EC3352 DIGITAL SYSTEM DESIGN

COMBINATIONAL LOGIC CIRCUITS

Combinational Logic

- Logic circuits for digital systems may be combinational or sequential.
- A combinational circuit consists of input variables, logic gates, and output variables.

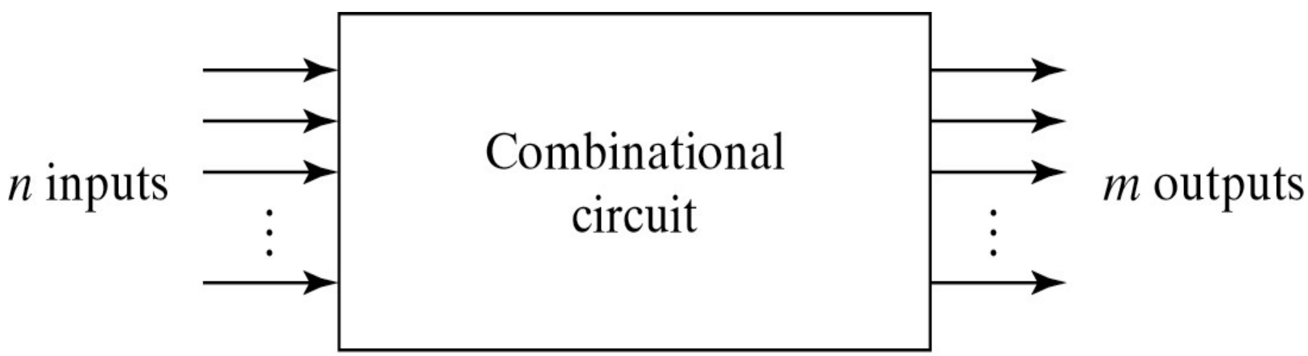


Fig. 4-1 Block Diagram of Combinational Circuit

4-2. Analysis procedure

- To obtain the output Boolean functions from a logic diagram, proceed as follows:

1. Label all gate outputs that are a function of input variables with arbitrary symbols. Determine the Boolean functions for each gate output.
2. Label the gates that are a function of input variables and previously labeled gates with other arbitrary symbols. Find the Boolean functions for these gates.

4-2. Analysis procedure

3. Repeat the process outlined in step 2 until the outputs of the circuit are obtained.

4. By repeated substitution of previously defined functions, obtain the output Boolean functions in terms of input variables.

Example

F_2 = AB + AC + BC; T_1 = A + B + C; T_2 = ABC; $T_3 = F_2'T_1$;

$F_1 = T_3 + T_2$

$F_1 = T_3 + T_2 = F_2'T_1$ + ABC = A'BC' + A'B'C + AB'C' + ABC

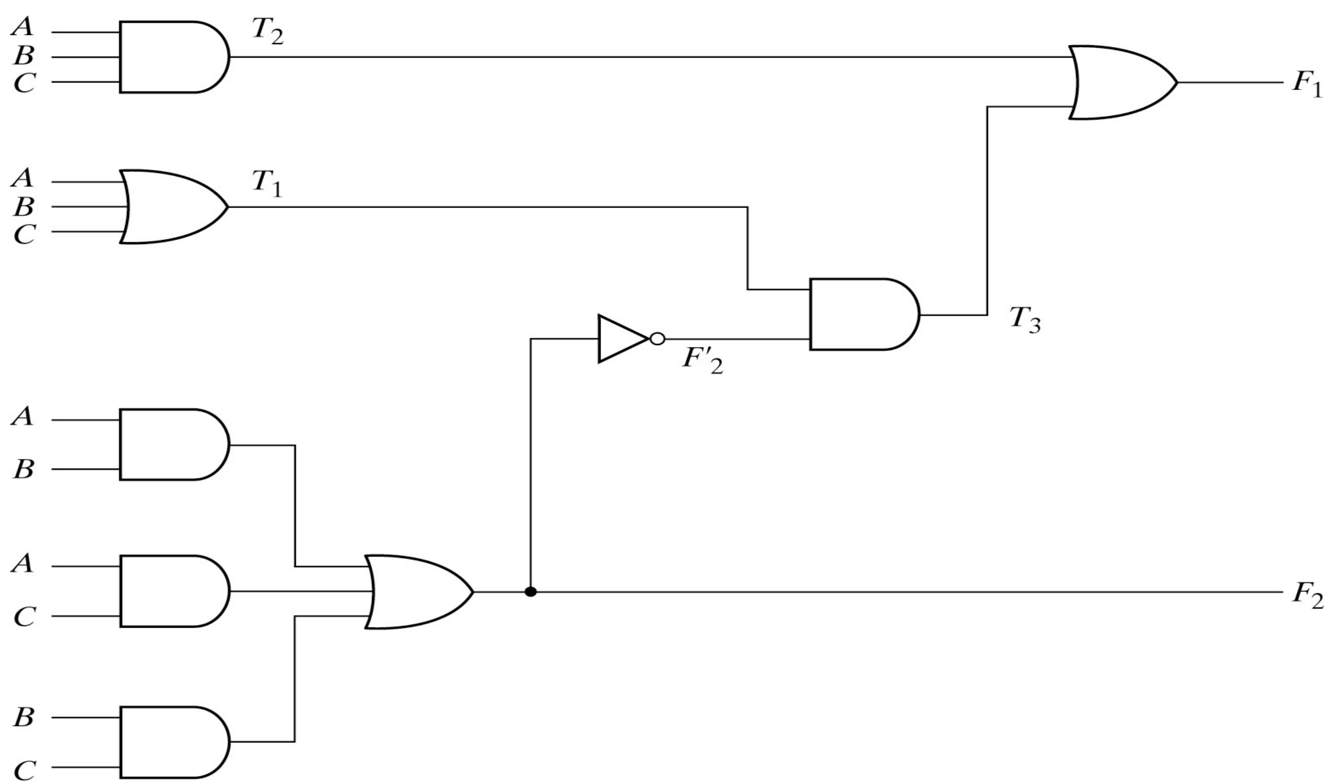


Fig. 4-2 Logic Diagram for Analysis Example

Derive truth table from logic diagram

- We can derive the truth table in Table 4-1 by using the circuit of Fig.4-2.

Table 4-1
Truth Table for the Logic Diagram of Fig. 4-2

A	B	C	F_2	F_2'	T_1	T_2	T_3	F_1
0	0	0	0	1	0	0	0	0
0	0	1	0	1	1	0	1	1
0	1	0	0	1	1	0	1	1
0	1	1	1	0	1	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	1	0	1	0	0	0
1	1	0	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1

4-3. Design procedure

1. Table4-2 is a Code-Conversion example, first, we can list the relation of the BCD and Excess-3 codes in the truth table.

Table 4-2
Truth Table for Code-Conversion Example

Input BCD				Output Excess-3 Code			
A	*B*	*C*	*D*	*w*	*x*	*y*	*z*
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

Karnaugh map

2. For each symbol of the Excess-3 code, we use 1's to draw the map for simplifying Boolean function.

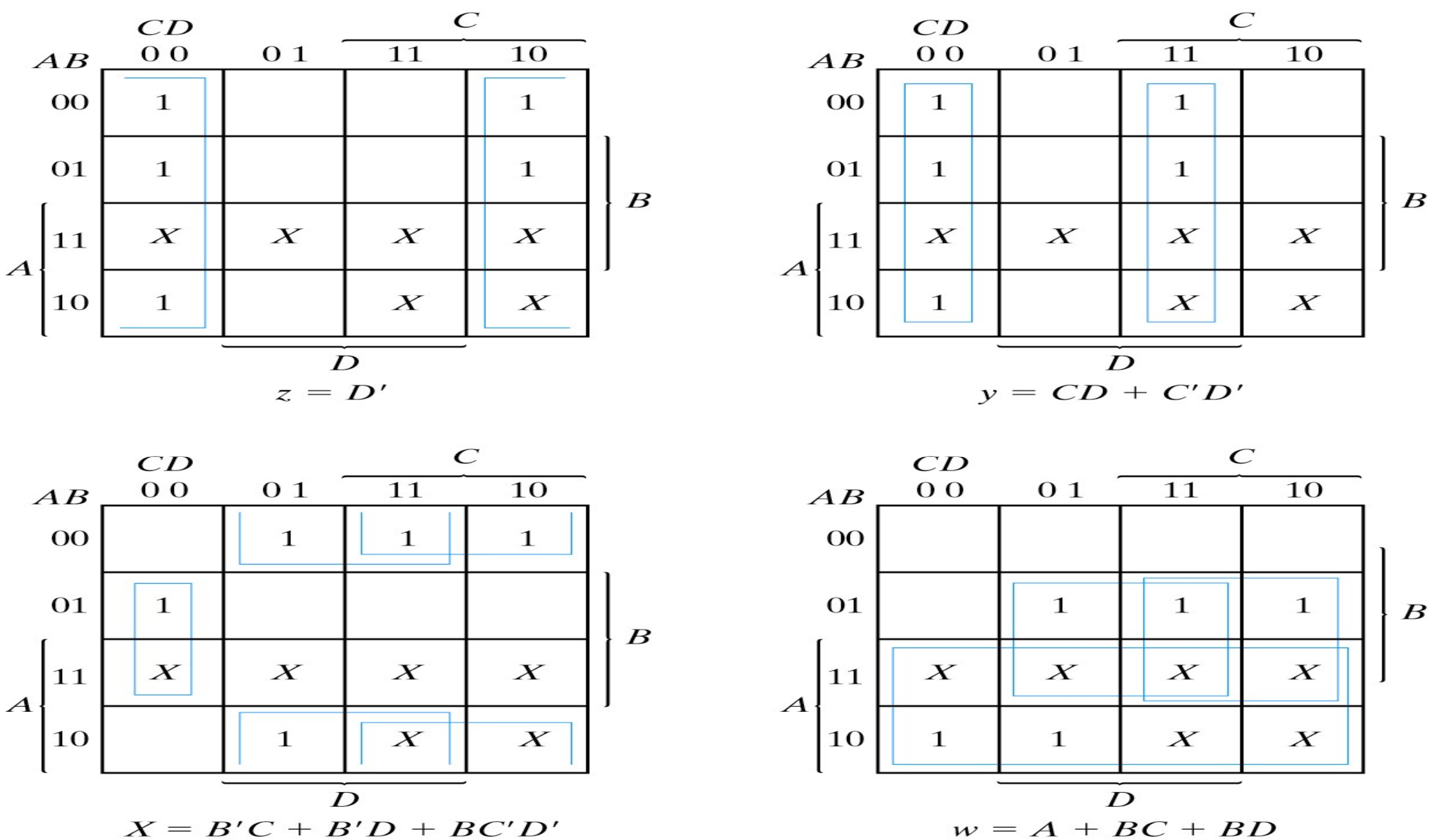


Fig. 4-3 Maps for BCD to Excess-3 Code Converter

Circuit implementation

$z = D'$; $y = CD + C'D' = CD + (C + D)'$

$x = B'C + B'D + BC'D' = B'(C + D) + B(C + D)'$

$w = A + BC + BD = A + B(C + D)$

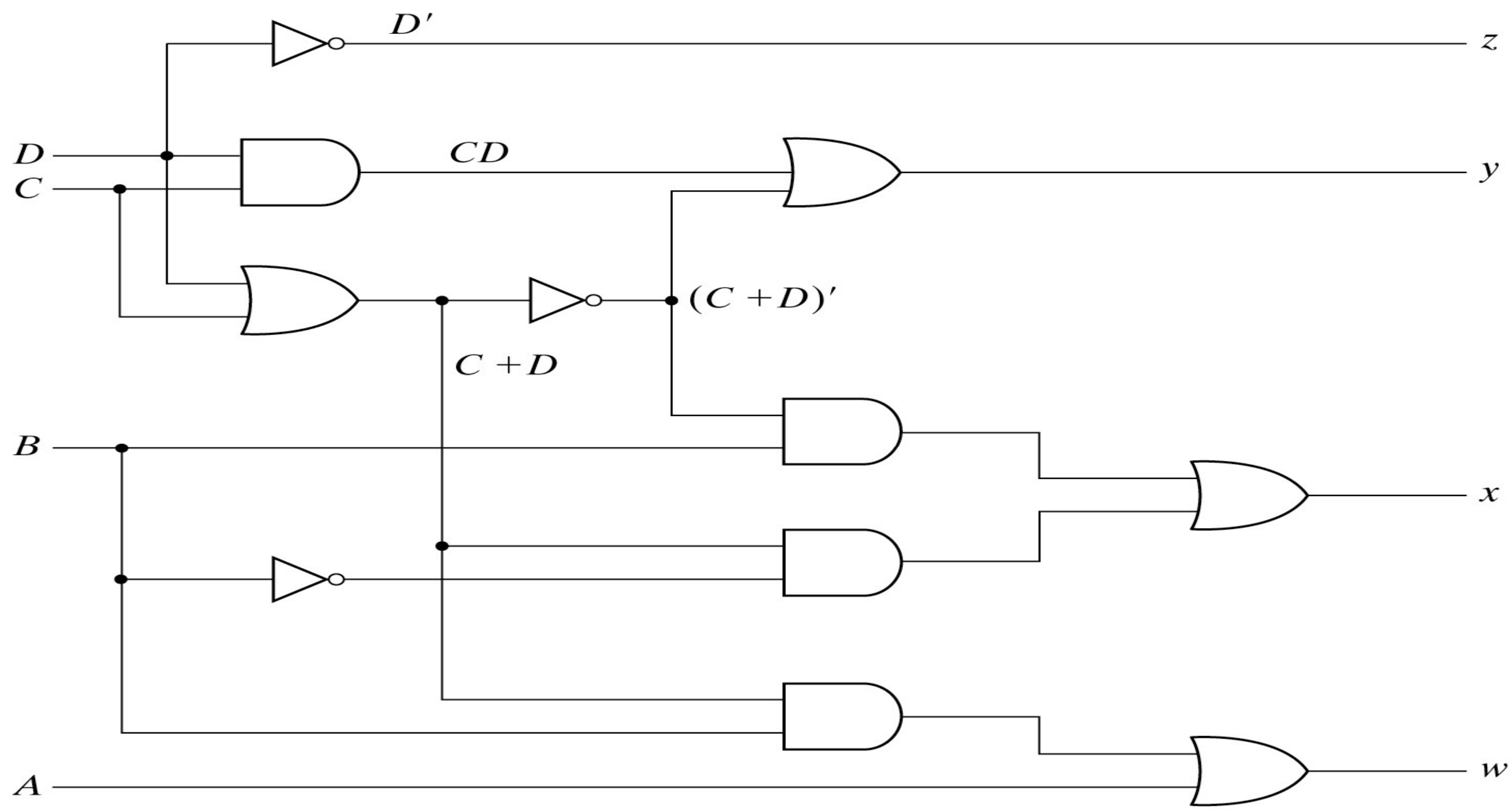


Fig. 4-4 Logic Diagram for BCD to Excess-3 Code Converter

4-4. Binary Adder-Subtractor

- A combinational circuit that performs the addition of two bits is called a half adder.
- The truth table for the half adder is listed below:

Table 4-3
Half Adder

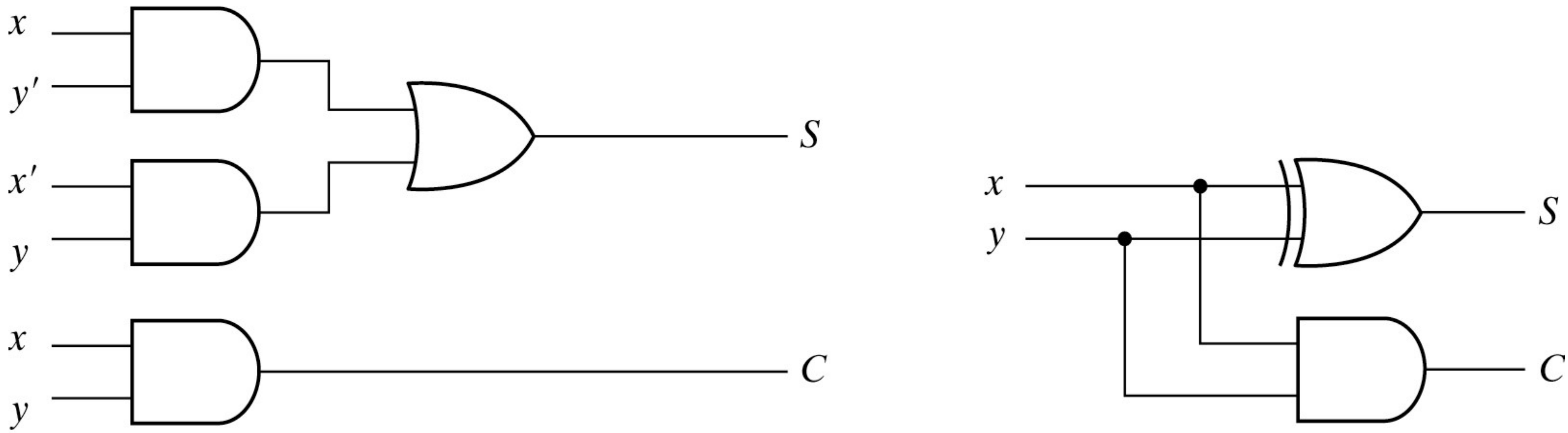
x	y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

S: Sum
C: Carry

$S = x'y + xy'$

$C = xy$

Implementation of Half-Adder



(a) $S = xy' + x'y$
$C = xy$

(b) $S = x \oplus y$
$C = xy$

Fig. 4-5 Implementation of Half-Adder

Full-Adder

- One that performs the addition of three bits(two significant bits and a previous carry) is a full adder.

Table 4-4
Full Adder

x	*y*	*z*	*C*	*S*
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Simplified Expressions

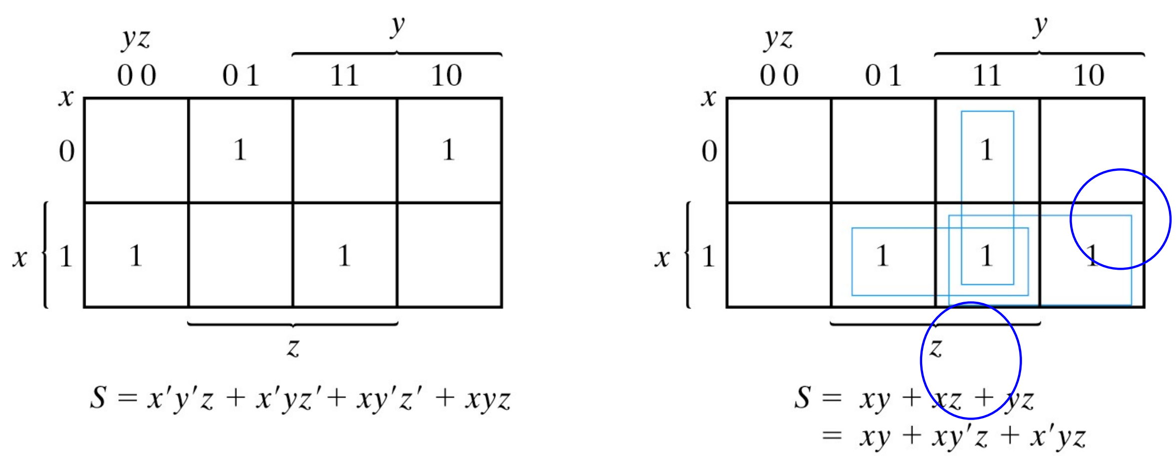


Fig. 4-6 Maps for Full Adder

C

$S = x'y'z + x'yz' + xy'z' + xyz$

$C = xy + xz + yz$

Full adder implemented in SOP

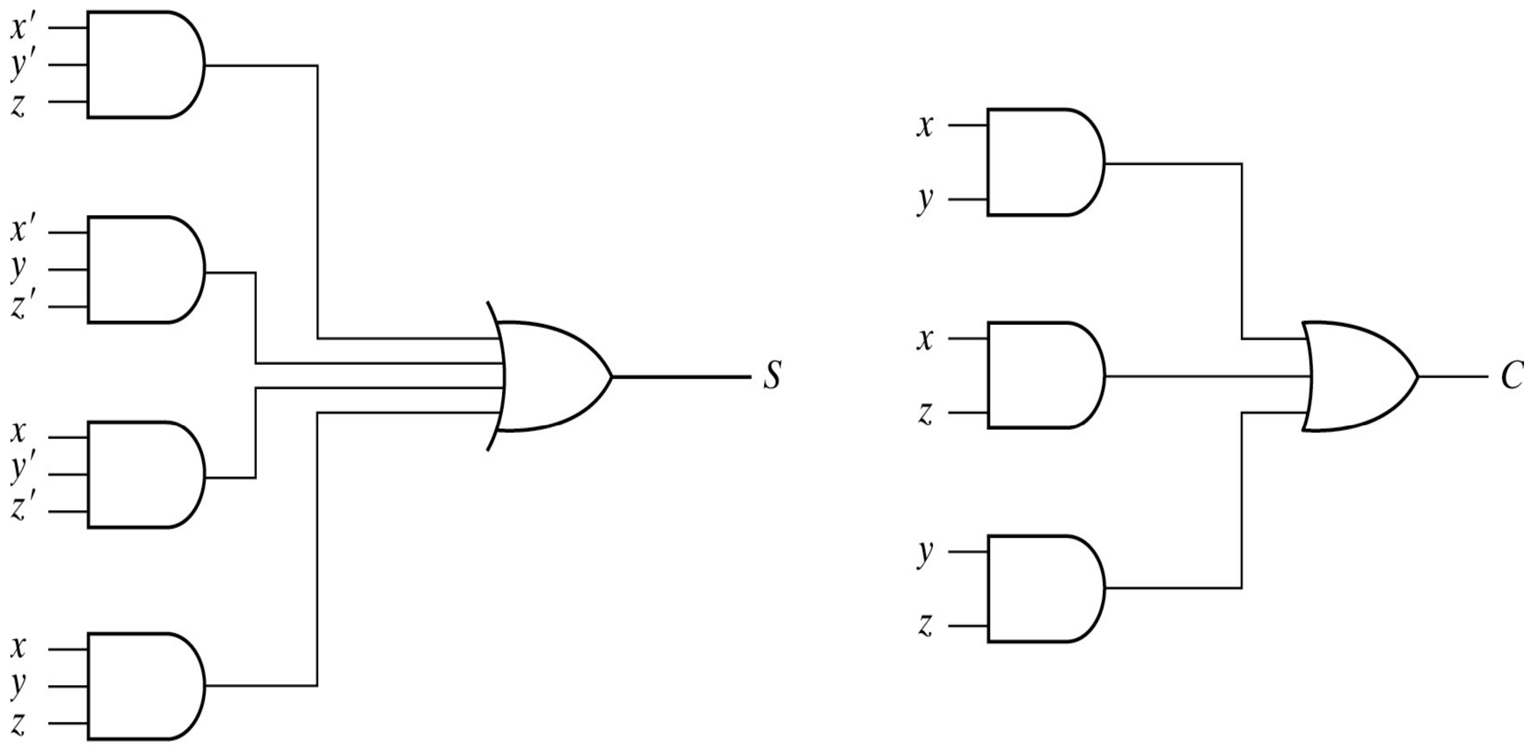


Fig. 4-7 Implementation of Full Adder in Sum of Products

Another implementation

- Full-adder can also implemented with two half adders and one OR gate (Carry Look-Ahead adder).

$$S = z \oplus (x \oplus y)$$
$$= z'(xy' + x'y) + z(xy' + x'y)'$$
$$= xy'z' + x'yz' + xyz + x'y'z$$
$$C = z(xy' + x'y) + xy = xy'z + x'yz + xy$$

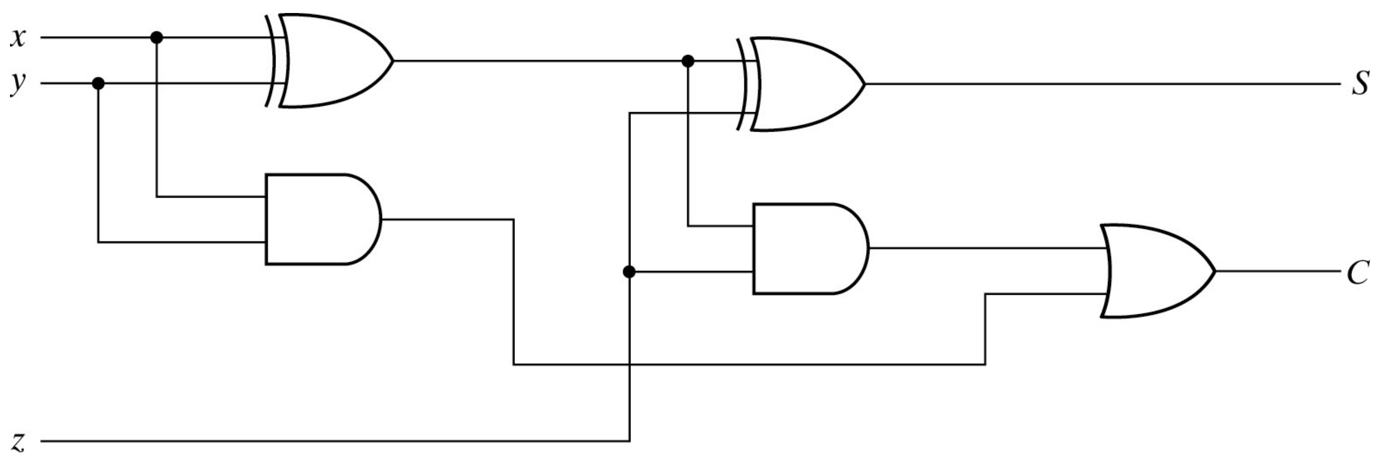


Fig. 4-8 Implementation of Full Adder with Two Half Adders and an OR Gate

Binary adder

- This is also called Ripple Carry Adder ,because of the construction with full adders are connected in cascade.

Subscript i:	3	2	1	0	
Input carry	0	1	1	0	C_i
Augend	1	0	1	1	A_i
Addend	0	0	1	1	B_i
Sum	1	1	1	0	S_i
Output carry	0	0	1	1	C_{i+1}

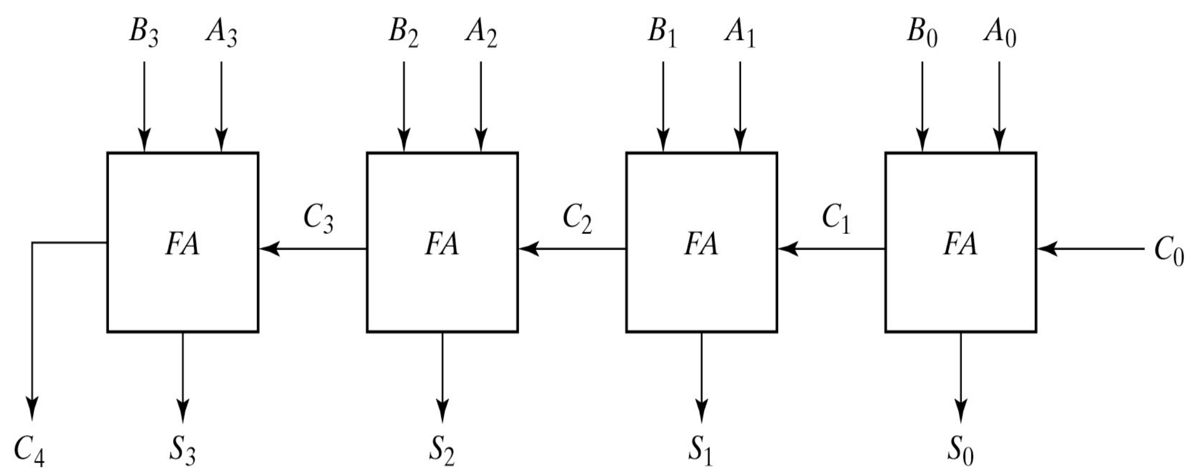


Fig. 4-9 4-Bit Adder

Carry Propagation

- Fig.4-9 causes a unstable factor on carry bit, and produces a longest propagation delay.
- The signal from C_i to the output carry C_{i+1}, propagates through an AND and OR gates, so, for an n-bit RCA, there are 2n gate levels for the carry to propagate from input to output.

Carry Propagation

- Because the propagation delay will affect the output signals on different time, so the signals are given enough time to get the precise and stable outputs.
- The most widely used technique employs the principle of carry look-ahead to improve the speed of the algorithm.

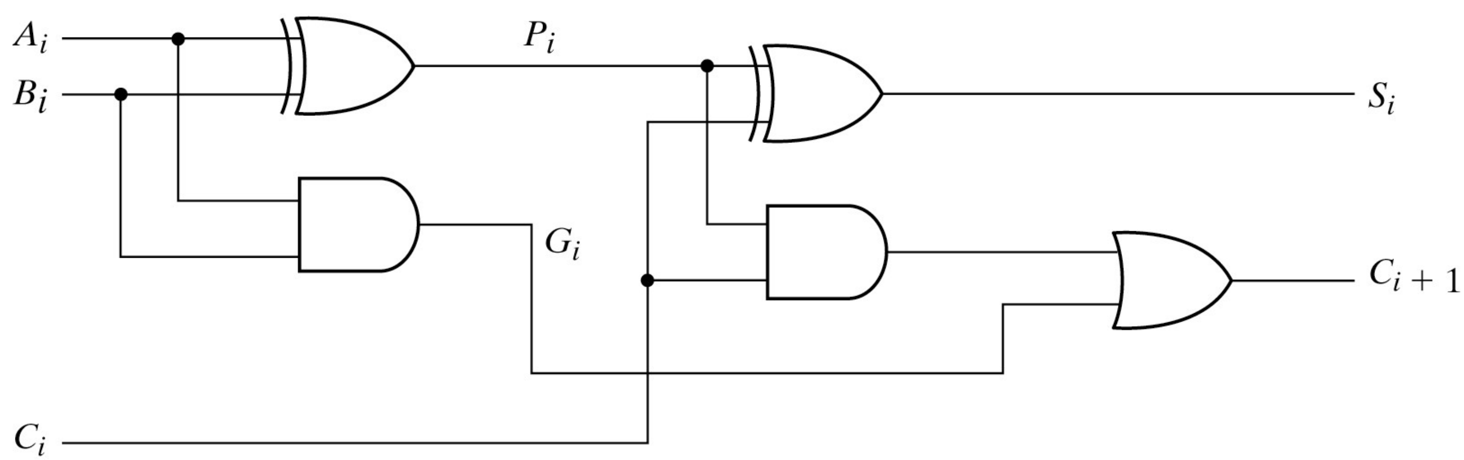


Fig. 4-10 Full Adder with P and G Shown

Boolean functions

$P_i = A_i \oplus B_i$ steady state value

$G_i = A_iB_i$ steady state value

Output sum and carry

$S_i = P_i \oplus C_i$

$C_{i+1} = G_i + P_iC_i$

G_i : carry generate P_i : carry propagate

C_0 = input carry

$C_1 = G_0 + P_0C_0$

$C_2 = G_1 + P_1C_1 = G_1 + P_1G_0 + P_1P_0C_0$

$C_3 = G_2 + P_2C_2 = G_2 + P_2G_1 + P_2P_1G_0 + P_2P_1P_0C_0$

- C_3 does not have to wait for C_2 and C_1 to propagate.

Logic diagram of carry look-ahead generator

- C_3 is propagated at the same time as C_2 and C_1.

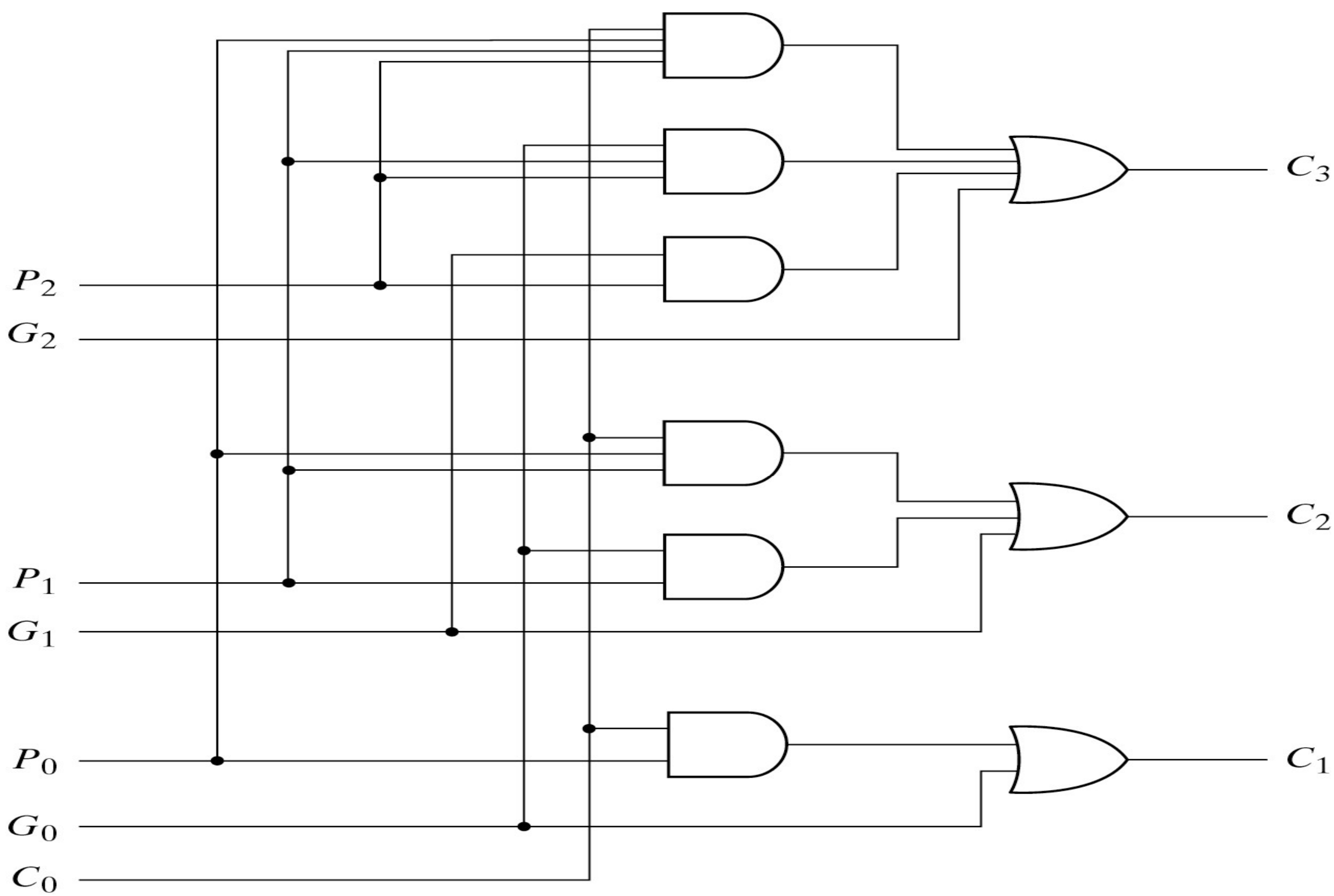


Fig. 4-11 Logic Diagram of Carry Lookahead Generator

4-bit adder with carry lookahead

- Delay time of n-bit CLAA = XOR + (AND + OR) + XOR

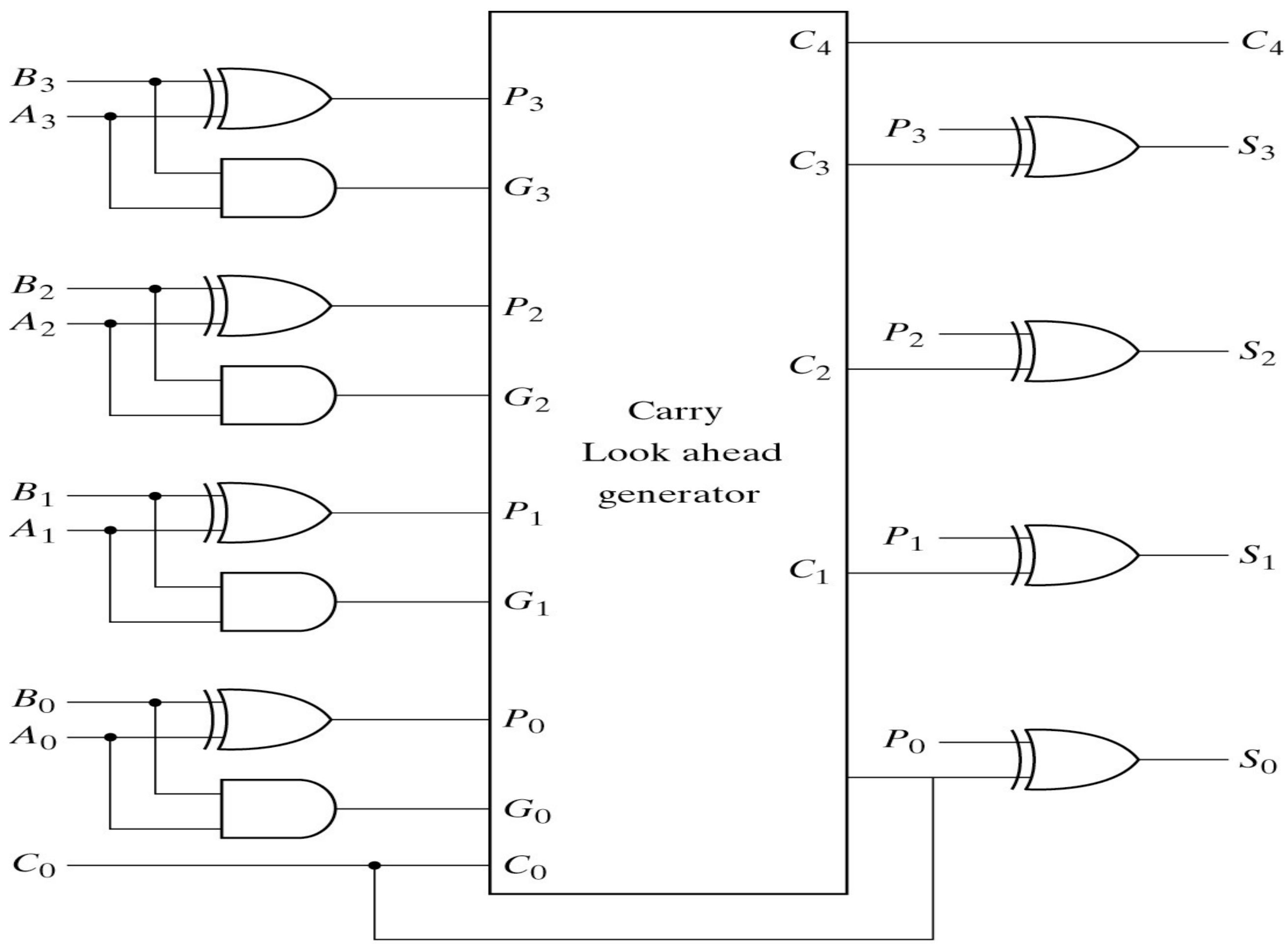


Fig. 4-12 4-Bit Adder with Carry Lookahead

Binary subtractor

M = 1→subtractor ; M = 0→adder

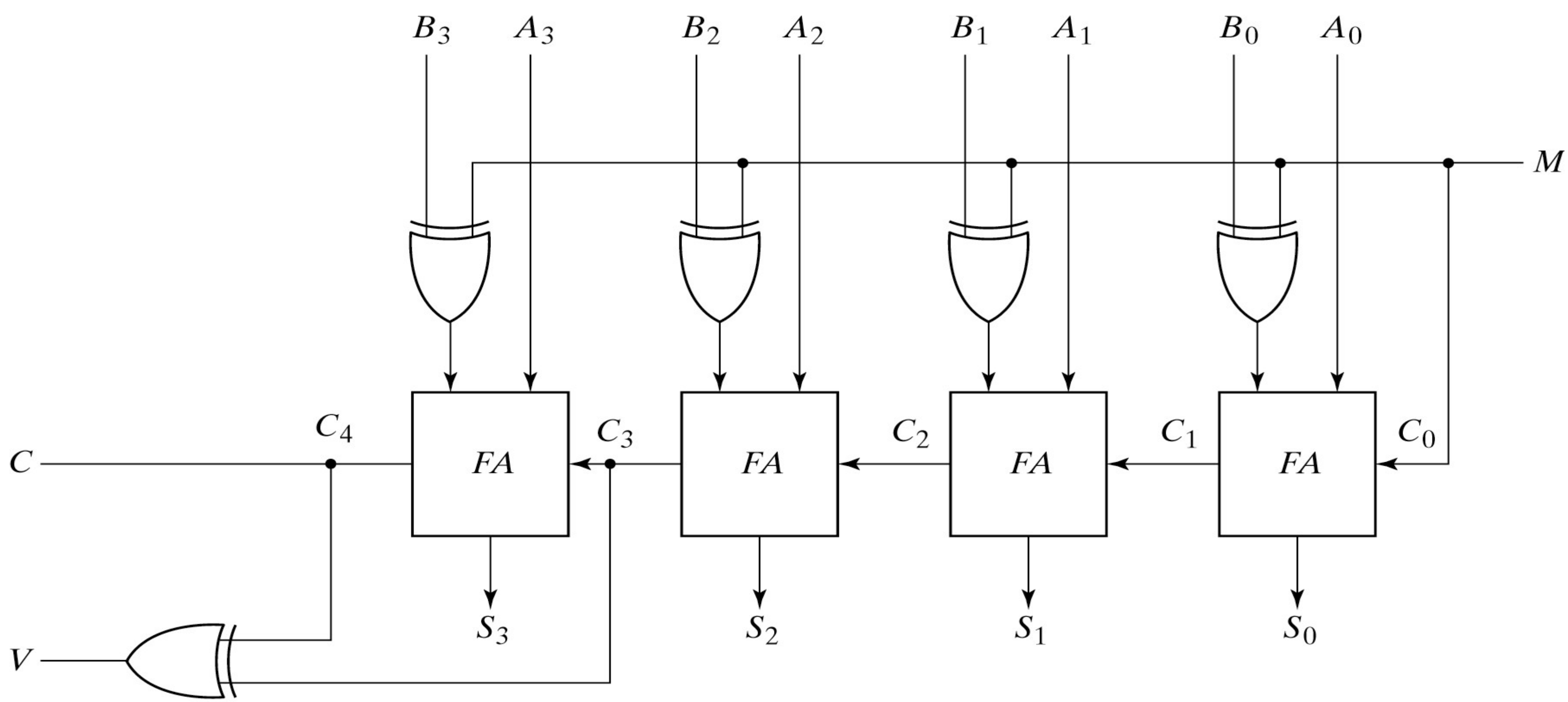


Fig. 4-13 4-Bit Adder Subtractor

Overflow

- It is worth noting Fig.4-13 that binary numbers in the signed-complement system are added and subtracted by the same basic addition and subtraction rules as unsigned numbers.
- Overflow is a problem in digital computers because the number of bits that hold the number is finite and a result that contains n+1 bits cannot be accommodated.

Overflow on signed and unsigned

- When two unsigned numbers are added, an overflow is detected from the end carry out of the MSB position.
- When two signed numbers are added, the sign bit is treated as part of the number and the end carry does not indicate an overflow.
- An overflow cann't occur after an addition if one number is positive and the other is negative.
- An overflow may occur if the two numbers added are both positive or both negative.

4-5 Decimal adder

BCD adder can't exceed 9 on each input digit. K is the carry.

Table 4-5
Derivation of BCD Adder

Binary Sum					BCD Sum					Decimal
K	Z_8	Z_4	Z_2	Z_1	C	S_8	S_4	S_2	S_1	
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	2
0	0	0	1	1	0	0	0	1	1	3
0	0	1	0	0	0	0	1	0	0	4
0	0	1	0	1	0	0	1	0	1	5
0	0	1	1	0	0	0	1	1	0	6
0	0	1	1	1	0	0	1	1	1	7
0	1	0	0	0	0	1	0	0	0	8
0	1	0	0	1	0	1	0	0	1	9
0	1	0	1	0	1	0	0	0	0	10
0	1	0	1	1	1	0	0	0	1	11
0	1	1	0	0	1	0	0	1	0	12
0	1	1	0	1	1	0	0	1	1	13
0	1	1	1	0	1	0	1	0	0	14
0	1	1	1	1	1	0	1	0	1	15
1	0	0	0	0	1	0	1	1	0	16
1	0	0	0	1	1	0	1	1	1	17
1	0	0	1	0	1	1	0	0	0	18
1	0	0	1	1	1	1	0	0	1	19

Rules of BCD adder

- When the binary sum is greater than 1001, we obtain a non-valid BCD representation.
- The addition of binary 6(0110) to the binary sum converts it to the correct BCD representation and also produces an output carry as required.
- To distinguish them from binary 1000 and 1001, which also have a 1 in position Z_8, we specify further that either Z_4 or Z_2 must have a 1.

$$C = K + Z_8 Z_4 + Z_8 Z_2$$

Implementation of BCD adder

- A decimal parallel adder that adds n decimal digits needs n BCD adder stages.
- The output carry from one stage must be connected to the input carry of the next higher-order stage.

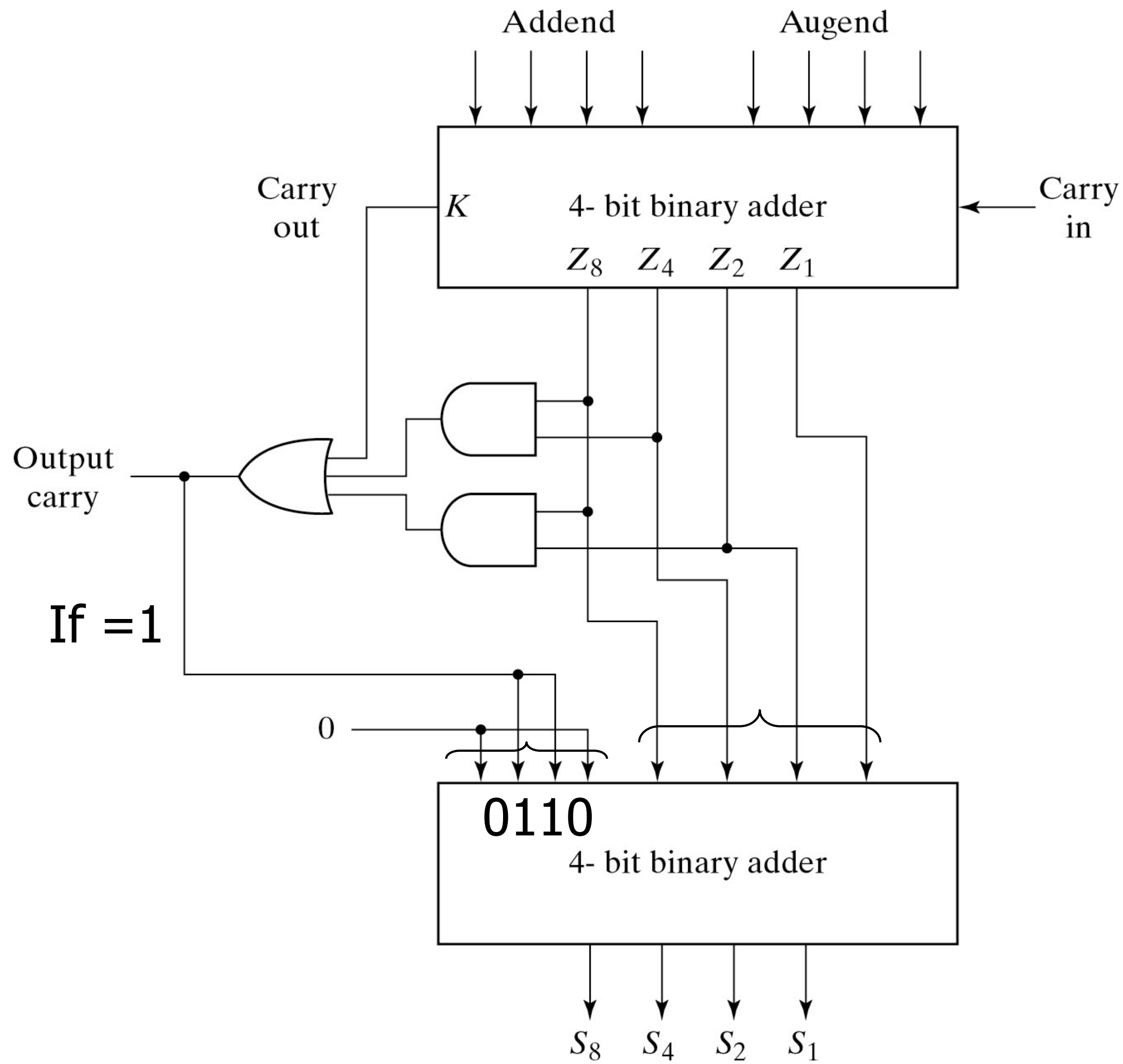


Fig. 4-14 Block Diagram of a BCD Adder

4-6. Binary multiplier

- Usually there are more bits in the partial products and it is necessary to use full adders to produce the sum of the partial products.

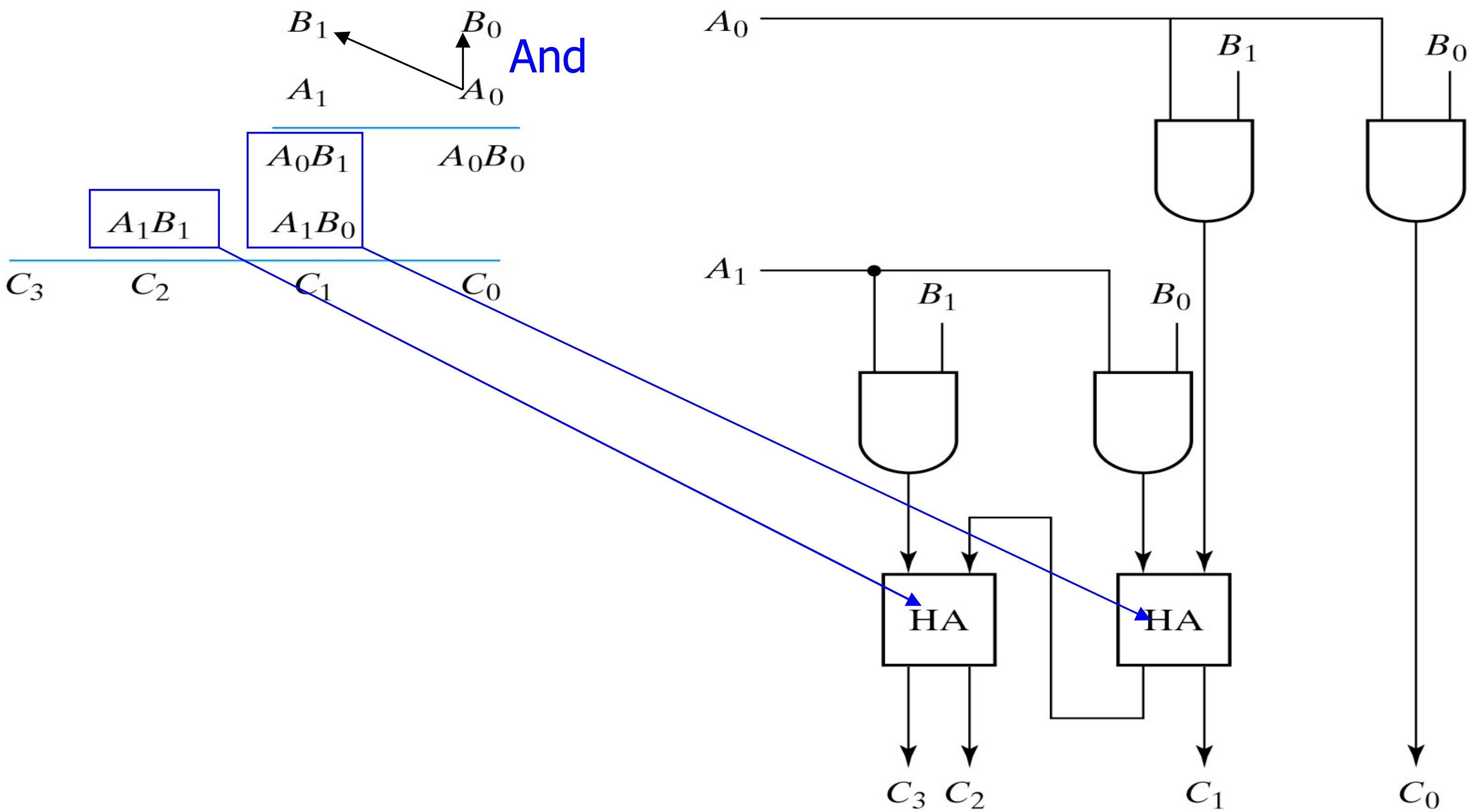


Fig. 4-15 2-Bit by 2-Bit Binary Multiplier

4-bit by 3-bit binary multiplier

- For J multiplier bits and K multiplicand bits we need (J X K) AND gates and (J − 1) K-bit adders to produce a product of J+K bits.
- K=4 and J=3, we need 12 AND gates and two 4-bit adders.

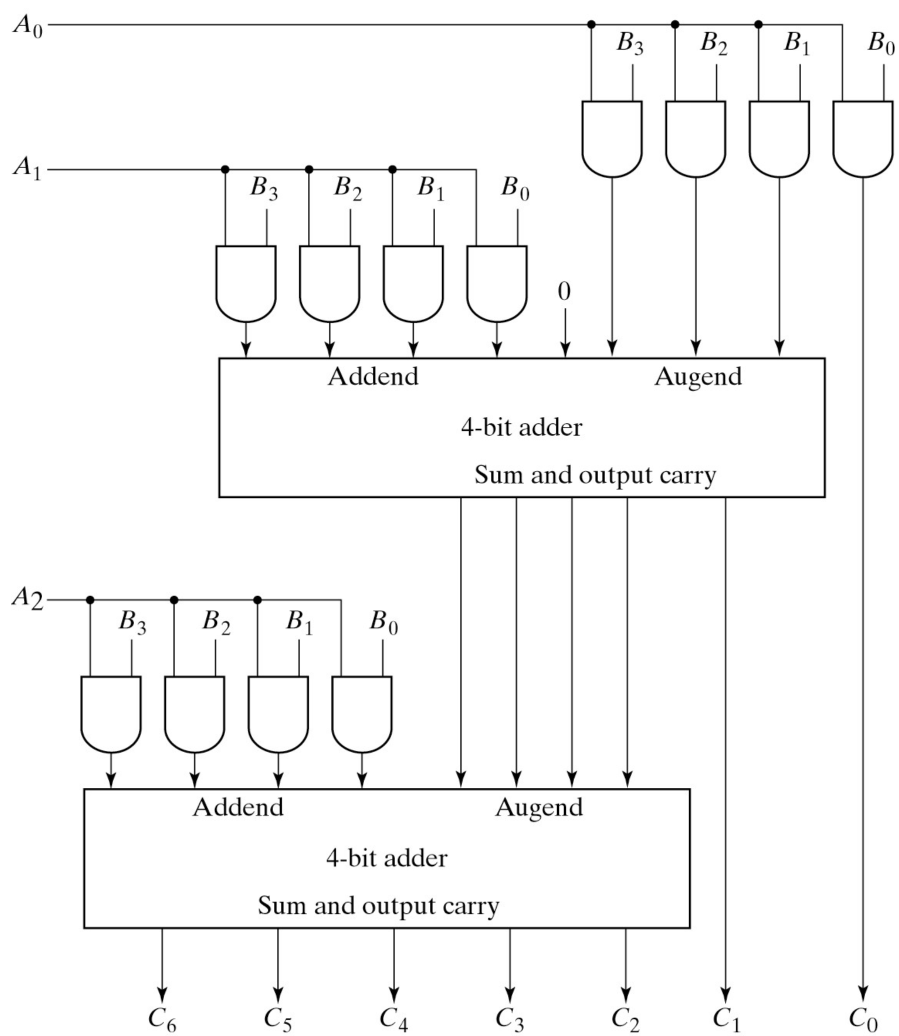


Fig. 4-16 4-Bit by 3-Bit Binary Multiplier

4-7. Magnitude comparator

- The equality relation of each pair of bits can be expressed logically with an exclusive-NOR function as:

  $A = A_3A_2A_1A_0$; $B = B_3B_2B_1B_0$

  $x_i = A_iB_i + A_i'B_i'$ for i = 0, 1, 2, 3

  $(A = B) = x_3x_2x_1x_0$

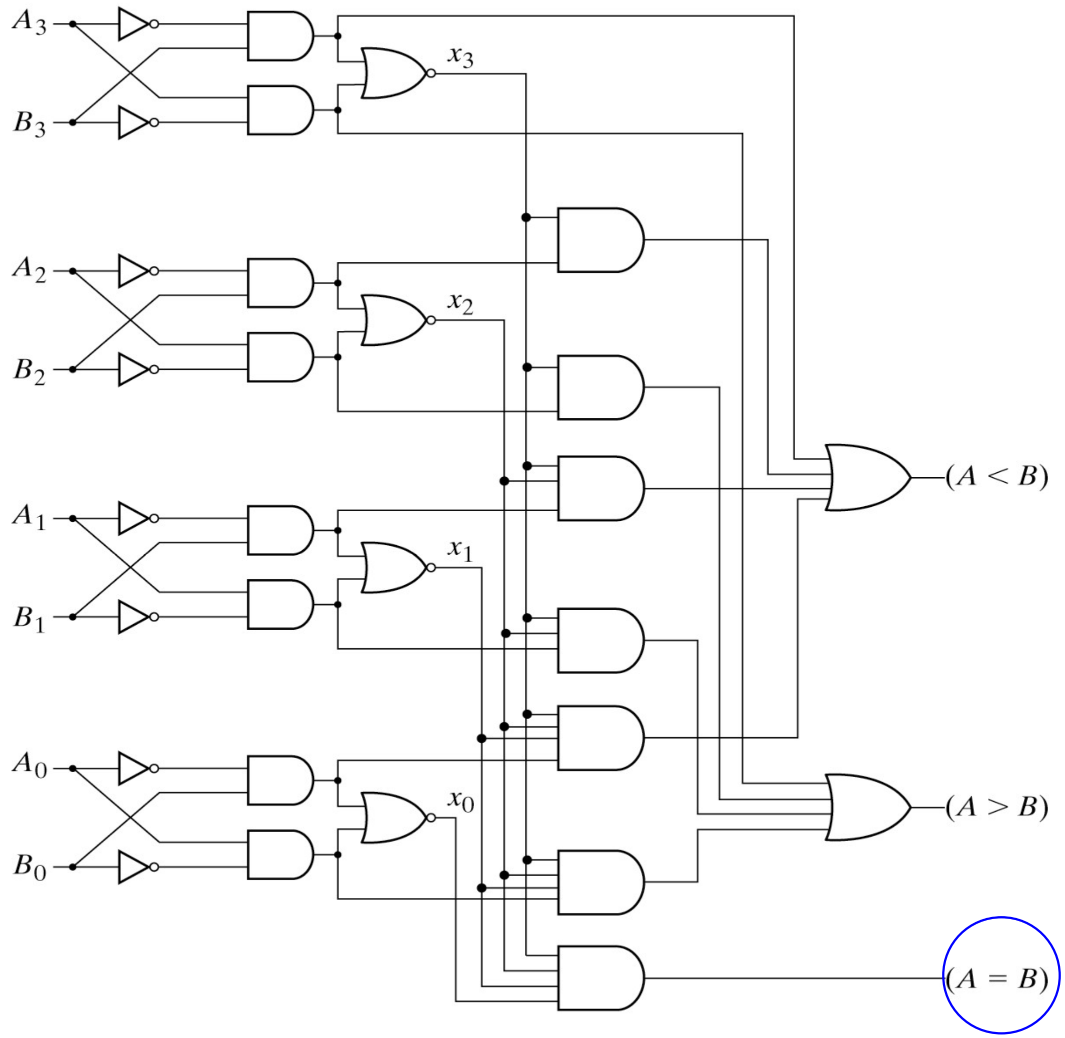


Fig. 4-17 4-Bit Magnitude Comparator

Magnitude comparator

- We inspect the relative magnitudes of pairs of MSB. If equal, we compare the next lower significant pair of digits until a pair of unequal digits is reached.
- If the corresponding digit of A is 1 and that of B is 0, we conclude that A>B.

(A>B)=
$A_3B'_3+x_3A_2B'_2+x_3x_2A_1B'_1+x_3x_2x_1A_0B'_0$
(A<B)=
$A'_3B_3+x_3A'_2B_2+x_3x_2A'_1B_1+x_3x_2x_1A'_0B_0$

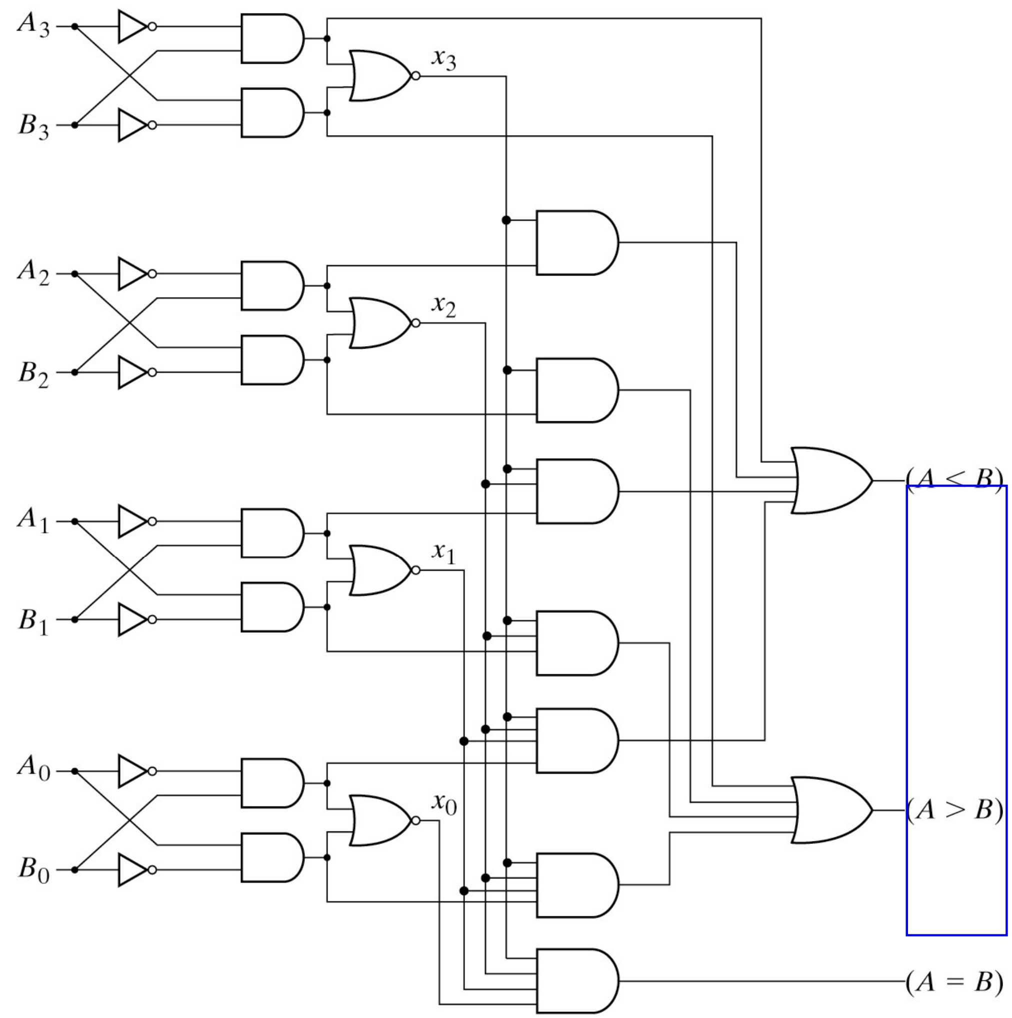


Fig. 4-17 4-Bit Magnitude Comparator

4-8. Decoders

- The decoder is called n-to-m-line decoder, where $m \leq 2^n$.
- the decoder is also used in conjunction with other code converters such as a BCD-to-seven_segment decoder.
- 3-to-8 line decoder: For each possible input combination, there are seven outputs that are equal to 0 and only one that is equal to 1.

Implementation and truth table

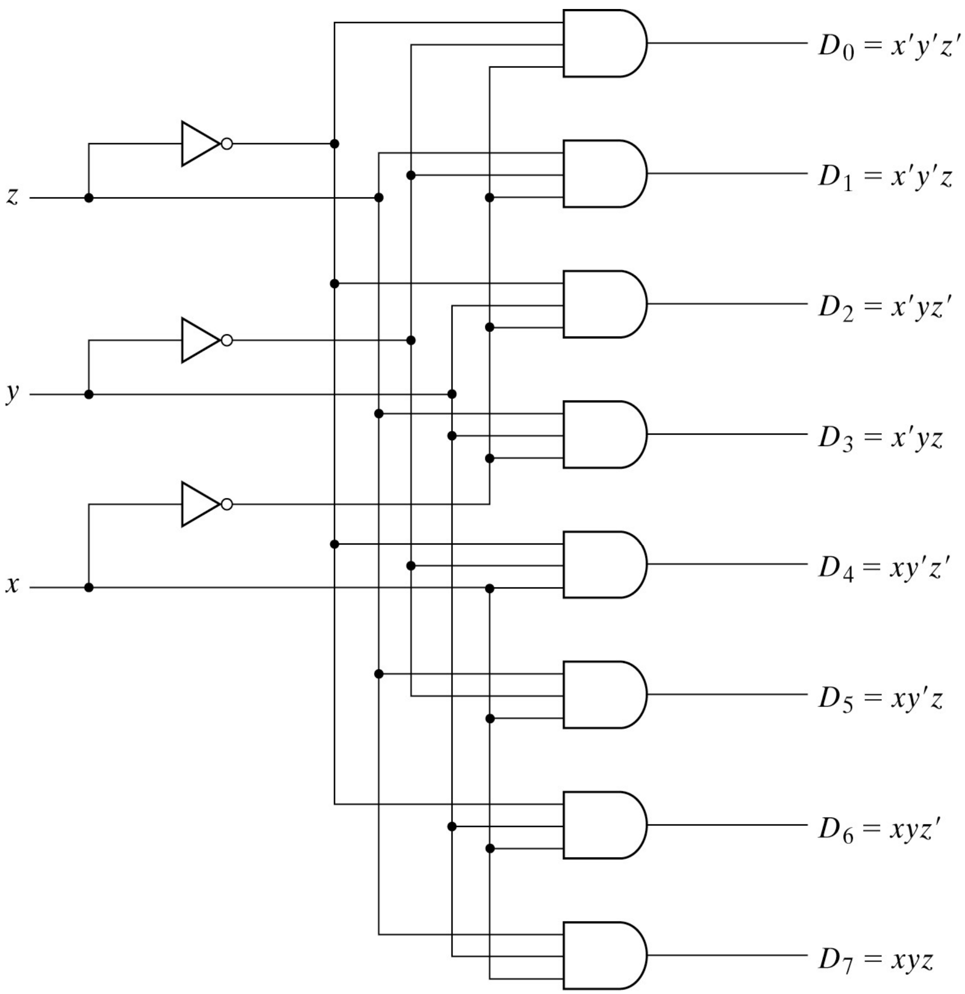


Fig. 4-18 3-to-8-Line Decoder

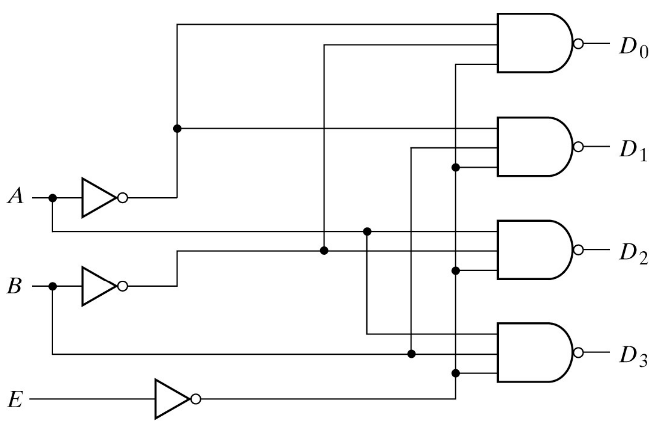
Table 4-6

Truth Table of a 3-to-8-Line Decoder

Inputs			Outputs							
x	y	z	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Decoder with enable input

- Some decoders are constructed with NAND gates, it becomes more economical to generate the decoder minterms in their complemented form.
- As indicated by the truth table , only one output can be equal to 0 at any given time, all other outputs are equal to 1.



(a) Logic diagram

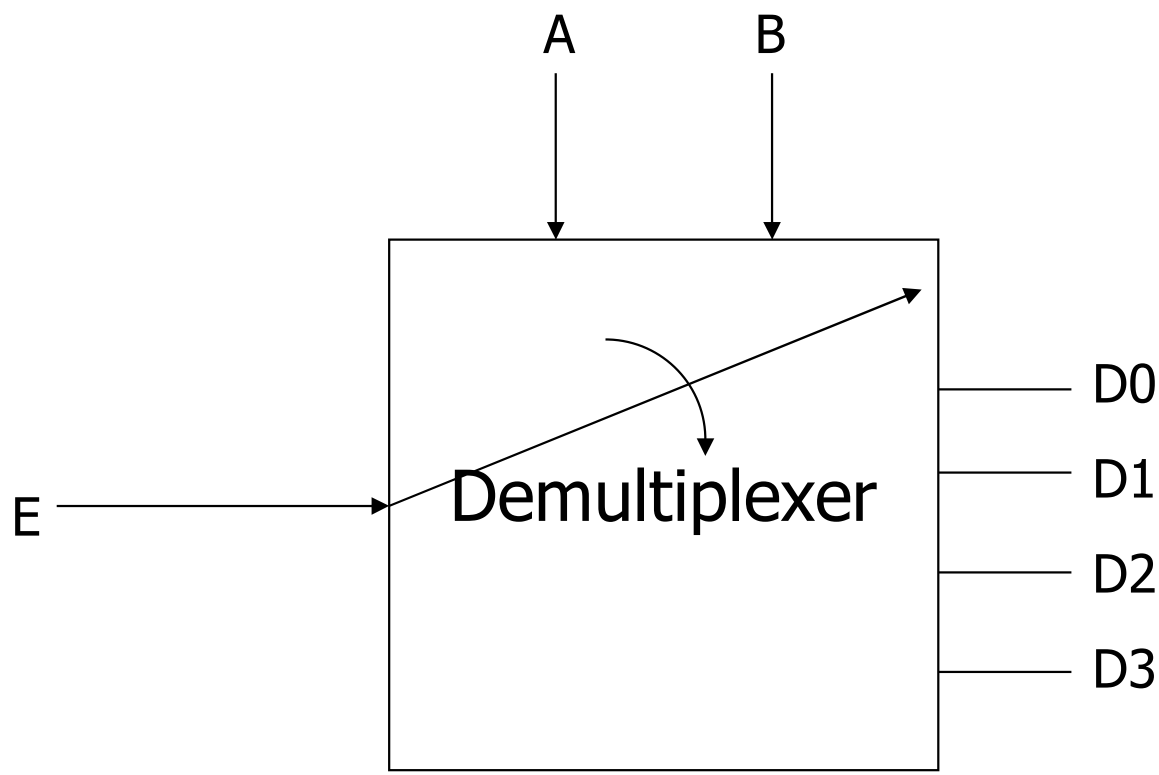
E	A	B	D_0	D_1	D_2	D_3
1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

(b) Truth table

Fig. 4-19 2-to-4-Line Decoder with Enable Input

Demultiplexer

- A decoder with an enable input is referred to as a decoder/demultiplexer.
- The truth table of demultiplexer is the same with decoder.



3-to-8 decoder with enable implement the 4-to-16 decoder

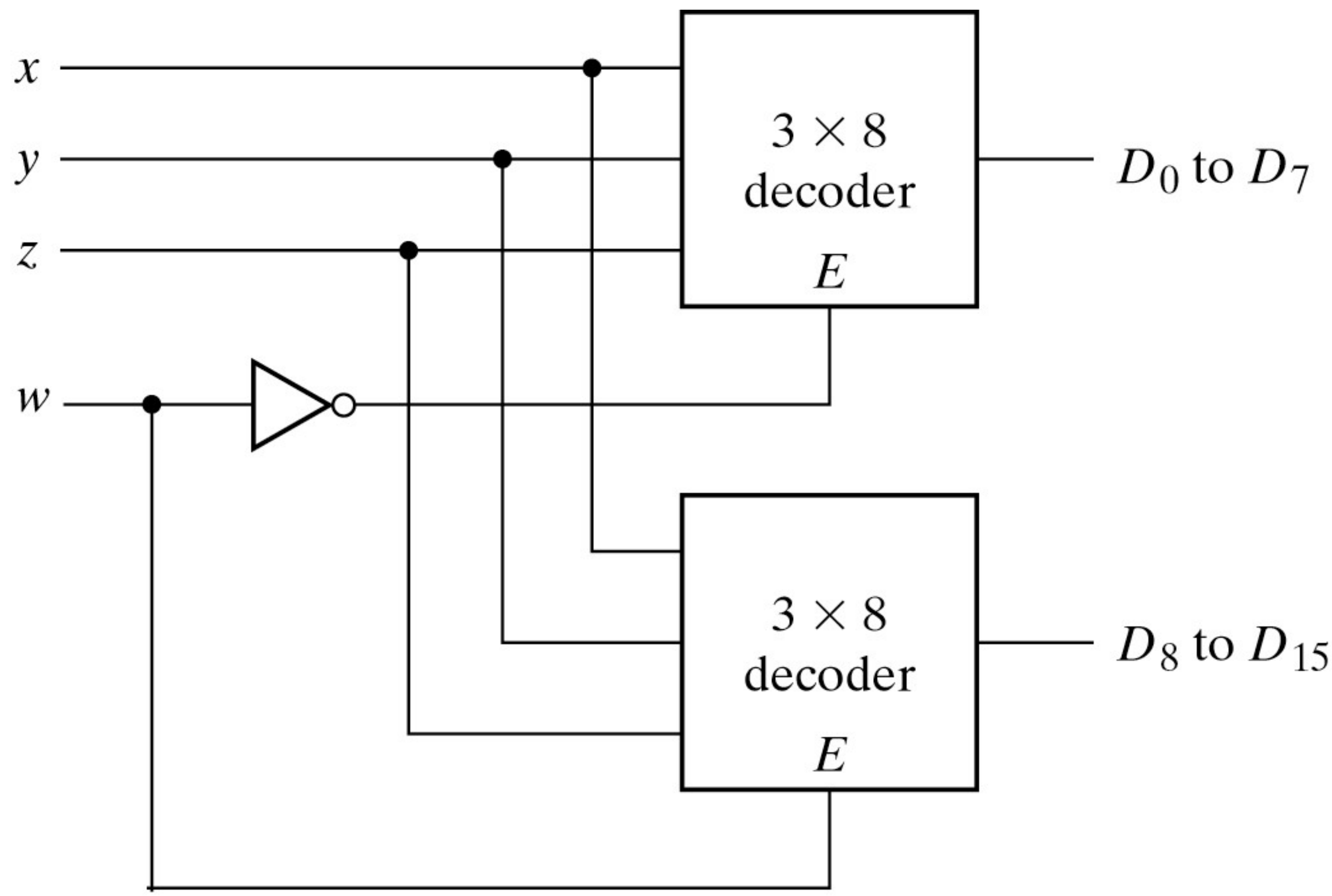


Fig. 4-20 4 × 16 Decoder Constructed with Two 3 × 8 Decoders

Implementation of a Full Adder with a Decoder

- From table 4-4, we obtain the functions for the combinational circuit in sum of minterms:

  $S(x, y, z) = \sum(1, 2, 4, 7)$

  $C(x, y, z) = \sum(3, 5, 6, 7)$

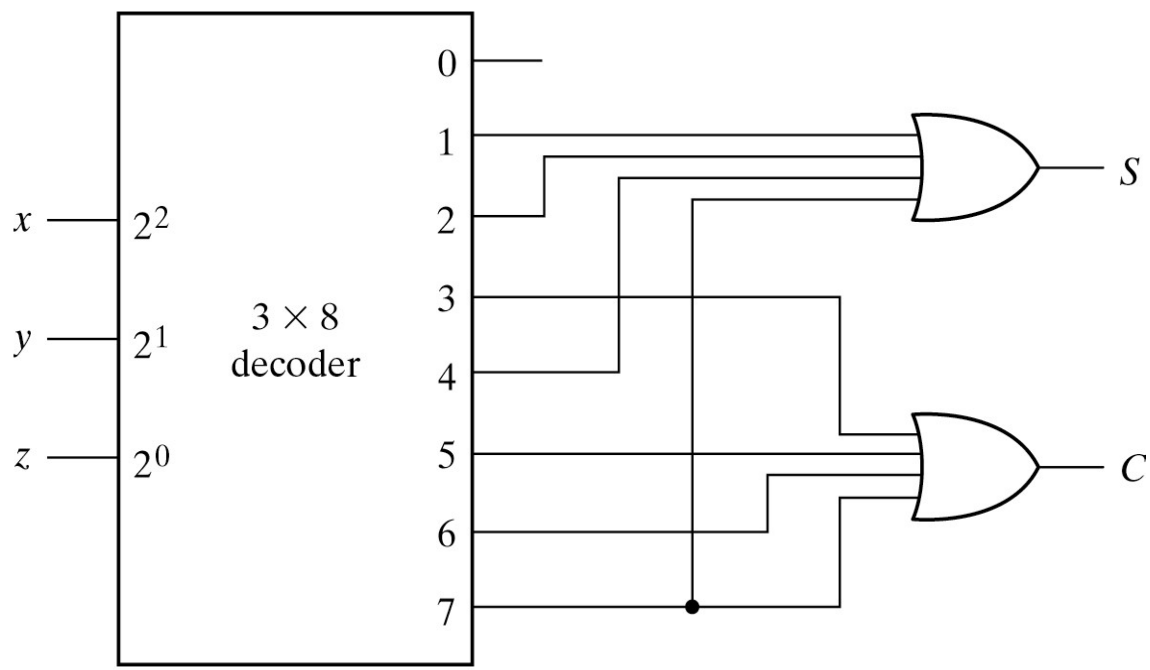


Fig. 4-21 Implementation of a Full Adder with a Decoder

4-9. Encoders

- An encoder is the inverse operation of a decoder.
- We can derive the Boolean functions by table 4-7

$$z = D_1 + D_3 + D_5 + D_7$$

$$y = D_2 + D_3 + D_6 + D_7$$

$$x = D_4 + D_5 + D_6 + D_7$$

Table 4-7
Truth Table of Octal-to-Binary Encoder

Inputs								Outputs		
D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	x	y	z
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

Priority encoder

- If two inputs are active simultaneously, the output produces an undefined combination. We can establish an input priority to ensure that only one input is encoded.
- Another ambiguity in the octal-to-binary encoder is that an output with all 0's is generated when all the inputs are 0; the output is the same as when D_0 is equal to 1.
- The discrepancy tables on Table 4-7 and Table 4-8 can resolve aforesaid condition by providing one more output to indicate that at least one input is equal to 1.

Priority encoder

V=0→no valid inputs

V=1→valid inputs

X's in output columns represent don't-care conditions

X's in the input columns are useful for representing a truth table in condensed form.

Instead of listing all 16 minterms of four variables.

Table 4-8
Truth Table of a Priority Encoder

Inputs				Outputs		
D_0	D_1	D_2	D_3	x	y	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

4-input priority encoder

- Implementation of table 4-8

$x = D_2 + D_3$

$y = D_3 + D_1D'_2$

$V = D_0 + D_1 + D_2 + D_3$

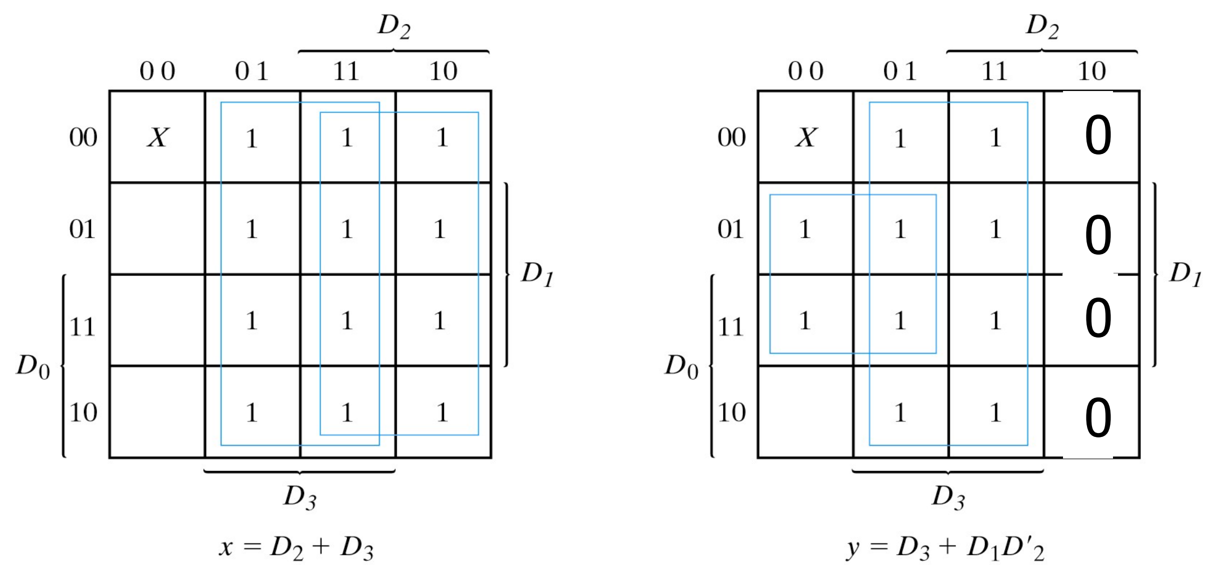


Fig. 4-22 Maps for a Priority Encoder

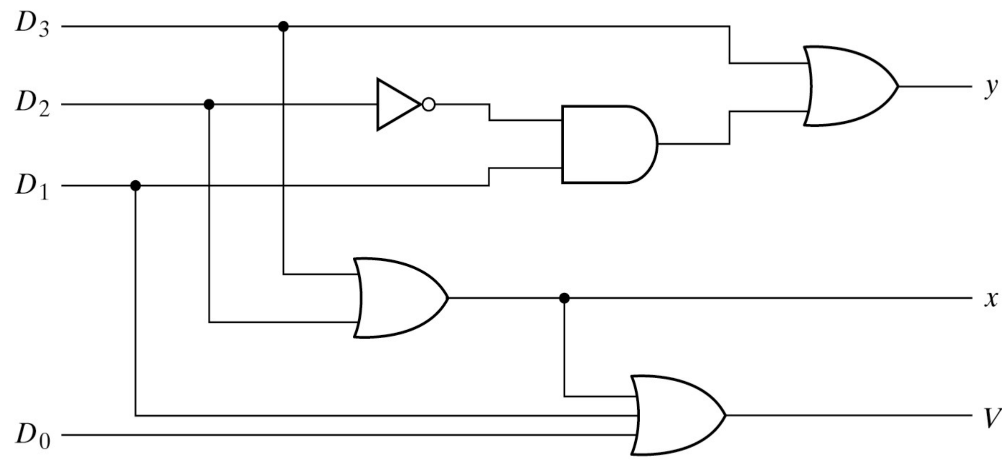


Fig. 4-23 4-Input Priority Encoder

4-10. Multiplexers

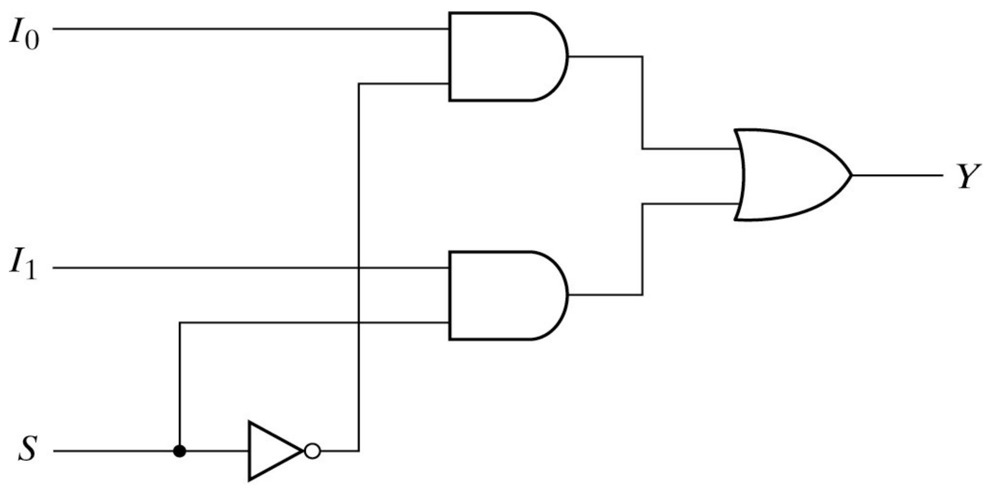
$S = 0, Y = I_0$

$S = 1, Y = I_1$

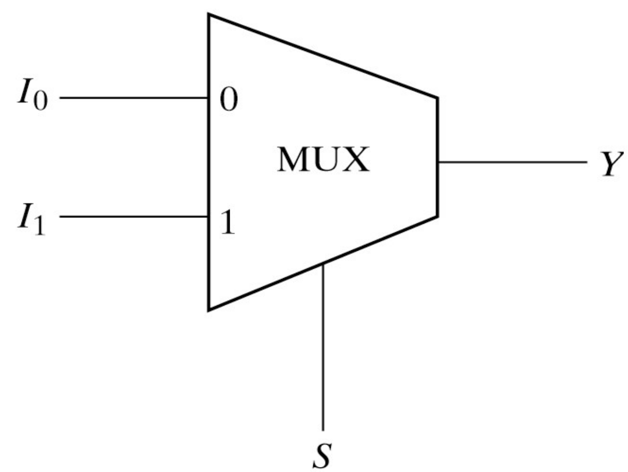
Truth Table→

S	Y
0	I_0
1	I_1

$Y = S'I_0 + SI_1$



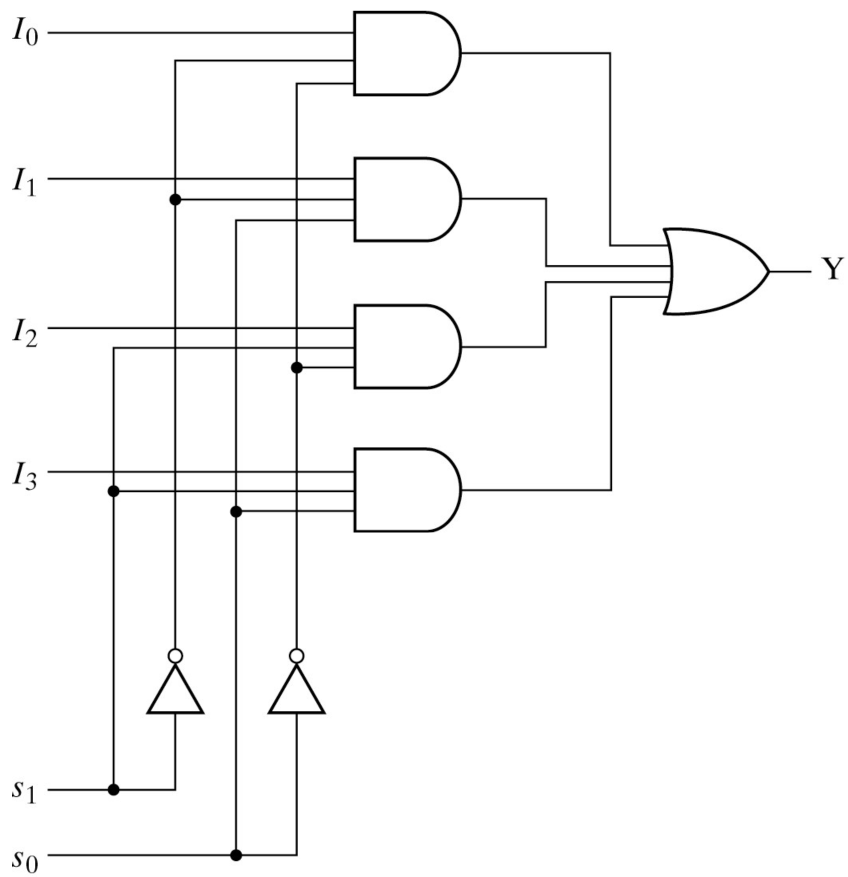
(a) Logic diagram



(b) Block diagram

Fig. 4-24 2-to-1-Line Multiplexer

4-to-1 Line Multiplexer



(a) Logic diagram

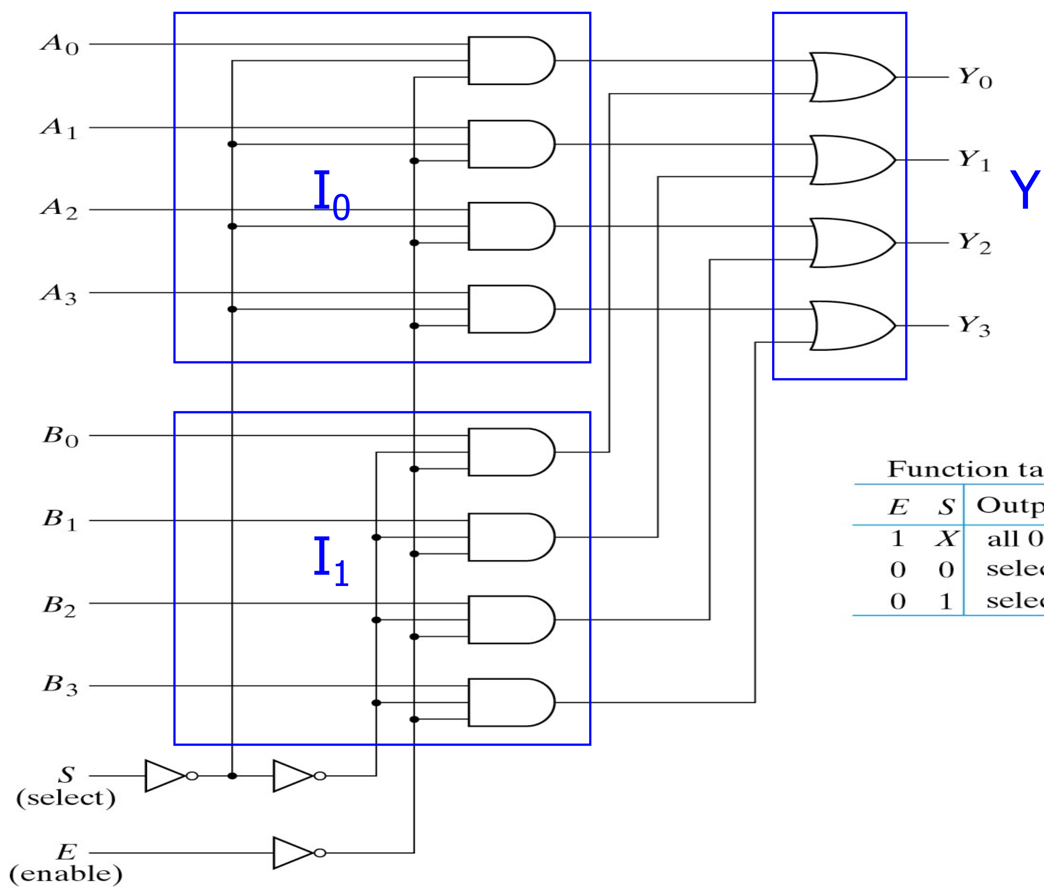
s_1	s_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

(b) Function table

Fig. 4-25 4-to-1-Line Multiplexer

Quadruple 2-to-1 Line Multiplexer

- Multiplexer circuits can be combined with common selection inputs to provide multiple-bit selection logic. Compare with Fig4-24.



Function table

E	S	Output Y
1	X	all 0's
0	0	select A
0	1	select B

Fig. 4-26 Quadruple 2-to-1-Line Multiplexer

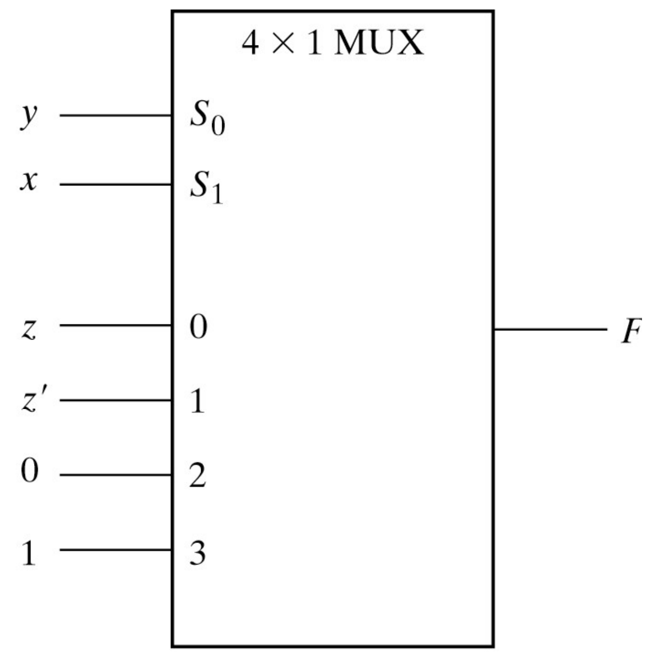
Boolean function implementation

- A more efficient method for implementing a Boolean function of n variables with a multiplexer that has n-1 selection inputs.

$$F(x, y, z) = \Sigma(1,2,6,7)$$

x	y	z	F	
0	0	0	0	$F = z$
0	0	1	1	
0	1	0	1	$F = z'$
0	1	1	0	
1	0	0	0	$F = 0$
1	0	1	0	
1	1	0	1	$F = 1$
1	1	1	1	

(a) Truth table



(b) Multiplexer implementation

Fig. 4-27 Implementing a Boolean Function with a Multiplexer

4-input function with a multiplexer

$F(A, B, C, D) = \Sigma(1, 3, 4, 11, 12, 13, 14, 15)$

A	B	C	D	F	
0	0	0	0	0	$F = D$
0	0	0	1	1	
0	0	1	0	0	$F = D$
0	0	1	1	1	
0	1	0	0	1	$F = D'$
0	1	0	1	0	
0	1	1	0	0	$F = 0$
0	1	1	1	0	
1	0	0	0	0	$F = 0$
1	0	0	1	0	
1	0	1	0	0	$F = D$
1	0	1	1	1	
1	1	0	0	1	$F = 1$
1	1	0	1	1	
1	1	1	0	1	$F = 1$
1	1	1	1	1	

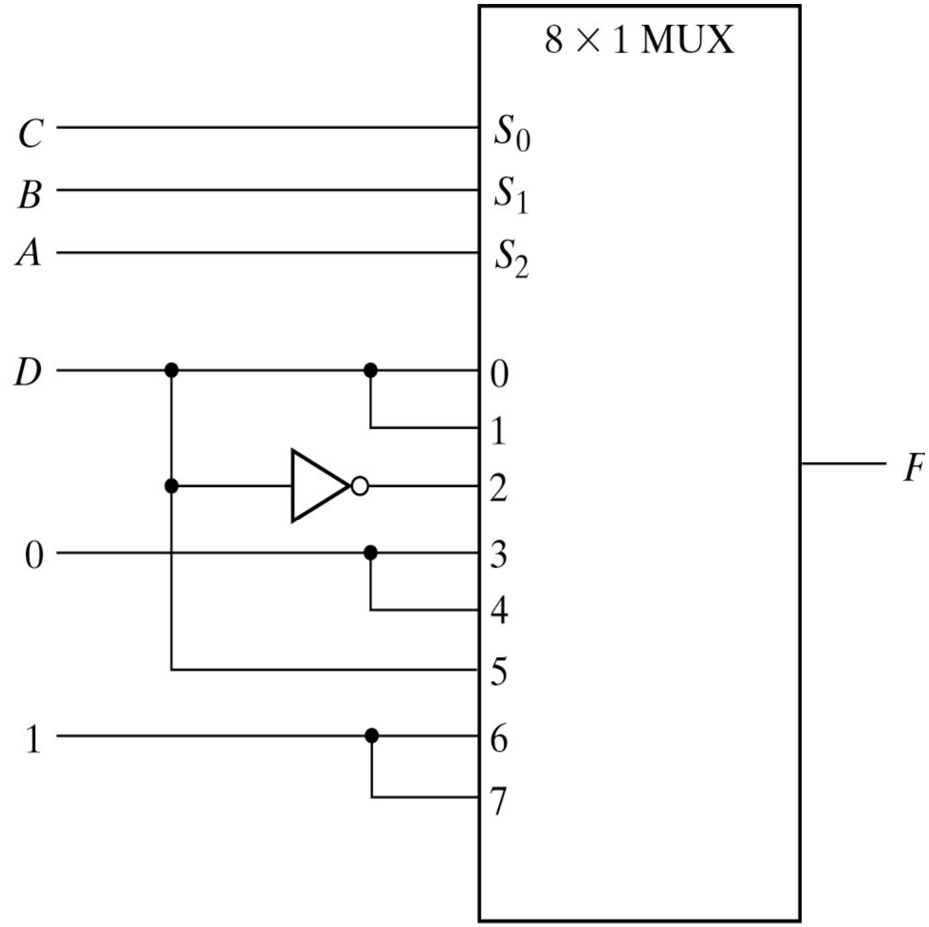


Fig. 4-28 Implementing a 4-Input Function with a Multiplexer

Three-State Gates

- A multiplexer can be constructed with three-state gates.

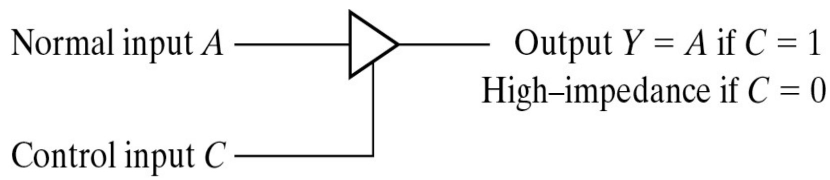


Fig. 4-29 Graphic Symbol for a Three-State Buffer

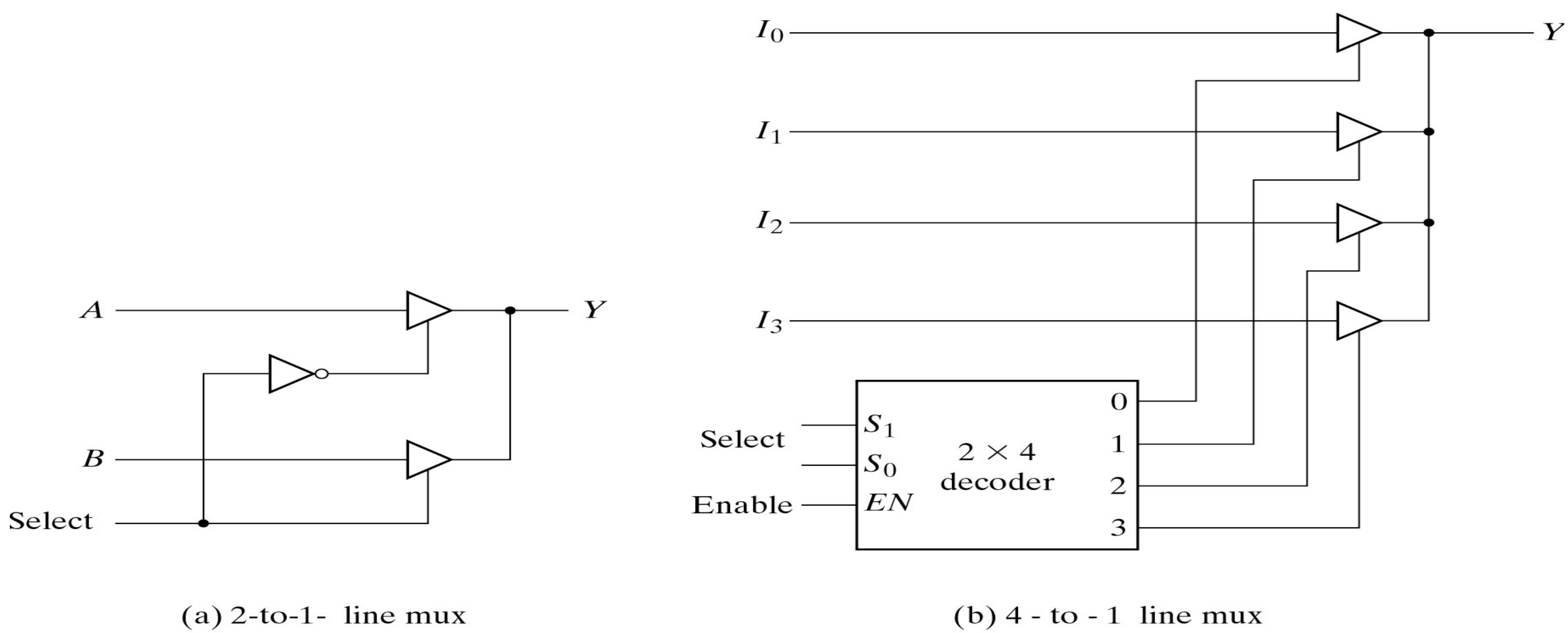


Fig. 4-30 Multiplexers with Three-State Gates

4-11. HDL for combinational circuits

- A module can be described in any one of the following modeling techniques:

1. Gate-level modeling using instantiation of primitive gates and user-defined modules.
2. Dataflow modeling using continuous assignment statements with keyword assign.
3. Behavioral modeling using procedural assignment statements with keyword always.

Gate-level Modeling

- A circuit is specified by its logic gates and their interconnection.
- Verilog recognizes 12 basic gates as predefined primitives.
- The logic values of each gate may be 1, 0, x(unknown), z(high-impedance).

Table 4-9
Truth Table for Predefined Primitive Gates

and	0	1	x	z
0	0	0	0	0
1	0	1	x	x
x	0	x	x	x
z	0	x	x	x

or	0	1	x	z
0	0	1	x	x
1	1	1	1	1
x	x	1	x	x
z	x	1	x	x

xor	0	1	x	z
0	0	1	x	x
1	1	0	x	x
x	x	x	x	x
z	x	x	x	x

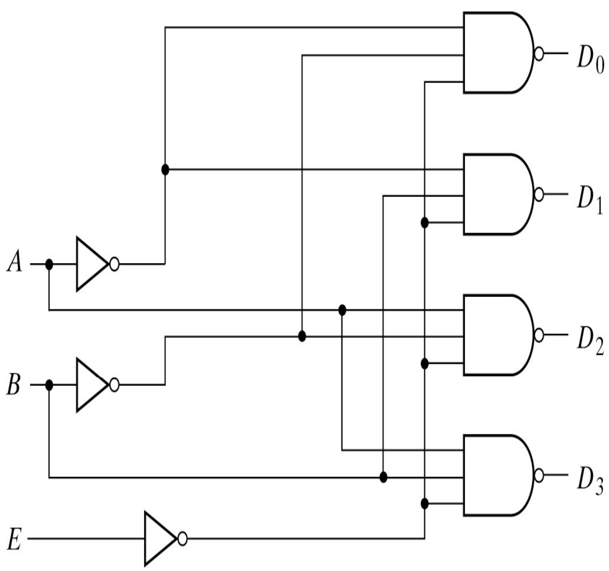
not	input	output
	0	1
	1	0
	x	x
	z	x

Gate-level description on Verilog code

The wire declaration is for internal

HDL Example 4-1

```
//Gate-level description of a 2-to-4-line decoder
//Figure 4-19
module decoder_gl (A,B,E,D);
   input A,B,E;
   output [0:3]D;
   wire Anot,Bnot,Enot;
   not
      n1 (Anot,A),
      n2 (Bnot,B),
      n3 (Enot,E);
   nand
      n4 (D[0],Anot,Bnot,Enot),
      n5 (D[1],Anot,B,Enot),
      n6 (D[2],A,Bnot,Enot),
      n7 (D[3],A,B,Enot);
endmodule
```



(a) Logic diagram

E	A	B	D_0	D_1	D_2	D_3
1	X	X	1	1	1	1
0	0	0	0	1	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	0

(b) Truth table

Fig. 4-19 2-to-4-Line Decoder with Enable Input

Design methodologies

- There are two basic types of design methodologies: top-down and bottom-up.
- Top-down: the top-level block is defined and then the sub-blocks necessary to build the top-level block are identified.(Fig.4-9 binary adder)
- Bottom-up: the building blocks are first identified and then combined to build the top-level block.(Example 4-2 4-bit adder)