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CE8602 STRUCTURAL ANALYSIS II

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SYLLABUS

UNIT I INFLUENCE LINES FOR DETERMINATE BEAMS 9

Influence lines for reactions in statically determinate beams – Influence lines for shear force and bending moment – Calculation of critical stress resultants due to concentrated and distributed moving loads – absolute maximum bending moment - influence lines for member forces in pin jointed plane frames.

UNIT II INFLUENCE LINES FOR INDETERMINATE BEAMS 9

Muller Breslau's principle– Influence line for Shearing force, Bending Moment and support reaction components of propped cantilever, continuous beams (Redundancy restricted to one), and fixed beams.

UNIT III ARCHES 9

Arches - Types of arches – Analysis of three hinged, two hinged and fixed arches - Parabolic and circular arches – Settlement and temperature effects.

UNIT IV CABLES AND SUSPENSION BRIDGES 9

Equilibrium of cable – length of cable - anchorage of suspension cables – stiffening girders - cables with three hinged stiffening girders – Influence lines for three hinged stiffening girders

UNIT V PLASTIC ANALYSIS 9

Plastic theory - Statically indeterminate structures – Plastic moment of resistance – Plastic modulus – Shape factor – Load factor – Plastic hinge and mechanism – collapse load - Static and kinematic methods – Upper and lower bound theorems - Plastic analysis of indeterminate beams and frames.

CE8602 STRUCTURAL ANALYSIS II

UNIT I INFLUENCE LINES FOR DETERMINATE BEAMS

Influence lines for reactions in statically determinate beams – Influence lines for shear force and bending moment – Calculation of critical stress resultants due to concentrated and distributed moving loads – absolute maximum bending moment - influence lines for member forces in pin jointed plane frames.

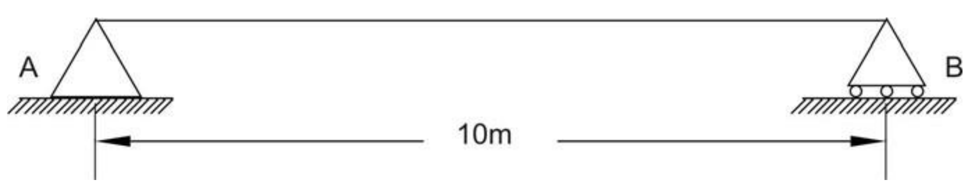
INTRODUCTION

An influence line is a diagram whose ordinates, which are plotted as a function of distance along the span, give the value of an internal force, a reaction, or a displacement at a particular point in a structure as a unit load move across the structure. An influence line is a curve the ordinate to which at any point equals the value of some particular function due to unit load acting at that point. An influence line represents the variation of the reaction, shear, moment, or deflection at a specific point in a member as a unit concentrated force moves over the member.

<u>INFLUENCE LINES FOR REACTIONS IN STATICALLY DETERMINATE BEAMS</u>

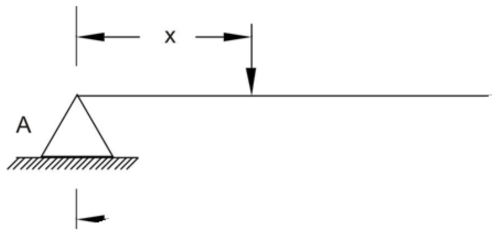
Problem:

1. Construct the influence line for the reaction at support B for the beam of span 10 m. The beam structure is shown in Fig.

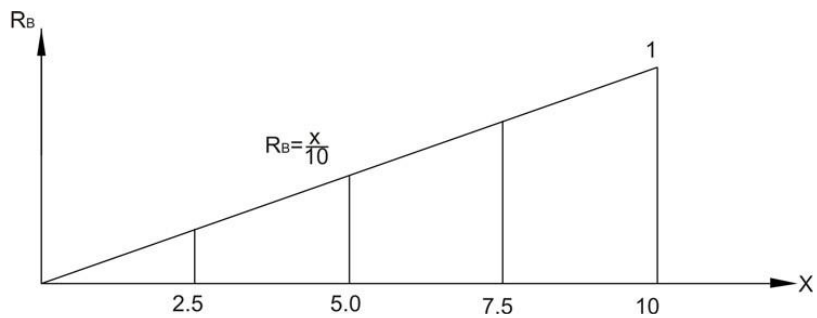


A unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows

$$\Sigma M_A = 0 : R_B \times 10 - 1 \times 2.5 = 0 \Rightarrow R_B = 0.25$$



the load can be placed at 5.0, 7.5 and 10 m. away from support A and reaction R_B can be computed and tabulated as given below.

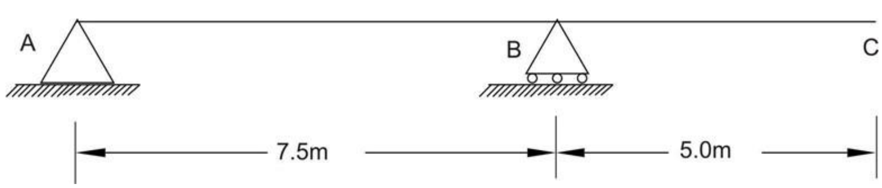


Influence Line Equation:

When the unit load is placed at any location between two supports from support A at distance x then the equation for reaction R_B can be written as

$$\boldsymbol{\Sigma M_A = 0 : R_B \times 10 - x = 0 \Rightarrow R_B = x/10}$$

2. Construct the influence line for support reaction at B for the given beam as shown in Fig.



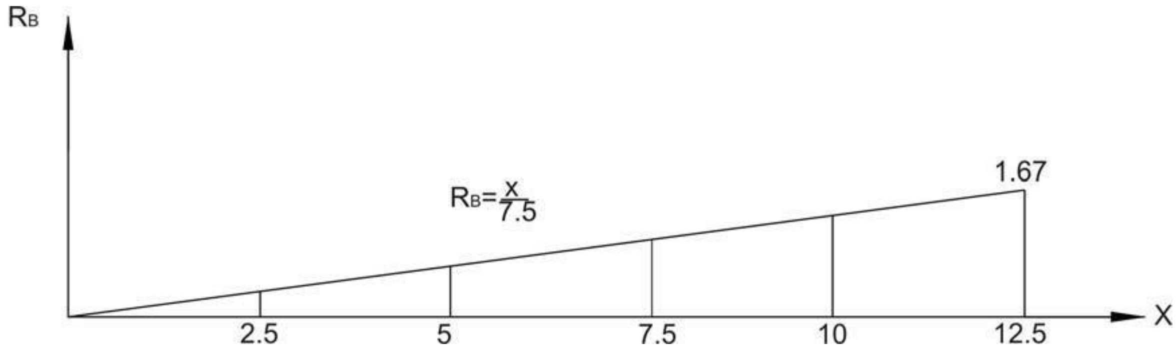
A unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows.

$$\boldsymbol{\Sigma M_A = 0 : R_B \times 7.5 - 1 \times 2.5 = 0 \Rightarrow R_B = 0.33}$$

Similarly one can place a unit load at distances 5.0 m and 7.5 m from support A and compute reaction at B. When the load is placed at 10.0 m from support A, then reaction at B can be computed using following equation.

Similarly a unit load can be placed at 12.5 and the reaction at B can be computed.

Graphical representation of influence line for R_B is shown in Fig.

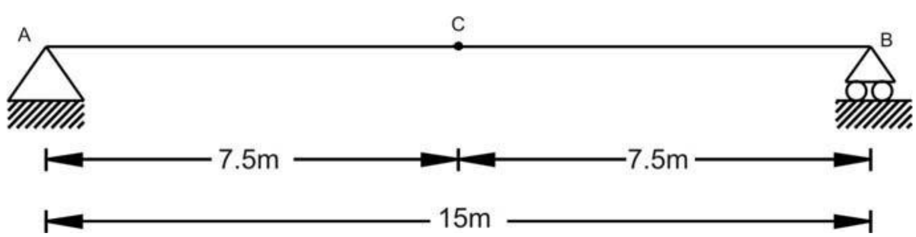


Applying the moment equation at A,

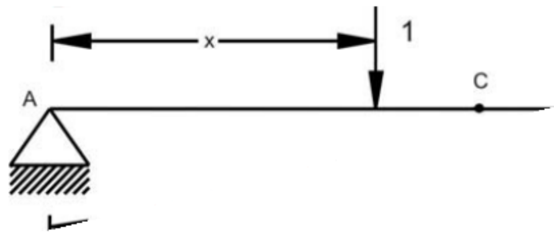
$$\mathbf{\Sigma M_A = 0 : R_B \times 7.5 - 1 = 0 \Rightarrow R_B = x/7.5}$$

<u>INFLUENCE LINES FOR SHEAR FORCE AND BENDING MOMENT</u>

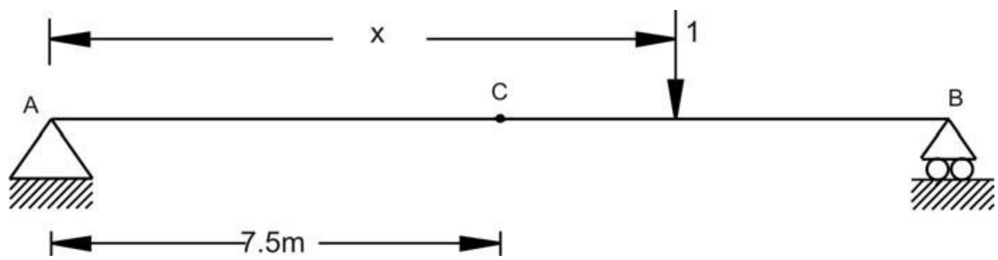
3. Construct the influence line for shearing point C of the beam.



Place a unit load at different location at distance x from support A and find the reactions at A and finally computer shear force taking section at C. The shear force at C should be carefully computed when unit load is placed before point C and after point C. The resultant values of shear force at C are tabulated as follows.

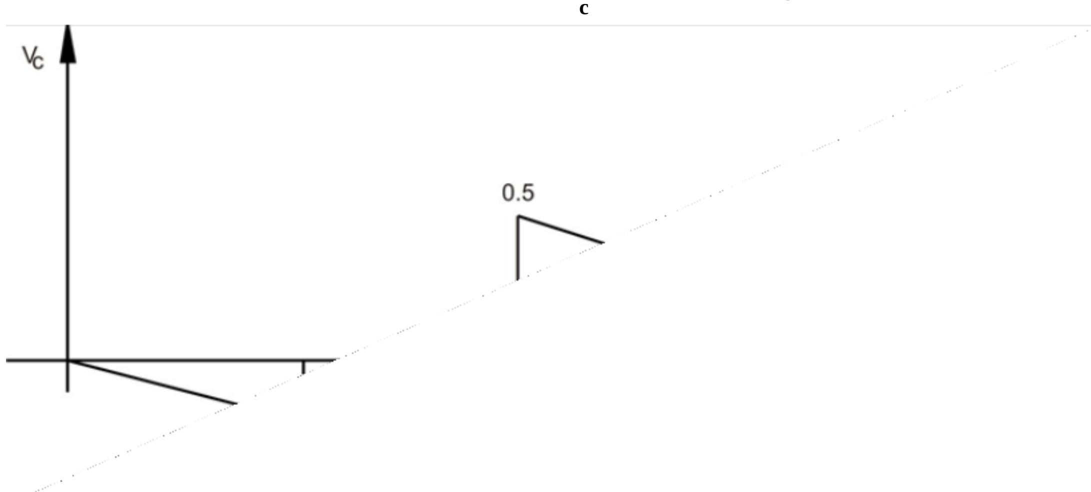


The beam structure – a unit load before section



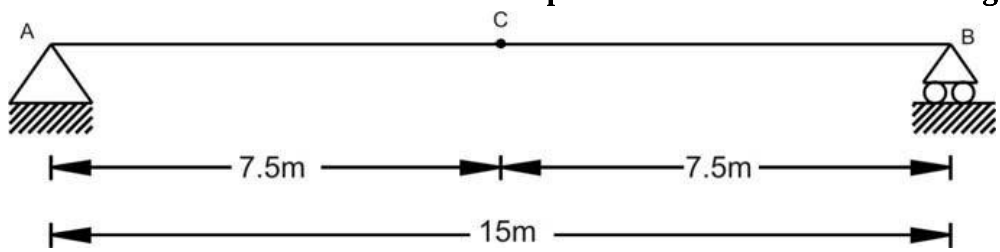
The beam structure - a unit load before section

Graphical representation of influence line for V_c is shown in Fig.



Influence line for shear point C

4. Construct the influence line for the moment at point C of the beam shown in Fig.



Beam structure

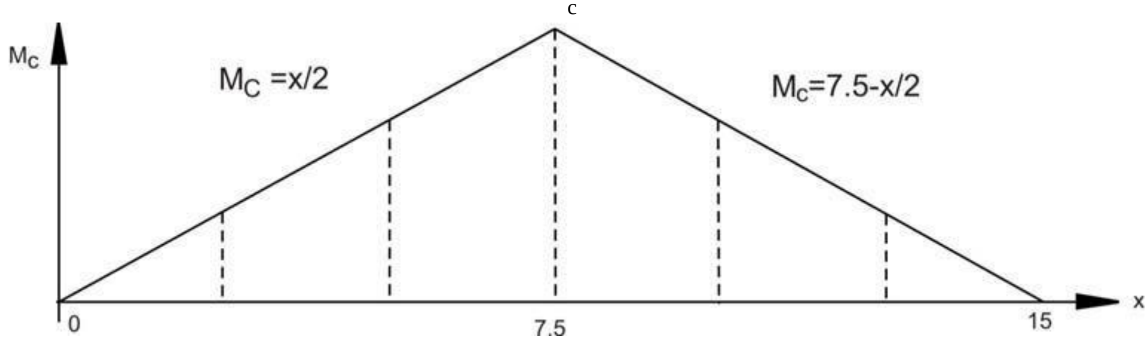
A unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example, we place the unit load at x=2.5 m from support A, then the support reaction at A will be 0.833 and support reaction B will be 0.167. Taking section at C and computation of moment at C can be given by

$$\mathbf{\Sigma M_c = 0 : - M_{c+}\ R_B\ x\ 7.5 - = 0 \Rightarrow - M_{c+}\ 0.167\ x\ 7.5 - = 0 \Rightarrow M_c = 1.25}$$

A unit load before section

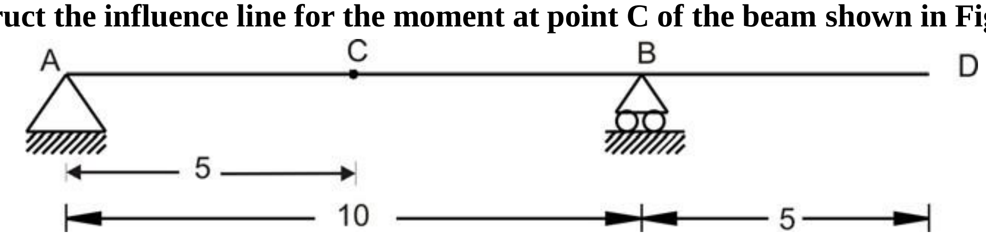
Similarly, compute the moment M_c for difference unit load position in the span.

Graphical representation of influence line for M_c is shown in Fig.



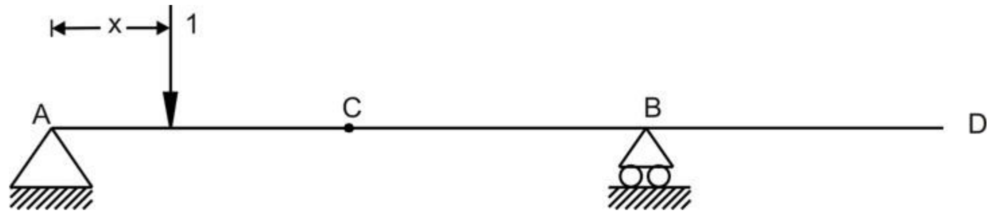
Influence line for moment at section C

5. Construct the influence line for the moment at point C of the beam shown in Fig.



Overhang beam structure

A unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example as shown in Figure 37.20, we place a unit load at 2.5 m from support A, then the support reaction at A will be 0.75 and support reaction B will be 0.25.

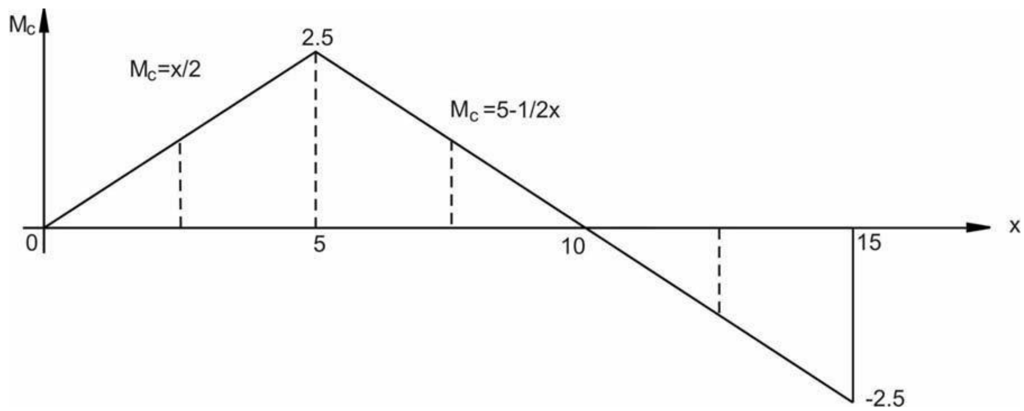


A unit load before section C

Taking section at C and computation of moment at C can be given by

$$\Sigma M_c = 0 : - M_{c+}\ R_B \times 5.0 - = 0 \Rightarrow - M_{c+}\ 0.25 \times 5.0 = 0 \Rightarrow M_c = 1.25$$

Graphical representation of influence line for M_c is shown in Fig.



Influence line of moment at section C

There will be two influence line equations for the section before point C and after point C. When a unit load is placed before point C then the moment equation for given Fig. can be given by

$$\Sigma M_c = 0 : M_c + 1(5.0 - x) - (1 - x/10)x5.0 = 0 \Rightarrow M_c = x/2, \text{ where } 0 \le x \le 5.0$$

A unit load before section C

When a unit load is placed after point C then the moment equation for given Fig. can be given by

$$\Sigma M_c = 0 : M_c - (1 - x/10) \times 5.0 = 0 \Rightarrow M_c = 5 - x/2, \text{ where } 5 < x \le 15$$

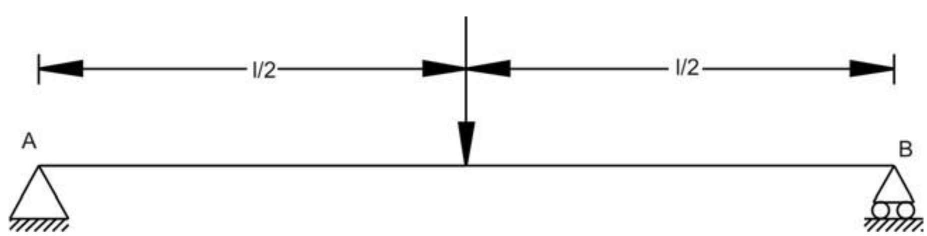
A unit load after section C

CALCULATION OF CRITICAL STRESS RESULTANTS DUE TO CONCENTRATED AND DISTRIBUTED MOVING LOADS

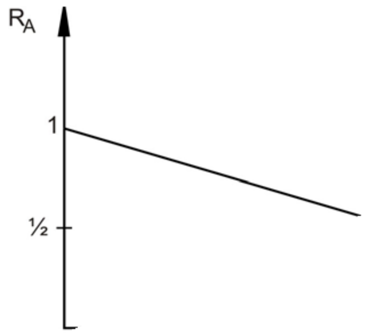
Generally in beams/girders are main load carrying components in structural systems. Hence it is necessary to construct the influence line for the reaction, shear or moment at any specified point in beam to check for criticality. Let us assume that there are two kinds of load acting on the beam. They are concentrated load and uniformly distributed load (UDL).

Concentrated load

Let us say, point load P is moving on beam from A to B. Looking at the position, we need to find out what will be the influence line for reaction B for this load. Hence, to generalize our approach, like earlier examples, let us assume that unit load is moving from A to B and influence line for reaction A can be plotted as shown in Fig. Now we want to know, if load P is at the center of span then what will be the value of reaction A? From Fig., we can find that for the load position of P, influence line of unit load gives value of 0.5. Hence, reaction A will be 0.5xP. Similarly, for various load positions and load value, reactions A can be computed.



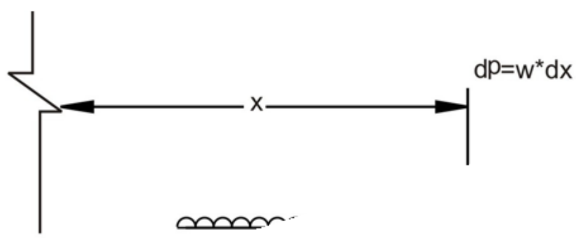
Beam structure



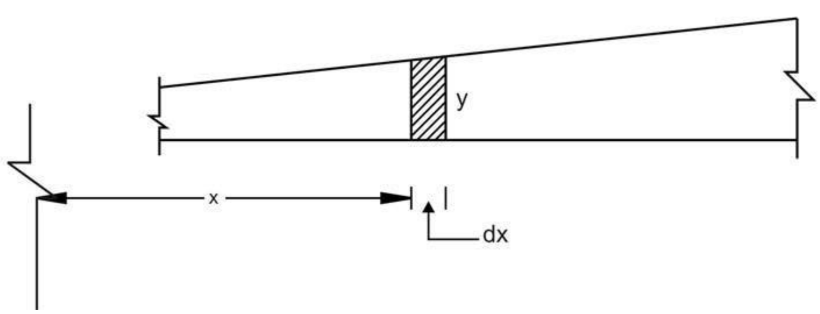
Influence line for support reaction at A

Uniformly Distributed Load

Beam is loaded with uniformly distributed load (UDL) and our objective is to find influence line for reaction A so that we can generalize the approach. For UDL of w on span, considering for segment of dx, the concentrated load dP can be given by w.dx acting at x. Let us assume that beam's influence line ordinate for some function (reaction, shear, and moment) is y as shown in Fig. In that case, the value of function is given by (dP)(y) = (w.dx).y. For computation of the effect of all these concentrated loads, we have to integrate over the entire length of the beam. Hence, we can say that it will be ∫w.y.dx = w ∫y.dx. The term ∫y.dx is equivalent to area under the influence line.

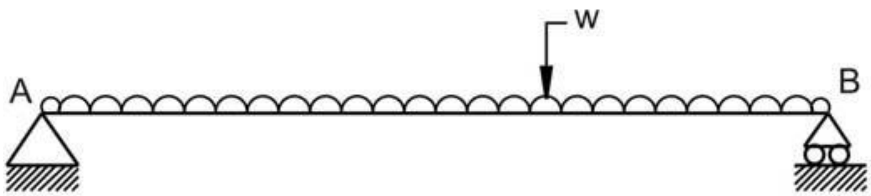


Uniformly distributed load on beam

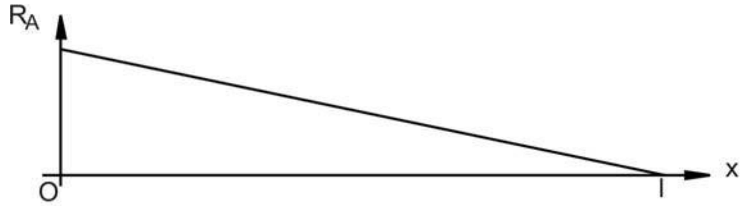


Segment of influence line diagram

For a given example of UDL on beam as shown in Fig, the influence line for reaction A can be given by area covered by the influence line for unit load into UDL value. i.e. [0.5x (1)xl] w = 0.5 w.l.

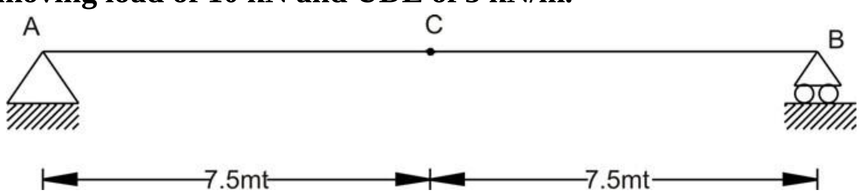


UDL on simply supported beam



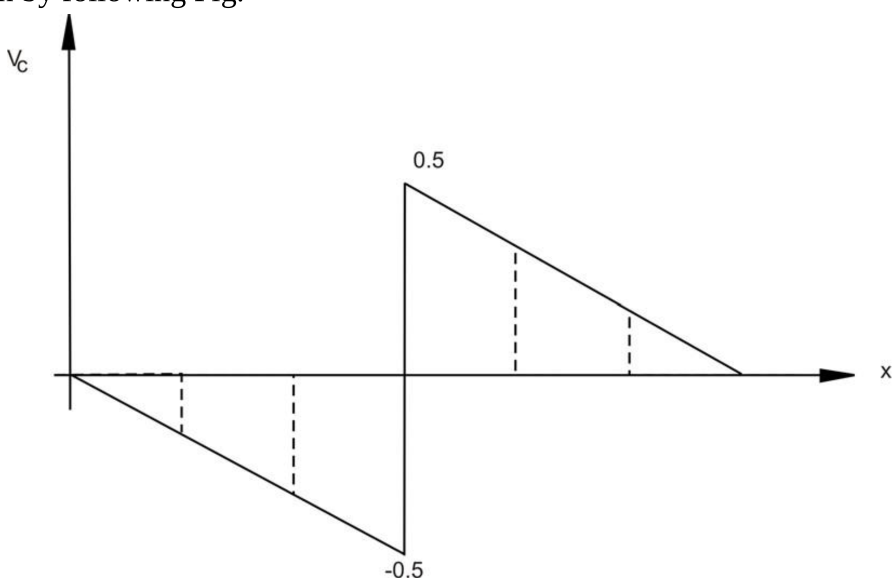
Influence line for support reaction at A.

6. Find the maximum positive live shear at point C when the beam is loaded with a concentrated moving load of 10 kN and UDL of 5 kN/m.



Simply supported beam

As discussed earlier for unit load moving on beam from A to B, the influence line for the shear at C can be given by following Fig.



Influence line for shear at section C.

Concentrated load: As shown in Fig., the maximum live shear force at C will be when the concentrated load 10 kN is located just before C or just after C. Our aim is to find positive live shear and hence, we will put 10 kN just after C. In that case,

$$V_c = 0.5 \times 10 = 5 \text{ kN.}$$

UDL: As shown in Fig., the maximum positive live shear force at C will be when the UDL 5 kN/m is acting between x = 7.5 and x = 15.

$$V_c = [0.5 \times (15 - 7.5)\ (0.5)] \times 5 = 9.375$$

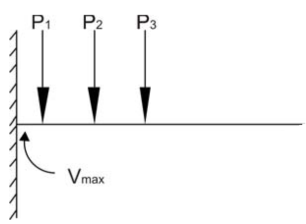
Total maximum Shear at C:

$$(V_c)\ max = 5 + 9.375 = 14.375.$$

ABSOLUTE MAXIMUM MOMENT

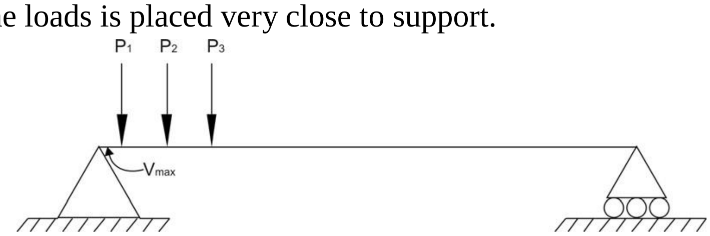
From design point of view it is necessary to know the critical location of the point in the beam and the position of the loading on the beam to find maximum shear and moment induced by the loads.

Maximum Shear: As shown in the Fig., for the cantilever beam, absolute maximum shear will occur at a point located very near to fixed end of the beam. After placing the load as close as to fixed support, find the shear at the section close to fixed end.



Absolute maximum shear case – cantilever beam

Similarly for the simply supported beam, as shown in Fig., the absolute maximum shear will occur when one of the loads is placed very close to support.



Absolute maximum shear – simply supported beam

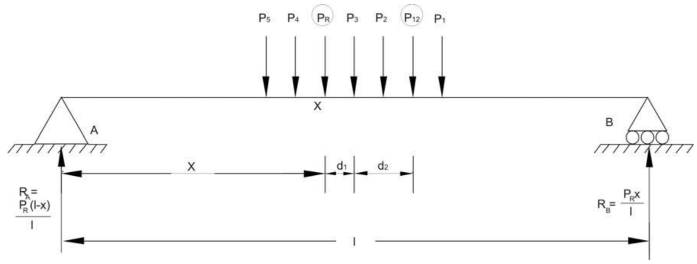
Moment:
The absolute maximum bending moment in case of cantilever beam will occur where the maximum shear has occurred, but the loading position will be at the free end as shown in Fig.

Absolute maximum moment – cantilever beam

The absolute maximum bending moment in the case of simply supported beam, one cannot obtain by direct inspection. However, we can identify position analytically. In this regard, we need to prove an important proposition.

Proposition:
When a series of wheel loads crosses a beam, simply supported ends, the maximum bending moment under any given wheel occurs when its axis and the center of gravity of the load system on span are equidistant from the center of the span.

Let us assume that load P_1, $P_{2,}$ P_3 etc. are spaced shown in Fig. and traveling from left to right. Assume P_R to be resultant of the loads, which are on the beam, located in such way that it nearer to P_3 at a distance of d_1 as shown in Fig.



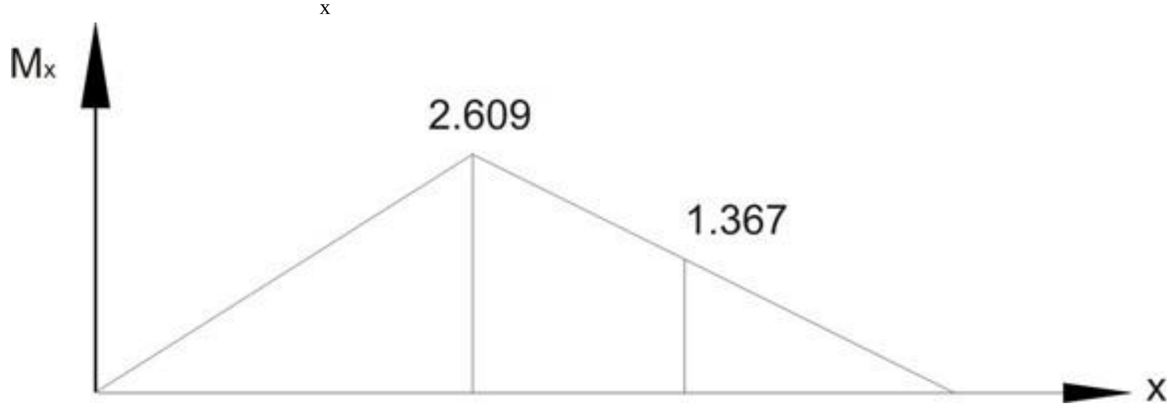
Absolute maximum moment case – simply supported beam

If P_{12} is resultant of P_1 and $P_{2,}$ and distance from P_3 is d_2.

6. The beam is loaded with two loads 25 kN each spaced at 2.5 m is traveling on the beam having span of 10 m. Find the absolute maximum moment.
When the load of 25kN and center of gravity of loads are equidistant from the center of span then absolute bending moment will occur. Hence, place the load on the beam as shown in Fig.

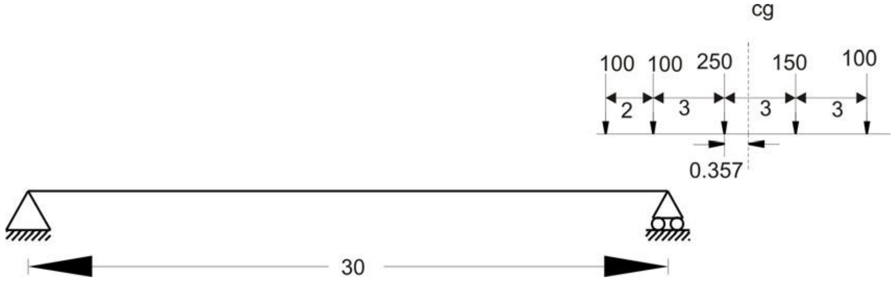
Simply supported beam

The influence line for M_x is shown in Fig.



Influence line for moment at X

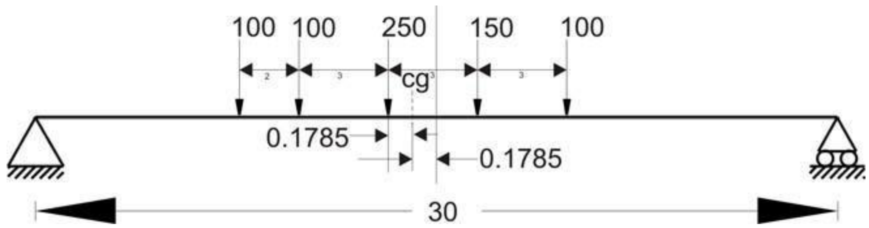
Compute the absolute maximum bending moment for the beam having span of 30 m and loaded with a series of concentrated loads moving across the span as shown in Fig.



Simply supported beam

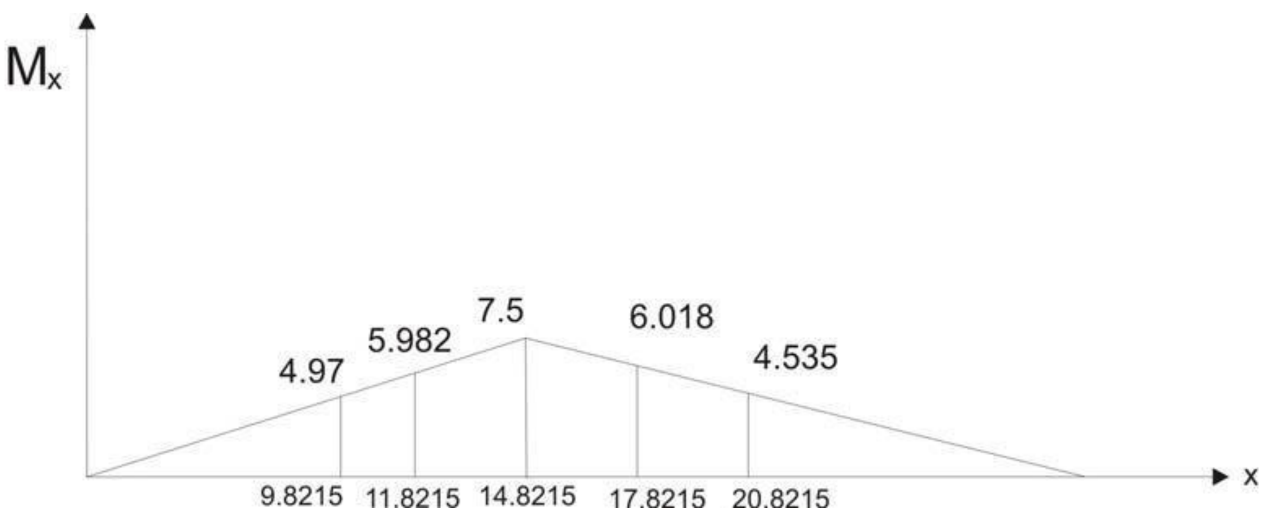
$$\frac{100(2)+ 250(5)+150(8)+ 100(11)}{100+100+250\ 150+100} = \mathbf{5.357m.}$$

Now place the loads as shown in Fig.



Simply supported beam with load positions

Also, draw the influence line as shown in Fig.

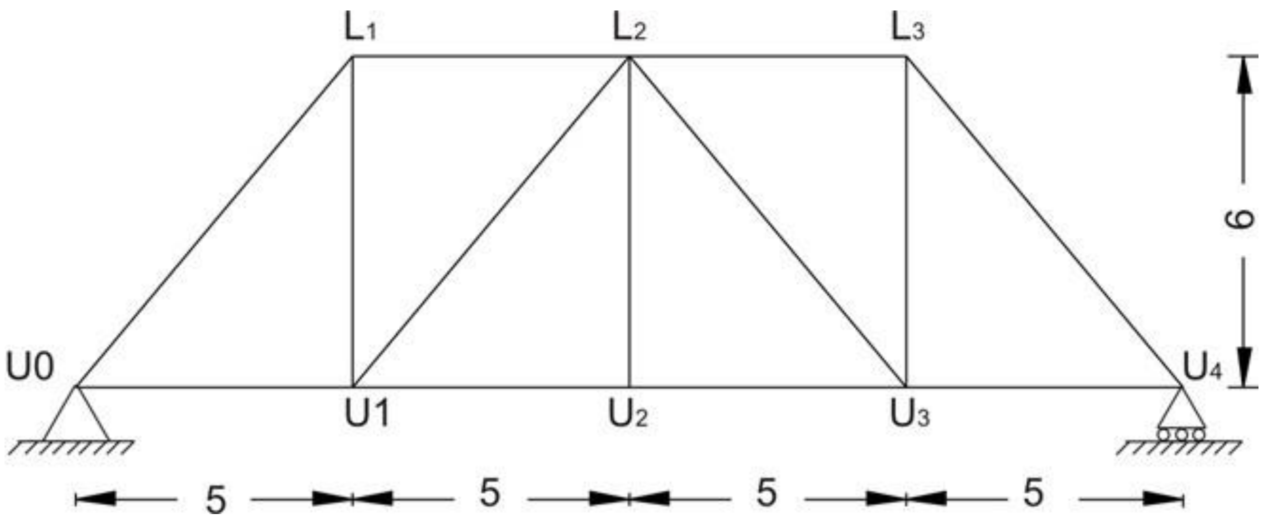


Influence Line for moment at section X

M_x = 100(4.97) + 100(5.982) + 250(7.5) + 150(6.018) + 100(4.535) = 4326.4kN.m

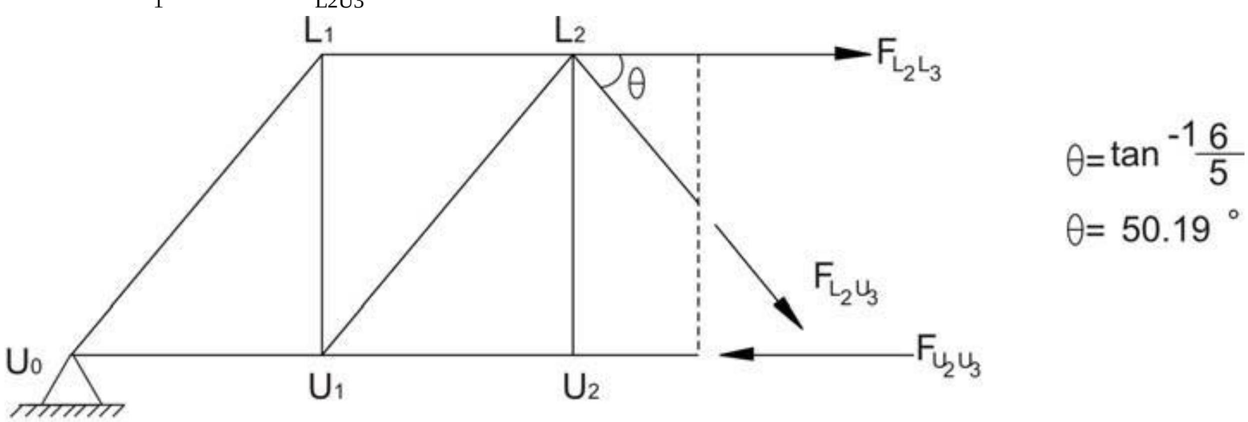
INFLUENCE LINES FOR MEMBER FORCES IN PIN JOINTED PLANE FRAMES.

7. Construct the influence line for the force in member GB of the bridge truss shown in Fig.



Bridge Truss

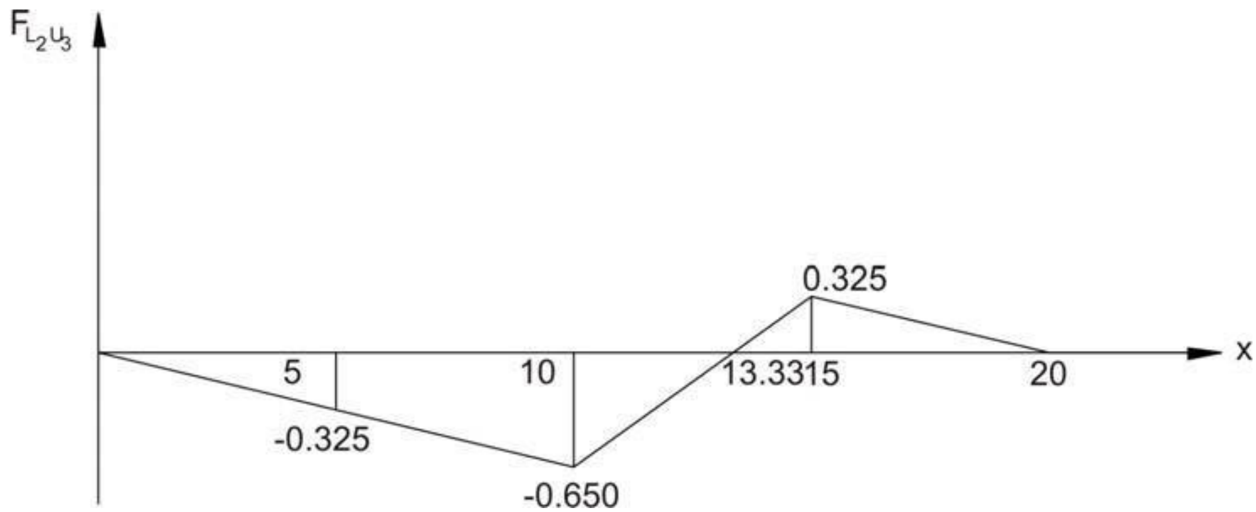
In this case, successive joints L_0, L_1, L_2, L_3, and L_4 are loaded with a unit load and the force F_{L2U3} in the member L_2U_3 are using the method of sections. Fig. shows a case where the joint load is applied at L_1 and force F_{L2U3} is calculated.



$$\sum F_y = 0 ; \quad 0.75 - 1.0 + F_{L_2U_3} \sin 50.19^\circ = 0$$

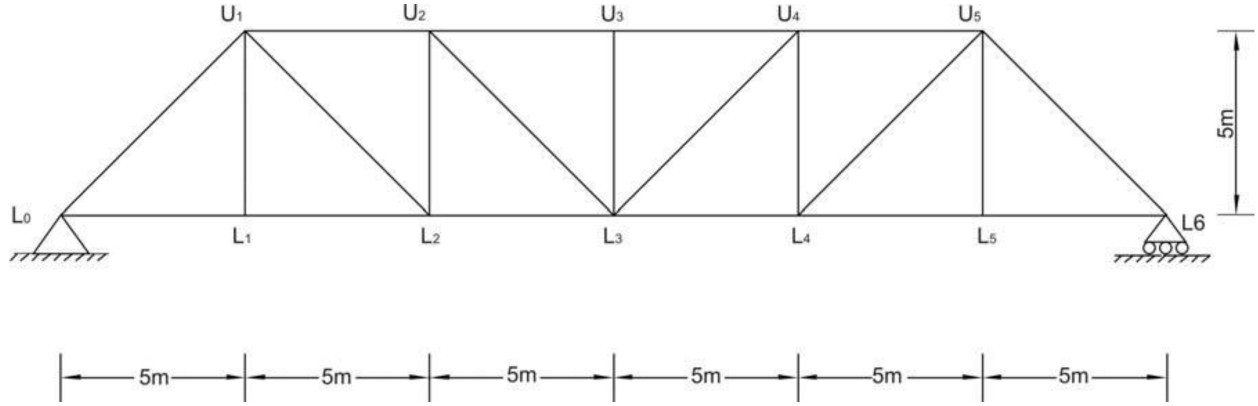
$$F_{L_2U_3} = -\,0.325$$

MEMBER FORCE F_{L2U3} CALCULATION USING METHOD OF SECTIONS.



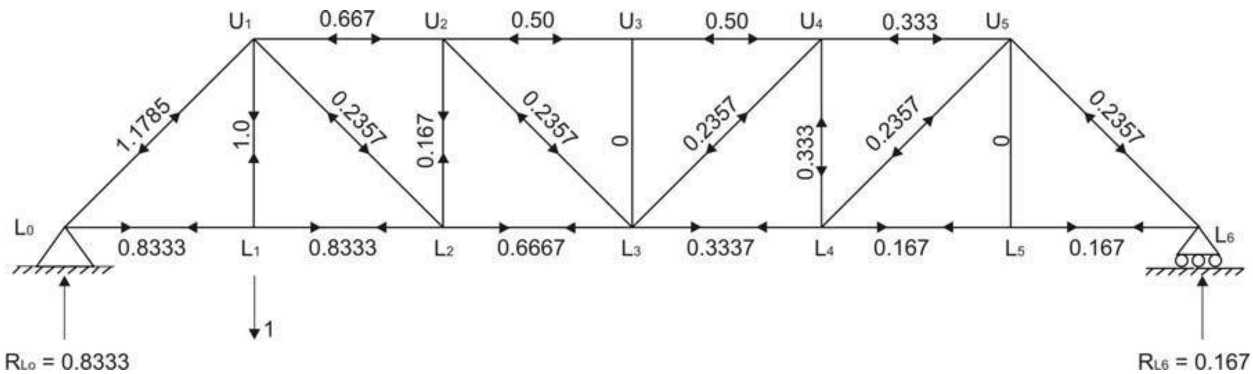
INFLUENCE LINE FOR MEMBER FORCE F_{L2U3}

8. Tabulate the influence line values for all the members of the bridge truss shown in Fig.

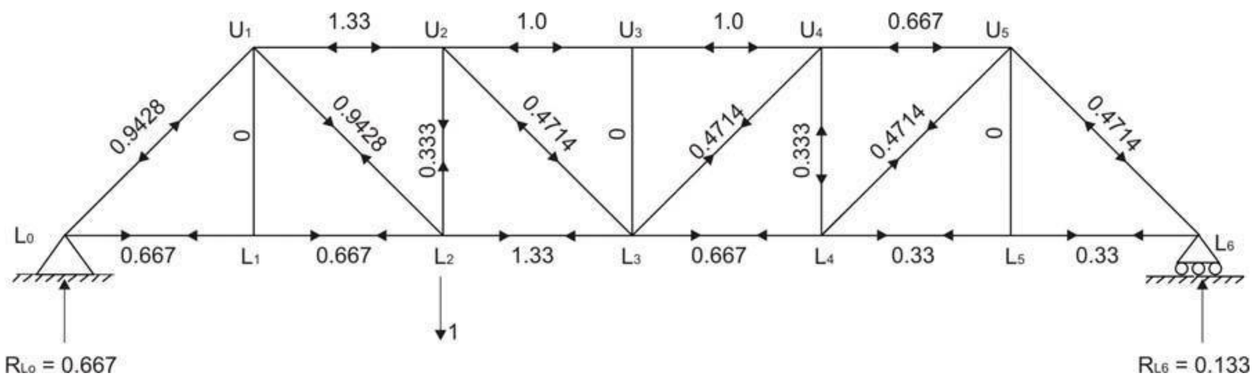


Bridge Truss

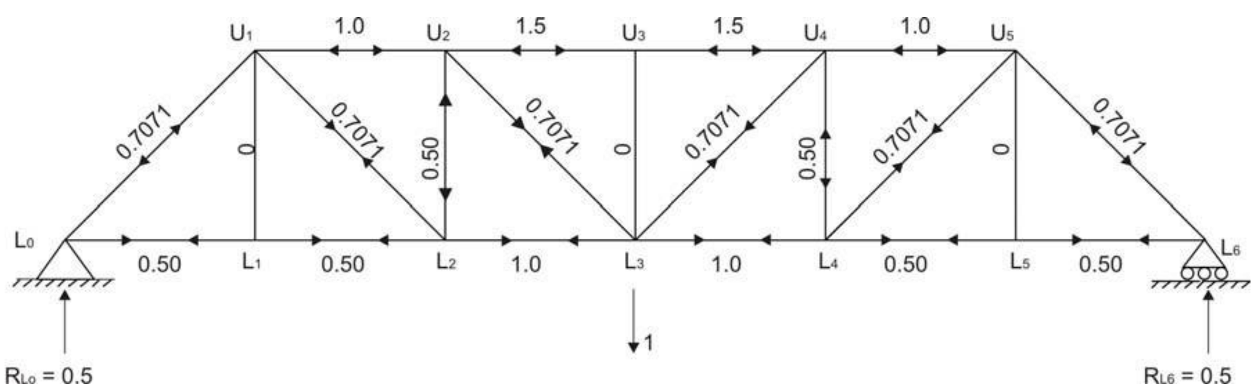
To construct the influence line for all the members of the bridge truss, hence it is necessary to place a unit load at each lower joints and find the forces in the members. Typical cases where the unit load is applied at $L_{1,}$ L_2 and L_3 are shown in Fig. and forces in the members are computed using method of joints and are tabulated below.



MEMBER FORCES CALCULATION WHEN UNIT LOAD IS APPLIED AT L_1



MEMBER FORCES CALCULATION WHEN UNIT LOAD IS APPLIED AT L_2



MEMBER FORCES CALCULATION WHEN UNIT LOAD IS APPLIED AT L_3

Member	Member force due to unit load at:						
	L_0	L_1	L_2	L_3	L_4	L_5	L_6
L_0L_1	0	0.8333	0.6667	0.5	0.3333	0.1678	0
L_1L_2	0	0.8333	0.6667	0.5	0.3333	0.1678	0
L_2L_3	0	0.6667	1.3333	1.0	0.6667	0.3336	0
L_3L_4	0	0.3336	0.6667	1.0	1.3333	0.6667	0
L_4L_5	0	0.1678	0.3333	0.5	0.6667	0.8333	0
L_5L_6	0	0.1678	0.3333	0.5	0.6667	0.8333	0
U_1U_2	0	-0.6667	-1.333	-1.0	-0.6667	-0.333	0
U_2U_3	0	-0.50	-1.000	-1.5	-1.0	-0.50	0
U_3U_4	0	-0.50	-1.000	-1,5	-1.0	-0.50	0
U_4U_5	0	-0.333	-0.6667	-1.0	-1.333	-0.6667	0
L_0U_1	0	-1.1785	-0.9428	-0.7071	-0.4714	-0.2357	0
L_1U_1	0	1	0	0	0	0	0
L_2U_1	0	-0.2357	0.9428	0.7071	0.4714	0.2357	0
L_2U_2	0	0.167	0.3333	-0.50	-0.3333	-0.3333	0
L_3U_2	0	-0.2357	-0.4714	0.7071	0.4714	0.2357	0
L_3U_3	0	0	0	0	0	0	0
L_3U_4	0	0.2357	0.4714	0.7071	-0.4714	-0.2357	0
L_4U_4	0	-03333	-0.3333	-0.50	0.3333	0.167	0
L_4U_5	0	0.2357	0.4714	0.7071	0.9428	-0.2357	0
L_5U_5	0	0	0	0	0	1	0
L_6U_5	0	-0.2357	-0.4714	-0.7071	-0.9428	-1.1785	0

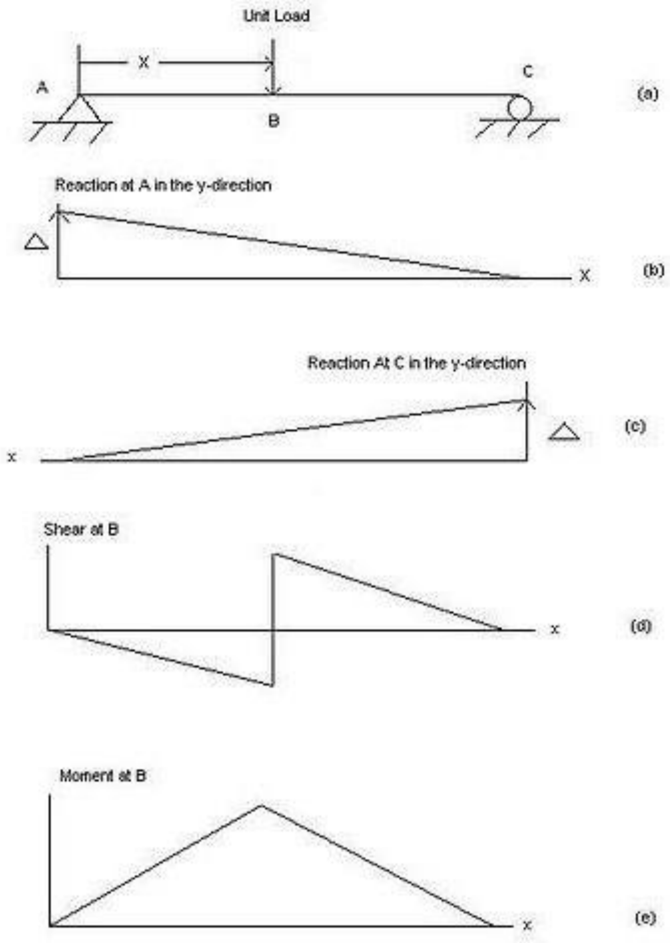
UNIT-II

MOVING LOADS AND INFLUENCE LINES

In engineering, an influence line graphs the variation of a function (such as the shear felt in a structure member) at a specific point on a beam or truss caused by a unit load placed at any point along the structure. Some of the common functions studied with influence lines include reactions (the forces that the structure's supports must apply in order for the structure to remain static), shear, moment, and deflection. Influence lines are important in the designing beams and trusses used in bridges, crane rails, conveyor belts, floor girders, and other structures where loads will move along their span. The influence lines show where a load will create the maximum effect for any of the functions studied.

Influence lines are both scalar and additive. This means that they can be used even when the load that will be applied is not a unit load or if there are multiple loads applied. To find the effect of any non-unit load on a structure, the ordinate results obtained by the influence line are multiplied by the magnitude of the actual load to be applied. The entire influence line can be scaled, or just the maximum and minimum effects experienced along the line. The scaled maximum and minimum are the critical magnitudes that must be designed for in the beam or truss.

In cases where multiple loads may be in effect, the influence lines for the individual loads may be added together in order to obtain the total effect felt by the structure at a given point. When adding the influence lines together, it is necessary to include the appropriate offsets due to the spacing of loads across the structure. For example, if it is known that load A will be three feet in front of load B, then the effect of A at x feet along the structure must be added to the effect of B at $(x - 3)$ feet along the structure—not the effect of B at x feet along the structure Many loads are distributed rather than concentrated. Influence lines can be used with either concentrated or distributed loadings. For a concentrated (or point) load, a unit point load is moved along the structure. For a distributed load of a given width, a unit-distributed load of the same width is moved along the structure, noting that as the load nears the ends and moves off the structure only part of the total load is carried by the structure. The effect of the distributed unit load can also be obtained by integrating the point load's influence line over the corresponding length of the structure.



When designing a beam or truss, it is necessary to design for the scenarios causing the maximum expected reactions, shears, and moments within the structure members in order to ensure that no member will fail during the life of the structure. When dealing with dead loads (loads that never move, such as the weight of the structure itself), this is relatively easy because the loads are easy to predict and plan for. For live loads (any load that will be moved during the life of the structure, such as furniture and people), it becomes much harder to predict where the loads will be or how concentrated or distributed they will be throughout the life of the structure.

Influence lines graph the response of a beam or truss as a unit load travels across it. The influence line allows the designers to discover quickly where to place a live load in order to calculate the maximum resulting response for each of the following functions: reaction, shear, or moment. The designer can then scale the influence line by the greatest expected load to calculate the maximum response of each function for which the beam or truss must be designed. Influence lines can also be used to find the responses of other functions (such as deflection or axial force) to the applied unit load, but these uses of influence lines is less common.

Influence Lines

The major difference between shear and moment diagrams as compared to influence lines is that shear and bending moment diagrams show the variation of the shear and the moment over the entire structure for loads at a fixed position. An influence line for shear or moment shows the variation of the function at one section cause by a moving load. Influence lines for functions of deterministic structures consist of a set of straight lines. The shape of influence lines for truss members is a bit more deceptive. What we have looked at is quantitative influence lines. These have numerical values and can be computed. Qualitative influence lines are based on a principle by Heinrich Müller Breslau, which states: *" The deflected shape of a structure represents to some scale the influence line for a function such as reaction, shear or moment, if the function in question is allowed to act through a small distance. "* In other words, is that the structure draws its own influence lines from the deflection curves. The shape of the influence lines can be created by deflecting the location in question by a moment, or shear or displacement to get idea of the behavior of the influence line. Realizing that the supports are zero values or poles.

Müller's principle for statically determinate structures is useful, but for indeterminated structures it is of great value. You can get an idea of the behavior of the shear and moment at a point in the beam.

Using influence lines to calculate values From the previous examples of a twenty foot beam for the reactions, shear, and moment. We can use the values from the influence lines to calculate the shear and moment at a point.

R_{Ay} = $\sum$ (F_i)* Value of the influence line of R_{Ay} @ location of the force

V_{11} = $\sum$ (F_i)* Value of the influence line of V_{11} @ location of the force

M_{11} = $\sum$ (F_i)* Value of the influence line of M_{11} @ location of the force

If we are looking at the forces due to uniform loads over the beam at point. The shear or moment is equal to the area under the influence line times the distributed load.

R_{Ay} = $\sum$ (w_i)* Area of the influence line of R_{Ay} over which w covers

V_{11} = $\sum$ (w_i)* Area of the influence line of V_{11} over which w covers

M_{11} = $\sum$ (w_i)* Area of the influence line of M_{11} over which w covers

For moving set of loads the influence lines can be used to calculate the maximum function. This can be done by moving the loads over the influence line find where they will generate the largest value for the particular point. Panels or floating floor

The method can be extend to deal with floor joist and floating floors in which we deal with panels, which are simple beam elements acting on the floor joist. You will need to find the fore as function of the intersection. You are going to find the moment and the shear as you move across the surface of the beam.

An example problem is used to show how this can be used to find the shear and moment at a point for a moving load. This technique is similar to that used in truss members.

Methods for constructing influence lines

There are three methods used for constructing the influence line. The first is to tabulate the influence values for multiple points along the structure, then use those points to create the influence line. The second is to determine the influence-line equations that apply to the structure, thereby solving for all points along the influence line in terms of *x*, where *x* is the number of feet from the start of the structure to the point where the unit load is applied. The third method is called the Müller-Breslau principle. It creates a qualitative influence line. This influence line will still provide the designer with an accurate idea of where the unit load will produce the largest response of a function at the point being studied, but it cannot be used directly to calculate what the magnitude that response will be, whereas the influence lines produced by the first two methods can.

Influence-line equations

It is possible to create equations defining the influence line across the entire span of a structure. This is done by solving for the reaction, shear, or moment at the point A caused by a unit load placed at *x* feet along the structure instead of a specific distance. This method is similar to the tabulated values method, but rather than obtaining a numeric solution, the outcome is an equation in terms of *x*. It is important to understanding where the slope of the influence line changes for this method because the influence-line equation will change for each linear section of the influence line. Therefore, the complete equation will be a piecewise linear function which has a separate influence-line equation for each linear section of the influence line.

Müller-Breslau Principle

The Müller-Breslau Principle can be utilized to draw qualitative influence lines, which are directly proportional to the actual influence line.” Instead of moving a unit load along a beam, the Müller-Breslau Principle finds the deflected shape of the beam caused by first releasing the beam at the point being studied, and then applying the function (reaction, shear, or moment) being studied to that point. The principle states that the influence line of a function will have a scaled shape that is the same as the deflected shape of the beam when the beam is acted upon by the function.

In order to understand how the beam will deflect under the function, it is necessary to remove the beam's capacity to resist the function. Below are explanations of how to find the influence lines of a simply supported, rigid beam

When determining the reaction caused at a support, the support is replaced with a roller, which cannot resist a vertical reaction. Then an upward (positive) reaction is applied to the point where the support was. Since the support has been removed, the beam will rotate upwards, and since the beam is rigid, it will create a triangle with the point at the second support. If the beam extends beyond the second support as a cantilever, a similar triangle will be formed below the cantilevers position. This means that the reaction's influence line will be a straight, sloping line with a value of zero at the location of the second support.

When determining the shear caused at some point B along the beam, the beam must be cut and a roller-guide (which is able to resist moments but not shear) must be inserted at point B. Then, by applying a positive shear to that point, it can be seen that the left side will rotate down, but the right side will rotate up. This creates a discontinuous influence line which reaches zero at the supports and whose slope is equal on either side of the discontinuity. If point B is at a support, then the deflection between point B and any other supports will still create a triangle, but if the beam is cantilevered, then the entire cantilevered side will move up or down creating a rectangle.

When determining the moment caused by at some point B along the beam, a hinge will be placed at point B, releasing it to moments but resisting shear. Then when a positive moment is placed at point B, both sides of the beam will rotate up. This will create a continuous influence line, but the slopes will be equal and opposite on either side of the hinge at point B. Since the beam is simply supported, its end supports (pins) cannot resist moment; therefore, it can be observed that the supports will never experience moments in a static situation regardless of where the load is placed.

The Müller-Breslau Principle can only produce qualitative influence lines. This means that engineers can use it to determine where to place a load to incur the maximum of a function, but the magnitude of that maximum cannot be calculated from the influence line. Instead, the engineer must use statics to solve for the functions value in that loading case.

For example, the influence line for the support reaction at A of the structure shown in Figure 1, is found by applying a unit load at several points (See Figure 2) on the structure and determining what the resulting reaction will be at A. This can be done by solving the support reaction Y_A as a function of the position of a downward acting unit load. One such equation can be found by summing moments at Support B.

Figure 1 - Beam structure for influence line example

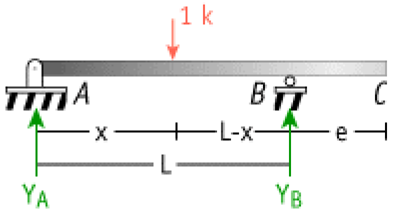


Figure 2 - Beam structure showing application of unit load

$\Sigma M_B = 0$ *(Assume counter-clockwise positive moment)*

$-Y_A(L)+1(L-x) = 0$

$Y_A = (L-x)/L = 1 - (x/L)$

The graph of this equation is the influence line for the support reaction at A (See Figure 3). The graph illustrates that if the unit load was applied at A, the reaction at A would be equal to unity. Similarly, if the unit load was applied at B, the reaction at A would be equal to 0, and if the unit load was applied at C, the reaction at A would be equal to -e/L.

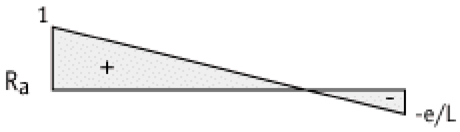


Figure 3 - Influence line for the support reaction at A

Once an understanding is gained on how these equations and the influence lines they produce are developed, some general properties of influence lines for statically determinate structures can be stated.

1. For a statically determinate structure the influence line will consist of only straight line segments between critical ordinate values.
2. The influence line for a shear force at a given location will contain a translational discontinuity at this location. The summation of the positive and negative shear forces at this location is equal to unity.
3. Except at an internal hinge location, the slope to the shear force influence line will be the same on each side of the critical section since the bending moment is continuous at the critical section.

4. The influence line for a bending moment will contain a unit rotational discontinuity at the point where the bending moment is being evaluated.
5. To determine the location for positioning a single concentrated load to produce maximum magnitude for a particular function (reaction, shear, axial, or bending moment) place the load at the location of the maximum ordinate to the influence line. The value for the particular function will be equal to the magnitude of the concentrated load, multiplied by the ordinate value of the influence line at that point.
6. To determine the location for positioning a uniform load of constant intensity to produce the maximum magnitude for a particular function, place the load along those portions of the structure for which the ordinates to the influence line have the same algebraic sign. The value for the particular function will be equal to the magnitude of the uniform load, multiplied by the area under the influence diagram between the beginning and ending points of the uniform load.

There are two methods that can be used to plot an influence line for any function. In the first, the approach described above, is to write an equation for the function being determined, e.g., the equation for the shear, moment, or axial force induced at a point due to the application of a unit load at any other location on the structure. The second approach, which uses the Müller Breslau Principle, can be utilized to draw qualitative influence lines, which are directly proportional to the actual influence line.

The following examples demonstrate how to determine the influence lines for reactions, shear, and bending moments of beams and frames using both methods described above.

For example, the influence line for the support reaction at A of the structure shown in Figure 1, is found by applying a unit load at several points (See Figure 2) on the structure and determining what the resulting reaction will be at A. This can be done by solving the support reaction Y_A as a function of the position of a downward acting unit load. One such equation can be found by summing moments at Support B.

Figure 1 - Beam structure for influence line example

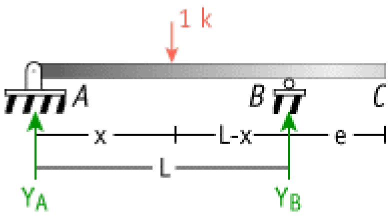


Figure 2 - Beam structure showing application of unit load

$\Sigma M_B = 0$ *(Assume counter-clockwise positive moment)*

$-Y_A(L)+1(L-x) = 0$

$Y_A = (L-x)/L = 1 - (x/L)$

The graph of this equation is the influence line for the support reaction at A (See Figure 3). The graph illustrates that if the unit load was applied at A, the reaction at A would be equal to unity. Similarly, if the unit load was applied at B, the reaction at A would be equal to 0, and if the unit load was applied at C, the reaction at A would be equal to -e/L.

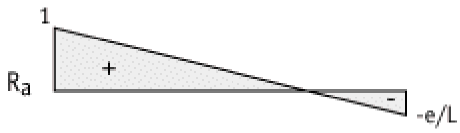


Figure 3 - Influence line for the support reaction at A

PROBLEM

Draw the influence lines for the reactions Y_A, Y_C, and the shear and bending moment at point B, of the simply supported beam shown by developing the equations for the respective influence lines.

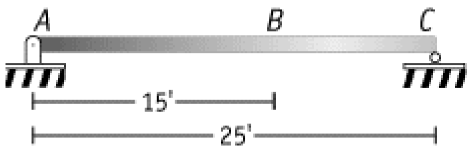


Figure 1 - Beam structure to analyze

- ### Reaction Y_A

The influence line for a reaction at a support is found by independently applying a unit load at several points on the structure and determining, through statics, what the resulting reaction at the support will be for each case. In this example, one such equation for the influence line of Y_A can be found by summing moments around Support C.

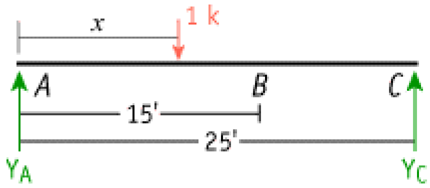


Figure 2 - Application of unit load

$\Sigma M_C = 0$ *(Assume counter-clockwise positive moment)*

$-Y_A(25)+1(25-x) = 0$

$Y_A = (25-x)/25 = 1 - (x/25)$

The graph of this equation is the influence line for Y_A (See Figure 3). This figure illustrates that if the unit load is applied at A, the reaction at A will be equal to unity. Similarly, if the unit load is applied at B, the reaction at A will be equal to 1-(15/25)=0.4, and if the unit load is applied at C, the reaction at A will be equal to 0.

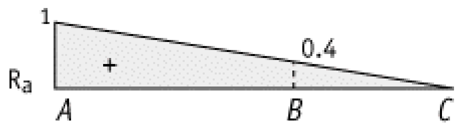


Figure 3 - Influence line for Y_A, the support reaction at A

The fact that Y_A=1 when the unit load is applied at A and zero when the unit load is applied at C can be used to quickly generate the influence line diagram. Plotting these two values at A and C, respectively, and connecting them with a straight line will yield the the influence line for Y_A. The structure is statically determinate, therefore, the resulting function is a straight line.

Reaction at C

The equation for the influence line of the support reaction at C is found by developing an equation that relates the reaction to the position of a downward acting unit load applied at all locations on the structure. This equation is found by summing the moments around support A.

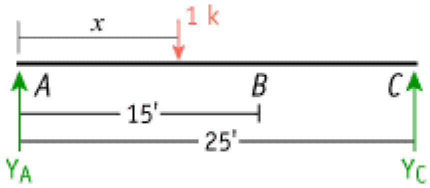


Figure 4 - Application of unit load

$\Sigma M_A = 0$ *(Assume counter-clockwise positive moment)*

$Y_C(25)-1(x) = 0$

$Y_C = x/25$

The graph of this equation is the influence line for Y_C. This shows that if the unit load is applied at C, the reaction at C will be equal to unity. Similarly, if the unit load is applied at B, the reaction at C will be equal to 15/25=0.6. And, if the unit load is applied at A, the reaction at C will equal to 0.

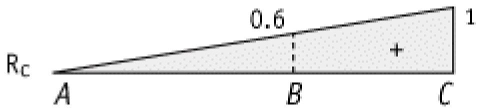


Figure 5 - Influence line for the reaction at support C

The fact that Y_C=1 when the unit load is applied at C and zero when the unit load is applied at A can be used to quickly generate the influence line diagram. Plotting these two values at A and C, respectively, and connecting them with a straight line will yield the the influence line for Y_C. Notice, since the structure is statically determinate, the resulting function is a straight line.

- Shear at B

The influence line for the shear at point B can be found by developing equations for the shear at the section using statics. This can be accomplished as follows:

a) if the load moves from B to C, the shear diagram will be as shown in Fig. 6 below, this demonstrates that the shear at B will equal Y_A as long as the load is located to the right of B, i.e., $V_B = Y_A$. One can also calculate the shear at B from the Free Body Diagram (FBD) shown in Fig. 7.

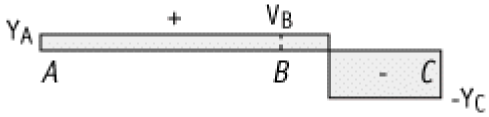


Figure 6 - Shear diagram for load located between B and C

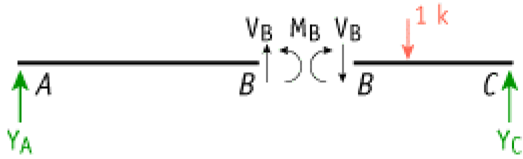


Figure 7 - Free body diagram for section at B with a load located between B and C

b) if the load moves from A to B, the shear diagram will be as shown in Fig. 8, below, this demonstrates that the shear at B will equal $-Y_C$ as long as the load is located to the left of B, i.e., $V_B = - Y_C$. One can also calculate the shear at B from the FBD shown in Fig. 9.

Figure 8 - Shear diagram for load located between A and B

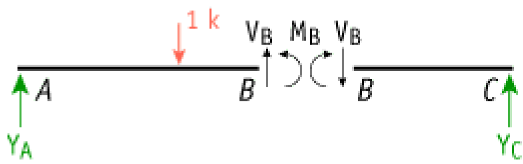


Figure 9 - Free body diagram for section at B with a load located between A and B

The influence line for the Shear at point B is then constructed by drawing the influence line for Y_A and negative Y_C. Then highlight the portion that represents the sides over which the load was moving. In this case, highlight the the part from B to C on Y_A and from A to B on $-Y_C$. Notice that at point B, the summation of the absolute values of the positive and negative shear is equal to 1.

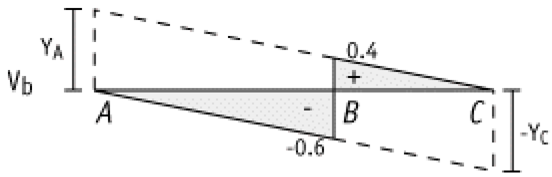


Figure 10 - Influence line for shear at point B

- Moment at B

The influence line for the moment at point B can be found by using statics to develop equations for the moment at the point of interest, due to a unit load acting at any location on the structure. This can be accomplished as follows.

a) if the load is at a location between B and C, the moment at B can be calculated by using the FBD shown in Fig. 7 above, e.g., at B, $M_B = 15\ Y_A$ - notice that this relation is valid if and only if the load is moving from B to C.

b) if the load is at a location between A and B, the moment at B can be calculated by using the FBD shown in Fig. 9 above, e.g., at B, $M_B = 10\ Y_C$ - notice that this relation is valid if and only if the load is moving from A to B.

The influence line for the Moment at point B is then constructed by magnifying the influence lines for Y_A and Y_C by 15 and 10, respectively, as shown below. Having plotted the functions, 15 Y_A and 10 Y_C, highlight the portion from B to C of the function 15 Y_A and from A to B on the function 10 Y_C. These are the two portions what correspond to the correct moment relations as explained above. The two functions must intersect above point B. The value of the function at B then equals (1 x 10 x 15)/25 = 6. This represents the moment at B if the load was positioned at B.

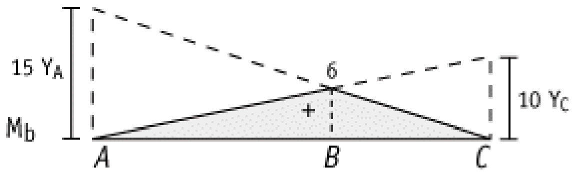


Figure 11 - Influence line for moment at point B

InfluenceLines

Qualitative Influence Lines using the Müller Breslau Principle

- Müller Breslau Principle

The Müller Breslau Principle is another alternative available to qualitatively develop the influence lines for different functions. The Müller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

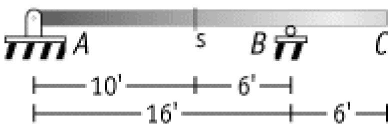


Figure 1 - Beam structure to analyze

For example, to obtain the influence line for the support reaction at A for the beam shown in Figure 1, above, remove the support corresponding to the reaction and apply a force in the positive direction that will cause a unit displacement in the direction of Y_A. The resulting deflected shape will be proportional to the true influence line for this reaction. i.e., for the support reaction at A. The deflected shape due to a unit displacement at A is shown below. Notice that the deflected shape is linear, i.e., the beam rotates as a rigid body without any curvature. This is true only for statically determinate systems.

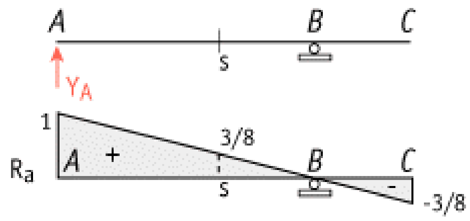


Fig. 2 -Support removed, unit load applied, and resulting influence line for support reaction at A

Similarly, to construct the influence line for the support reaction Y_B, remove the support at B and apply a vertical force that induces a unit displacement at B. The resulting deflected shape is the qualitative influence line for the support reaction Y_B.

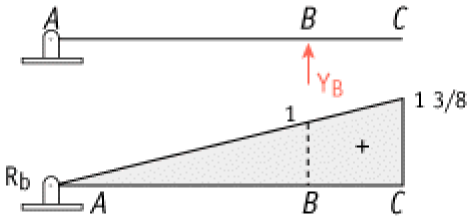


Fig. 3 - Support removed, unit load applied, and resulting influence line for support reaction at B

Once again, notice that the influence line is linear, since the structure is statically determinate.

This principle will be now be extended to develop the influence lines for other functions.

- Shear at s

To determine the qualitative influence line for the shear at s, remove the shear resistance of the beam at this section by inserting a roller guide, i.e., a support that does not resist shear, but maintains axial force and bending moment resistance.

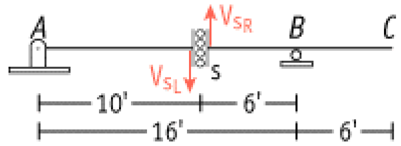


Figure 4 - Structure with shear capacity removed at s

Removing the shear resistance will then allow the ends on each side of the section to move perpendicular to the beam axis of the structure at this section. Next, apply a shear force, i.e., V_{s-R} and V_{s-L} that will result in the relative vertical displacement between the two ends to equal unity. The magnitude of these forces are proportional to the location of the section and the span of the beam. In this case,

V_{s-L} = 1/16 x 10 = 10/16 = 5/8

V_{s-R} = 1/16 x 6 = 6/16 = 3/8

The final influence line for V_s is shown below.

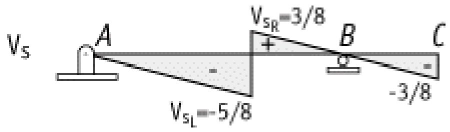


Figure 5 - Influence line for shear at s

- Shear just to the left side of B

The shear just to the left side of support B can be constructed using the ideas explained above. Simply imagine that section s in the previous example is moved just to the left of B. By doing this, the magnitude of the positive shear decreases until it reaches zero, while the negative shear increases to 1.

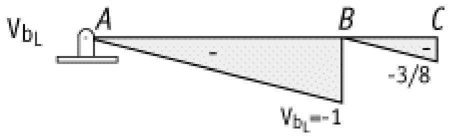


Figure 6 - Influence line for shear just to the left of B

- Shear just to the right side of B

To plot the influence line for the shear just to the right side of support B, $V_{b\text{-}R}$, release the shear just to the right of the support by introducing the type of roller shown in Fig. 7, below. The resulting deflected shape represents the influence line for $V_{b\text{-}R}$. Notice that no deflection occurs between A and B, since neither of those supports were removed and hence the deflections at A and B must remain zero. The deflected shape between B and C is a straight line that represents the motion of a rigid body.

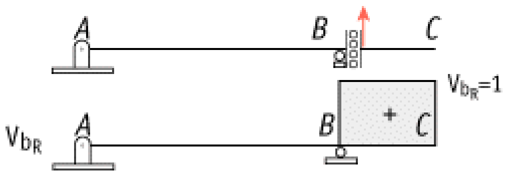


Figure 7 - Structure with shear capacity removed at just to the right of B and the resulting influence line

- Moment at s

To obtain a qualitative influence line for the bending moment at a section, remove the moment restraint at the section, but maintain axial and shear force resistance. The moment resistance is eliminated by inserting a hinge in the structure at the section location. Apply equal and opposite moments respectively on the right and left sides of the hinge that will introduce a unit relative rotation between the two tangents of the deflected shape at the hinge. The corresponding elastic curve for the beam, under these conditions, is the influence line for the bending moment at the section. The resulting influence line is shown below.

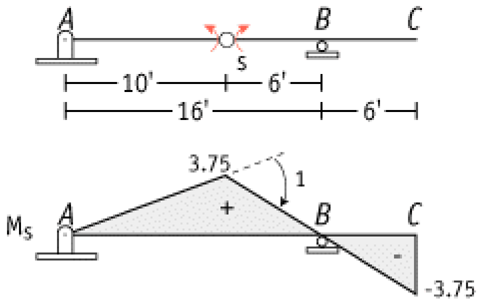


Figure 8 - Structure with moment capacity removed at s and the resulting influence line

The values of the moments shown in Figure 8, above, are calculated as follows:

a. when the unit load is applied at s, the moment at s is Y_A x 10 = 3/8 x 10 = 3.75 (see the influence line for Y_A, Figure 2, above, for the value of Y_A with a unit load applied at s)
b. when the unit load is applied at C, the moment at s is Y_A x 10 = -3/8 x 10 = -3.75 (again, see the influence line for Y_A for the value of Y_A with a unit load applied at C)

Following the general properties of influence lines, given in the Introduction, these two values are plotted on the beam at the locations where the load is applied and the resulting influence line is constructed.

- Moment at B

The qualitative influence line for the bending moment at B is obtained by introducing a hinge at support B and applying a moment that introduces a unit relative rotation. Notice that no deflection occurs between supports A and B since neither of the supports were removed. Therefore, the only portion that will rotate is part BC as shown in Fig. 9, below.

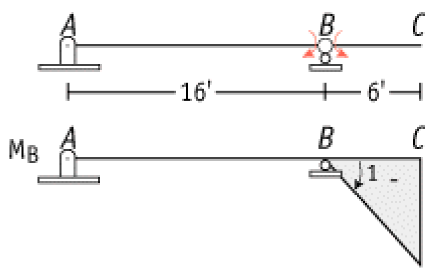


Figure 9 - Structure with moment capacity removed at B and the resulting influence line

- Shear and moment envelopes due to uniform dead and live loads

The shear and moment envelopes are graphs which show the variation in the minimum and maximum values for the function along the structure due to the application of all possible loading conditions. The diagrams are obtained by superimposing the individual diagrams for the function based on each loading condition. The resulting diagram that shows the upper and lower bounds for the function along the structure due to the loading conditions is called the envelope.

The loading conditions, also referred to as load cases, are determined by examining the influence lines and interpreting where loads must be placed to result in the maximum values. To calculate the maximum positive and negative values of a function, the dead load must be applied over the entire beam, while the live load is placed over either the respective positive or negative portions of the influence line. The value for the function will be equal to the magnitude of the uniform load, multiplied by the area under the influence line diagram between the beginning and ending points of the uniform load.

For example, to develop the shear and moment envelopes for the beam shown in Figure 1, first sketch the influence lines for the shear and moment at various locations. The influence lines for $V_{a\text{-}R}$, $V_{b\text{-}L}$, $V_{b\text{-}R}$, M_b, V_s, and M_s are shown in Fig. 10.

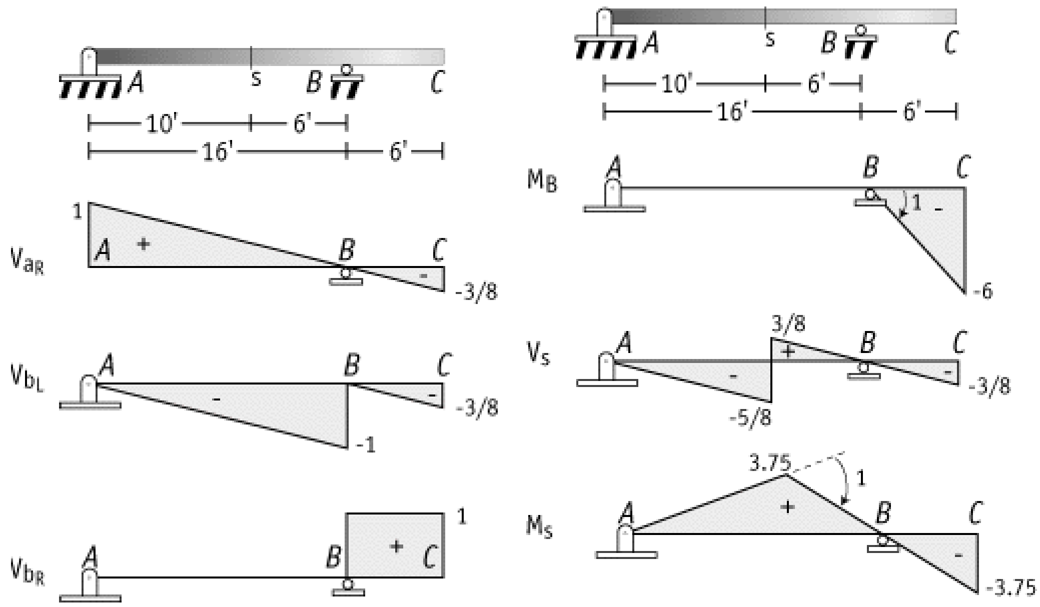


Figure 10 - Influence lines

These influence lines are used to determine where to place the uniform live load to yield the maximum positive and negative values for the different functions. For example;

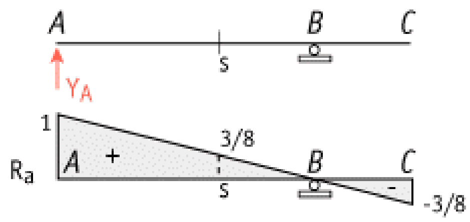


Fig. 11 - Support removed, unit load applied, and resulting influence line for support reaction at A

 The maximum value for the positive reaction at A, assuming no partial loading, will occur when the uniform load is applied on the beam from A to B (load case 1)

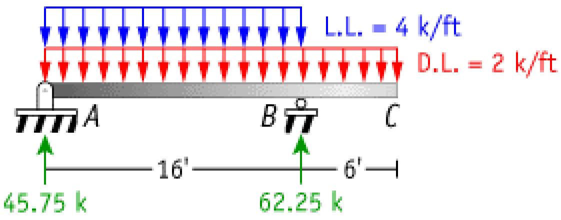


Figure 12 - Load case 1

 The maximum negative value for the reaction at A will occur if a uniform load is placed on the beam from B to C (load case 2)

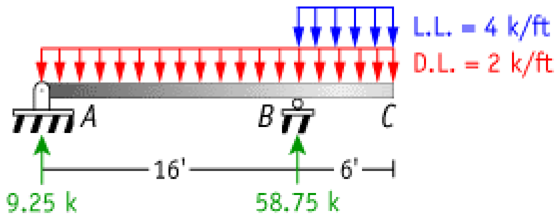


Figure 13 - Load case 2

 Load case 1 is also used for:

- maximum positive value of the shear at the right of support A
- maximum positive moment M_s

 Load case 2 is also used for:

- maximum positive value of the shear at the right of support B
- maximum negative moments at support B and M_s

 Load case 3 is required for:

- maximum positive reaction at B
- maximum negative shear on the left side of B

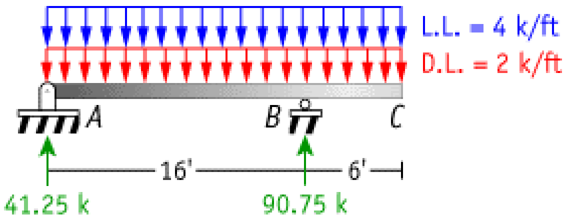


Figure 14 - Load case 3

 Load case 4 is required for the maximum positive shear force at section s

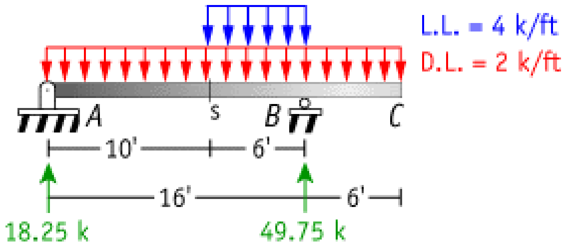


Figure 15 - Load case 4

 Load case 5 is required for the maximum negative shear force at section s

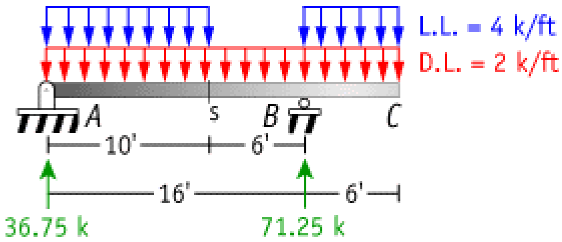


Figure 16 - Load case 5

To develop the shear and moment envelopes, construct the shear and moment diagrams for each load case. The envelope is the area that is enclosed by superimposing all of these diagrams. The maximum positive and negative values can then be determined by looking at the maximum and minimum values of the envelope at each point.

Individual shear diagrams for each load case;

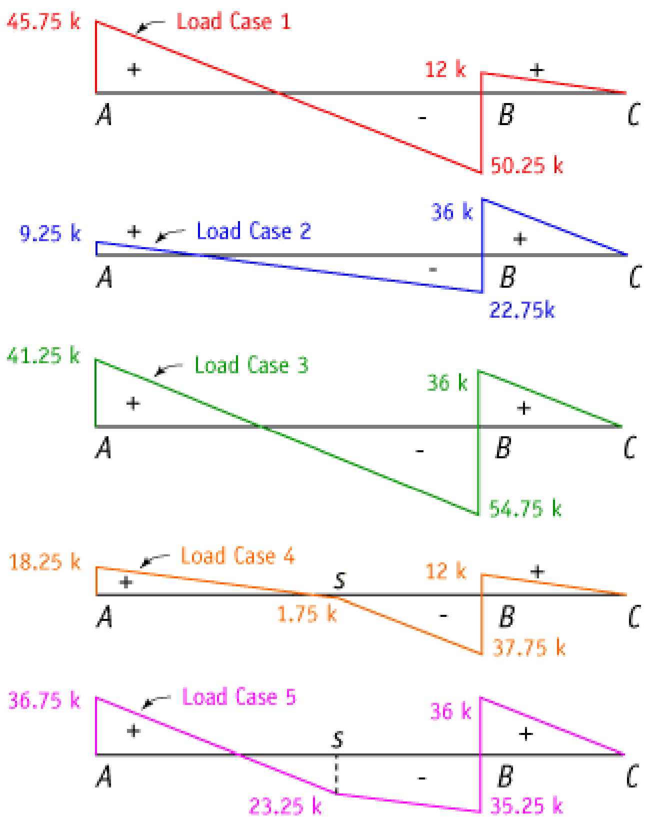


Figure 17 - Individual shear diagrams

Superimpose all of these diagrams together to determine the final shear envelope.

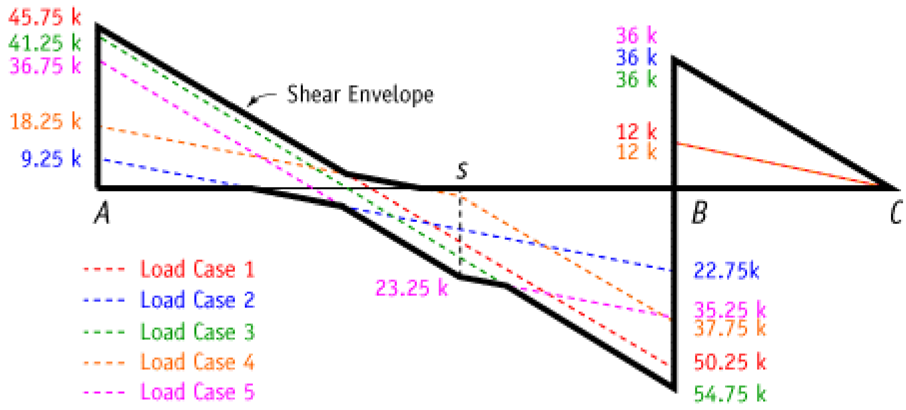


Figure 18 - Resulting superimposed shear envelope

Individual moment diagrams for each load case;

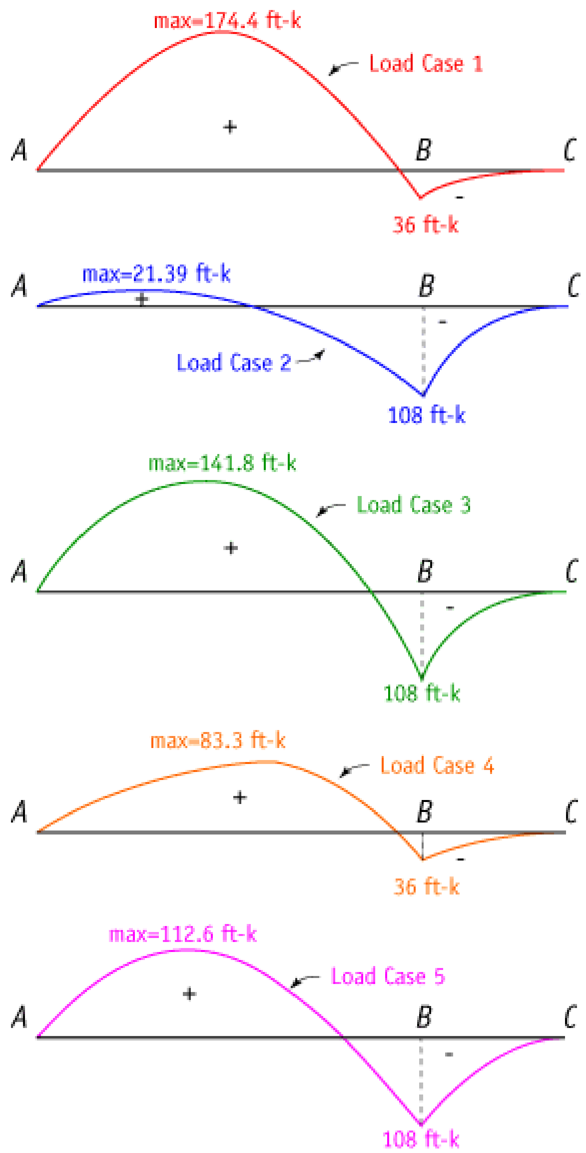


Figure 19 - Individual moment diagrams

Superimpose all of these diagrams together to determine the final moment envelope.

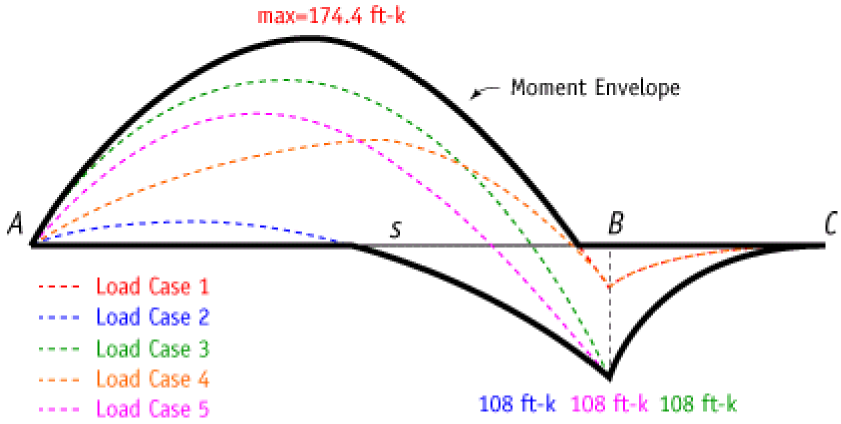


Figure 20 - Resulting superimposed moment envelope

Qualitative Influence Lines for a Statically Determinate Continuous Beam

problem

Draw the qualitative influence lines for the vertical reactions at the supports, the shear and moments at sections s1 and s2, and the shear at the left and right of support B of the continuous beam shown.

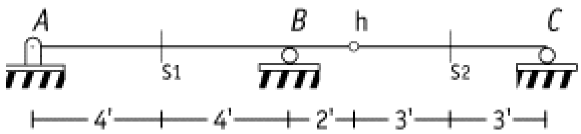


Figure 1 - Beam structure to analyze

- Reactions at A, B, and C

Qualitative influence lines for the support reactions at A, B, and C are found by using the Müller Breslau Principle for reactions, i.e., apply a force which will introduce a unit displacement in the structure at each support. The resulting deflected shape will be proportional to the influence line for the support reactions.

The resulting influence lines for the support reactions at A, B, and C are shown in Figure 2, below.

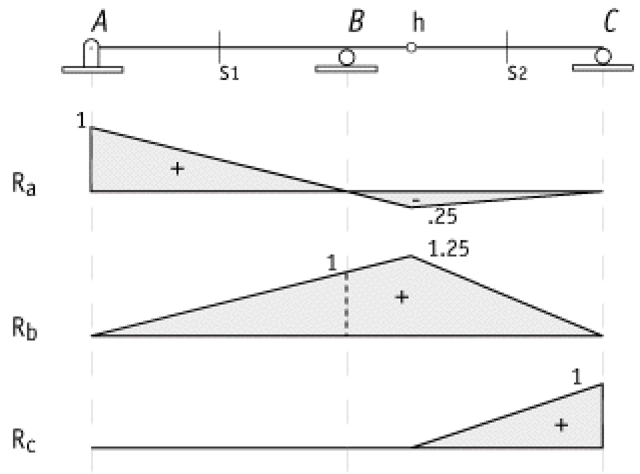


Figure 2 - Influence lines for the reactions at A, B, and C

Note: Beam BC does not experience internal forces or reactions when the load moves from A to h. In other words, influence lines for beam hC will be zero as long as the load is located between A and h. This can also be explained by the fact that portion hC of the beam is supported by beam ABh as shown in Figure 3, below.

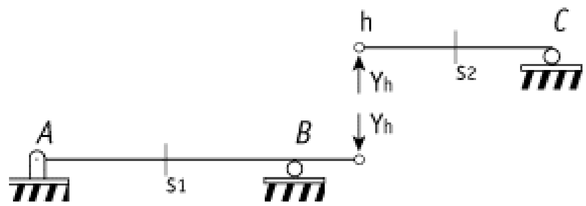


Figure 3 - Beam hC supported by beam ABh

Therefore, the force Y_h required to maintain equilibrium in portion hC when the load from h to C is provided by portion ABh. This force, Y_h, is equal to zero when the load moves between A an h, and hence, no shear or moment will be induced in portion hC.

- Shear and moment at section S_1 and S_2

To determine the shear at s_1, remove the shear resistance of the beam at the section by inserting a support that does not resist shear, but maintains axial force and bending moment resistance (see the inserted support in Figure 4). Removing the shear resistance will allow the ends on each side of the section to move perpendicular to the beam axis of the structure at this section. Next, apply shear forces on each side of the section to induce a relative displacement between the two ends that will equal unity. Since the section is cut at the midspan, the magnitude of each force is equal to 1/2.

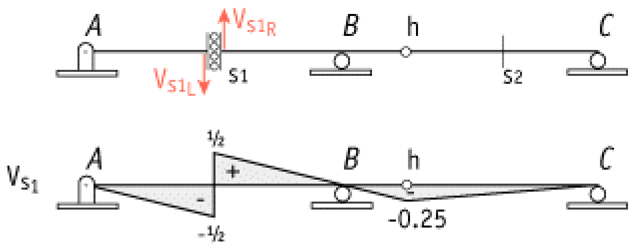


Figure 4 - Structure with shear capacity removed at s1 and resulting influence line

For the moment at s_1, remove the moment restraint at the section, but maintain axial and shear force resistance. The moment resistance is eliminated by inserting a hinge in the structure at the section location. Apply equal and opposite moments on the right and left sides of the hinge that will introduce a unit relative rotation between the two tangents of the deflected shape at the hinge. The corresponding elastic curve for the beam, under these conditions, is the influence line for the bending moment at the section.

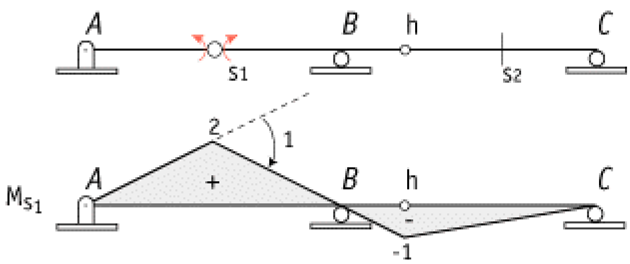


Figure 5 - Structure with moment capacity removed at s1 and resulting influence line

The value of the moment shown in Figure 5, above, is equal to the value of R_a when a unit load is applied at s_1, multiplied by the distance from A to s_1. M_{s1} = 1/2 x 4 = 2.

The influence lines for the shear and moment at section s_2 can be constructed following a similar procedure. Notice that when the load is located between A and h, the magnitudes of the influence lines are zero for the shear and moment at s_1. The was explained previously in the discussion of the influence line for the support reaction at C (see Figures 2 and 3).

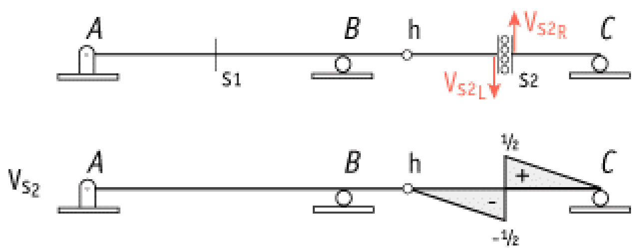


Figure 6 - Structure with shear capacity removed at s2 and resulting influence line

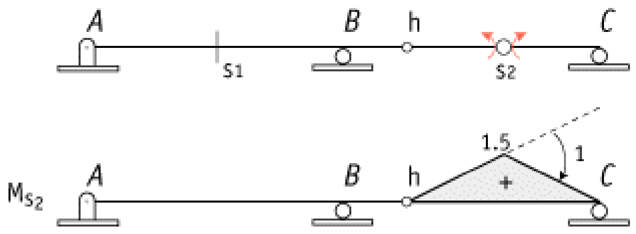


Figure 7 - Structure with moment capacity removed at s2 and resulting influence line

- Shear at the left and right of B

Since the shear at B occurs on both sides of a support, it is necessary to independently determine the shear for each side.

To plot the influence line for $V_{b\text{-}L}$, follow the instructions outlined above for plotting the influence line for the shear at s_1. To construct the shear just to the left of support B, imagine that the section s_1 has been moved to the left of B. In this case, the positive ordinates of the influence line between A and B will decrease to zero while the negative ordinates will increase to 1 (see Figure 8).

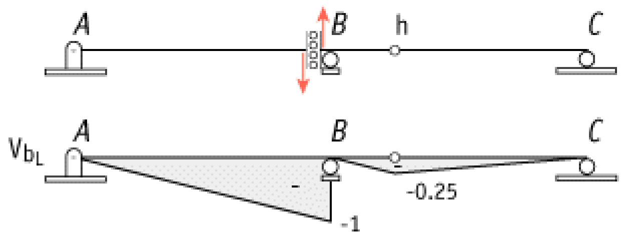


Figure 8 - Structure with shear capacity removed at the left of B and the resulting influence line

The influence line for the shear forces just to the right of support B, $V_{b\text{-}R}$, is represented by the resulting deflected shape of the beam induced by shear forces acting just to the right of support B. Notice that the portion of the beam from B to h moves as a rigid body (see explanation in the Simple Beam with a Cantilever example) while the influence line varies linearly from h to C. This is due to the fact that the deflection at C is zero and the assumption that the deflection of a statically determinate system is linear.

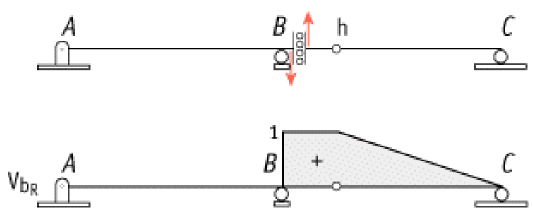


Figure 9 - Structure with shear capacity removed at the right of B and the resulting ILline

2.A single rolling load of 100 kN moves on a girder of span 20m.(a) Construct the influence lines for (i) shear force and (ii) bending moment for a section 5m from the left support. (b) Construct the influence lines for points at which the maximum shears and maximumbending moment develop. Determine these values.

Solution:

(a) To find maximum shear force and bending moment at 5m from the left support:

For the ILD for shear,

IL ordinate to the right of D $= \frac{l-x}{l} = \frac{20-5}{} = 0.75$

For the IL for bending moment, IL ordinate at D =

$\frac{x(l-x)}{l} = \frac{5*15}{20} = 3.75\ m$

(i) **Maximum positive shear force**

By inspection of the ILD for shear force, it is evident that maximum positive shear force occurs when the load is placed just to the right of D.

Maximum positive shear force = load * ordinate = 100 * 7.5

At D,

SF_{max} + = 75 kN. (ii) **Maximum negative shear force**

Maximum negative shear force occurs when the load is placed just to the left D.

Maximum negative shear force = load * ordinate = 100 * 0.25

At D, SF_{max} = -25 kN.

(iii) **Maximum bending moment**

Maximum bending moment occurs when the load is placed on the section D itself.

Maximum bending moment = load * ordinate = 100 * 3.75 = 375 kNm

(b) Maximum positive shear force will occur at A. Maximum negative shear force will occur at B. Maximum bending moment will occur at mid span.

The ILs are sketched in fig.

(i) Positive shear force

Maximum positive shear force occurs when the load is placed at A.

Maximum positive shear force = load * ordinate = 100*1

SF_{maxmax} + = 100 kN

(ii) Negative shear force

Maximum negative shear force occurs when the load is placed at B.

Maximum negative shear force = load * ordinate = 100 * (-1)

SF_{maxmax} = - 100 kN

(iii) Maximum bending moment

Maximum bending moment occurs when the load is at mid span

Maximum bending moment = load * ordinate = 100 * 5 = 500 kNm

3.Draw the ILD for shear force and bending moment for a section at 5m from the left hand support of a simply supported beam, 20m long. Hence, calculate the maximum bending moment and shear force at the section, due to a uniformly distributed rolling load of length 8m and intensity 10 kN/m run.(Apr/May 05)

Solution:

(a) Maximum bending moment:

Maximum bending moment at a D due to a udl shorter than the span occurs when the section divides the load in the same ratio as it divides the span.

In the above fig. $\frac{A_1D}{B_1D} = \frac{AD}{BD} = 0.25, \quad A_1D = 2M, B_1D = 6M$

Ordinates:

Ordinate under A_1 = (3.75/5)*3 = 2.25

Ordinate under B_1 = (3.75/15)*9 = 2.25

Maximum bending moment = Intensity of load * Area of ILd under the load

$= 10 * \frac{(3.75 + 2.25) * 8}{}$

At D, M_{max} = 240 kNm

(b) Maximum positive shear force

Maximum positive shear force occurs when the tail of the UDL is at D as it traverses from left to right.

Ordinate under B_1 = $\frac{0.75}{15} * (15 - 8) = 0.$

(c) Maximum negative shear force

Maximum negative shear force occurs when the head of the UDL is at D as it traverses from left to right.

Maximum negative shear force = Intensity of load * Area of ILD under the load

= 10(1/2*0.25*5)

Negative SF_{max} = 6.25 kN.

Begg's deformeter

Begg's deformeter is a device to carry out indirect model analysis on structures. It has the facility to apply displacement corresponding to moment, shear or thrust at any desired point in the model. In addition, it provides facility to measure accurately the consequent displacements all over the model.

'dummy length' in models tested with Begg's deformeter.

Dummy length is the additional length (of about 10 to 12 mm) left at the extremities of the model to enable any desired connection to be made with the gauges.

Three types of connections possible with the model used with Begg's deformeter.

(i) Hinged connection

(ii) Fixed connection

(iii) Floating connection

Use of a micrometer microscope in model analysis with Begg's deformeter

Micrometer microscope is an instrument used to measure the displacements of any point in the x and y directions of a model during tests with Begg's deformeter.

Name the types of rolling loads for which the absolute maximum bending moment occurs at the mid span of a beam.

Types of rolling loads:

(i) Single concentrated load

(ii) Udl longer than the span

(iii) Udl shorter than the span

Absolute maximum bending moment in a beam

When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section. The maximum of these maximum bending moments will usually occur near or at the mid span. This maximum of maximum bending moment is called the absolute maximum bending moment, $M_{maxmax.}$

The portal frame in fig. is hinged at D and is on rollers at A. Sketch the influence line for bending moment at B.

To get the influence line diagram for M_B, we shall introduce a hinge at B (and remove the resistance to bending moment). Now we get a unit rotation between BA and BC at B.

BC cannot rotate since column CD will prevent the rotation. BA would rotate freely (with zero moment). For θ =1 at B, displacement at A = 3m. The displaced position shows the influence line for M_B as shown in fig.

UNIT- III- ARCHES

Arches - Types of arches – Analysis of three hinged, two hinged and fixed arches - Parabolic and circular arches – Settlement and temperature effects.

Arch

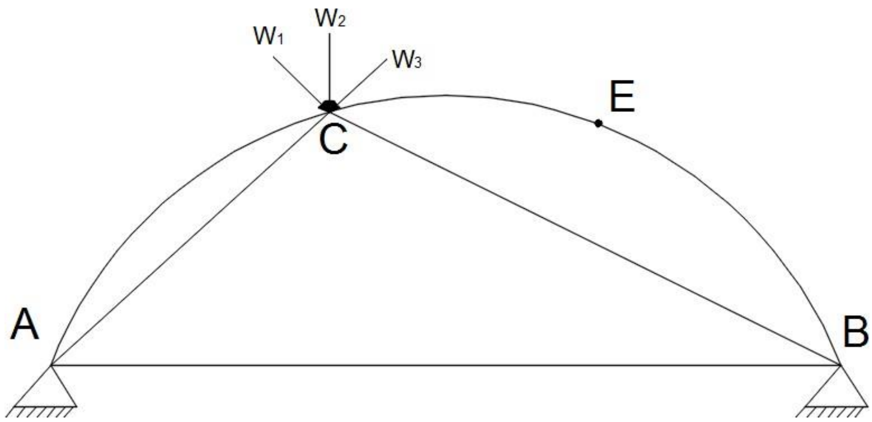
An arch is a curved beam in which horizontal movement at the support is wholly or partially prevented. Hence there will be horizontal thrust induced at the supports. The shape of an arch doesn't change with loading and therefore some bending may occur.

Types of Arches

On the basis of material used arches may be classified into and steel arches, reinforced concrete arches, masonry arches etc.

.**Linear arch**

If an arch is to take loads, say W1, W2, and W3 and a vector diagram and funicular polygon are plotted as shown; the funicular polygon is known as the linear arch or theoretical arch.



The polar distance 'ot' represents the horizontal thrust.

The links AC, CD, DE and EB will br under compression and there will be no bending moment. If an arch of this shape ACDEB is provided, there will be no bending moment.

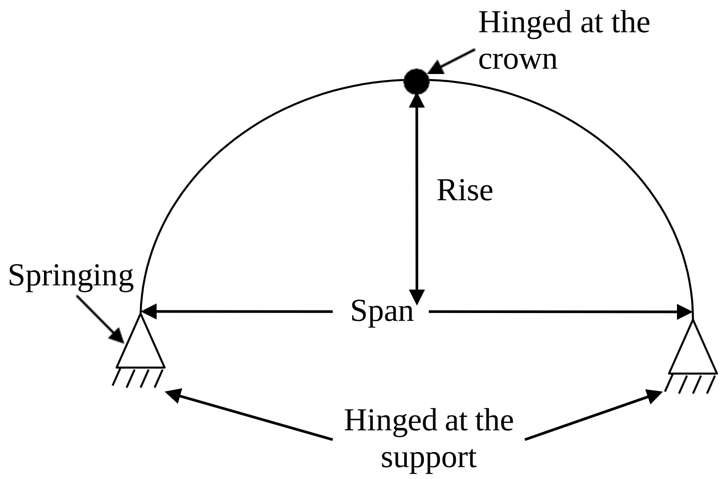
Eddy's theorem.

Eddy's theorem states that "The bending moment at any section of an arch is proportional to the vertical intercept between the linear arch (or theoretical arch) and the center line of the actual arch".

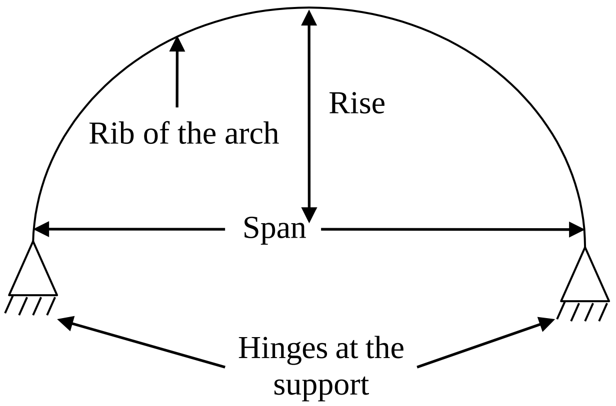
BMx = ordinate O2 O3 * scale factor

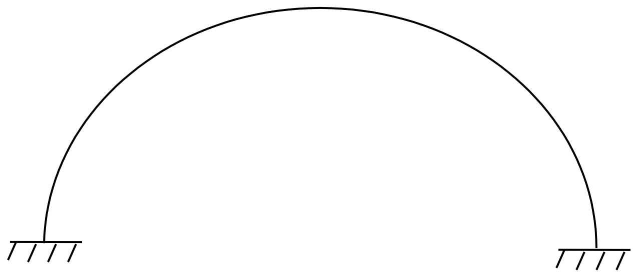
On the basis of structural behavior arches are classified as :

Three hinged arches:- Hinged at the supports and the crown.



Two hinged arches:- Hinged only at the support





The supports are fixed

A 3-hinged arch is a statically determinate structure. A 2-hinged arch is an indeterminate structure of degree of indeterminacy equal to 1. A fixed arch is a statically indeterminate structure. The degree of indeterminacy is 3.

Depending upon the type of space between the loaded area and the rib arches can be classified as open arch or closed arch (solid arch).

Analysis of 3-hinged arches

It is the process of determining external reactions at the support and internal quantities such as normal thrust, shear and bending moment at any section in the arch.

Procedure to find reactions at the supports

1. Sketch the arch with the loads and reactions at the support.

2. Apply equilibrium conditions namely $\sum F_x = 0, \ \sum F_y = 0$ and $\sum M = 0$

3. Apply the condition that BM about the hinge at the crown is zero (Moment of all the forces either to the left or to the right of the crown).

4. Solve for unknown quantities.

Let us take a section X of an arch. Let θ be the inclination of the tangent at X. if H is the horizontal thrust and V the net vertical shear at X, from the free body of the RHS of the arch, it is clear that V and H will have normal and radial components given by,

$$N = H \cos \theta + V \sin \theta$$
$$R = V \cos\theta - H \sin \theta$$

The normal thrust and radial shear in an arch rib.

Parabolic arches are preferable to carry distributed loads. Because, both, the shape of the arch and the shape of the bending moment diagram are parabolic. Hence the intercept between the theoretical arch and actual arch is zero everywhere. Hence, the bending moment at every section of the arch will be zero. The arch will be under pure compression that will be economical.

Difference between the basic action of an arch and a suspension cable.

An arch is essentially a compression member, which can also take bending moments and shears. Bending moment and shears will be absent if the arch is parabolic and the loading uniformly distributed. A cable can take only tension. A suspension bridge will therefore have a cable and a stiffening girder. The girder will take the bending moment and shears in the bridge and the cable, only tension. Because of the thrust in cables and arches, the bending moments are considerably reduced. If the load on the girder in uniform. The bridge will have only cable tension and no bending moment on the girder.

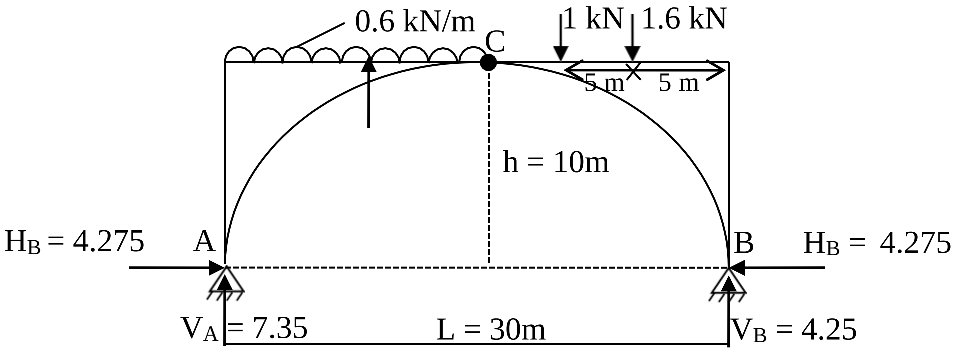
Under what conditions will the bending moment in an arch be zero throughout

The bending moment in an arch throughout the span will be zero, if

(i) The arch is parabolic and

(ii) The arch carries udl throughout the span

1. **A 3-hinged arch has a span of 30m and a rise of 10m. The arch carries UDL of 0.6 kN/m on the left half of the span. It also carries 2 concentrated loads of 1.6 kN and 1 kN at 5 m and 10 m from the 'right' end. Determine the reactions at the support. (Sketch not given).**



$$\sum F_x = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B \qquad \text{------ (1)}$$

To find vertical reaction.

$$\sum F_y = 0$$

$$V_A + V_B = 0.6\,x15+1+1.6$$

$$= 11.6 \qquad \text{..........(2)}$$

$$\sum M_A = 0$$

$$-V_B\ x\,30 + 1.6\,x\,25 + 1x\,20 + (0.6\,x15)\,7.5 = 0$$

$$V_B = 4.25\,\text{kN}$$

$$V_A = 4.25 = 11.6$$

$$A_A = 7.35\,\text{kN}$$

To find horizontal reaction.

$$M_C = 0$$

$$-1x5 - 1.6x10 + 4.25x15 - H_Bx10 = 0$$

$$H_B = 4.275\text{kN}$$

$$H_A = 4.275\text{kN}$$

OR

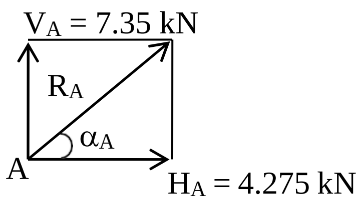
$M_C = 0$

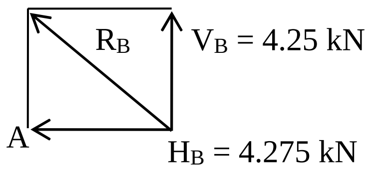
$$7.375x15 - H_A x10 - (0.6x15)7.5$$

$$H_A = 4.275kN$$

$$H_B = 4.275kN$$

To find total reaction





$$R_A = \sqrt{H_A^{\,2} = V_A^{\,2}}$$

$$\sqrt{4.275^2 + 7.35^2}$$

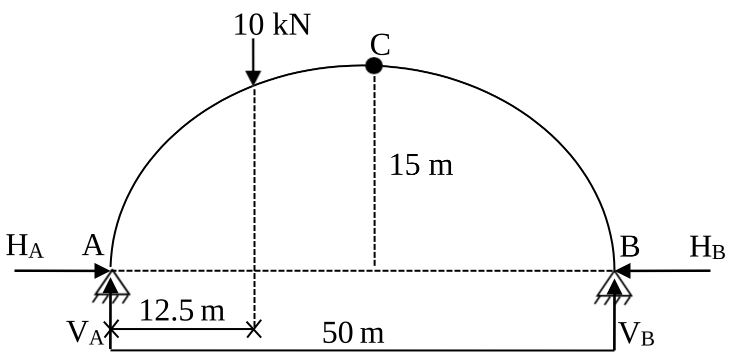
$$= 8.5kN$$

$$\alpha_A = \tan^{-1}\left(\frac{V_A}{H_A}\right) = 59^0.82$$

$$R_B = \sqrt{H_B^{\,2} + V_B^{\,2} = 6.02kN}$$

$$\alpha_B = \tan^{-1}\left(\frac{V_B}{H_B}\right) = 44.83$$

2. A 3-hinged parabolic arch of span 50m and rise 15m carries a load of 10kN at quarter span as shown in figure. Calculate total reaction at the hinges.



$$\sum F_x = 0$$

$$H_A = H_B$$

To find vertical reaction.

$$\sum Fy = 0$$

$$V_A + V_B = 10 \qquad \text{------ (1)}$$

$$\sum M_A = 0$$

$$-V_B \times 50 + 10 \times 12.5 = 0$$

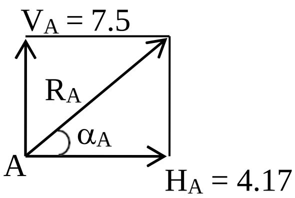
$$V_B = 2.5\,kN \qquad V_A = 7.5\,kN$$

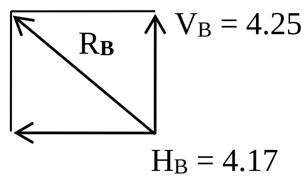
To find Horizontal reaction

$$M_C = 0$$

$$V_B \times 25 - H_B \times 15 = 0$$

To find total reaction.





$$H_B = 4.17\,kN = H_A$$

$$R_A = \sqrt{4.17^2 + 7.5^2}$$

$$R_A = 8.581kN$$

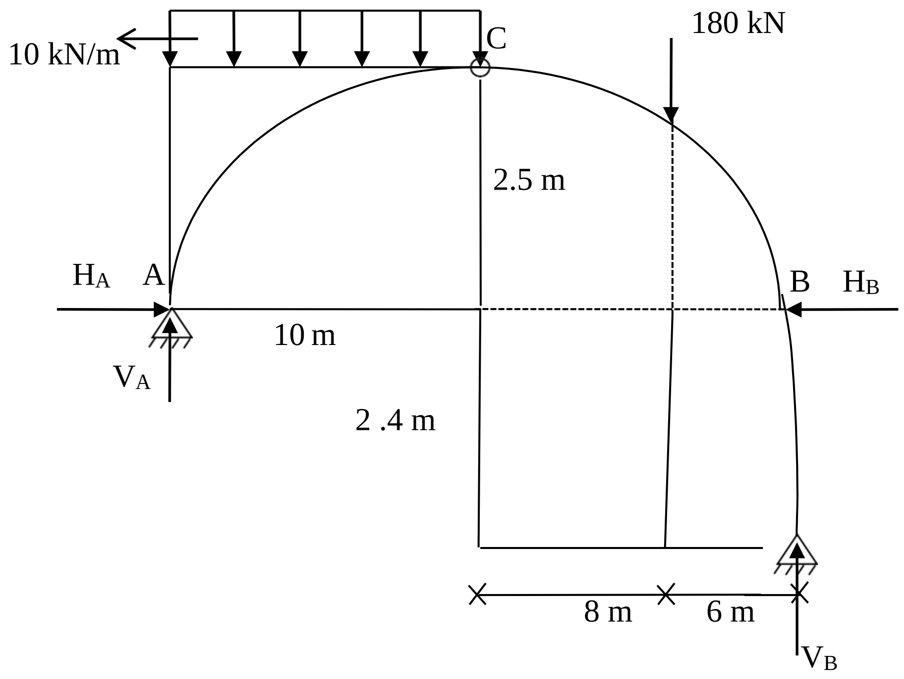
$$\alpha_A = \tan^{-1}\left(\frac{V_A}{H_A}\right) = 60^0.92$$

$$R_B = \sqrt{H_A^{\ 2} + V_B^{\ 2}}$$

$$R_B = 4.861kN$$

$$\alpha_B = \tan^{-1}\left(\frac{V_B}{H_B}\right) = 30^0.94$$

3. Determine the reaction components at supports A and B for 3-hinged arch shown in fig.



To find Horizontal reaction

$$\sum F_x = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B \qquad \text{------ (1)}$$

To find vertical reaction.

$$\sum F_y = 0$$

$$V_A + V_B = 180 + 10 \text{ x} 10$$

$$V_A + V_B = 280 \qquad \text{..........(2)}$$

$$\sum M_A = 0$$

$$-V_B \text{x} 24 + H_B \text{ x } 2.4 + 180 \text{ x} 18 + 10 \text{ x } 10 \text{ x } 5 = 0$$

$$2.4 H_B - 24 V_B = -3740 \qquad \text{------ (3)}$$

$$H_B - 10 V_B = -1558.33$$

$$M_C = 0$$

$$-180 \times 8 - V_B \times 14 - H_B \times 4.9 = 0$$

$$H_B \times 4.9 - V_B 14 = -1440 \qquad \text{------ (4)}$$

$$-H_B + 2.857 V_B = +293.87$$

Adding 2 and 3

$$-10V_B + 2.857V_B = -1558.33 + 293.87$$

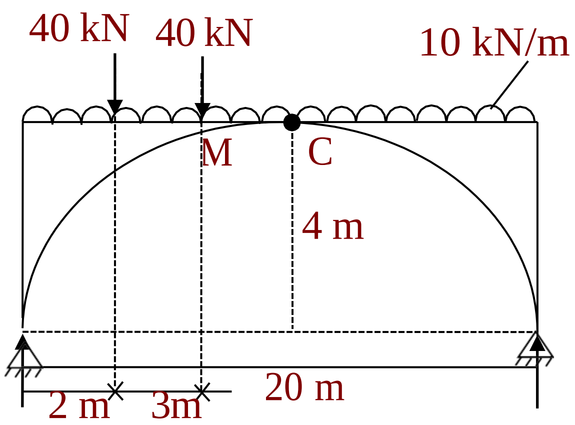
$$V_B = 177kN$$

$$V_A = 103kN$$

$$H_B - 10 \times 177 = -1558.33$$

$$H_B = 211.67kN = H_A$$

4. A symmetrical 3-hinged parabolic arch has a span of 20m. It carries UDL of intensity 10 kNm over the entire span and 2 point loads of 40 kN each at 2m and 5m from left support. Compute the reactions. Also find BM, radial shear and normal thrust at a section 4m from left end take central rise as 4m.



$$\sum F_x = 0$$

$$H_A - H_B = 0 \qquad \text{..........(1)}$$

$$H_A = H_B$$

$$\sum F_y = 0$$

$$V_A + V_B - 40 - 40 - 10 \text{ x } 20 = 0$$

..........(2)

$$V_A + V_B = 280$$

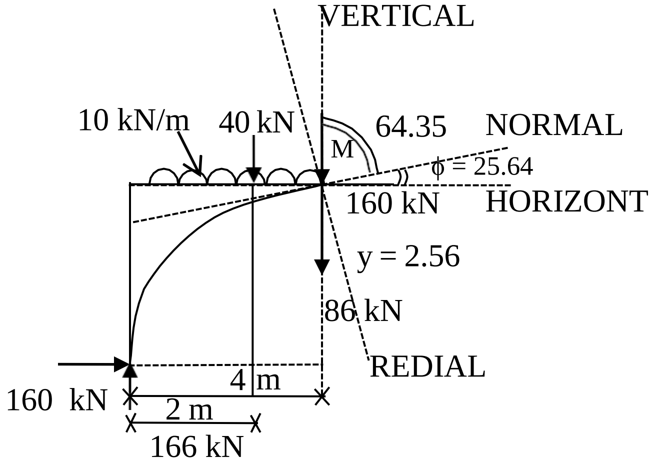
$$\sum M_A = 0$$

$$M_c = 0$$

$$-(10 \text{ x} 10)5 - H_B \text{ x } 4 + 114 \text{ x} 10 = 0$$

$$H_B = 160 \text{ kN}$$

$$H_A = 160 \text{ kN}$$



BM at M

= - 160 x 2.56

+ 166 x 4 – 40 x 2

- (10 x 4)2

= + 94.4 kNm

$$y = \frac{4hx}{L^2}(L - x)$$

$$= \frac{4 \text{ x } 4 \text{ x } 4}{20^2}(20 - 4)$$

$$y = 2.56\text{m}$$

$$\tan\phi = \frac{4h}{L^2}(L - 2x)$$

$$= \frac{4 \text{ x } 4}{20^2}(20 - 2 \text{ x } 4)$$

$\phi = 25^0.64$

Normal thrust = N = + 160 Cos 25.64

+ 86 Cos 64.36

= 181.46 kN

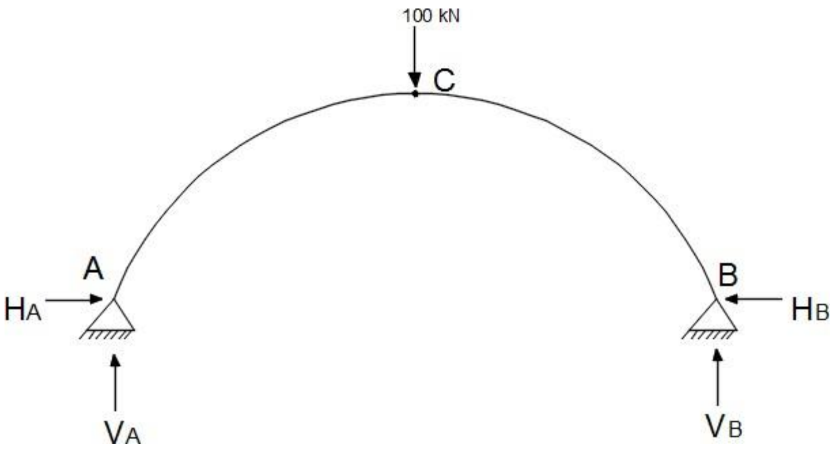
S = 160 Sin 25.64

- 86 x Sin 64.36

S = - 8.29 kN

5. A three-hinged semicircular arch carries a point load of 100 kN at the crown. The radius of the arch is 4m. Find the horizontal reactions at the supports.

$V_A = V_B = 50$ kN

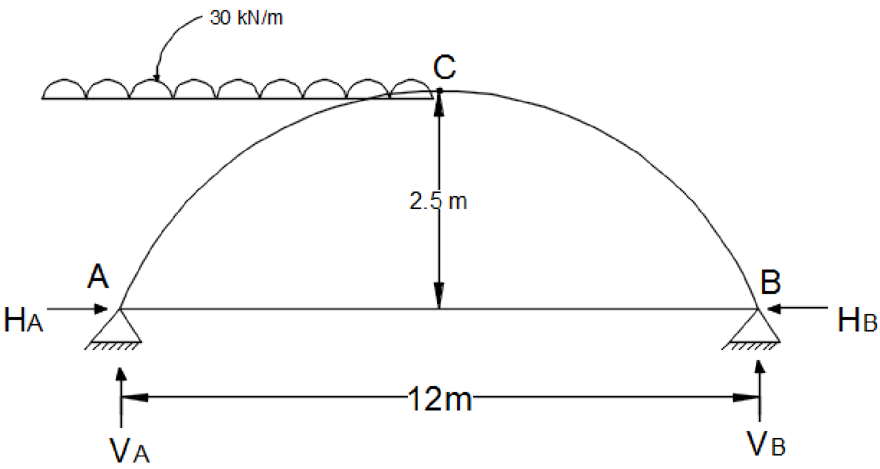


Equating the moment about C to Zero, $V_A * 4 - H*4 = 0$

$H = V_A$

Horizontal reaction, H = 50 kN

6. Determine H, V_A and V_B in the semicircular arch shown in fig



Equating moments about A to Zero,

$V_B * 12 – 12 * 9 = 0$;

$V_B = 9$ kN and $V_A = 3$ kN Equating moments to the left of C to zero,

$H = V_A = 3$ kN;

H= 3 kN

7. Distinguish between two hinged and three hinged arches.

SI. NO	Two hinged arches	Three hinged arches
1.	Statically indeterminate to first degree	Statically determinate
2.	Might develop temperature stresses.	Increase in temperature causes increases
3.	Structurally more efficient.	Easy to analyse. But, in construction, the central hinge may involve additional expenditure.
4.	Will develop stresses due to sinking of supports	Since this is determinate, no stresses due to support sinking

Rib – shorting in the case of arches.

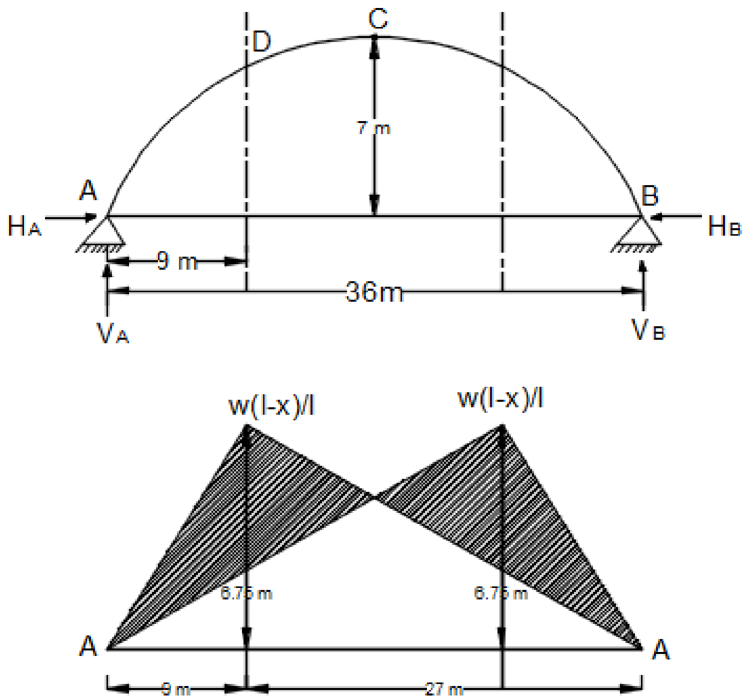
In a 2-hinged arch, the normal thrust, which is a compressive force along the axis of the arch, will shorten then rib of the arch. This is turn will release part of the horizontal thrust.

Normally, this effect is not considered in the analysis (in the case of two hinged arches). Depending upon the important of the work we can either take into account or omit the effect of rib shortening. This will be done by considering (or omitting) strain energy due to axial compression along with the strain energy due to bending in evaluating H.

Effect of yielding of support in the case of an arch.

Yielding of supports has no effect in the case of a 3 hinged arch which is determinate.

8. A three-hinged parabolic arch has a horizontal span of 36m with a central rise of 6m. A point load of 40 kN moves across the span from the left to the right. What is the absolute maximum positive bending moment that wills occur in the arch?



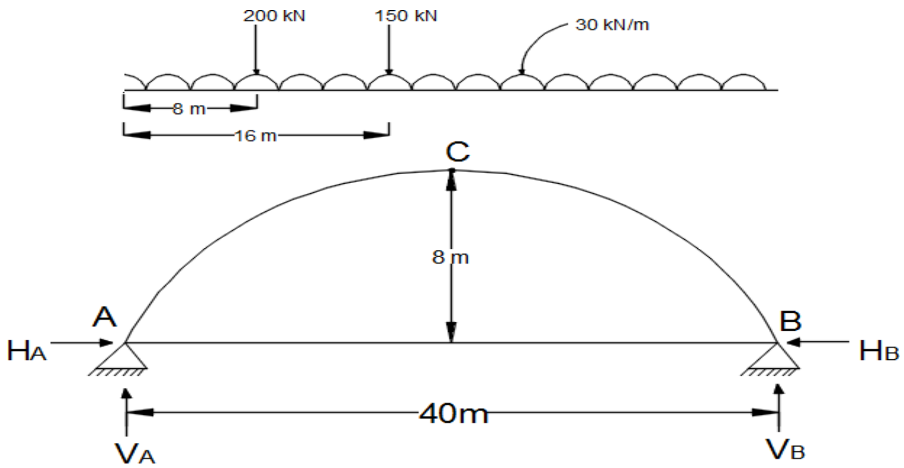
For a single concentrated load moving from one end to the other, absolute maximum positive bending moment

= 0.096wl = 0.096*40 * 36=138.24 kNm

This occurs at 0.211 l = 0.211 * 36 = 7.596 m from the ends.

Absolute maximum positive bending moment = 138.24 kNm at 7.596 m from the ends.

9. A 3 hinged arch of span 40m and rise 8m carries concentrated loads of 200 kN and 150 kN at a distance of 8m and 16m from the left end and an udl of 50 kN/m on the right half of the span. Find the horizontal thrust.



Solution:

(a) Vertical reactions V_A and V_B :

Taking moments about A,

$200(8) + 150(16) + 50 * 20 * (20 + 20/2) – V_B (40) = 0$

$1600 + 2400 + 30000 – 40 V_B = 0$

$V_B = 850$ kN

V_A = Total load – V_B = 200 + 150 + 50 * 20 – 850 = 500 KN

(b) Horizontal thrust (H)

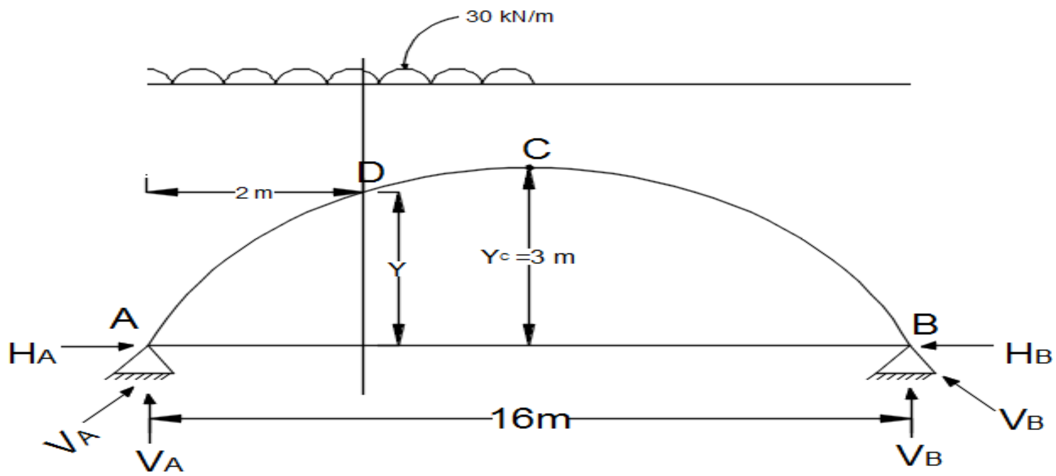
Taking moments about C,

$-H \times 8 + V_A (20) – 200 (20 – 8) – 150 (20 – 16) = 0$

$-8H + 500 * 20 – 200 (12) – 150 (4) = 0$

H = 875 kN

10. A parabolic 3-hinged arch carries a udl of 30kN/m on the left half of the span. It has a span of 16m and central rise of 3m. Determine the resultant reaction at supports. Find the bending moment, normal thrust and radial shear at xx, and 2m from left support.



(1) Reaction at A nd B;

(i) Vertical components of reactions; Taking moments about A,

$-V_B (16) + 30 \times 8^2 /2 = 0$

$- V_B (16) + 30 * 32 = 0$

$V_B = 60$ kN

V_A = Total load – V_B = 30 * 8 – 60 kN

$V_A = 180$ kN

(ii) Horizontal components of reactions at A and

Taking moments about the crown point C, $V_A * 8 - 30 * 8 * 8/2 - H_A * y_C = 0$

$180 * 8 - 30 * 32 = H_A * 3$

$H_A = 160$ kN

$H_B = H_A$ = since $\sum H = 0$

$H_B = 160$ kN

(iii) Resultants reactions at A and B;

$$R_A = \sqrt{V_A^2 + H_A^2}$$
$$= \sqrt{180^2 + 160^2}$$
$$= 240.83\ kN$$
$$R_B = \sqrt{V_B^2 + H_B^2}$$
$$= \sqrt{60^2 + 160^2}$$
$$= 170.88\ kN$$

(2) Bending moment at x = 2m from A:

Bending moment = V_A (2) – 30 * 2 *1 – H_A(y) ---------- (1)

Where, y = Rise of the arch at x = 2m from 'A':

For parabolic arches,

$$y = \frac{4r}{l^2} * x(l - x)$$

at a distance of 'x' from the support Where, r = rise of the arch at Crown Point = 3m

$$y = \frac{4 * 3}{16^2} * 2(16 - 2)$$

Substitute in (1)y = 1.3125 m at x = 2m from 'A'.

Bending moment at x = 2m from A = 180 (2) – 30 * 2 * 1 – 160 * 1.3125

Bending moment at xx = 90 kN-m

(3) Radial shear force at x = 2m from A

Shear force, $R_X = V_X \cos\theta - H \sin\theta$

Where, V = Net vertical shear force at x = 2m from A

$= V_A$ - w (2) = 180 – 30 * 2

V = 120 kN

H = Horizontal shear force = 160 kN

θ = 29º21'

R = 120 cos 29º21' – 160 sin 29º21'

R = 26.15 kN

(4) Normal thrust at x = 2m from A:

Normal thrust $P_N = V_X \sin\theta + H \cos\theta$ = 120 sin 29º21' + 160 cos 29º21'

P_N = 198.28 kN.

TWO HINGED ARCH:

11. **A two hinged parabolic arch has a span of 32 m and a central rise of 7 m. Calculate the max. positive and negative bending moment at section distance 10m from the left support, due to single point load of 10 kN rolling from left to right.**

$$Maximum\ Postive\ BM = \frac{x(l-x)}{l}$$

$$Maximum\ Postive\ BM = \frac{10(32-10)}{32} = 6.87\ m$$

Rise of arch of section of 10m from left support.

$$Y = \frac{4Y_c}{l^2}x(l-x)$$

$$Y = \frac{4*7}{32^2}*10(32-10)$$

$$Y = 5.94\ m$$

Maximum negative bending:

$$Moment\ ordinate = \frac{25\,l_y}{128y_c}$$

$$= \frac{25*32*5.94}{128*7}$$

$$= 5.3\ m$$

i). max. positive bending moment at D:

Max. +tive BM occurs under the point load.

$$k = \frac{x}{l} = \frac{10}{32}$$

$$k = 0.31$$

$$IL\ ordinary\ of\ D = 6.87 - Z$$

Where

$$Z = \frac{5l}{8Y_C}\text{K}(1-k)(1+K-K^2)\text{Y}$$

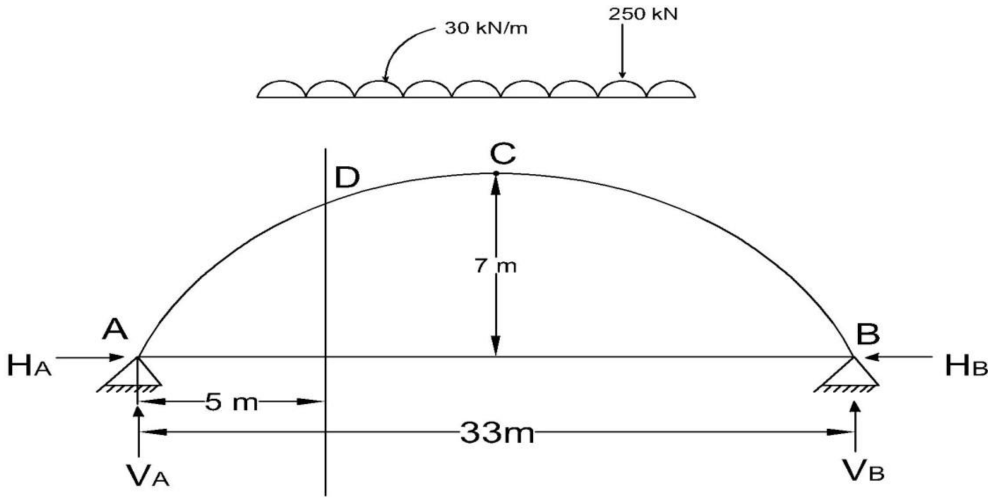
$$= \frac{5*32}{8*7}*0.31(1-0.31)*(1+0.31-0.31^2)*5.94$$

$$= 2.37\ m$$

Bending moment D = 2.37*10

= 23.7 KN-m

12. **A two hinged parabolic arch of span 33 m and rise 7 m carries a UDL of 45 kN per meter on the whole span and a point load of 250 kN at a distance of 7m from the right end. Find the horizontal thrust, bending moment, normal thrust and radial shear at section 5 m from the left end.**



$\sum M_A = 0$

$\left[45 * 33 * \frac{33}{2}\right] + (250 * 26) - [V_B * 33] = 0$

$V_B = 940\ kN$

$V_A + V_B = Total\ load$

$V_A = [45 * 33] + [250] - 940$

$V_A = 795\ kN$

$\sum M_C = 0(left)$

$(V_A * 16.5) - \left[45 * 16.5 * \frac{16.5}{2}\right] - 7H_A = 0$

$H_A = 999\ kN$

$H_B = +999\ kN$

$\sum H = 0$

$H_A + H_B = 0$

$R_A = \sqrt{V_A{}^2 + H_A{}^2}$

$= \sqrt{795^2 + 999^2}$

$R_A = 1277\ kN$

$\theta_A = \tan^{-1}\frac{V_A}{H_A}$

$= \tan^{-1}\frac{795}{999}$

$Q_A = 38.51^0$

$R_B = \sqrt{V_B{}^2 + H_B{}^2}$

$= \sqrt{940^2 + 999^2}$

$R_B = 1372\ kN$

$\theta_A = \tan^{-1}\frac{V_B}{H_B}$

$= \tan^{-1}\frac{940}{999}$

$Q_B = 43.26^0$

BMD:

$$Y_D = \frac{4Y_C}{l^2} * (l - x)$$
$$= \frac{4 * 7}{33^2} * (5 * 33 - 5)$$
$$= 3.60\ m$$

BMD

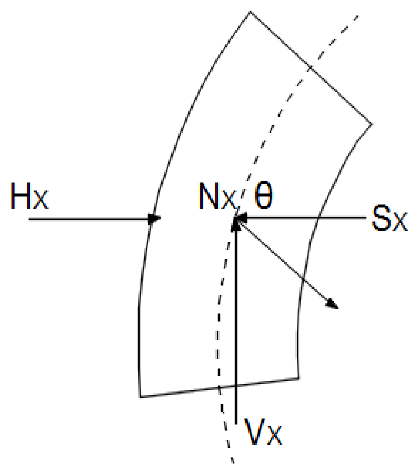
$$x = 5\ m = (V_A * 5) - \left[45 * 5 * \frac{5}{2}\right] - (H_A * 3.6)$$
$$= (795 * 5) - (45 * 5 * 2.5) - (999 * 3.6)$$
$$BMD = \ -183.9\ kN - m$$

Radial shear at D:

$$(S_x)_D = V_x \cos\theta - H_x \sin\theta$$
$$H(X = 5m) = 999\ kN$$
$$\theta = \ \tan^{-1}\left[\frac{4Y_C}{l^2} * (l - x)\right]$$
$$\theta = \ \tan^{-1}\left[\frac{4 * 7}{33^2} * (33 - (2 * 5)\right]$$
$$\theta = \ 30.60^0$$
$$(S_x)_D = \ 570(\cos 30 * 60^0) - 999(\sin 30 - 60)$$
$$(S_x = 5m)_D = \ -18\ kN$$

Normal thrust (N_x):

$$[N_x = 5m]_D = V_x \sin\theta + H_x \cos\theta$$
$$= 570(\sin 30.6^0) + 999(\cos 30.60^0)$$
$$[N_x = 5m]_D = 1150\ kN$$



CIRCULAR ARCH

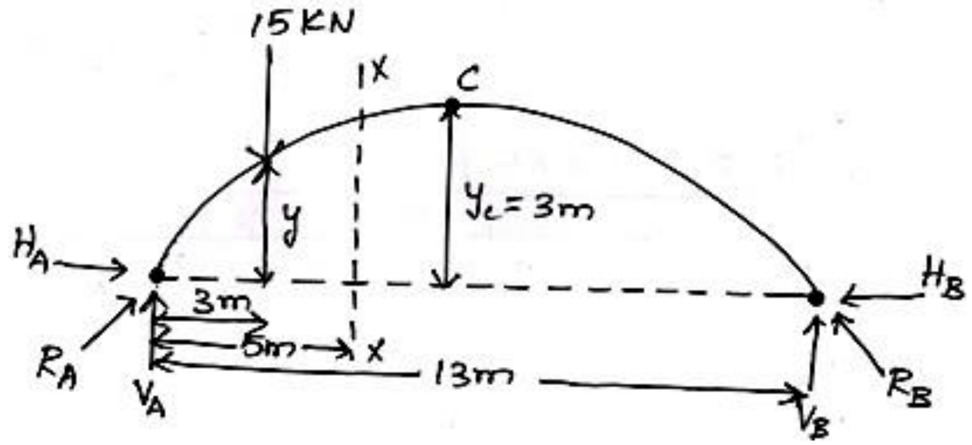
13. **A symmetrical three hinged circular arch has a span of 13m and a rise to the central hinge of 3m. It carries a vertical load of 15 kN at 3m from the left hand end. Find**

i). The reaction at the support

ii). Magnitude of the thrust at the springing.

iii). B.M at 5m from the left hand hinge

iv). The max. Positive and negative B.M.



$Step1; Vertical\ reaction: -$

$\sum V = 0$

$V_A + V_B = 15$

$Taking\ moment\ about\ A,$

$-V_B * 13 + (15 * 3) = 0$

$V_B = 3.46\ kN$

$V_A + 3.46 = 15$

$V_A = 11.54\ kN$

$Step2 : Horizontal\ thrust$

$Taking\ moment\ about\ C,$

$(V_A * 6.5) - H_A Y_C - (15 * 3.5) = 0$

$(11.54 * 6.5) - 3H_A - (15 * 3.5) = 0$

$H_A = +7.5kN$

$Step3 : Resultant\ Reaction$

$R_A = \sqrt{V_A{}^2 + H_A{}^2}$

$R_A = \sqrt{11.54^2 + 7.5^2}$

$R_A = 13.76\ kN$

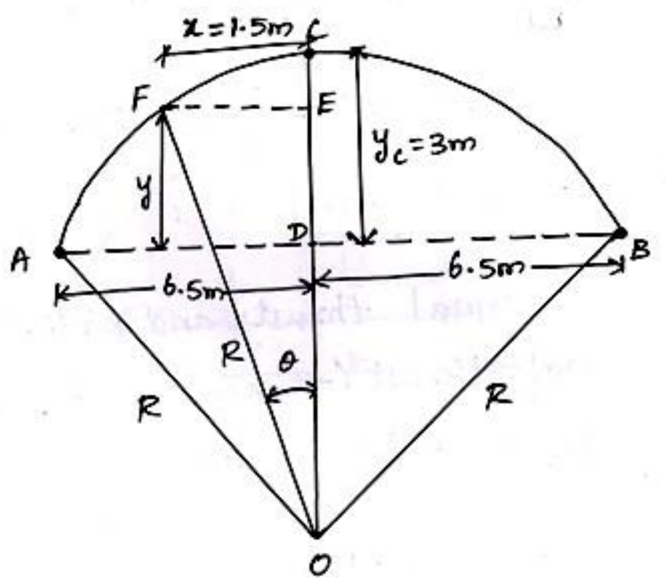
$R_B = \sqrt{V_B{}^2 + H_B{}^2}$

$R_A = \sqrt{3.46 + 7.5^2}$

$R_B = 8.26\ kN$

$Step\ 4: Bending\ moment\ at\ 5m\ from\ left\ support$

In the bending moment at x=5m from the left support, we find the radius and y value by using the formula.



To find the radius (R):-

$(2R - Y_c)Y_C = \left(\frac{l}{2}\right)^2$

$(2R - 3)3 = \left(\frac{13}{2}\right)^2$

$R = 8.54\ m$

$To\ find\ y: -$

$In\ \Delta\ OFE,$

$R^2 = X^2 + (R - Y_C + Y)^2$

$R^2 = 1.5^2 + (R - 3 + Y)^2$

$8.54^2 = 1.5^2 + (8.54 - 3 + Y)^2$

$70.68 = (5.54 + Y)^2$

$8.41 = 5.54 + y$

$y = 2.86m\ at\ x = 1.m\ from\ centre$

$BM = V_A * 5 - H_A * Y - 15 * 2$

$BM = 11.54 * 3 - 7.5 * 2.86 - 15 * 2$

$BM = 6.25\ kN - m$

$\boldsymbol{Step\ 5: Normal\ thrust\ and\ Radial\ shear: -}$

i). Normal thrust (at x=5m from A)

$N_x = \mathrm{V}\sin\theta + \mathrm{H}\cos\theta$

Here,

$\theta = \tan^{-1}\frac{FE}{DE}$

$\theta = \tan^{-1}\frac{1.5}{5.4}$

$\theta = 10^0 7^0$

$V = Net\ Vertical\ shear\ force\ at\ x = 5m\ from\ A.$

$= V_A - 15$

$= 11.54 - 15$

$V = -3.46\ kN$

$N_x = (-3.46 * \sin 10^0 7^0) + (7.5 * \cos 10^0 7^0)$

$N_x = 6.77\ kN$

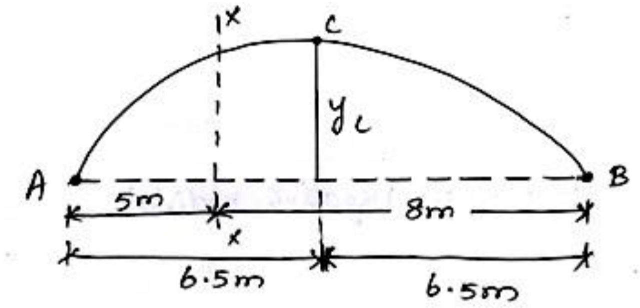
$iii).Radial\ shear$

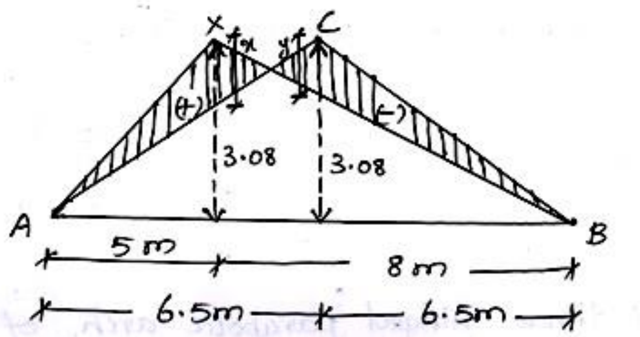
$R_x = \text{V Cos}\theta - \text{H Sin}\,\theta$

$R_x = (-3.46 * \cos 10^0 7^0) + (7.5 * \sin 10^0 7^0)$

$R_x = -4.72\ kN$

$Step\ 6$:$Maximum\ bending\ moment$





$Maximum\ positive\ (sagging) ordinate = \frac{x(l - x)}{l}$

$= \frac{5(13 - 5)}{13}$

$= 3.08$

$Net\ maximum\ positive\ ordinate, x = 3.08 - \frac{3.08}{6.5} * 5$

$x = 0.71$

$maximum\ positive\ moment = load * ordinate$

$= 15 * 0.71$

$= 10.65\ kN - m$

$$Maximum\ negative\ (hogging) ordinate = \frac{x(l-x)}{l}$$
$$= \frac{5(13-5)}{13}$$
$$= 3.08$$

$$Net\ maximum\ positive\ ordinate, x = 3.08 - \frac{3.08}{6.5} * 6.5$$
$$x = 0.58$$
$$maximum\ positive\ moment = load * ordinate$$
$$= 15 * 0.58$$
$$= 8.7\ kN - m$$

<u>FIXED ARCH</u>

14. **A Fixed arch shown in fig carries loads as indicated Determine i). Resultant reactions at end support. Ii). Bending moment, shear (radial) and normal thrust at D, 5m from A.**

<u>Vertical reaction:</u>

$$\sum V = 0$$
$$V_A + V_B = 200 + 20 + 32$$
$$V_A + V_B = 252 - - - - - - - - - - (1)$$
Taking moment about B:
$$V_A * 20 - 20 * 10 * \frac{10}{2} - 32 * 3 - 20 * 17 = 0$$
$$V_A = +87.8\ kN$$
$$V_B = 252 - 87.8$$
$$V_B = 164.2\ kN$$

Horizontal reaction:
$$\sum H = 0$$
$$H_A = H_B$$
Taking moment about C:
$$20 * 10 * \frac{10}{2} - V_B * 10 + 5H_B = 0$$
$$H_B = +128.4\ kN$$

Resultant Reaction:
$$R_A = \sqrt{V_A{}^2 + H_A{}^2}$$
$$R_A = \sqrt{87.8^2 + 128.4^2}$$
$$R_A = 155.5\ kN$$
$$R_B = \sqrt{V_B{}^2 + H_B{}^2}$$

$R_B = \sqrt{164.2^2 + 128.4^2}$
$R_B = 208.44\ kN$

Maximum B.M:
$B.M\ at\ D = V_A * 5 - H_A * Y_D - 20 * 2$
$B.M\ at\ D = 399 - 128.4 Y_D$
$$Y_D = \frac{4Y_C x(l-x)}{l^2}$$
$$Y_D = \frac{4 * 5(20-5)}{20^2}$$
$Y_D = 3.75\ m$
$B.M\ at\ D = -82.5\ kN - m$

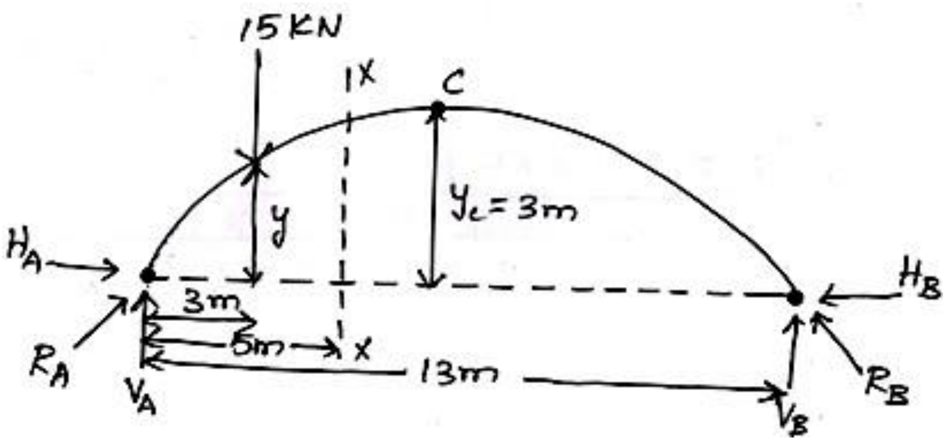
Radial shear force at x=5 from A:
$S.F, \quad R_x = V_x \cos\theta - H \sin\theta$
$V = vertical\ shear\ force\ at\ x = 5m\ from\ A$
$= 87.8 - 20$
$V = 67.8\ kN$
$H = Horizontal\ shear\ force = 128.4\ kN$
$$\theta = \tan^{-1}\left(\frac{4r}{l^2}(l-2x)\right)$$
$$\theta = \tan^{-1}\left(\frac{4*5}{20^2}(20-2*5)\right)$$
$\theta = 45^0$
$R = 67.8 \cos 45 - 128.4 \sin 45$
$R = -42.85\ kN$

Normal thrust at x=5 from A
$N_x = V_x \sin\theta + H \cos\theta$
$= 67.8\ \mathrm{Sin}\ 45 + 128.4 \cos 45$
$N_x = 138.73\ kN$

<u>SETTLEMENT AND TEMPERATURE EFFECTS</u>

15. A symmetrical three hinged circular arch has a span of 13m and a rise to the central hinge of 3m. It carries a vertical load of 15 kN at 3m from the left hand end. Find

i). The reaction at the support
ii). Magnitude of the thrust at the springing.
iii). B.M at 5m from the left hand hinge
iv). The max. Positive and negative B.M.



$\boldsymbol{Step1; Vertical\ reaction:-}$

$\sum V = 0$

$V_A + V_B = 15$

$Taking\ moment\ about\ A,$

$-V_B * 13 + (15 * 3) = 0$

$V_B = 3.46\ kN$

$V_A + 3.46 = 15$

$V_A = 11.54\ kN$

$\boldsymbol{Step2: Horizontal\ thrust}$

$Taking\ moment\ about\ C,$

$(V_A * 6.5) - H_A Y_C - (15 * 3.5) = 0$

$(11.54 * 6.5) - 3H_A - (15 * 3.5) = 0$

$H_A = +7.5kN$

$\boldsymbol{Step3: Resultant\ Reaction}$

$R_A = \sqrt{{V_A}^2 + {H_A}^2}$

$R_A = \sqrt{11.54^2 + 7.5^2}$

$R_A = 13.76\ kN$

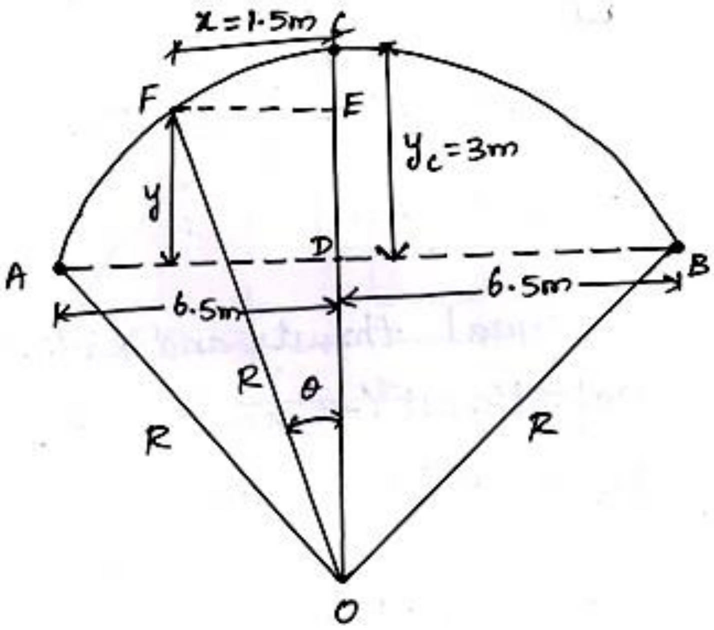
$R_B = \sqrt{{V_B}^2 + {H_B}^2}$

$R_A = \sqrt{3.46 + 7.5^2}$

$R_B = 8.26\ kN$

$Step\ 4: Bending\ moment\ at\ 5m\ from\ left\ support$

In the bending moment at x=5m from the left support, we find the radius and y value by using the formula.



To find the radius (R):-

$(2R - Y_C)Y_C = \left(\frac{l}{2}\right)^2$

$(2R - 3)3 = \left(\frac{13}{2}\right)^2$

$R = 8.54\ m$

$To\ find\ y: -$

$In\ \Delta\ OFE,$

$R^2 = X^2 + (R - Y_C + Y)^2$

$R^2 = 1.5^2 + (R - 3 + Y)^2$

$8.54^2 = 1.5^2 + (8.54 - 3 + Y)^2$

$70.68 = (5.54 + Y)^2$

$8.41 = 5.54 + y$

$y = 2.86m\ at\ x = 1.m\ from\ centre$

$BM = V_A * 5 - H_A * Y - 15 * 2$

$BM = 11.54 * 3 - 7.5 * 2.86 - 15 * 2$

$BM = 6.25\ kN - m$

$\boldsymbol{Step\ 5: Normal\ thrust\ and\ Radial\ shear: -}$

i). Normal thrust (at x=5m from A)

$N_x = \text{V} \sin\theta + \text{H} \cos\theta$

Here,

$\theta = \tan^{-1}\frac{FE}{DE}$

$\theta = \tan^{-1}\frac{1.5}{5.4}$

$\theta = 10^0 7^0$

$V = Net\ Vertical\ shear\ force\ at\ x = 5m\ from\ A.$

$= V_A - 15$

$= 11.54 - 15$

$V = -3.46\ kN$

$N_x = (-3.46 * \sin 10^0 7^0) + (7.5 * \cos 10^0 7^0)$
$N_x = 6.77\ kN$

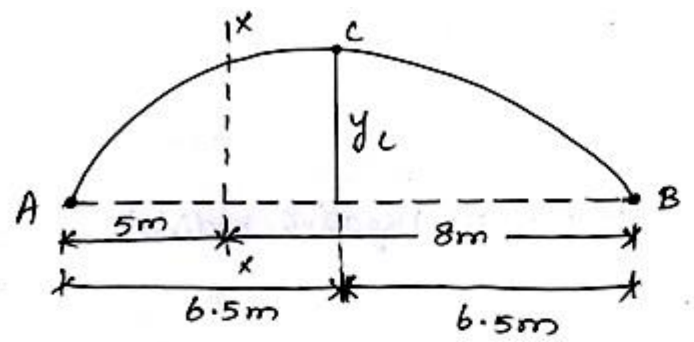
iii).Radial shear

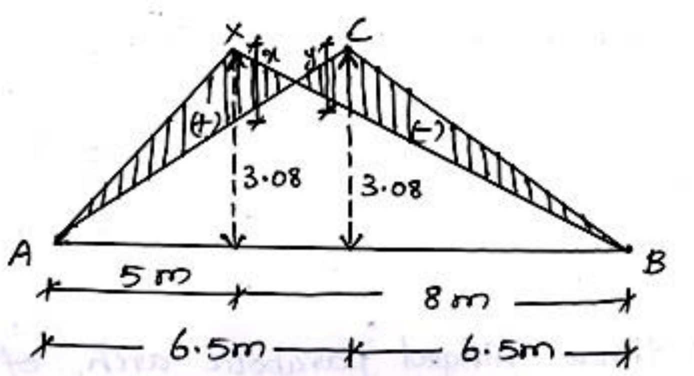
$R_x = \text{V Cos}\theta - \text{H Sin}\theta$

$R_x = (-3.46 * \cos 10^0 7^0) + (7.5 * \sin 10^0 7^0)$

$R_x = -4.72\ kN$

Step 6:Maximum bending moment





$Maximum\ positive\ (sagging) ordinate = \dfrac{x(l-x)}{l}$

$= \dfrac{5(13-5)}{13}$

$= 3.08$

$Net\ maximum\ positive\ ordinate, x = 3.08 - \dfrac{3.08}{6.5} * 5$

$x = 0.71$

$maximum\ positive\ moment = load * ordinate$

$= 15 * 0.71$

$= 10.65\ kN - m$

$Maximum\ negative\ (hogging) ordinate = \dfrac{x(l-x)}{l}$

$$= \frac{5(13-5)}{13}$$

$$= 3.08$$

$$Net\ maximum\ positive\ ordinate, x = 3.08 - \frac{3.08}{6.5} * 6.5$$

$$x = 0.58$$

$$maximum\ positive\ moment = load * ordinate$$

$$= 15 * 0.58$$

$$= 8.7\ kN - m$$

16. **A Parabolic 3hinged arch shown in fig carries loads as indicated Determine i). Resultant reactions at end support. ii). Bending moment, shear (radial) and normal thrust at D, 5m from A.**

<u>Vertical reaction:</u>

$$\sum V = 0$$

$$V_A + V_B = 200 + 20 + 32$$

$$V_A + V_B = 252 ---------- (1)$$

Taking moment about B:

$$V_A * 20 - 20 * 10 * \frac{10}{2} - 32 * 3 - 20 * 17 = 0$$

$$V_A = +87.8\ kN$$

$$V_B = 252 - 87.8$$

$$V_B = 164.2\ kN$$

Horizontal reaction:

$$\sum H = 0$$

$$H_A = H_B$$

Taking moment about C:

$$20 * 10 * \frac{10}{2} - V_B * 10 + 5H_B = 0$$

$$H_B = +128.4\ kN$$

Resultant Reaction:

$$R_A = \sqrt{{V_A}^2 + {H_A}^2}$$

$$R_A = \sqrt{87.8^2 + 128.4^2}$$

$$R_A = 155.5\ kN$$

$$R_B = \sqrt{{V_B}^2 + {H_B}^2}$$

$$R_B = \sqrt{164.2^2 + 128.4^2}$$

$$R_B = 208.44\ kN$$

Maximum B.M:

$B.M\ at\ D = V_A * 5 - H_A * Y_D - 20 * 2$

$B.M\ at\ D = 399 - 128.4Y_D$

$Y_D = \frac{4Y_C x(l-x)}{l^2}$

$Y_D = \frac{4 * 5(20-5)}{20^2}$

$Y_D = 3.75\ m$

$B.M\ at\ D = -82.5\ kN - m$

Radial shear force at x=5 from A:

$S.F, \quad R_x = V_x \cos\theta - H \sin\theta$

$V = vertical\ shear\ force\ at\ x = 5m\ from\ A$

$= 87.8 - 20$

$V = 67.8\ kN$

$H = Horizontal\ shear\ force = 128.4\ kN$

$\theta = \tan^{-1}\left(\frac{4r}{l^2}(l-2x)\right)$

$\theta = \tan^{-1}\left(\frac{4*5}{20^2}(20-2*5)\right)$

$\theta = 45^0$

$R = 67.8 \cos 45 - 128.4 \sin 45$

$R = -42.85\ kN$

Normal thrust at x=5 from A

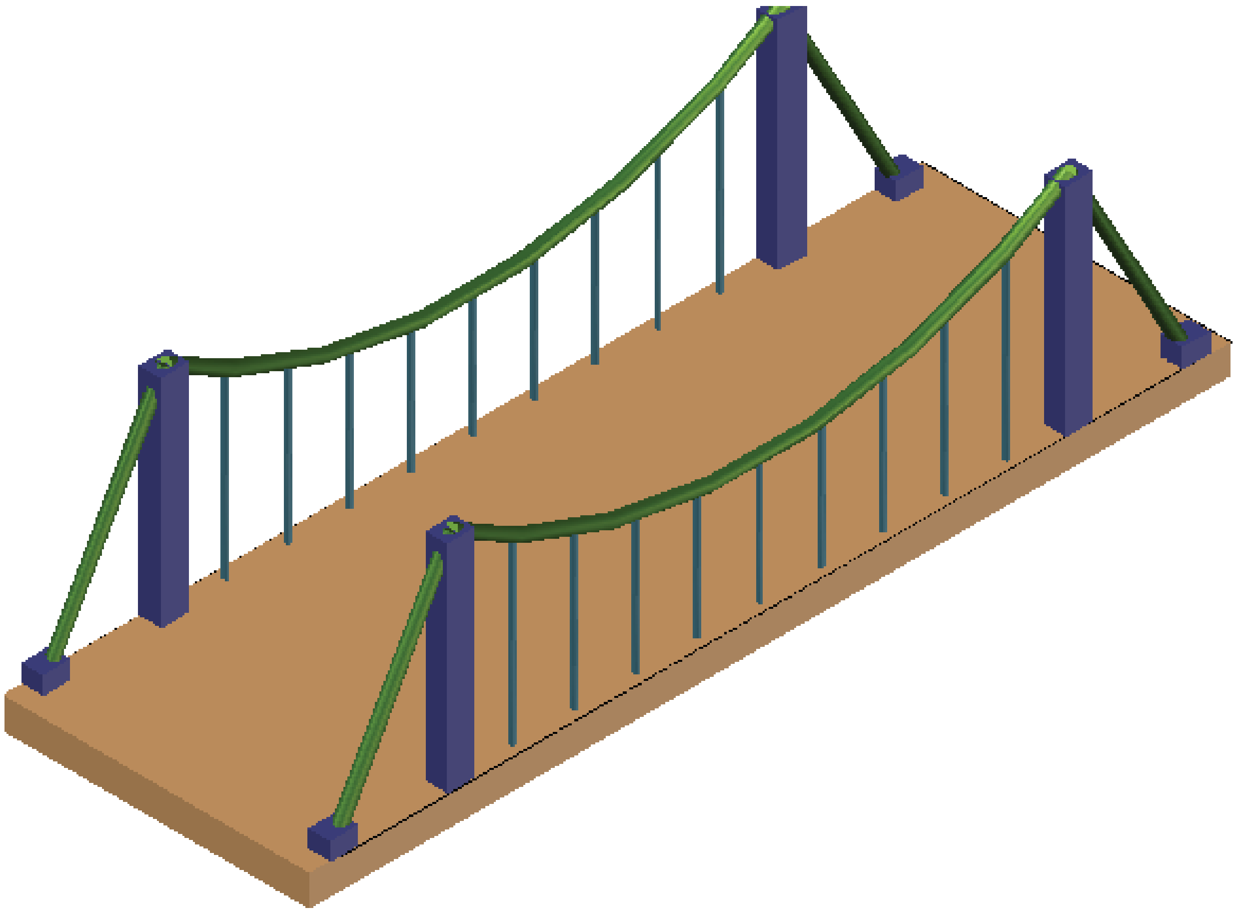
$N_x = V_x \sin\theta + H\cos\theta$

$= 67.8\ \mathrm{Sin}\ 45 + 128.4 \cos 45$

$N_x = 138.73\ kN$

UNIT – IV

CABLES AND SUSPENSION BRIDGES



Components and their functions

1. Suspension cable:

   This is the main load bearing member. Suspension cable will have a central dip of 1/10 to1/15 of the horizontal span. Suspension cables are flexible and hence can change their shape under the load systems since cables cannot take any bending moment and can take only direct tension.

2. Suspenders:

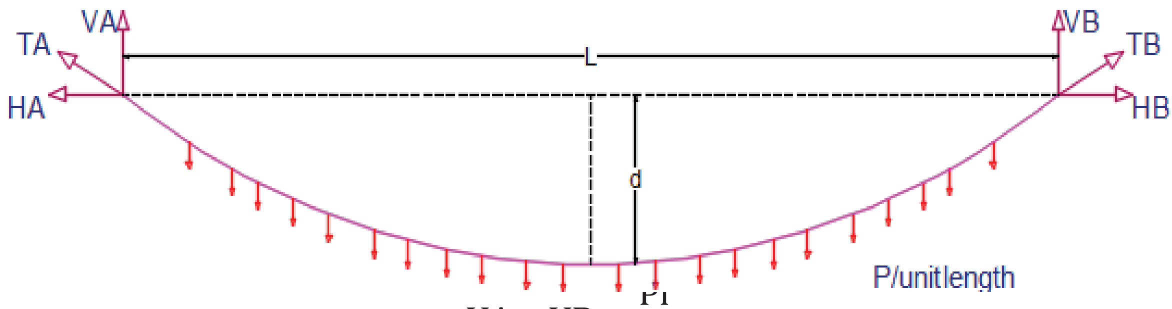
   The stiffening girder with deck slab of roadway is suspended from the suspension cables by means of suspenders or hangers. Suspenders are closely spaced and transfer a part of the traffic load on the deck slab to the suspension cable as a uniformly distributed load.

3. Supporting towers:

   Each suspension cable is supported on 2 towers or pylons on either side. A guide pulley or saddle placed on rollers is usually provided on the towers for passing the suspension cable on them.

REACTIONS, TENSION AND LENGTH OF SUSPENSION CABLE

a) Supports at the same level



$$VA = VB = \frac{Pl}{2}$$

Taking moment about C,

$$\left(VA \times \frac{l}{2}\right) - \left(P \times \frac{l}{2} \times \frac{l}{4}\right) - (H \times d) = 0$$

$$\left(\frac{Pl}{2} \times \frac{l}{2}\right) - \left(P \times \frac{l}{2} \times \frac{l}{4}\right) - (H \times d) = 0$$

$$H = \left(\frac{Pl^2}{8d}\right)$$

$$Mx = (VA \times x) - \left(\frac{Px^2}{2}\right) - (H \times d1) = 0$$

$$Mx = \left(\frac{Pl}{2} \times x\right) - \left(\frac{Px^2}{2}\right) - (H \times d1) = 0$$

$$d1 = \frac{4d}{l^2}[lx - x^2]$$

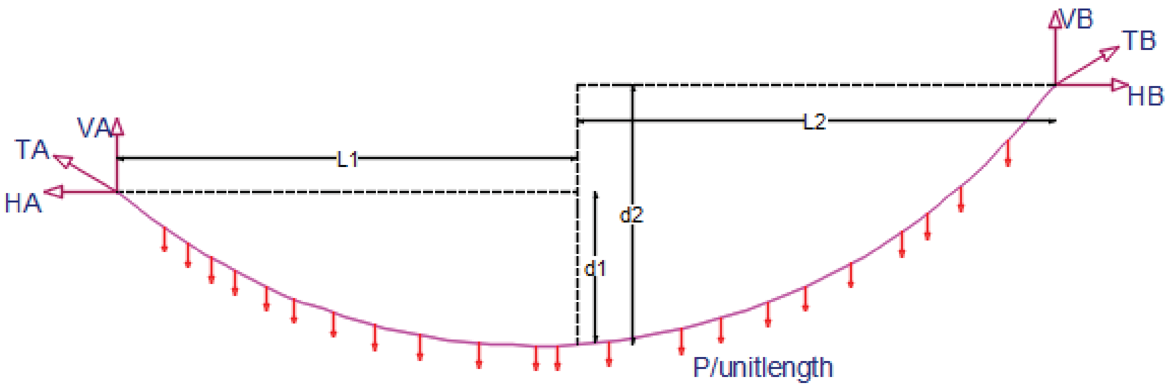
$$TA = TB = \sqrt{V^2 + H^2}$$

$$\tan\theta = \frac{H}{V} = \frac{4d}{l^2}[l - 2x]$$

$$\theta = \tan^{-1}\left[\frac{l}{4d}\right]$$

$$S = l + \frac{8d^2}{3l}$$

b) Supports at the different level



$$VA = Pl1; VB = Pl2$$

$$H = \left(\frac{Pl1^2}{2d1}\right)\ (or)\left(\frac{Pl2^2}{2d2}\right)$$

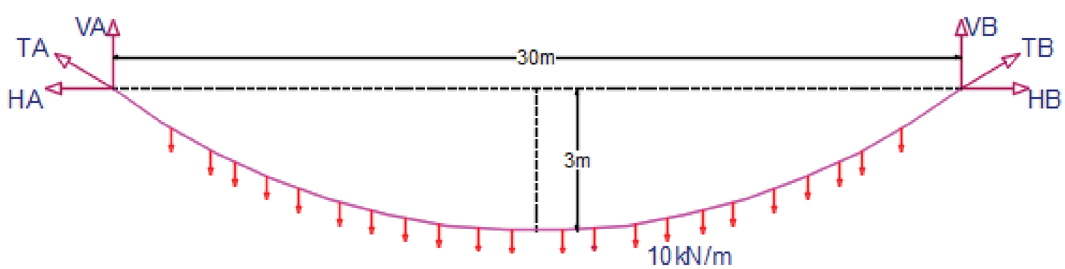
$$T = \sqrt{V^2 + H^2}$$

$$\left(\frac{l1}{l2}\right) = \sqrt{\frac{d1}{d2}}$$

$$S = l + \frac{2d1^2}{3l1} + \frac{2d2^2}{3l2}$$

Problem

A suspension cable having supports at the same level has a span of 30m and a maximum dip of 3m. The cable is loaded with a uniformly distributed load of 10kN/m throughout its length. Find the maximum tension in the cable.



$$VA = VB = \frac{Pl}{2} = \frac{10 \times 30}{2} = 150kN$$

$$H = \left(\frac{Pl^2}{8d}\right) = \frac{10 \times 30^2}{8 \times 3} = 375kN$$

$$T_{max} = \sqrt{V^2 + H^2}$$

$$T_{max} = \sqrt{150^2 + 375^2} = 403.9 \text{ kN}$$

$$S = l + \frac{8d^2}{3l}$$

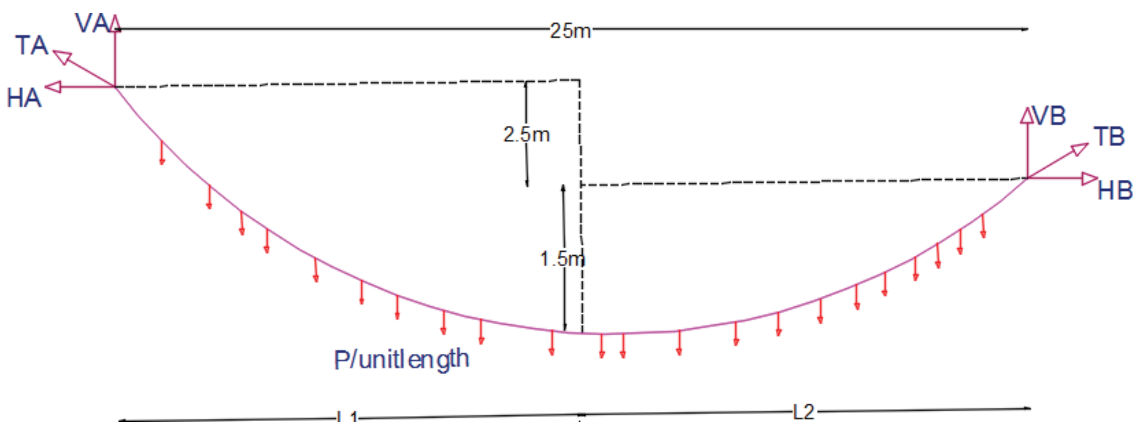
$$S = 30 + \frac{8 \times 3^2}{3 \times 30} = 30.8\text{m}$$

$$\theta = \tan^{-1}\left[\frac{l}{4d}\right]$$

$$\theta = \tan^{-1}\left[\frac{30}{4 \times 3}\right] = 68.2$$

Problem

A suspension cable is supported at 2 points 25m apart. The left support is 2,5m above the right support. The cable is loaded with an udl of 10 kN/m throughout the span. The maximum dip in the cable from the left support is 4m. Find the maximum and minimum tension in the cable.



$$\left(\frac{l1}{l2}\right) = \sqrt{\frac{d1}{d2}}$$

$$l1 = \sqrt{\frac{4}{1.5}} \times l2 = 1.6329l2$$

$$l1 + l2 = l$$

$$1.6329l2 + l2 = 25$$

$$l2 = 9.495m$$

$$l1 = 15.505m$$

$$VA = Pl1 = 10 \times 15.505 = 155.05kN$$

$$VB = Pl2 = —10 \times 9.495 = 94.95kN$$

$$H = \left(\frac{Pl1^2}{2d1}\right) = \left(\frac{10 \times 15.505^2}{2 \times 4}\right) = 300.51kN$$

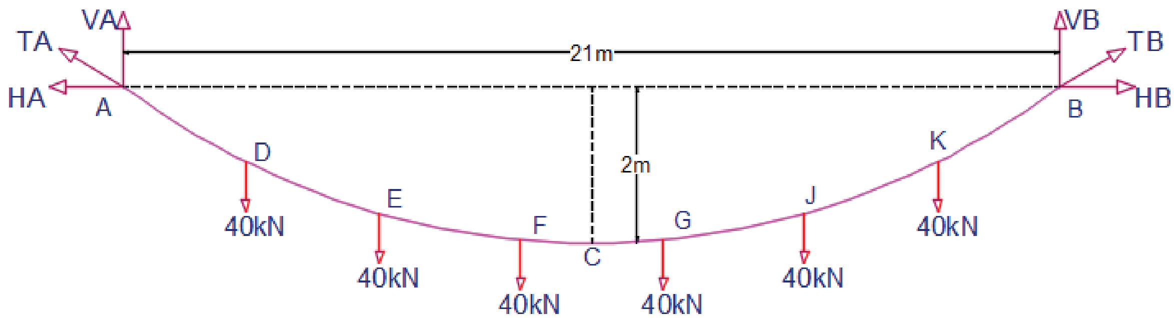
$$H = \left(\frac{Pl2^2}{2d2}\right) = \left(\frac{10 \times 9.495^2}{2 \times 1.5}\right) = 300.51kN$$

$$TA = \sqrt{VA^2 + H^2} = \sqrt{155.05^2 + 300.51^2} = 338.15kN$$

$$TB = \sqrt{VB^2 + H^2} = \sqrt{94.95^2 + 300.51^2} = 315.15kN$$

Problem

A cable of horizontal span 21m is to be used to support six equal loads of 40kN each at 3m spacing. The central dip of the cable is limited to 2m. Find the length of the cable required and also its sectional area if the safe tensile stress is 750N/mm^2



$$VA = VB = \frac{\text{total load}}{2} = \frac{6 \times 40}{2} = 120kN$$

To find the horizontal pull

$$(VA \times 10.5) — (40 \times 7.5) — (40 \times 4.5) — (40 \times 1.5) — (H \times 2) = 0$$

$$H = 360kN$$

Equating moments about D to zero,

$$(120 \times 3) — (360 \times d1) = 0$$

$$d1 = 1m$$

Equating moments about E to zero,

$$(120 \times 6) - (40 \times 3) - (360 \times d2) = 0$$

$$d2 = 1.667m$$

Equating moments about F to zero,

$$(120 \times 9) - (40 \times 6) - (40 \times 3) - (360 \times d3) = 0$$

$$d3 = 2m$$

Length

$$AD = \sqrt{3^2 + 1^2} = 3.162m$$

$$DE = \sqrt{3^2 + (1.667 - 1)^2} = 3.073m$$

$$EF = \sqrt{3^2 + (2 - 1.667)^2} = 3.018m$$

$$S = 2(3.162 + 3.073 + 3.018 + 1.5) = 21.506m$$

$$T_{max} = \sqrt{V^2 + H^2}$$

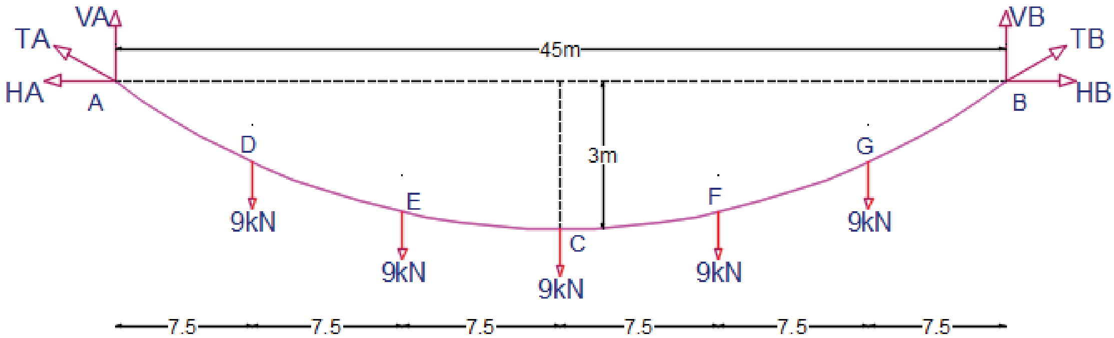
$$T_{max} = \sqrt{120^2 + 360^2} = 379.47 \text{ kN}$$

$$T_{max} = \sigma \times A$$

$$A = \frac{T_{max}}{\sigma} = \frac{379.47 \times 1000}{750} = 505.96 \text{ mm}^2$$

Problem

A cable is used to support five equal and equidistant loads over a span of 45m. Find the length of the cable required and its sectional area if the safe tensile stress is $140N/mm^2$. The central dip of the cable is 3m and loads are 9kN each.



$$VA = VB = \frac{\text{total load}}{2} = \frac{5 \times 9}{2} = 22.5\text{kN}$$

To find the horizontal pull

$$(H \times 3) + (9 \times 7.5) + (9 \times 15) - (VB \times 22.5) = 0$$

$$H = 101.25\text{kN}$$

Equating moments about D to zero,

$$(22.5 \times 7.5) - (101.25 \times d1) = 0$$

$$d1 = 1.67\text{m}$$

Equating moments about E to zero,

$$(22.5 \times 15) - (9 \times 7.5) - (101.25 \times d2) = 0$$

$$d2 = 2.67\text{m}$$

Length

$$AD = \sqrt{7.5^2 + 1.67^2} = 7.683\text{m}$$

$$DE = \sqrt{7.5^2 + (2.667 - 1.667)^2} = 7.566\text{m}$$

$$EF = \sqrt{7.5^2 + (3 - 2.667)^2} = 7.507\text{m}$$

$$S = 2(7.683 + 7.566 + 7.507) = 45.512\text{m}$$

$$T_{max} = \sqrt{V^2 + H^2}$$

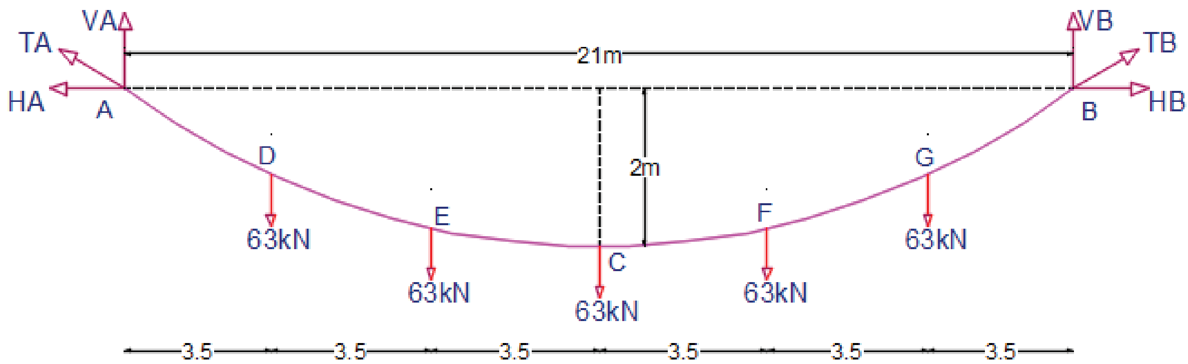
$$T_{max} = \sqrt{22.5^2 + 101.25^2} = 103.72\text{ kN}$$

$$T_{max} = \sigma \times A$$

$$A = \frac{T_{max}}{\sigma} = \frac{103.72 \times 1000}{140} = 740.85\ mm^2$$

Problem

A light suspension bridge is constructed to carry a pathway 3m broad over a channel 21m wide. The pathway is supported by six equidistant suspension rods. The cable has central dip of 2m. The lateral load on the platform is 10kN/m^2. Find the maximum tension in the cable.



$$\text{load} = 21 \times 3 \times 10 = 630\ \text{kN}$$

$$\text{load on each cable} = \frac{630}{2} = 315\ \text{kN}$$

$$\square\ \text{each point load on cable} = \frac{315}{5} = 63\ \text{kN}$$

$$VA = VB = \frac{\text{total load}}{2} = \frac{5 \times 63}{2} = 157.5\text{kN}$$

To find the horizontal pull

$$(VA \times 10.5) - (63 \times 3.5) - (63 \times 7) - (H \times 2) = 0$$

$$H = 496.125\text{kN}$$

Equating moments about D to zero,

$$(157.5 \times 3.5) - (496.125 \times d1) = 0$$

$$d1 = 1.11\text{m}$$

Equating moments about E to zero,

$$(157.5 \times 7) - (63 \times 3.5) - (496.125 \times d2) = 0$$

$$d2 = 1.78\text{m}$$

Length

$$AD = \sqrt{3.5^2 + 1.11^2} = 3.672m$$

$$DE = \sqrt{3.5^2 + (1.78 - 1.11)^2} = 3.56m$$

$$EF = \sqrt{3.5^2 + (2 - 1.78)^2} = 3.507m$$

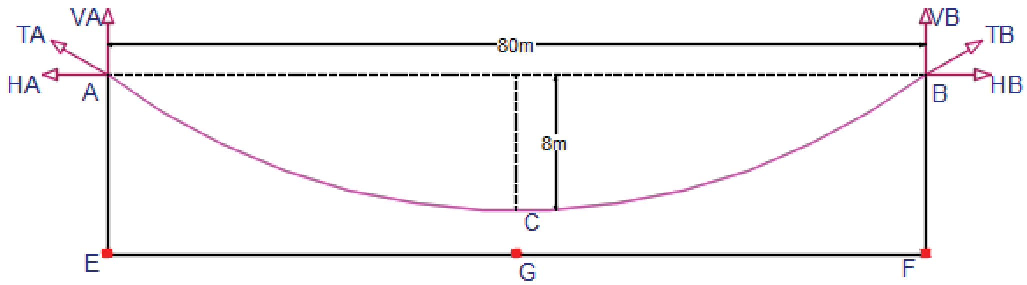
$$S = 2(3.672 + 3.56 + 3.507) = 21.478m$$

$$T_{max} = \sqrt{V^2 + H^2}$$

$$T_{max} = \sqrt{157.5^2 + 496.125^2} = 520.5\ kN$$

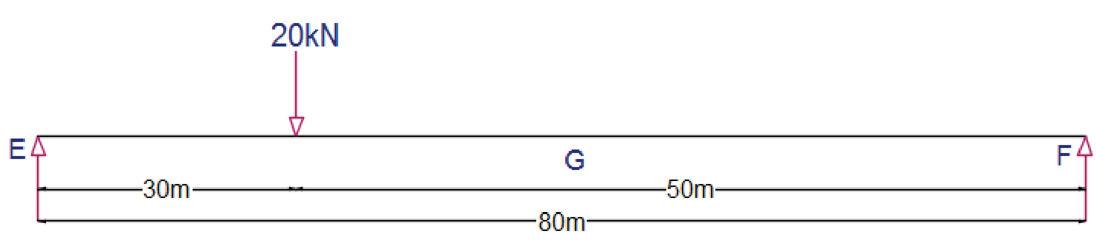
Problem

A suspension bridge cable of span 80m and central dip 8m is suspended from the same level at two towers. the bridge cable is stiffened by a three hinged stiffening girder which carries a single concentrated load of 20 kN at a point of 30m from one end. Sketch the SFD for the girder.



A and B are supports. E,G, F are the hinged points . C is the mid point of the cable.

the applied load is shown in fig.



taking moment about F,

$$(VE \times 80) - (20 \times 50) = 0$$

$$VE = 12.5\ kN$$

$$VF = 7.5\ kN$$

$$\text{moment at C}, \mu c = VF \times \frac{span}{2}$$

$$\mu c = 7.5 \times \frac{80}{2}$$

$$\mu c = 300\ kNm$$

$$H = \frac{\mu c}{d} = \frac{300}{8} = 37.5kN$$

Bending moment

Let 20 kN load is applied at I,

$$B.M\ at\ I = (VF \times 50) — (H \times y) = 0$$

$$y = \frac{4d}{l^2}x[l — x]$$

$$y = \frac{4 \times 8}{80^2} \times 50[80 — 50] = 7.5m$$

$$B.M\ at\ I = (7.5 \times 50) — (300 \times 7.5) = 93.75\ kN$$

Shear force

$$S.F\ at\ I = Vb — H\tan\theta$$

$$Vb = \text{net vertical shear at I}$$

$$Vb = VE — 20 = —7.5kN$$

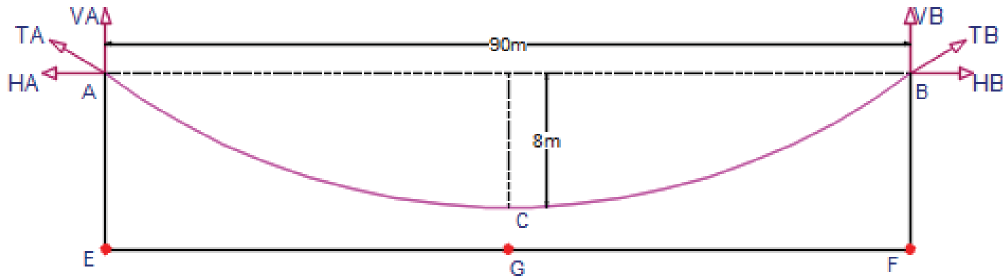
$$\tan\theta = \frac{4d}{l^2}[l — 2x]$$

$$\tan\theta = \frac{4 \times 8}{80^2}[80 — 2 \times 30] = 0.1$$

$$S.F\ at\ I = —7.5 — (37.5 \times 0.1) = —11.25kN$$

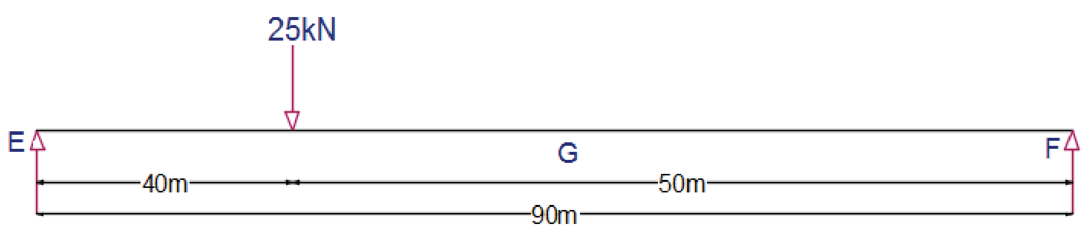
Problem

A suspension bridge cable of span 90m and central dip 6m is suspended from the same level at two towers. the bridge cable is stiffened by a three hinged stiffening girder which carries a single concentrated load of 25 kN at a point of 40m from one end. Sketch the SFD for the girder.



A and B are supports. E,G, F are the hinged points . C is the mid point of the cable.

the applied load is shown in fig.



taking moment about F,

$$(VE \times 90) — (25 \times 50) = 0$$

$$VE = 13.89\ kN$$

$$VF = 11.11\ kN$$

$$\text{moment at C}, \mu c = VF \times \frac{span}{2}$$

$$\mu c = 11.11 \times \frac{90}{2}$$

$$\mu c = 499.95\ kNm$$

$$H = \frac{\mu c}{d} = \frac{499.95}{6} = 83.325kN$$

Bending moment

Let 25 kN load is applied at I,

$$\text{B.M at I} = (VF \times 50) - (H \times y) = 0$$

$$y = \frac{4d}{l^2} x[l - x]$$

$$y = \frac{4 \times 6}{90^2} \times 50[90 - 50] = 5.93\text{m}$$

$$\text{B.M at I} = (11.11 \times 50) - (83.325 \times 5.93) = 61.38 \text{ kN}$$

Shear force

$$\text{S.F at I} = Vb - H\tan\theta$$

$$Vb = \text{net vertical shear at I}$$

$$Vb = VE - 25 = -11.11\text{kN}$$

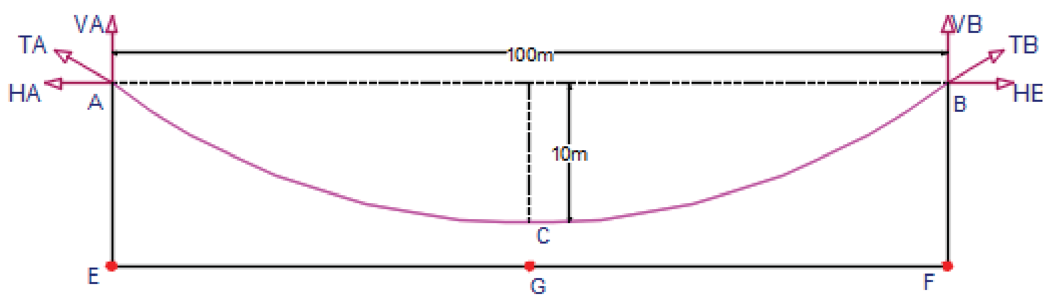
$$\tan\theta = \frac{4d}{l^2}[l - 2x]$$

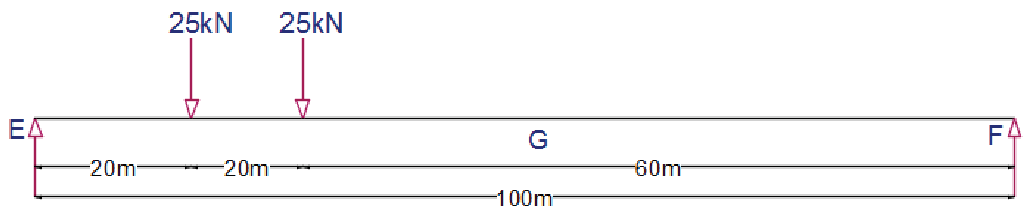
$$\tan\theta = \frac{4 \times 6}{90^2}[90 - 2 \times 40] = 0.03$$

$$\text{S.F at I} = -11.11 - (83.325 \times 0.03) = -13.61\text{kN}$$

Problem

A three hinged stiffening girder of a suspension bridge of 100m span subjected to two point loads 10 kN each placed at 20m and 40m, respectively from the left hand hinge. Determine the bending moment and shear force in the girder at section 30m from each end. Also determine the tension in the cable which has central dip of 10m.





taking moment about F,

$$(VE \times 100) - (10 \times 60) - (10 \times 40) = 0$$

$$VE = 14 \text{ kN}$$

$$VF = 6 \text{ kN}$$

$$\text{moment at C, } \mu c = VF \times \frac{\text{span}}{2}$$

$$\mu c = 6 \times \frac{1000}{2}$$

$$\mu c = 300 \text{ kNm}$$

$$H = \frac{\mu c}{d} = \frac{300}{10} = 30\text{kN}$$

Bending moment at 30m from left hinge

$$B.M \text{ at } G = (VE \times 30) - (H \times y) - (10 \times 10)$$

$$y = \frac{4d}{l^2} x[l - x]$$

$$y = \frac{4 \times 10}{100^2} \times 30[100 - 30] = 8.4\text{m}$$

$$B.M \text{ at } G = (14 \times 30) - (30 \times 8.4) - (10 \times 10) = 68\text{kN}$$

$$B.M \text{ at } I = (VF \times 30) - (H \times y)$$

$$B.M \text{ at } I = (6 \times 30) - (30 \times 8.4) = -72 \text{ kN}$$

Shear force

$$S.F \text{ at } G = Vb - H\tan\theta$$

$$Vb = \text{net vertical shear at } I$$

$$Vb = VE - 10 = 4kN$$

$$\tan\theta = \frac{4d}{l^2}[l - 2x]$$

$$\tan\theta = \frac{4 \times 10}{100^2}[100 - 2 \times 30] = 0.16$$

$$S.F \text{ at } G = 4 - (30 \times 0.16) = 0.8kN$$

$$S.F \text{ at } I = -4 + (30 \times 0.16) = 0.8kN$$

BEAMS CURVED IN PLAN

Arched beams have initial curvature. The curvature is visible only in elevation. in plan they would appear straight.bur beams also has curved in plan. Eg: curved beams, ring beams, supporting water tanks, silos etc.

Curved beams, in addition to bending moment and shear the torsion moment also exist.

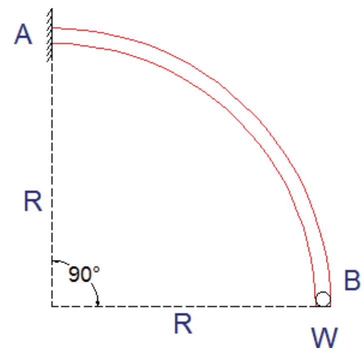
$$U = \int \frac{M^2 ds}{2EI} + \int \frac{T^2 ds}{2GJ}$$

And displacement

$$\eth = \frac{dU}{dp}$$

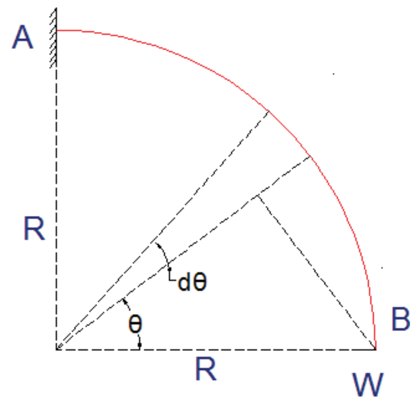
Problem:

A curved beam in the form of a quadrant of a circle of radius R and having a uniform cross section is in a horizontal plane. It is fixed at A and free at B as shown I fig. it carries a vertical concentrated load W at free end B. compute the shear force, bending moment and twisting moment values and sketch variations of the above quantities. Also determine the vertical deflection of the free end B.



The given cantilever is a statically determinate structure.

Consider any point X on the beam at an angle θ from OB



$$CX = R(1 - \cos\theta)$$

i) Shear force at the section X,

$$F_\theta = W$$

F_θ is independent of θ, and uniform through out.

ii) Bending moment at the section X,

$$M_\theta = -W(CB)$$
$$M_\theta = -W.R \sin\theta$$
$$\text{At } \theta = 0,\ M_B = 0$$
$$\text{At } \theta = \frac{\pi}{2},\ M_B = -WR$$

iii) Twisting moment at the section X,

$$T_\theta = -W(CX)$$
$$T_\theta = -WR(1 - \cos\theta)$$
$$\text{At } \theta = 0,\ T_B = -WR(1 - \cos 0) = 0$$
$$\text{At } \theta = \frac{\pi}{2},\ T_B = -WR\left(1 - \cos\frac{\pi}{2}\right) = -WR$$

iv) Deflection at the free endB

$$\text{strain energy}, U = \int \frac{M_\theta^2}{2EI} ds + \int \frac{T_\theta^2}{2GJ}$$

$$U = \frac{1}{2EI}\int_0^{\frac{\pi}{2}} (-WR\sin\theta)^2 R d\theta + \frac{1}{2GJ}\int_0^{\frac{\pi}{2}} [-WR(1-\cos\theta)]^2 R d\theta$$

$$U = \frac{1}{2EI}\int_0^{\frac{\pi}{2}} W^2 R^2 \sin^2\theta R d\theta + \frac{1}{2GJ}\int_0^{\frac{\pi}{2}} W^2 R^2 (1+\cos^2\theta - 2\cos\theta) R d\theta$$

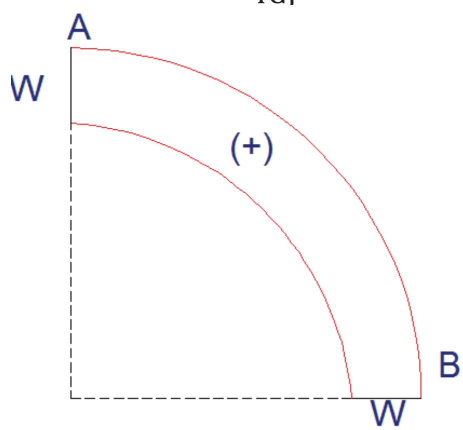
$$U = \frac{1}{2EI}\int_0^{\frac{\pi}{2}} W^2 R^3 \left(\frac{1-\cos2\theta}{2}\right) d\theta + \frac{1}{2GJ}\int_0^{\frac{\pi}{2}} W^2 R^3 \left(1 + \left(\frac{1-\cos2\theta}{2}\right) - 2\cos\theta\right) d\theta$$

$$U = \frac{W^2R^3}{4EI}\left[\theta - \frac{\sin2\theta}{2}\right]_0^{\frac{\pi}{2}} + \frac{W^2R^3}{4GJ}\left[3\theta + \frac{\sin2\theta}{2} - 4\sin\theta\right]_0^{\frac{\pi}{2}}$$

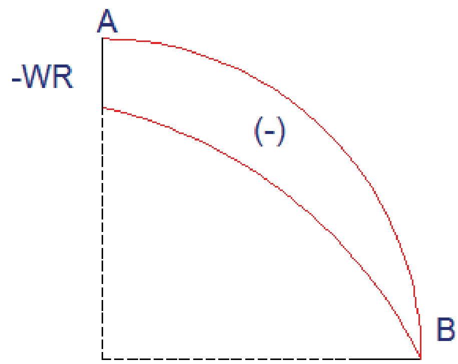
$$U = \frac{\pi W^2R^3}{8EI} + \frac{W^2R^3}{8GJ}[3\pi - 8]$$

$$\delta_B = \frac{dU}{dW}$$

$$\delta_B = \frac{\pi WR^3}{4EI} + \frac{WR^3}{4GJ}[3\pi - 8]$$



SFD



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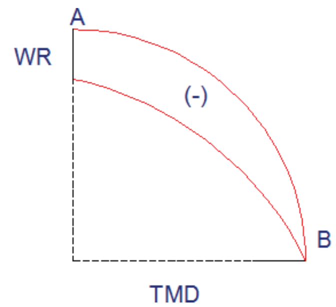
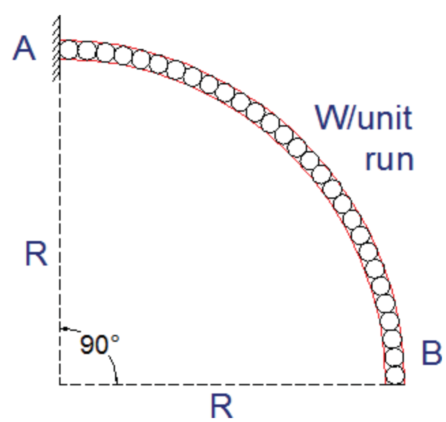


Problem:

A curved beam AB of uniform cross section is horizontal in plan and in the form of a quadrant of a circle of radius R. the beam is fixed at A and free at B. it carries a uniformly distributed load of W/unit run over the entire length of yhe beam as shown. Calculate the shaer force and bending moment and torsional moment values at A and B and sketch the variations same. Also deermine the deflection at the free end B.



The given cantilever is a statically determinate structure.

Consider any point X on the beam at an angle θ from OB

i) Shear force at the section X,

$$F_\theta = W.R\theta$$

$$\text{At } \theta = 0,\ F_B = 0$$

$$\text{At } \theta = \frac{\pi}{2},\ F_A = WR\frac{\pi}{2} = \frac{\pi WR}{2}$$

ii) Bending moment at the section X,

The bending moment due to load will be negative since tension will occur at the top.
Let us consider the bending moment M_θ at X, at θ from the free end.
The bending moment due to load on an element Rdθ at an angleφ from OX is given by

$$dM_\theta = -W.R\, d\phi\, R\sin\phi$$

$$M_\theta = -\int_0^\theta w\, Rd\emptyset\, R\sin\emptyset$$

$$M_\theta = -wR^2 \int_0^\theta \sin\emptyset\, d\emptyset$$

$$M_\theta = -wR^2(1 - \cos\theta)$$

At $\theta = 0$, $M_B = 0$

$$\text{At } \theta = \frac{\pi}{2}, M_{\text{Æ}} = -WR^2$$

iii) Twisting moment at the section X,

The twisting momet due to the udl will be negative since the twist would be anti clockwise an the right face.

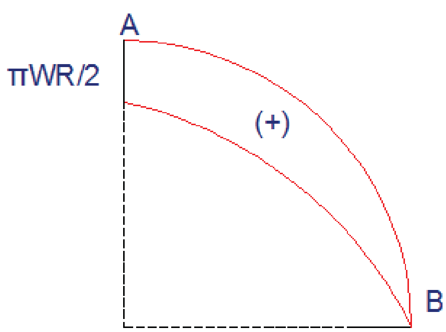
$$T_\theta = -\int_0^\theta w\, Rd\emptyset\, R(1 - \cos\emptyset)$$

$$T_\theta = -wR^2 \int_0^\theta (1 - \cos\emptyset)\, d\emptyset$$

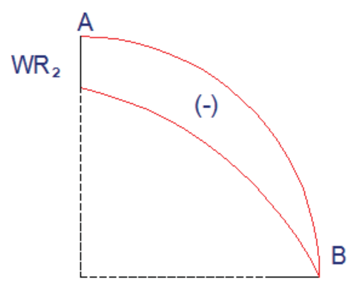
$$T_\theta = -wR^2(\theta - \sin\theta)$$

At $\theta = 0$, $T_B = 0$

$$\text{At } \theta = \frac{\pi}{2}, T_{\text{Æ}} = -WR^2\left(\frac{\pi}{2} - 1\right)$$



SFD



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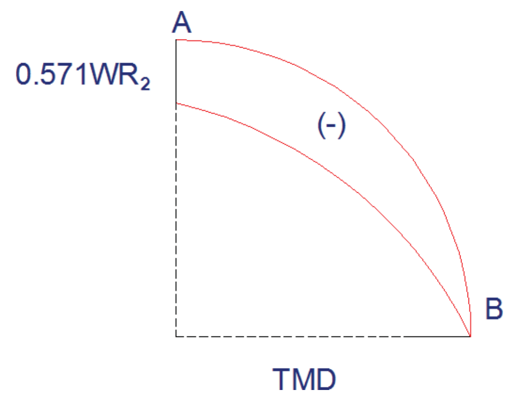
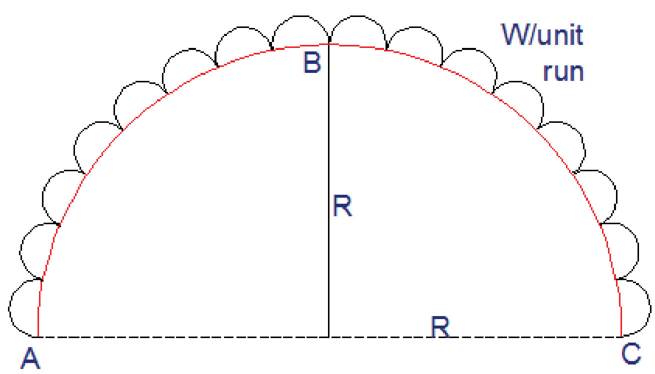
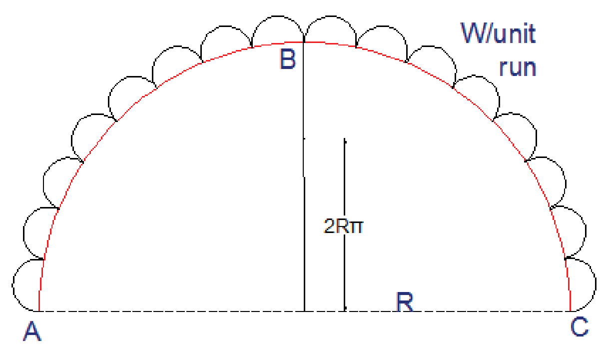


Problem:

A curved beam semicircular in plan and supported on three equally spaced supports. The beam carries a udl of wl unit of the circular length. Analyse the beam and sketch the bending moment and twisting moment diagrams.



The curved beam is shown in fig. the XX and YY axes are as shown and ZZ is vertical axis, the unknown reactions are VA,VB and VC. The end supported do not exert an moment reaction. There are three equations of static equilibrium equations. Hence the structure is externally determinate.



total load = w × ɎR

taking moments of all forces at B

$$2(VA \times R) - \pi wR\left(R - \frac{2R}{\pi}\right) = 0$$

$$VA = wR\left(\frac{\pi - 2}{2}\right)$$

$$VB = 2wR$$

Bending and twisting moment:

Consider a section X located at an angle θ with OA.

Take segment Rdϕ at an angle ϕ from x.

Bending moment at x is

$$M_\theta = VA \times AN - \int_0^\theta w\, Rd\phi\, R\sin\phi$$

$$M_\theta = wR\left(\frac{\pi - 2}{2}\right) R\sin\theta - \int_0^\theta w\, Rd\phi\, R\sin\phi$$

$$M_\theta = wR^2\left[\frac{\pi - 2}{2}\sin\theta - (1 - \cos\theta)\right]$$

At $\theta = 0$, $M_A = 0$

$$\text{At } \theta = \frac{\pi}{2},\ M_B = -0.429WR^2$$

Maximum bending moment might occur between A and B

Maximum bending moment is

$$\frac{dM_\theta}{d\theta} = 0$$

$$\frac{d}{d\theta} wR^2\left[\frac{\pi - 2}{2}\sin\theta - (1 - \cos\theta)\right] = 0$$

$$\left[\frac{\pi - 2}{2}\cos\theta - \sin\theta)\right] = 0$$

$$\tan\theta = \frac{\pi - 2}{2}$$

$$\theta = 29.43$$

$$M_{max} = 0.1515WR^2$$

Point of contraflexure

At point of contraflexure

$$M_\theta = 0$$

$$M_\theta = wR^2\left[\frac{\pi - 2}{2}\sin\theta - (1 - \cos\theta)\right] = 0$$

$$\left[\frac{\pi - 2}{2}\sin\theta - (1 - \cos\theta)\right] = 0$$

$$\frac{(1 - \cos\theta)}{\sin\theta} = \left[\frac{\pi - 2}{2}\right]$$

$$\theta = 59.27$$

Twisting moment

$$T_\theta = -V_A \times XN - \int_0^\theta w\,Rd\varnothing\,R(1 - \cos\varnothing)$$

$$T_\theta = wR^2\left[-\left(\frac{\pi - 2}{2}\right)(1 - \cos\theta) + \theta - \sin\theta\right] = 0$$

At $\theta = 0$, $T_A = 0$

At $\theta = \frac{\pi}{2}$, $T_B = 0$

Maximum torsional moment is

$$\frac{dT_\theta}{d\theta} = 0$$

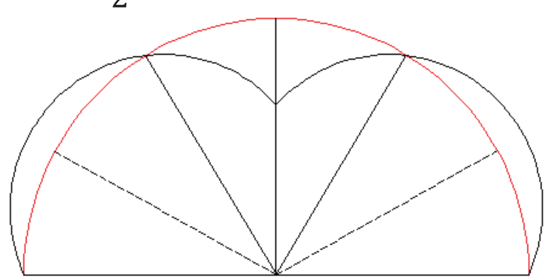
$$\frac{d}{d\theta} wR^2\left[-\left(\frac{\pi - 2}{2}\right)(1 - \cos\theta) + \theta - \sin\theta\right] = 0$$

$$\left[\frac{\pi - 2}{2}\cos\theta - \sin\theta)\right] = 0$$

$$\frac{(1 - \cos\theta)}{\sin\theta} = \left[\frac{\pi - 2}{2}\right]$$

$$\theta = 59.27$$

$$T_{max} = wR^2\left[-\left(\frac{\pi - 2}{2}\right)(1 - \cos\theta) + \theta - \sin\theta\right] = -0.1043wR^2$$



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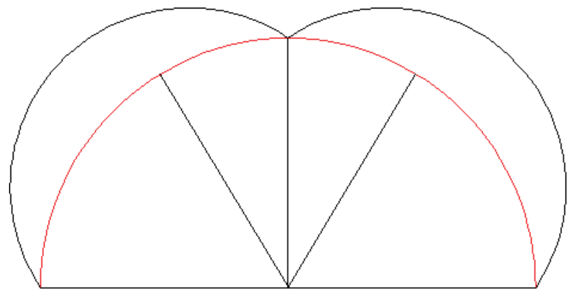
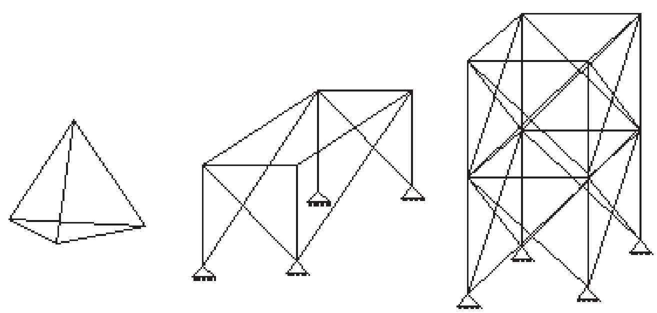


SPACE TRUSSES

Space trusses are more common in practice than we care to think. A space truss or space frame is a three dimensional assemblage of line members each member being joined at its ends, either to the foundation or to other members by friction less ball and socket joints. The simplest space frame consists of six member joined to form a tetrahedron. Beginning with a six member tetrahedron, a stable space frame can be constructed by successive addition of three new members and a new joint.



In frame trusses, the relationship between the number of members (m) and number of joints (j) is given by

$$m=(3j-6)$$

If the truss or frame has less number of members than what is given by the the above equation then the trusses or frame will be unstable. If it more members, the frame will be internally statically indeterminate.

Tension coefficient method

The force per unit length of a member is usually known as the tension coefficient of the member. The tension coefficient of the member is defined as the pull or tension in that member divided by its length. Thus

$$t = \frac{T}{L}$$

where,

t- Tension coefficient of the member

T – Tension or pull in the member

L – Length of the member

$$T_x = T_{ij}\cos\theta$$

$$T_x = T_{ij}\frac{x_{ij}}{l_{ij}}$$

$$T_x = \left(\frac{T_{ij}}{l_{ij}}\right) x_{ij}$$

$$x_{ij} = x_j - x_i$$

$$T_x = t_{ij}x_{ij}$$

$$T_y = t_{ij}y_{ij}$$

$$l_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}$$

Analysis procedure using tension coefficients

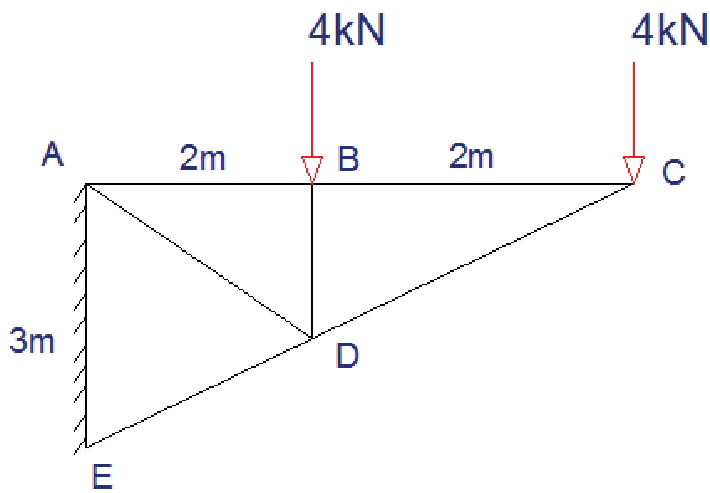
1. List the coordinates of each joint(node) of the truss.
2. Determine the projected length x_{ij} and y_{ij} of each member of the truss. Determine the length l_{ij} of each member.
3. Resolve the applied forces at each joint in the X and Y directions. Determine the support reactions and their X and Y components.
4. Identify a node with only two unknown member forces and apply the equations of equilibrium. The solution yields the tension coefficients for the members at the node.
5. Select the next joint with only two unknown and apply the equations of equilibrium and obtain the tension coefficients.
6. Repeat the next step 5 till the tension coefficients of all the members are obtained.

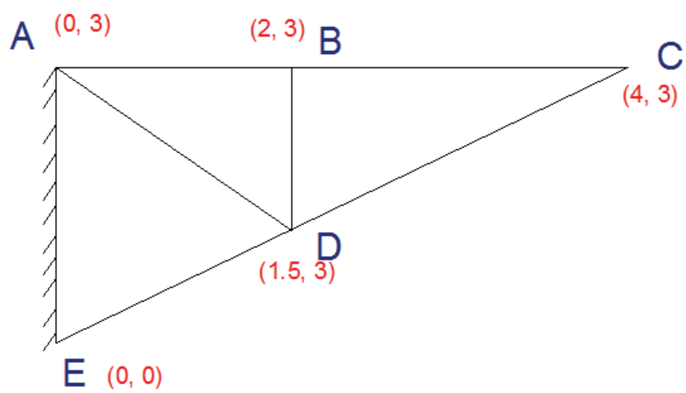
7. Compute the member forces from the tension coefficients obtained as above, using the tension formula.

$$T_{ij} = t_{ij}x_{ij}$$

Problem:

Using tension coefficient method analyse the cantilever plane truss shown and find the member forces.



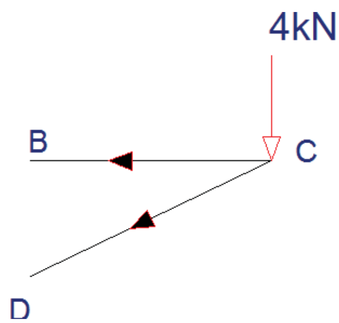


S.no	**Member**	x_i	x_j	x_{ij} **(xj - xi)**	y_i	y_j	y_{ij} **(yj - yi)**	l_{ij} $\sqrt{x_{ij}^2 + y_{ij}^2}$
1	AB	0	2	2	3	3	0	2
2	BC	2	4	2	3	3	0	2
3	CD	4	2	-2	3	1.5	-1.5	2.5

4	DE	2	0	-2	1.5	0	-1.5	2.5
5	AD	0	2	2	3	1.5	-1.5	2.5
6	BD	2	2	0	3	1.5	-1.5	1.5

Calculation of Tension coefficients

Joint C,



$$\Sigma H = 0$$

$$(x_{cb} \times t_{cb}) + (x_{cd} \times t_{cd}) = 0$$

$$-2\ t_{cb}\ -2\ t_{cd} = 0$$

$$t_{cb} = t_{cd}$$

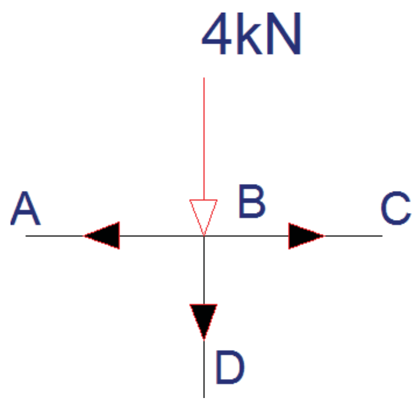
$$\Sigma V = 0$$

$$(y_{cb} \times t_{cb}) + (y_{cd} \times t_{cd})\ -4 = 0$$

$$-1.5\ t_{cd} = 4$$

$$t_{cd} = -2.67\ kN/m$$

Joint B,



$\Sigma H = 0$

$(x_{ba}\times t_{ba}) + (x_{bc}\times t_{bc}) = 0$

$-2\ t_{ba} + 2\ t_{bc} = 0$

$t_{ba} = t_{bc}$

$t_{ba} = 2.67$ kN/m

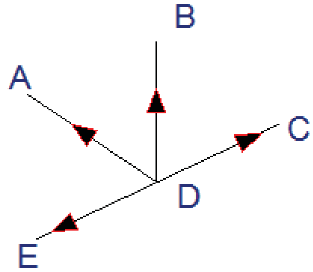
$\Sigma V = 0$

$(y_{bd}\times t_{bd}) - 4 = 0$

$-1.5\ t_{bd} = 4$

$t_{bd} = -2.67$ kN/m

Joint D,



$\Sigma H = 0$

$(x_{da}\times t_{da}) + (x_{de}\times t_{de}) + (x_{dc}\times t_{dc}) = 0$

$-2\ t_{da} - 2\ t_{de} + (2\times -2.67) = 0$

$t_{da} + t_{de} = -2.67$ ……… Eq1

$\Sigma V = 0$

$(y_{da} \times t_{da}) + (y_{de} \times t_{de}) + (y_{dc} \times t_{dc}) + (y_{bd} \times t_{bd}) = 0$

$t_{da} - t_{de} = 5.34$ Eq2

solving equations 1 and 2 we get

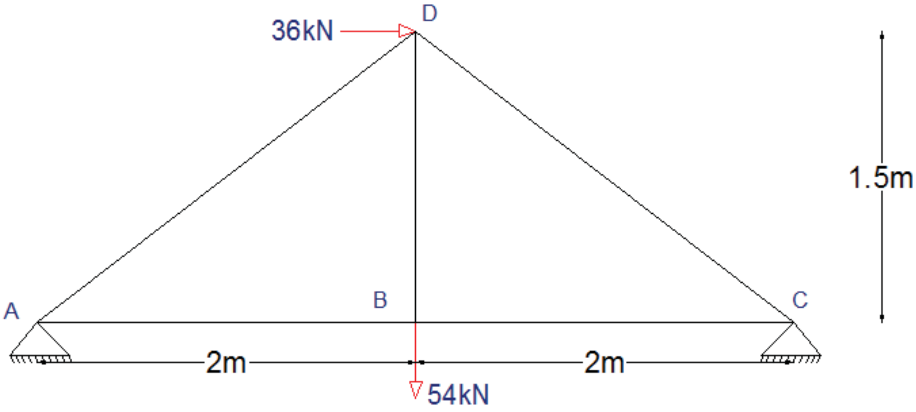
$t_{da} = 1.335$ kN/m

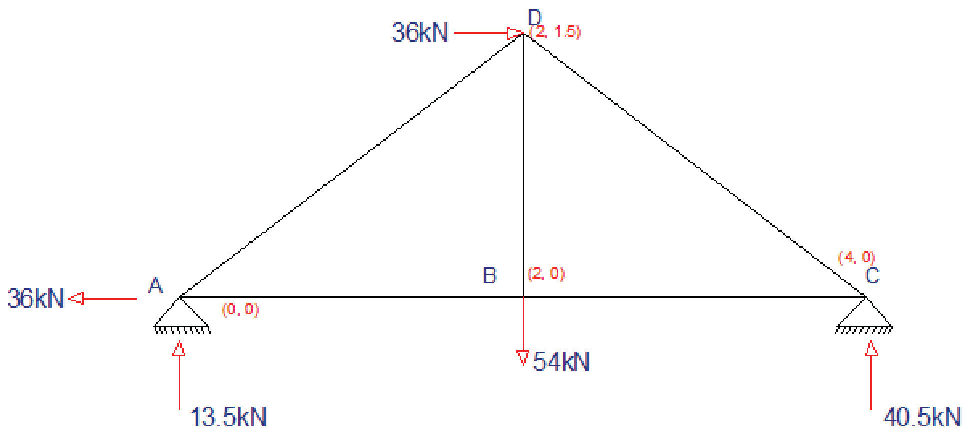
$t_{de} = -4$ kN/m

S.no	**Member**	**t_{ij}**	**l_{ij}** $\sqrt{x_{ij}^2 + y_{ij}^2}$	**T_{ij}**	**Nature**
1	AB	2.67	2	5.34	Tensile
2	BC	2.67	2	5.34	Tensile
3	CD	-2.67	2.5	-6.675	compressive
4	DE	-4	2.5	-10	compressive
5	AD	1.335	2.5	3.33	Tensile
6	BD	-2.67	1.5	-4	compressive

Problem:

Using tension coefficient method analyse the plane truss shown and find the member forces.





Taking moment about A,

$$(VC \times 4) — (36 \times 1.5) — (54 \times 2) = 0$$

$$VC = 40.5 \text{ kN}$$

$$VA = 13.5 \text{ kN}$$

$$\Sigma H = 0$$

$$HA = 36 \text{ kN}$$

S.no	Member	x_i	x_j	x_{ij} (xj - xi)	y_i	y_j	y_{ij} (yj - yi)	l_{ij} $\sqrt{x_{ij}^2 + y_{ij}^2}$
1	AB	0	2	2	0	0	0	2
2	BC	2	4	2	0	0	0	2
3	CD	4	2	-2	0	1.5	1.5	2.5
4	DA	2	0	-2	1.5	0	-1.5	2.5
5	BD	2	2	0	0	1.5	1.5	1.5

Calculation of Tension coefficients

Joint A,

$\Sigma H = 0$

$(x_{ad} \times t_{ad}) + (x_{ab} \times t_{ab}) - 36 = 0$

$2\, t_{ad} + 2\, t_{ab} = 36$

$t_{ad} + t_{ab} = 18$

$\Sigma V = 0$

$(y_{ad} \times t_{ad}) + (y_{ab} \times t_{ab}) + 13.5 = 0$

$1.5\, t_{ad} = -13.5$

$t_{ad} = -9$ kN/m

$t_{ab} = 27$ kN/m

Joint B,

$\Sigma H = 0$

$(x_{ba} \times t_{ba}) + (x_{bc} \times t_{bc}) = 0$

$-2\, t_{ba} + 2\, t_{bc} = 0$

$t_{ba} = t_{bc}$

$t_{bc} = 27$ kN/m

$\Sigma V = 0$

$(y_{bd} \times t_{bd}) - 54 = 0$

$1.5\ t_{bd} = 54$

$t_{bd} = 36$ kN/m

Joint C,

$\Sigma H = 0$

$(x_{cd} \times t_{cd}) + (x_{cb} \times t_{cb}) = 0$

$-2\ t_{cd} - 2\ t_{cb} = 0$

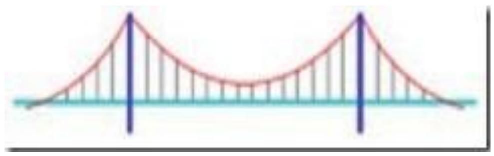
$t_{cd} = -27$ kN/m

S.no	**Member**	t_{ij}	l_{ij} $\sqrt{x_{ij}^2 + y_{ij}^2}$	T_{ij}	**Nature**
1	AB	27	2	54	Tensile
2	BC	27	2	54	Tensile
3	CD	-27	2.5	-67.5	compressive
4	DA	-9	2.5	-22.5	compressive
5	BD	36	1.5	54	Tensile

PART-A

1. What are cable structures?

Long span structures subjected to tension and uses suspension cables for supports. Examples of cable structures are suspension bridges, cable stayed roof.



Suspension bridge – cable structure

2. What is the true shape of cable structures?

Cable structures especially the cable of a suspension bridge is in the form of a catenary. Catenary is the shape assumed by a string / cable freely suspended between two points.

3. What is the nature of force in the cables?

Cables of cable structures have only tension and no compression or bending.

4. What is a catenary?

Catenary is the shape taken up by a cable or rope freely suspended between two supports and under its own self weight.

5. Mention the different types of cable structures.

Cable structures are mainly of two types: (a) Cable over a guide pulley

(b) Cable over a saddle

6. Briefly explain cable over a guide pulley.

Cable over a guide pulley has the following properties:

Ø Tension in the suspension cable = Tension in the anchor cable

Ø The supporting tower will be subjected to vertical pressure and bending due to net horizontal cable tension.

7. Briefly explain cable over saddle.

Cable over saddle has the following properties:

Ø Horizontal component of tension in the suspension cable = Horizontal component of tension in the anchor cable

Ø The supporting tower will be subjected to only vertical pressure due to cable tension.

8. What are the main functions of stiffening girders in suspension bridges?

Stiffening girders have the following functions.

Ø They help in keeping the cables in shape

Ø They resist part of shear force and bending moment due to live loads.

9. What is the degree of indeterminacy of a suspension bridge with two hinged stiffening girder?

The two hinged stiffening girder has one degree of indeterminacy.

10. Differentiate between plane truss and space truss.

Plane truss:

Ø All members lie in one plane

Ø All joints are assumed to be hinged.

Space truss:

Ø This is a three dimensional truss

Ø All joints are assumed to be ball and socketed.

11. Define tension coefficient of a truss member.

The tension coefficient for a member of a truss is defined as the pull or tension in the member divided by its length, i. e. the force in the member per unit length.

12. Give some examples of beams curved in plan.

Curved beams are found in the following structures.

Ø Beams in a bridge negotiating a curve

Ø Ring beams supporting a water tank

Ø Beams supporting corner lintels

Ø Beams in ramps

13. What are the forces developed in beams curved in plan?

Beams curved in plan will have the following forces developed in them:

Ø Shear forces

Ø Torsional moments

14. What are the significant features of circular beams on equally spaced supports?

Ø Slope on either side of any support will be zero.

Ø Torsional moment on every support will be zero

15. Give the expression for calculating equivalent UDL on a girder.

Equivalent UDL on a girder is given by:

$$We = \frac{Total\ load\ on\ girder}{Span\ of\ girder}$$

16. Give the range of central dip of a cable.

The central dip of a cable ranges from 1/10 to 1/12 of the span.

17. Give the expression for determining the tension T in the cable.

The tension developed in the cable is given by

$$T = \sqrt{H^2 + V^2}$$

Where, H = horizontal component and V = vertical component.

18. Give the types of significant cable structures

Linear structures:

Ø Suspension bridges

Ø Cable-stayed beams or trusses

Ø Cable trusses

Ø Straight tensioned cables

Three-dimensional structures:

Ø 3D cable trusses

Ø Tensegrity structures

Ø Tensairity structures

19. What are cables made of?

Cables can be of mild steel, high strength steel, stainless steel, or polyester fibres. Structural cables are made of a series of small strands twisted or bound together to form a much larger cable. Steel cables are either spiral strand, where circular rods are twisted together or locked coil strand, where individual interlocking steel strands form the cable (often with a spiral strand core).

Spiral strand is slightly weaker than locked coil strand. Steel spiral strand cables have a Young's modulus, E of 150 ± 10 kN/mm^2 and come in sizes from 3 to 90 mm diameter. Spiral strand suffers from construction stretch, where the strands compact when the cable is loaded.

UNIT – V

PLASTIC ANALYSIS

Assumptions made to evaluate the fully plastic moment of a section

1. Plane transverse sections remain plane and normal to the longitudinal axis after bending, the effect of shear being neglected.
2. Modulus of elasticity has the same value in tension and compression.
3. The material is homogeneous and isotropic in both the elastic and plastic state
4. There is no resultant axial force on the beam
5. The cross section of the beam is symmetrical about an axis through its centroid parallel to the plane of bending.
6. Longitudinal fibers are free to expand and contract without affecting the fibers in the lateral direction.

$$M_P = \sigma_y \times z_p$$

M_P – plastic moment

σ_y – yield stress

z_p – plastic modulus of the section

Shape factor:

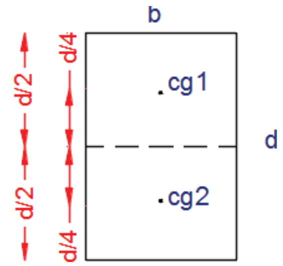
It is defined as the ratio between plastic moment of a section to the yield moment of the section.

$$S = \frac{M_p}{M_y}$$

$$S = \frac{\sigma_y \times Z_p}{\sigma_y \times Z}$$

$$S = \frac{Z_p}{Z}$$

Derive the shape factor for a rectangular section



$$S = \frac{Z_p}{Z}$$

$$Z_p = \frac{A}{2}(y1 + y2)$$

$$Z_p = \frac{bd}{2}\left(\frac{d}{4} + \frac{d}{4}\right)$$

$$Z_p = \frac{bd}{2}\left(\frac{d}{2}\right) = \frac{bd^2}{4}$$

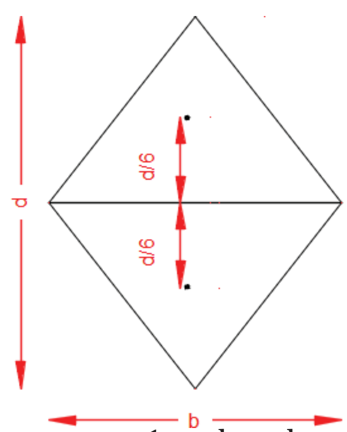
$$Z = \frac{bd^2}{6}$$

$$S = \frac{Z_p}{Z}$$

$$S = \frac{bd^2}{4} \times \frac{6}{bd^2}$$

$$S = \frac{6}{4} = 1.5$$

Derive the shape factor for a diamond section



$$y1 = \frac{1}{3} \times \frac{d}{2} = \frac{d}{6}$$

$$y2 = \frac{1}{3} \times \frac{d}{2} = \frac{d}{6}$$

$$Z_p = \frac{A}{2}(y1 + y2)$$

$$Z_p = \frac{1}{2} \times b \times d \times \frac{1}{2}\left(\frac{d}{6} + \frac{d}{6}\right)$$

$$Z_p = \frac{bd}{4}\left(\frac{d}{3}\right) = \frac{bd^2}{12}$$

$$Z = \frac{I}{y}$$

$$I = \frac{bd^3}{12}$$

$$I = \left\{\frac{1}{12} \times b \times \left(\frac{d}{2}\right)^3\right\} \times 2$$

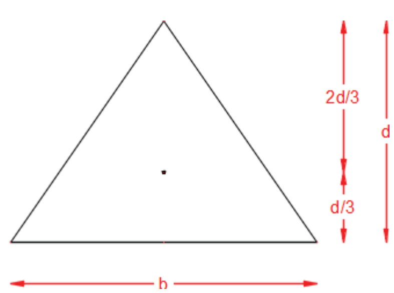
$$I = \frac{bd^3}{48}$$

$$Z = \frac{bd^3}{48} \times \frac{2}{d} = \frac{bd^2}{24}$$

$$S = \frac{bd^2}{12} \times \frac{24}{bd^2}$$

$$S = 2$$

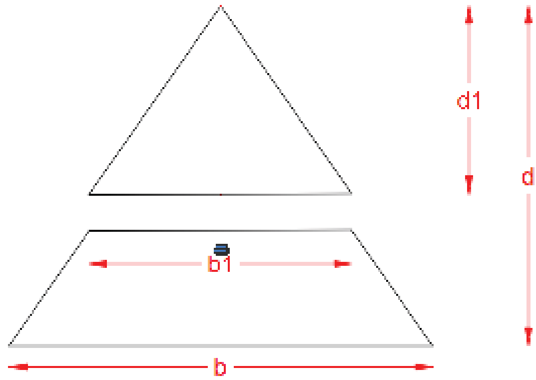
Derive the shape factor for a triangular section



$$Z = \frac{I}{y}$$

$$I = \frac{bd^3}{36}$$

$$Z = \frac{bd^3}{36} \times \frac{3}{2d} = \frac{bd^2}{24}$$



$$\frac{b1}{b} = \frac{d1}{d}$$

$$b1 = \frac{d1}{d} \times b$$

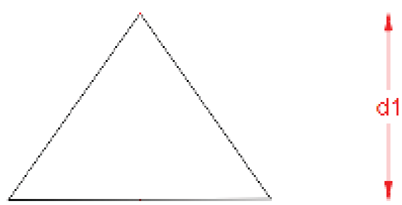
$$A1 = A2 = \frac{A}{2}$$

$$\frac{1}{2} \times b1 \times d1 = \frac{1}{2} \times \frac{1}{2} \times b \times d$$

$$d1^2 = \frac{d^2}{2}$$

$$d1 = \frac{d}{\sqrt{2}}$$

$$b1 = \frac{b}{\sqrt{2}}$$



$$y1 = \frac{1}{3} \times \frac{d}{\sqrt{2}}$$

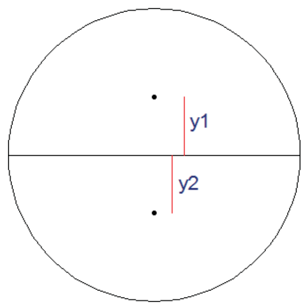
$$y2 = \frac{(\sqrt{2}b + 2b)}{(\frac{b}{\sqrt{2}} + b)} \times \frac{0.293d}{3}$$

$$Z_p = \frac{bd}{4}(0.236d + 0.155d)$$

$$Z_p = 0.09775bd^2$$

$$S = \frac{Z_p}{Z} = 2.346$$

Derive the shape factor for a circular section



$$S = \frac{Z_p}{Z}$$

$$Z = \frac{I}{y} = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$$

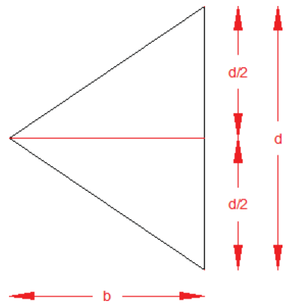
$$Z_p = \frac{A}{2}(y1 + y2)$$

$$y1 = y2 = \frac{4r}{3\pi} = \frac{2d}{3\pi}$$

$$Z_p = \frac{\pi \times d^2}{8}\left(\frac{2d}{3\pi} + \frac{2d}{3\pi}\right) = \frac{d^3}{6}$$

$$S = \frac{Z_p}{Z} = \frac{d^3}{6} \times \frac{32}{\pi d^3} = 1.697$$

Derive the shape factor for a triangular section



$$S = \frac{Z_p}{Z}$$

$$Z = \frac{I}{y} = = \frac{1}{12} \times b \times \left(\frac{d}{2}\right)^3 \times 2 \times \frac{2}{d} = \frac{bd^2}{24}$$

$$Z_p = \frac{A}{2}(y1 + y2)$$

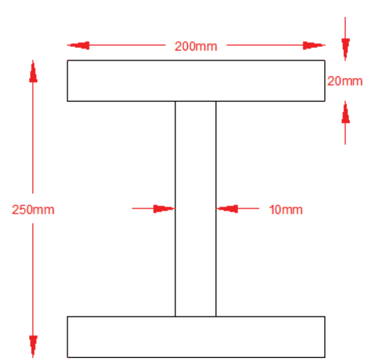
$$Z_p = \frac{1}{2} \times b \times d \times \frac{1}{2}\left(\frac{d}{6} + \frac{d}{6}\right)$$

$$Z_p = \frac{bd}{4}\left(\frac{d}{3}\right) = \frac{bd^2}{12}$$

$$S = \frac{Z_p}{Z} = \frac{bd^2}{12} \times \frac{24}{bd^2} = 2$$

A mild steel I section 200mm wide and 250mm deep has a flange thickness of 20mm and a web thicknees of 10mm. calculate the shape factor. Find the fully plastic moment if σ_y = 252N/mm^2.

Solution:



$$Z = \frac{I}{y}$$

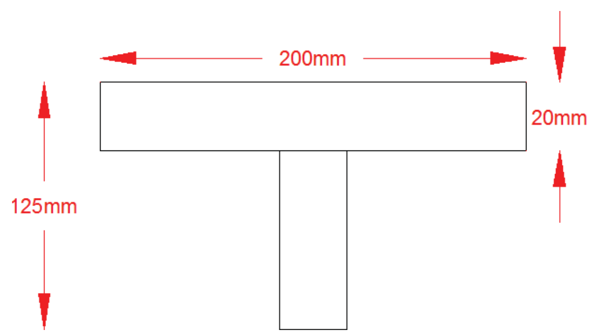
$$I = \frac{bd^3}{12}$$

$$I = \frac{200 \times 250^3}{12} - \frac{190 \times 210^3}{12}$$

$$I = 113.78 \times 10^6 \text{ mm}^4$$

$$y = \frac{250}{2} = 125 \text{ mm}$$

$$Z = \frac{113.78 \times 10^6}{125} = 910.27 \times 10^3 \text{ mm}^3$$



$$Z_p = \frac{A}{2}(y1 + y2)$$

$$y1 = y2 = \frac{(a1y1 + a2y2)}{(a1 + a2)}$$

$$y1 = y2 = \frac{(105 \times 10 \times (\frac{105}{2})) + (200 \times 20 \times (105 + \frac{20}{2}))}{(105 \times 10) + (200 \times 20)} = 102\text{mm}$$

$$Z_p = \frac{2(200 \times 20) + (210 \times 10)}{2}(102 + 102)$$

$$Zp = 1030.2 \times 10^3 \text{ mm}^3$$

$$S = \frac{Z_p}{Z} = 1.132$$

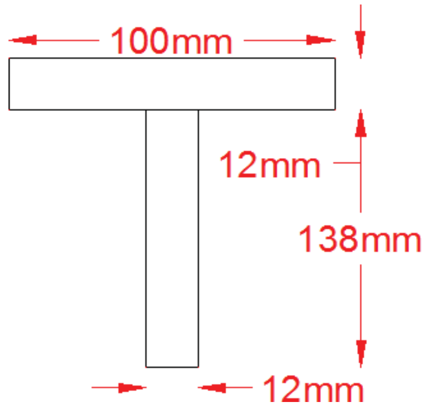
$$M_P = \sigma_y \times z_p$$

$$M_P = 252 \times 1030.2 \times 10^3$$

$$M_P = 259.6 \text{ kNm}$$

Determine the shape factor of a T section beam of flange dimension 100 x 12 mm and web dimension 138 x 12 mm thick.

Solution:



$$\bar{y} = \frac{(a1y1 + a2y2)}{(a1 + a2)}$$

$$\bar{y} = \frac{(138 \times 12 \times (\frac{138}{2})) + (100 \times 12 \times (138 + \frac{12}{2}))}{(138 \times 12) + (100 \times 12)} = 100.51\text{mm}$$

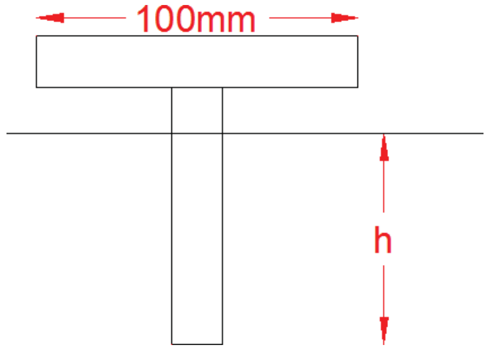
$$I = \frac{bd^3}{12} + a\bar{h}^2$$

$$I = \{\frac{12 \times 138^3}{12} + (12 \times 138)(\frac{138}{2} - 100.51)^2\} + \{\frac{100 \times 12^3}{12} + (100 \times 12)(144 - 100.51)^2\}$$

$$I = 6556337.57 \text{ mm}^4$$

$$Z = \frac{I}{\bar{y}} = \frac{6556337.57}{100.51} = 65230.7\text{mm}^3$$

Equal area axis



$$\frac{A}{2} = (100 \times 12) + (12 \times h)$$

$$\frac{(100 \times 12) + (12 \times 138)}{2} = 1200 + (12 \times h)$$

H= 19 mm

$$y1 = \frac{(a1y1 + a2y2)}{(a1 + a2)}$$

$$y1 = \frac{(100 \times 12 \times (19 + \frac{12}{2})) + (19 \times 12 \times (\frac{19}{2}))}{(100 \times 12) + (19 \times 12)} = 22.52mm$$

$$y2 = \frac{119}{2} = 59.5mm$$

$$Z_p = \frac{A}{2}(y1 + y2)$$

$$Z_p = \frac{2856}{2}(22.52 + 59.5) = 117138.84mm^3$$

$$S = \frac{Z_p}{Z} = 1.08$$

Plastic hinge:

Fully plastic moment is considered to have developed at any section of a structure subjected to a system of loads, when the section is completely yielded or plastified. The fibers on one side of the equal area axis of the section are in compression and tend to contract. The fibers on the other side of the axis are in tension and tend to expand. The section acts like a hinge. This hinge is known as a plastic hinge.

The plastic hinge is defined as an yielded zone due to bending in a structural member, at which large rotations can take place at a section at a constant plastic moment, $M_{P.}$

Mechanism:

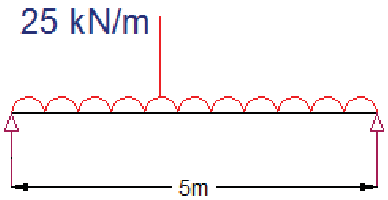
A stable structure shall be able to resist displacement. There has to be a force corresponding to any displacement. A displacement without resistance is called a rigid body displacement. Structures have elastic displacement. Mechanisms have rigid body displacements.

Types of mechanism:

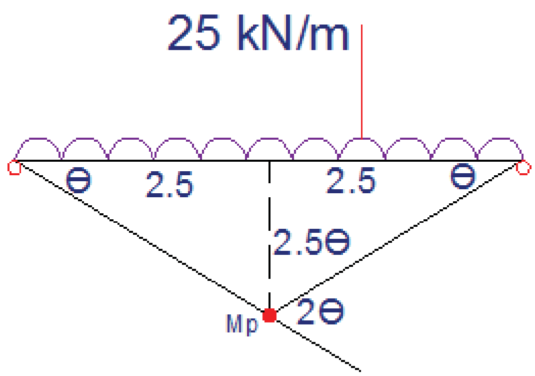
1. Independent mechanism
2. Combined (or) composite mechanism

Problem

A simply supported beam of span 5m is to be designed for a udl of 25kN/m. design a suitable I section using plastic theory, assuming yield stress in steel as f_y = 250N/mm^2.



Solution:



$$IWD = Mp \times 2\theta$$

$$EWD = \text{load} \times \text{area}$$

$$EWD = 25 \times \frac{1}{2} \times 5 \times 2.5\theta$$

$$EWD = 156.25\theta$$

$$IWD = EWD$$

$$Mp \times 2\theta = 156.25\theta$$

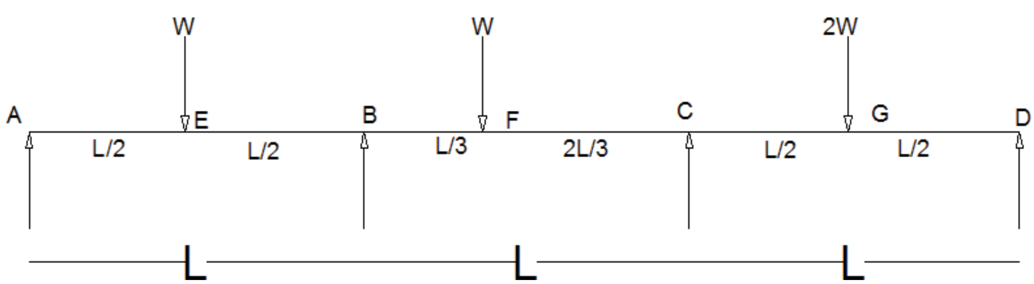
$$Mp = 78.125 \text{ kNm}$$

$$M_P = \sigma_y \times z_p$$

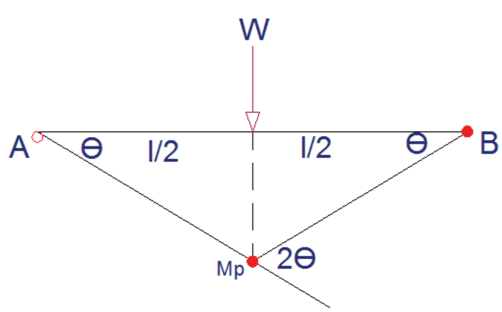
$$Zp = \frac{M_p}{\sigma y} = 312.5 \times 10^3 \text{ mm}^3$$

Problem

Determine the collapse load 'W' for a three span continuous beam of constant plastic moment Mp, loaded as shown in fig.



Mechanism 1:



$$IWD = (Mp \times 2\theta) + Mp\ \theta = 3Mp\ \theta$$

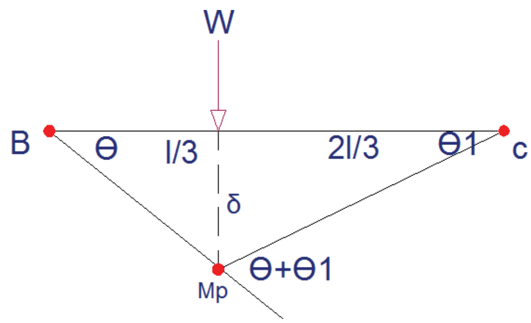
$$EWD = W \times \frac{l}{2} \times \theta$$

IWD = EWD

$$3Mp\theta = W \times \frac{l}{2} \times \theta$$

$$W = \frac{6Mp}{l}$$

Mechanism 2:



$$ð = \frac{l}{3} \times \theta = \frac{2l}{3} \times \theta 1$$

$$\theta = \frac{\theta 1}{2}$$

IWD = Mp θ+ Mp(θ+ θ1) + Mp θ1 = 3Mp θ

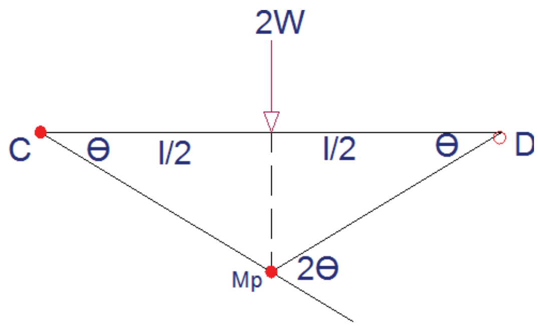
$$EWD = W \times \frac{l}{3} \times \theta$$

IWD = EWD

$$3Mp\theta = W \times \frac{l}{3} \times \theta$$

$$W = \frac{9Mp}{l}$$

Mechanism 3:



IWD = (Mp × 2θ) + Mp θ = 3Mp θ

$$EWD = 2W \times \frac{l}{2} \times \theta$$

IWD = EWD

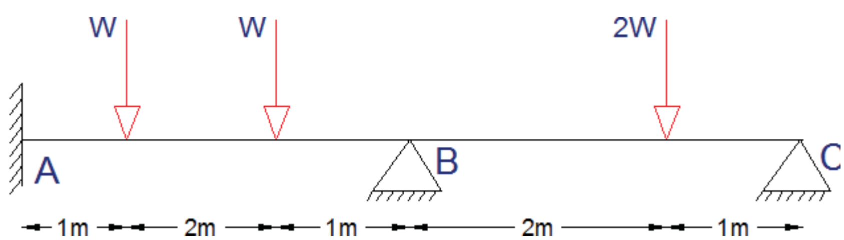
$$3Mp\theta = 2W \times \frac{l}{2} \times \theta$$

$$W = \frac{3Mp}{l}$$

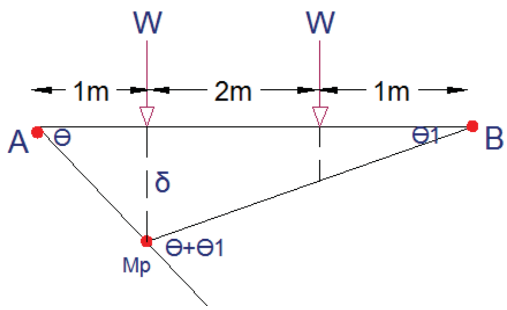
 the collapse load is $\frac{3Mp}{l}$

Problem

Determine the collapse load 'W' of the beam loaded as shown in fig.



Mechanism 1:



$$ð = 1\theta = 3\theta1$$

$$\theta1 = \frac{\theta}{3}$$

$$IWD = Mp\ \theta + Mp(\theta + \theta1) + Mp\ \theta1$$

$$IWD = 2Mp\ \theta + 2Mp\ \theta1$$

$$IWD = 2MP\theta + 2MP\frac{\theta}{3} = \frac{8MP\theta}{3}$$

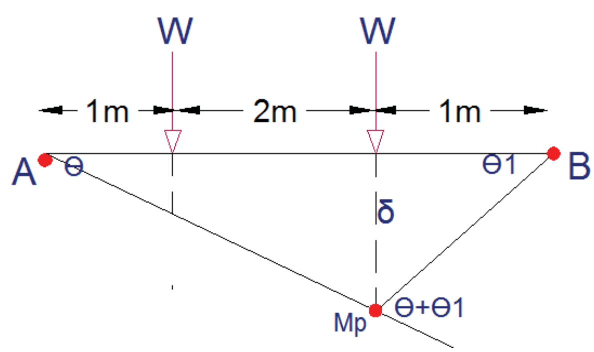
$$EWD = W\theta + W \times \frac{\theta}{3} = \frac{4W\theta}{3}$$

$$IWD = EWD$$

$$\frac{8MP\theta}{3} = \frac{4W\theta}{3}$$

$$W = 2MP$$

Mechanism 2:



$$ð = 3\theta = 1\theta1$$

$$\theta1 = 3\theta$$

$$IWD = Mp\ \theta + Mp(\theta + \theta 1) + Mp\ \theta 1$$

$$IWD = 2Mp\ \theta + 2Mp\ \theta 1$$

$$IWD = 2MP\theta + 2MP(3\theta) = 8MP\theta$$

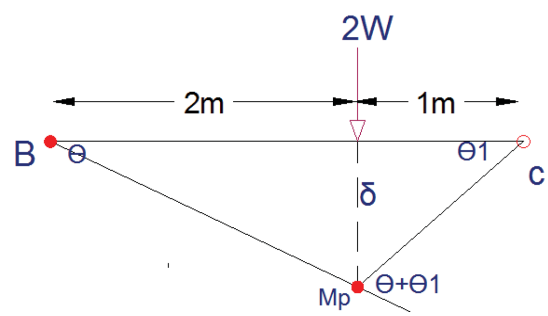
$$EWD = W\theta + W\theta 1 = 4W\theta$$

$$IWD = EWD$$

$$8MP\theta = 4W\theta$$

$$W = 2MP$$

Mechanism 3:



$$ð = 2\theta = 1\theta 1$$

$$\theta 1 = 2\theta$$

$$IWD = Mp\ \theta + Mp(\theta + \theta 1)$$

$$IWD = 2Mp\ \theta + Mp\ \theta 1$$

$$IWD = 2MP\theta + MP(2\theta) = 4MP\theta$$

$$EWD = 2W \times 2\theta = 4W\theta$$

$$IWD = EWD$$

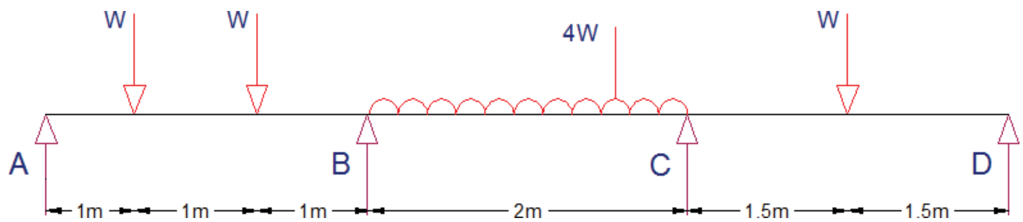
$$4MP\theta = 4W\theta$$

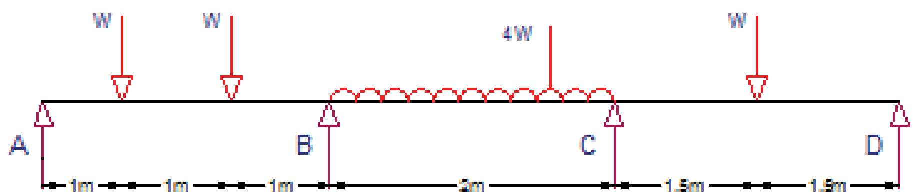
$$W = MP$$

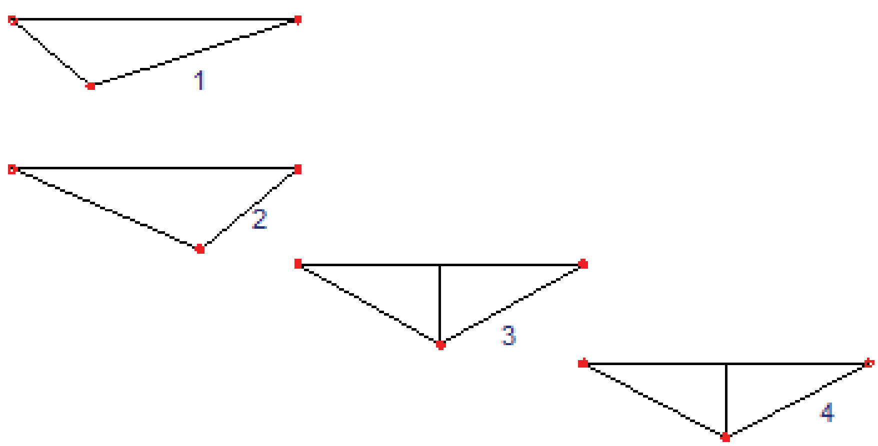
 the collapse load is W = MP

Problem

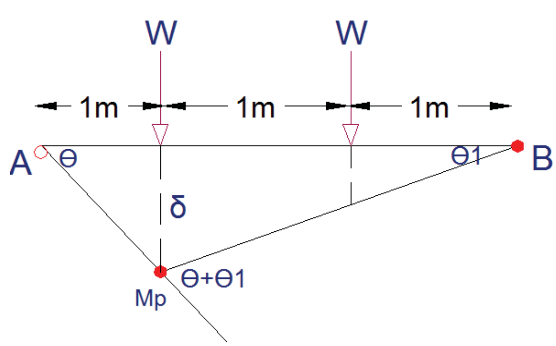
Determine the collapse load 'W' for the continuous beam loaded as shown in fig. has uniform plastic moment Mp.







Mechanism 1:



$$ð = 1θ = 2θ1$$

$$θ1 = \frac{θ}{23}$$

IWD = Mp(θ+ θ1) + Mp θ1

IWD = Mp θ+ 2Mp θ1

$$IWD = MPθ + 2MP\frac{θ}{2} = 2MPθ$$

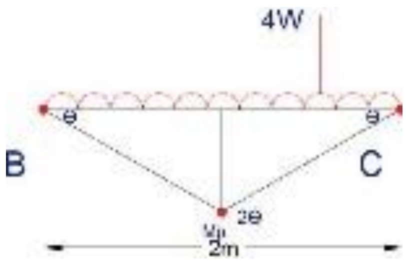
$$EWD = Wθ + W \times \frac{θ}{2} = \frac{3Wθ}{2}$$

IWD = EWD

$$2MPθ = \frac{3Wθ}{2}$$

$$W = \frac{4MP}{3}$$

Mechanism 2:



$$ð = 1θ$$

IWD = Mpθ + Mp2 θ + Mp θ

IWD = 4Mp θ

$$IWD = MPθ + 2MP\frac{θ}{2} = 2MPθ$$

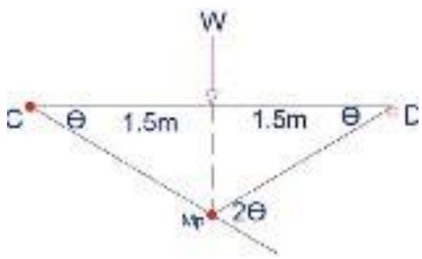
$$EWD = 2W \times \frac{1}{2} \times 2 \times 1θ = 2Wθ$$

IWD = EWD

$$2MPθ = 2Wθ$$

$$W = MP$$

Mechanism 3:



$$IWD = (Mp \times 2\theta) + Mp\ \theta = 3Mp\ \theta$$

$$EWD = W \times 1.5 \times \theta$$

$$IWD = EWD$$

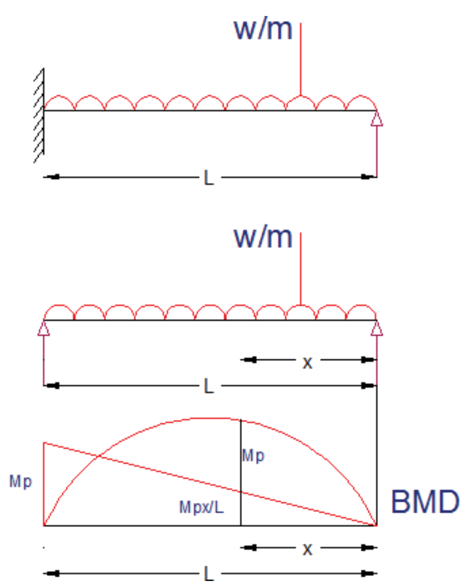
$$3Mp\theta = W \times 1.5 \times \theta$$

$$W = 2MP$$

 the collapse load is $\frac{4Mp}{3}$

Problem

Analyse a propped cantilever of length 'L' and subjected to UDL of w/m length for the entire span and find the collapse load.



$$RA = RB = \frac{w}{2}$$

Taking moment about XX axis at a distance x from B.

$$Mx = \frac{wx}{2} - \frac{wx^2}{2l}$$

$$Mp + \frac{Mpx}{l} = \frac{wx}{2} - \frac{wx^2}{2l}$$

$$Mp[1 + \frac{x}{l}] = \frac{w}{2}[x - \frac{x^2}{l}]$$

$$Mp = \frac{w}{2}\frac{(lx - x^2)}{(l + x)}$$

$$\frac{dMp}{dx} = 0$$

$$\frac{dMp}{dx} = \frac{(l + x)(l - 2x) - (lx - x^2)(1)}{(l + x)^2} = 0$$

$$(l + x)(l - 2x) - (lx - x^2) = 0$$

$$(l^2 + xl - 2xl - 2x^2 - lx + x^2) = 0$$

$$(l^2 - 2xl - x^2) = 0$$

$$(x^2 + 2xl - l^2) = 0$$

$$x = \frac{-2l \pm \sqrt{8l^2}}{2}$$

$$x = 0.414l$$

$$0.586l \times \theta = 0.414l \times \theta 1$$

$$\theta 1 = 1.4155 \times \theta$$

$$EWD = \frac{w}{l} \times \frac{1}{2} \times l \times 0.586l \times \theta = 0.293wl\theta$$

$$IWD = Mp\ \theta + 2.4155Mp\ \theta = 3.4155Mp\ \theta$$

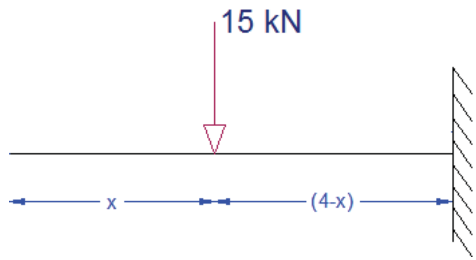
$$IWD = EWD$$

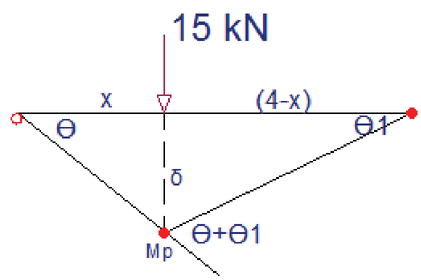
$$0.293wl\theta = 3.4155Mp\ \theta$$

$$W = \frac{11.656Mp}{l}$$

Problem

A uniform beam of span 4m and fully plastic moment Mp is simply supported at one end and rigidly clamped at other end. A concentrated load of 15 kN may be applied anywhere within the span. Find the smallest value of Mp such that collapse would first occur when the load is in its most unfavorable position.





$$ð = x\,\theta = (4 - x)\theta 1$$

$$\theta 1 = \frac{x\theta}{(4 - x)}$$

$$IWD = Mp\,\theta 1 + Mp\,(\theta + \theta 1)$$

$$IWD = 2Mp\,\theta 1 + Mp\,\theta$$

$$IWD = Mp\,\theta + 2Mp\frac{x\theta}{(4 - x)}$$

$$IWD = Mp\,\theta\,[1 + \frac{2x}{(4 - x)}]$$

$$IWD = Mp\,\theta\,\frac{(4 + x)}{(4 - x)}$$

$$EWD = 15\,X\,\theta$$

IWD = EWD

$$Mp\ \theta \frac{(4+x)}{(4-x)} = 15x\theta$$

$$Mp = \frac{(60x - 15x^2)}{(4+x)}$$

$$\frac{dMp}{dx} = 0$$

$$\frac{dMp}{dx} = \frac{(4+x)(60-30x) - (60x-15x^2)(1)}{(4+x)^2} = 0$$

$$(4+x)(60-30x) - (60x - 15x^2) = 0$$

$$(x^2 + 8x - 16) = 0$$

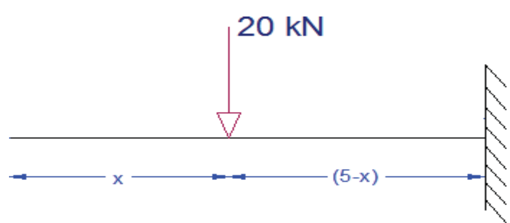
$$x = 1.66m$$

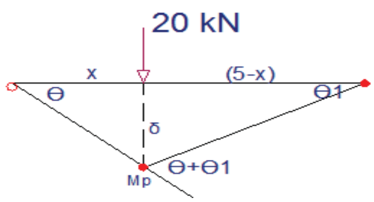
$$Mp = \frac{(60(1.66) - 15(1.66)^2)}{(4+1.66)}$$

Mp = 10.29 kNm

Problem

A uniform beam of span 5m and fully plastic moment Mp is simply supported at one end and rigidly clamped at other end. A concentrated load of 20 kN may be applied anywhere within the span. Find the smallest value of Mp such that collapse would first occur when the load is in its most unfavorable position.





$$ð = x\,\theta = (5 - x)\theta 1$$

$$\theta 1 = \frac{x\theta}{(5 - x)}$$

$$IWD = Mp\,\theta 1 + Mp\,(\theta + \theta 1)$$

$$IWD = 2Mp\,\theta 1 + Mp\,\theta$$

$$IWD = Mp\,\theta + 2Mp\frac{x\theta}{(5 - x)}$$

$$IWD = Mp\,\theta\,[1 + \frac{2x}{(5 - x)}]$$

$$IWD = Mp\,\theta\,\frac{(5 + x)}{(5 - x)}$$

$$EWD = 20\,X\,\theta$$

$$IWD = EWD$$

$$Mp\,\theta\frac{(5 + x)}{(5 - x)} = 20x\theta$$

$$Mp = \frac{(100x - 20x^2)}{(5 + x)}$$

$$\frac{dMp}{dx} = 0$$

$$\frac{dMp}{dx} = \frac{(5 + x)(100 - 40x) - (100x - 20x^2)(1)}{(5 + x)^2} = 0$$

$$(5 + x)(100 - 40x) - (100x - 20x^2) = 0$$

$$(x^2 + 10x - 25) = 0$$

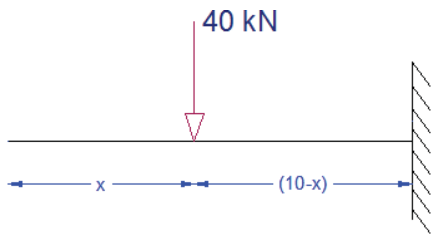
$$x = 2.071m$$

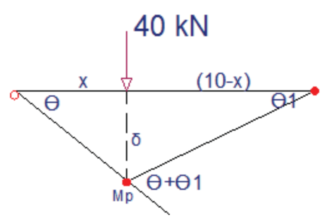
$$Mp = \frac{(100(2.071) - 20(2.071)^2)}{(5 + 2.071)}$$

$$Mp = 17.16\ kNm$$

Problem

A uniform beam of span 10m and fully plastic moment Mp is simply supported at one end and rigidly clamped at other end. A concentrated load of 40 kN may be applied anywhere within the span. Find the smallest value of Mp such that collapse would first occur when the load is in its most unfavorable position.





$$ð = x\,\theta = (10 - x)\theta 1$$

$$\theta 1 = \frac{x\theta}{(10 - x)}$$

$$IWD = Mp\ \theta 1 + Mp\ (\theta + \theta 1)$$

$$IWD = 2Mp\ \theta 1 + Mp\ \theta$$

$$IWD = Mp\ \theta + 2Mp\frac{x\theta}{(10 - x)}$$

$$IWD = Mp\ \theta\ [1 + \frac{2x}{(10 - x)}]$$

$$IWD = Mp\ \theta\ \frac{(10 + x)}{(10 - x)}$$

$$EWD = 40\ X\ \theta$$

$$IWD = EWD$$

$$Mp\ \theta\ \frac{(10 + x)}{(10 - x)} = 40x\theta$$

$$Mp = \frac{(400x - 40x^2)}{(10 + x)}$$

$$\frac{dMp}{dx} = 0$$

$$\frac{dMp}{dx} = \frac{(10 + x)(400 - 80x) - (400x - 40x^2)(1)}{(10 + x)^2} = 0$$

$$(10 + x)(400 - 80x) - (400x - 40x^2) = 0$$

$$(x^2 + 20x - 100) = 0$$

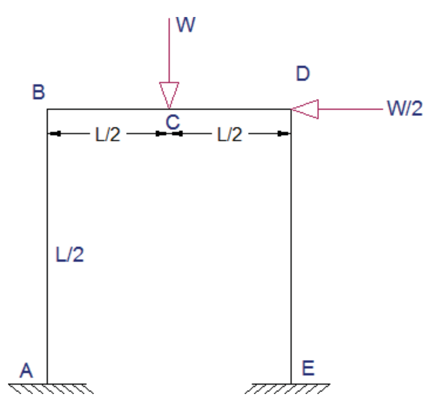
$$x = 4.14m$$

$$Mp = \frac{(400(4.14) - 40(4.14)^2)}{(10 + 4.14)}$$

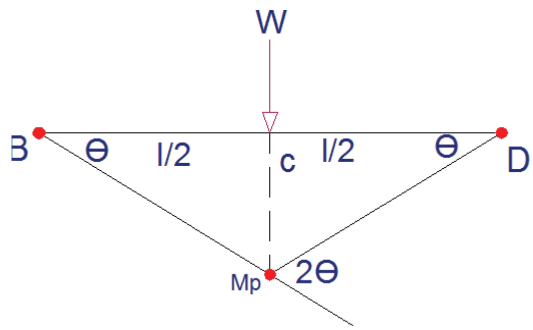
Mp = 68.63 kNm

Problem

A rectangular portal frame of span L and height L/2 is fixed to the ground at both ends and has a uniform section throughout with its fully plastic moment of resistance equal to My. It is loaded with a point load W at the centre of span as well as a horizontal free W/2 at its top right corner. Calculate the value of W at collapse of the frame.



Beam mechanism



IWD = Mp θ+ (Mp × 2θ) + Mp θ = 4Mp θ

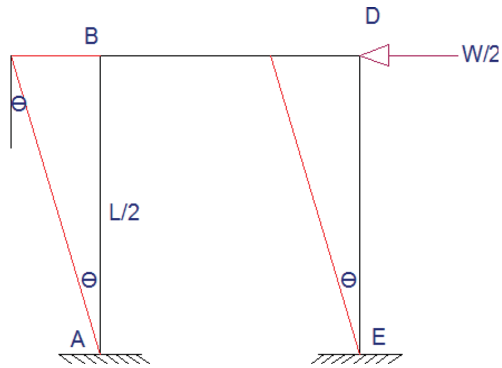
$$EWD = W \times \frac{l}{2} \times \theta$$

IWD = EWD

$$4Mp\theta = W \times \frac{l}{2} \times \theta$$

$$W = \frac{8Mp}{l}$$

Sway mechanism



IWD = Mp θ+ Mp θ + Mp θ+ Mp θ = 4Mp θ

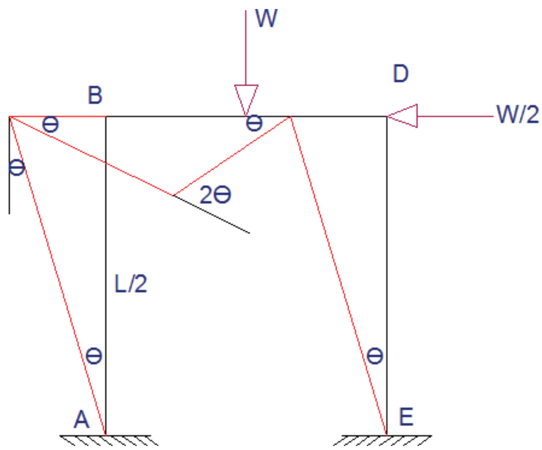
$$EWD = \frac{W}{2} \times \frac{l}{2} \times \theta$$

IWD = EWD

$$4Mp\theta = \frac{W}{2} \times \frac{l}{2} \times \theta$$

$$W = \frac{16Mp}{l}$$

Combined mechanism



$$IWD = Mp\,\theta + Mp\,\theta + Mp\,\theta + Mp\,(2\theta) + Mp\,\theta = 6Mp\,\theta$$

$$EWD = \left\{\frac{W}{2} \times \frac{l}{2} \times \theta\right\} + \left\{W \times \frac{l}{2} \times \theta\right\} = \frac{3}{4} \times Wl \times \theta$$

IWD = EWD

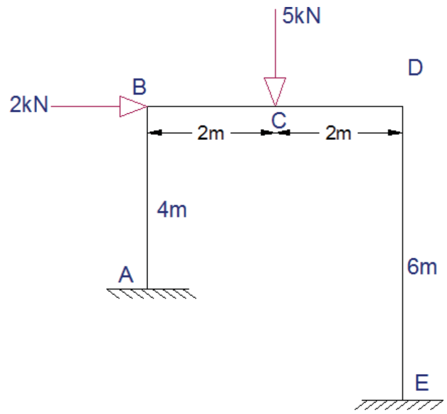
$$6Mp\theta = \frac{3}{4} \times Wl \times \theta$$

$$W = \frac{8Mp}{l}$$

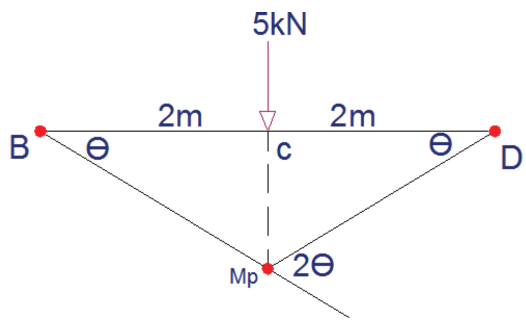
 therefore the critical load is $\frac{8Mp}{l}$

Problem

Find the fully plastic moment required for the frame shown in fig if all the members have same value of M_P



Beam mechanism



$$IWD = Mp\,\theta + (Mp \times 2\theta) + Mp\,\theta = 4Mp\,\theta$$

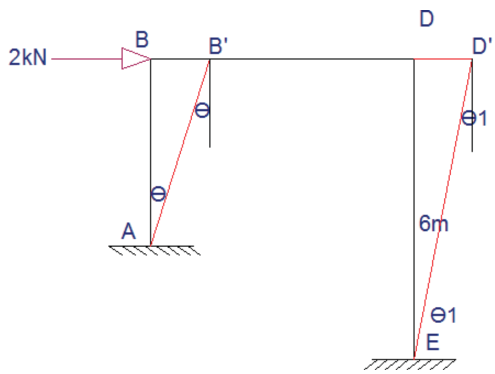
$$EWD = 5 \times 2 \times \theta = 10\,\theta$$

$$IWD = EWD$$

$$4Mp\theta = 10\,\theta$$

$$Mp = 2.5$$

Sway mechanism



$$\Delta = 4\,\theta = 6\,\theta 1$$

$$\theta 1 = \frac{4}{6}\theta$$

$$IWD = Mp\ \theta + Mp\ \theta + Mp\ \theta 1 + Mp\ \theta 1$$

$$IWD = \frac{10}{3} \times Mp \times \theta$$

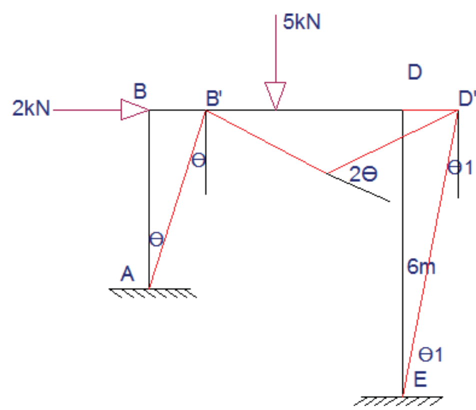
$$EWD = 2 \times 4\,\theta$$

$$IWD = EWD$$

$$\frac{10}{3} \times Mp \times \theta = 8\,\theta$$

$$Mp = 2.4$$

Combined mechanism



$$\Delta = 4\ \theta = 6\ \theta 1$$

$$\theta 1 = \frac{4}{6}\theta$$

$$IWD = Mp\ \theta + Mp\ 2\theta + Mp\ \theta 1 + Mp\ \theta + Mp\ \theta 1$$

$$IWD = \frac{160}{3} \times Mp \times \theta$$

$$EWD = (2 \times 4\ \theta) + (5 \times 2\theta) = 18\ \theta$$

$$IWD = EWD$$

$$\frac{16}{3} \times Mp \times \theta = 18\ \theta$$

$$Mp = 3.38$$

PART-A

1. What is a plastic hinge?

When a section attains full plastic moment Mp, it acts as hinge which is called a plastic hinge. It is defined as the yielded zone due to bending at which large rotations can occur with a constant value of plastic moment Mp.

2. What is a mechanism?

When a n-degree indeterminate structure develops n plastic hinges, it becomes determinate and the formation of an additional hinge will reduce the structure to a mechanism. Once a structure becomes a mechanism, it will collapse.

3. What is difference between plastic hinge and mechanical hinge?

Plastic hinges modify the behavior of structures in the same way as mechanical hinges. The only difference is that plastic hinges permit rotation with a constant resisting moment equal to the plastic moment Mp. At mechanical hinges, the resisting moment is equal to zero.

4. Define collapse load.

The load that causes the (n + 1) the hinge to form a mechanism is called collapse load where n is the degree of statically indeterminacy. Once the structure becomes a mechanism

5. List out the assumptions made for plastic analysis.

The assumptions for plastic analysis are:

Ø Plane transverse sections remain plane and normal to the longitudinal axis before and after bending.

Ø Effect of shear is neglected.

Ø The material is homogeneous and isotropic both in the elastic and plastic state.

Ø Modulus of elasticity has the same value both in tension and compression.

Ø There is no resultant axial force in the beam.

Ø The cross-section of the beam is symmetrical about an axis through its centroid and parallel to the plane of bending.

6. Define shape factor.

Shape factor (S) is defined as the ratio of plastic moment of the section to the yield moment of the section.

$$S = \frac{M_p}{M_y}$$

$$S = \frac{\sigma_y \times Z_p}{\sigma_y \times Z}$$

$$S = \frac{Z_p}{Z}$$

Where Mp = Plastic moment

M = Yield moment

Zp = Plastic section modulus

Z = Elastic section modulus

7. List out the shape factors for the following sections.

(a) Rectangular section, S = 1.5

(b) Triangular section, S = 2.346 (c) Circular section, S = 1.697 (d) Diamond section, S = 2

8. Mention the section having maximum shape factor.

The section having maximum shape factor is a triangular section, S = 2.345.

9. Define load factor.

Load factor is defined as the ratio of collapse load to working load and is given by

$$\lambda = \frac{W_c}{W}$$

10. State upper bound theory.

Upper bound theory states that of all the assumed mechanisms the exact collapse mechanism is that which requires a minimum load.

11. State lower bound theory.

Lower bound theory states that the collapse load is determined by assuming suitable moment distribution diagram. The moment distribution diagram is drawn in such a way that the conditions of equilibrium are satisfied.

12. What are the different types of mechanisms?

The different types of mechanisms are:

Ø Beam mechanism

Ø Column mechanism

Ø Panel or sway mechanism

Ø Cable mechanism

Ø Combined or composite mechanism

13. Mention the types of frames.

Frames are broadly of two types:

(a) Symmetric frames

(b) Un-symmetric frames

14. What are symmetric frames and how they analyzed?

Symmetric frames are frames having the same support conditions, lengths and loading conditions on the columns and beams of the frame. Symmetric frames can be analyzed by:

(a) Beam mechanism

(b) Column mechanism

15. What are unsymmetrical frames and how are they analyzed?

Un-symmetric frames have different support conditions, lengths and loading conditions on its columns and beams. These frames can be analyzed by:

(a) Beam mechanism

(b) Column mechanism

(c) Panel or sway mechanism

(d) Combined mechanism

16. Define plastic modulus of a section Zp.

The plastic modulus of a section is the first moment of the area above and below the equal area axis. It is the resisting modulus of a fully plasticized section.

$Z_p = A/2 (y_1 + y_2)$

17. How is the shape factor of a hollow circular section related to the shape factor of a ordinary circular section?

The shape factor of a hollow circular section = A factor K x shape factor of ordinary circular section.

SF of hollow circular section = SF of circular section x {(1 – c3)/ (1 – c4)}

18. Give the governing equation for bending.

The governing equation for bending is given by

M/I =σ /y

Where M = Bending moment

I = Moment of inertia

y = c. g. distance

19. Give the theorems for determining the collapse load.

The two theorems for the determination of collapse load are:

(a) Static Method [Lower bound Theorem]

(b) Kinematic Method [Upper bound Theorem]