



**EC8093 Digital Image Processing**  
Image Processing Fundamentals  
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What is an image?

What is pixel?

What is image processing?

What is the purpose of image processing?

What is digital image?

What is digital image processing?

What is Digital Image Processing?

### **Digital Image**

**a two-dimensional function**

**$x$  and  $y$  are spatial coordinates**

**The amplitude of  $f$  is called intensity or gray level at the point  $(x, y)$**

### **Digital Image Processing**

**Process digital images by means of computer, it covers low-, mid-, and high-level processes**

**low-level: inputs and outputs are images**

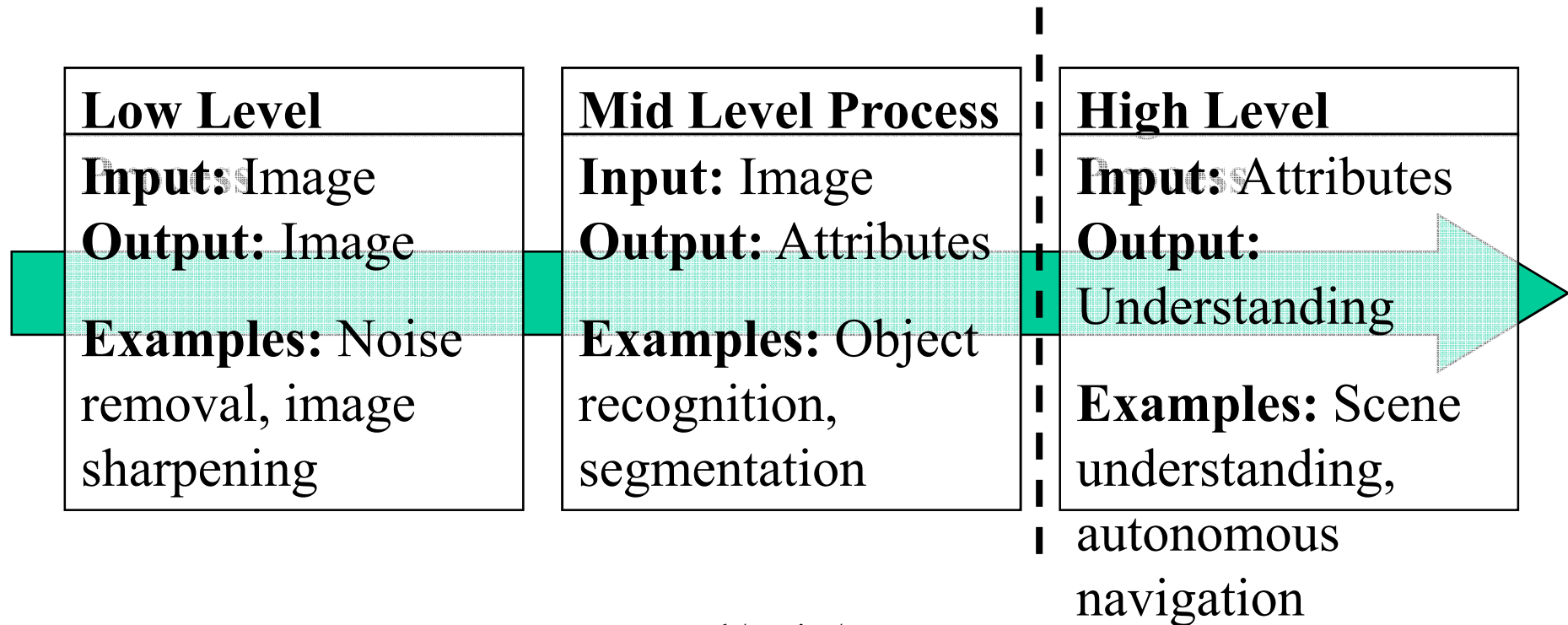
**mid-level: outputs are attributes extracted from input images**

**high-level: an ensemble of recognition of individual objects**

### **Pixel**

**the elements of a digital image**

# What is DIP?



# Representing Digital Images

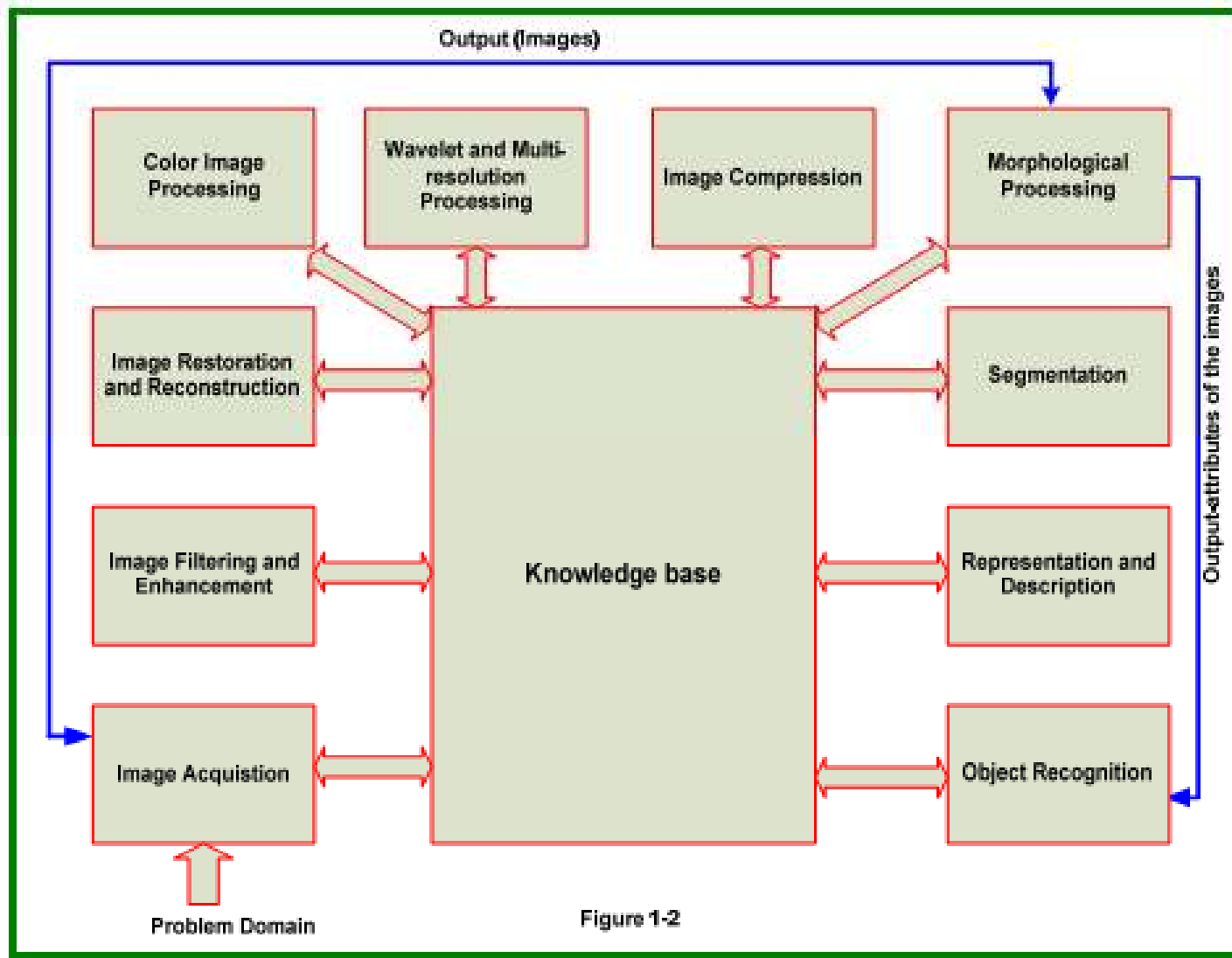
- The representation of an  $M \times N$  numerical array as

$$f(x, y) = \begin{bmatrix} f(0, 0) & f(0, 1) & \dots & f(0, N-1) \\ f(1, 0) & f(1, 1) & \dots & f(1, N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1, 0) & f(M-1, 1) & \dots & f(M-1, N-1) \end{bmatrix}$$

digital image is composed of  $M$  rows and  $N$  columns of pixels each storing a value Pixel values are most Before we discuss image acquisition recall that a digital image is composed of  $M$  rows and  $N$  columns of pixels each storing a value Pixel values are most often grey levels in the range 0-255(black-white)

We will see later on that images can easily be represented as matrices often grey levels in the range 0-255(black-white)

## Fundamental Steps in Digital Image Processing



### Step 1. Image Acquisition:

- In this step, the image is captured by a **sensor** (such as a monochrome or color TV camera) and **digitized**, if the output of the camera or sensor is not already in digital form- an **analog-to-digital** converter (**ADC**) digitizes it.

- **Camera:**

Camera consists of 2 parts:

- A **lens** that collects the appropriate type of radiation emitted from the object of interest and that forms an image of the real object.
- **Semiconductor device** – so called charged coupled device or **CCD** which converts the **irradiance** at the image plan into an **electrical signal**.
- **Frame Grabber**
- Frame Grabber only needs circuits to **digitize the electrical signal** (standard video signal) from imaging sensor to store the image in the memory (RAM) of the computer.

### Step 2. Image Enhancement:

- **Image Enhancement** is the process of manipulating an image so that the result is more suitable than the original for specific applications. Enhancement techniques are so varied, and use so many different image processing approaches.

### Step 3. Image Restoration:

- **Improving** the appearance of the image.
- Tend to be **mathematical or probabilities models** of image degradation.

### Step 4. Color Image Processing:

- Use the **color** of the image to extract features of interest in an image.

### Step 5. Wavelets:

- Used in image data **compression** and **pyramidal representation**.



### Step 6. Compression:

- Techniques Compression for
  - Reducing the storage required to **save** an image.
  - Reducing the size of the image to **transmit** it ("JPEG Standard"), with suitable bandwidth required for transmission.

### Step 7. Morphological Processing:

- Morphological Processing are the **tools** for extracting image' components that are useful in the representation and description of shape.

### Step 8. Image Segmentation:

- Computer tries to separate **objects** from the image background.
- It is one of the most **difficult tasks** in DIP.
- **Segmentation kinds:**
  - **Autonomous Segmentation.**
  - **Rugged Segmentation** (long process to get successful solution).
  - **Erratic Segmentation.**

### Step 9. Representation and Description:

- Representation makes a decision whether the data should be represented as a **boundary** or as a **complete region**:
  - **Boundary Representation** focuses on **external** shape characteristics, such as corners and inflections.
  - **Region Representation** focuses on **internal** properties, such as texture or skeleton shape.

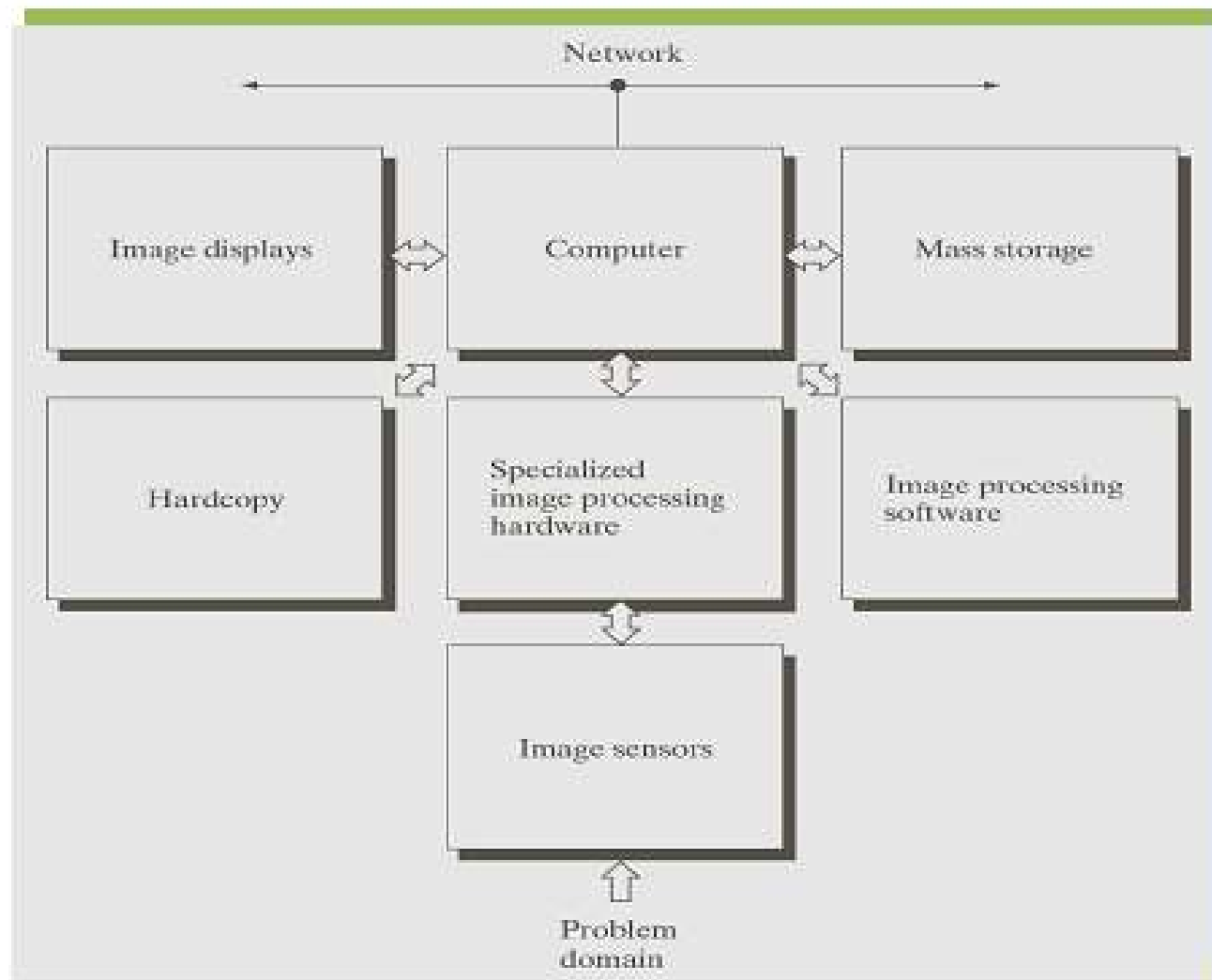
### Step 10. Recognition and Interpretation:

- Recognition is the process that assigns **label** to an object based on the information provided by its **descriptors**.

### Step 11. Knowledge base:

- The Knowledge base also controls the **interaction between modules**. The knowledge about a problem is coded into an image processing system in the form of a Knowledge base.

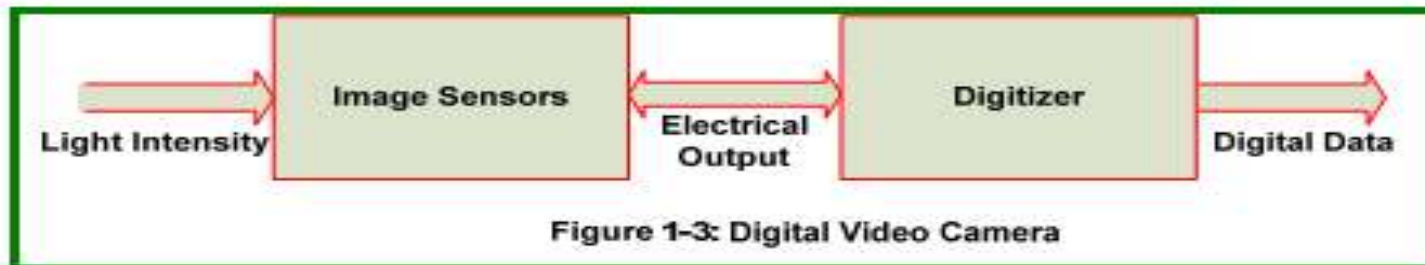
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## Components of a Digital Image Processing System

### 1. Image Sensors

- Physical device that is sensitive to the **energy** radiated by the object.
- Digitizer that converts the output of the physical sensing device into digital form.



### 2. Specialized Image Processing Hardware

- Specialized Image Processing Hardware usually consists of the digitizers and hardware that performs other primitive operations, such as arithmetic logic unit (ALU). Speed is the most important parameter (30 frames /sec).

### 3. Specialized Image Processing Software

- Specialized Image Processing Software is specialized modules that perform specific tasks.

### 4. Computer: Image processing system: from computer to a supercomputer.

### 5. Mass Storage Capability:

- It is a must in image processing applications (image size of 1024x1024 pixels, with intensity level for each pixel : 8 bits, requires one Megabyte for saving)
- **Mass Storage categories:**
  - **Short-term storage** for use during processing.
  - **On-line storage** for relatively fast operations.
  - **Archival storage** for infrequent access.

### 6. Image Displays: Flat screen TV monitors.

### 7. Hardcopy: devices for recording images: laser printers, film cameras, CD-ROM disk, others.

### 8. Networking: the key parameter is the bandwidth.

# Human Visual Perception

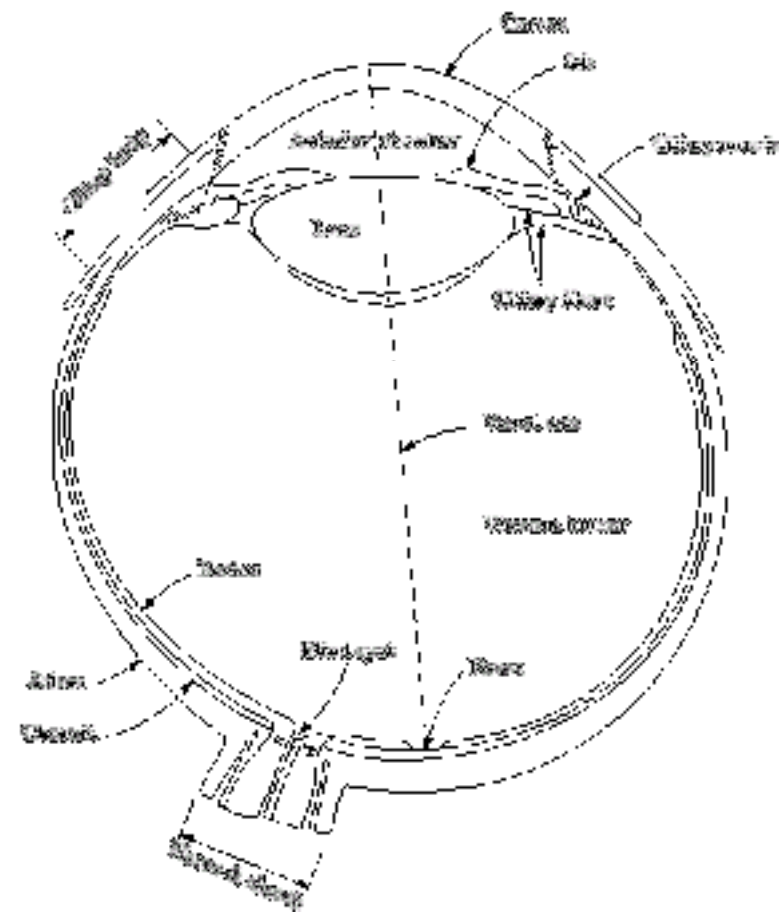


FIGURE 32.1  
Simplified  
Diagram of the  
Structure of the  
Human Eye

- Elements of visual perception
- Mechanics of the human visual system
- Structure of the eye
- Image formation in the eye
- Brightness adaptation and discrimination

# The Human Eye

- Diameter: 20 mm
- 3 membranes enclose the eye
  - Cornea & sclera
  - Choroid
  - Retina

# The Choroid

- The choroid contains blood vessels for eye nutrition and is heavily pigmented to reduce extraneous light entrance and backscatter.
- It is divided into the ciliary body and the iris diaphragm, which controls the amount of light that enters the pupil (2 mm ~ 8 mm).

# The Lens

- The lens is made up of fibrous cells and is suspended by fibers that attach it to the ciliary body.
- It is slightly yellow and absorbs approx. 8% of the visible light spectrum.
- It Contains 60-70% water, 6% fat, more protein



# The Retina

- The retina lines the inner most membrane (portion.)
- Photo Receptors: Specialized type of neuron in retina, it convert light in to signals.
- Discrete light receptors are distributed over the surface of the retina:
  - cones (6-7 million per eye) and
  - rods (75-150 million per eye)

# Cones

- Cones are located in the fovea (centre portion of the retina) and are sensitive to color.
- Each one is connected to its own nerve end.
- 6 and 7 Millions cones in the eye.
- Cone vision is called *photopic* (or bright-light vision).

# Rods

- Rods are giving a general, overall picture of the field of view and are not involved in color vision.
- Several rods are connected to a single nerve and are sensitive to low levels of illumination (*scotopic* or dim-light vision).
- Ie not sensitive to colour.
- No of rods 75 to 150 millions
- Object appear bright in day light and appear dim during moon light is called dim light vision.

# Receptor Distribution

- The distribution of receptors is radially symmetric about the fovea.
- Cones are most dense in the center of the fovea while rods increase in density from the center out to approximately 20% off axis and then decrease.

# Cones & Rods

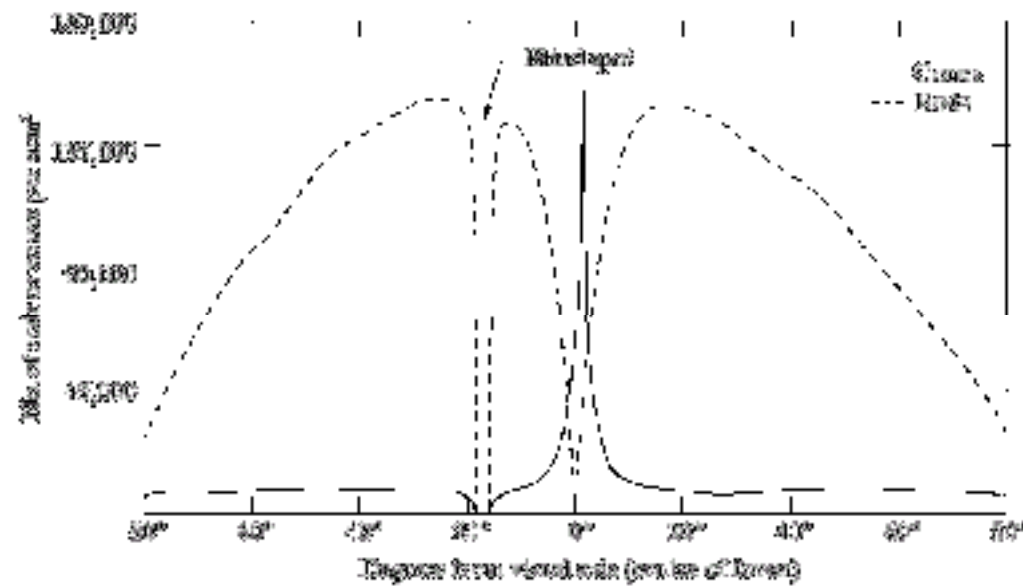


FIGURE 2.2  
Distribution of  
rod and cone  
photoreceptors.

# The Fovea

- The fovea is circular (1.5 mm in diameter) but can be assumed to be a square sensor array (1.5 mm x 1.5 mm).
- The density of cones: 150,000 elements/mm<sup>2</sup>  
~ 337,000 for the fovea.
- A CCD imaging chip of medium resolution needs 5 mm x 5 mm for this number of elements

# Image Formation in the Eye

- The eye lens (if compared to an optical lens) is flexible.
- It gets controlled by the fibers of the ciliary body and to focus on distant objects it gets flatter (and vice versa).
- To focus on distance objects-Muscles flatten lens
- To focus on close objects lens get thicken.

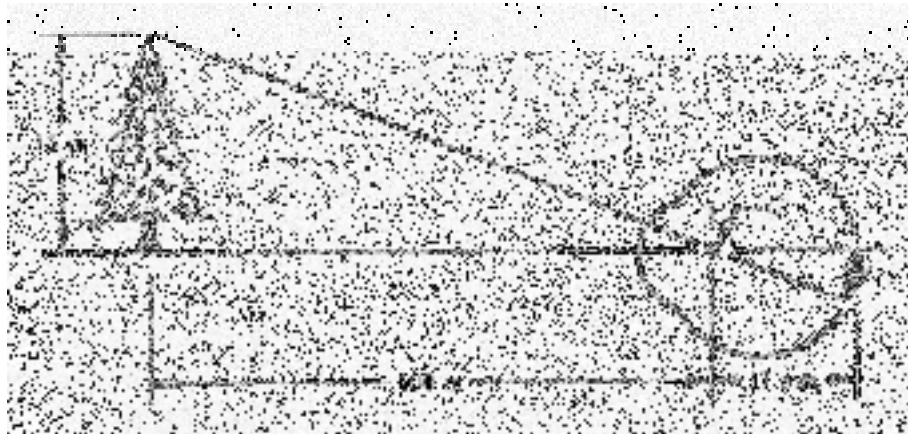
# Image Formation in the Eye

- Distance between the center of the lens and the retina (*focal length*):
  - varies from 17 mm to 14 mm (refractive power of lens goes from minimum to maximum).
- Objects farther than 3 m use minimum refractive lens powers (and vice versa).



# Image Formation in the Eye

- Example:
  - Calculation of retinal image of an object



$$\frac{15}{100} = \frac{x}{17}$$

$$x = 2.55mm$$

# Image Formation in the Eye

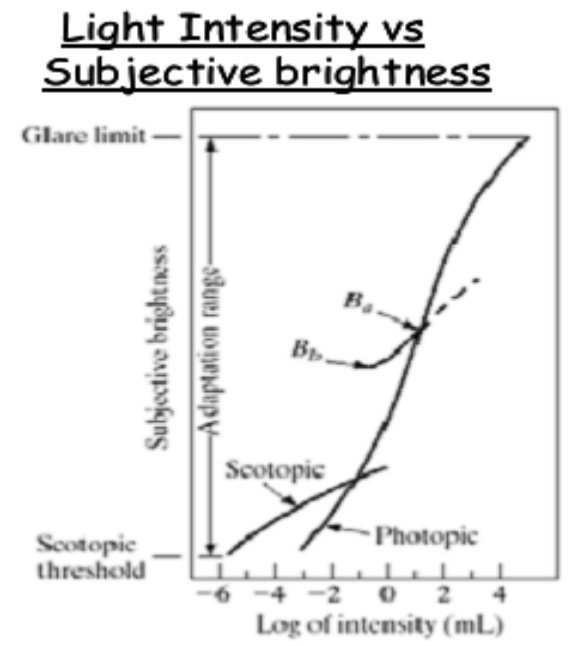
- **Perception** (ability to see or become aware of something through the sense) takes place by the relative excitation of light receptors.
- These receptors transform radiant energy into electrical impulses that are ultimately decoded by the brain.

# Brightness Adaptation & Discrimination

- Range of light intensity levels to which HVS (human visual system) can adapt: on the order of  $10^{10}$ .
- Subjective brightness (i.e. intensity as perceived by the HVS) is a logarithmic function of the light intensity incident on the eye.

# Brightness Adaptation & Discrimination

- The HVS cannot operate over such a range simultaneously.
- For any given set of conditions, the current sensitivity level of HVS is called the **brightness adaptation level**

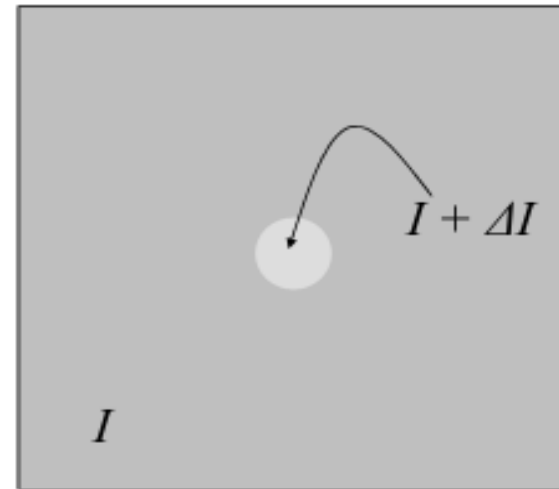


# Brightness Discrimination

- The eye also discriminates between **changes** in brightness at any specific adaptation level.

## Classic Experiment

- Weber Ratio:  $\Delta I_c / I$*
- $\Delta I_c$  is the incremental illumination discriminable 50% of the time
- Small Weber Ratio represents "good" brightness discrimination



I : background illumination

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# Brightness Adaptation & Discrimination

- Small values of Weber ratio mean good brightness discrimination (and vice versa).
- At low levels of illumination brightness discrimination is poor (rods) and it improves significantly as background illumination increases (cones).

# Brightness Adaptation & Discrimination

- The typical observer can distinguish(1-2) one to two dozen different intensity changes
  - i.e. the number of different intensities a person can see at any one point in a monochrome image



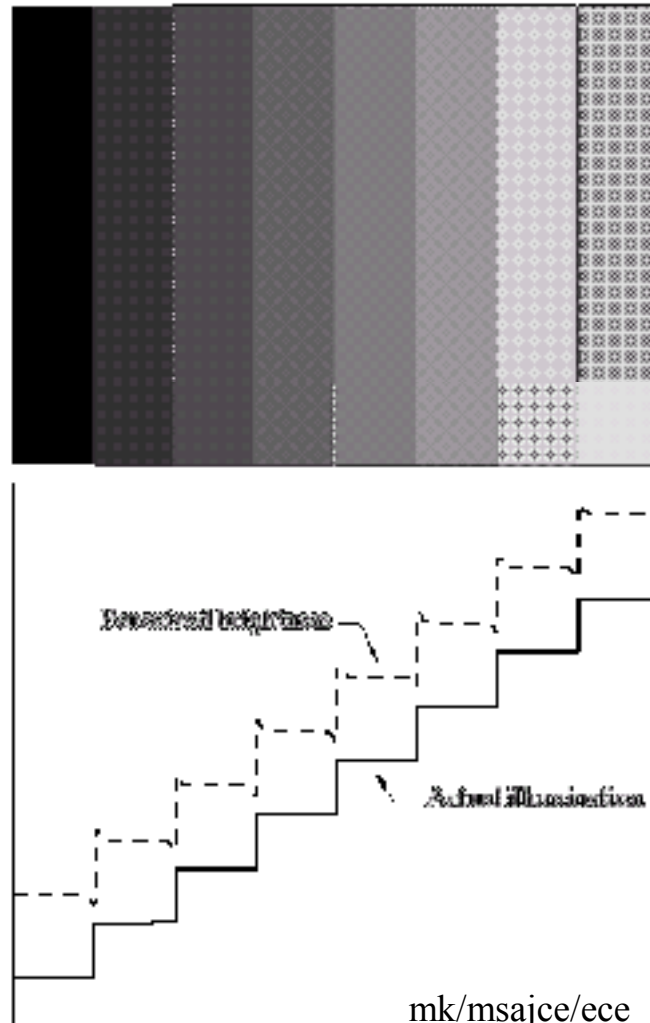
# Brightness Adaptation & Discrimination

- Overall intensity discrimination is broad due to different set of incremental changes to be detected at each new adaptation level.
- Perceived brightness is not a simple function of intensity
  - Scalloped effect, Mach band pattern
  - Simultaneous contrast

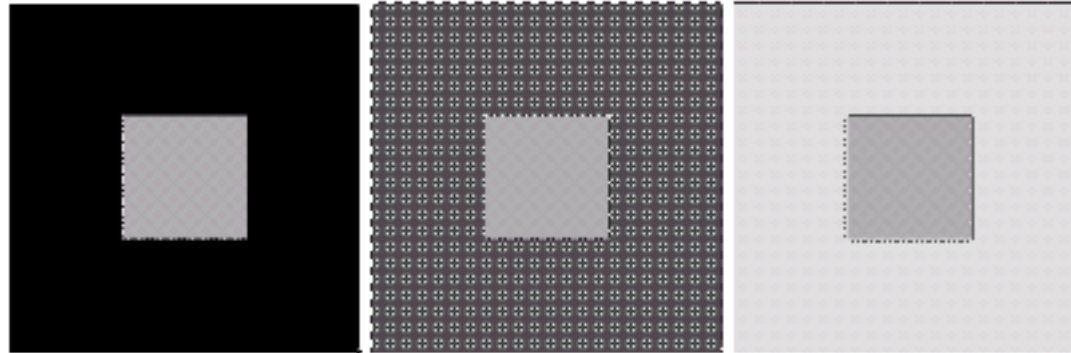
# Mach Band Effect

## First Phenomenon

- Visual system tends to under or overshoot around the boundary of two regions of different intensity
- *Mach Bands*



**FIGURE 2.7**  
 (a) An example showing that perceived brightness is not a simple function of intensity. The relative vertical position between the two profiles in (b) have no special significance; they were chosen for clarity.



**FIGURE 3.1** Examples of simultaneous contrast. All the inner squares have the same luminance, but they appear progressively darker as the background becomes lighter.

## Second Phenomenon

- *Simultaneous Contrast*



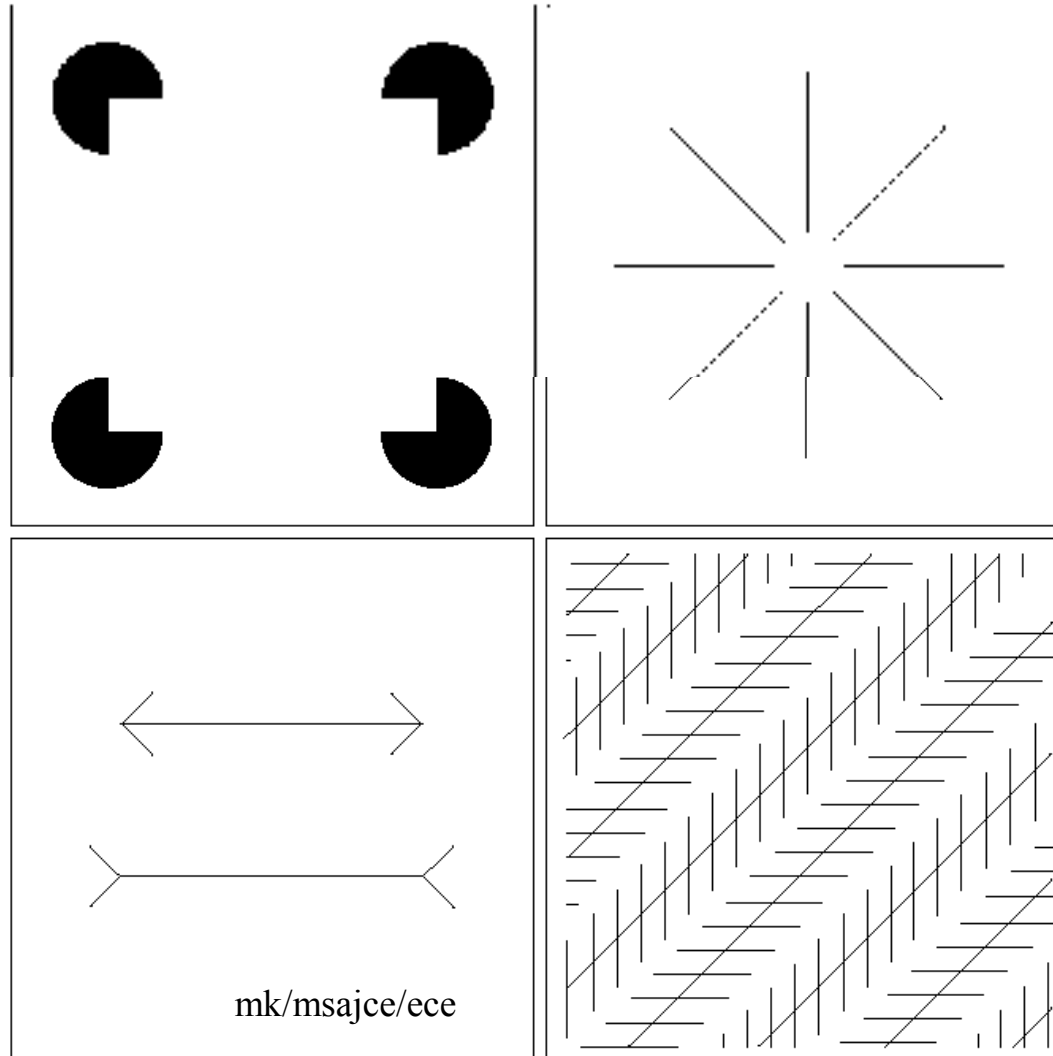
All the inner squares have intensity

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# Illusions



**FIGURE 3.10** Some well-known optical illusions.

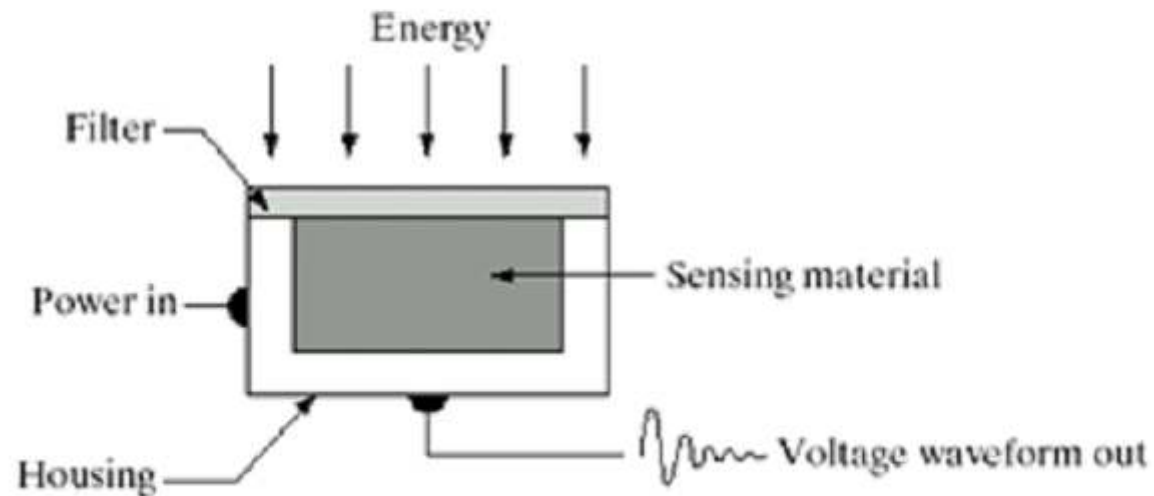


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## Image Sensing and Acquisition

There are 3 principal sensor arrangements (produce an electrical output proportional to light intensity).

(i) Single imaging Sensor (ii) Line sensor (iii) Array sensor

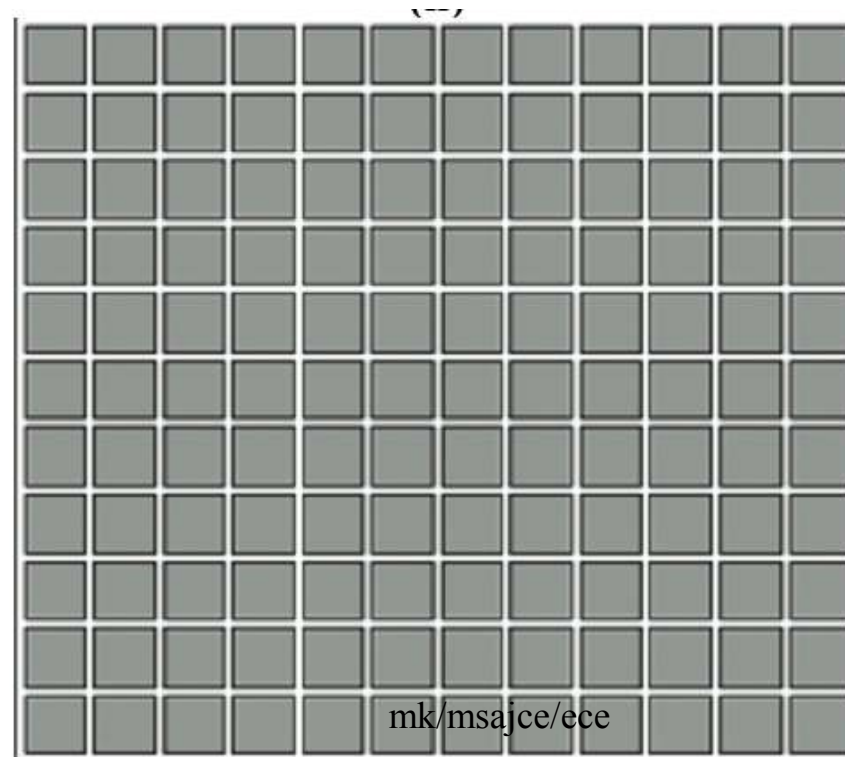


(i)

## Line Sensor



## Array Sensor

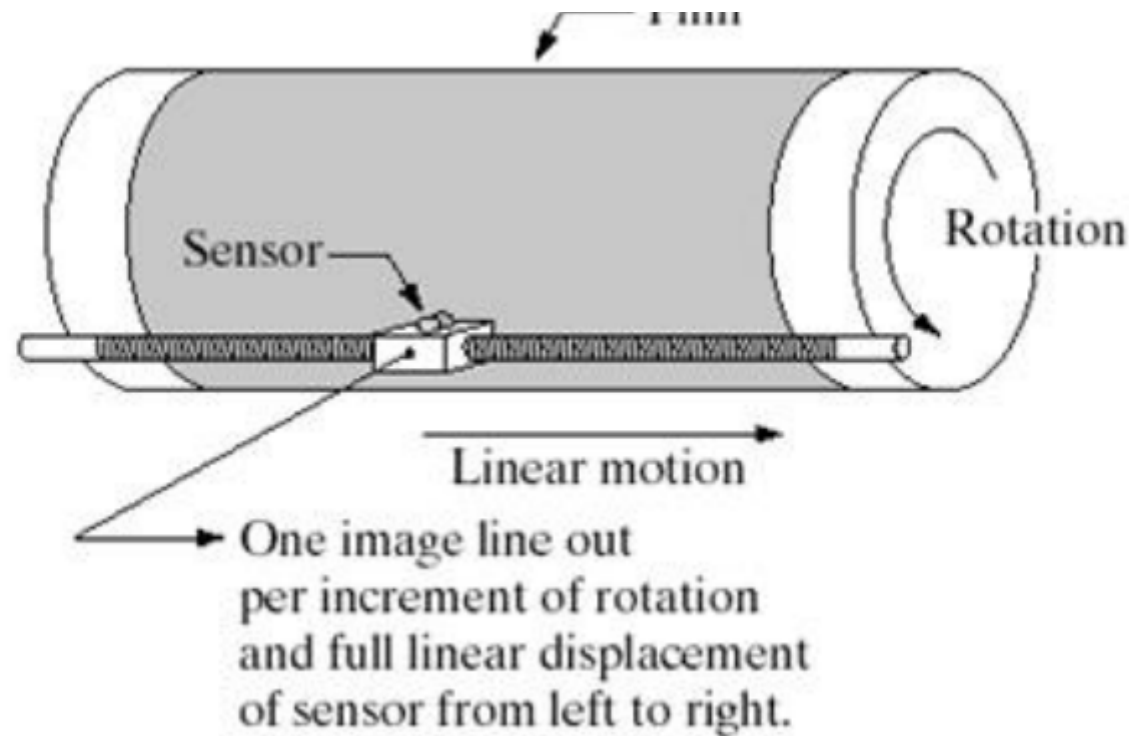


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(iii)

## **Image Acquisition using a single sensor**

The most common sensor of this type is the photodiode, which is constructed of silicon materials and whose output voltage waveform is proportional to light. The use of a filter in front of a sensor improves selectivity. For example, a green (pass) filter in front of a light sensor favours light in the green band of the color spectrum. As a consequence, the sensor output will be stronger for green light than for other components in the visible spectrum.

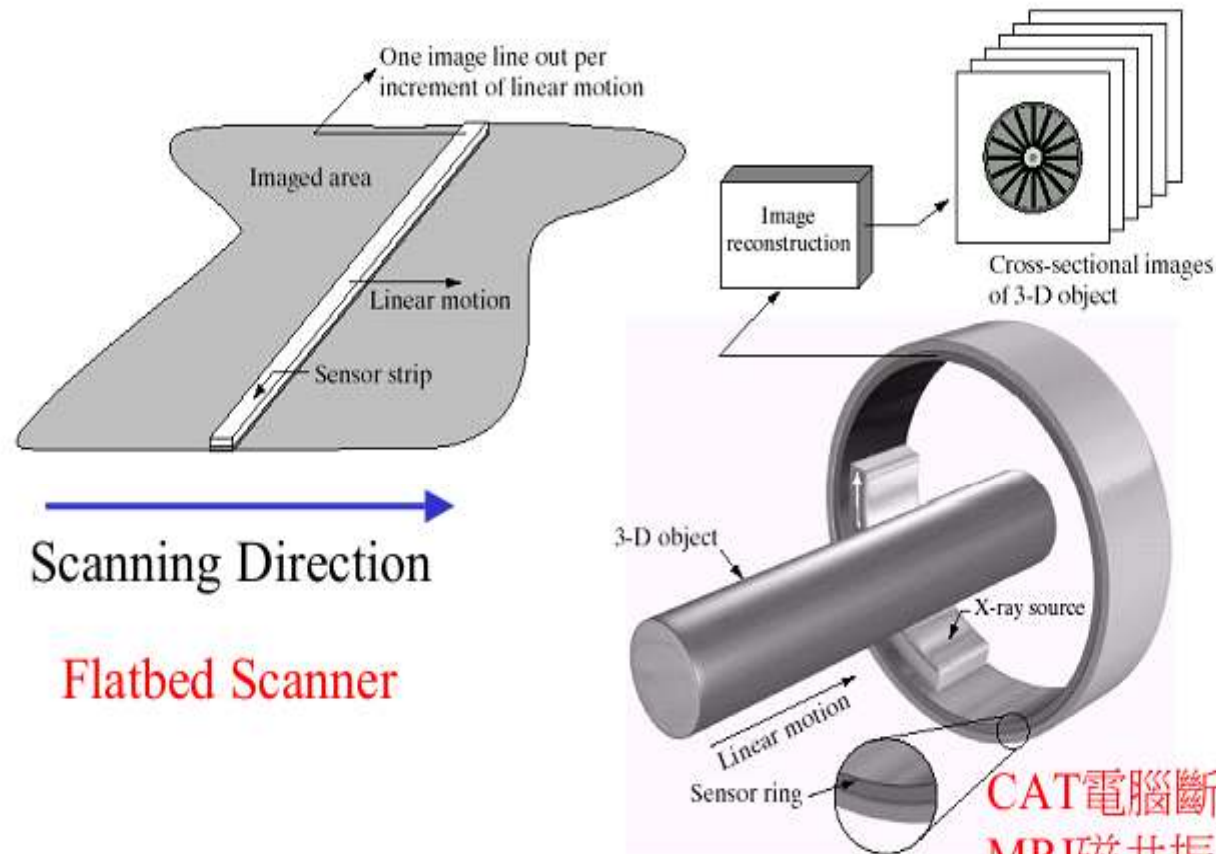


**Fig: Combining a single sensor with motion to generate a 2-D image**

In order to generate a 2-D image using a single sensor, there have to be relative displacements in both the x- and y-directions between the sensor and the area to be imaged. An arrangement used in high precision scanning, where a film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension. The single sensor is mounted on a lead screw that provides motion in the perpendicular direction. Since mechanical motion can be controlled with high precision, this method is an inexpensive (but slow) way to obtain high-resolution images.



## 2.3.2 Image acquisition using sensor strips



CAT電腦斷層掃描  
MRI磁共振造影  
PET正子放射斷層掃描

a b

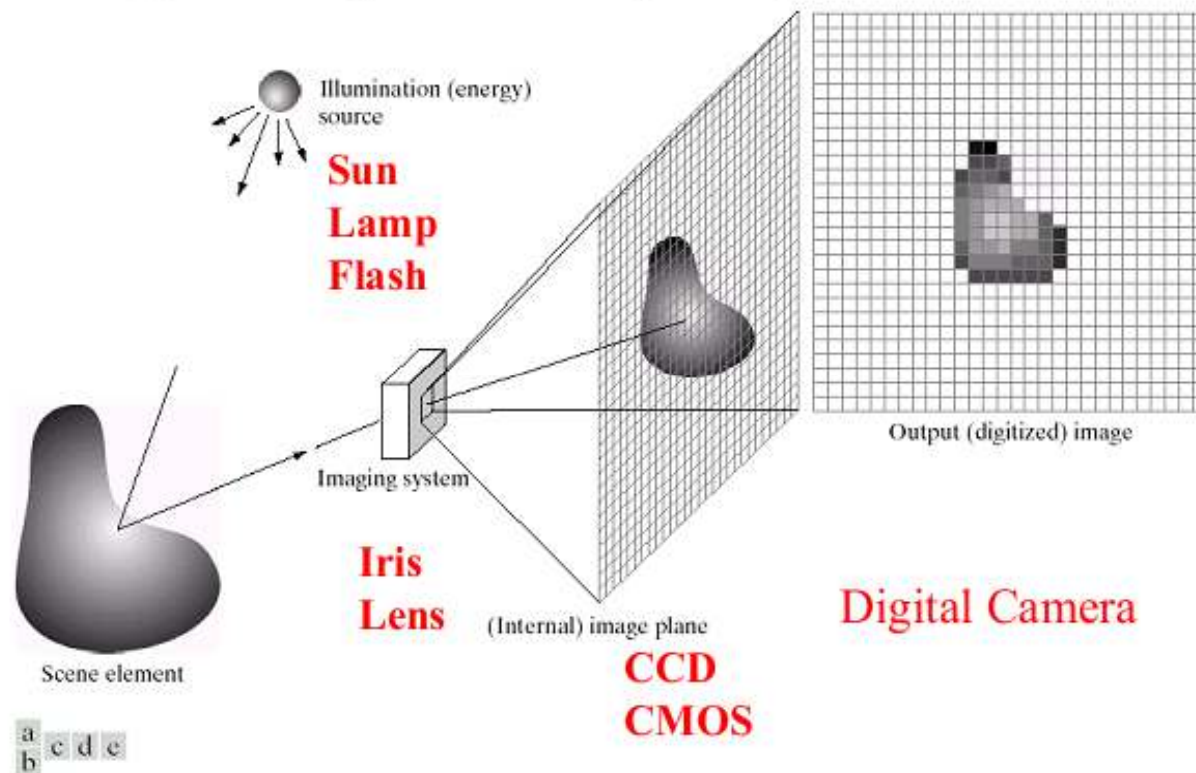
**FIGURE 2.14** (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

The strip provides imaging elements in one direction. Motion perpendicular to the strip provides imaging in the other direction. This is the type of arrangement used in most flatbed scanners. Sensing devices with 4000 or more in-line sensors are possible. In-line sensors are used routinely in airborne imaging applications, in which the imaging system is mounted on an aircraft that flies at a constant altitude and speed over the geographical area to be imaged. One-dimensional

strips mounted in a ring configuration are used in medical and industrial imaging to obtain cross-sectional (“slice”) images of 3-D objects. A rotating X-ray source provides illumination and the portion of the sensors opposite the source collect the X-ray energy that pass through the object (the sensors obviously have to be sensitive to X-ray energy). This is the basis for medical and industrial computerized axial tomography (CAT) imaging.

### 2.3.3 Image acquisition using sensor arrays

- Charged Couple Diode (CCD)



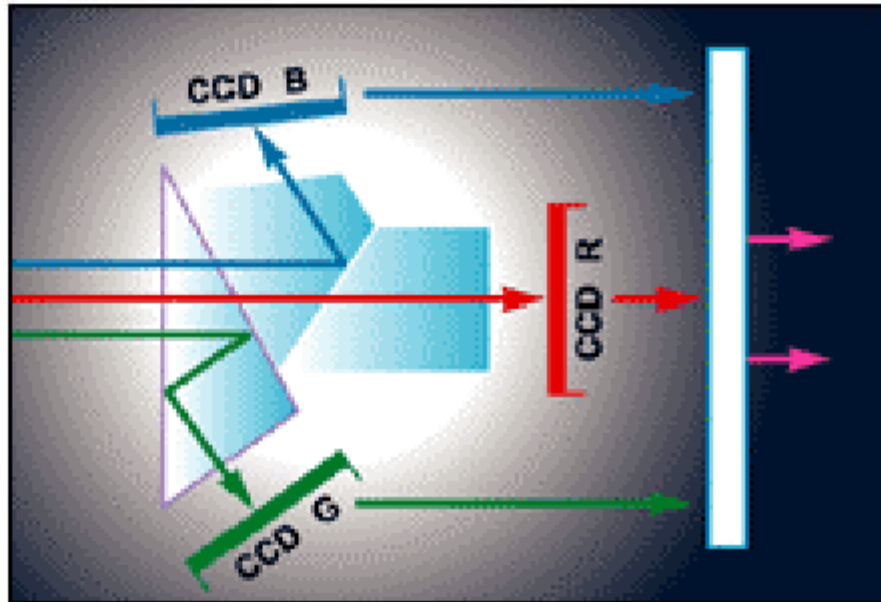
**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

This type of arrangement is found in digital cameras. A typical sensor for these cameras is a CCD array, which can be manufactured with a broad range of sensing properties and can be packaged in rugged arrays of  $4000 * 4000$  elements or more. CCD sensors are used widely in digital cameras and other light sensing instruments. The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor, a property that is used in astronomical and other applications requiring low noise images.

The first function performed by the imaging system is to collect the incoming energy and focus it onto an image plane. If the illumination is light, the front end of the imaging system is a lens, which projects the viewed scene onto the lens focal plane. The sensor array, which is coincident with the focal plane, produces outputs proportional to the integral of the light received at each sensor.

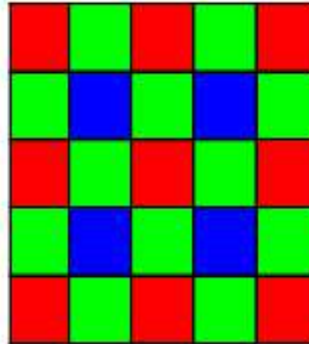
## 2.3 Image Sensing and Acquisition

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## *Color Filter Array (CFA)*





## A Simple Image Formation Model

An image is defined by two dimensional function  $f(x,y)$ . The value or amplitude of  $f$  at spatial coordinates  $(x,y)$  is a positive scalar quantity. When an image is generated from a physical process, its value are proportional to energy radiated by physical source. As a consequence,  $f(x,y)$  must be nonzero and finite; that is,

$$0 < f(x,y) < \infty$$

The function  $f(x,y)$  may be characterized by two components: (1) the amount of source illumination incident on the scene being viewed and (2) the amount of illumination reflected by the objects in the scene. These are called illumination and reflectance components denoted by  $i(x,y)$  and  $r(x,y)$  respectively. The two function combine as product to form  $f(x,y)$ :

$$f(x,y) = i(x,y) r(x,y)$$

Where  $0 < i(x,y) < \infty$  and  $0 < r(x,y) < 1$   $r(x,y)=0$  means total absorption  $r(x,y)=1$  means total reflectance

We call the intensity of a monochrome image at any coordinates  $(x,y)$  the gray level ( $l$ ) of the image at that point. That is  $l=f(x,y)$ .

The interval of  $l$  ranges from  $[0,L-1]$ . Where  $l=0$  indicates black and  $l=L-1$  indicates white. All the intermediate values are shades of gray varying from black to white.



# A Simple Image Formation Model

$$f(x, y) = i(x, y) \cdot r(x, y)$$

$$0 < f(x, y) < \infty$$

$f(x, y)$ : intensity at the point  $(x, y)$

$i(x, y)$ : illumination at the point  $(x, y)$

(the amount of source illumination incident on the scene)

$r(x, y)$ : reflectance/transmissivity at the point  $(x, y)$

(the amount of illumination reflected/transmitted by the object)

where  $0 < i(x, y) < \infty$  and  $0 < r(x, y) < 1$

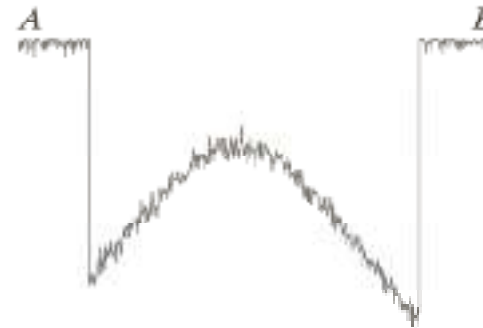
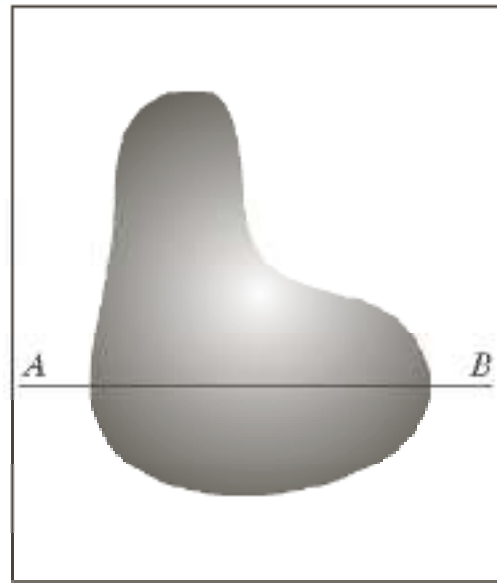
Where  $0 < i(x, y) < \infty$  and  $0 < r(x, y) < 1$   $r(x, y) = 0$  means total absorption  $r(x, y) = 1$  means total reflectance

## Image Sampling and Quantization

Sampling and quantization are the two important processes used to convert continuous analog image into digital image. Image sampling refers to discretization of spatial coordinates (along x axis) whereas quantization refers to discretization of gray level values (amplitude (along y axis)).

(Given a continuous image,  $f(x,y)$ , digitizing the coordinate values is called sampling and digitizing the amplitude (intensity) values is called quantization.)

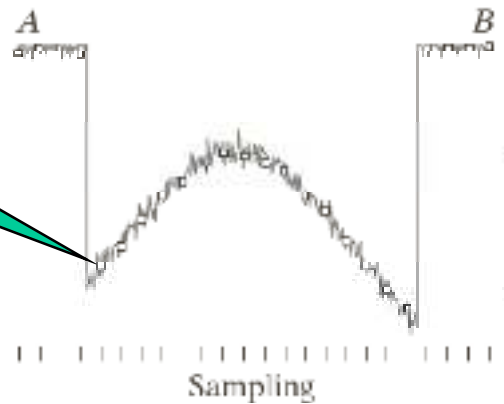
# Image Sampling and Quantization



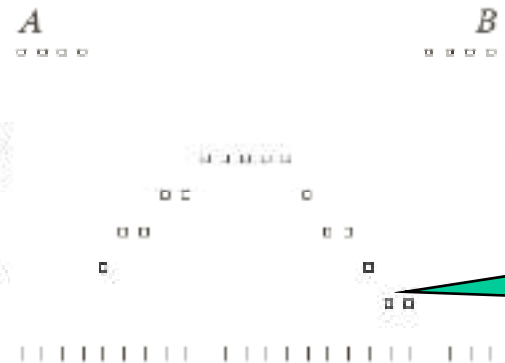
a	b
c	d

**FIGURE 2.16**  
Generating a digital image.  
(a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Digitizing the  
coordinate  
values

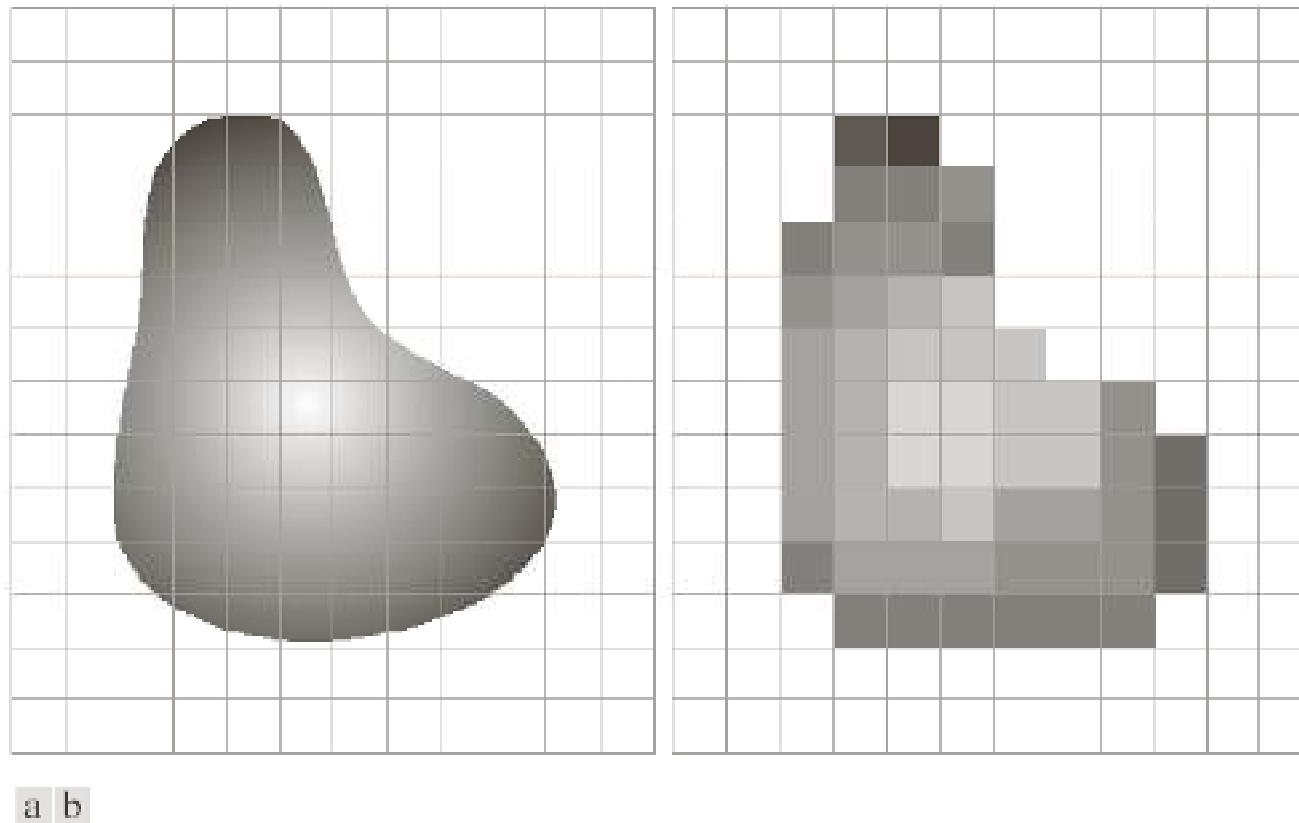


Quantization



Digitizing the  
amplitude  
values

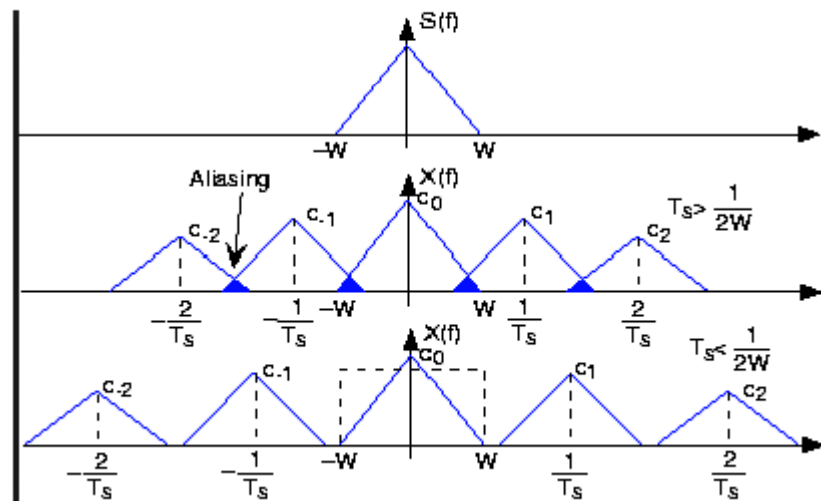
# Image Sampling and Quantization



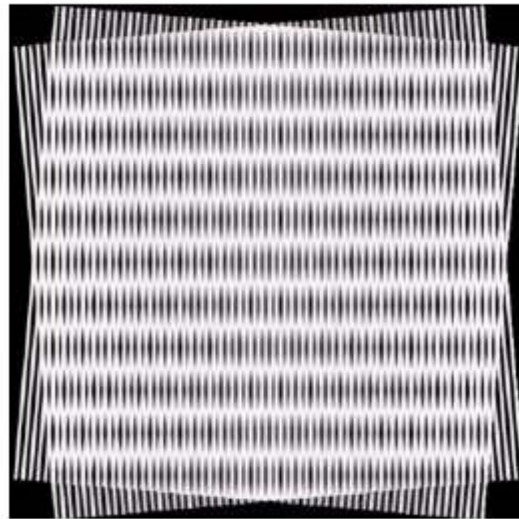
**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

## Aliasing and Moire Patterns

Shannon sampling theorem tells us that, if the function is sampled at a rate equal to or greater than twice its highest frequency ( $f_s \geq f_m$ ) it is possible to recover completely the original function from the samples. If the function is undersampled, then a phenomenon called **aliasing (distortion)** (If two pattern or spectrum overlap, the overlapped portion is called



A **moiré pattern** is a secondary and visually evident superimposed pattern created, for example, when two identical (usually transparent) patterns on a flat or curved surface (such as closely spaced straight lines drawn radiating from a point or taking the form of a grid) are overlaid while displaced or rotated a small amount from one another.



**Fig: Illustration of the moire effect**

## **Zooming and Shrinking Digital Image**

Zooming may be viewed as oversampling and shrinking may be viewed as undersampling.

Zooming is a method of increasing the size of a given image. Zooming requires two steps: creation of new pixel locations, and the assigning new grey level values to those new locations

# Representing Digital Images

- The representation of an  $M \times N$  numerical array as

$$f(x, y) = \begin{bmatrix} f(0, 0) & f(0, 1) & \dots & f(0, N-1) \\ f(1, 0) & f(1, 1) & \dots & f(1, N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1, 0) & f(M-1, 1) & \dots & f(M-1, N-1) \end{bmatrix}$$



# Representing Digital Images

- The representation of an  $M \times N$  numerical array as

$$f(x, y) = \begin{bmatrix} f(0, 0) & f(0, 1) & \dots & f(0, N-1) \\ f(1, 0) & f(1, 1) & \dots & f(1, N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1, 0) & f(M-1, 1) & \dots & f(M-1, N-1) \end{bmatrix}$$

# Representing Digital Images

- The representation of an  $M \times N$  numerical array in MATLAB

$$f(x, y) = \begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

## Outline of the Lecture

- Neighbourhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries
- Distance Measures

# Image coordinate system

However, in digital image processing, the discrete form of the image is often used. Discrete images are usually represented in the fourth quadrant of the Cartesian coordinate system. A discrete image  $f(x, y)$ , of dimension  $3 \times 3$ , is shown in Fig. 3.2(a).

Many programming environments including MATLAB start with an index of (1, 1). The equivalent representation of the given matrix is shown in Fig. 3.2(b).

The coordinates used for discrete image is, by default, the fourth quadrant of the Cartesian system.

	$y = 0$	$y = 1$	$y = 2$		$y = 1$	$y = 2$	$y = 3$
$x = 0$	$f(0, 0)$	$f(0, 1)$	$f(0, 2)$	$x = 1$	$f(1, 1)$	$f(1, 2)$	$f(1, 3)$
$x = 1$	$f(1, 0)$	$f(1, 1)$	$f(1, 2)$	$x = 2$	$f(2, 1)$	$f(2, 2)$	$f(2, 3)$
$x = 2$	$f(2, 0)$	$f(2, 1)$	$f(2, 2)$	$x = 3$	$f(3, 1)$	$f(3, 2)$	$f(3, 3)$
	(a)				(b)		

**Fig. 3.2** Discrete image (a) Image in the fourth quadrant of Cartesian coordinate system  
(b) Image coordinates as handled by software environments such as MATLAB

# Image Topology

Image topology is a branch of image processing that deals with the fundamental properties of the image such as image neighbourhood, paths among pixels, boundary, and connected components. It characterizes the image with topological properties such as neighbourhood, adjacency, and connectivity. *Neighbourhood* is fundamental to understanding image topology. In the simplest case, the neighbours of a given reference pixel are those pixels with which the given reference pixel shares its edges and corners.



In  $N_4(p)$ , the reference pixel  $p(x, y)$  at the coordinate position  $(x, y)$  has two horizontal and two vertical pixels as neighbours. This is shown graphically in Fig. 3.3.

$$\begin{pmatrix} 0 & X & 0 \\ X & p(x, y) & X \\ 0 & X & 0 \end{pmatrix}$$

Fig. 3.3 4-Neighbourhood  $N_4(p)$

In  $N_4(p)$ , the reference pixel  $p(x, y)$  at the coordinate position  $(x, y)$  has two horizontal and two vertical pixels as neighbours. This is shown graphically in Fig. 3.3.

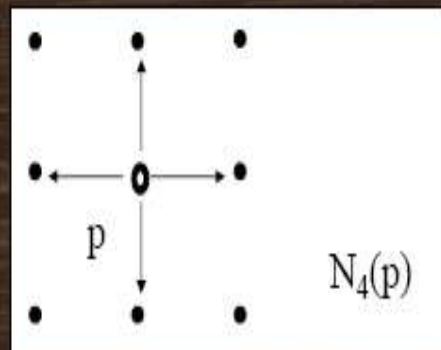
### Neighbors of a Pixel

#### 1. $N_4(p)$ : 4-neighbors of $p$ .

- Any pixel  $p(x, y)$  has two vertical and two horizontal neighbors, given by  $(x+1, y)$ ,  $(x-1, y)$ ,  $(x, y+1)$ ,  $(x, y-1)$
- This set of pixels are called the 4-neighbors of  $P$ , and is denoted by  $N_4(P)$
- Each of them is at a unit distance from  $P$ .

#### Neighbors of a pixel

- a. 4-neighbors of a pixel  $p$  are its vertical and horizontal neighbors denoted by  $N_4(p)$



$$\begin{pmatrix} 0 & X & 0 \\ X & p(x, y) & X \\ 0 & X & 0 \end{pmatrix}$$

Fig. 3.3 4-Neighbourhood  $N_4(p)$

# Diagonal Neighbours

## 2. $N_D(p)$

- This set of pixels, called 4-neighbors and denoted by  $N_D(p)$ .
- $N_D(p)$ : four diagonal neighbors of  $p$  have coordinates:  
 $(x+1, y+1)$ ,  $(x+1, y-1)$ ,  $(x-1, y+1)$ ,  $(x-1, y-1)$
- Each of them are at Euclidean distance of 1.414 from  $P$ .

## 3. $N_8(p)$ : 8-neighbors of $p$ .

- $N_4(P)$  and  $N_D(p)$  together are called 8-neighbors of  $p$ , denoted by  $N_8(p)$ .
- $N_8 = N_4 \cup N_D$
- Some of the points in the  $N_4$ ,  $N_D$  and  $N_8$  may fall outside image when  $P$  lies on the border of image.

$F(x-1, y-1)$	$F(x-1, y)$	$F(x-1, y+1)$
$F(x, y-1)$	$F(x, y)$	$F(x, y+1)$
$F(x+1, y-1)$	$F(x+1, y)$	$F(x+1, y+1)$

$N_8(p)$



$N_D$	$N_4$	$N_D$
$N_4$	P	$N_4$
$N_D$	$N_4$	$N_D$

- $N_4$  - 4-neighbors
- $N_D$  - diagonal neighbors
- $N_8$  - 8-neighbors ( $N_4 \cup N_D$ )

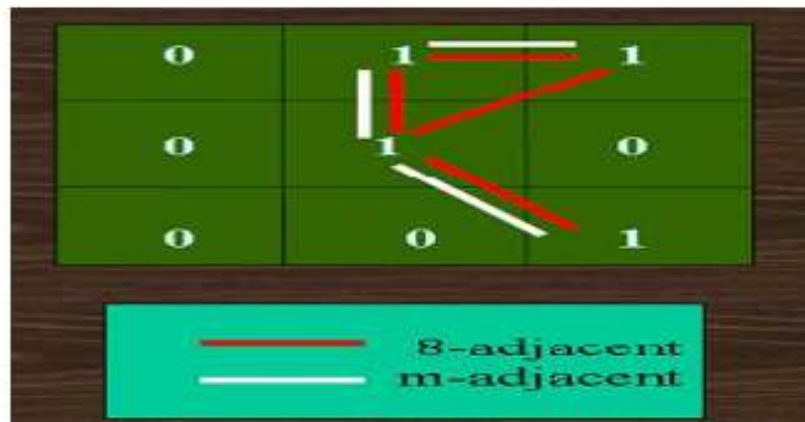
## Adjacency

- Two pixels are **connected** if they are neighbors and their gray levels satisfy some specified criterion of similarity.
- For example, in a binary image two pixels are connected if they are 4-neighbors and have same value (0/1)
- Let **v**: a set of intensity values used to *define adjacency* and *connectivity*.
- In a **binary Image** **v={1}**, if we are referring to adjacency of pixels with value 1.
- In a **Gray scale image**, the idea is the same, but **v** typically contains more elements, for example **v= {180, 181, 182,.....,200}**.
- If the possible intensity values 0 to 255, **v** set could be any subset of these 256 values.

## Types of adjacency

- 1. 4-adjacency:** Two pixels **p** and **q** with values from **v** are **4-adjacent** if **q** is in the set  **$N_4(p)$** .
- 2. 8-adjacency:** Two pixels **p** and **q** with values from **v** are **8-adjacent** if **q** is in the set  **$N_8(p)$** .
- 3. m-adjacency (mixed):** two pixels **p** and **q** with values from **v** are **m-adjacent** if:
  - ▶ **q** is in  **$N_4(p)$**  **or**
  - ▶ **q** is in  **$N_D(p)$**  **and**
  - ▶ The set  **$N_4(p) \cap N_4(q)$**  has no pixel whose values are from **v** (**No intersection**).
  - **Mixed adjacency** is a modification of 8-adjacency "introduced to eliminate the ambiguities that often arise when 8-adjacency is used. (eliminate multiple path connection)
  - Pixel arrangement as shown in figure for  **$v = \{1\}$**

*Example:*





# Connectivity

The relationship between two or more pixels is defined by pixel connectivity. Connectivity information is used to establish the boundaries of the objects. The pixels  $p$  and  $q$  are said to be connected if certain conditions on pixel brightness specified by the set  $V$  and spatial adjacency are satisfied. For a binary image, this set  $V$  will be  $\{0, 1\}$  and for grey scale images,  $V$  might be any range of grey levels.

**4-Connectivity** The pixels  $p$  and  $q$  are said to be in 4-connectivity when both have the same values as specified by the set  $V$  and if  $q$  is said to be in the set  $N_4(p)$ . This implies any path from  $p$  to  $q$  on which every other pixel is 4-connected to the next pixel.

**8-Connectivity** It is assumed that the pixels  $p$  and  $q$  share a common grey scale value. The pixels  $p$  and  $q$  are said to be in 8-connectivity if  $q$  is in the set  $N_8(p)$ .

**Mixed connectivity** Mixed connectivity is also known as  $m$ -connectivity. Two pixels  $p$  and  $q$  are said to be in  $m$ -connectivity when

1.  $q$  is in  $N_4(p)$  or
2.  $q$  is in  $N_D(p)$  and the intersection of  $N_4(p)$  and  $N_4(q)$  is empty.

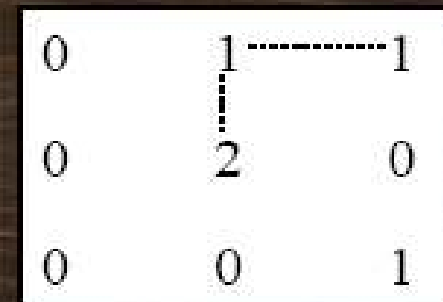
## Connectivity :

To determine whether the pixels are adjacent in some sense.

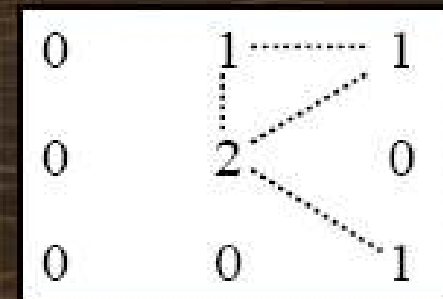
Let  $V$  be the set of gray-level values used to define connectivity; then Two pixels  $p, q$  that have values from the set  $V$  are:

- a. 4-connected, if  $q$  is in the set  $N_4(p)$
- b. 8-connected, if  $q$  is in the set  $N_8(p)$
- c.  $m$ -connected, iff
  - i.  $q$  is in  $N_4(p)$  or
  - ii.  $q$  is in  $N_D(p)$  and the set  $N_4(p) \cap N_4(q)$  is empty

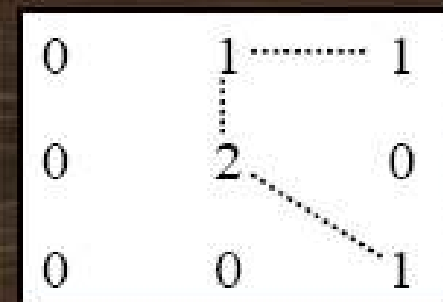
$$V = \{1, 2\}$$



a.



b.



c.

8-Connectivity is shown as lines. Here, a multiple path or loop is present. In  $m$ -connectivity, there are no such multiple paths. The  $m$ -connectivity for the image in Fig.

## Path

- A *digital path* (or curve) from pixel **p** with coordinate **(x,y)** to pixel **q** with coordinate **(s,t)** is a sequence of *distinct* pixels with coordinates **(x<sub>0</sub>, y<sub>0</sub>)**, **(x<sub>1</sub>, y<sub>1</sub>)**, ..., **(x<sub>n</sub>, y<sub>n</sub>)**, where **(x<sub>0</sub>, y<sub>0</sub>) = (x,y)**, **(x<sub>n</sub>, y<sub>n</sub>) = (s,t)**
- **(x<sub>i</sub>, y<sub>i</sub>)** is adjacent pixel **(x<sub>i-1</sub>, y<sub>i-1</sub>)** for **1 ≤ i ≤ n**,
- **n**- The *length* of the path.
- If **(x<sub>0</sub>, y<sub>0</sub>) = (x<sub>n</sub>, y<sub>n</sub>)**:the path is *closed path*.
- We can define 4-,8-, or *m-paths* depending on the type of adjacency specified.



**Reflexive** For any element  $a$  in the set  $A$ , if the relation  $aRa$  holds, this is known as a reflexive relation.

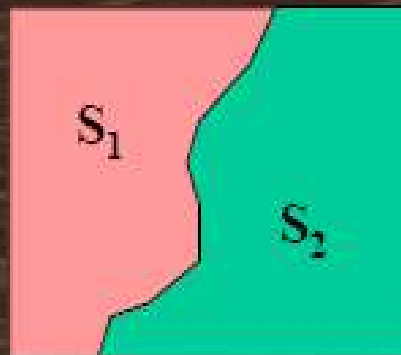
**Symmetric** If  $aRb$  implies that  $bRa$  also exists, this is known as a symmetric relation.

**Transitive** If the relations  $aRb$  and  $bRc$  exist, it implies that the relationship  $aRc$  also exists. This is called the transitivity property.

If all these three properties hold, the relationship is called an *equivalence relation*. The relationships can be expressed as a matrix. Assume that the set  $A$  is divided into  $K$  disjoint sets,

# Adjacency/Connectivity

- Pixel  $p$  is *adjacent* to pixel  $q$  if they are connected.
- Two *image subsets*  $S_1$  and  $S_2$  are adjacent if some pixel in  $S_1$  is adjacent to some pixel in  $S_2$





# Connected Components

- If  $p$  and  $q$  are pixels of an image subset  $S$  then  $p$  is *connected* to  $q$  in  $S$  if there is a path from  $p$  to  $q$  consisting entirely of pixels in  $S$ .
- For every pixel  $p$  in  $S$ , the set of pixels in  $S$  that are connected to  $p$  is called a *connected component* of  $S$ .
- If  $S$  has only one connected component then  $S$  is called *Connected Set*.

## Region

- Let **R** to be a subset of pixels in an image, we call a **R** a region of the image. If **R** is a *connected* set.
- Region that are not adjacent are said to be disjoint.
- Example*: the two regions (of 1s) in figure, are adjacent only if 8-adjacency is used.

1	1	1	} <b>R<sub>i</sub></b>
1	0	1	
0	1	0	
0	0	1	} <b>R<sub>j</sub></b>
1	1	1	
1	1	1	

- 4-path* between the two regions does not exist. (so their union is not a connected set).

## Distance Measures

The distance between the pixels  $p$  and  $q$  in an image can be given by distance measures such as Euclidian distance,  $D_4$  distance, and  $D_8$  distance. Consider three pixels  $p$ ,  $q$ , and  $z$ . If the coordinates of the pixels are  $P(x, y)$ ,  $Q(s, t)$ , and  $Z(u, w)$  as shown in Fig. 3.9, the distances between the pixels can be calculated.

The distance function can be called metric if the following properties are satisfied:

1.  $D(p, q)$  is well-defined and finite for all  $p$  and  $q$ .
2.  $D(p, q) \geq 0$  if  $p = q$ , then  $D(p, q) = 0$ .
3. The distance  $D(p, q) = D(q, p)$ .
4.  $D(p, q) + D(q, z) \geq D(p, z)$ . This is called the property of triangular inequality.

The Euclidean distance between the pixels  $p$  and  $q$ , with coordinates  $(x, y)$  and  $(s, t)$ , respectively, can be defined as

$$D_e(p, q) = \sqrt{(x - s)^2 + (y - t)^2}$$

0	1	1	1(z)
1	0	0	1
1	1	1	1(q)
1	1	1	1
(p)			

Fig. 3.9 Sample image



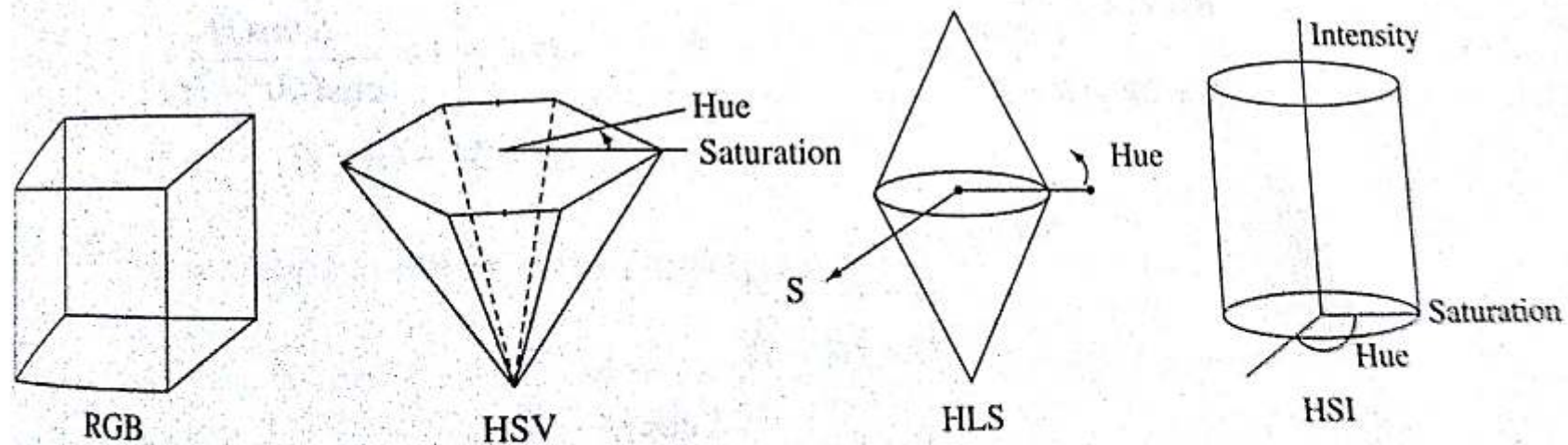
### 3 COLOUR

Colour spaces or colour models are used interchangeably in image processing. However, there is a subtle difference between these terms. Colour space is a mathematical, virtual model and allows representation, creation, visualization, and reproduction of colours. Colours are represented as a tuple of numbers (mostly three numbers and as four numbers in the CMYK model). A set of numbers that define a colour is called a colour space.

### 10.3.1 RGB Colour Model

This is the most common format, where the colours are represented in a cube. Each point in the cube is represented by a colour. The origin is represented by black, whereas the opposite corner is represented by white. This model is used in TV, cameras, scanners, and computer monitors. The lines connecting the primaries represent the various shades of the given colour.

Some of the colour shapes are shown in Fig. 10.4.



**Fig. 10.4** Shapes of colour models

### 10.3.2 HSI Colour Model

The human perception of colour closely resembles the HSI colour model. Here  $H$  represents hue,  $S$  represents saturation, and  $I$  represents intensity. The component  $I$  is the average of the R, G, and B components, and hue is expressed as an angle.

RGB can be converted to HSI using a set of formulae as follows:

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases}$$

with

$$\theta = \cos^{-1} \left\{ \frac{0.5[(R - G) + (R - B)]}{[(R - G)^2 + (R - B)(G - B)]^{\frac{1}{2}}} \right\}$$

$$S = 1 - \frac{3}{(R + G + B)} [\min(R, G, B)]$$

$$I = \frac{1}{3}(R + G + B)$$



Similarly, the HSI coordinates can be converted to RGB as follows:  
Divide  $0^\circ$ – $360^\circ$  into three sectors as

1. sector 1 if  $H$  is between  $0^\circ$  and  $120^\circ$ ,
2. sector 2 if  $H$  is between  $120^\circ$  and  $240^\circ$ , and
3. sector 3 if  $H$  is between  $240^\circ$  and  $360^\circ$ .

Based on the sector, compute the components as follows:

Sector 1:

$$B = I(1 - S)$$

$$R = I \left( 1 + \frac{S \cos H}{\cos(60^\circ - H)} \right)$$

$$G = 3I - (R + B)$$

Sector 2:

$$H = H - 120$$

$$R = I(1 - S)$$

$$G = I \left[ 1 + \frac{S \cos H}{\cos(60^\circ - H)} \right]$$

$$B = 3I - (R + G)$$

Sector 3:

$$H = H - 240$$

$$G = I(1 - S)$$

$$B = I \left[ 1 + \frac{S \cos H}{\cos(60^\circ - H)} \right]$$

$$R = 3I - (G + B)$$