

Classification a) Broad Classification

1) Free water or gravitational water
2) Held water
   - i) Structural water
   - ii) Adsorbed water
   - iii) Capillary water

b) Phenomenological Basis
   - i) Ground water
   - ii) Capillary water
   - iii) Adsorbed water
   - iv) Infiltrated water

c) Structural Aspect
   - i) Pore water
   - ii) Solvate water
   - iii) Adsorbed water
   - iv) Structural water



1. Pore water
2. Adsorbed water
3. Solvate water
4. Structural water
5. Solid Particle.

Ground Water

Subsurface water that fills the voids continuously and is subjected to no forces other than gravity. This water is also known as gravitational water or force water.

Capillary Water

It is the water lifted by surface tension above the free ground water surface. Capillary water fills all the pores in the soil to a certain distance above the water table. This is known as zone of capillary saturation.

Adsorbed Water

It comprises of i) Hygroscopic water
ii) Film water

i) Hygroscopic Water

The soil particles freely absorb from atmosphere by the physical forces of attraction and is held by forces of adhesion is known as hygroscopic water.

sand - 1% ; silt - 6% ; clay - 16%

ii) Film Water

The film forms because of condensation of aqueous vapours. Film moisture is also held by molecular forces of high intensity but not as high as in the case of hygroscopic film. The greater surface of soil, the more is the film moisture that can be contained.

Infiltrated Water

It is that portion of surface precipitation which soaks into ground, moving downwards through air containing zones.

Pore Water

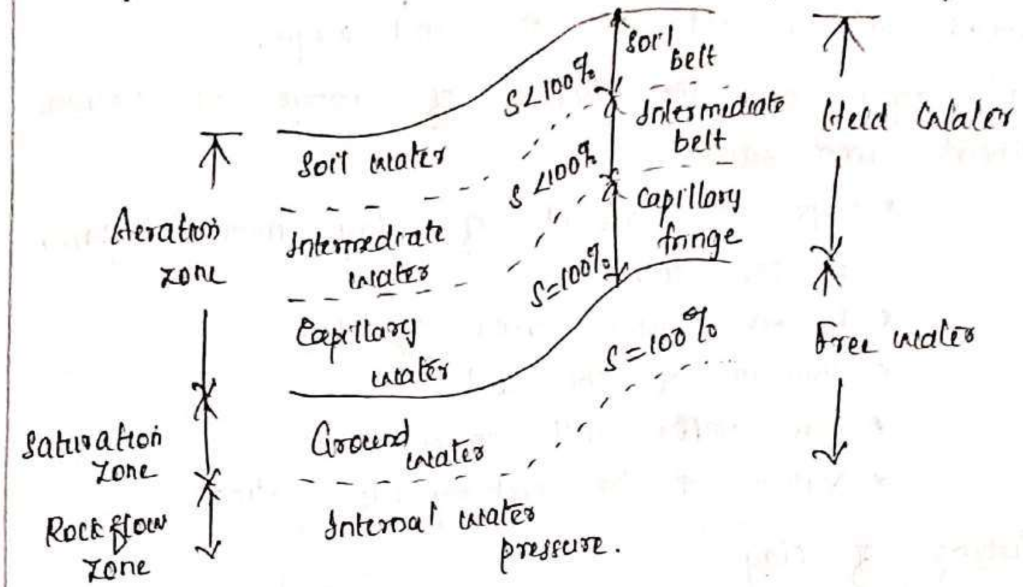
It is essentially free of strong soil attractive forces. The capillary water and gravitational water may be considered as the two types of pore water.

Solvate Water

It is the water which forms a hydration shell around soil grains. It is subject to polar, electrostatic and ionic binding forces. (2)

Structural Water

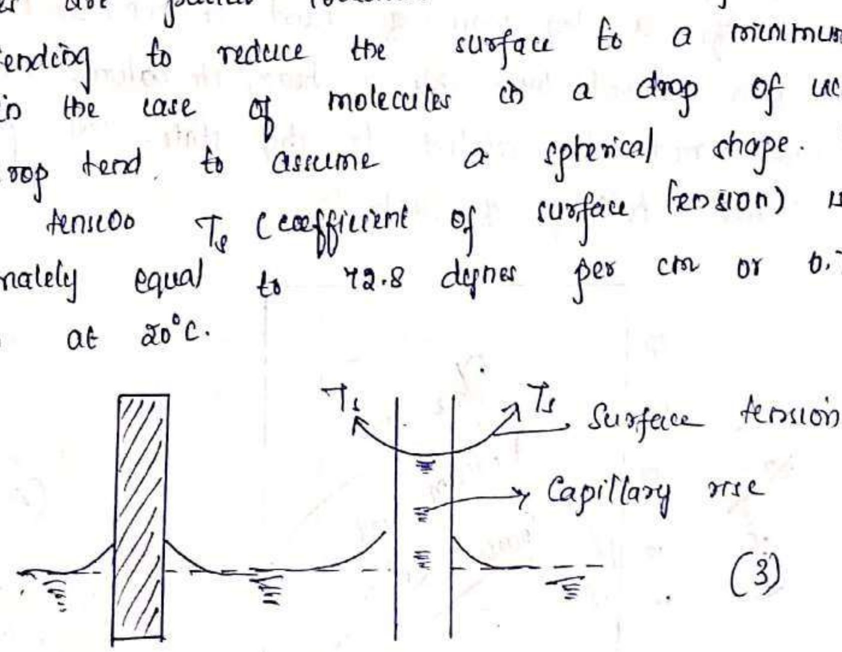
It is the water chemically combined in the crystal structure of the soil mineral, under loading encountered in soil engineering, the structural water cannot be seperated or removed and is therefore unimportant.



Soil and Water Zones.

Surface Tension

Surface tension of water is the property which exists in the surface film of water tending to contract the contained volume into a form having a minimum superficial area possible. The molecules on surface of a liquid are attracted by others molecules on the surface and inside the body of the liquid. Because there is no pull from outside, the surface molecules are pulled towards the inside of the liquid mass tending to reduce the surface to a minimum. Thus, in the case of molecules in a drop of water the drop tend to assume a spherical shape. The surface tension T_s (coefficient of surface tension) is approximately equal to 72.8 dynes per cm or 0.728×10^{-6} kN/cm at 20°C.



Shrinkage and swelling of soils.

Soils undergo a volume change where the water content is changed. Shrinkage takes place due to decrease in water content in the soils, where a saturated soils is allowed to dry, a meniscus develops in each void at the soil surface.

For clayey soils, the degree of change in volume depends upon factors.

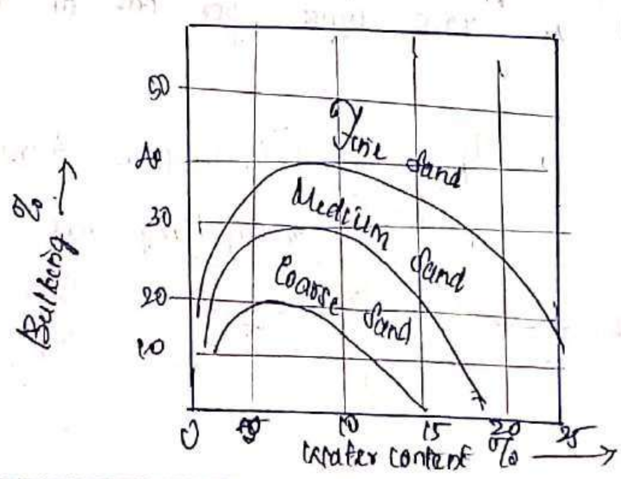
* Type and amount of clay minerals present in the soil.
* Specific surface area of clay.
* Structure of the soil
* Pore water salt concentration
* Valence of the exchangeable cation.

Slaking of Clay

Consider a mass of clayey soil which has been dried well below the shrinkage limit, thus attaining the minimum volume. When this mass of soil is suddenly immersed in water, it will cause it slaking, resulting in its disintegration into a soft wet mass. Such slaking is due to the entry of air into the void space during the drying of the soil below the shrinkage limit.

Bulking of Sand

If a dry mass of sand is moistured slightly and then dumped loose into a heap, its volume will increase considerably relative to dry state. This phenomenon is called bulking of sand.



(4)

Stress Conditions in soil: Effective and neutral pressures

Total Stress / Unit Pressure

At any plane in a soil mass, the total stress or unit pressure σ is the total load per unit area. This pressure may be due to i) Self weight of soil
ii) Over burden on the soil

The total pressure consist of two distinct components
i) Intergranular pressure or effective pressure
ii) Neutral pressure or pore pressure.

Effective Pressure (σ')

It is the pressure transmitted from particle through their point of contact through the soil mass above the plane, such a pressure, also termed as intergranular pressure, is effective in decreasing the voids ratio of the soil mass and in mobilising its shear strength.

Neutral Pressure

The neutral pressure or the pore water pressure or pore pressure is the pressure transmitted through the pore fluid.

$$u = \gamma_w \cdot h_p$$

γ_w - unit weight of water
h_p - pressure head of water

Effective stress, σ' = Total stress (σ) − Neutral stress (u).

$$\sigma' = \sigma - u$$

Effective pressure for different condition.

i) Submerged soil mass
ii) Soil mass with surcharge
iii) Saturated soil with capillary fringe
iv) Partially saturated soil.

i) <u>Submerged Soil Mass</u>.

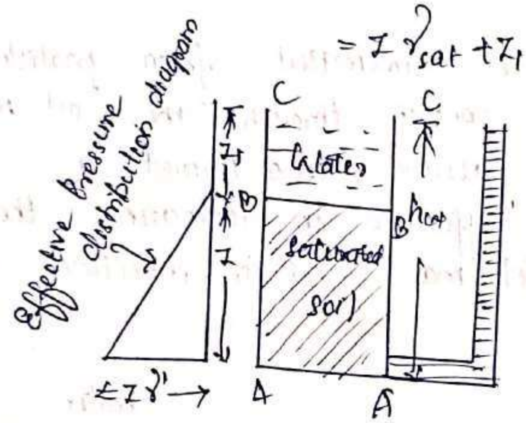
The fig shows a saturated soil mass of depth Z, submerged under water of height Z_1 above its top level. If a piezometric tube is inserted at level AA, water will rise in it upto level CC. Now total pressure at AA is given by

Total stress, $\sigma = Z\gamma_{sat} + Z_1\gamma_w$

Pore pressure, $u = \gamma_w \cdot h_w$

$$\sigma' = \sigma - u = Z\gamma_{sat} + Z_1\gamma_w - h_w\gamma_w$$

$$= Z\gamma_{sat} + Z_1\gamma_w - (Z+Z_1)\gamma_w$$



$$\sigma' = Z(\gamma_{sat} - \gamma_w)$$

$$\sigma' = Z\gamma'$$

ii) <u>Soil Mass with Surcharge</u>

Let us consider a moist soil mass of height Z_1 above a saturated mass of height Z.

$$\sigma' = \sigma = q + Z_1\gamma$$

At level AA, $\sigma = q + Z_1\gamma + Z\gamma_{sat}$

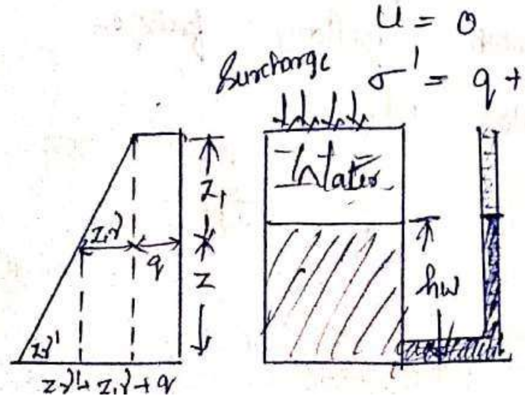
$$u = h_w\gamma_w$$
$$= Z\gamma_w$$

$$\sigma' = \sigma - u$$
$$= q + Z_1\gamma + Z\gamma'$$

At level BB, $\sigma = q + Z_1\gamma$

$$u = 0$$

Surcharge $\sigma' = q + Z_1\gamma$



(6)

iii) Saturated soil with capillary fringe

$$\sigma' = Z\gamma' + Z_1\gamma_{sat}.$$

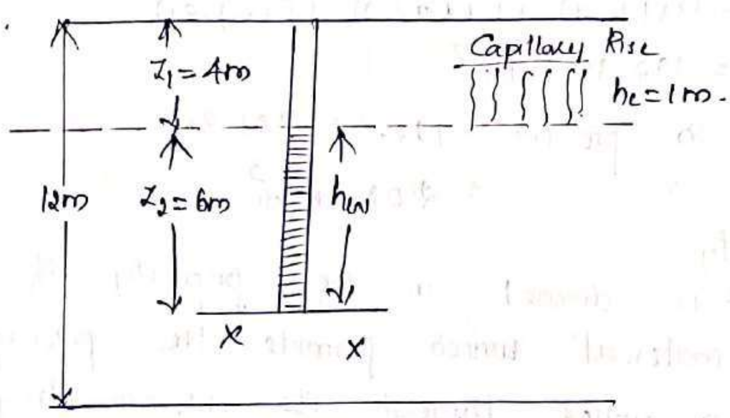
iv) Partially saturated soil

$$\sigma' = \sigma - u.$$

Problems

1) The water table in a certain area is at a depth of 4m below the ground surface. To a depth of 12m, the soil consists of very fine sand having an average voids ratio of 0.7. Above the water table the sand has an average degree of saturation of 50%. Calculate the effective pressure on the horizontal plane at a depth 10m below the ground surface. What will be the increase in the effective pressure if the soil gets saturated by capillary upto a height of 1m above the water table? Assume $G = 2.65$.

Soln:



Height of sand layer above water table, $Z_1 = 4m$

Height of saturated layer $= 12 - 4 = 8m.$

Depth of point X, where pressure is to be computed $= 10m$

Height of saturated layer above $X = Z_2 = 10 - 4 = 6m.$

Now, $\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.7} = 15.29\ kN/m^3.$

i) For sand above water table

$$e = \frac{wG}{S_r} \Rightarrow w = \frac{eS_r}{G}$$

$$= \frac{0.7 \times 0.5}{2.65} = 0.132$$

(7)

$\gamma_1 = \gamma_d (1+w) = 15.29 \times 1.132 = 17.31 \text{ kN/m}^3$

ii) for saturated sand below water table

$w_{sat} = \frac{e}{G} = \frac{0.7}{2.65} = 2.64$

$\gamma_2 = \gamma_d (1 + w_{sat}) = 15.29 \times 1.264 = 19.33 \text{ kN/m}^3$

$\gamma_2' = 19.33 - 9.81 = 9.52 \text{ kN/m}^3$.

Effective pressure at X

$\sigma = Z_1 \gamma_1 + Z_2 \gamma_2$

$= 4 \times 17.31 + 6 \times 19.33 = 185.22 \text{ kN/m}^2$.

$u = h_w \gamma_w = 6 \times 9.81 = 58.85 \text{ kN/m}^2$

$\sigma' = \sigma - u$

$= 126.36 \text{ kN/m}^2$.

Effective stress at x after capillary rise

$\sigma' = 3\gamma_1 + (6+1)\gamma_2' + h_c \gamma_w$

$= (3 \times 17.31) + (7 \times 9.52) + (1 \times 9.81)$

$= 128.38 \text{ kN/m}^2$.

Increase in pressure $= 128.38 - 126.36$

$= 2.02 \text{ kN/m}^2$.

<u>Permeability</u>

It is defined as the property of a porous material which permits the passage or seepage of water through its interconnecting voids.

A material having continuous voids is called permeable.

The study of seepage of water through soil is important for the following engineering problems.

i) Determination of rate of settlement of a saturated compressible soil layer.

ii) Calculation of seepage through the body of earth dams, and stability of slopes.

3) Calculation of uplift pressure under hydraulic structures and their safety against piping.

4) Ground water flow towards well and drainage of soil.

Darcy's Law

The law of flow of water through soil was first studied by Darcy (1856) who demonstrated experimentally that for laminar flow conditions in a saturated soil, the rate of flow or the discharge per unit time is proportional to the hydraulic gradient.

$$q = kiA \quad \text{——— (1)}$$

$$v = \frac{q}{A} = ki \quad \text{——— (2)}.$$

where q - discharge per unit time

A - total cross sectional area of soil mass, perpendicular to the direction of flow

i - hydraulic gradient.

k - Darcy's coefficient of permeability.

v - velocity of flow or average discharge velocity.

If a soil sample of length L and cross sectional area A is subjected to differential head of water, $h_1 - h_2$, the hydraulic gradient i will be equal to $\frac{h_1 - h_2}{L}$,

$$q = k.\frac{h_1 - h_2}{L}.A$$

when hydraulic gradient is unity, k is equal to V

Coefficient of permeability.

It is defined as the average velocity of flow that will occur through the total cross sectional area of soil under unit hydraulic gradient. Its unit is cm/sec.

Seepage Velocity :

It is defined as the rate of discharge of percolating water per unit cross sectional area of voids perpendicular to the direction of flow.

$$q = VA = V_s A_s .$$

$$V_s = V . \frac{A}{A_s} \quad ; \quad \text{But} \quad \frac{A_v}{A} = \frac{V_v}{V} = n .$$

$$= V . \frac{A}{A_v} = V . \frac{1}{n} = \frac{V}{n} = \frac{1+e}{e} . n .$$

Validity of Darcy's Law

Reynold found that the flow is laminar so long as the ~~critical~~ velocity of flow is less than a lower critical velocity V_c, expressed in terms of Reynolds number as follows:

$$\frac{V_c \, d \, \rho_w}{\eta g} = 2000 .$$

V_c - lower critical velocity in the pipe (cm/sec)

d - diameter of pipe (cm)

ρ_w - density of water (g/ml)

η = viscosity of water (g sec / cm^2)

Factors affecting permeability.

i) Effect of size and shape of particles

Permeability varies approximately as the square of the grain size.

$$k = C D_{10}^2$$

$C \rightarrow$ constant, approximately equal to 100

$D_{10} \rightarrow$ in cm.

ii) Effect of properties of pore fluid.

Permeability is directly proportional to the unit weight of water and inversely proportional to its viscosity. Though the unit weight of water does not change much with the change in temperature there is great variation in viscosity with temperature.

$$\frac{k_1}{k_2} = \frac{\eta_2}{\eta_1}$$

It is usual to convert the permeability result to a standard temperature (27°C) for comparison purposes, by expression.

$$k_{27} = k \cdot \frac{\eta}{\eta_{27}}$$

(iii) Effects of voids ratio

The effects of voids ratio on the values of permeability can be expressed as

$$\frac{k_1}{k_2} = \left[\frac{C_1 e_1^3}{1+e_1}\right] \div \left[\frac{C_2 e_2^3}{1+e_2}\right]$$

iv) Effect of structural arrangement of particles and stratification

The structural arrangement of the particle may vary, at the same voids ratio, depending upon the method of deposition or compacting the soil mass. The structure may be entirely different for a disturbed sample as compared to an undisturbed sample which may possess stratification.

v) Effect of degree of saturation and other foreign matter

The permeability is greatly reduced if air is entrapped in the voids thus reducing its degree of saturation. The dissolved air in the pore fluid may get liberated, thus changing the permeability.

vi) Effect of adsorbed water

The adsorbed water surrounding the fine soil particles is not free to move and reduces the effective pore space available for the passage of water.

Coefficient of absolute permeability.

The coefficient of absolute permeability is defined by the expression,

$$k = k \left(\frac{\eta}{\gamma_w} \right)$$

$$k = C \left(\frac{e^3}{1+e} \right) D^2$$

The above equation indicates that the coefficient of absolute permeability is independent of the properties of water and it depends on the properties of soil mass.

k has the dimensions of area, mm^2, cm^2, m^2.

Determination of Coefficient of Permeability

The coefficient of permeability can be determined by the following methods.

a) Laboratory Methods.
   i) Constant head permeability test
   ii) Falling head permeability test

b) Field Methods.
   i) Pumping out test
   ii) Pumping in test

c) Indirect Methods
   i) Computation from grain size or specific surface
   ii) Horizontal capillarity test
   iii) Consolidation test data.

Empirical Formulae to determine the coefficient of permeability.

i) Loudon's formula

$$k = 100 D_m^2 \quad ; D_m \rightarrow \text{grain size in cm}$$

ii) Allen Hazen's formula

$$k = C D_{10}^2 \quad ; C \rightarrow \text{Constant} \simeq 100 \; ; D_{10} \text{ in cm}$$

iii) Terzaghi's formula

$$k = 200 D_e^2 e^2 \quad ; D_e \rightarrow \text{effective grain size.}$$

iv) Kozeny's formula

$$k = \frac{1}{k_k \eta S_s^2} \times \frac{n^3}{1-n^2}$$

S_s - specific surface of particles (cm^2/cm^3)

η - viscosity ($g\ sec/cm^2$)

k_k - constant, equal to 5 for spherical particles.

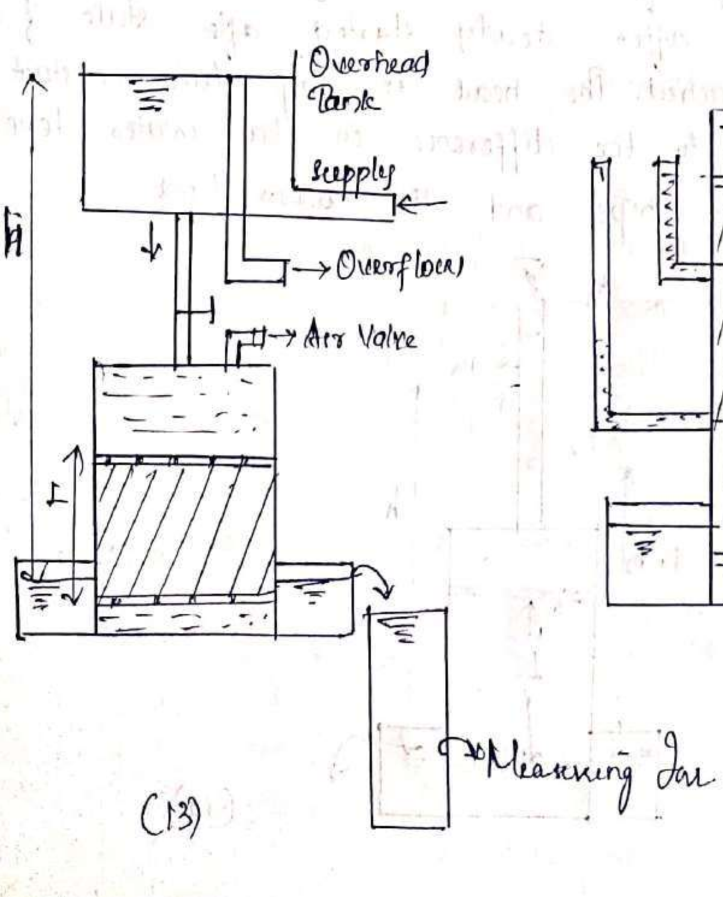
v) Louden's formula

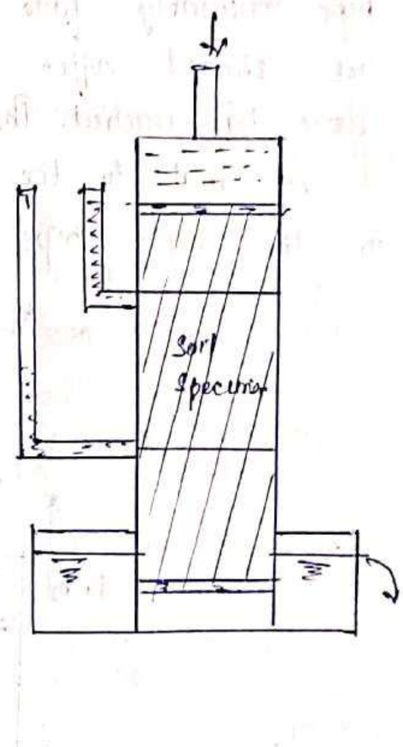
$$\log_{10}(k S_s^2) = a + bn$$

$a, b \rightarrow$ constants $= 1.365$ & 5.15 @ $10^\circ C$

Constant head permeability Test

Water flows from the overhead tank consisting of three tubes; the inlet tube, the overflow tube and the outlet tube. The constant hydraulic gradient 'i' causing the flow is the head h (i.e., difference in the water levels of the overhead and bottom tanks) divided by the length L of the sample. If the length of the sample is large, the head lost over a length of specimen is measured by inserting piezometric tubes.





If Q is the total quantity of flow in a time interval t, we have from Darcy's law

$$q = \frac{Q}{t} = kiA$$

$$k = \frac{Q}{t} \frac{1}{i.A}$$

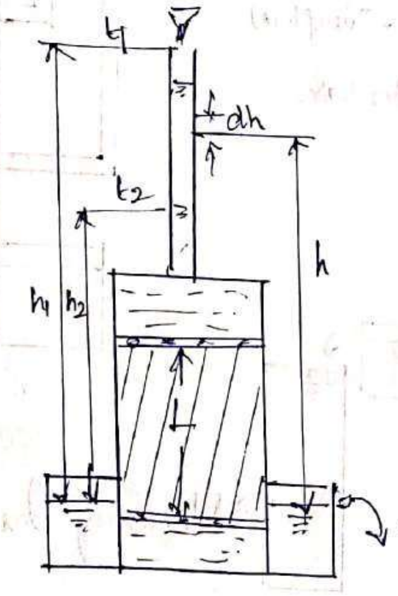
$$= \frac{Q}{t} \cdot \frac{L}{h} \cdot \frac{1}{A}$$; $A \rightarrow$ cross sectional area

When steady flow is reached, the total quantity of water Q in time t collected in a measuring jar.

Falling head permeability Test

The constant head permeability test is used for coarse grained soil only where a reasonable discharge can be collected in a given time. However the falling head test is used for relatively less permeable soils where the discharge is small.

A stand pipe of known cross sectional area 'a' is fitted over the permeameter and water is allowed to run down. The water level in the stand pipe constantly falls as water flows. Observations are started after steady ~~started~~ after state of flow has reached. The head at any time instant t is equal to the difference in the water level in the stand pipe and the bottom tank.



Let h_1 and h_2 be heads at time intervals t_1 and t_2 ($t_2 > t_1$) respectively. Let h be the head at any intermediate time interval t and $-dh$ be the change in the head in a smaller time interval dt (minus sign has been used since h decreases as t increases)

Hence, from Darcy's law, the rate of flow q is given by

$$q = \left(\frac{-dh \cdot a}{dt}\right) = -kiA$$

$i \rightarrow$ hydraulic gradient at time $t = \frac{h}{L}$

$$\frac{k \cdot h}{L} A = \frac{-dh}{dt} \cdot a \quad \text{or} \quad \frac{Ak}{aL} dt = \frac{-dh}{h}$$

Integrating between two limits.

$$\frac{Ak}{aL} \int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{dh}{h} = \int_{h_2}^{h_1} \frac{dh}{h}$$

$$\text{or} \quad \frac{Ak}{aL}(t_2 - t_1) = \log_e \frac{h_1}{h_2}$$

Denoting $t_2 - t_1 = t$, we get

$$k = \frac{aL}{At} \log_e\left(\frac{h_1}{h_2}\right) = 2.3 \frac{aL}{At} \log_{10}\left(\frac{h_1}{h_2}\right).$$

Problems

1) Calculate the coefficient of permeability of a soil sample, 6 cm in height and 50 cm² in cross sectional area, if a quantity of water equal to 430 ml passed down in 10 minutes, under an effective constant head of 40 cm. On oven drying, the test specimen has mass of 498 g. Taking the specific gravity of soil solids as 2.65, Calculate the seepage velocity of water during the test.

Soln:

$Q = 430$ ml ; $t = 10 \times 60 = 600$ s. ; $A = 50\,cm^2$; $L = 6\,cm$; $h = 40\,cm$

$$k = \frac{Q}{t} \cdot \frac{L}{h} \cdot \frac{1}{A} = \frac{430}{600} \times \frac{6}{40} \times \frac{1}{50} = 2.15 \times 10^{-3}\ cm/sec.$$

$$= 2.15 \times 10^{-3} \times 864$$

$$= 1.86\ m/d$$

[1 cm/sec = 864 m/day).

$$V = \frac{q}{A} = \frac{430}{600 \times 50} = 1.435 \times 10^{-2}\ cm/sec.$$

$$\rho_d = \frac{M_d}{V} = \frac{498}{50 \times 6} = 1.66\ g/cm^3$$

$$e = \frac{G \rho_w}{\rho_d} - 1 = \frac{2.65 \times 1}{1.66} - 1$$

$$= 0.595$$

$$\therefore n = \frac{e}{1+e} = \frac{0.595}{1+0.595} = 0.373$$

$$V_s = \frac{v}{n} = \frac{1.435 \times 10^{-2}}{0.373} = 3.85 \times 10^{-2}\ cm/s.$$

2) In a falling head permeameter test, the initial head ($t=0$) is 40 cm. The head drops by 5 cm in 10 minutes. Calculate the time required to run the test for the final head to be at 20 cm. If the sample is 6 cm in height and 50 cm² in cross sectional area, calculate the coefficient of permeability, taking area of stand pipe = 0.5 cm².

Soln:

In a time interval $t = 10$ minutes, the head drops from initial value of $h_1 = 40$ to $h_2 = 40-5 = 35$ cm.

$$k = 2.3 \frac{aL}{At} \log_{10}\left(\frac{h_1}{h_2}\right)$$

$$t = \frac{2.3\, aL}{Ak} \log_{10}\left(\frac{h_1}{h_2}\right)$$

$$= m \log_{10}\left(\frac{h_1}{h_2}\right)$$

$$m = \frac{2.3\, aL}{Ak}$$

$$t = m \log_{10} \frac{h_1}{h_2}$$

$$10 = m \cdot \log_{10}\left(\frac{40}{35}\right)$$

$$m = \frac{10}{\log_{10}\left(\frac{40}{35}\right)} = \frac{10}{0.058}$$

(16)

$$= 172.5\ \text{units}.$$

$$\therefore t = m \log_{10}\left(\frac{h_1}{h_2}\right)$$

$$= 172.5 \log_{10}\left(\frac{h_1}{h_2}\right)$$

Now, let the time interval required for the head to drop from initial value of $h_1 = 40$ cm to a final value of $h_2 = 20$ cm be 't' minutes

$$t = 172.5 \log_{10}\left(\frac{40}{20}\right) \qquad \log_{10}\left(\frac{40}{20}\right) = 0.301$$

$$= 51.9 \text{ mins.}$$

$$\therefore k = \frac{2.3\, aL}{Am.} \qquad \left[\because m = \frac{2.3\, aL}{Ak}\right]$$

$$= \frac{2.3 \times 0.5 \times 6}{50 \times 172.5 \times 60} \text{ cm/s.} \qquad \rightarrow (t \rightarrow \text{sec}).$$

$$= 1.335 \times 10^{-5} \text{ cm/sec}$$

3) A constant head permeability test was run on a sand sample 16 cm in length and 60 cm² in cross sectional area. Porosity was $n_1 = 40\%$. Under a constant head of 30 cm, the discharge was found to be 45 cm³ in 18 seconds. Calculate the coefficient of permeability. Also, determine the discharge velocity and seepage velocity during the test. Estimate the permeability of the sand for a porosity of $n_2 = 35\%$.

Soln:

$$K = \frac{Q}{t} \cdot \frac{L}{h} \cdot \frac{1}{A} = \frac{45}{18} \times \frac{16}{30} \times \frac{1}{60}.$$

$$= 2.22 \times 10^{-2} \text{ cm/s.}$$

Discharge velocity, $v = ki = k \cdot \frac{h}{l}$

$$= 2.22 \times 10^{-2} \times \frac{30}{60}$$

$$= 4.17 \times 10^{-2} \text{ cm/s.}$$

Seepage velocity, $v_s = \frac{v}{n}$

$$= \frac{4.7 \times 10^{-2}}{0.4}$$

$$= 10.42 \times 10^{-2} \text{ cm/s.}$$

$$\frac{k_1}{k_2} = \frac{e_1^3}{1+e_1} \cdot \frac{1+e_2}{e_2^3}$$

$$= \frac{n_1^3}{(1-n_1)^2} \Big/ \frac{n_2^3}{(1-n_2)^2}$$

$$k_2 = k_1 \cdot \frac{\frac{n_2^3}{(1-n_2)^3}}{\frac{n_1^3}{(1-n_1)^2}}$$

$$= 2.22\times10^{-2} \times \frac{\frac{0.35^3}{(1-0.35)^2}}{\frac{0.4^3}{(1-0.4)^2}}$$

$$= 1.26\times10^{-2} \text{ cm/s.}$$

Permeability of stratified soil deposits:

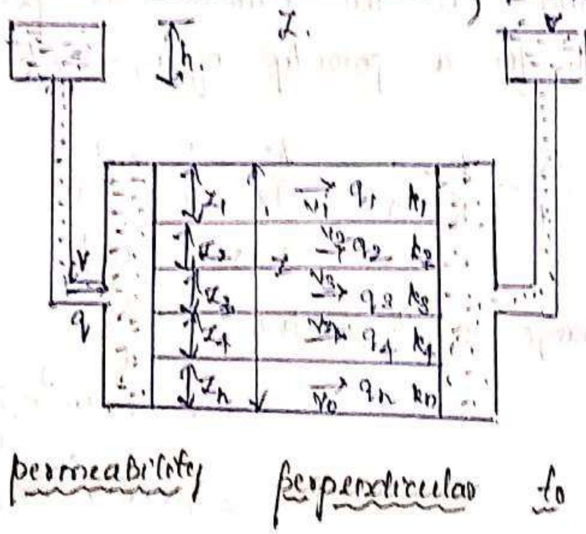
i) Average permeability parallel to the bedding plane.

$$q = q_1 + q_2 + \dots + q_n$$

$$q = k_x i Z$$

$$= k_1 i Z_1 + k_2 i Z_2 + \dots + k_n i Z_n$$

$$k_x = \frac{k_1 Z_1 + k_2 Z_2 + \dots + k_n Z_n}{Z} \; ; \; Z = Z_1 + Z_2 + \dots + Z_n$$



ii) Average permeability perpendicular to the bedding plane.

$$h = h_1 + h_2 + \dots + h_n$$

$$h_1 = i_1 Z_1 \; ; \; h_2 = i_2 Z_2 \; \dots \; h_n = i_n Z_n$$

(18) $$h = i_1 Z_1 + i_2 Z_2 + \dots + i_n Z_n \quad \text{—— (1)}$$

$$v = k_z i = k_z \frac{h}{Z}$$

$$i_1 = \frac{v}{k_1}, \; i_2 = \frac{v}{k_2}; \; \dots$$

Sub. in ①

$$\frac{vZ}{k_z} = \frac{vZ_1}{k_1} + \frac{vZ_2}{k_2} + \dots + \frac{vZ_n}{k_n}.$$

$$k_z = \frac{Z}{\frac{Z_1}{k_1} + \frac{Z_2}{k_2} + \dots + \frac{Z_n}{k_n}}.$$

Problems.

1) A stratified soil deposit consists of four layers of equal thickness. The coefficient of permeability of the second, third and fourth layers are respectively $1/3^{rd}$, $1/2$ and twice of the coefficient of permeability of the top layer. Compute the average permeabilities of the deposit, parallel and perpendicular to the direction of the stratification in terms of the permeability of the top layer.

Soln:

Let the thickness of the top layer be Z and its permeability be k.

$\therefore$ Total thickness of deposit $= 4Z$

Now $k_1 = k$; $k_2 = 1/3\, k$; $k_3 = k/2$; $k_4 = 2k$

$$k_x = \frac{Z.k + Zk/3 + Zk/2 + Z.2k}{4Z}$$

$$= \frac{23}{24} k$$

$$k_z = \frac{4Z}{\frac{Z}{k} + \frac{3}{k}Z + \frac{2}{k}Z + \frac{Z}{2k}}$$

$$= \frac{8}{13} k.$$

Water hydraulics

Aquifer:

Aquifers are permeable formations having structure which permit appreciable quantity of water to move through them under ordinary field conditions.

Types i) Confined aquifer

ii) Unconfined aquifer.

i) Confined aquifer or artesian aquifer:

It is the one in which ground water is confined under pressure greater than atmospheric by overlying relatively impermeable strata.

ii) Unconfined aquifer or water table aquifer

It is the one in which a water table serves as the upper surface of zone of saturation. It is also sometimes known as free, phreatic or non artesian aquifer.

Specific Yield of an aquifer:

It is defined as the ratio, expressed as percentage of the volume of water which after being saturated can be drained by gravity of the total volume of the aquifer.

$$S_y = \frac{\text{Volume of water drained by gravity}}{\text{Total volume}}$$

$$= \frac{V_{wy}}{V} \times 100.$$

Specific Retention

It is the ratio expressed as a percentage of the volume of water it will retain after saturation against the force of gravity to its own volume.

$$S_R = \frac{V_{wR}}{V} \times 100$$

V_{wR} - Volume of water retained.

Storage Coefficient

The water yielding capacity of a confined aquifer can be expressed in terms of its storage coefficient. It is defined as the volume of water that an aquifer releases per unit surface area of aquifer for unit change in the component of head normal to that surface. Its value ranges from 0.00005 to 0.005.

Coefficient of Permeability (K)

It is defined as the velocity of flow which will occur through the cross sectional area of the soil under a unit hydraulic gradient.

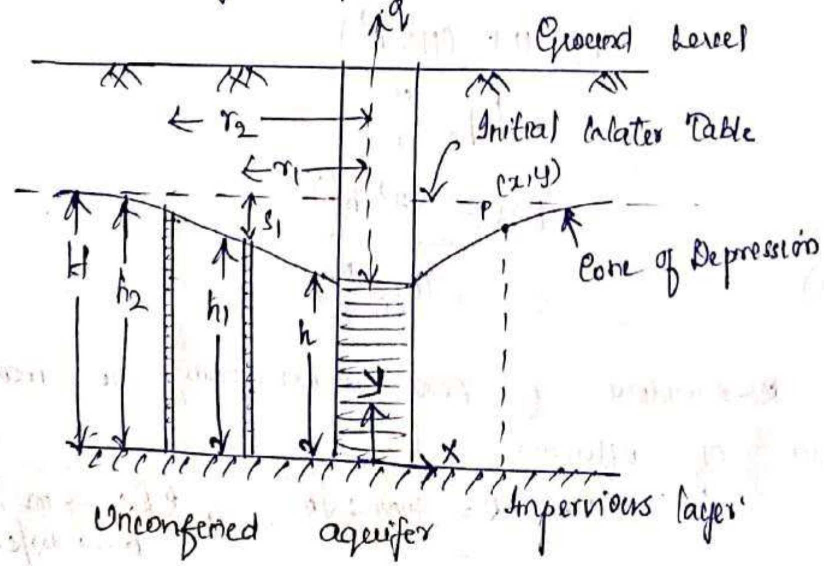
Coefficient of transmissibility (T)

It is defined as the rate of flow of water (in m^3/day) through a vertical strip of aquifer of unit width (1m) and extending the full saturation height under unit hydraulic gradient.

$$T = b \times K \rightarrow b = \text{aquifer thickness}$$
$$K = \text{coefficient of permeability.}$$

Steady Radial Flow to a well: Dupuit's Theory

When a well is penetrated into an extensive homogeneous aquifer, the water table initially remains horizontal in the well. When the well is pumped, water is removed from the aquifer and the water table or the piezometric surface, depending upon the type of the aquifer, is lowered resulting in a parabolic depression in the water table or the piezometric surface. The depression is called the cone of depression or the drawdown curve.



At any point, away from the well, the drawdown 's' is the vertical distance by which the water table or the piezometric surface is lowered.

1) Unconfined Aquifer

Let r - radius of the well

H - Thickness of the aquifer measured from the impermeable layer to the initial level of water table

s - draw down of the well

h - depth of water in the well measured above the impermeable layer.

Considering the origin of coordinates at a point O, at the centre of the well at its bottom, let the coordinates of any point P on the draw down curve be (x, y). Then from Darcy's law,

Discharge, $q = k A_x \cdot i_x$.

A_x - area of cross section of the saturated part of the aquifer at $P = (2\pi x) y = 2\pi xy$.

i_x - hydraulic gradient at $P = \frac{dy}{dx}$

$\therefore q = k(2\pi xy)\frac{dy}{dx}$ (or) $q \cdot \frac{dx}{x} = 2\pi k y\, dy$

Integrating.

$$q \int_r^R \frac{dx}{x} = 2\pi k \int_h^H y\, dy$$

$$q (\log_e x)_r^R = 2\pi k \left(\frac{y^2}{2}\right)_h^H$$

$$q = \frac{\pi k (H^2 - h^2)}{\log_e \frac{R}{r}}$$

$$= 1.36 k \frac{(H^2 - h^2)}{\log_e \frac{R}{r}}$$

(22)

$R \rightarrow$ radius of zero drawdown or maximum radius of influence.

Also, $R = 3000 s\sqrt{k}$, R & $s \rightarrow m$.

$k \rightarrow m/s$

Assumptions and Limitations of Dupuit's theory

1) The velocity of flow is proportional to the tangent of the hydraulic gradient instead of its sine.

2) The flow is horizontal and uniform everywhere in the vertical section.

3) Aquifer is homogeneous, isotropic and of infinite areal extent.

4) The well penetrates and receives water from the entire thickness of the aquifer.

5) The coefficient of transmissibility is constant at all places and at all times.

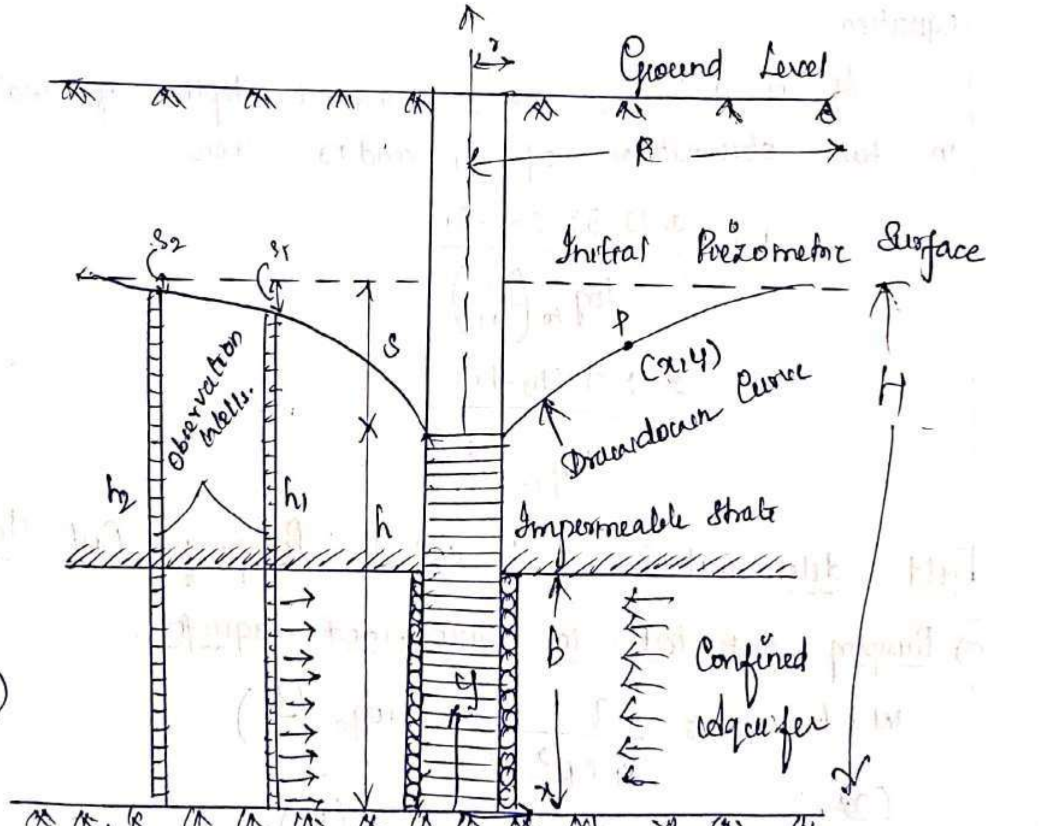
6) Natural ground water regime affecting an aquifer remains constant with time.

7) Flow is laminar and Darcy's law is valid.

2) Confined Aquifer.

Let (x,y) be the coordinates of any point P on the drawdown curve measured with respect to the origin O. Then from Darcy's law, flow crossing a vertical plane through P is given by $q = k\, i_x A_x$.

A_x - cross sectional area of the flow measured as $A = 2\pi x b$.



Confined aquifer.

(23)

b - thickness of confined aquifer.

i_x - hydraulic gradient at $P = \frac{dy}{dx}$.

$$\therefore\ q = k\left(\frac{dy}{dx}\right)(2\pi x b) \quad \text{or}$$

$$q\,\frac{dx}{x} = 2\pi k\, b\, dy.$$

Integrating between the limits (R, r) for x and (H, h) for y, we get

$$q\int_{r}^{R}\frac{dx}{x} = 2\pi k b\int_{h}^{H} dy$$

$$q\left[\log_e x\right]_{r}^{R} = 2\pi k b\left[y\right]_{h}^{H}$$

$$q = \frac{2\pi k b\,(H-h)}{\log_e \frac{R}{r}} = \frac{2.72\, bk\,(H-h)}{\log_{10}\frac{R}{r}}$$

$$= \frac{2\pi T s}{\log_e \frac{R}{r}} = \frac{2.72\, T s}{\log_{10}\frac{R}{r}}$$

T - coefficient of transmissibility = bk

$s \rightarrow$ drawdown at the well.

This is known as equilibrium equation or Thiem equation.

If h_1 and h_2 are measured depths of water in two observations of r_1 and r_2 then

$$q = \frac{2.72\, bk\,(h_2-h_1)}{\log_{10}\left(\frac{r_2}{r_1}\right)}$$

$$= \frac{2.72\, T\,(h_2-h_1)}{\log_{10}\left(\frac{r_2}{r_1}\right)}$$

Field determination of k and T : Pumping Out Test

a) Pumping out test in unconfined aquifer.

W.k.t, $k = \frac{q}{\pi (H^2-h^2)}\cdot \log_e\left(\frac{R}{r}\right)$

(24)

$$= \frac{q}{1.36\,(H^2-h^2)}\cdot \log_{10}\left(\frac{R}{r}\right)$$

and $k = \frac{q}{\pi (h_2^2 - h_1^2)} \log_e \left(\frac{r_2}{r_1}\right)$.

$$= \frac{q}{1.36 (h_2^2 - h_1^2)} \log_{10} \left(\frac{r_2}{r_1}\right)$$

b) Pumping out test in unconfined aquifer.

w.k.t. $k = \frac{q}{2\pi b (H-h)} \log_e \left(\frac{R}{r}\right)$

$$= \frac{q}{2.72 \, b \, (H-h)} \log_{10} \left(\frac{R}{r}\right).$$

& $k = \frac{q}{2\pi b (h_2 - h_1)} \log_e \left(\frac{r_2}{r_1}\right)$

$$= \frac{q}{2.72 \, b \, (h_2 - h_1)} \log_{10} \left(\frac{r_2}{r_1}\right)$$

From ground water point of view, however the field practice is to determine the coefficient of transmissibility T, the draw down are observed at various observation wells.

From confined aquifer fig.

Let s_1 - drawdown in observation well $1 = H - h_1$

s_2 — draw down in observation well $2 = H - h_2$.

$\therefore h_2 - h_1 = (H - s_2) - (H - s_1) = s_1 - s_2$.

w.k.t $q = \frac{2.72 \, T \, (h_2 - h_1)}{\log_{10}\left(\frac{r_2}{r_1}\right)} = \frac{2.72 \, T \, (s_1 - s_2)}{\log_{10}\left(\frac{r_2}{r_1}\right)}$

$$T = \frac{q}{2.72 \, (s_1 - s_2)} \log_{10} \left(\frac{r_2}{r_1}\right)$$

Choosing $r_2 = 10 r_1$, we find $\log_{10} \frac{r_2}{r_1} = 1$

$$\therefore T = \frac{q}{2.72 \, (s_1 - s_2)}$$

$$= \frac{q}{2.72 \, \Delta s} \quad \Delta s - \text{difference in drawdown.}$$

Also $k = \frac{T}{b}$.

Problem.

1) In order to determine the field permeability of a free aquifer, pumping out test was performed and following observations were made.

Diameter of well = 20cm; Discharge from well = 240 m^3/hr.

R.L of original water surface, before pumping started = 240.5 m.

R.L of water in the well at constant pumping = 235.6 m

R.L of impervious layer = 210 m.

R.L of water in observation well = 239.8 m.

Radial distance of observation well from the tube well = 50 m

Calculate k. Also calculate i) the error in k if observations are not taken in the observation well and (ii) radius of influence is assumed to be 300 m. (iii) actual radius of influence based on the observations of observation well.

Soln:

We have $q = \dfrac{1.36\,k(h_2^2 - h_1^2)}{\log_{10}\left(\frac{r_2}{r_1}\right)}$

$r = D/2$

$r_1 = 10cm = 0.1\,m$; $r_2 = 50m$.

$h_1 = 235.6 - 210 = 25.6m$

$h_2 = 239.8 - 210 = 29.8m$

$(h_2 + h_1) = 29.8 + 25.6 = 55.4m$

$h_2 - h_1 = 29.8 - 25.6 = 4.2m$.

$q = 240\ m^3/hour$.

$$240 = \frac{1.36\,k(55.4) \times 4.2}{\log_{10}\left(\frac{50}{0.1}\right)}$$

$$k = \frac{240}{1.36 \times 55.4 \times 4.2}\ \log_{10} 500$$

$= 2.045\ m/hr$.

$= 49.15\ m/day$.

(26)

b) If k is computed on the basis of assumed radius of influence,

$$q = \frac{1.36k(H^2-h^2)}{\log_{10}\left(\frac{R}{r}\right)}$$

$$\frac{R}{r} = \frac{300}{0.1} = 3000 \; ; \quad H = 240.5 - 210 = 30.5m$$

$$h = 235.6 - 210 = 25.6m.$$

$$H+h = 30.5 + 25.6 = 56.1m.$$

$$H-h = 30.5 - 25.6 = 4.9m.$$

$$k = \frac{240}{13.6 \times 56.1 \times 4.9} \log_{10} 3000$$

$$= 2.23 \; m/hour = 53.4 \; m/day.$$

$$\therefore \% \text{ error} = \frac{53.4 - 49.15}{49.15} \times 100$$

$$= 8.6\%.$$

c) Calculation of actual radius of influence

$$\log_{10}\left(\frac{R}{r}\right) = \frac{1.36k(H^2-h^2)}{q} \; ; \quad k = 2.045 \; m/hr.$$

$$H+h = 56.1m \; , \quad H-h = 4.9m \; ; \quad q = 240 \; m^3/hr.$$

$$\log_{10}\left(\frac{R}{r}\right) = \frac{1.36 \times 2.045 \times 56.1 \times 4.9}{240}$$

$$= 3.185$$

$$\frac{R}{r} = 1531$$

$$\Rightarrow R = 1531 \times r = 1531 \times 0.1 = 153m.$$

Total Two Dimensional Flow: Laplace Equation

The quantity of water flowing through a saturated soil mass, as well as the distribution of water pressure can be estimated by the theory of flow of fluids through porous medium. While computing these quantities with the help of theoretical analysis that follow the following assumptions.

1) The saturated porous medium is incompressible. The size of the pore spaces does not change with time regardless of water pressure. (27)

2) The seeping water flows under a hydraulic gradient which is due only to gravity head loss, or Darcy's law for flow through porous medium is valid.

3) There is no change in the degree of saturation in the zone of soil through which water seeps and the quantity of water flowing into any element of volume is equal to the quantity which flows out in the same length of time.

4) The hydraulic boundary conditions at entry and exit are known.

5) Water is incompressible.

We shall first derive the Laplace equation for two dimensional flow.

Consider an element of soil of size $\Delta x, \Delta y$ and of unit thickness perpendicular to the plane of the paper. Let v_x and v_y be the entry velocity components in x and y directions. Then $\left(v_x + \frac{\partial v_x}{\partial x} \Delta x\right)$ and $\left(v_y + \frac{\partial v_y}{\partial y} \cdot \Delta y\right)$ will be the corresponding velocity components at the exit of the element.

According to assumption 3 stated above, the quantity of water entering is equal to the quantity of water leaving it

$$v_x (\Delta y . 1) + v_y (\Delta x . 1) = \left(v_x + \frac{\partial v_x}{\partial x} . \Delta x\right)(\Delta y . 1) + \left(v_y + \frac{\partial v_y}{\partial y} . \Delta y\right)(\Delta x . 1)$$

From which $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$ → (1)

This is known as continuity equation.

According to assumption 2

$$v_x = k_x \cdot i_x = k_x \cdot \frac{\partial h}{\partial x}$$ and

$$v_y = k_y \cdot i_y = k_y \frac{\partial h}{\partial y}$$

where $h \rightarrow$ hydraulic head under which water flows.
k_x and $k_y \rightarrow$ coefficients of permeability in x and y direction

substituting this in ①

$$\frac{\partial^2 (k_x h)}{\partial x^2} + \frac{\partial^2 (k_y h)}{\partial y^2} = 0.$$

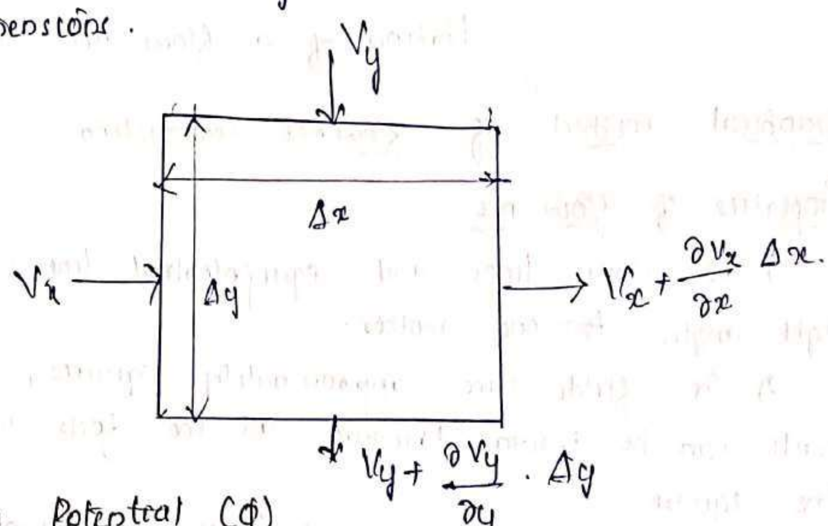
For an isotropic soil, $k_x = k_y = k$.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0.$$

substituting $\phi = kh$ = velocity potential

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$$

This is the laplace equation of flow in two dimensions.



Velocity Potential (ϕ)

It may be defined as a scalar function of space and time such that its derivative with respect to any direction gives the fluid velocity in that direction.

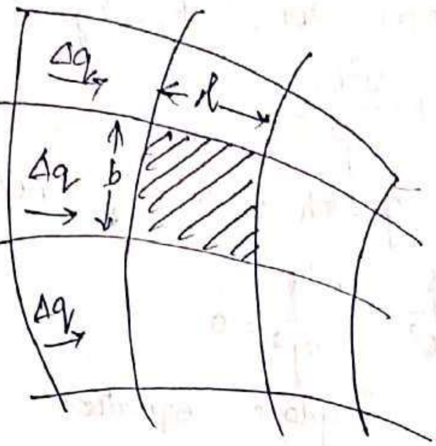
$$\therefore \quad \phi = kh$$

$$\therefore \frac{\partial \phi}{\partial x} = k \cdot \frac{\partial h}{\partial x} = k i_x = V_x$$

$$\text{IIIly} \quad \frac{\partial \phi}{\partial y} = k \cdot \frac{\partial h}{\partial y} = k \cdot i_y = V_y .$$

The solution gives two sets of curves known as equipotential lines and stream lines (flow lines) mutually orthogonal to each other.

The direction of seepage is always perpendicular to the equipotential lines. The path along which the individual particles of water seep through the soil are called <u>stream lines</u> or <u>flow lines</u>



Portion of a flow net.

<u>Graphical method of flow net construction</u>

<u>Properties of flow net</u>

1) The flow lines and equipotential lines meet at right angles to one another.

2) The fields are approximately squares, so that a circle can be drawn touching all the four sides of the square.

3) The quantity of water flowing through each flow channel is the same. Similarly the same potential drop occurs between two successive equipotential lines.

4) Smaller the dimensions of the field, greater will be the hydraulic gradient and velocity of flow through it.

5) In a homogenous soil, every transition in the shape of the curves is smooth, being either elliptical or parabolic shape

Arthur Casagrande (1937) gave the following excellent hints for the flow net sketching.

1) Use every opportunity to study the appearance of well constructed flow nets. When the picture is sufficiently absorbed in your mind, try to draw the same flow net without net in a satisfactory manner.

2) Four or five flow channels are usually sufficient for the first attempts; the use of too many flow channels may distract the attention from essential features

__Applications of flow net.__

1) __Determination of seepage__

The portion between any two successive flow lines is known as flow channel. The portion enclosed between two successive equipotential lines and successive flow lines is known as field such as that shown (Hatch).

Let b and l be the width and length of the field. Δh - head drop through the field;

Δq - discharge passing through the flow channel, H - total hydraulic head.

From Darcy's law

$$\Delta q = k \cdot \frac{\Delta h}{l} (b \times l)$$

If N_d - total number of potential drops in the complete flow net then, $\Delta h = \frac{H}{N_d}$

Hence $\Delta q = k \cdot \frac{H}{N_d} \left(\frac{b}{l}\right)$.

The total discharge through the complete flow net is given by

$$q = \Sigma \Delta q = k \cdot \frac{H}{N_d} \left(\frac{b}{l}\right) \cdot N_f$$

$$= kH \cdot \frac{N_f}{N_d} \frac{b}{l}.$$

(81)

$$b = kH \frac{N_f}{N_d}$$ [If field is square $b = l$]

ii) <u>Determination of hydrostatic pressure</u>

The hydrostatic pressure at any point within the soil mass is given by

$u = h_w \gamma_w$. $u \to$ hydrostatic pressure
h_w - piezometric head.

iii) <u>Determination of Seepage Pressure</u>

The hydraulic potential 'h' at any point located after n potential drops, each of value Δh is given by

$h = H - n\Delta h$.

iv) <u>Determination of exit gradient</u>

The exit gradient is the hydraulic gradient at the downstream end of the flow line where the percolating water leaves the soil mass and emerges into the free water at downstream.

$$i_e = \frac{\Delta h}{l}$$

$i_e \to$ exit gradient.
$\Delta h \to$ Potential drop
$l \to$ average length

Problems

1) For a homogeneous earth dam 52m high and 2m free board, a flow net was constructed and following results were obtained.

Number of equipotential drops = 25
Number of flow channels = 4. The dam has a horizontal filter of 6m length at its downstream end. Calculate the discharge per unit length of dam if the coefficient of permeability of dam material is 3×10^{-3} cm/sec.

Soln:

$q = kH \cdot \frac{N_f}{N_d}$; $H = 52-2$
$= 50m$

$k = 3\times10^{-3}$ cm/sec $= 3\times10^{-5}$ m/sec

$N_f = 4$; $N_d = 25$

$q = 3\times10^{-5} \times 50 \times \frac{4}{25}$

$= 24\times10^{-5}$ m^3/sec/m

$= 0.00024$ m^3/m length.

UNIT-III

STRESS DISTRIBUTION AND SETTLEMENT:

Stress distribution in homogeneous and isotropic medium - Boussinesq theory (Point load, line load and strip) Use of Newmark's influence chart - components of settlement - Immediate and consolidation settlement - Factors influencing settlement - Terzaghi's one dimensional consolidation theory - Computation of rate of settlement - $\sqrt{t}$ and log t methods - e - log p relationship consolidation settlement - N-C clays - O.C clays - Computation.

Stress Distribution:

Consider stresses within a soil mass due to its own weight. Stresses due to self weight are sometimes known as geostatic stresses.

Let us take soil mass to be bounded by horizontal plane (ground surface) xy and the z-axis be directed downwards. Under this condition, soil mass is said to be semi infinite. Where there is no external loading, the ground plane becomes a principal plane since it has shear loading.

Within soil mass,

$$\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

$\sigma_z = \gamma z$ ← From equilibrium equation

$\gamma \rightarrow$ unit weight of soil.

$\sigma_z \rightarrow$ vertical stress at a point within and soil mass, situated at a depth z below ground surface.

From compatibility equations,

$$\sigma_x = \sigma_y = \frac{\mu}{1-\mu}\gamma z.$$

$$\sigma_x = \sigma_y = K_0 \gamma z \qquad K_0 = \frac{\mu}{1-\mu}.$$

μ - Poisson's ratio.

K_0 - coefficient of lateral pressure @ rest.

At certain point within soil mass the stresses are caused due to both surface loadings as well as due to self weight of soil above it.

Total stress = stress due to soil + stress due to surface load

Stress Tensor

$$\begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

σ - Normal stress
τ - Shear stress.

Strain Tensor

$$\begin{bmatrix} \epsilon_x & \gamma_{yx}/2 & \gamma_{zx}/2 \\ \gamma_{xy}/2 & \epsilon_y & \gamma_{xy}/2 \\ \gamma_{xz}/2 & \gamma_{yz}/2 & \epsilon_z \end{bmatrix}$$

ϵ - direct strain
γ - shearing strain

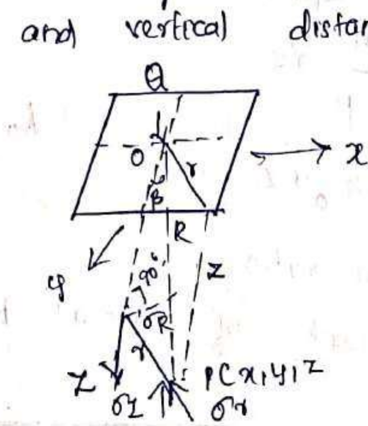
Boussinesq Equations

Concentrated Forces

Assumptions

* Soil mass is an elastic medium for which modulus of elasticity E is constant.
* Soil mass is homogeneous, it has identical properties at every point in its identical directions.
* Soil mass is isotropic, it has identical elastic properties in all directions through any point of it.
* Soil mass is semi infinite, it extends infinitely in all directions below level surface.

Let us find stress components at a point P in soil mass, having coordinates x, y, z having radial distance r and vertical distance z from point O



σ_R - Polar radial stress

(2)

Using logarithmic stress function, Boussinesq showed that polar radial stress

$$\sigma_R = \frac{3}{2}\frac{Q}{\pi}\frac{\cos\beta}{R^2}$$

R- Polar radial coordinate of point P

$$R = \sqrt{r^2+z^2} = \sqrt{x^2+y^2+z^2}$$

$$\cos\beta = \frac{z}{R}$$

σ_z - Vertical stress; τ_{rz} - tangential stress

$$\sigma_z = \sigma_R \cos^2\beta$$

$$= \frac{3}{2}\cdot\frac{Q}{\pi}\frac{\cos\beta}{R^2}$$

$$= \frac{3}{2}\cdot\frac{Q}{\pi}\frac{z^3}{R^5} = \frac{3Q}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

Multiply and divide by z^2

$$\sigma_z = \frac{3Q}{2\pi}\cdot\frac{z^2}{z^2}\cdot\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$= \frac{3Q}{2\pi z^2}\cdot\frac{z^5}{(r^2+z^2)^{5/2}} = \frac{3Q}{2\pi z^2}\frac{z^5}{(z^2)^{5/2}\left[1+\frac{r^2}{z^2}\right]^{5/2}}$$

$$= \frac{3Q}{2\pi z^2}\frac{\cancel{z^5}}{\cancel{z^5}\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$= \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$\tau_{rz} = \frac{1}{2}\sigma_R \sin 2\beta \qquad \because \sin 2\beta = 2\sin\beta\cos\beta$$

$$= \frac{1}{2}\sigma_R(2\sin\beta\cos\beta)$$

$$= \frac{3}{2}\frac{Q}{\pi}\frac{\cos\beta}{R^2}\sin\beta\cos\beta$$

$$= \frac{3Q}{2\pi R^2}\sin\beta\cos^2\beta$$

$$= \frac{3Q}{2\pi R^2}\cdot\frac{r}{R}\cdot\frac{z^2}{R^2}$$

$$= \frac{3Qrz^2}{2\pi R^5} = \frac{3Q}{2\pi}\cdot\frac{rz^2}{(r^2+z^2)^{5/2}}$$

$$= \frac{3Q r}{2\pi} \cdot \frac{z^7}{(z^2)^{5/2}\left[1+\frac{r^2}{z^2}\right]^{5/2}}$$

$$= \frac{3Q r}{2\pi z^3}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

Boussinesq influence factor (K_B)

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$= K_B \cdot \frac{Q}{z^2}.$$

$$\boxed{K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}}$$

Influence factor - dimensionless.

Intensity of vertical pressure, directly below point load ($r=0$)

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} \quad \because r=0$$

$$= \frac{3Q}{2\pi z^2}$$

$$\therefore \sigma_z = 0.4775\frac{Q}{z^2}$$ (@ point directly below point load).

Pressure distribution Diagrams

By means of Boussinesq's stress distribution theory, the following vertical pressure distribution diagrams can be prepared.

* Stress isobar or isobar diagram
* Vertical pressure distribution on a horizontal plane.
* Vertical pressure distribution on a vertical line.

Isobars

It is a curve or contour connecting all points below ground surface of equal vertical pressure.

It is a spatial, curved surface of the shape of a bulb, because vertical pressure on a given horizontal plane is same in all directions at points located at equal radial distances around axis of loading.

The zone in a loaded soil mass bounded by an isobar of given vertical pressure intensity is called a pressure bulb.

The vertical pressure at every point on surface of pressure bulb is same.

Suppose an isobar of value $\sigma_z = 0.25\,Q$ per unit area is to be plotted.

$$\sigma_z = k_B \cdot \frac{Q}{z^2}$$

$$0.25Q = k_B \cdot \frac{Q}{z^2}$$

$$k_B = 0.25\,z^2$$

Vertical pressure distribution on a horizontal plane.

The vertical pressure distribution on any horizontal plane at a depth z below ground surface due to concentrated load is given by

$$\sigma_z = k_B \cdot \frac{Q}{z^2}$$

$z \rightarrow$ is known

$r \rightarrow$ horizontal distance

Below the load, i.e., $r=0$

$$\sigma_z = 0.4775 \cdot \frac{Q}{z^2}$$

r/z	k_B	σ_z
0	0.4775	Maximum
0.5	0.2733	57% of maximum
1	0.0844	17.7% of maximum
2	0.0085	1.8% of maximum.

Vertical Pressure under a uniformly loaded circular area

Boussinesq equation for vertical stress due to single concentrated load can now be extended to find vertical pressure on any point on vertical axis passing through the centre of uniformly loaded circular area

The fig. shows a uniformly loaded circular area of radius a and, load intensity q per unit area.

Consider an elementary ring of radius r and width δ_r on the loaded area. If elementary ring is further divided into small parts, each of area δ_A, the load on each elementary area will be $q\delta_A$.

This load may be considered as point load. Vertical pressure at point P, situated at depth z on vertical axis through centre of area is given by

$$\delta\sigma_z = \frac{3}{2\pi}(q\cdot\delta_A)\cdot\frac{z^3}{(r^2+z^2)^{5/2}}.$$

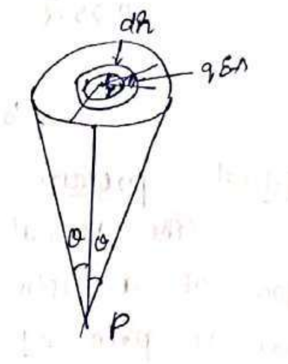
Integrating over entire ring of radius r, vertical stress $\Delta\sigma_z$ is given by

$$\Delta\sigma_z = \frac{3q}{2\pi}(\Sigma\delta_A)\cdot\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$= \frac{3q}{2\pi}\Sigma\,\delta(\pi r^2)\cdot\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$= \frac{3q}{\cancel{2\pi}}\,\frac{\cancel{2\pi} r\,\delta r\,z^3}{(r^2+z^2)^{5/2}}$$

$$= 3qr\,\delta_r\cdot\frac{z^3}{(r^2+z^2)^{5/2}}$$



The total vertical pressure σ_z due to entire loaded area is given by integrating above expression between limits r=0 to r=a

$$\sigma_z = 3qz^3\int_0^a \frac{r\,dr}{(r^2+z^2)^{5/2}}$$

Put $r^2+z^2=n^2$

Differentiate w.r.t. r

$$2r\,dr+0 = 2n\,dn$$

$$r\,dr = n\,dn$$

when $r=0$; $n=z$ & when $r=a$; $n=\sqrt{a^2+z^2}$

$$\sigma_z = 3qz^3\int_z^{\sqrt{a^2+z^2}}\frac{n\,dn}{(n^2)^{5/2}}$$

$$= 3qz^3\int_z^{\sqrt{a^2+z^2}}\frac{n\,dn}{n^5}$$

$$= 3qz^3\int_z^{\sqrt{a^2+z^2}}\frac{dn}{n^4}$$

$$= 3qz^3\int_z^{\sqrt{a^2+z^2}} n^{-4}\,dn.$$

Differentiation

$x^n = nx^{n-1}$

$x^{-4} = -4n^{-4-1}$

Integration

$x^n = \frac{x^{n+1}}{n+1}$

$n^{-4} = \frac{n^{-4+1}}{-4+1}$

$= \frac{n^{-3}}{-3}$.

$$= \frac{-3}{3} q z^3 \left[\frac{1}{r^3} \right]_z^{\sqrt{a^2+z^2}}$$

$$= -q z^3 \left[\frac{1}{r^3} \right]_z^{\sqrt{a^2+z^2}} = -q z^3 \left[\frac{1}{(a^2+z^2)^{3/2}} - \frac{1}{z^3} \right]$$

$$= q z^3 \left[\frac{1}{z^3} - \frac{1}{(a^2+z^2)^{3/2}} \right] = q \left[1 - \frac{z^3}{(a^2+z^2)^{3/2}} \right]$$

$$= q \left[1 - \frac{z^3}{z^3 \left[1 + \frac{a^2}{z^2} \right]^{3/2}} \right]$$

$$= q \left[1 - \left\{ \frac{1}{1 + \frac{a^2}{z^2}} \right\}^{3/2} \right]$$

$$= k_B \cdot q.$$

k_B - Boussinesq influence factor for uniformly distributed circular load.

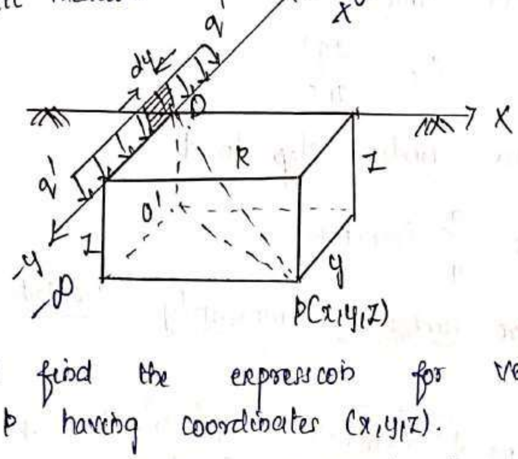
$$k_B = 1 - \left[\frac{1}{1 + \frac{a^2}{z^2}} \right]^{3/2}.$$

If θ is the angle which the line joining the point P, makes with outer edge of loading.

$$\sigma_z = q\,[1 - \cos^2\theta)$$

Vertical pressure due to line load

Let us consider an infinitely long line load of intensity q per unit length acting on the surface of a semi infinite elastic medium.



Let us find the expression for vertical stress at any point P having coordinates (x,y,z).

The radial distance of point $P = r$

$$r = \sqrt{x^2+y^2}.$$

The polar distance of point $P = R$.

$$R = \sqrt{r^2 + z^2} = \sqrt{x^2 + y^2 + z^2}$$

Consider a small length dy along line load. The elementary load in this length will be equal to $q' dy$, which may be considered as concentrated load.

Vertical stress, $\Delta\sigma_z = \dfrac{3(q' dy) z^3}{2\pi R^5}$.

$$= \frac{3q' dy\, z^3}{2\pi (x^2 + y^2 + z^2)^{5/2}}$$

$$\sigma_z = \int_{-\infty}^{\infty} \frac{3q' z^3\, dy}{2\pi (x^2 + y^2 + z^2)^{5/2}}$$

$$= 2\int_{0}^{\infty} \frac{3q' z^3\, dy}{2\pi (x^2 + y^2 + z^2)^{5/2}}$$

$$= \frac{2q' z^3}{\pi (x^2 + z^2)^2}$$

$$= \frac{2q'}{\pi z^4} \cdot \frac{z^3}{\left(1 + \frac{x^2}{z^2}\right)^2}$$

$$\sigma_z = \frac{2q'}{\pi z} \cdot \frac{1}{\left[1 + \left(\frac{x}{z}\right)^2\right]^2}$$

When P is situated be vertically below line load, at depth z we have $x = 0$

$$\therefore \sigma_z = \frac{2q'}{\pi z}.$$

Vertical pressure under strip load

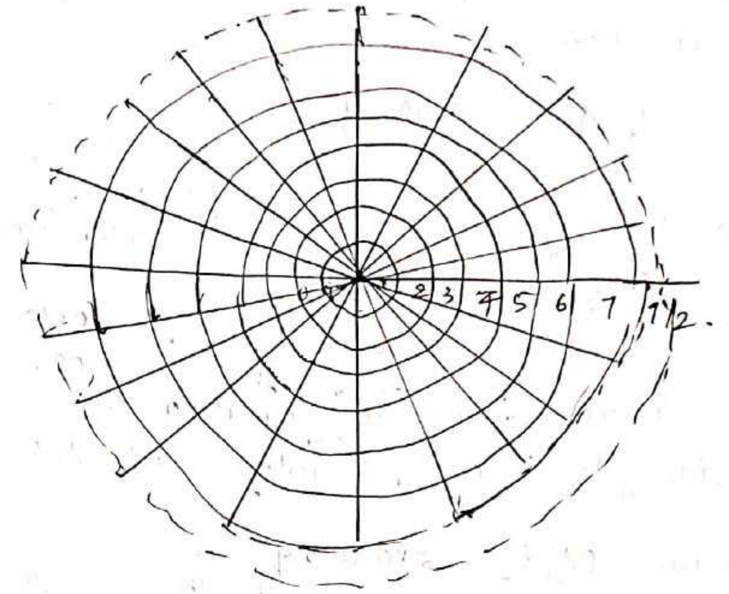
$$\sigma_z = \frac{q}{\pi}(\theta + \sin\theta)$$

Vertical pressure under a uniformly loaded rectangular area

$$\sigma_z = \frac{q}{4\pi}\left[\frac{2mn\sqrt{(m^2+n^2+1)}}{m^2+n^2+m^2n^2+1} \cdot \frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2mn\sqrt{(m^2+n^2+1)}}{m^2+n^2-m^2n^2+1}\right]$$

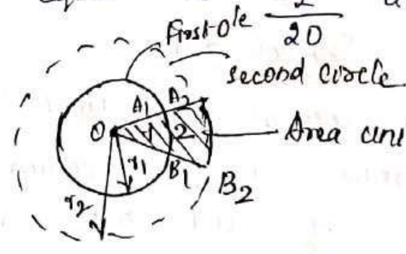
Newmark's Influence Chart (Accurate Method)

A chart consisting of number of circles and radiating lines is so prepared that the influence of each area unit is the same at centre of con circles i.e., each area unit causes the equal vertical stress at the centre of diagram.



Newmark's Influence Chart

Let a uniformly loaded circular area of radius r_1 cm be divided into 20 sectors (area units). If q is the intensity of loading and σ_z is vertical pressure at depth z below centre of area, each unit such as OA_1B_1 exerts a pressure equal to $\frac{\sigma_z}{20}$ at centre.



From formula of σ_z from uniformly loaded circular area

$$\sigma_z = q\left[1 - \left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{3/2}\right] \qquad (9)$$

Here $a = r$

$$\frac{\sigma_z}{20} = \frac{q}{20}\left[1 - \left\{\frac{1}{1+(r/z)^2}\right\}^{3/2}\right]$$

$$= i_f \cdot q$$

$$i_f \text{ - influence value } = \frac{1}{20}\left[1 - \left\{\frac{1}{1+(r_1/z)^2}\right\}^{3/2}\right]$$

If i_f be made equal to an arbitrary fixed value say 0.005, we have

$$\frac{\sigma_z}{20} = i_f \cdot q = 0.005\,q.$$

$$\frac{q}{20}\left[1 - \left\{\frac{1}{1+(r_1/z)^2}\right\}^{3/2}\right] = 0.005\,q.$$

$$i_f = \frac{1}{\text{No. of radial lines} \times \text{No. of concentric circle}} = \frac{1}{20 \times 10} = 0.005$$

r_1 can be found from the equation when z is known. The pressure due to second concentric circle A_1B_1, A_2B_2 is $0.005q$.

Total pressure $OA_2B_2 = 2 \times 0.005\,q$.

$|||^{ly}$ the radii of $3^{rd}, 4^{th}, 5^{th}, 6^{th}, 7^{th}, 8^{th}$ and 9^{th} circles can be calculated.

The radius of 10^{th} circle is given by

$$\frac{q}{20}\left[1 - \left\{\frac{1}{1+(r_{10}/z)^2}\right\}^{3/2}\right] = 10 \times 0.005\,q$$

$$= \frac{q}{20}$$

which means $r_{10} = \infty$

Vertical pressure $\sigma_z = \sigma_A = 0.005q \times N_A$

N_A - No. of area units under loaded area.

One dimensional Consolidation

When a compression load is applied to soil mass, a decrease in its volume takes place. The decrease in volume of soil mass under stress is known as compression.

The property of soil mass pertaining to its susceptibility to decrease in volume under pressure is known as compressibility

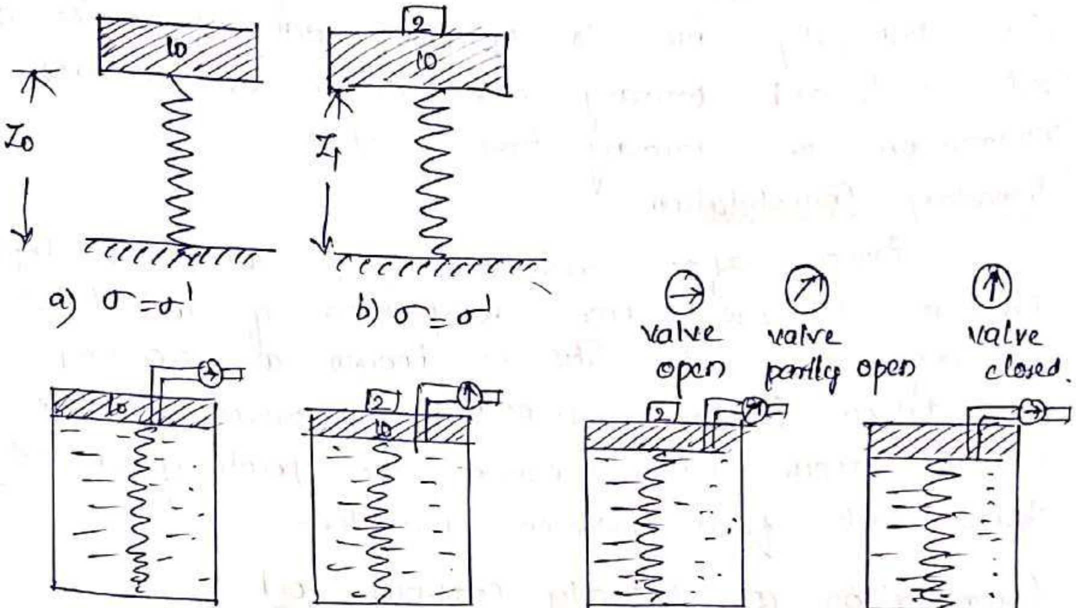
The compression that results from long term static load and consequent escape of pore water is known as consolidation.

According to Terzaghi: "Every process involving a decrease in water content of a saturated soil without replacement of water by air is called a process of consolidation."

Consolidation – decrease in water content

Compaction – Expulsion of air.

<u>Spring Analogy</u>



a) $\sigma = \sigma'$ b) $\sigma = \sigma'$

c) $\sigma = \sigma' = 10$ d) $\sigma = \sigma' + \bar{u}$ e) $\sigma = (10+\Delta\sigma') + (2-\Delta\sigma')$ f) $\sigma = \sigma' + \bar{u}$

Total Pressure = Pressure in spring + Pressure in water

<u>Pore Pressure</u> $(\bar{u})$

The pressure that builds up in pore water due to load increment on soil is termed as ~~exp~~ excess pore pressure / excess hydrostatic pressure or hydrodynamic pressure $(\bar{u})$. because it is in excess of initial pressure in water under static condition.

The excess hydrostatic pressure forces the water to drain out of voids. As water starts escaping from voids, the excess hydrostatic pressure in water gets gradually dissipated and the pressure increment is shifted as an increase in effective pressure on soil solids and soil ~~mass~~ decrease in volume.

When whole of pressure increment or consolidation pressure is carried as an increase in effective pressure on solids no more water escapes from voids and a condition of equilibrium is attained.

Hydrodynamic lag

The delay caused in consolidation by the slow drainage of water out of saturated soil mass is called hydrodynamic lag.

Primary Consolidation

The reduction in volume of soil which is due principally due to squeezing out of water from voids is termed primary consolidation or primary compression or primary time effect.

Secondary Consolidation

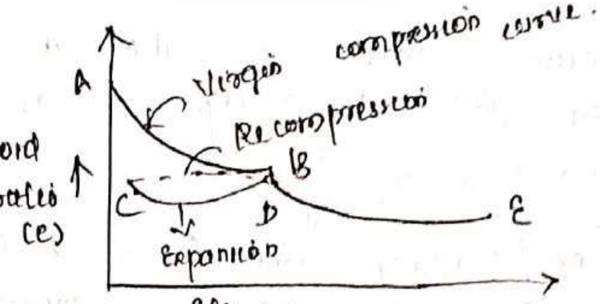
Even after reduction of all excess hydrostatic pressure to zero, some compression of soil takes place at very slow rate. This is known as secondary consolidation. During secondary compression, some of highly viscous water between the points of contact is forced out from between particles.

Consolidation of laterally confined soil.

If a remoulded soil is laterally confined in a consolidometer, consisting of a metal ring and porous stones are placed both at its top and bottom faces, the compression or consolidation of soil sample takes place under a vertical pressure applied on the top of porous stones. These porous stones provide free drainage of water and air from or into soil sample.

Under a given applied pressure, a final settlement and equilibrium voids ratio is attained after certain time.

At the equilibrium stage, the applied pressure naturally becomes effective pressure σ' on soil. The pressure can there be increased and a new equilibrium voids ratio in the form of curve.



At any intermediate stage at B, the pressure is completely removed, then the sample expands as represented by the expansion curve BC.

During expansion, the sample never attains the original void ratio because of some permanent compression mainly due to some irreversible orientation undergone by soil particles under compression.

If soil is again put under compression, a recompression curve such as CD is obtained. The void ratio at D being always less than that of B at the same pressure.

On further pressure increments, the curve DE is obtained. The portion AB of the curve represents the compression of soil which has not been subjected in past to pressures greater than those which are being applied for the present compression. Such a curve is called virgin compression curve and curve DE is virgin curve.

$\left.\begin{array}{l}\sigma' - \text{abscissa} \\ e - \text{ordinate}\end{array}\right\}$ – semi log plot – Virgin compression curve – straight line.

It can be expressed by

$$e = e_0 - C_c \log_{10} \frac{\sigma'}{\sigma_0'}$$

e_0 – initial voids ratio corresponding to initial pressure σ_0'

e – void ratio at increased pressure σ'

C_c – compression index (dimensionless)

The compression index represents slope of linear portion of the pressure – voids ratio curve and remains constant within a fairly large range of pressure.

$$C_c = \frac{e_0 - e}{\log_{10} \frac{\sigma'}{\sigma_0'}} = \frac{\Delta e}{\Delta \log_{10} \sigma'}$$

$$\Delta_e = C_c \log_{10} \left(\frac{\sigma_0' + \Delta\sigma'}{\sigma_0'} \right) \qquad (13)$$

The expansion curve is also a fairly straight line on semilog plot and is expressed as

$$e_0 = e + C_s \log_{10} \frac{\sigma'}{\sigma_0'}$$

C_s – Expansion / swelling index. It is a measure of volume increase due to removal of pressure.

Skempton conducted consolidated tests on number of clays and gave following equation.

$$C_c = 0.007 \,(w_L - 10\%)$$

C_c of remoulded sample

$$C_c = 0.009 \,(w_L - 10\%)$$

Clough gave equation for precompressed soils.

$$C_c = 0.3\,(e_0 - 0.27)$$

e_0 - initial void ratio.

<u>Coefficient of Compressibility (a_v)</u>

It is defined as the decrease in void ratio per unit increase of pressure.

$$a_v = \frac{-\Delta e}{\Delta \sigma'} = \frac{e_0 - e}{\sigma' - \sigma_0'}$$

For a given difference in pressure, value of coefficient of compressibility decreases as the pressure increases.

<u>Coefficient of volume change (m_v)</u>

It is defined as the change in volume of a soil per unit of initial volume due to given unit ~~decrease~~ increase in the pressure.

$$m_v = -\frac{\Delta e}{1+e_0} \cdot \frac{1}{\Delta \sigma'}$$

$$\frac{-\Delta e}{\Delta \sigma'} = a_v \Rightarrow m_v = \frac{a_v}{1+e_0}$$

When soil is laterally confined, change in volume is proportional to change in thickness ΔH and initial volume is proportional to initial thickness H_0

$$m_v = \frac{-\Delta H}{H_0} \cdot \frac{1}{\Delta \sigma'}$$

Settlement

The two types of settlement are

i) Initial settlement ii) Consolidation settlement.

i) Initial settlement

The change in thickness, ΔH due to pressure increment is given by.

$$\Delta H = -m_v H_0 \Delta\sigma'$$

(-) sign - decrease of void ratio / thickness with increase in pressure.

ii) Consolidation settlement

a) Using coefficient of volume change (m_v)

b) Using void ratio.

a) Final settlement using coefficient of volume change (m_v).

ρ_f - consolidation settlement

$= m_v H \Delta\sigma'$

This is an assumption that pressure increment is transmitted uniformly over thickness H.

In practical cases, under a finite surface loading, intensity of $\Delta\sigma'$ decreases with depth of layer in a non-linear manner. In such circumstances, consolidation settlement $\Delta\rho_f$ of an element of thickness dz is calculated under an average effective pressure increment $\Delta\sigma'$.

$$\Delta\rho_f = m_v \Delta\sigma' dz$$

Integrating for total thickness H of layer.

$$\rho_f = \int_0^H m_v \Delta\sigma' dz.$$

m_v and $\Delta\sigma'$ are variable.

The numerical integration can be performed by dividing the total thickness H into no. of thin layers and $\Delta\sigma'$ at mid height of each layer may be considered to represent a constant average pressure increment for the layer. Settlement of each layer can then be calculated.

Total Settlement = Sum of individual settlements of various thin layers.

$$\frac{\Delta H}{H} = \frac{e_0 - e}{1+e_0}$$

$$\rho_f = \Delta H = \frac{e_0 - e}{1+e_0} \cdot H$$

Consolidation of undisturbed specimen

OCR - Over Consolidation Ratio.

Soil deposits may be divided into 3 classes.

i) Pre consolidated or over consolidated - OCR > 1
U > 100

ii) Normally consolidated - OCR = 1 ; U = 100

iii) Under consolidated - OCR < 1 ; U < 100

Clay - Precompressed | Preconsolidated | Over consolidated

If it has been subjected to pressure in excess of its present over burden pressure. The temporary overburden pressure. The temporary overburden pressure is known as preconsolidation pressure.

Normally Consolidated soil

It is one which has never been subjected to an effective pressure greater than existing overburden pressure and which is completely consolidated by existing over-burden.

Under - Consolidated Soil

A soil which is not fully consolidated under the existing overburden pressure is called an under consolidated soil.

Terzaghi's Theory of one dimensional consolidation

Assumption

* The soil is homogeneous and fully saturated.
* Soil mass and water are incompressible.
* Deformation of soil is entirely due to change in volume.
* Darcy's law for velocity of flow of water through soil is perfectly valid.
* Coefficient of permeability is constant during consolidation.
* Load is applied in one direction only and deformation occurs only in direction of load application. i.e., soil is restrained against lateral deformation.
* Excess pore water drains out only in vertical direction.
* Boundary is free surface offering no resistance to flow of water from soil.

* Change in thickness during consolidation is insignificant.
* Time lag in consolidation is entirely due to permeability of soil and thus secondary consolidation is disregarded.

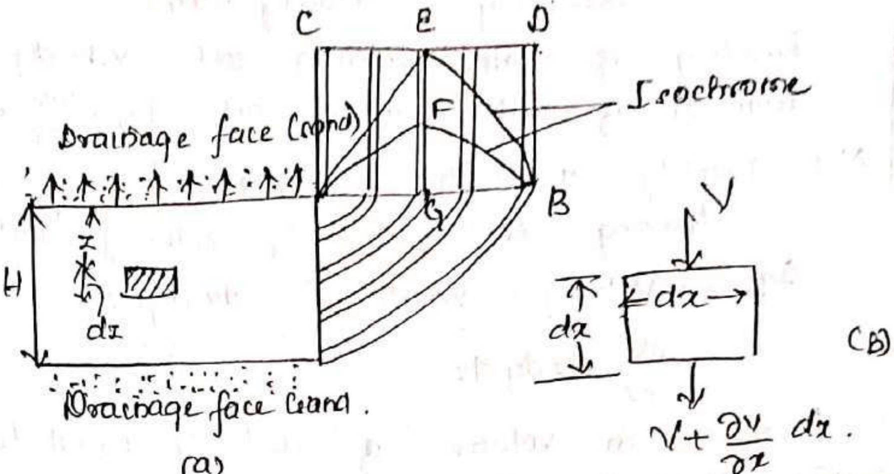


Fig (a) shows a clay layer of thickness H sandwiched between two layers of sand which serves as drainage faces.

When layer is subjected to pressure increment $\Delta\sigma$, excess hydrostatic pressure is set up in clay layer.

At time $t_0 = \Delta\sigma = \bar{u}$ – CED.

At time t_f $\bar{u} = 0$ – AGB

At any time t, $\Delta\sigma = \Delta\sigma' + \bar{u}$ – AFB

$$\bar{u} = h\gamma_w$$

Hydraulic head, $h = \frac{\bar{u}}{\gamma_w}$ ——— ①.

Hydraulic gradient, $i = \frac{\partial h}{\partial z} = \frac{\partial}{\partial z}\left(\frac{\bar{u}}{\gamma_w}\right)$

$$i = \frac{1}{\gamma_w}\frac{\partial \bar{u}}{\partial z} \text{ ——— ②}$$

Thus, rate of change of $\bar{u}$ along depth of layer represents hydraulic gradient.

Darcy's law, $v = ki$

$$= \frac{k}{\gamma_w}\cdot\frac{\partial \bar{u}}{\partial z} \text{ ——— ③}$$

The rate of change of velocity is

$$\frac{\partial v}{\partial z} = \frac{u}{\gamma_w}\cdot\frac{\partial^2 \bar{u}}{\partial z^2} \text{ ——— ④}$$

Consider a small soil element of size dx, dz & width dy perpendicular to xz plane.

v – velocity of water @ entry

$v + \frac{\partial v}{\partial z} \cdot dz$ – velocity of water @ exit

Quantity = velocity × area.

Quantity of water entering soil $= v\,dx\,dy$

Quantity of water leaving soil $= \left[v + \frac{\partial v}{\partial z}\,dz\right] dx\,dy$

Net quantity of water squeezed out of soil = Quantity leaving soil – quantity entering soil.

$$\Delta q = -v\,dx\,dy + v\,dx\,dy + \frac{\partial v}{\partial z}\,dx\,dy\,dz$$

$$= \frac{\partial v}{\partial z} \cdot dx\,dy\,dz \quad \text{———} \quad (5)$$

Decrease in volume of soil is equal to volume of water squeezed out.

$$\Delta V = -m_v V_0 \cdot \Delta\sigma' \quad \text{———} \quad (6)$$

$$V_0 = dx\,dy\,dz.$$

Change of volume per unit time

$$\frac{\partial(\Delta V)}{\partial t} = -m_v\,dx\,dy\,dz \cdot \frac{\partial(\Delta\sigma')}{\partial t} \quad \text{———} \quad (7)$$

Equating (5) & (7)

$$\Delta q = \frac{\partial(\Delta V)}{\partial t}$$

$$\frac{\partial v}{\partial z} = -m_v \frac{\partial(\Delta\sigma')}{\partial t} \quad \text{———} \quad (8)$$

Combining (4) and (8)

$$\frac{k}{\gamma_w} \cdot \frac{\partial^2 \bar{u}}{\partial z^2} = -m_v \cdot \frac{\partial(\Delta\sigma')}{\partial t}$$

$$\Delta\sigma = \Delta\sigma' + \bar{u} \Rightarrow \Delta\sigma - \text{constant}$$

$$\frac{\partial(\Delta\sigma')}{\partial t} = -\frac{\partial \bar{u}}{\partial t}.$$

$$\frac{k}{\gamma_w} \cdot \frac{\partial^2 \bar{u}}{\partial z^2} = -m_v \frac{\partial \bar{u}}{\partial t}$$

$$\frac{\partial \bar{u}}{\partial t} = \frac{-k}{m_v \gamma_w} \cdot \frac{\partial^2 \bar{u}}{\partial z^2}$$

$$\frac{\partial \bar{u}}{\partial t} = C_v \cdot \frac{\partial^2 \bar{u}}{\partial z^2}$$

$$C_v - \frac{k}{m_v \gamma_w} = \frac{k(1+e_0)}{a_v \gamma_w}$$

– Coefficient of consolidation

– cm^2/sec

This is the basic differential equation of consolidation which relates rate of change of excess hydrostatic pressure to rate of expulsion of excess pore water from unit volume of soil during some time interval.

The solution of consolidation equation is

$$T_V = \frac{k}{m_V \gamma_w} \cdot \frac{t}{d^2} \Rightarrow T_V = \frac{C_V t}{d^2}$$

$$= \frac{k(1+e_0)}{a_V \gamma_w} \cdot \frac{t}{d^2}$$

$$\frac{T_V}{C_V} = \text{constant}$$

$$t \propto d^2$$

T_V - Time factor ; t - thickness of clay layer.

Degree of consolidation.

$$U = \frac{\text{Excess pore pressure dissipated}}{\text{Initial excess}} \times 100$$

$$= \frac{\text{Initial excess} - \text{Present excess}}{\text{Initial excess}} \times 100$$

If initial excess and present excess is same, there $U=0$
No consolidation

If present excess is 0

$U = 100\%$ — full consolidation

$$U = \frac{\text{Present settlement}}{\text{Ultimate settlement}} \times 100.$$

If $U < 60\%$, $T_V = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$

If $U > 60\%$, $T_V = 1.7813 - 0.9332 \log_{10}(100 - U\%)$.

Laboratory Consolidation Test

Apparatus - Consolidometer

* Consists of loading frame and consolidation cell.
* Porous stones are put on top and bottom ends of specimen
  - Floating ring cell
  - Fixed ring cell

Fixed Ring Cell	Floating Ring Cell
1) Only top porous stone is permitted to move downwards as specimen compresses.	Both top and bottom porous stones are free to compress the compressed specimen towards middle.
2) Permeability of specimen at any stage can be directly measured.	Permeability of specimen at any stage cannot be directly measured.
	Smaller effects of friction between specimen ring and soil specimen.

Vertical compression of specimen is measured by means of dial gauge.

After completion of consolidation under desired maximum vertical pressure, specimen is unloaded and allowed to swell. Final dial reading is recorded and specimen is taken out. Test data are used to determine

1) Voids ratio and coefficient of volume change
2) Coefficient of consolidation
3) Coefficient of permeability.

1) Determination of voids ratio and coefficient of volume change

Two methods i) Height of solids method
ii) Change in voids ratio method

Change in voids ratio — Only for fully saturated specimen
Height of solids - Both saturated as well as unsaturated specimen.

i) Height of solids method

$$\text{Height of solids}, H_s = \frac{M_d}{G A \rho_w} = \frac{W_d}{GA}$$

H_s - Height of solids (cm)
M_d - Mass of dried specimen (g)
W_d - Weight of dried specimen (g)
A - cross sectional area of specimen (cm²)
G - Specific gravity of soil.

$$\text{Void Ratio}, \quad e = \frac{H - H_s}{H_s}.$$

H - Specimen height at equilibrium.

$$H = H_0 + \Sigma \Delta H = H_1 + \Delta H$$

H_0 - Initial height of specimen.
ΔH - Change in specimen thickness under any pressure increment
H_1 - Height of specimen at beginning of load of increment.

ii) Change in voids ratio method.

$$e_f = w_f G.$$

e_f = Final void ratio
w_f - Final water content

$eS_r = wG$
If $S_r = 1$
$e = wG$

$$\frac{\Delta e}{1+e} = \frac{\Delta H}{H}$$

$$\Delta e = \frac{1+e_f}{H_f} \cdot \Delta H$$

H_f - Final height of specimen.

Coefficient of volume change

$$m_v = \frac{-\Delta e}{1+e_0} \cdot \frac{1}{\Delta \sigma'}$$

$$= \frac{-\Delta H}{H_0} \cdot \frac{1}{\Delta \sigma'}$$

(ii) Determination of coefficient of consolidation

Two methods

i) Square root of time fitting method.

ii) Logarithm of time fitting method.

i) Square root of time fitting method.

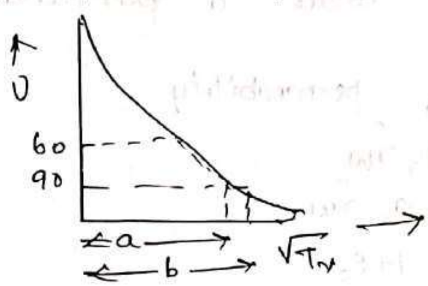


Fig. shows theoretical characteristic curve between degree of consolidation and $\sqrt{T_v}$.

Upto $U = 60\%$, curve is straight.

Abscissa $= \sqrt{t}$ R - dial reading

Ordinate $= R$ t - Time

R_0 - Initial dial reading.

R_c - Corrected zero reading.

Consolidation between R_0 & R_c - Initial consolidation

From R_c, B is drawn

$$B = 1.15 A$$

Intersection of B with consolidation curve $= 0.90\%$

$$(T_v)_{90} = 0.848$$

$$C_v = \frac{0.848\, d^2}{t_{90}}.$$

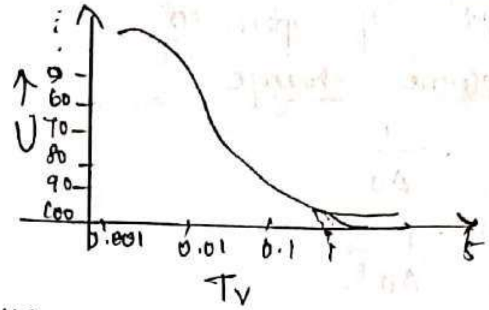
From this C_v can be calculated.

(ii) Logarithm of time fitting method (Casagrande)

Abscissa - $\log_{10} T_v$

Ordinate = $U\%$

Semi log plot of lab time consolidation curve



From this graph, C_v can be calculated.

(iii) Determination of coefficient of permeability.

A falling head permeability test can be performed on consolidation specimen attaching a stand pipe to fixed ring consolidometer when the consolidation of specimen is complete under a particular pressure increment

Coefficient of permeability

$$k = C_v\, m_v\, \gamma_w$$

$$k = \frac{C_v\, a_v\, \gamma_w}{1+e_0}$$

Knowing C_v and m_v, k can be calculated.

Problems.

1) An undisturbed sample of clay 24mm thick, consolidated 50% in 20 minutes, when tested in laboratory with drainage allowed at top and bottom. The clay layer, from which sample was obtained is 4m thick in field. How much time will it take to consolidate 50% with double drainage? If clay stratum has only single drainage, calculate the time to consolidate 50%. Assume uniform distribution of consolidatic pressure.

Soln:

Same degree of consolidation, T_v - same

Both soils are same, C_v - same

$$T_v = C_v \frac{t}{d^2}$$

$$t \propto d^2$$

Field - 2
Lab - 1

a) Double drainage

$$t \propto d^2$$

$$\frac{t_2}{t_1} \propto \left(\frac{d_2}{d_1}\right)^2$$

$t_1 = 20$ min

$t_2 = ?$

$U = 50\%$

Double drainage $= \frac{d}{2}$

$d_1 = \frac{24}{2}$ mm

$= 12$ mm $= 0.012$ m

$d_2 = \frac{4}{2} = 2$ m

$$t_2 = t_1 \left(\frac{d_2}{d_1}\right)^2 = 20\left[\frac{2}{0.012}\right]^2 = 555,566.67 \text{ min}$$

$$= \frac{555566.67}{24 \times 60} \text{ days}$$

$$= 386 \text{ days}$$

b) Single drainage

$$t \propto d^2$$

$$t_2 = t_1 \left(\frac{d_2}{d_1}\right)^2$$

$d = H$ — Single

$= \frac{H}{2}$ — Double.

$t_1 = 20$ mins

$d_1 = \frac{24}{2}$ mm $= 12$ mm $= 0.012$ m (double)

$d_2 = 4$ m (single)

$$t_2 = 20\left[\frac{4}{0.012}\right]^2 = 2,222,220 \text{ min}$$

$$= 1544 \text{ days}$$

$T_{\text{single drainage}} = 4 \times T_{\text{double drainage}}$

$$T_s = 4 T_D$$

2) An undisturbed sample of clay stratum 2m thick was tested in laboratory and average value of coefficient of consolidation was found to be 2×10^{-4} cm²/sec. If a structure is built on clay stratum, how long it will take to attain half the ultimate settlement under load of structure? Assume double drainage.

Soln:

$C_v = 2 \times 10^{-4}$ cm²/sec

$H = 2$m

$U = 50\%$.

Double drainage

$t = ?$

$$T_v = \frac{C_v t}{d^2} \Rightarrow t = \frac{T_v d^2}{C_v}$$

$d = \frac{H}{2} = \frac{2}{2} = 1m = 100\ cm.$

$U < 60\%, \quad T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$

$= \frac{\pi}{4}\left(\frac{50}{100}\right)^2$

$= 0.197.$

$t = \frac{0.197 \times (100)^2}{2\times10^4}$

$= \frac{9850000\ sec}{24\times60\times60}$

$= 114\ days$

Time to attain half the ultimate settlement $= 114$ days.

3) Two clay specimens A and B of thickness 2cm and 3cm, have equilibrium void ratio 0.68 and 0.72 respectively under a pressure of 200 kN/m². If the equilibrium void ratio of the two soils reduced to 0.5 and 0.62 respectively when pressure was increased to 400 kN/m², find ratio of coefficient of permeability of two specimens. The time required by specimen A to reach 40% degree of consolidation is 1/4 of that required by specimen B for reaching 40% degree of consolidation.

Soln:

$U = 40\%$; $t_A = \frac{1}{4} t_B$

$H_A = 2cm = d_A \quad H_B = 3cm = d_B$

$\sigma = 200\ kN/m^2 \quad , \quad e_A = 0.68 \quad , \quad e_B = 0.72$

$\sigma = 400\ kN/m^2 \quad , \quad e_A = 0.5 \quad , \quad e_B = 0.62$

$C_v = \frac{k}{m_v \gamma_w} \qquad \gamma_{wA} = \gamma_{wB}$

$$\frac{C_{VA}}{C_{VB}} = \frac{\frac{k_A}{m_{VA}\gamma_{wA}}}{\frac{k_B}{m_{VA}\gamma_{wB}}} = \frac{k_A}{k_B}\cdot\frac{m_{VB}}{m_{VA}}$$

Coefficient of permeability - k

$$\frac{k_A}{k_B} = \frac{C_{VA}}{C_{VB}} \cdot \frac{m_{VA}}{m_{VB}}$$

$$m_v = \frac{\frac{\Delta e}{1+e_0}}{\Delta\sigma'} \qquad T_v = \frac{C_v t}{d^2}$$

$$\frac{T_{VA}}{T_{VB}} = \frac{C_{VA}\, t_A}{C_{VB}\, t_B} \cdot \left(\frac{d_B}{d_A}\right)^2$$

$U = 40\%$ - $T_{VA} = T_{VB}$

$$\frac{C_{VA}}{C_{VB}} = \frac{t_B}{t_A}\left(\frac{d_B}{d_A}\right)^2 \qquad t_B = 4t_A$$

$$= \frac{4t_A}{t_A}\left(\frac{3}{2}\right)^2$$

$$= 9$$

$$m_{VA} = \frac{0.68-0.5}{1+0.68} \Big/ (400-200)$$

$$= 5.36\times10^{-4}\ m^2/kN.$$

$$m_{VA} = \frac{0.72-0.62}{1+0.72} \Big/ 200 \qquad \frac{m_{VA}}{m_{VB}} = 1.842$$

$$= 2.91\times10^{-4}\ m^2/kN$$

$$\frac{m_{VA}}{m_{VB}} = \frac{5.36\times10^{-4}}{2.91\times10^{-4}} = 1.842.$$

$$\frac{k_A}{k_B} = \frac{C_{VA}}{C_{VB}} \cdot \frac{m_{VA}}{m_{VB}} = 9\times1.842$$

$$= 16.58$$

4) The loading period for a new building extended from July 1980 to July 1982. In July 1985, the average measured settlement was found to be 6.78 cm. If it is known that ultimate settlement will be about 25 cm, estimate settlement in July 1991. Assume double drainage to occur.

Soln:

July 1981 - July 1985 - 4 years

$t_1 = 4$ yrs $= 6.78$ cm

Ultimate settlement $= 25$ cm

Degree of consolidation

$$U = \frac{6.78}{25}$$

4yrs, $U = 27.12\%$.

July 1981 – July 1991 – 10 yrs

$$U_{10} = \frac{\rho_{40}}{25} \quad ; \quad \rho \text{ - settlement}$$

$$= 0.04\,\rho_0$$

$$T_V = \frac{C_V t}{d^2} \quad ; \quad T_V = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$$

$$\frac{\frac{C_{V_1} t_1}{d_1^2}}{\frac{C_{V_2} t_2}{d_2^2}} = \frac{\frac{\pi}{4}\left(\frac{U_1}{100}\right)^2}{\frac{\pi}{4}\left(\frac{U_2}{100}\right)^2}$$

$$\frac{t_1}{t_2} = \left(\frac{U_1}{U_2}\right)^2$$

$$\frac{4}{10} = \left(\frac{0.2712}{0.04\,\rho_{10}}\right)^2$$

$$\rho_{10}^2 = \left(\frac{0.2712}{0.04}\right)^2 \times \frac{10}{4}$$

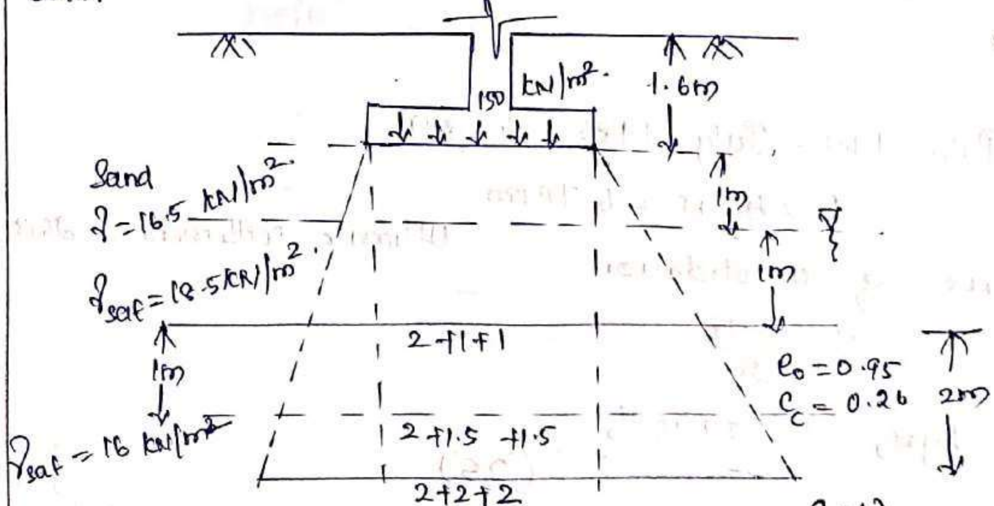
$$\rho_{10} = 10.72 \text{ cm}.$$

5) A building column has a footing area of 2m x 3m and transmits a pressure increment of 150 kN/m² at its base embedded 1.6m below ground level. Assuming pressure distribution of 2 vertical to 1 horizontal, Determine consolidation settlement @ middle of clay layer. Consider pressure variation across thickness of clay layer also. Given the following

i) For sand, $\gamma = 16.5$ kN/m³ & $\gamma_{sat} = 18.5$ kN/m³.

ii) For clay, $\gamma_{sat} = 16$ kN/m³, $e_0 = 0.95$, $C_c = 0.26$.

Soln:



Initial pressure σ_0' @ centre of clay $= \sigma - u$

$= (2.6 \times 16.5) + (18.5 \times 1) + (16 \times 1) - (1 \times 9.81) - (1 \times 9.81)$

$= (2.6 \times 16.5) + (18.5 - 9.81) + (16 - 9.81)$

$= 57.78\ kN/m^2$

$\sigma_0' = 57.78\ kN/m^2$.

Pressure increase @ top, middle and bottom of clay layer

Area $= 2 \times 3\,m$.

$$(\Delta\sigma)_t = \frac{150 \times 2 \times 3}{(2+2)(3+2)} = 45\ kN/m^2.$$

$$(\Delta\sigma)_m = \frac{150 \times 2 \times 3}{(2+3)(3+3)} = 30\ kN/m^2.$$

$$(\Delta\sigma)_b = \frac{150 \times 2 \times 3}{(2+4)(3+4)} = 21.43\ kN/m^2.$$

Average pressure is found by simpson's rule.

$$\Delta\sigma = \frac{1}{6}\left[\Delta\sigma_t + 4\,\Delta\sigma_m + \Delta\sigma_b\right]$$

$$= \frac{1}{6}\left[45 + (4 \times 30) + 21.43\right)$$

$$= 31.07\ kN/m^2.$$

Settlement @ middle of the clay layer.

$$\rho_f = \frac{C_c}{1+e_0} H \log_{10}\left(\frac{\sigma_0' + \Delta\sigma}{\sigma_0}\right)$$

$$= \frac{0.26}{1+0.95} \times 2 \times \log_{10}\left(\frac{57.78 + 31.07}{57.78}\right)$$

$$\rho_f = 0.0498\ m.$$

Secondary Consolidation

When excess pore pressure due to consolidation has been dissipated, the change in void ratio continuous but at reduced rate. This phenomenon is secondary consolidation. It is very small, so it is neglected.

1. **What are the assumptions of Boussinesq Equations?**
    - The soil mass in homogeneous, that is all its constituent parts or elements are similar and it has identical properties at every point in it in identical directions.
    - The soil mass is isotropic, that is it has identical elastic properties in all directions through any point of it.
    - The soil mass is semi-infinite, that is it extends infinitely in all directions below a level surface.

2. **Define isobar. (N/D‟11) (A/M‟11)**

    An isobar is a curved or contour connecting all points below the ground surface of equal vertical pressure.

3. **Define pressure bulb or Stress isobar. (A/M‟09, 16)**

    The zone in a loaded soil mass bounded by an isobar of given vertical pressure intensity is called a pressure bulb.

4. **Write down the Boussinesq equations of vertical pressure due to concentrated load. (N/D‟12, 16)**

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

Q = point load

z = depth of stress acting

r = distance from the axis of load

5. **Define Influence Value. (Nov 2013)**

Newmarks chart consists of number of circles and sectors. Total elements for a nine circle chart will be 200. It is denoted by N. The value of 1/N (i.e., 1/200 or 0.005) is said to be the _influence value‘ (or _influence factor‘) for the chart. Each mesh may thus be understood to represent an influence area

6. **What is the principle behind the Newmark‟s influence chart? (A/M‟17)**

    Newmark‟s Influence Chart is an illustration used to determine the vertical pressure at any point below a uniformly loaded flexible area of soil of any shape. This method, like others, was derived by integration of Boussinesq‘s equation for a point load.

7. **Define consolidation and write its stages. (N/D"11) (A/M"16)**

The process of gradual compression due to the expulsion of pore water under steady pressure is referred to as ‗Consolidation'. This is a time dependent phenomenon, especially occurs in clays.

(i) Primary consolidation (ii) Secondary consolidation (iii) Tertiary compression

8. **What are the factors which influence the compression behavior of soil? (N/D"15)**

The compressibility of a soil depends on the

(i) Structural arrangement of the soil particles,
(ii) Degree to which adjacent particles are bonded, in fine-grained soils.
(iii) Pressure on the soil.

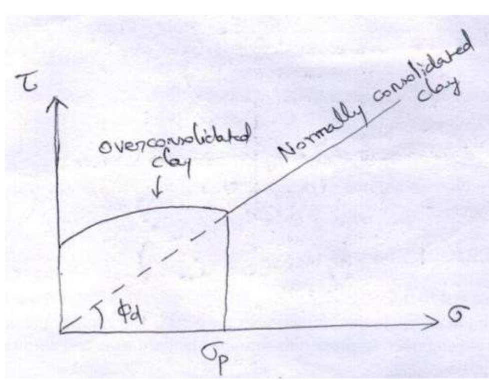
9. **Write any four assumptions of Terzaghi"s theory of one dimensional consolidation. (A/M"10)**

1. The soil is homogeneous (k_z is independent of z).
2. The soil is completely saturated (S = 100%).
3. The soil grains and water are virtually incompressible (γ_w is constant and volume change of soil is only due to change in void ratio).
4. The compression is one-dimensional (u varies with z only).

10. **Define isochrones.(A/M"09)**

In a consolidation curve of a soil, the hydrostatic excess pressure will be maximum at the the middle and it is zero at the top and bottom. The distribution of the hydrostatic excess pressure with depth is sinusoidal at other instants of time, as shown by dotted lines. These curves are called –Isochrones‖.

11. **Draw the consolidation curves for normally consolidated clay and over consolidated clay. (A/M"12, 14)**



12. List out the uses of Influence charts in soil mechanics. (A/M'' 12)

- Many loaded areas have to be drawn; alternatively, many influence charts have to be drawn.
- For each different depth, counting of the influence meshes must be done. Considerable amount of guesswork may be required in estimating the influence units partially covered by the loaded area.
- It can be used for loaded area of any shape and that it is relatively rapid. This makes it attractive.
- This is applicable to a semi-infinite, homogeneous, isotropic and elastic soil mass (and not for a stratified soil).

Part B

1.A water tank is supported by a ring foundation having outer diameter of 10m and inner diameter of 7.5m. the ring foundation transmits uniform load intensity of 160 kN/m2.Determine the vertical stress induced at depth of4 m, below the centre of ring foundation, using (i) Boussinesque analysis and (ii)Westergaard's analysis, taking μ= 0. (CO 3) (BTL-K5) (AUC Apr / May 2010)

2.Explain with a neat sketch the Terzhaghi's one dimensional consolidation theory. (CO 3) (BTL-K2) (AUC Nov/Dec 2012)

3.The load from a continuous footing of width 2m,which may be considered to be strip load of considerable length, is 200 kN/m2. Determine the maximum principal stress at 1.5m depth below the footing, if the point lies (i)directly below the centre of the footing, (ii)directly below the edge of the footing and (iii)0.8m away from the edge of the footing. (CO 3) (BTL-K5) (AUC May/June 2012)

4.What are different components of settlement? Explain in detail. (CO 3) (BTL-K1) (AUC May/June 2012)

5.Explain the Newmark's influence chart in detail. (CO 3) (BTL-K2) (AUC Apr / May 2011)

6.a)Explain how will you determine pre-consolidation pressure? (CO 3) (BTL- K2) (AUC Apr / May 2011)

b)Explain how will you determine coefficient of compression index (CC)from an oedomoter test? (CO 3) (BTL-K2) (AUC Apr / May 2011)

7.Develop Boussinesque equations to find intensity of vertical pressure and tangential stress when a concentrated load is acting on the soil. (CO 3) (BTL-K6) (AUC Nov/Dec 2010)

8.Explain in detail the laboratory determination of co-efficient of consolidation. (CO 3) (BTL-K2) (AUC Apr / May 2009)

UNIT- IV

SHEAR STRENGTH

Shear strength of cohesive and cohesionless soils – Mohr- Coulomb failure theory – Shear strength – Direct shear, Triaxial compression, UCC and vane shear tests – Pore pressure parameters – Factors influences shear strength of soil.

Shear Strength

When soil is loaded, shear stresses are induced in it. When shear stress reaches its limiting value, shear deformation takes place, leading to failure of the soil mass.

The failure may be in the form of

- Sinking of a footing.
- Movement of a wedge of soil behind retaining wall forcing it to move out.
- Slide in earth embankment.

Shear Strength – Definition

The shear strength of soil is the resistance to deformation by continuous shear displacement of soil particles upon the action of shear stress. The failure conditions for a soil may be expressed in terms of limiting shear stress called shear strength.

The shearing resistance of soil is constituted basically of following components.

* Structural Resistance to displacement of soil because of interlocking of particles.

* Frictional Resistance to translocation between individual soil particles at their contact point.

* Cohesion or adhesion between surface of soil particles.

Shear strength of cohesionless soils.

The shear strength in cohesionless soil results from intergranular friction alone, while in all other soils it results from both internal friction as well as cohesion. However plastic undrained clay does not possess internal friction.

Mohr's Stress Circle

Through a point in loaded soil mass, innumerous planes pass and stress components on each plane depends upon direction of plane.

On every plane, there will be three typical planes, mutually orthogonal to each other, on which stress is wholly normal and no shear stress acts. These planes are called principal planes and normal stress acting on these planes are called principal stresses.

The plane which is devoid of shear is principal plane.

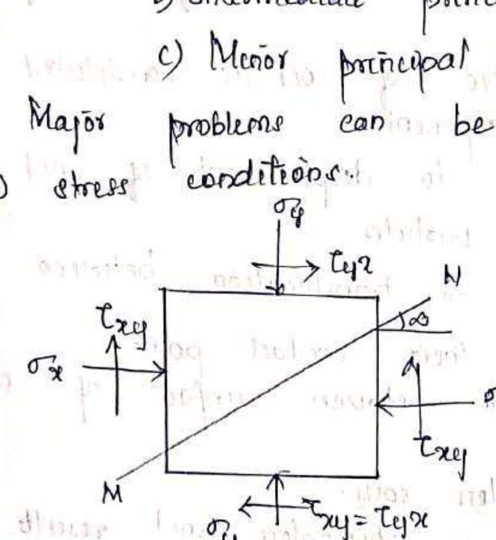
In the order of decreasing magnitude of normal stress, these planes are called

a) Major principal planes
b) Intermediate principal planes
c) Minor principal planes.

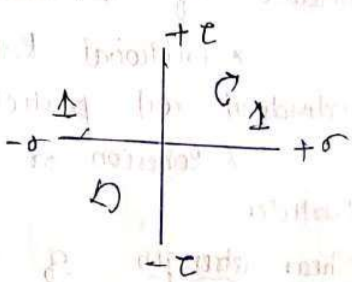
The corresponding normal stresses on them are

a) Major principal stresses (σ_1)
b) Intermediate principal stresses (σ_2)
c) Minor principal stresses (σ_3).

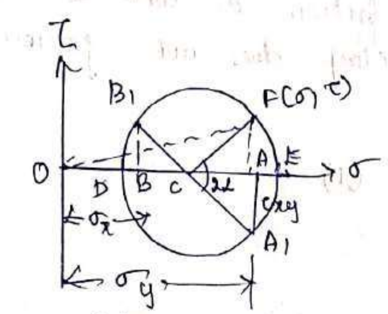
Major problems can be approximated by considering 2D stress conditions.



a) Soil Element

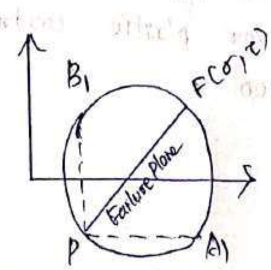


b) Sign convention



(2)

c) Mohr Circle



P - Pole or Origin of planes

Fig. (a) shows a soil element subjected to 2D stress system.

From equilibrium of element, the following expressions were found for normal stress σ and shearing stress τ on any plane MN inclined at α with x direction.

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \left[\frac{\sigma_y - \sigma_x}{2}\right]\cos 2\alpha + \tau_{xy}\sin 2\alpha \quad \text{—— (1)}$$

$$\tau = \frac{\sigma_y - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha \quad \text{—— (2)}$$

when $\sigma_y > \sigma_x$.

σ_x - normal stress on plane ⊥r to x axis

σ_y - normal stress on plane ⊥ to y axis

$\tau_{xy} = \tau_{yx} \Rightarrow$ shear stress on these two plane.

From (1)

$$\sigma - \frac{\sigma_y + \sigma_x}{2} = \left[\frac{\sigma_y - \sigma_x}{2}\right]\cos 2\alpha + \tau_{xy}\sin 2\alpha \quad \text{—— (3)}$$

squaring & adding (3) & (2).

$$\left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 = \left[\frac{\sigma_y - \sigma_x}{2}\cos 2\alpha + \tau_{xy}\sin 2\alpha\right]^2$$

$$= \left[\frac{\sigma_y - \sigma_x}{2}\cos 2\alpha\right]^2 + \left[\tau_{xy}\sin 2\alpha\right]^2 + 2\left[\frac{\sigma_y - \sigma_x}{2}\cos 2\alpha \cdot \tau_{xy}\sin 2\alpha\right]$$

$$= \left[\frac{\sigma_y - \sigma_x}{2}\right]^2\cos^2 2\alpha + \tau_{xy}^2\sin^2 2\alpha + (\sigma_y - \sigma_x)\tau_{xy}\sin 2\alpha\cos 2\alpha$$

$$\tau^2 = \left[\frac{\sigma_y - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha\right]^2$$

$$= \left(\frac{\sigma_y - \sigma_x}{2}\right)^2\sin^2 2\alpha + \tau_{xy}^2\cos^2 2\alpha - \tau_{xy}(\sigma_y - \sigma_x)\sin 2\alpha\cos 2\alpha$$

Adding

$$\left[\sigma - \frac{\sigma_y + \sigma_x}{2}\right]^2 + \tau^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2\left[\underbrace{\sin^2 2\alpha + \cos^2 2\alpha}_{1}\right] + \tau_{xy}^2\left[\underbrace{\sin^2 2\alpha + \cos^2 2\alpha}_{1}\right] + (\sigma_y - \sigma_x)\tau_{xy}\sin 2\alpha\cos 2\alpha - (\sigma_y - \sigma_x)\tau_{xy}\sin 2\alpha\cos 2\alpha.$$

$$\left[\sigma - \frac{\sigma_y+\sigma_x}{2}\right]^2 + \tau^2 = \left[\frac{\sigma_y-\sigma_x}{2}\right]^2 + \tau_{xy}^2 \quad \text{--- (1)}$$

Equation (1) is the equation of circle like

$x^2+y^2 = R^2$ $\qquad (x-a)^2 + (y-b)^2 = R^2$

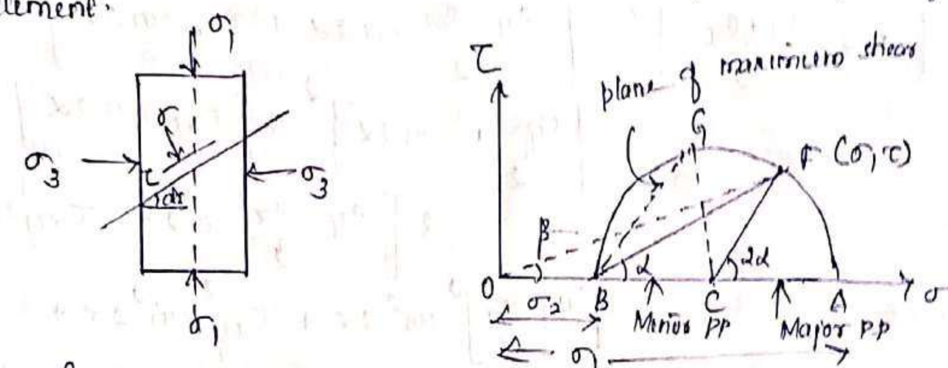
Coordinates $(a, b) = \left[\frac{\sigma_y+\sigma_x}{2}, 0\right]$

Radius $R^2 = \left[\frac{\sigma_y-\sigma_x}{2}\right]^2 + \tau_{xy}^2$

$$R = \sqrt{\left[\frac{\sigma_y-\sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

Coordinates of points on circle represents normal and shearing stress on inclined planes at a given point. This circle is known as Mohr's circle of stress.

Let us take the case of soil element whose sides are principal planes - consider the state of stress where only normal stresses are acting on faces of element.



Expression for σ, τ are

$$\sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\alpha$$

$$\tau = \frac{\sigma_1-\sigma_3}{2}\sin 2\alpha$$

Resultant stress $= \sqrt{\sigma^2+\tau^2}$

Angle of obliquity $\beta = \tan^{-1}\left(\frac{\tau}{\sigma}\right)$

Max. shear stress, $\tau_{max} = \left(\frac{\sigma_1-\sigma_3}{2}\right)$

It occurs on plane $\alpha = 45^0$.

At max. shear stress,

Normal stress $= \frac{\sigma_1+\sigma_3}{2}$

Mohr - Coulomb Failure Theory

Essential points of Mohr's strength theory are:

* Materials fail essentially by shear. The critical shear stress causing failure depends upon the properties of material as well as on normal stress on failure plane.

* The ultimate strength of material is determined by stresses on potential failure plane.

* When the material is subjected to 3D principal stress $(\sigma_1, \sigma_2, \sigma_3)$ the intermediate principal stress does not have any influence on strength of material. i.e., the failure criterion is independent of intermediate principal stress.

The theory can be expressed algebraically by equation

$$\tau_f = s = F(\sigma) \longrightarrow \text{Mohr}$$

$\tau_f = s =$ shear stress on failure plane at failure
= shear resistance of material.

$F(\sigma)$ = Function of normal stress.

If normal stress and shear stress corresponding to failure are plotted, then a curve is obtained. That ~~occur~~ curve is called strength envelope.

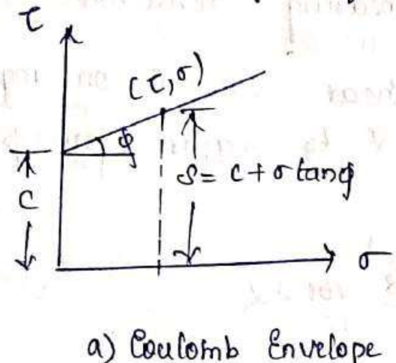
Coulomb defined the function $F(\sigma)$ as a linear function of σ and gave equation

$$s = c + \sigma \tan \phi \longrightarrow \text{Coulomb}.$$

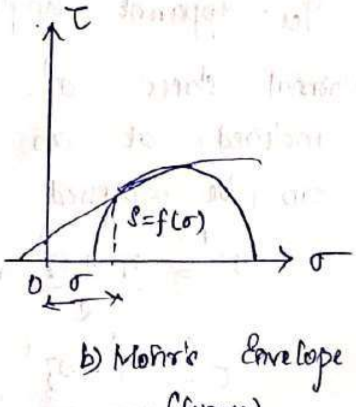
c & ϕ - Empirical constants.

c - cohesion - intercept on shear axis

ϕ - angle of internal friction / shearing resistance - slope of straight line



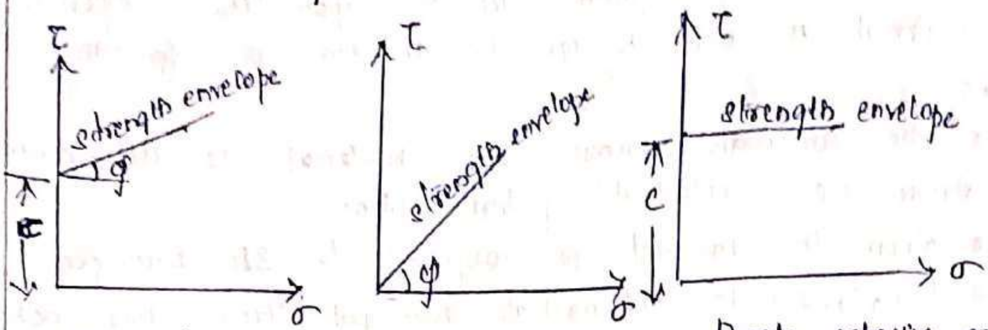
a) Coulomb Envelope
(straight line)



b) Mohr's Envelope
(Curve)

From fig (a)
shear strength ∝ Normal stress

From fig. (b)
shear stress & normal stress relation is not linear



Cohesionless soil — Cohesive soil (Pure friction) — Purely cohesive soil.

Effective Stress Principle

In equation $s = c + \sigma \tan\phi$, it is assumed that total normal stress governs shear strength of soil. This assumption is not always correct.

Extensive tests on remoulded clays have sustained beyond doubt Terzaghi's early concept that effective normal stresses control shearing resistance of soils.

$$\tau_f = c' + \sigma' \tan\phi'$$
$$= c' + (\sigma - u)\tan\phi'$$

c' - effective cohesion concept.

ϕ' - effective angle of shearing resistance

In terms of total stresses

$$\tau_f = c_u + \sigma \tan\phi_u.$$

c_u - apparent cohesion.

ϕ_u - apparent angle of shearing resistance.

Normal stress σ' and shear stress τ on any plane inclined at an angle α to major principal plane can be expressed by

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\alpha$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{2}\sin 2\alpha$$

σ_1' - effective major principal stress

σ_3' - effective minor principal stress

substituting values of σ'

$$\tau_f = c' + \tan\phi' \left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \right]$$

Most dangerous plane - failure occurs - $(\tau_f - \tau_\alpha)$ between shear strength and shear stress is minimum.

$$\tau_f - \tau = c' + \frac{\sigma_1' + \sigma_3'}{2} \tan\phi' + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \tan\phi' - \frac{\sigma_1' - \sigma_3'}{2} \sin 2\alpha$$

Differentiate w.r.t. α

$$\frac{d}{d\alpha}(\tau_f - \tau) = \frac{\sigma_1' - \sigma_3'}{2} \tan\phi' \left[-2 \sin 2\alpha\right] - \frac{\sigma_1' - \sigma_3'}{2} \left[2 \cos 2\alpha\right]$$

$$= -(\sigma_1' - \sigma_3') \tan\phi' \sin 2\alpha - (\sigma_1' - \sigma_3') \cos 2\alpha$$

For minimum $(\tau_f - \tau)$,

$$\frac{d}{d\alpha}(\tau_f - \tau) = 0$$

$$(\sigma_1' - \sigma_3') \cos 2\alpha = -(\sigma_1' - \sigma_3') \tan\phi' \sin 2\alpha$$

$$\tan\phi = \frac{-\cos 2\alpha}{\sin 2\alpha} = -\cot 2\alpha$$

$$\cot - \tan\phi = \cot 2\alpha$$

$$\cot(90° + \phi) = \cot 2\alpha$$

$$2\alpha = 90° + \phi$$

$$\alpha = \frac{90° + \phi}{2} = 45° + \frac{\phi}{2}$$

It can also be derived from Mohr's circle.

Measurement of shear strength

The measurement of shear strength of soil involves certain test observations at failure with the help of which failure envelope or strength envelope can be plotted.

Laboratory Test

i) Direct shear test

ii) Triaxial shear test

iii) Unconfined compression test

iv) Vane shear test

Depending on drainage conditions

* Undrained test or quick test
* Consolidated drained test
* Drained Test.

Undrained Test

No drainage of water is permitted. There is no dissipation of pore pressure during entire test

Direct shear test - Drainage not permitted during application of both normal stress & shear stress.

Triaxial compression test - Drainage not permitted during period of both pole pressure & deviated pressure

Drained Test

Drainage is permitted throughout the test during application of both normal and shear stresses so that full consolidation occurs and no excess pore pressure is set up at any stage of test.

Consolidated undrained Test

Drainage is permitted under initially applied normal stress only and full primary consolidation or softening is allowed to take place. No drainage is allowed afterwards.

C & ϕ - vary with drainage conditions.

Direct shear test - allowed to consolidate fully under applied normal stress and shear for high rate of strain to prevent dissipation of pore pressure during shearing.

Triaxial compression test - Allowed to consolidate fully under applied self pressure and then pore water outlet is closed and specimen subjected to increasing deviated stress at high rate of strain

Direct Shear Test.

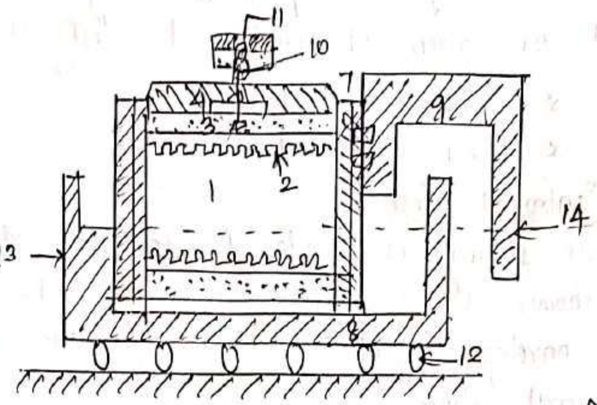
* Simple and commonly used test.
* Shear box apparatus

Apparatus

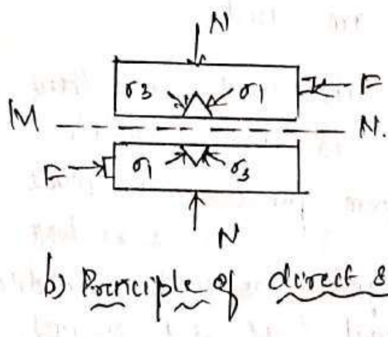
* The apparatus consists of two piece shear box of square or circular cross section

* The lower half of the box is rigidly held in position in a container which rest over slides or rollers and can be pushed forward at a constant rate by geared jack, driven either by electric motor or by hand.
* The upper half of the box is placed against proving ring.
* The soil sample is compacted in the shear box and is held between metal grids and porous stones.

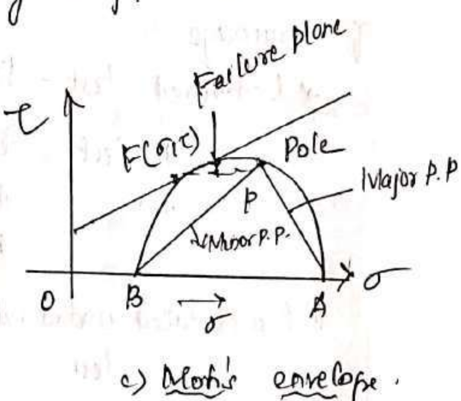
Upper box half – Upper half of specimen.
Lower box – Lower half of specimen.
Point – centre of specimen.



1. Soil specimen. 2. Metal Guide 3. Porous Stones 4. Loading pad 5. Upper Part 6. Lower part 7. screws to fix two holes of shear box. 8. Container for shear box 9. U-Arm 10. Steel Ball 11. Loading yoke 12. Rollers 13. Shear force applied by jack 14. Shear resistance measured by proving ring.



b) Principle of direct shear box



c) Mohr's envelope.

Procedure

* Normal Pressure is applied on the specimen from loading yoke bearing upon steel bolt of pressure pad.
* When shearing force is applied to lower box through geared jack, the movement of lower part of box is transmitted through specimen to upper box and hence on proving ring.

* Deformation of proving ring indicates shear force.
* The volume change during consolidation and during shearing process is measured by mounting a dial guage at top of box.
* The soil specimen can be compacted in shear box by clamping both the parts together with the help of two screws.
* These screws are removed before shearing force is applied.
* Metal grids placed above top and below, bottom of specimen may be perforated if drained test is required or plain if undrained test is required.
  * Strain controlled test
  * Stress controlled test.

Strain Controlled Test

Shear strain is made to interest at constant rate. Fig (b) shows strain controlled shear test. Fig (c) shows failure envelope plotted as a function of shear stress and normal stress.

Stress Controlled Test

There is an arrangement to increase the shear stress at a desired rate and measure the shearing strain.

Test can be performed under all three conditions of drainage.

* Undrained test - Plain guides are used.
* Drained test (slow) - Perforated grids are used and then sheared slowly so that complete dissipation of pore pressure takes place (2-5) days.
* Consolidated undrained test - Perforated grids are used. Consolidated under normal load and sheared quickly in about 5-10 mins.

Advantages

* Simple test
* The relatively thin thickness of sample permits quick drainage and quick dissipation of pore pressure developed during test

Disadvantages

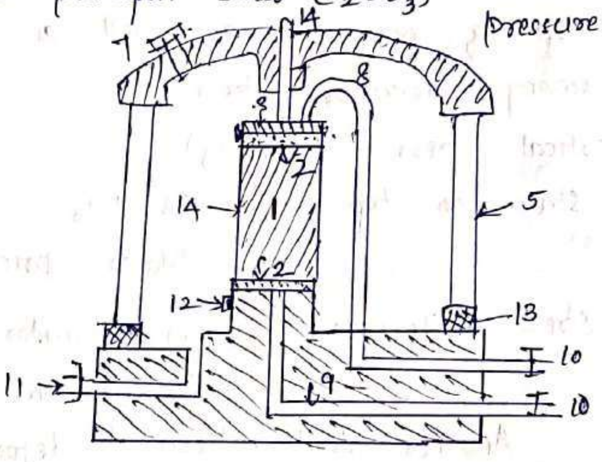
* Stress conditions across soil sample are very complex.
* As test progresses, the area under shear gradually decreases.
* As compared to triaxial test, there is little control on drainage of soil.
* The plane of shear failure is predetermined which may not be weakest one.
* There is effect of lateral restraint by side walls of shear box.

Triaxial Compression Test.

The solid specimen, cylindrical in shape, is subjected to direct stresses acting in three mutually perpendicular directions.

Major Principal stress (σ_1) - Vertical direction

Other Minor two principal stress ($\sigma_2 = \sigma_3$) - Horizontal direction (Fluid pressure round the specimen).



1. Soil specimen 2. Porous disc 3. Top cap 4. Rubber Membrane 5. Perspex cylinder 6. Loading ram 7. Air release valve 8. Top drainage tube 9. Bottom drainage tube 10. Connections for drainage 11. Cell fluid inlet 12. Rubber rings 13. Sealing ring 14. Axial load through proving ring.

Apparatus

* High pressure cylindrical cell - Perspex or other transparent material - Fitted between base and top cap.
* 3 outlet connections are provided through base
  * Cell fluid inlet
  * Pore water outlet from bottom of specimen.
  * Drainage outlet from top of specimen.

* A seperate compression is used to apply fluid pressure in the cell.

* Pore pressure developed in specimen during test can be measured with help of seperate pore pressure measuring equipment such as Bishop's Apparatus.

* A stainless steel piston running through centre of top cap applies the vertical compressive load (deviator stress) on specimen under list.

* The load is applied through a proving ring, with the help of mechanically operated load frame.

* Depending on drainage conditions of test, solid non porous or porous disc are placed on top and bottom and rubber membrane is sealed on to three end caps.

* Length of specimen = 2 to $2\frac{1}{2}$ times its diameter.

Cell pressure $\sigma_3 = (\sigma_2)$ acts all all round specimen it also acts on top of specimen as well as vertical piston meant for among deviator stress.

$$\text{Vertical stress} = (\sigma_1 - \sigma_3).$$

$$\text{Total stress on top} = (\sigma_1 - \sigma_3) + \sigma_3$$

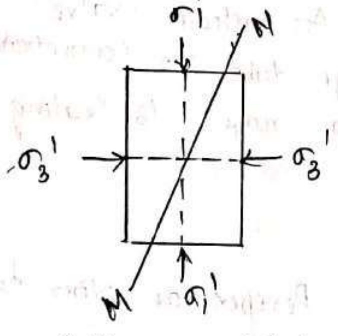
$$= \sigma_1 - \text{Major principal stress.}$$

$$\text{stress difference } (\sigma_1 - \sigma_3) = \text{Deviator stress}$$

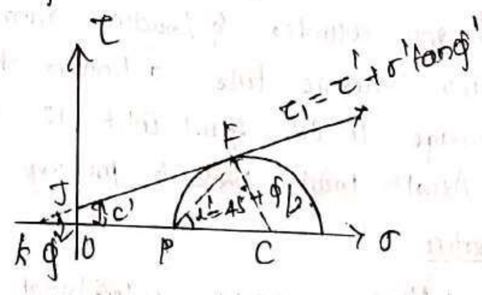
Record on proving ring dial

Another dial - Vertical deformation.

<u>Stress condition in soil specimen during triaxial testing.</u>



a) Stress conditions



b) Failure envelope in triaxial compression test

Minor principal stress = Intermediate principal stress

Effective minor principal stress = Cell pressure − Pore pressure

Major principal stress = Deviator stress + cell pressure.

Failure plane inclined at an angle α' to major principal plane.

$$\alpha' = 45^\circ + \frac{\phi'}{2}$$

FC = Radius of Mohr's circle $= \frac{1}{2}(\sigma_1' - \sigma_3')$

$OC = \frac{1}{2}(\sigma_1' + \sigma_3')$

$OK = c' \cot\phi'$

$$\sin\phi' = \frac{FC}{KC} = \frac{FC}{KO + OC}$$

$$= \frac{\frac{\sigma_1' - \sigma_3'}{2}}{c'\cot\phi' + \frac{\sigma_1' + \sigma_3'}{2}}$$

$$\sin\phi' = \frac{\sigma_1' - \sigma_3'}{2c'\cot\phi' + (\sigma_1' + \sigma_3')}$$

$$(\sigma_1' - \sigma_3') = 2c'\cot\phi' \sin\phi' + (\sigma_1' + \sigma_3')\sin\phi'.$$

$$= 2c' \frac{\cos\phi'}{\cancel{\sin\phi'}} \cancel{\sin\phi'} + (\sigma_1' + \sigma_3')\sin\phi'$$

$$= 2c'\cos\phi' + (\sigma_1' + \sigma_3')\sin\phi'$$

$$\sigma_1' - \sigma_1'\sin\phi' = 2c'\cos\phi' + \sigma_3'\sin\phi' + \sigma_3'$$

$$\sigma_1'(1 - \sin\phi') = 2c'\cos\phi' + \sigma_3'(1 + \sin\phi')$$

$$\sigma_1' = 2c'\frac{\cos\phi'}{(1 - \sin\phi')} + \sigma_3'\frac{(1 + \sin\phi')}{(1 - \sin\phi')}$$

$$= 2c'\tan\left(45 + \frac{\phi'}{2}\right) + \sigma_3'\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$= \sigma_3'\tan^2\alpha' + 2c'\tan\alpha'$$

$$\sigma_1' = \sigma_3' N_\phi' + 2c'\sqrt{N_\phi}.$$

$$N_\phi' = \tan^2\alpha' = \tan^2(45^\circ + \phi/2).$$

In terms of total stress,

$$\sigma_1 = \sigma_3\tan^2\alpha + 2C_u\tan\alpha$$

$$\alpha = 45^\circ + \frac{\phi_u}{2}.$$

The calculation of deviator stress must be done on basis of changed area of cross section at failure, or during any stage of test. The area A_f at failure, or during any stage of test can be found by relation

$$A_2 = \frac{V_1 \pm \Delta V}{L_1 - \Delta L}$$

V_1 - Initial volume of specimen.
L_1 - Initial length of specimen.
ΔV - Change in volume of specimen.
ΔL - Change in length of specimen.

Deviator stress σ_d.

$$\sigma_d = \frac{\text{Additional axial load}}{A_2}$$

σ_3 = Fluid pressure.

$\sigma_1 = \sigma_3 + \sigma_d$.

Advantages

* Shear tests under all three drainage conditions can be performed with complete control.
* Precise measurement of pore pressure and volume change during test are possible.
* Stress distribution on failure plane is uniform
* State of stress within specimen during any stage of test as well as at failure is completely determined.

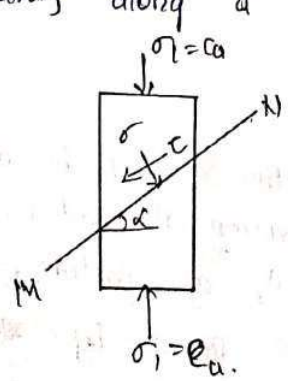
Unconfined Compression Test

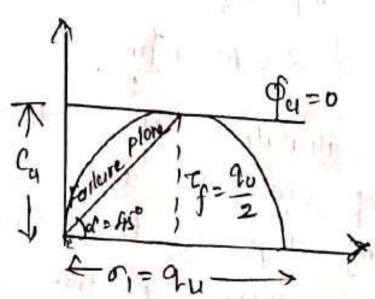
It is a special case of triaxial compression test in which $\sigma_2 = \sigma_3 = 0$

Cell pressure = confining pressure.

Absence of confining pressure - Uniaxial test - Unconfined compression test.

Cylindrical specimen of of soil is subjected to major principal stress σ_1 till specimen fails due to shearing along a critical plane or failure.





Apparatus.

* Small load frame fitted with proving ring to measure vertical stress applied to soil specimen.

* Deformation is measured with dial gauge.

$\sigma_3 = 0$; Mohr's circle passes through origin which is also a pole.

$$\sigma_1 = 2C_u \tan\alpha = 2C_u \tan\left(45^\circ + \frac{\phi_u}{2}\right)$$

Unknowns - C_u & ϕ_u - can't be found by unconfined test.

It gives the value of σ_1

This test is generally applicable to saturated clays for which the apparent angle of shearing resistance ϕ_u is zero.

$$\sigma_1 = 2C_u$$

When Mohr's circle is drawn its radius is equal to $\sigma_1/2 = C_u$.

$$\sigma = \frac{\sigma_1}{2} = \frac{q_u}{2} \quad ; \quad \tau_f = \frac{\sigma_1}{2} = \frac{q_u}{2} = C_u$$

$$\tau_f = C_u = \frac{q_u}{2}$$

q_u - unconfined compression strength at failure.

Compression stress is calculated on the basis of changed cross sectional area A_2 at failure.

$$A_2 = \frac{V}{L_1 - \Delta L} = \frac{A_1}{1 - \frac{\Delta L}{L_1}} - A_2.$$

V - Initial volume of specimen

L_1 - Initial length of specimen

ΔL - Change in length at failure.

Vane Shear Test (Quick Test)

- Used in laboratory and in field.
- To determine undrained shear strength of cohesive soil.
- The tester consists of 4 thin steel plates, called vanes welded orthogonally to steel rod.
- A torque measuring arrangement such as a calibrated torsion spring is attached to the rod which is rotated by worm gear and worm wheel arrangement.
- After pushing the vanes gently into soil, the torque rod is rotated at uniform speed.

- The rotation of spring in degrees is indicated by a pointer moving on graduated dial attached to worm wheel shaft.

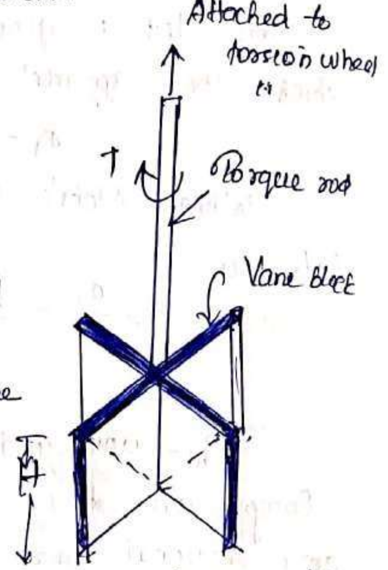
- The torque T is then calculated by multiplying dial reading with spring constant.

- A typical laboratory vane is 20 mm high and 12 mm in diameter with Blade thickness from 0.5 to 1 mm.

Blades - High tensile steel.

Field shear vane - 10 to 20 cm.

Blade thickness - 2.5 m.



τ_f - unit strength of soil

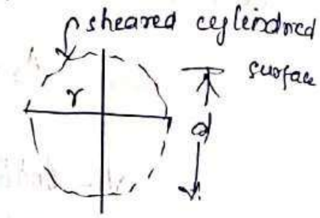
H - Height of vane

d - diameter of vane.

__Case 1__

Vane is pushed in soil with its top and below surface of soil so that both top and bottom ends partake in shearing of soil.

$$T_f = \pi d H \tau_f \cdot \frac{d}{2} + 2\int_0^{d/2} (2\pi r dr)\tau_f \cdot r$$



$$= \pi d H \tau_f \frac{d}{2} + 4\pi\tau_f \int_0^{d/2} r^2 dr$$

$$\int_0^{d/2} r^2 dr = \left[\frac{r^3}{3}\right]_0^{d/2} = \frac{d^3}{8\times 3}$$

$$\therefore\ T_f = \pi d H \tau_f \frac{d}{2} + 4\pi\tau_f \frac{d^3}{8\times 3}$$

$$= \pi\tau_f\left[\frac{d^2 H}{2} + \frac{d^3}{6}\right]$$

$$= \pi d^2 \tau_f\left[\frac{H}{2} + \frac{d}{6}\right]$$

__Case 2__

The vane is pushed inside the soil with its top end flushed with surface of soil so that only Bottom end partakes in shearing the soil.

$$T_f = (\pi d H \tau_f) d/2 + \int_0^{d/2} 2\pi r\, dr\, \tau_f\, r.$$

$$= \frac{\pi d^2}{2} H \tau_f + 2\pi\tau_f \frac{d^3}{8\times 3}.$$

$$= \frac{\pi d^2}{2} H \tau_f + \pi \tau_f \frac{d^3}{12}$$

$$= \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{12} \right]$$

Knowing T_f, H, d, shear strength τ_f can be determined.

Problems

1) A direct shear test was carried out on cohesive soil sample under the following results are obtained.

Normal stress (kN/m³)	Shear stress (kN/m³)
150	110
250	120

What would be deviated stress at failure of the triaxial shear test was carried on the same soil with a cell pressure of 150 kN/m².

Soln:

Coulomb's shear strength equation is given by

$$\tau_f = C + \sigma \tan\phi$$

$$110 = C + 150 \tan\phi$$

$$120 = C + 250 \tan\phi$$

Solving; $C = 95$ kN/m².

$$\tan\phi = 0.1$$

$$\phi = 5.71°.$$

Deviated stress at failure

$$\sigma_d = \sigma_1 - \sigma_3$$

$$\sigma_1 = \sigma_3 \tan^2(45 + \phi/2) + 2C \tan(45 + \phi/2)$$

$$= 150 \tan^2\left(45 + \frac{5.71}{2}\right) + 2 \times 95 \tan\left(45 + \frac{5.71}{2}\right)$$

$$= 393.09 \text{ kN/m}^2.$$

$$\sigma_d = \sigma_1 - \sigma_3.$$

$$= 393.09 - 150$$

$$= 243.1 \text{ kN/m}^2.$$

2) A consolidated undrained soil test was conducted on a dry sample on following results were obtained. Determine the shear strength parameters w.r.t i) Total stress concept ii) Effective stress concept

Cell Pressure (kN/m²)	Deviated stress @ failure (kN/m²)	Pore water pressure @ failure (kN/m²)
200	118	110
400	240	220
600	352	320

Soln:

σ_3	σ_d	u	$\sigma_1 = \sigma_3 + \sigma_d$	$\sigma_1' = \sigma_1 - u$	$\sigma_3' = \sigma_3 - u$
200	118	110	318	208	90
400	240	220	640	420	180
600	352	320	952	632	280.

$$\sigma_1 = \sigma_3 \tan^2(45 + \phi/2) + 2c \tan(45 + \phi/2)$$

$318 = 200 \tan^2\alpha + 2c \tan\alpha$

$640 = 400 \tan^2\alpha + 2c \tan\alpha$

$322 = 200 \tan^2\alpha$

$\alpha = 51.76°$.

$45 + \phi/2 = 51.76$

$\phi/2 = 6.76 \Rightarrow \phi = 13.52°$.

$318 = (200 \times 1.61) + 2c \tan 51.76$

$c = -1.5$

$$\sigma_1' = \sigma_3' \tan^2(45 + \phi/2) + 2c' \tan(45 + \phi/2)$$

$208 = 90 \tan^2\alpha' + 2c' \tan\alpha'$

$420 = 180 \tan^2\alpha' + 2c' \tan\alpha'$

$212 = 90 \tan^2\alpha'$

$\alpha' = 56.91°$

$45 + \phi/2 = 56.91$.

$\phi' = 23.82°$.

$208 = 90 \tan^2(56.91) + 2c' \tan(56.91)$

$c' = -1.3$

3) An unconfined compression test was conducted on an undisturbed sample of clay. The sample has a diameter of 38mm and length 76mm. Load at failure was 30N and axial deformation of sample is 11mm. Determine the undrained shear strength parameters if failure plane makes an angle of 50° with horizontal.

Soln:

Initial length of sample = 76mm.

Diameter of sample = 38mm.

Initial area of c/s, $A = \frac{\pi d^2}{4} = \frac{\pi \times 38^2}{4} = 1134.11\ mm^2$.

Change in length, $\Delta l = 11mm$

Axial strain @ failure $= \frac{\Delta l}{l} = \frac{11}{76} = 0.145$.

Area of c/s @ failure, $A_f = \frac{A}{1-\epsilon} = \frac{1134.11}{1-0.145}$

$= 1326.44\ mm^2$.

$$q_u = \frac{P}{A_f} = \frac{30}{1326.44} = 0.023\ N/mm^2.$$

$\alpha = 50°$

$45 + \phi/2 = 50°$

$\phi = 10°$ $\quad \because \sigma_3 = 0.$

$$\sigma_1 = \cancel{\sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right)} + 2c \tan\left(45 + \phi/2\right)$$

$$\sigma_1 = q_u = 2c \tan(45 + \phi/2)$$

$$2c = \frac{0.023}{\tan 50}$$

$$c = 0.0096\ N/mm^2.$$

4) An unconfined compression test is conducted on a saturated clay specimen of 40mm ϕ and 90mm length measured on its sides. The specimen has cone edged and its length b/w apex of cone is 80mm. The specimen fails under an axial compression load of 460 N with axial deformation of 10mm. Calculate the unconfined compressive strength of clay.

Soln:

Length of sample on its sides = 90 mm

Length b/w apex of cone = 80mm

Diameter of sample = 40 mm

Actual length $= 90 - \frac{2 \times 5}{3}$; Area $= \frac{\pi d^2}{4}$

$= 1256.64\ mm^2$

$= 86.67 mm$

$\Delta L = 10$

Strain $= \frac{\Delta L}{L} = \frac{10}{86.67} = 0.1154$

$A_f = \frac{A}{1-\epsilon} = \frac{1256.64}{1-0.1154}$

$= 1420.5\ mm^2$.

$P = 460\ kN$

$q_u = \frac{P}{A_f} = 0.324\ N/mm^2$

$= \frac{0.324 \times 10^6}{10^3}\ kN/m^2$.

$C_u = \frac{q_u}{2} = \frac{324}{2}$

$= 162\ kN/m^2$.

5) In a vane shear test conducted in a soft clay deposit, failure occured at a torque of 42 Nm. Afterwards, the vane was allowed to rotate rapidly and the test was repeated in the rebounded soil. The torque at failure in the rebounded soil was 17Nm. Calculate the sensitivity of soil in both cases. In both cases, the vane was pushed completely inside the soil. The height and diameter of vane was 100mm and 80mm respectively.

Soln: Sensitivity of clay $= \frac{\text{cohesion in undisturbed state}}{\text{cohesion in remoulded state}}$

$= \frac{C_{undisturbed}}{C_{remoulded}}$

$= \frac{q_u/2\ (u)}{q_u/2\ (r)}$

Height of vane (H) = 100 mm
Diameter (d) = 80 mm

$T_f = 17\ Nm = 17000\ Nmm$.

In this problem, both top and bottom ends particle in shearing the soil.

$$T_f = \pi d^2 . \tau_f \left[\frac{H}{2} + \frac{d}{6} \right]$$

Case 1

For natural soil, $T = 42\ Nm$
$= 42000\ N/mm$

$$42000 = \pi \times 80^2 \times \tau_f \left[\frac{100}{2} + \frac{80}{6} \right]$$

$$\tau_f = 0.033\ N/mm^2.$$

Case 2

For remoulded state

$$17000 = \pi \times 80^2 \times \tau_f \left[\frac{100}{2} + \frac{80}{6} \right]$$

$$\tau_f = 0.0134\ N/mm^2.$$

$\therefore$ Sensitivity $= \frac{0.033}{0.0134} = 2.54$

Hyxotrophic Effect.

If the soil is disturbed, its shear strength decreases. If we leave the soil, it regain its shear strength after some times. This is known as hyxotrophic effect.

Skempton's pore pressure parameters.

The change in the pore pressure due to change in the applied stress, during an undrained shear, may be explained in terms of empirical coefficients called pore pressure parameters.

A pore pressure parameter may be defined as a dimensionless number that indicates the fraction of total stress increment that show up an excess pore pressure for condition of no drainage.

Let us consider a small mass subjected to increase in 3 principal stresses $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_3$. resulting in volume decrease Δv and a consequent increase in pore pressure of Δu

The increase in effective stress

$$\Delta\sigma_1' = \Delta\sigma_1 - u$$
$$\Delta\sigma_2' = \Delta\sigma_2 - u$$
$$\Delta\sigma_3' = \Delta\sigma_3 - u$$

$\epsilon_1, \epsilon_2, \epsilon_3$ - strains in 3 directions.

Young's Modulus, $E = \frac{\text{stress}}{\text{strain}}$.

$$E\epsilon_1 = \Delta\sigma_1' - \mu(\Delta\sigma_2' + \Delta\sigma_3')$$
$$E\epsilon_2 = \Delta\sigma_2' - \mu(\Delta\sigma_3' + \Delta\sigma_1')$$
$$E\epsilon_3 = \Delta\sigma_3' - \mu(\Delta\sigma_1' + \Delta\sigma_2')$$

$$E(\epsilon_1+\epsilon_2+\epsilon_3) = E\cdot\epsilon_V = E\cdot\frac{\Delta V}{V}$$
$$= \Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3' - \mu[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_2' + \Delta\sigma_3' + \Delta\sigma_3' + \Delta\sigma_1']$$
$$= \Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3' - 2\mu[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3']$$
$$= (1-2\mu)(\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3')$$
$$E\frac{\Delta V}{V} = (1-2\mu)(\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3')$$
$$\frac{\Delta V}{V} = \frac{(1-2\mu)}{E}(\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3')$$

Multiply and divide by 3

$$\frac{\Delta V}{V} = \frac{3(1-2\mu)}{E}\cdot\frac{1}{3}[\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3']$$

$\frac{3(1-2\mu)}{E} = C_c$ = compressibility of soil skeleton.

Substituting the values of effective stress in terms of total stresses,

$$\Delta\sigma_1' = \Delta\sigma - u.$$
$$\frac{\Delta V}{V} = \frac{C_c}{3}[\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u]$$
$$= C_c\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right]$$

$n \rightarrow$ Porosity.

Volume of voids $= nV$.

of the pore fluid is assumed to show a linear relationship b/w volume change and stress and it's coefficient of volume compressibility is represented by C_v, the change in volume of pore fluid ΔV_w due to increase in pore pressure Δu under the condition of no drainage is given by

$$\Delta V_w = nVC_v \Delta u.$$

The decrease in volume of soil skeleton is almost entirely due to decrease in volume of voids.

$$nVC_v \Delta u = VC_c \left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right]$$

$$\Delta u = \frac{VC_c}{nVC_v}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right].$$

$$nVC_v \Delta u + C_c \Delta u = \frac{VC_c}{3}\left[\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3\right]$$

$$\Delta u(nVC_v + VC_c) = \frac{VC_c}{3}\left[\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3\right]$$

$$\Delta u = \frac{VC_c}{(nVC_v + VC_c)}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\right]$$

$$= \frac{1}{1 + nV\frac{C_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\right].$$

In a conventional triaxial test

$$\Delta\sigma_2 = \Delta\sigma_3.$$

$$\Delta u = \frac{1}{1 + \frac{nC_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + 2\Delta\sigma_3)\right].$$

$$= \frac{1}{1 + \frac{nC_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + 3\Delta\sigma_3 - \Delta\sigma_3]\right)$$

$$\Delta u = \frac{1}{1 + \frac{nC_v}{C_c}}\left[\Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)\right] \quad \text{——— (1)}$$

This equation can be written in the form of

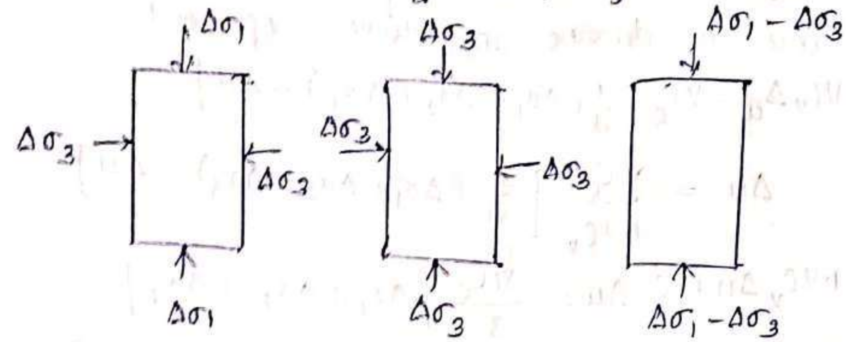
$$\Delta u = B(\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)) \quad \text{——— (2)}$$

A & $B \rightarrow$ Skempton's pore pressure parameters.

The parameters are to be determined experimentally. In an undrained triaxial test, stress changes are usually made in two stages.

a) an increase in cell pressure $\Delta\sigma_3$ resulting in an all round change in stress.

b) an increase in axial load resulting in change in deviator stress $\Delta\sigma_d = (\Delta\sigma_1 - \sigma_3)$



$(u) = (\Delta u_1) + (\Delta u_2)$.

Let Δu_1 - change in pore pressure during first stage of test when cell pressure is applied.

Δu_2 - change in pore pressure when deviator stress is applied.

$$\Delta u = \Delta u_1 + \Delta u_2 \quad \text{———} \quad (3)$$

Comparing (2) & (3)

$$\Delta u_1 + \Delta u_2 = B\Delta\sigma_3 + AB(\Delta\sigma_1 - \Delta\sigma_3)$$

$$\Delta u_1 = B\Delta\sigma_3 \;; \quad \Delta u_2 = AB(\Delta\sigma_1 - \Delta\sigma_3)$$

$$B = \frac{\Delta u_1}{\Delta\sigma_3} \;; \quad \cancel{B} = \bar{A}(\Delta\sigma_1 - \Delta\sigma_3)$$

$$\bar{A}_1 = \frac{\Delta u_2}{\Delta\sigma_1 - \Delta\sigma_3}$$

From equation (1) and (2)

$$B = \frac{1}{1 + \frac{n C_v}{C_c}}$$

<u>Factors affecting A and B</u>

$$C_v \ll C_c$$

Fully saturated soil - $\frac{C_v}{C_c} = 1 \quad \therefore B = 1$

Perfectly dry soil - $\frac{C_v}{C_c} = \infty \quad \therefore B = 0$

Partially saturated soils $= 0 < B < 1$

Proctor's optimum water content and density.

$B = 0.1$ to 0.5.

The coefficient A varies with stresses and strains. It depends on whether total stresses are increasing or decreasing.

Preconsolidation reduces A.

Other factors affecting A – Type of shear.

Sample disturbance Environment (Temp & nature of fluid).

<u>Determination of parameters A & B:</u>

B is determined in lab by measuring change in pore pressure Δu_1 due to change in all cell pressure $\Delta\sigma_3$ in first part of test.

$$B = \frac{\Delta u_1}{\Delta \sigma_3}.$$

A is measured during second stage of test when deviator stress $(\Delta\sigma_1 - \Delta\sigma_3)$ is applied at constant cell pressure, when Δu_2 is measured.

$$\bar{A} = A \cdot B = \frac{\Delta u_2}{\Delta\sigma_1 - \Delta\sigma_3}.$$

$$A = \frac{\Delta u - \Delta\sigma_3}{\Delta\sigma_1 - \Delta\sigma_3}.$$

For usual undrained triaxial test, $\Delta\sigma_3 = 0$; when deviator stress is applied.

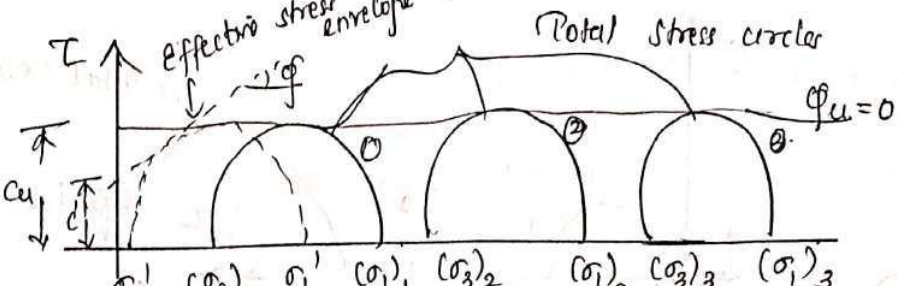
$$A = \frac{\Delta u}{\Delta\sigma_1}$$

<u>Shear strength of cohesive soils.</u>

a) <u>Undrained test on saturated cohesive soils.</u>

It is carried on undisturbed sample of clay, silt and peat to determine strength of natural ground.

It is also carried out on remoulded samples of clay to measure its sensitivity.



For 1^{st} circle

Diameter = $(\sigma_1)_1 - (\sigma_3)_1$

u - pore pressure measured @ failure

Diameter of effective stress circle $= (\sigma_1')_1 - (\sigma_3')_1$

$= [(\sigma_1)_1' - u) - [(\sigma_3)_1' - u]$

$= (\sigma_1)_1 - (\sigma_3)_1$

Diameter of effective stress circle = Diameter of total stress circle.

Major effective principal stress does not change.

B=1 both major and minor principal axis effective stress are independent of magnitude of cell pressure applied.

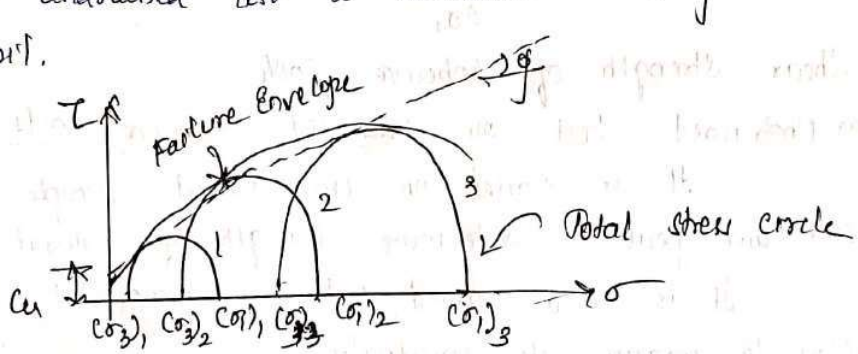
∴ We get only one Mohr circle in terms of effective stress, for all identical specimens tested under increased pressure.

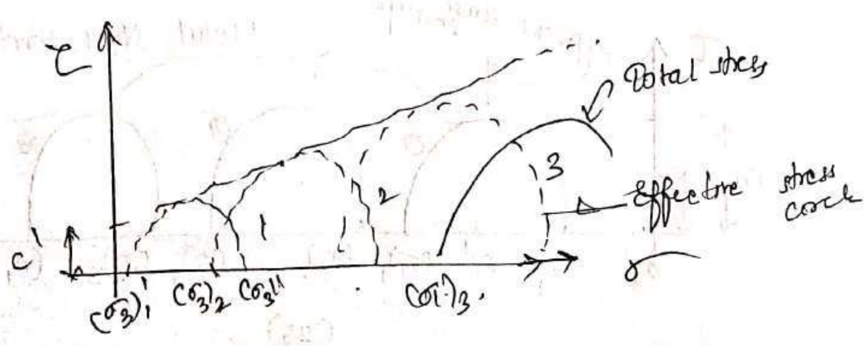
Deviator stress $\sigma_d = \sigma_1' - \sigma_3'$ ($\phi_u = 0$, $c_u = d$.)

Effective stress envelope cannot be obtained from this test.

b) <u>Undrained test on partly saturated cohesive soil.</u>

In case of earth embankments, which are compacted at optimum water content, the soil remains partly saturated soil and it is necessary to conduct undrained test to determine shear parameter of soil.





(26)

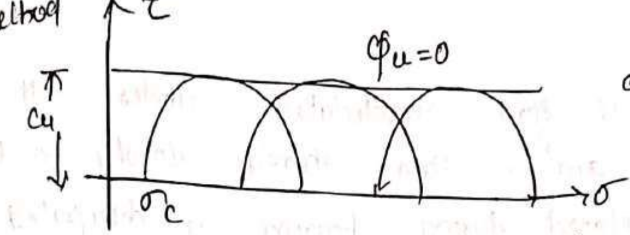
As all pressure increased, deviator stress @ failure also increases, though this increase in deviator stress becomes smaller as the air in soil voids is compressed and dissolved.

The increase in deviator stress later ceases when large stresses cause full saturation.

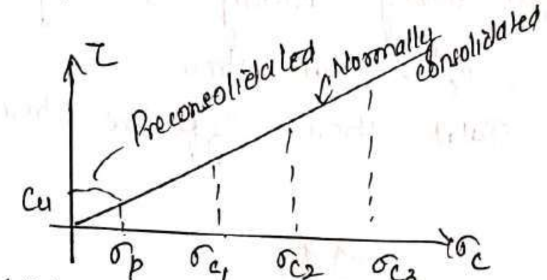
Due to this, failure envelope in terms of total stress is non linear.

Failure envelope in terms of effective stress is very closely a straight line over wide range of pressure.

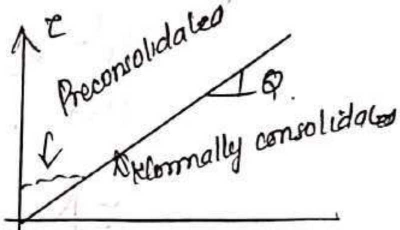
I Method



a) Failure envelope for preconsolidated pressure σ_c

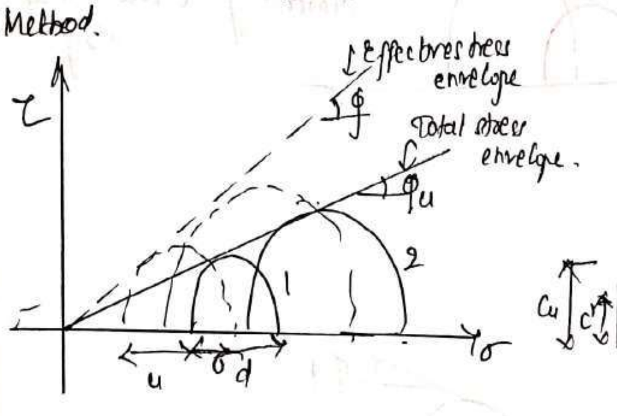


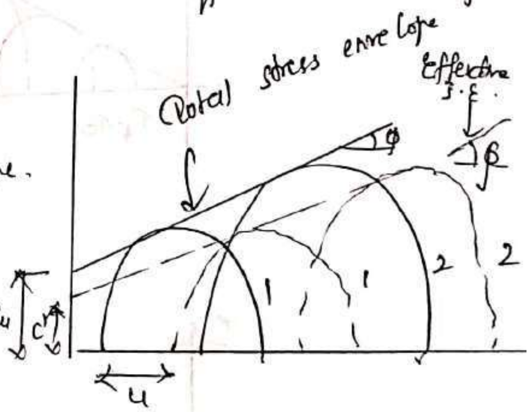
b) Variation of cu with σ



c) Effective stress envelope.

II Method.





$\tau_f = \sigma \tan \phi_{cu}$

Normally consolidated

$\tau_f = c_{cu} + \sigma \tan \phi_u$.

Preconsolidated.

d) Consolidate undrained test on partly saturated cohesive soils.

* To examine the effect on c' and φ' of flooding foundation strata and earth fill materials by applying back pressure to pore space to ensure full saturation

Shear strength - independent of change in cell pressure.

Incomplete saturation will mean that unique value of c_{cu} and ϕ_{cu} will only be obtained if no change in cell pressure is made after consolidation stage, before sample is sheared.

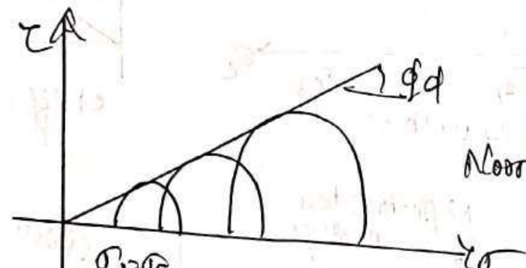
c' and ϕ' are determined by measuring pore pressure and getting value of effective stress @ failure.

e) Drained Test

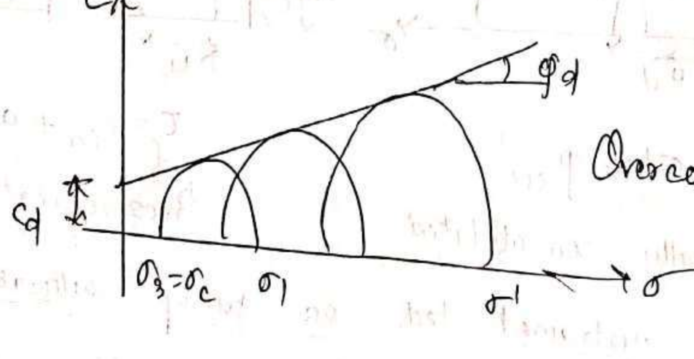
Specimen is first consolidated under cell pressure ($\sigma_c = \sigma_3$) and is then sheared slowly so that pore pressure developed during shearing is dissipated.

$\sigma_3' = \sigma_c$; σ_c' - Axial stress

$u = 0$, Total stress = Effective stress.



Normally consolidated clay



Overconsolidated clay

UNIT - V

SLOPE STABILITY

Infinite slopes and finite slopes - Friction circle method - Use of stability number - Guidelines for location of critical slope surface in cohesive c-φ soil - slope protection measures.

Stability of slopes

Earth embankments are commonly required for railways, roadways, earth dams, river training works. The stability of these embankments or slopes as they are commonly called, should be very thoroughly analysed since their failure may lead to loss of human life as well as collosal economic loss.

Failure of mass of soil located beneath a slope is called slide. It involved a downward and outward movement of entire mass of soil that participates in failure.

Failure of slopes takes place mainly due to

i) Action of gravitational force.

ii) Seepage forces within the soil

iii) Due to excavation or under cutting of its foot

iv) Due to gradual disintegration of structure of soil.

Analysis of stability of slope consists of two parts.

i) Determination of the most severly stressed internal surface and the magnitude of shearing stress to which it is subjected.

ii) Determination of shearing strength along this surface.

The shearing stress to which any slope can be subjected depends upon unit weight of material and geometry of slope, while shear strength which can be mobilised to resist shearing stress depends on character of soil, its density and drainage conditions.

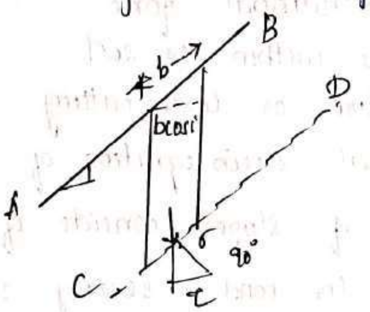
Slope

Infinite slope	Finite slope
Boundary surface of semi finite soil mass	slope of limited extent
↓	↓
soil properties for all identical depth below surface are constant	Inclined force of earth dam, embankment
↓	↓
Slope extending to infinity do not exist in nature	All man made slopes.

Stability analysis of infinite slopes.

Fig. shows an infinite slope AB, inclined at angle 'i' to to horizontal. Soil properties and the soil stress on any plane parallel to slope surfaces are identical and failure of slope involves a sliding of soil mass along a plane parallel to slope at some depth.

CD – Failure plane at depth z below surface.



Consider a prism of soil, of inclined length b along slope and depth z upto critical surface.

Horizontal length of prism = $b \cos i$

Volume / unit length of prism = $z\, b \cos i$

Weight of prism = $W = \gamma z\, b \cos i$

Vertical stress σ_z on surface CD is given by

$$\sigma_z = \frac{W}{b} = \gamma z \cos i$$

σ – stress component normal to surface CD

τ – stress component horizontal to surface CD

$$\sigma = \sigma_z \cos i = \gamma z \cos^2 i$$

$$\tau = \sigma_z \sin i = \gamma z \cos i \sin i$$

τ - shear stress which is resisted by shear strength.

Factor of safety against sliding due to shear

$$F = \frac{\tau_f}{\tau}$$

τ_f consists of both cohesion and internal friction

Two cases: i) cohesionless soil

ii) Cohesive soil.

1) <u>Cohesionless soil</u>

$$\tau_f = \sigma \tan\phi$$

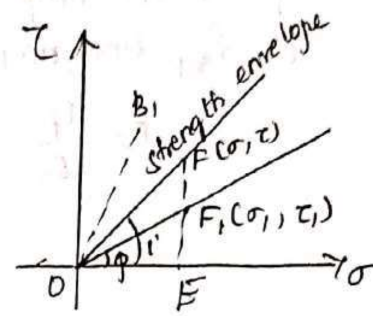
$$\frac{\sigma}{\tau} = \frac{\gamma z \cos^2 i}{\gamma z \cos i \sin i}$$

$$= \frac{\cos i}{\sin i} = \cot i = \text{constant}$$

$$\sigma = \tau \cot i$$

$$\tau = \frac{\sigma}{\cot i}$$

$$\tau = \sigma \cdot \tan i$$



$\tau < \tau_f$ or $i < \phi$ - Failure not occur.

Factor of safety

$$F = \frac{\tau_f}{\tau} = \frac{\tan\phi}{\tan i}$$

$$F = \frac{\tan\phi}{\tan i}$$

<u>Submerged slope</u>.

If slope is submerged, the bulk unit weight γ should be replaced by submerged unit weight γ'.

$$\sigma = \gamma' z \cos^2 i \qquad \tau_f = \sigma \tan\phi$$

$$\tau = \gamma' z \cos i \sin i \qquad = \gamma' z \cos^2 i \tan\phi$$

$$F = \frac{\tau_f}{\tau} = \frac{\gamma' z \cos^2 i \tan\phi}{\gamma' z \cos i \sin i} = \cot i \cdot \tan\phi$$

$$F = \frac{\tan\phi}{\tan i}$$

<u>Steady seepage along the slope</u>

$$|\kappa| = \gamma_{sat} - z\, b \cos i$$

$$\sigma_z = \frac{W}{b}$$

$$= \gamma_{sat} - z \cos i$$

$\sigma = \sigma_z \cos i = \gamma_{sat}\, z \cos^2 i$

$\tau = \sigma_z \sin i = \gamma_{sat}\, z \cos i \sin i$

In addition to these, there is an upward force u due to seeping water.

$u = \gamma_w z \cos^2 i$

$F = \frac{\tau_f}{\tau}$ $\qquad \tau_f = \sigma' \tan\phi$

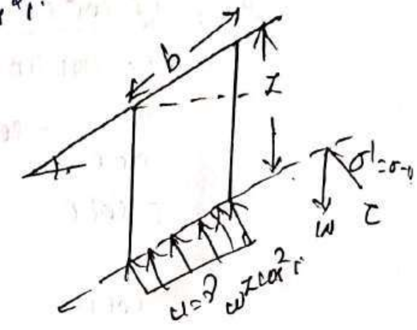
$\sigma' = \sigma - u = \gamma_{sat} z \cos^2 i - \gamma_w z \cos^2 i$

$\tau_f = \gamma' z \cos^2 i \tan\phi$ $\qquad = \gamma' z \cos^2 i$

$\tau = \gamma_{sat} \cdot z \cos i \sin i$

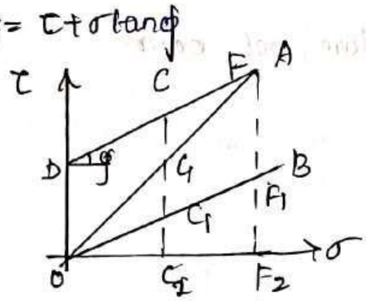
$$F = \frac{\tau_f}{\tau} = \frac{\gamma' z \cos^2 i \tan\phi}{\gamma_{sat} \cdot z \cdot \cos i \sin i}$$

$$= \frac{\gamma'}{\gamma_{sat}} \frac{\tan\phi}{\tan i}$$



(ii) Cohesive soil

$\tau_f = c + \sigma \tan\phi$



Slope angle $\leq \phi$, No critical state of stress - slope stable

If $i > \phi$ - it will cut strength envelope at some point F and a state of incipient failure is reached because shear stress corresponding to depth represented by point F equal to shear strength τ_f.

For any depth, $z < F$, $\tau < \tau_f$ - slope - stable

$i > \phi$ - slope is stable upto limited depth known as critical depth H_c

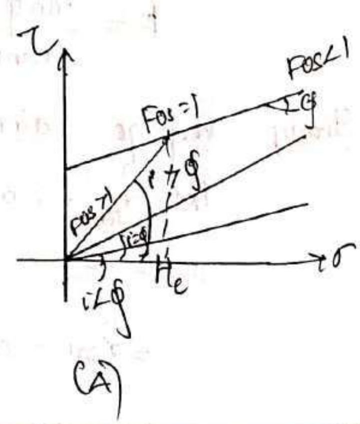
$F = \frac{\tau_f}{\tau}$

$= \frac{c + \sigma \tan\phi}{\tau}$

$\sigma = \gamma z \cos^2 i$

$\tau = \gamma z \cos i \sin i$

$$F = \frac{c + \gamma z \cos^2 i \tan\phi}{\gamma z \cos i \sin i}$$



(A)

$$F = \frac{c}{\gamma z \cos i \sin i} + \frac{\gamma z \cos^2 i \tan\phi}{\gamma z \cos i \sin i}$$

$$= \frac{c}{\gamma z \cos i \sin i} + \frac{\tan\phi}{\tan i}.$$

For non cohesive soil, $c = 0 \Rightarrow F = \frac{\tan\phi}{\tan i}$

For critical depth, $z = H_c$, $\tau_f = \tau$

$$c + \gamma H_c \cos^2 i \tan\phi = \gamma H_c \cos i \sin i$$

$$c = \gamma H_c \cos i \sin i - \gamma H_c \cos^2 i \tan\phi$$

$$= \gamma H_c \frac{\cos i}{\cos i} \cos i \sin i - \gamma H_c \cos^2 i \tan\phi$$

$$= \gamma H_c \cos^2 i \tan i - \gamma H_c \cos^2 i \tan\phi$$

$$H_c = \frac{c}{\gamma \cos^2 i (\tan i - \tan\phi)}$$

For given values of i & ϕ

$$H_c \propto \text{cohesion}$$

$$\frac{c}{\gamma H_c} = \cos^2 i (\tan i - \tan\phi)$$

Stability number $(S_n) = \frac{c}{\gamma H_c} \rightarrow$ Dimensionless quantity.

F_c – FOS w.r.t. cohesion

C_m – Mobilised cohesion at depth H

$$C_m = \frac{c}{F_c}$$

$$S_n = \frac{c}{\gamma H_c} = \frac{C_m}{\gamma H} = \frac{c}{F_c \gamma H} = (\tan i - \tan\phi)\cos^2 i$$

$$F_c = \frac{c}{C_m} \; ; \; F_c = \frac{H_c}{H}$$

Submerged slope:

$$F = \frac{c + \gamma' z \cos^2 i \tan\phi}{\gamma' z \cos i \sin i}$$

$$H_c = \frac{c}{\gamma'} \cdot \frac{1}{(\tan i - \tan\phi)\cos^2 i}$$

Steady seepage along the slope

$$F = \frac{c + \gamma' z \cos^2 i \tan\phi}{\gamma_{sat}\, z \cos i \sin i}.$$

$$F = \frac{c}{\gamma_{sat}\, z \cos i \sin i} + \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi}{\tan i}$$

For critical height, $z = H_c$, $F = 1$

$$\gamma_{sat} H_c \cos i \sin i = c + \gamma' H_c \cos^2 i \tan\phi$$

$$c = \gamma_{sat} H_c \cos i \sin i - \gamma' H_c \cos^2 i \tan\phi$$

$$= \gamma_{sat} H_c \cos^2 i \tan i - \gamma' H_c \cos^2 i \tan\phi$$

$$H_c = \frac{c}{\cos^2 i \,[\gamma_{sat} \tan i - \gamma' \tan\phi]}.$$

Problems

1) A long natural slope of cohesionless soil is inclined at 12° to horizontal. Taking $\phi = 30°$. Determine FOS of slope. If slope is completely submerged what will be FOS?

Soln:

$$F = \frac{\tan\phi}{\tan i} \qquad \phi = 30°;\; i = 12°$$

$$= \frac{\tan 30°}{\tan 12°} = 2.72$$

Submergence: $F = \frac{\tan\phi}{\tan i} = 2.72.$

2) A long natural slope of sandy soil ($\phi = 25°$) is inclined at 10° to horizontal. The water table is at the surface and seepage is parallel to slope. If saturated unit weight of soil is $19.5\ kN/m^3$, determine FOS of slope.

Soln:

$$F = \frac{\gamma' \tan\phi}{\gamma_{sat} \tan i} = \frac{(19.5 - 9.81) \tan 25}{19.5 \times \tan 10°}$$

$$= 1.31.$$

3) A long natural slope in a $c-\phi$ soil is inclined at 12° to horizontal. The water table is at the surface and the seepage is parallel to slope. If a plane slip has developed at a depth of 4m, determine FOS. Take $c = 8\ kN/m^2$, $\phi = 22°$ & $\gamma_{sat} = 19\ kN/m^3$

Soln:

$$F = \frac{c + \gamma' z \cos^2 i \tan\phi}{\gamma_{sat}\, z \cos i \sin i}$$

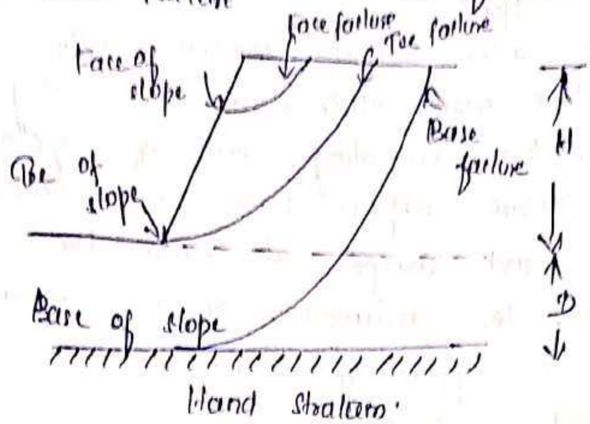
(6) $$= \frac{8 + (19 - 9.81) \times 4 \cos^2 12° \tan 22°}{19 \times 4 \cos 12° \cdot \sin 12°} = 1.44.$$

Stability Analysis of Finite Slopes

Failure of finite slopes occur along a surface which is a curve. In stability computations, the curve corresponding representing real surface of sliding is usually replaced by an arc of a circle.

Two basic types of failure

* Slope failure — Face failure, Toe failure
* Base failure



Depth Factor, $D_f = \frac{H+D}{H}$

$D_f > 1 \rightarrow$ Base failure

$D_f = 1 \rightarrow$ Toe failure

$D_f < 1 \rightarrow$ Face failure.

Face Failure

When slope angle β is very high. Soil in the upper part of slope is relatively weaker.

Toe Failure

When It occurs in steep slopes.

* It happens when soil mass above and below base is homogeneous.

Base Failure

* Soil below toe is relatively weak and soft.
* The slope is flat.

Types of slip surfaces or failure surfaces

The rupture of a finite slope may takes place along one of the following failure surface.

* Planar failure surface
* Circular failure surface
* Non circular failure surface.

Planar Failure Surface

- Soil deposit on embankment with a specific plane of weakness. In composite earth dams with sloping cores, planes of weakness within the bank may consist of 2 or 3 planar surfaces.

Circular failure surface

In most cases, actual failure surfaces are curved. The rupture mass slide down a sliding surface in a definite pattern resembling that of cycloid. Generally, the failure surfaces have arcs somewhat flatter at ends and sharper at centre. For simple idealised problems, the assumption of a circular failure surface is sufficiently accurate.

Non circular Failure Surface

It occurs in many practical cases. It may arise in homogeneous dams having one or more of the following.

i) Foundation of infinite depth.
ii) Rigid boundary planes of maximum or zero shear.
iii) Presence of relatively stronger or weaker layer.

Non homogeneous earth dams

i) Presence of soft layer in foundation.
ii) Use of different type of soil or rock in dam with varying strength or pore pressure condition.
iii) Use of drainage blankets to facilitate dissipation of pore pressure.

Methods of Analysis

Stability of finite slope can be investigated by no. of methods.

i) Fellenius method or Swedish circle method or Slip circle method.
ii) Friction circle method
iii) Culmann's method
iv) Bishop's method.

(c)

Swedish
~~Swedish~~ Slip Circle Method / Fellenius Method.

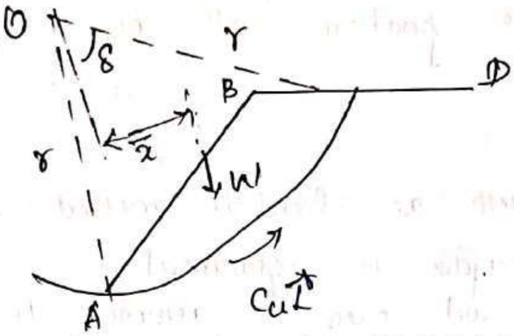
This method was developed at ~~Swedish~~ Swedish Geotechnical Commission headed by Fellenius. In this method, slip surface is assumed to be cylindrical. i.e., arc of circle in section.

Two cases

i) Analysis of purely cohesive soil ($C\phi_u = 0$ analysis)

ii) Analysis of a soil possessing both cohesion and friction ($C c$-ϕ analysis)

i) Cohesive soil ($C\phi_u = 0$ analysis)



The method consist in assuming no. of trial slip circles and finding FOS of each.

The circle corresponding to minimum FOS is called critical slip circle.

AD - Trial slip circle ; r - radius ; O - centre of rotation; W - weight of soil of wedge ABDA of unit thickness, acting through its centroid ; $\bar{x}$ - distance of line of action of W from vertical line passing through centre of rotation.

Cu - unit cohesion ; $\hat{L}$ - length of slip arc AD

Driving moment, $M_D = W\bar{x}$

$$\hat{L} = \frac{2\pi r\delta}{360}$$

Shear resistance developed along slip surface $= C_u\hat{L}$

Resisting moment, $M_R = rC_u\hat{L}$.

$$\text{FOS, } F = \frac{M_R}{M_D} = \frac{rC_u\hat{L}}{W\bar{x}}.$$

Cm - Mobilised shear resistance of soil

F = 1

$W\bar{x} = rC_m\hat{L}$

$$C_m = \frac{W\bar{x} \cdot \frac{1}{r}}{\hat{L}}$$

$$F = \frac{c_u}{c_m} = \frac{c_u \hat{L} r}{W \bar{x}},$$

$\bar{x}$ from O can be determined by dividing wedge into no. of vertical slices and dividing algebraic sum of moment of weight of each slice by weight of wedge.

Tension Crack

If a tension crack of depth $Z_0 = \frac{2c}{\gamma}$ develops, water enter into crack, exerting hydrostatic pressure force P_w acting on the portion DE at the height $Z_0/3$ from E. That portion will be ineffective in resisting slide.

(II) c – φ analysis.

It is known as Swedish method of slices.

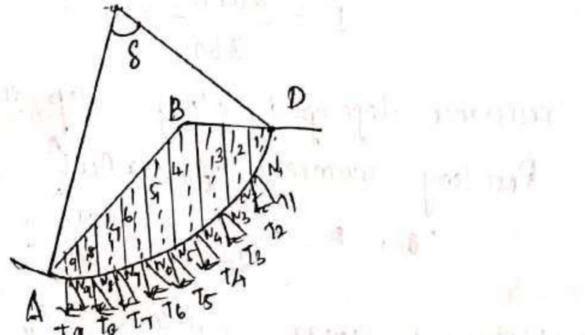
i) The slip surface is cylindrical

ii) The sliding soil mass is assumed to consists of a number of vertical slices

iii) The forces of interaction between adjacent slices are neglected.

Let AD be a slip circle of radius r, centre O and central angle ∠AOD = δ

Let sliding soil mass ABDA be divided into no. of vertical slices 1,2,.... The weights $W_1, W_2, \ldots$ of slices 1,2,.... acting through centre of gravity of respective slices are resolved into normal components $N_1, N_2 \ldots$ & tangential components $T_1, T_2, \ldots$



Taking moments about centre of rotation O,

Driving moment

$$M_D = T_1 r + T_2 r + \ldots$$
$$= r\left[T_1 + T_2 + \ldots\right] = r \Sigma T \qquad (10)$$

Restoring Moment, $M_R = \Sigma c \cdot \Delta l \cdot r + (N_1 \tan\phi + N_2 \tan\phi + \dots) r$

$= c r \Sigma \Delta l + (N_1 + N_2 + \dots) r \tan\phi$

$= r (c\hat{l} + \Sigma N \tan\phi)$

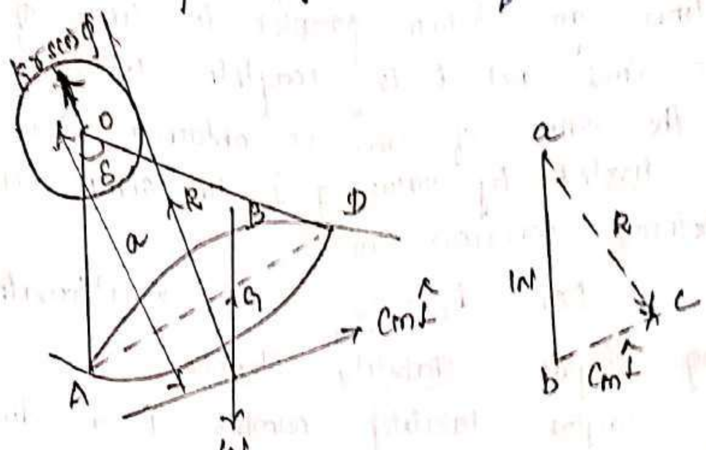
$\hat{l}$ - length of arc AD

For against sliding, $F = \frac{M_R}{M_D} = \frac{c\hat{l} + \Sigma N \tan\phi}{\Sigma T}$

This method is not only applicable to homogeneous soils but also to stratified soils, fully or partially submerged soils with considerations of seepage forces and pore pressures that may exist, and also non-uniform slopes.

Friction Circle Method.

The slip surface is assumed to be cylindrical i.e., arc of circle in section. The sliding soil mass is assumed to be acted upon by three forces keeping it in equilibrium.



Weight W of the sliding soil mass ABDA acting vertically through its centre of gravity. The resultant cohesive force, $C_m\hat{L}$ acting parallel to chord AD and at a distance a from centre of rotation O where

$a = r\frac{\hat{L}}{L}$ $\Rightarrow$ $\hat{L}$ - Length of arc AD

L - Length of chord AD

The resultant reaction R passing through point of intersection of the above two forces and tangential to the friction circle.

Procedure

i) With centre O and radius r, the slip circle with AD is constructed. The friction circle is drawn with centre O and radius $kr\sin\phi$.

$k=1$

2) A vertical line is drawn through centroid of section ABDA to get line of action of weight W

3) Chord AD is drawn, A line is drawn parallel to chord AD and at distance $a = r\frac{\hat{l}}{l}$ from O, to get line of action of resultant cohesive force $C_m \hat{l}$. The length of the arc AD $\hat{l}$ is computed using equation $\hat{l} = \frac{\pi r \delta}{180}$. The length of chord AD, $\hat{l}$ is obtained by measurement.

4) Through the point of intersection of lines of action of forces W and $C_m\hat{l}$, a line is drawn tangential to friction circle to get line of action of resultant reaction R.

5) Weight (W) of sliding soil mass ABDA is computed and plotted to scale. Through the ends of vector representing W, lines are drawn parallel to lines of action of forces $C_m\hat{l}$ and R to complete triangle of forces.

The value of $C_m\hat{l}$ is obtained from force triangle and divided by value of $\bar{l}$ to obtain the value of mobilised cohesion C_m.

FOS : $F_c = \frac{C}{C_m}$ $\quad C \rightarrow$ ultimate cohesion

Using Taylor Stability Number

Taylor stability number is a dimension less quantity denoted by S_n.

$$S_n = \frac{C_m}{\gamma H}.$$

$C_m \rightarrow$ Mobilised cohesion on slip surface.

$\gamma \rightarrow$ unit weight of soil.

$H \rightarrow$ Height of slope.

$$F_c = \frac{c}{C_m} \Rightarrow C_m = \frac{c}{F_c}$$

$$S_n = \frac{c}{F_c \gamma H}$$ $\quad c \rightarrow$ unit ultimate cohesion

$F_n = F_c$

$$F_H = \frac{H_c}{H}$$

$$F_H \cdot H = H_c$$

$$S_n = \frac{c}{\gamma H_c}$$

$H_c \rightarrow$ critical height of slope.

S_n varies w.r.t. slope angle i and angle of shearing ϕ

FOS is applicable to both cohesion and friction, we have mobilised shear resistance given by

$$\tau_m = \frac{\tau_f}{F} = \frac{c + \sigma \tan\phi}{F}$$

While obtaining S_n from chart, mobilised angle of shearing resistance ϕ_m should be used.

$$\tan\phi_m = \frac{\tan\phi}{F}$$

$$\phi_m = \tan^{-1}\left(\frac{\tan\phi}{F}\right)$$

$$\phi_m \approx \frac{\phi}{F}$$

For cohesionless soil $(c=0)$, Taylor stability number $S_n = 0$ Taylor's chart is not applicable.

$$F = \frac{\tan\phi}{\tan i}$$ is independent of height of slope.

For cohesive soils, c & ϕ can be obtained from drained test should be used. Use of Taylor's stability number gives an approximate idea of long term stability, if seepage effect can be neglected and no change in water content can be assumed.

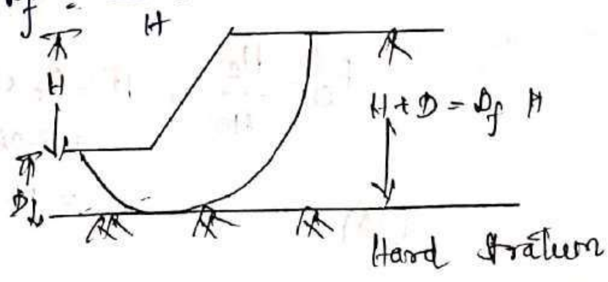
In case of fully submerged slopes, γ' should be used in expression for S_n.

When slope is saturated by capillary water, γ_{sat} should be used in expression S_n. S_n can be obtained from Taylor's chart, corresponding to weighted frictional angle ϕ_w

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \cdot \phi.$$

Taylor also determined stability number S_n for different values of slope angle i and depth factor D_f

$$D_f = \frac{H+D}{H}$$



(13)

Problems

1) Stability analysis of Swedish method of slices gave following values per remaining running meter for a 10m high embankment.

i) Total shearing force = 480 kN
ii) Total normal force = 1950 kN
iii) Total neutral force = 250 kN
iv) Length of arc = 22m.

If the properties of soil are $C = 24\ kN/m^2$ & $\phi = 6°$, Calculate FOS w.r.t to shear strength.

Soln:

$\Sigma T = 480\ kN$; $\Sigma N = 1950\ kN$

$\Sigma V = 250\ kN$; $C = 24\ kN/m^2$.

$\hat{L} = 22m$; $\phi = 6°$.

$$F = \frac{C\hat{L} + \Sigma(N-U)\tan\phi}{\Sigma T}$$

$$= \frac{24 \times 22 + (1950-250)\tan 6°}{480} = 1.47.$$

2) A slope 1 in 2 with height of 8m has following soil properties. $C = 28\ kN/m^2$, $\phi = 10°$, $\gamma = 18\ kN/m^3$, Calculate i) FOS w.r.t. cohesion

ii) Critical height of slope.

Soln:

If i is slope angle, $\tan i = 1/2$

$i = 26.6°$.

From Taylor stability chart, for $i = 26.6°$,

$\phi = 10°$, $S_n = 0.064$

$$S_n = \frac{C}{F_c \gamma H}$$

$$F_c = \frac{C}{S_n \gamma H} = \frac{28}{(0.064)(18)(8)}$$

$= 3.04$.

$$F_c = \frac{H_c}{H} \; ; \; H_c = F_c \times H$$

$= 3.04 \times 8$

$= 24.32\ m$.

(IA)

3) A 5m deep canal has side slopes of 1:1, Properties of soil are $C_u = 20 \, kN/m^2$, $\phi_u = 10°$, $e = 0.8$ and $G = 2.8$. If Taylor stability number is 0.108, determine F.O.S w.r.t cohesion, when the canal runs full. Also find the same in sudden draw down if Taylor's stability number for the condition is 0.137.

Soln:

$C_u = 20 \, kN/m^3$; $\phi_u = 10°$; $G = 2.8$

$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w = \frac{(2.8+0.8)}{(1+0.8)} \times 9.81$$

$$= 19.62 \; kN/m^3.$$

$$\gamma' = \gamma_{sat} - \gamma_w = 9.81 \; kN/m^3.$$

Case (i)

When canal runs full the side slopes are submerged.

$$S_n = \frac{c}{F_c \gamma' H} \quad \Bigg\| \quad F_c = \frac{\cancel{20}\, c}{S_n \gamma' H} = \frac{20}{(0.108 \times 9.81) \times 5}$$

$$F_c = 3.8.$$

case (ii)

sudden drawdown condition

$$S_n = 0.137.$$

$$F_c = \frac{c}{S_n \gamma' H} = 1.5$$

Slope Protection Measures

Slopes that are susceptible to sliding should be protected so that the area will be safe. Slopes which have failed recently are likely to fail under long term condition.

Slope have been protected by adopting some successful techniques. In general, protection measures involves:

i) Reducing the mass or loading which contribute to sliding.

ii) Improving the shearing strength along the anticipated zone of failure.

iii) Providing certain materials which will provide resistance to movement.

The protective measures to be adopted depends on different field conditions, types of soil in slope, the volume or depth of soil involving in sliding, ground water conditions, assessment of complete area which may require stabilization, the space available to undertake corrective measures, topographical conditions prevailing in the area and the possible changes that could due to vibratory measure undertaken.

* When base failure is anticipated, a berm may be provided near the toe.

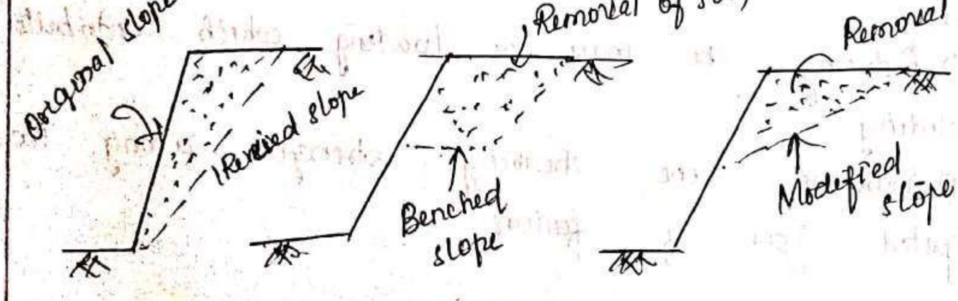
* If a zone near the toe is susceptible to erosion, a protective rock fill blanket followed by a rip rap can be provided.

* Soil shearing resistance of soil is reduced due to high ground water and excess pore water pressure.

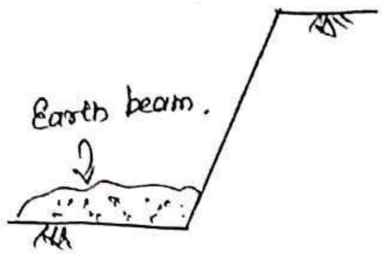
* This could be avoided by lowering the groundwater or ~~onto~~ intercepting the surface water.

* Driven piles are sometimes used to keep the moving part intact with the original ground. Sometimes driven piles, sheet piling and construction of retaining wall help by providing lateral support and increasing the resistance of slope to sliding.

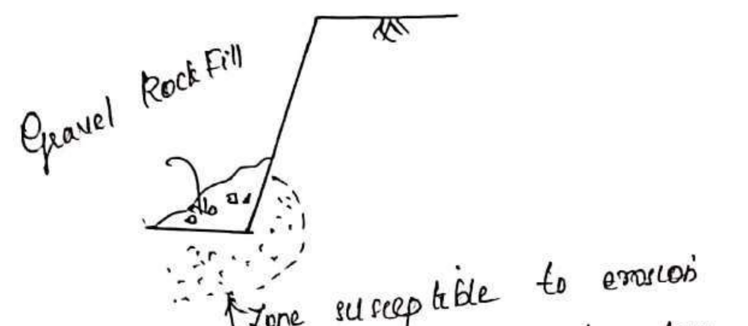
a) Slopes flattened or benched.



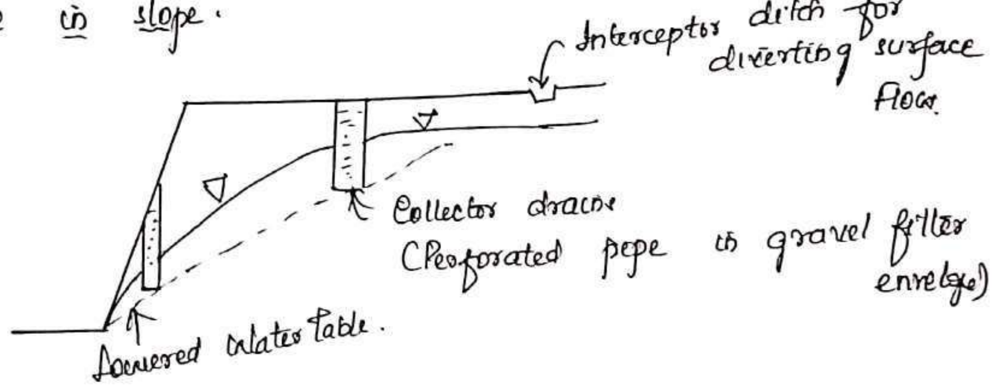
b) Beam provided at toe.



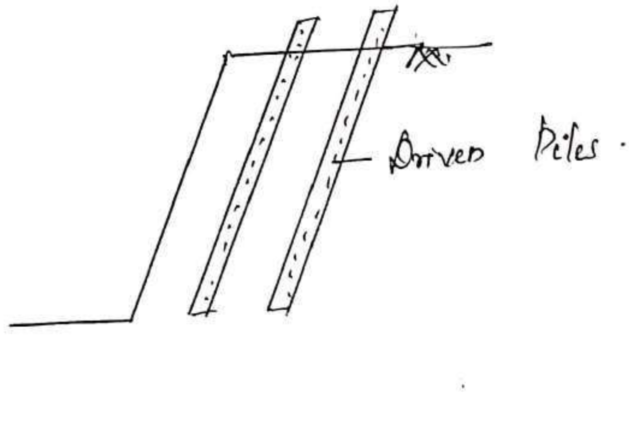
c) Protection against erosion provided at toe



d) Lowering of ground water table to reduce pore pressure in slope.



e) Use of driven or cast in place piles



——— x ———.