EC8701-ANTENNAS AND MICROWAVE ENGINEERING

UNIT I
INTRODUCTION TO MICROWAVE SYSTEMS AND ANTENNAS

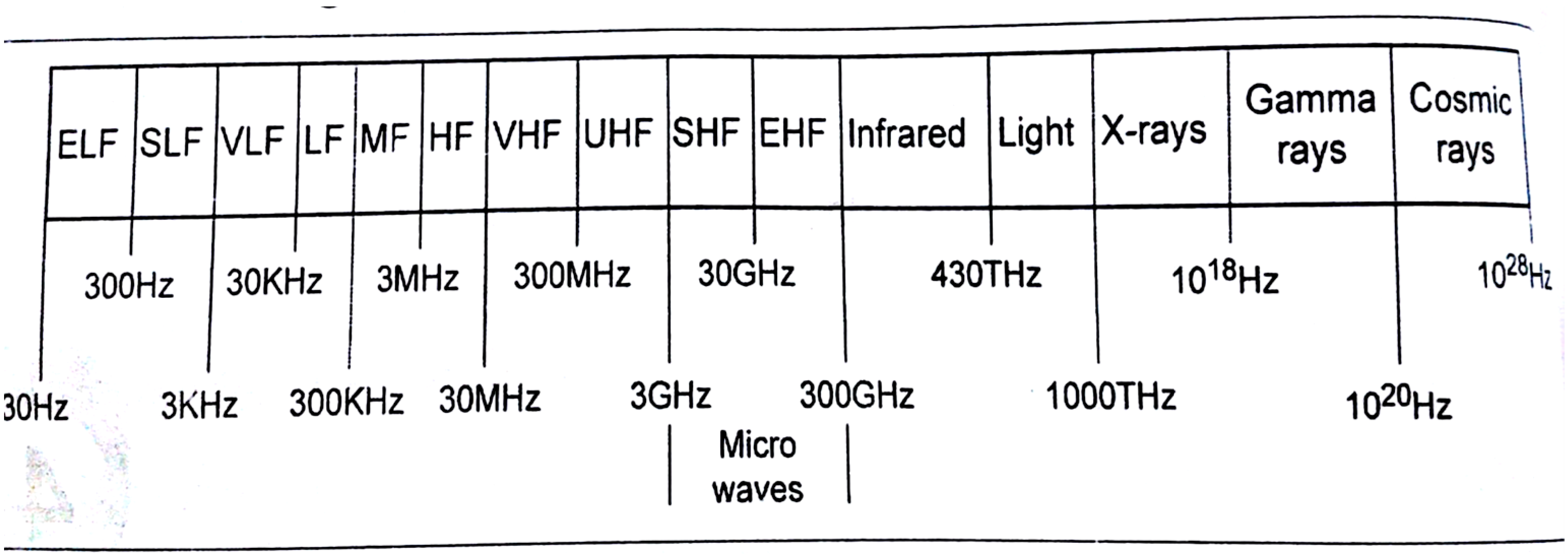
Microwave frequency bands, Physical concept of radiation, Near- and far-field regions, Fields and Power Radiated by an Antenna, Antenna Pattern Characteristics, Antenna Gain and Efficiency, Aperture Efficiency and Effective Area, Antenna Noise Temperature and G/T, Impedance matching, Friis transmission equation, Link budget and link margin, Noise Characterization of a microwave receiver.

MICROWAVE ENGINEERING
INTRODUCTION

- Microwave Frequencies
- The term *microwave* refers to alternating current signals with frequencies between **300 MHz (3×10^8 Hz) and 30 GHz (3×10^{10} Hz)**, with a corresponding electrical wavelength between 1 m and 1 cm
- Three major bands:

1. Ultra High Frequency (UHF) – 0.3 GHz to 3 GHz
2. Super High Frequency (SHF) – 3 GHz to 30 GHz
3. Extra High Frequency (EHF) – 30 GHz to 300 GHz

EM Spectrum


ELF
SLF
VLF
LF
MF
HF
VHF
UHF
SHF
EHF
Infrared
Light
X-rays
Gamma rays
Cosmic rays
300Hz
30KHz
3MHz
300MHz
30GHz
430THz
10^18Hz
10^28Hz
30Hz
3KHz
300KHz
30MHz
3GHz
300GHz
1000THz
10^20Hz
Micro waves

US New Military Microwave Bands

Designation	Frequency range in gigahertz
A band	0.100–0.250
B band	0.250–0.500
C band	0.500–1.000
D band	1.000–2.000
E band	2.000–3.000
F band	3.000–4.000
G band	4.000–6.000
H band	6.000– 8.000
I band	8.000– 10.000
J band	10.000– 20.000
K band	20.000– 40.000
L band	40.000– 60.000
M band	60.000–100.000

IEEE Microwave Frequency Bands

Designation	Frequency range in gigahertz
HF	0.003– 0.030
VHF	0.030– 0.300
UHF	0.300– 1.000
L band	1.000– 2.000
S band	2.000– 4.000
C band	4.000– 8.000
X band	8.000– 12.000
Ku band	12.000– 18.000
K band	18.000– 27.000
Ka band	27.000– 40.000
Millimeter	40.000–300.000
Submillimeter	>300.000

Microwave Applications

Microwave Applications

- Communication
  - Terrestrial
  - Satellite
- Radar
  - Civilian
    - Air traffic control
    - Aircraft navigation
    - Ship safety
    - Space vehicles
    - Remote sensing
    - Law enforcement
  - Military
    - Surveillance
    - Navigation
    - Guidance of weapons
    - Electronic warfare
    - C^3
- Industrial and Biomedical
  - Process control
  - Drying
  - Curing
  - Treatment of elastomers
  - Waste treatment
    - Nuclear
    - Cellulosic
  - Monitoring
  - Imaging
  - Hyperthermia
- Heating
  - Industrial
  - Household

Advantages

- Can carry large quantities of information (High Operating Frequency)
- High frequency → Low Wavelength → Small Antennas
- Easily propagated
- Fewer repeaters are necessary for amplification
- Increased bandwidth available

Disadvantages

- Difficult to analyze and design
- Measuring techniques are more difficult
- Difficult to implement conventional components at microwave frequencies (Resistors, Capacitors, Inductors ……)
- Transit time is more critical at microwave frequencies

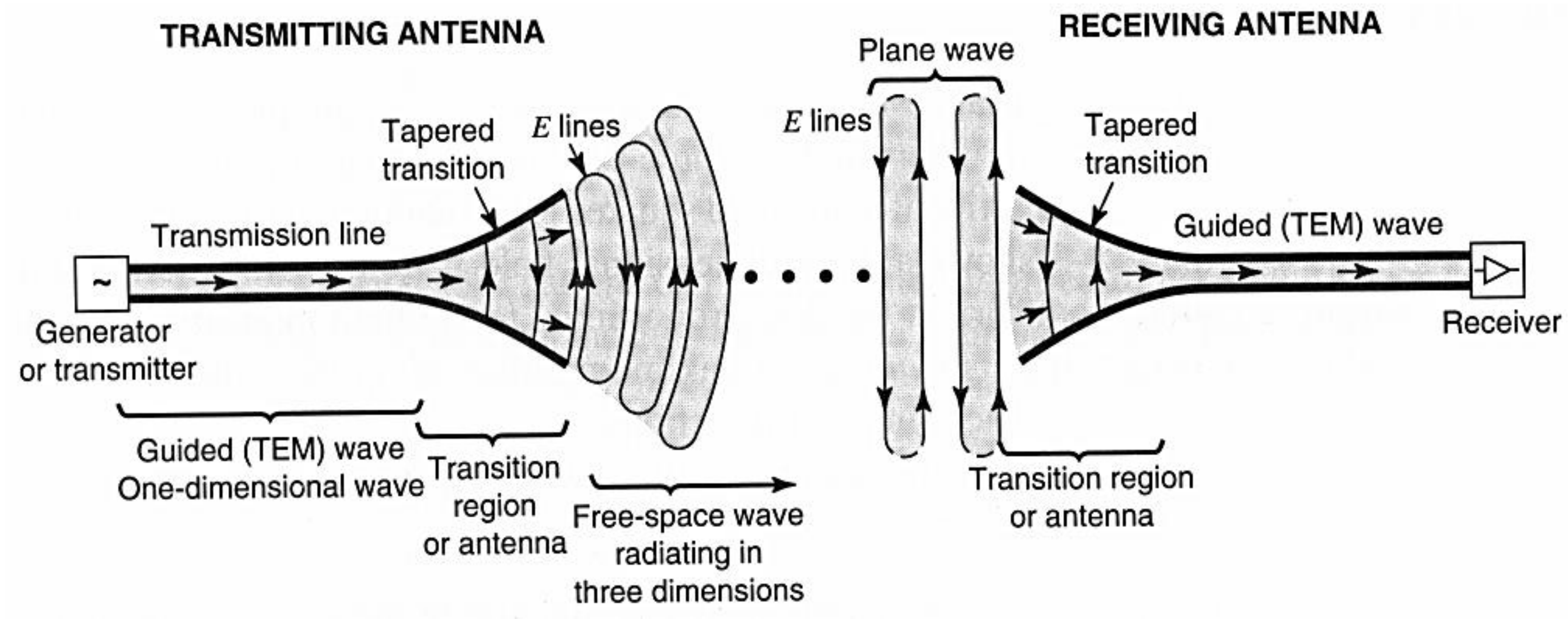
Antenna Basics

1. Introduction to antenna
2. Characteristics
3. Types

ANTENNA INTRODUCTION

An antenna is an electrical conductor or system of conductors

- Transmission - Radiates electromagnetic energy into free space
- Reception - Collects electromagnetic energy from free space



The role of antennas

Antennas serve four primary functions

- Spatial filter

  directionally-dependent sensitivity
- Polarization filter

  polarization-dependent sensitivity
- Impedance transformer

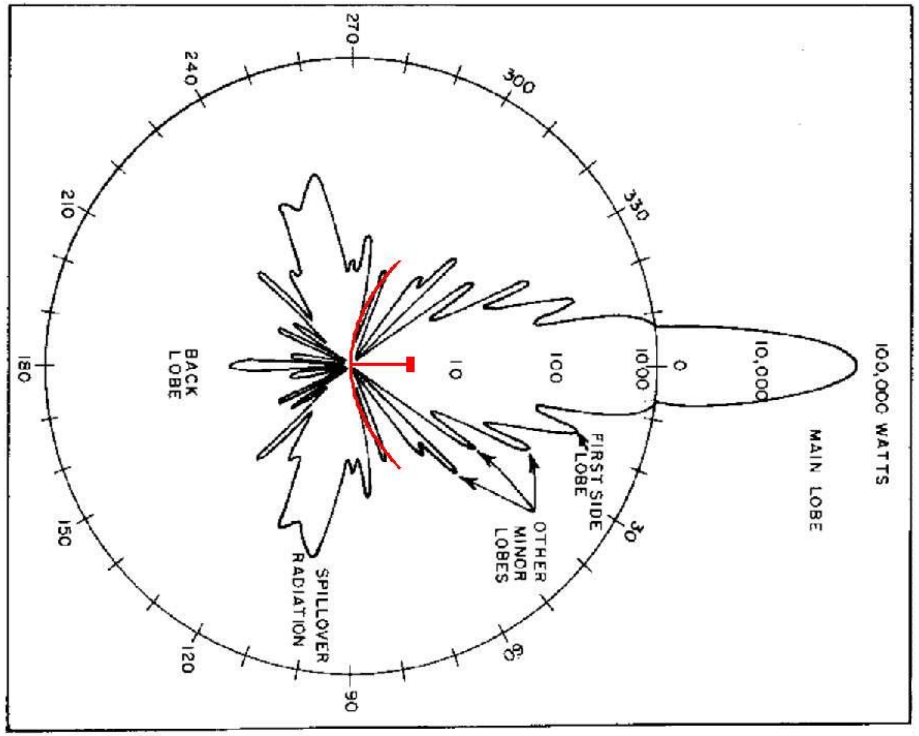
  transition between free space and transmission line
- Propagation mode adapter

  from free-space fields to guided waves
  (e.g., transmission line, waveguide)

Spatial filter

Antennas have the property of being more sensitive in one direction than in another which provides the ability to spatially filter signals from its environment.

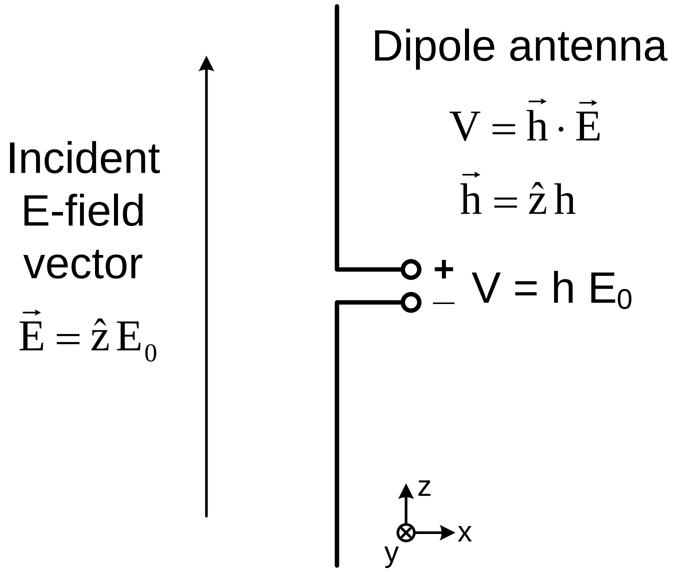
Directive antenna.

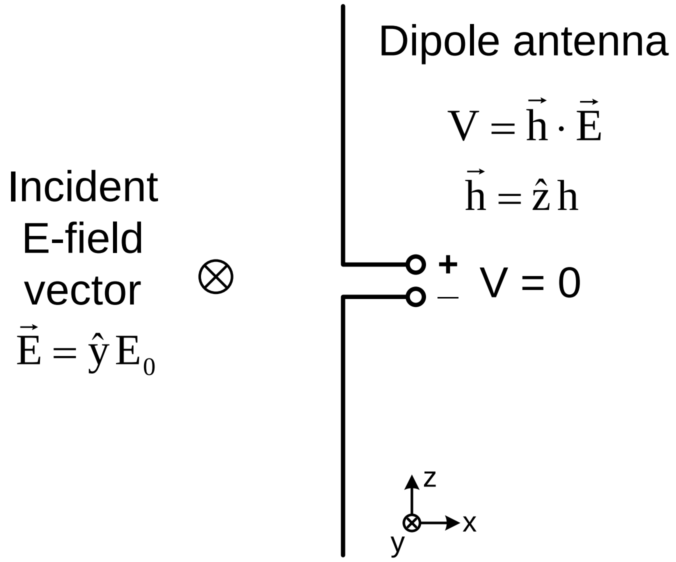


Radiation pattern of directive antenna.

Polarization filter

Antennas have the property of being more sensitive to one polarization than another which provides the ability to filter signals based on its polarization.





In this example, h is the antenna's effective height whose units are expressed in meters.

Impedance transformer

Intrinsic impedance of free-space, E/H

$$\eta_0 = \sqrt{\mu_0/\varepsilon_0}$$
$$= 120\pi$$
$$\cong 376.7\,\Omega$$

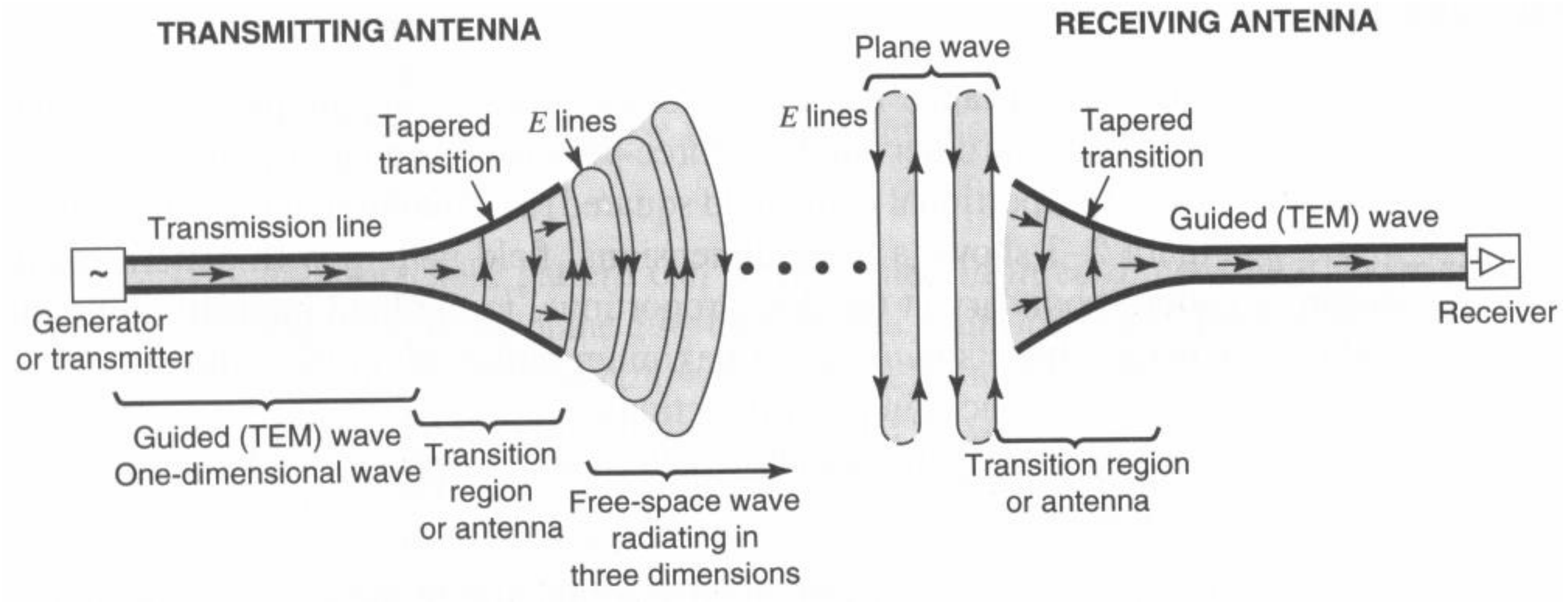
Characteristic impedance of transmission line, V/I

A typical value for Z_0 is 50 Ω.

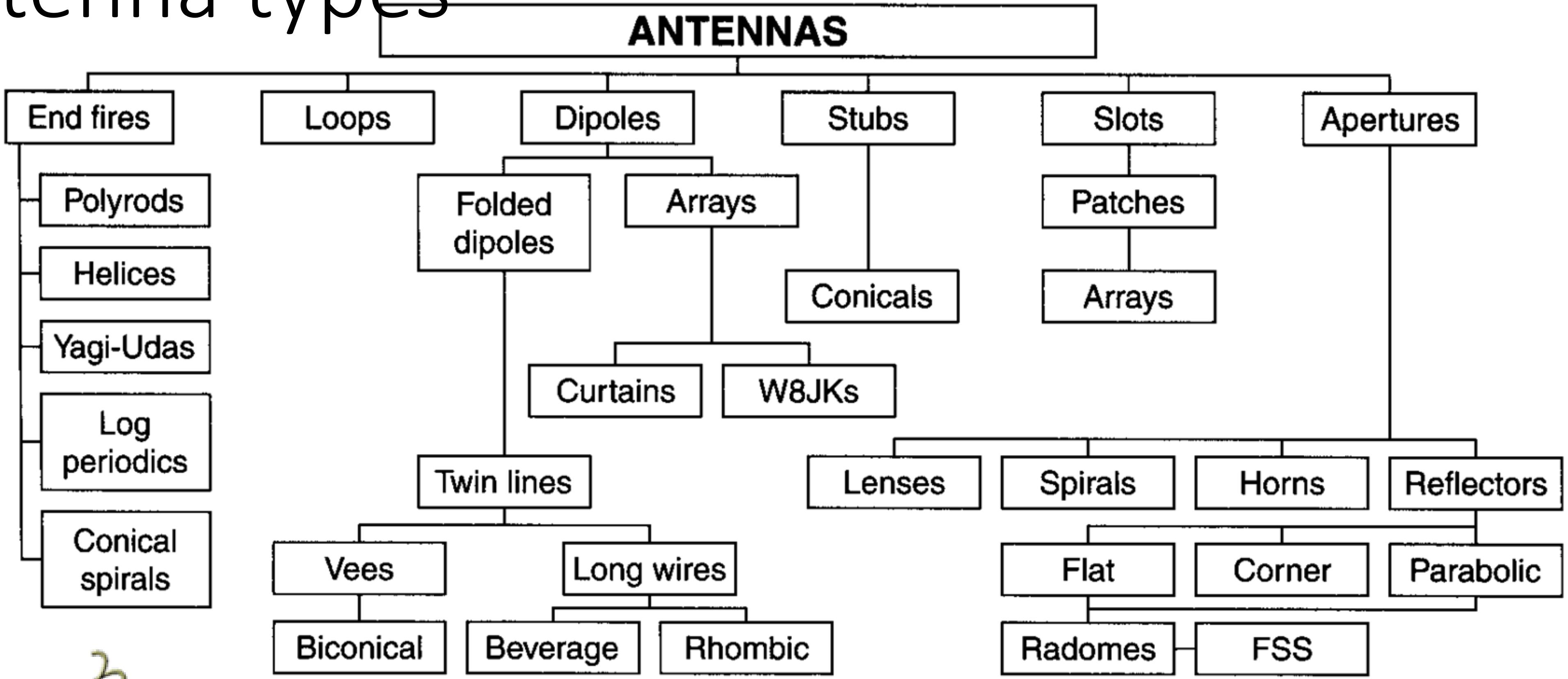
Clearly there is an impedance mismatch that must be addressed by the antenna.

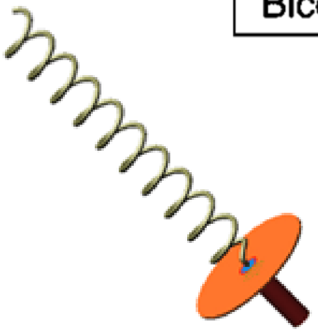
Propagation mode adapter

During both transmission and receive operations the antenna must provide the transition between these two propagation modes.

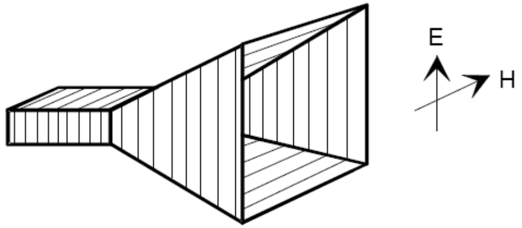


Antenna types

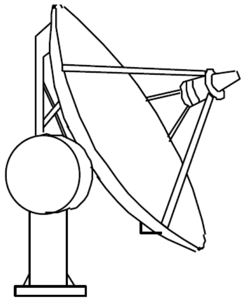




Helical antenna



Horn antenna



Parabolic reflector antenna

Antenna Characterization

- Directivity
- Power Pattern
- Antenna Gain
- Effective Area
- Antenna Efficiency

Radiation pattern

Radiation pattern – variation of the field intensity of an antenna as an angular function with respect to the axis

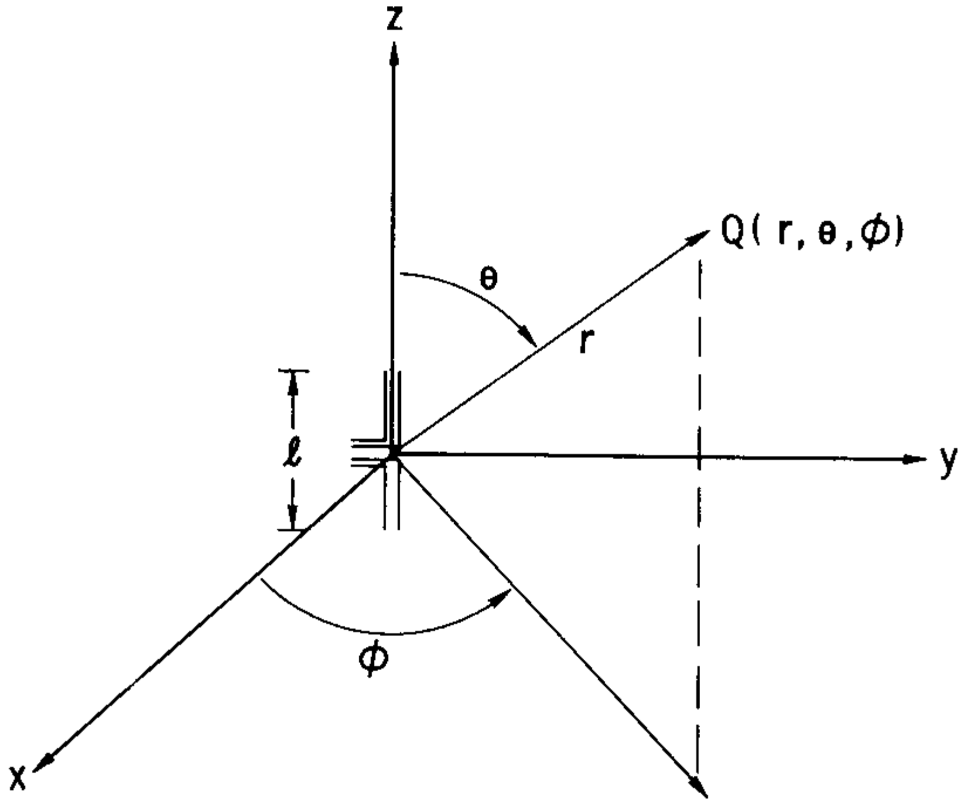
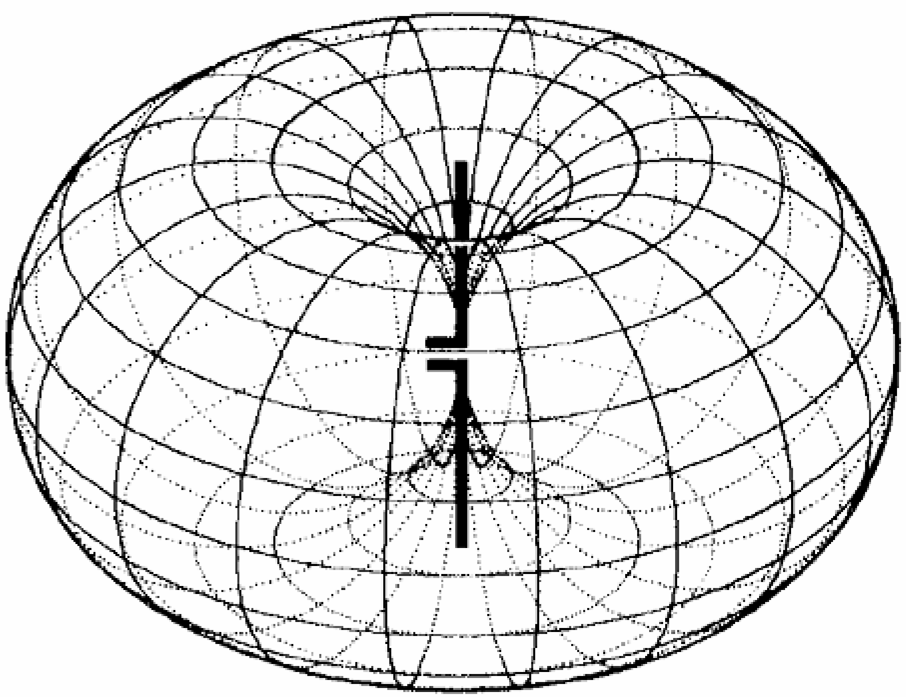


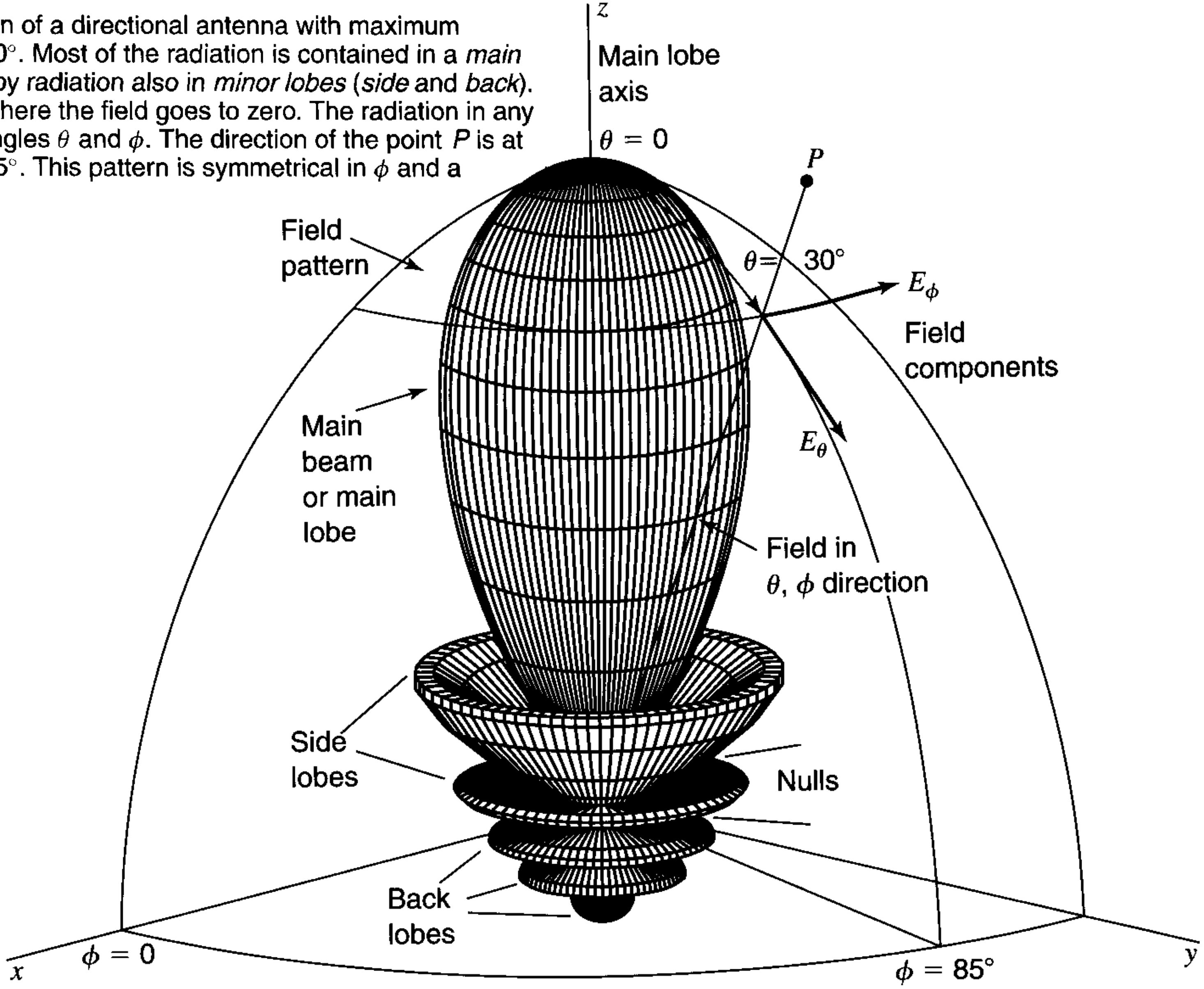
Fig. 3.7 Short dipole placed at the origin of a spherical coordinate system.



Three-dimensional representation of the radiation pattern of a dipole antenna

Radiation pattern

Figure 2–3
Three-dimensional field pattern of a directional antenna with maximum radiation in z-direction at $\theta = 0°$. Most of the radiation is contained in a *main beam* (or *lobe*) accompanied by radiation also in *minor lobes* (*side* and *back*). Between the lobes are *nulls* where the field goes to zero. The radiation in any direction is specified by the angles θ and ϕ. The direction of the point P is at the angles $\theta = 30°$ and $\phi = 85°$. This pattern is symmetrical in ϕ and a function only of θ.



Characteristics

Radiation pattern

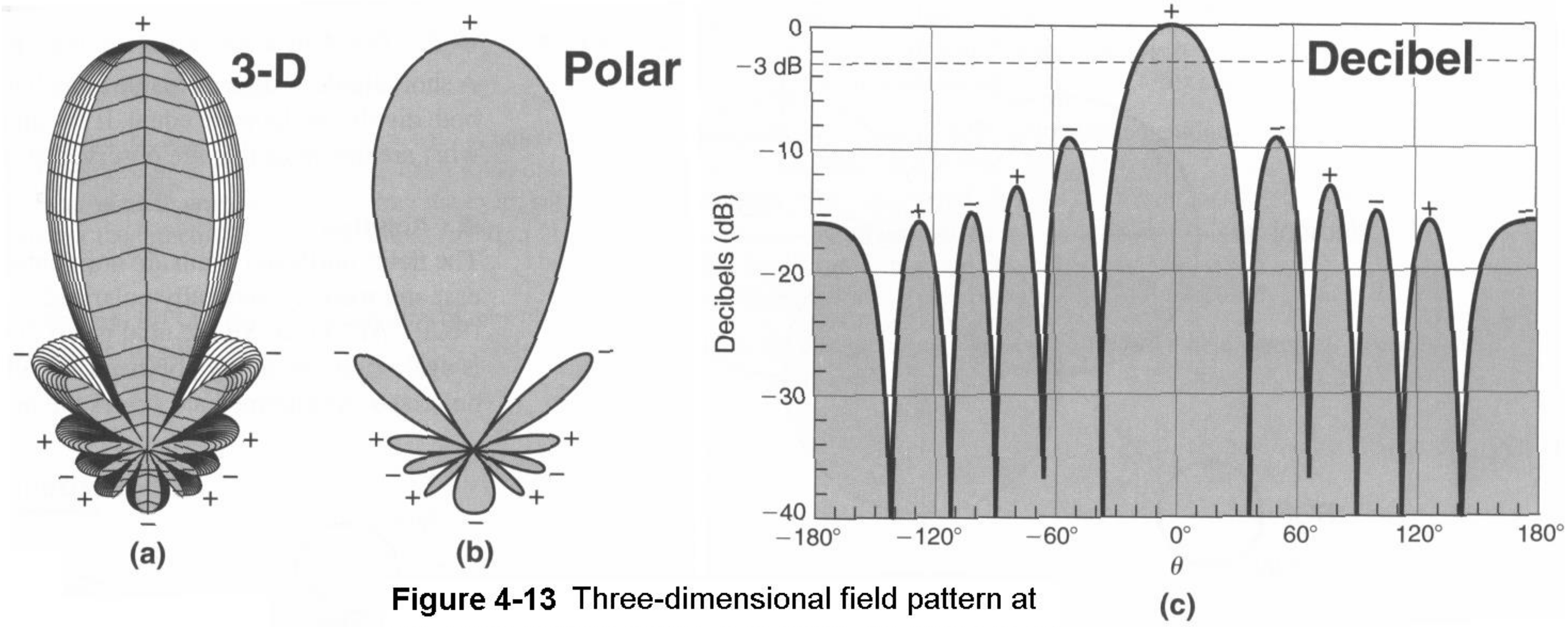


Figure 4-13 Three-dimensional field pattern at (a), polar pattern at (b), and decibel pattern at (c) showing alternate phasing (+ and -) of pattern lobes.

Fields from $\lambda/2$ Dipole

- To take account of the phase differences of the contributions from all the elements dl we need to integrate over the entire length of the antenna as shown by the figure (from Skilling, 1948)

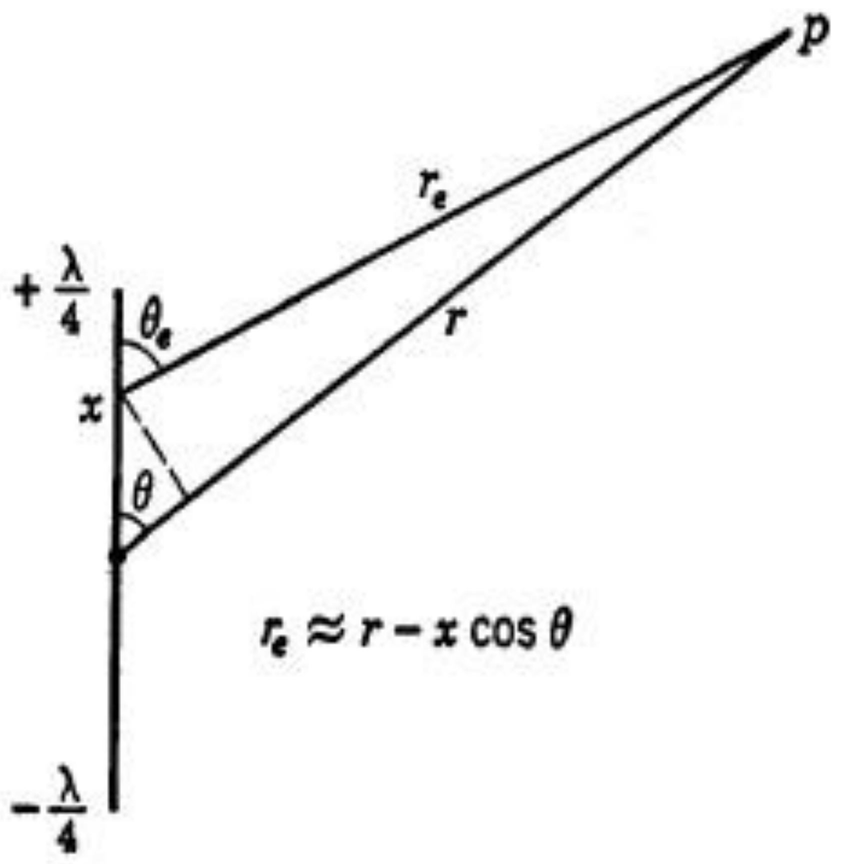
  $E_\theta = \int_{\pm\lambda/4} (\eta\ I_o \sin\theta_e / 2\ r_e\ \lambda) \cos kx\ \ \cos \omega[t-(r_e/c)]\ dx$

  - Integral is from $-\lambda/4$ to $\lambda/4$, i.e. over the antenna length

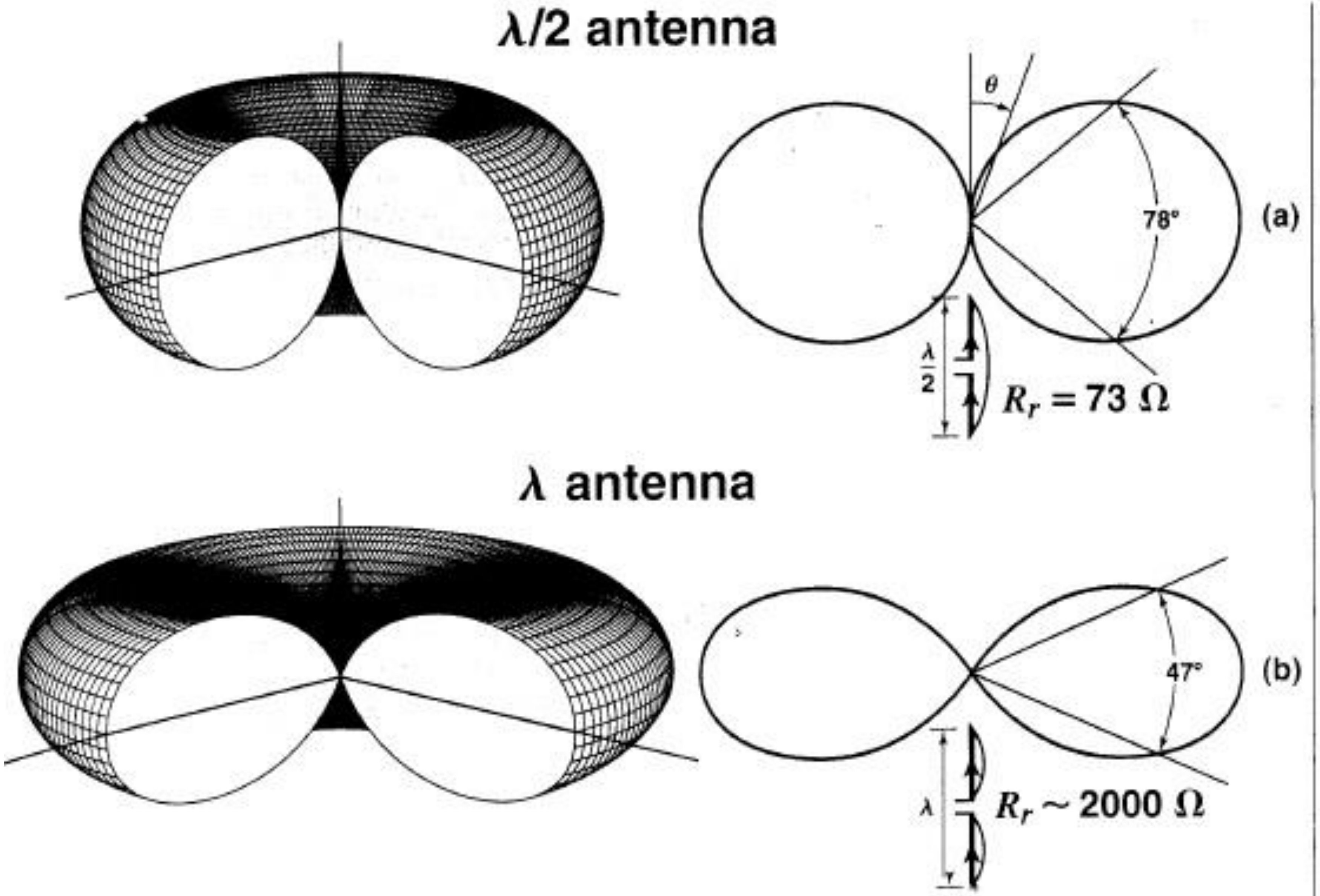
- Result of integration

  $E_\theta = (\eta I_o/2\pi\ r) \cos \omega[t-(r/c)]$
  $\{\cos [(\pi/2) \cos\theta] / \sin\theta\}$

- We know that $E_r = E_\phi = 0$ as for the Hertzian dipole

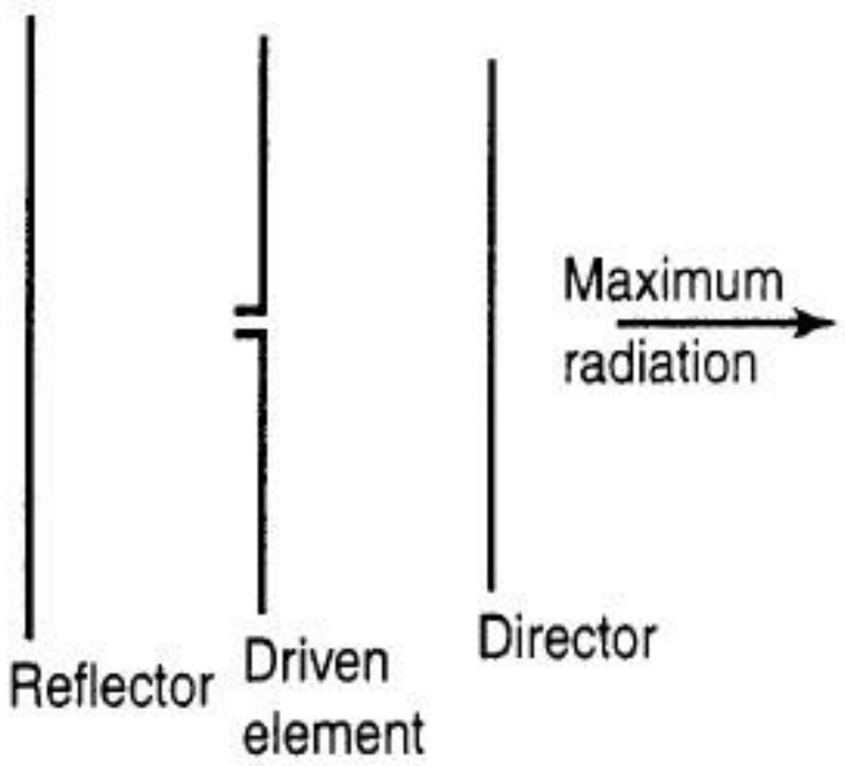


λ/2 and λ Dipole Antenna Pattern (E-field)

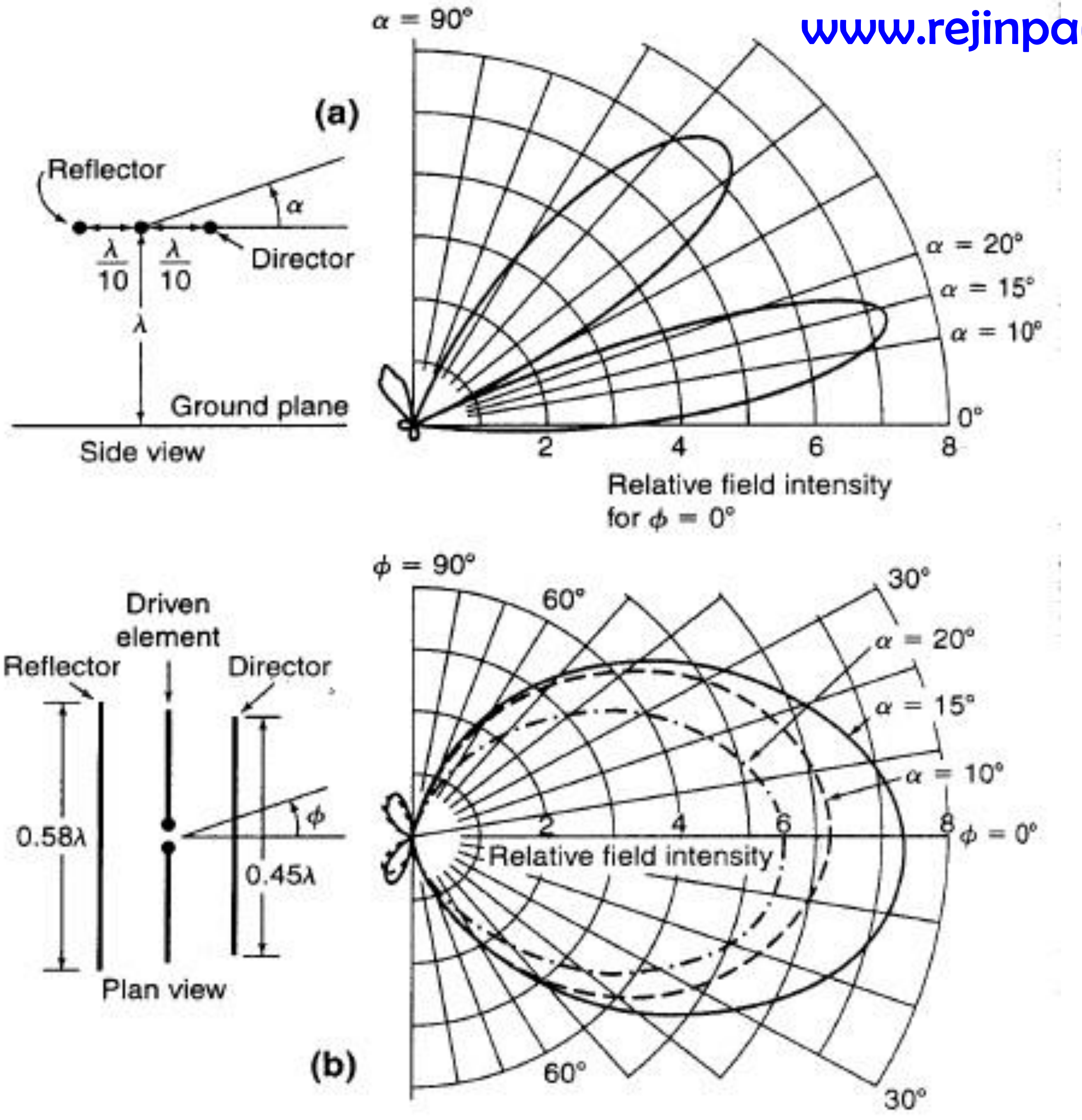


Yagi - Uda

- Driven element induces currents in parasitic elements
- When a parasitic element is slightly longer than $\lambda/2$, the element acts inductively and thus as a reflector -- current phased to reinforce radiation in the maximum direction and cancel in the opposite direction
- The director element is slightly shorter than $\lambda/2$, the element acts inductively and thus as a director -- current phased to reinforce radiation in the maximum direction and cancel in the opposite direction
- The elements are separated by ≈ $0.25\ \lambda$



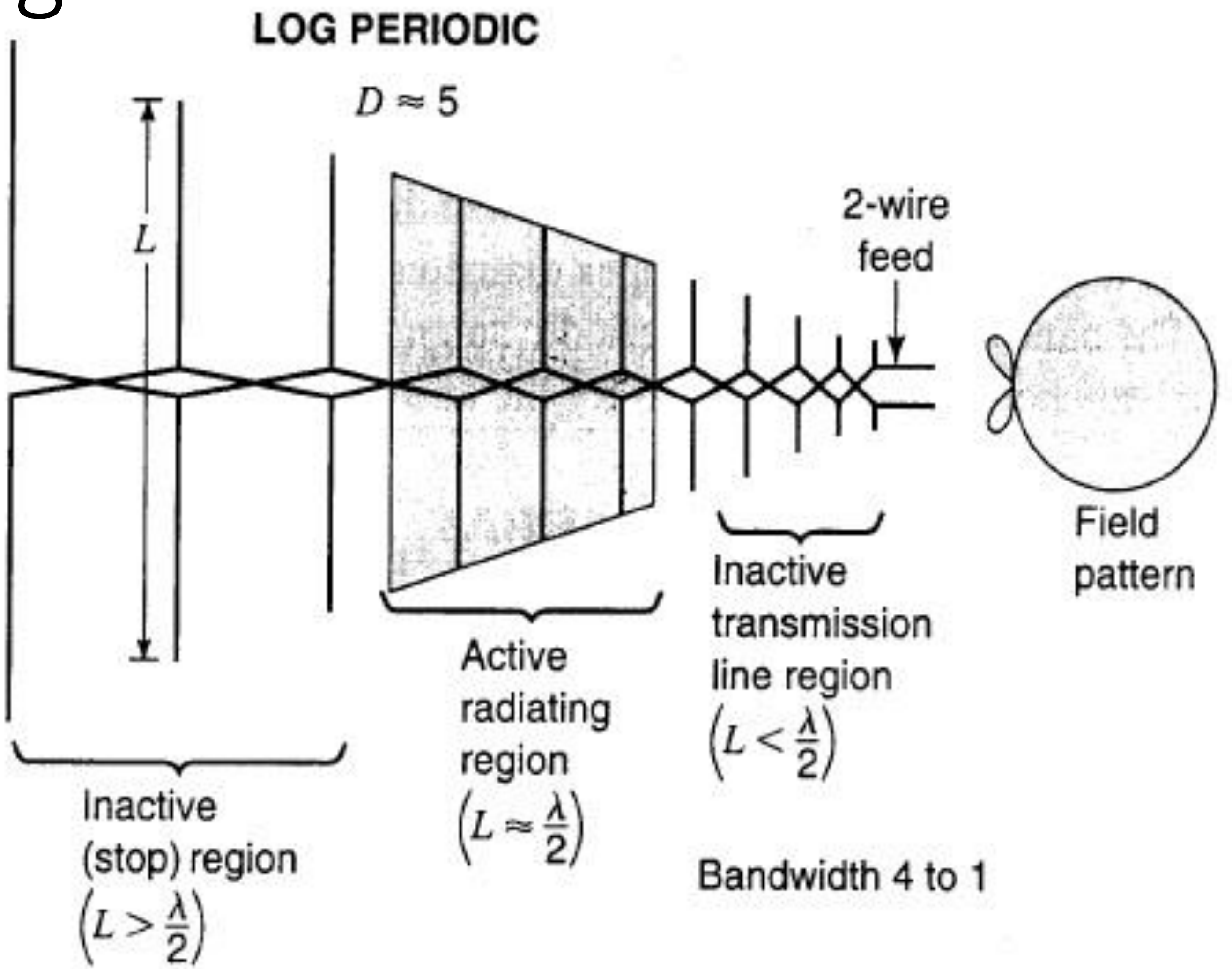
3 Element Yagi Antenna Pattern



2.4 GHz Yagi with 15dBi Gain

- G ≈ 1.66 * N (not dB)
- N = number of elements
- G ≈ 1.66 *3 = 5 = 7 dB
- G ≈ 1.66 * 16 = 27 = 16 dB

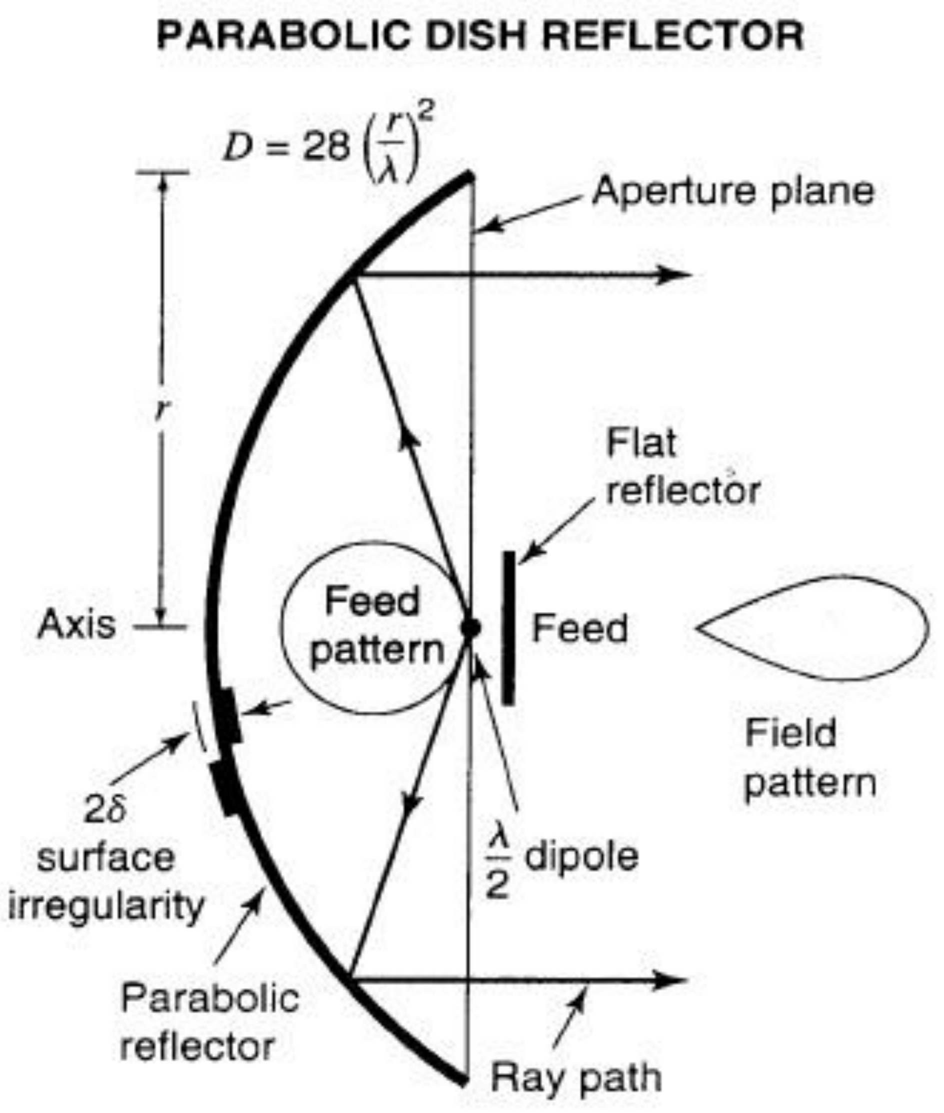
Log-Periodic Antennas



- A log periodic is an extension of the Yagi idea to a broad-band, perhaps 4 x in wavelength, antenna with a gain of ≈ 8 dB
- Log periodics are typically used in the HF to UHF bands

Parabolic Reflectors

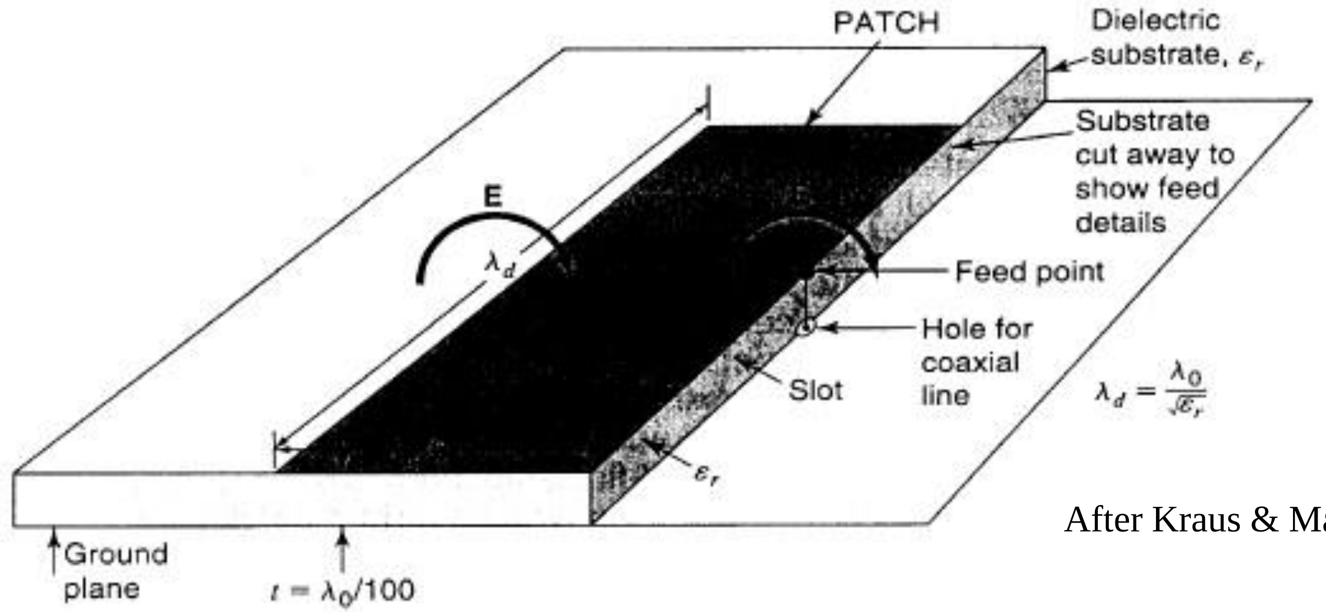
- A parabolic reflector operates much the same way a reflecting telescope does
- Reflections of rays from the feed point all contribute in phase to a plane wave leaving the antenna along the antenna bore sight (axis)
- Typically used at UHF and higher frequencies



Stanford's Big Dish

- 150 ft diameter dish on alt-azimuth mount made from parts of naval gun turrets
- Gain ≈ 4 π εA/λ^2
  ≈ 2 x 10^5 ≈ 53 dB
  for S-band (l ≈15 cm)

Patch Antennas



After Kraus & Marhefka, 2003

- Radiation is from two "slots" on left and right edges of patch where slot is region between patch and ground plane
- Length $\lambda_d = \lambda_o/\varepsilon_r^{1/2}$ Thickness typically ≈ 0.01 λ_o
- The big advantage is conformal, i.e. flat, shape and low weight
- Disadvantages: Low gain, Narrow bandwidth (overcome by fancy shapes and other heroic efforts), Becomes hard to feed when complex, e.g. for wide band operation

Patch Antenna Pattern

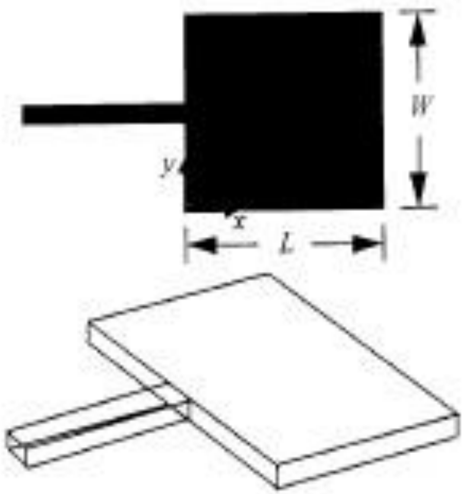


FIGURE 21.11. Half-wave patch antenna (conductor pattern and perspective view)

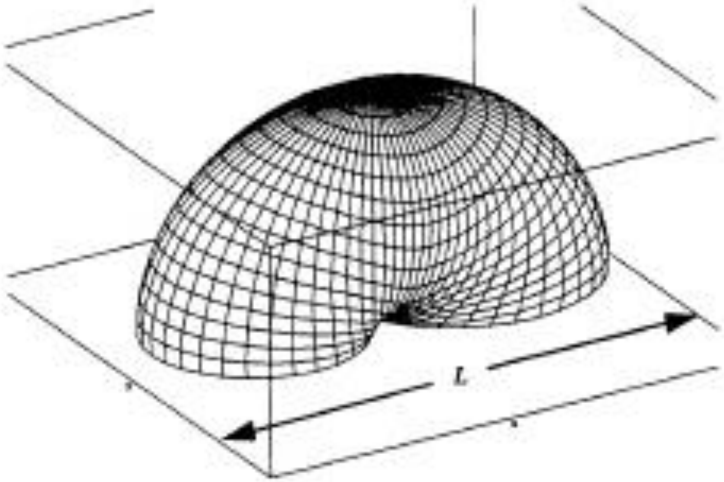
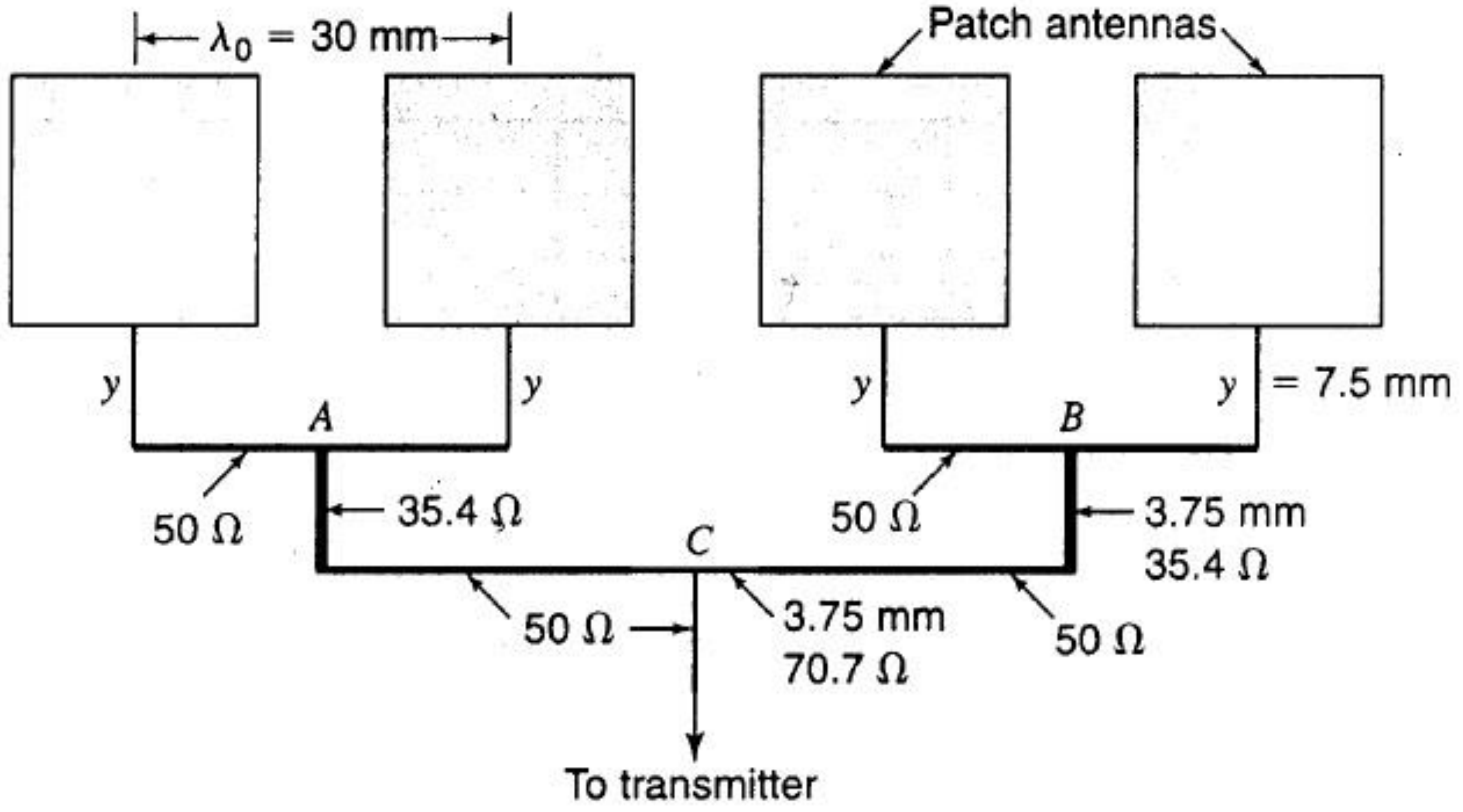


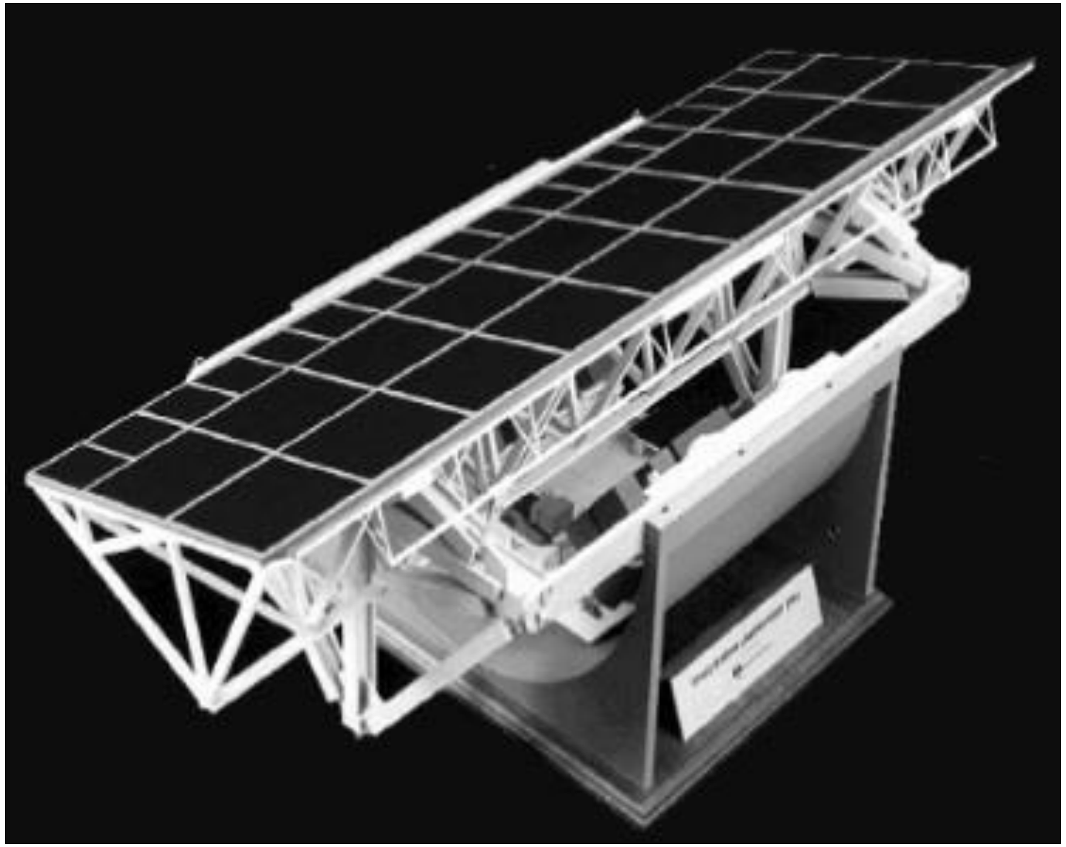
FIGURE 21.12. Typical radiation pattern for patch antenna of Figure 21.11

Array Antennas



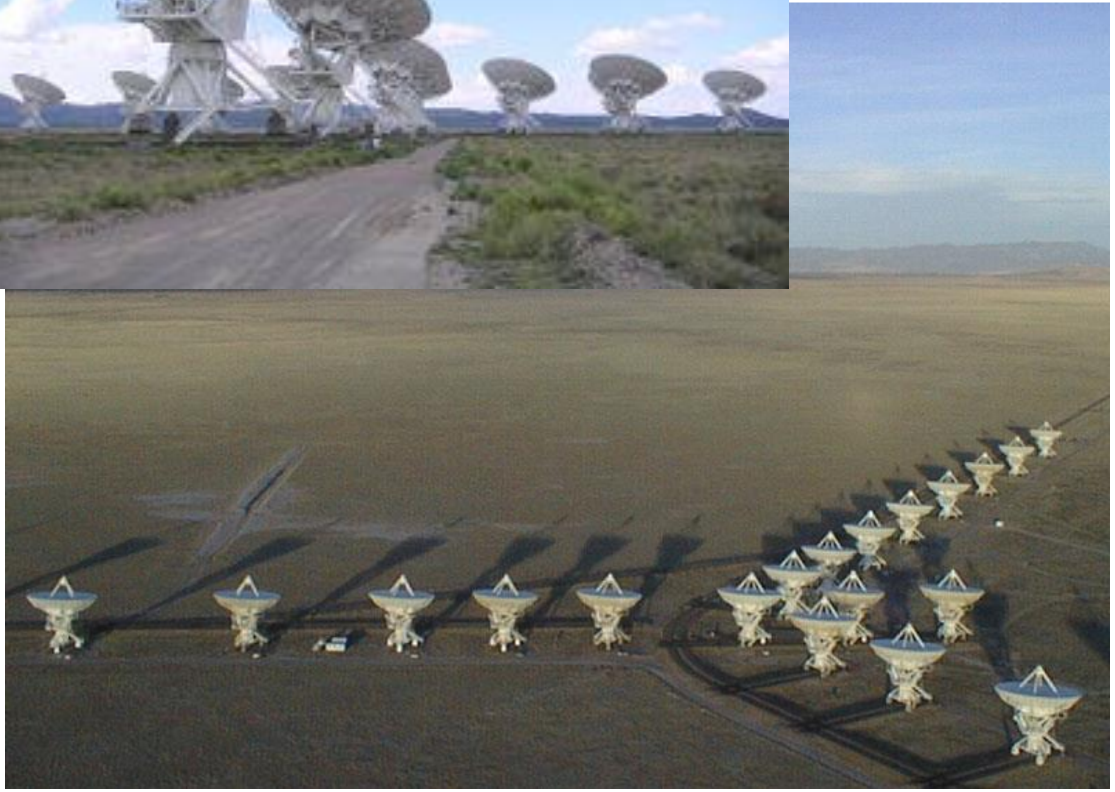
Patch Antenna Array for Space Craft

- The antenna is composed of two planar arrays, one for L-band and one for C-band.
- Each array is composed of a uniform grid of dual-polarized microstrip antenna radiators, with each polarization port fed by a separate corporate feed network.
- The overall size of the SIR-C antenna is 12.0 x 3.7 meters
- Used for synthetic aperture radar



Very Large Array

Organization: National Radio Astronomy Observatory
Location:Socorro NM
Wavelength:
radio 7 mm and larger
Number & Diameter
27 x 25 m
Angular resolution: 0.05 (7mm) to 700 arcsec



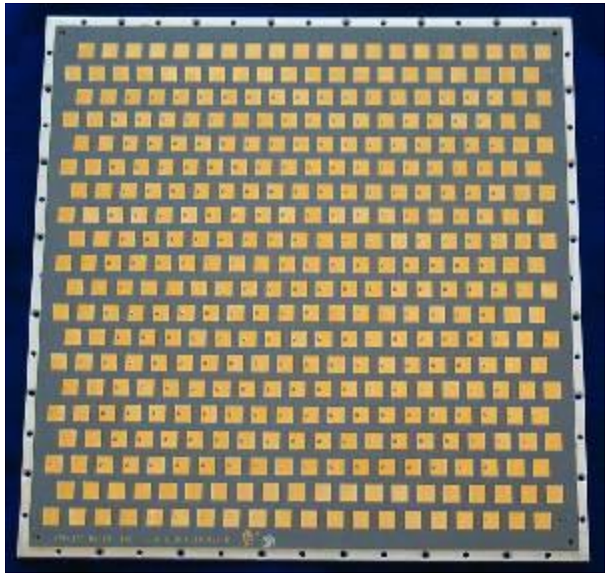
http://www.vla.nrao.edu/

Antenna arrays

Antenna array composed of several similar radiating elements (e.g., dipoles or horns).

Element spacing and the relative amplitudes and phases of the element excitation determine the array's radiative properties.

Linear array examples



Two-dimensional array of microstrip patch antennas

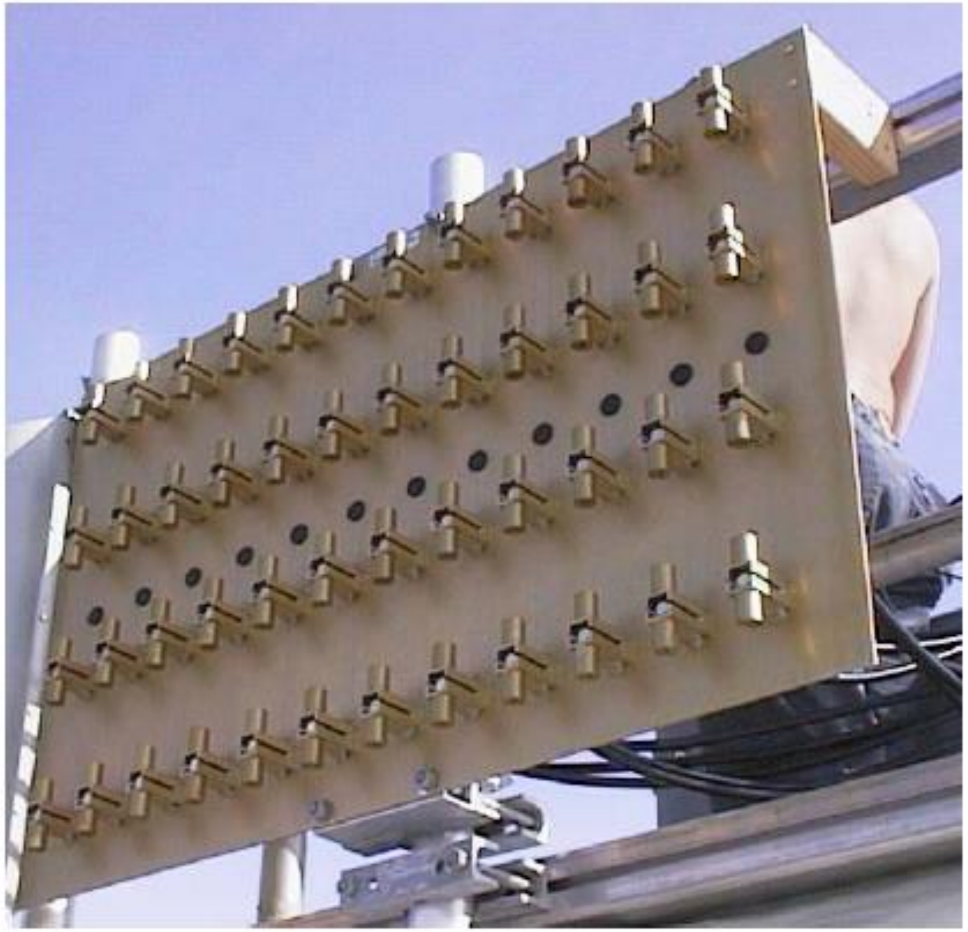
Satellite Antennas (TV)

Owens Valley Radio Observatory Array

New Mexico Very Large Array

[Sky & Telescope
Feb 1997 p. 30]

2 GHz adaptive antenna array



- A set of 48 2 GHz antennas
  - Source: Arraycomm

PHYSICAL CONCEPTS OF RADIATION

- Charge moving with uniform velocity along a **straight conductor does not radiate .**
- Charge moving back and forth –Harmonic motion

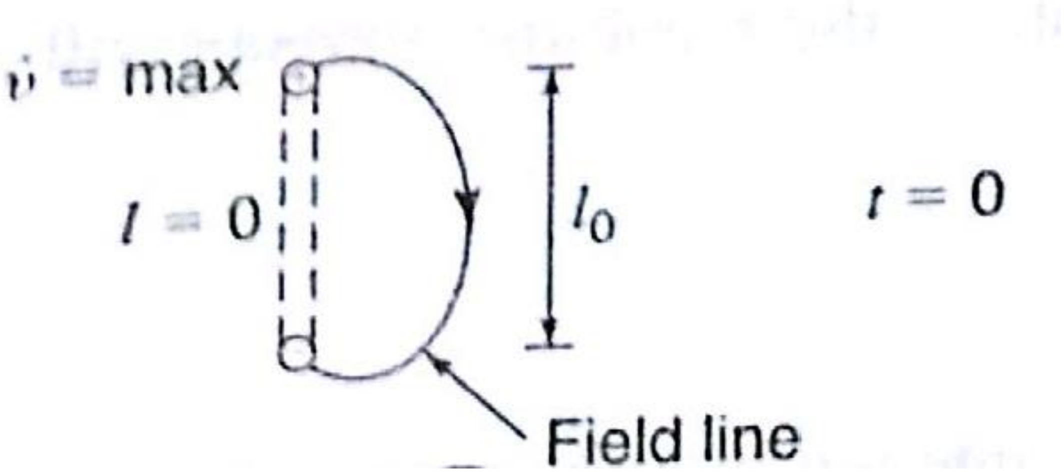
  - The conductor is subject to **acceleration and radiates.**

RADITAION FROM DIPOLE ANTENNA:

- Two equal charges of **opposite sign** of oscillating(up & down harmonic)
- The motion with instantaneous **separation "l"** (max separation l_0) – focusing attention on **electric filed**.
- Consider single electric filed.

www.rejinpaul.com

Single electric filed line



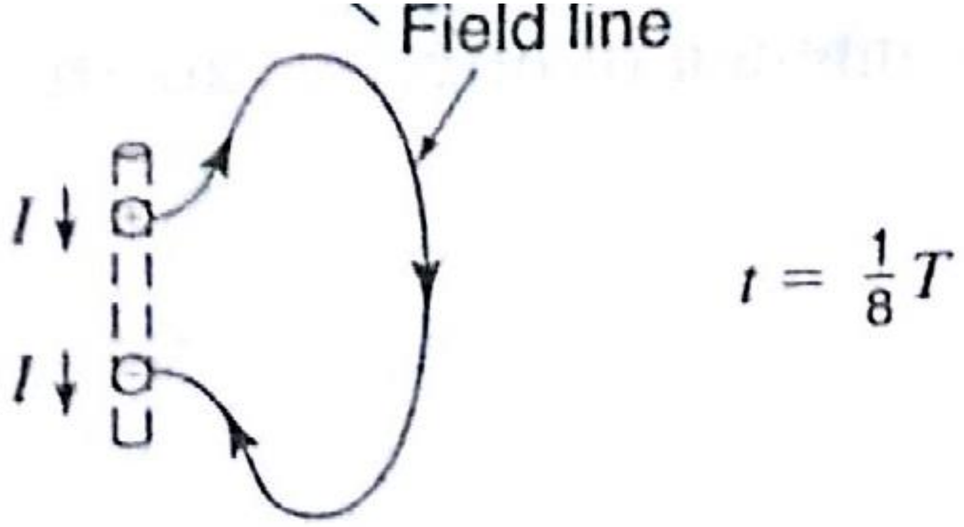
- At time **t=0**

-at minimum separation

-maximum acceleration v

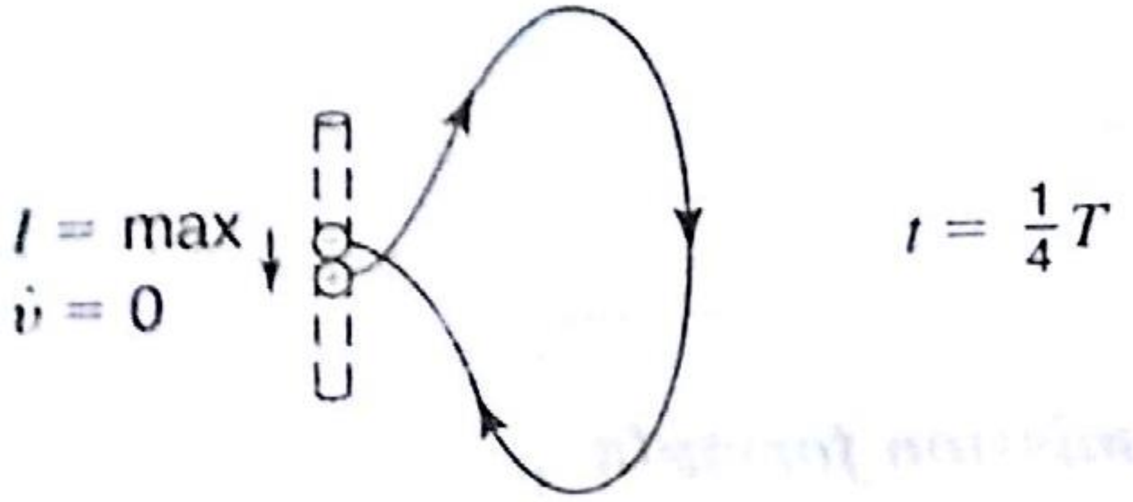
Reverse direction.

- current line l=0

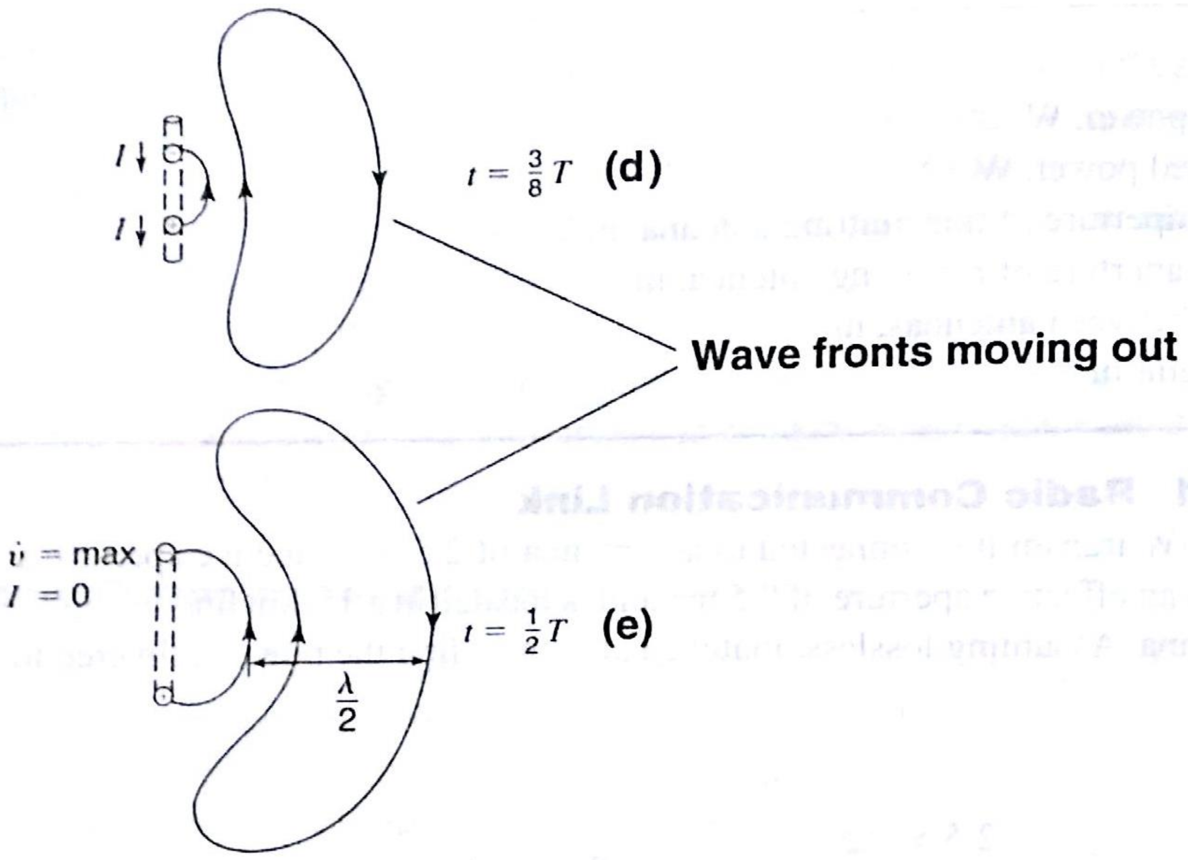


- At time **t=$\frac{1}{8}$ period**

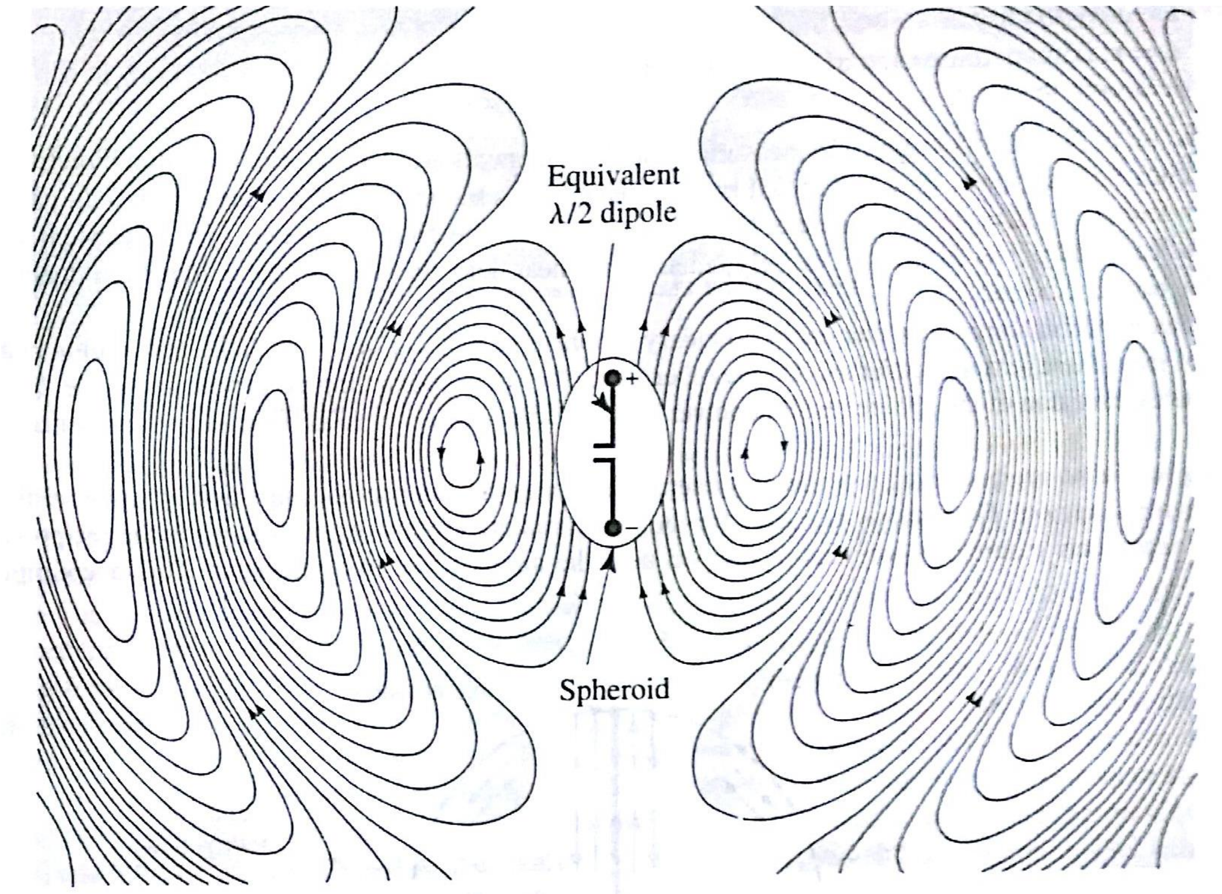
  -The charge moving each other direction.



- At time $t=\frac{1}{4}$ **period**
- **They pass the mid point**
- **The filed line detach and new one of opposite sign are formed.**
- **I= maximum =charge acceleration= 0**



- At time **$t=\frac{1}{2}$ period**
- **The filed continue to move out.**

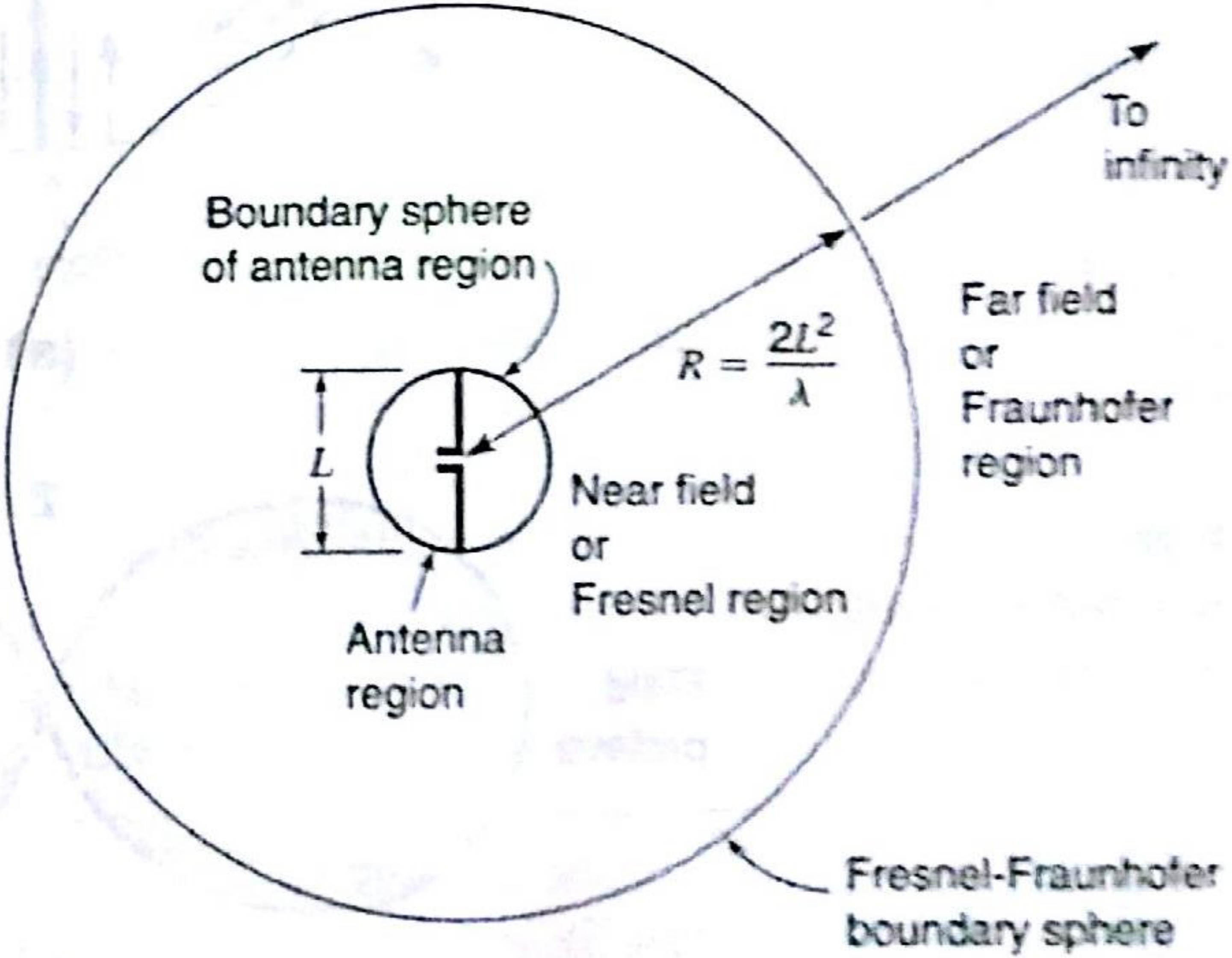


- **t=5 instant time**
- Image shows –capability of making ring smoke.
- Ring moves farther –size increases.
- Maintenance of shape -bigger ring size, lesser smoker density

Near- and far-field regions

ANTENNA REGION

- It classified into two types
  - Fresnel region –Near filed
  - Fraunhofer region –Far filed


To
infinity
Boundary sphere
of antenna region
Far field
or
Fraunhofer
region
$R = \frac{2L^2}{\lambda}$
L
Near field
or
Fresnel region
Antenna
region
Fresnel-Fraunhofer
boundary sphere

- The boundary between the two may be arbitrarily taken to be at **a** radius

$$\mathbf{R}=\frac{2L^2}{\lambda}$$

- Where

  L=Maximum dimension of the antenna

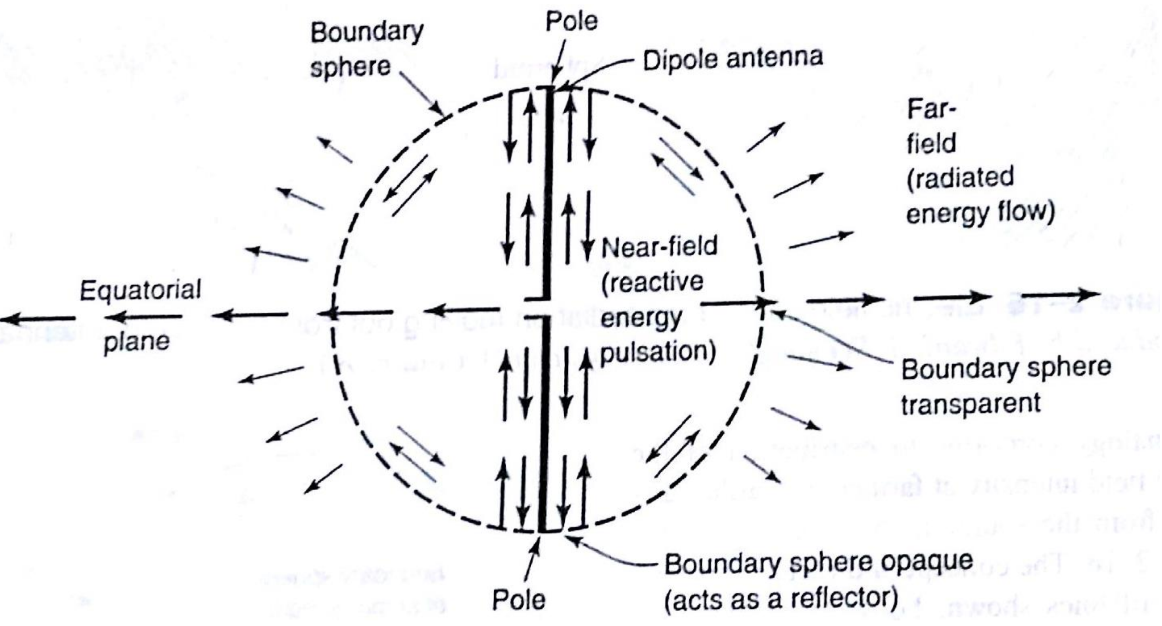
  λ=Wavelength

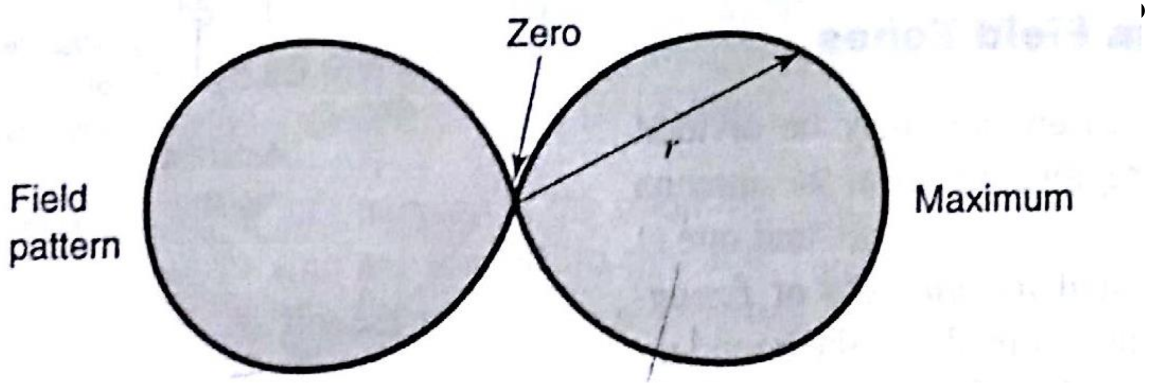
FAR (or) FRAUNHOFER REGION

- The filed components are transverse to the radial direction from the antenna.
- All the power flow is directed radially outward
- The shape of files pattern is “independent of the distance”

FRESNEL REGION –NEAR FILED

- longitudinal component of the electric filed
- Power flow is not entirely radial
- The shape of the filed pattern depends on the distance.



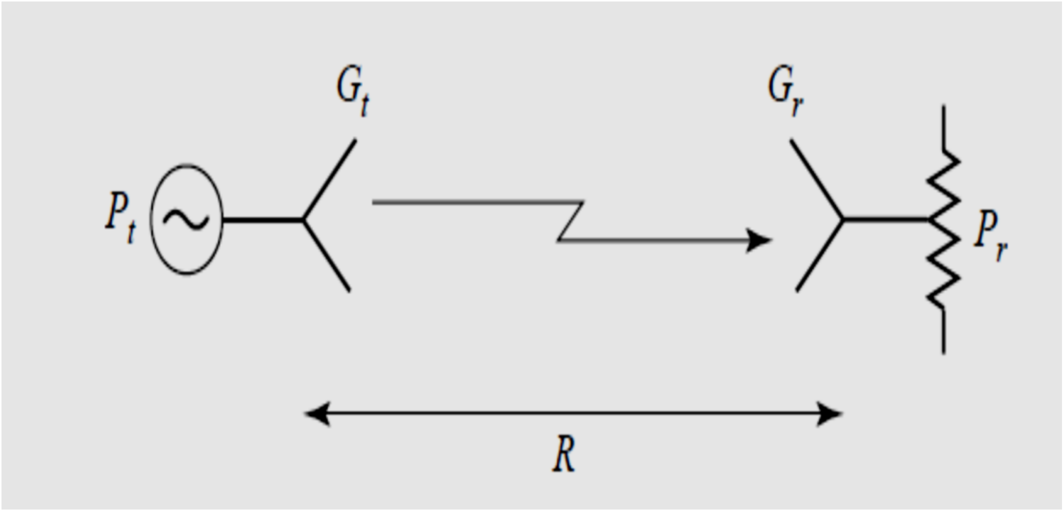


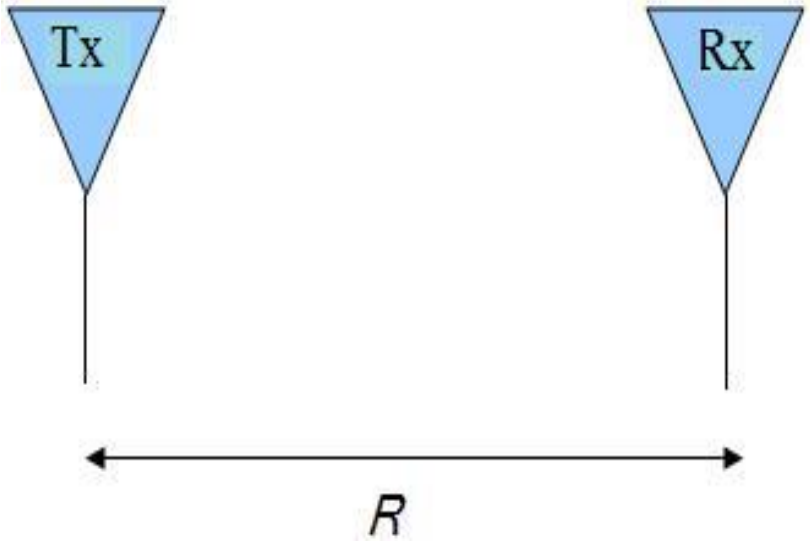
- Antenna in an imaginary boundary sphere.
- Near filed –reflector
- Wave expending perpendicular to the dipole in equatorial region-power leakage through the sphere
- Oscillating energy –outer flow equatorial region.
- outer flow-power radiated by an antenna, while reciprocating energy represent reactive power that is trapped near the antenna –resonator.

Friis Transmission equation

- A general radio system link,
  - The transmit power is P_t,
  - The transmit antenna gain is G_t ,
  - The receive antenna gain is G_r ,
  - The received power (delivered to a matched load) is P_r.
  - The transmit and receive antennas are separated by the distance ***R*.**

Basic radio system



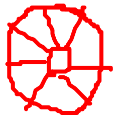


- The Friis Equation, consider **two antennas** in free space (no obstructions nearby) separated by a distance *R*:
- Assume that **(P_T)Watts of total power are delivered to the transmit antenna.**
- Assume that

 1.The transmit antenna is omnidirectional, lossless

 2. The receive antenna is in the far field of the transmit antenna.

- **The power density radiated** by an isotropic antenna *(D* =1 = 0 dB*)* at a distance *R* is given by



$$S_{avg} = \frac{P_T}{4\pi R^2}$$

- Able to recover all of the radiated power by integrating over a sphere of radius *R* surrounding the antenna
- The power is distributed isotropically, and the area of a sphere is $4\pi R^2$
- If the transmit antenna has a directivity greater than 0 dB

- **Directivity is defined** as the ratio of the actual radiation intensity to the equivalent isotropic radiation intensity.
- In addition, if the transmit antenna has losses→ Radiation efficiency factor→ Converting directivity to gain.
- Thus, the general expression for the power density radiated by an arbitrary transmit antenna is

$$S_{avg} = \frac{P_T}{4\pi R^2} G_T$$

- The gain term factors in the directionality and losses of a real antenna.
- Assume: The receive antenna has an **effective aperture given by A_e** Then the **power received P_r** by this antenna

$$P_r = A_e S_{avg}$$

$$P_r = \frac{P_T}{4\pi R^2} G_T A_e$$

- The effective aperture for any antenna can also be expressed as:

$$A_e = \frac{\lambda^2}{4\pi} G$$

- The resulting received power can be written as:

$$P_r = \frac{P_T G_T G_R \lambda^2}{(4\pi R)^2} \text{-----------------} (1)$$

- This is known as the ***Friis Transmission Formula.***
- It relates the free space path loss, antenna gains and wavelength to the received and transmit powers.

$$P_r = \frac{P_T G_T G_R c^2}{(4\pi R f)^2} \text{-----------------} (2)$$

(Since wavelength and frequency *f* are related by the speed of light *c)*

- Equ.(2) shows that more power is lost at higher frequencies.
- The path loss is higher for higher frequencies.(friss equ)
- The antennas are not polarization matched, the above received power could be multiplied by the Polarization Loss Factor (*PLF*) to properly account for this mismatch.
- Equ.(2) Includes polarization mismatch

$$P_r = (\mathrm{PLF}) . \frac{P_T G_T G_R c^2}{(4\pi R f)^2}$$

Effective isotropic radiated power (EIRP):

- The Friis formula, received power is proportional to the product $P_t G_t$
- These two factors—the transmit power and transmit antenna gain
  - **EIRP =$P_t G_t$W**
- For a given frequency, range, and receiver antenna gain, the received power is proportional to the EIRP of the transmitter and can only be increased by increasing the EIRP.
- This can be done by increasing the transmit power, or the transmit antenna gain, or both.

- In terms of decibel -Friis Transmission Formula:

$$P_r = \frac{P_T G_T G_R \lambda^2}{(4\pi R)^2}$$

- To convert this equation from linear units in Watts to decibels, we take the logarithm of both sides and multiply by 10

$$10 \log_{10} P_R = 10\, log_{10}\left(\frac{P_T G_T G_R \lambda^2}{(4\pi R)^2}\right)$$

- $(\text{i.e})\log_{10}(AB) = \log_{10}(A) + \log_{10}(B)$

Above equation ,

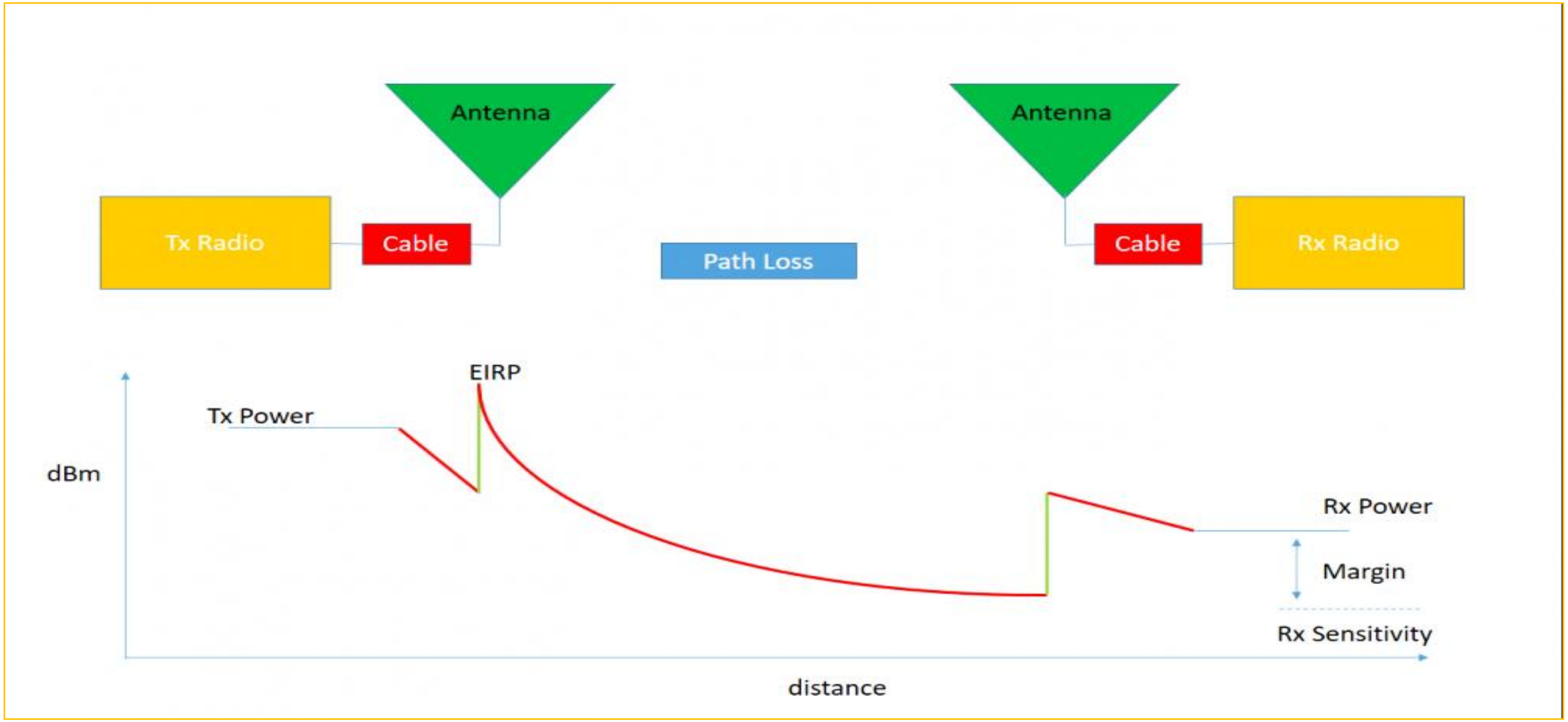
- $10 log_{10} P_R = 10 log_{10}(P_T) + 10 log_{10}(G_T) + 10 log_{10}(G_R) + 10 log_{10}\left(\frac{\lambda}{4\pi R}\right)^2$

- Using the definition of decibels, the above equation becomes a simple addition equation in dB:

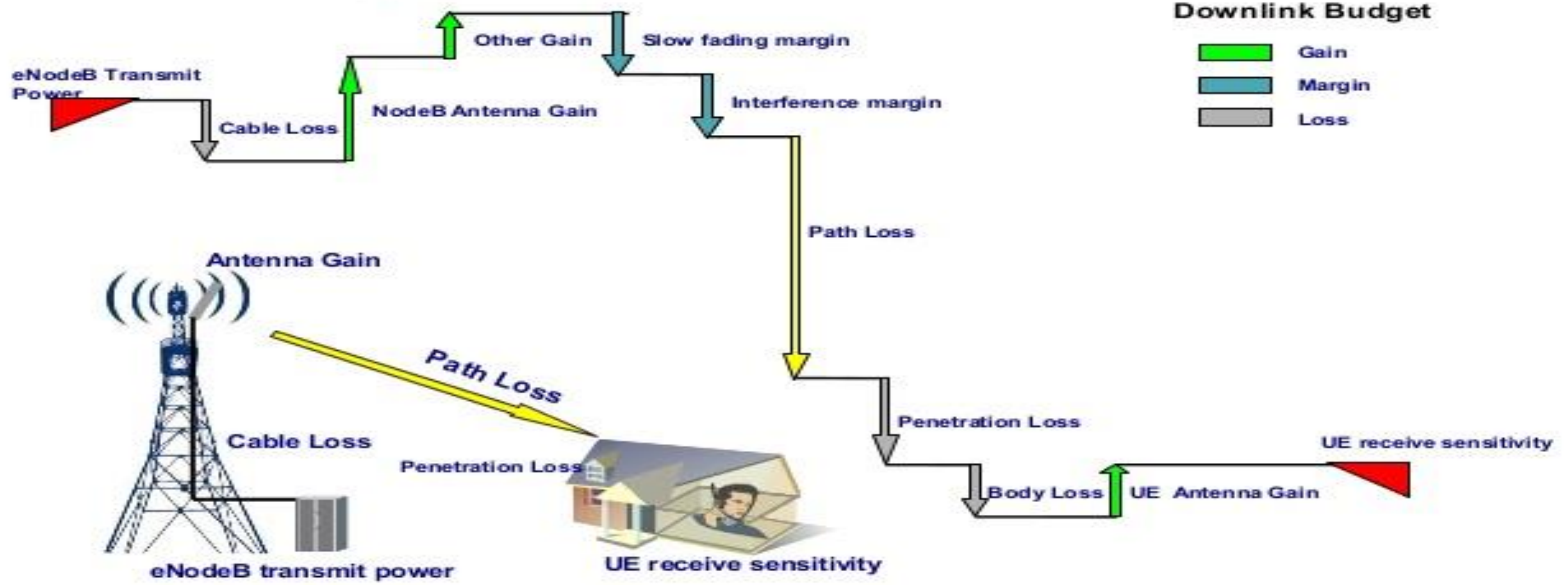
$$[P_R]_{dB}=[P_T]_{dB}+[G_T]_{dB}+[G_R]_{dB}+ [(\frac{\lambda}{4\pi R})^2]_{dB}$$

Link budget and link margin

Link budget


Antenna
Antenna
Tx Radio
Cable
Path Loss
Cable
Rx Radio
EIRP
Tx Power
dBm
Rx Power
Margin
Rx Sensitivity
distance

Link Budget Model: Downlink



Link budget

- **Link budget** is a way of quantifying the link performance.
- One of the terms in a **link budget is the *path loss***, accounting for the free-space reduction in signal strength with distance between the transmitter and receiver
- Path loss is defined (in dB) as

$$L_0 = 20 \log\left(\frac{4\pi R}{\lambda}\right) > 0$$

- Path loss **depends on wavelength** (frequency), which serves to provide a normalization for the units of distance
- we can write the remaining terms of the **Friis formula** as shown in the following link budget:

• Transmit power	P_t
• Transmit antenna line loss	(−) L_t
• Transmit antenna gain	P_t
• Path loss (free-space)	(−)L_0
• Atmospheric attenuation	(−) L_A
• Receive antenna gain	G_r
• Receive antenna line loss	(−)Lr
• Receive power	Pr

- Assuming that all of the above quantities are expressed in dB (or dBm, in the case of P_t

$$P_r \text{ (dB m)} = P_t - L_t + G_t - L_o - L_A + G_r - L_r$$

- Due to **impedance mismatch will reduce** the received power

by the factor $(1 - |\Gamma|^2)$

Impedance mismatch loss,

$$L_{imp}(\text{dB}) = -10 \log(1 - |\Gamma|^2) \geq 0, \quad (14.28)$$

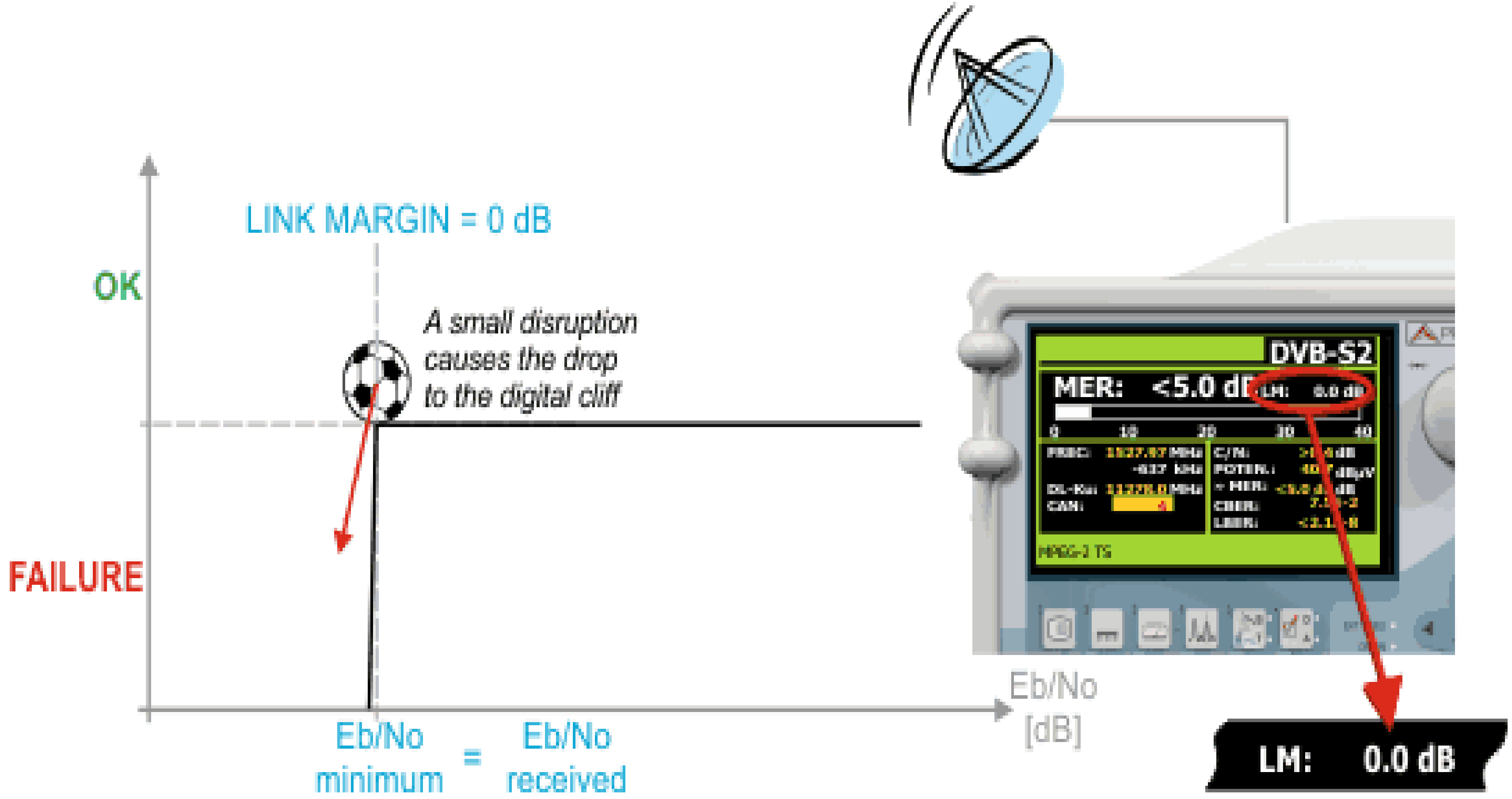
- It can be included in the link budget to account for the reduction in received power.

- Link budget relates to the **polarization matching :** both antennas to be polarized in the same **(tx&Rx)**
- If a transmit antenna is **vertically polarized**,

  for example,

- ❖ **Maximum power** will only be delivered to a **vertically polarized receiving antenna,**
- ❖ While **zero power** would be delivered to a **horizontally polarized** receive antenna,
- ❖ **Half the available power** would be delivered to a **circularly polarized** antenna.
- ❖ So Determine the ***polarization loss factor***

Link Margin


LINK MARGIN = 0 dB
OK
A small disruption causes the drop to the digital cliff
FAILURE
Eb/No minimum = Eb/No received
Eb/No [dB]
DVB-S2
MER: <5.0 dB
LM: 0.0 dB

Link Margin

- Referred to as ***fade margin***
- The received power level > the threshold level required for the minimum acceptable quality of service (mini. CNR, or mini SNR).
- This design allowance for received power is referred to as the ***link margin***

- It is defined as the **difference** between the design value of received power and the minimum threshold value of receive power.

$$\text{Link margin } (dB) = LM = P_r - P_r^{(\min)} > 0,$$

- where all **quantities are in dB**
- Link margin should be **a positive number(3 to 20 dB)**

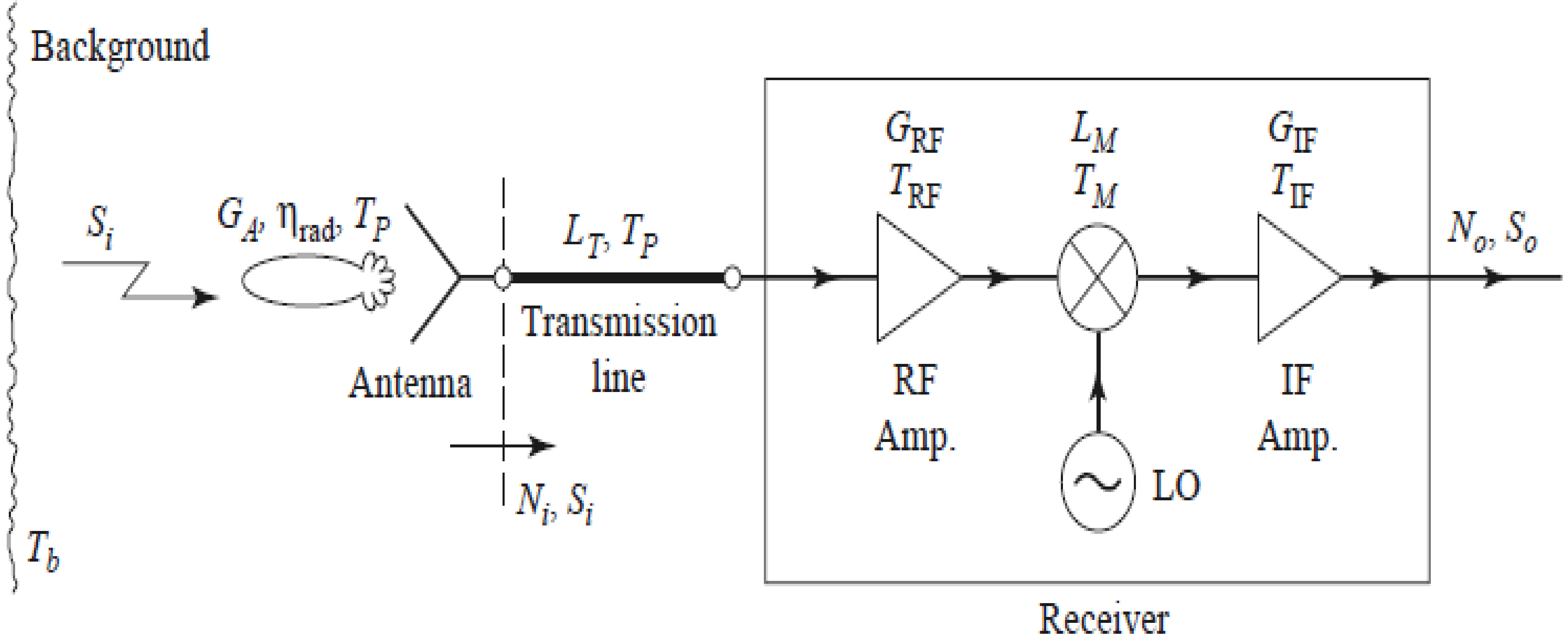
- link margin provides a level of robustness to the system to account **for variables**.
    - Signal fading due to weather,
    - Movement of a mobile user,
    - Multipath propagation problems,
    - Unpredictable effects
    - System performance and quality of service.
- It is used to account for fading effects is sometimes referred to as *fade margin*.

- Satellite links operating at frequencies above 10 GHz, for example, often require fade margins of 20 dB or more to account for attenuation **during heavy rain.**
- For a given communication system
  - ➢ Can be improved by increasing the received power
  - ➢ By reducing the minimum threshold power
  - ➢ Increasing link margin

∴Increase in cost and complexity, so **excessive increases in link margin are usually avoided**.

Noise Characterization of a Microwave Receiver

Noise analysis of a microwave receiver front end, including antenna and transmission line contributions.



- In this system **the total noise power** at the output of the receiver N_0,
  - Due to contributions from the antenna pattern,
  - The loss in the antenna,
  - The loss in the transmission line,
  - The receiver components.
- This noise power will **determine**
  - The **minimum detectable signal level** for the receiver end,
  - The **maximum range of the communication link**.

- The receiver components consist of

❖ RF amplifier with gain G_{RF}

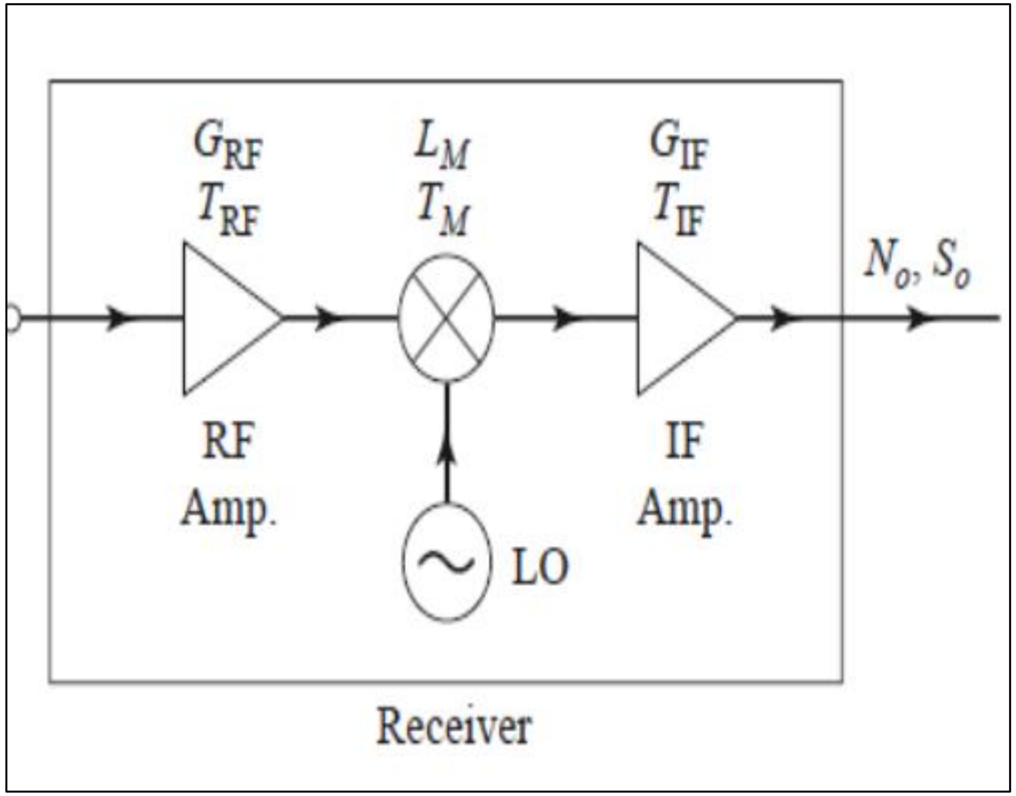
❖ Noise temperature T_{RF}

❖ A mixer with an RF-to-IF conversion loss factor L_M

❖ Noise temperature T_M,

❖ IF amplifier with gain G_{IF}

❖ Noise temperature T_{IF}



- The component ***noise temperatures*** can be related to noise figures as

$$T = (F - 1)T_0. \text{ -----(1)}$$

- The equivalent noise temperature of the receiver can be found as

$$T_{REC} = T_{RF} + \frac{T_M}{G_{RF}} + \frac{T_{IF} L_M}{G_{RF}} \text{ ----- -(2)}$$

- The transmission line connecting the antenna to the receiver has a loss L_T, and is at a physical temperature T_P

$$T_{TL} = (L_T - 1)T_P \text{-------(3)}$$

- **The noise temperature** of the transmission line (TL) and receiver (REC) cascade is

$$T_{TL+\mathrm{REC}} = T_{TL} + L_T T_{\mathrm{REC}}$$

Substitute equation (3) in above

$$T_{TL+\mathrm{REC}} = (L_T - 1)T_P + L_T T_{\mathrm{REC}}$$

- This noise temperature is defined at **the antenna terminals (the input to the transmission line).**

NOISE POWER

- If the antenna has a reasonably **high gain** with relatively **low side lobes**

Assume

- All noise power comes **via the main beam**, so that the noise temperature of the antenna is given

$$T_A = \eta_{rad} T_B + (1 - \eta_{rad})\, T_P$$

where

η_{rad} = Efficiency of the antenna,

T_P = Physical temperature,

T_B = Equivalent brightness temperature of the background(main beam)

- The noise power at the antenna terminals, which is also the noise power delivered to the transmission line, is

$$N_i = KBT_A = KB\,[\eta_{\text{rad}} T_B + (1 - \eta_{\text{rad}})\, T_P]$$

$$N_i == KB\,[\eta_{\text{rad}} T_B + (1 - \eta_{\text{rad}})\, T_P]$$

where B = system bandwidth

- If S_i is the received power at the antenna terminals→the input SNR at the antenna terminals is $\frac{S_i}{N_i}$
- The output signal power is

$$S_O = \frac{S_i G_{RF} G_{IF}}{L_T L_M} = S_i$$

$$S_O = S_i G_{SYS}$$

where G_{SYS} = defined as a system power gain.

The output noise power is

N_O = $(N_i + kBT_{TL++\mathrm{REC}})\ G_{SYS}$

= $KB(T_A + T_{TL++\mathrm{REC}})\ G_{SYS}$

= $KB[\eta_{\mathrm{rad}} T_b + (1 - \eta_{\mathrm{rad}})\ T_P + (L_T - 1)\ T_P + L_T\ T_{REC}]\ G_{SYS}$

$\boldsymbol{N_O = KBT_{SYS} G_{SYS}}$

where $\boldsymbol{T_{SYS}}$ has been defined as the overall system noise temperature

The output SNR is

$$\frac{S_O}{N_O} = \frac{S_i G_{SYS}}{kBT_{SYS} G_{SYS}}$$

$$= \frac{S_i}{kBT_{SYS}}$$

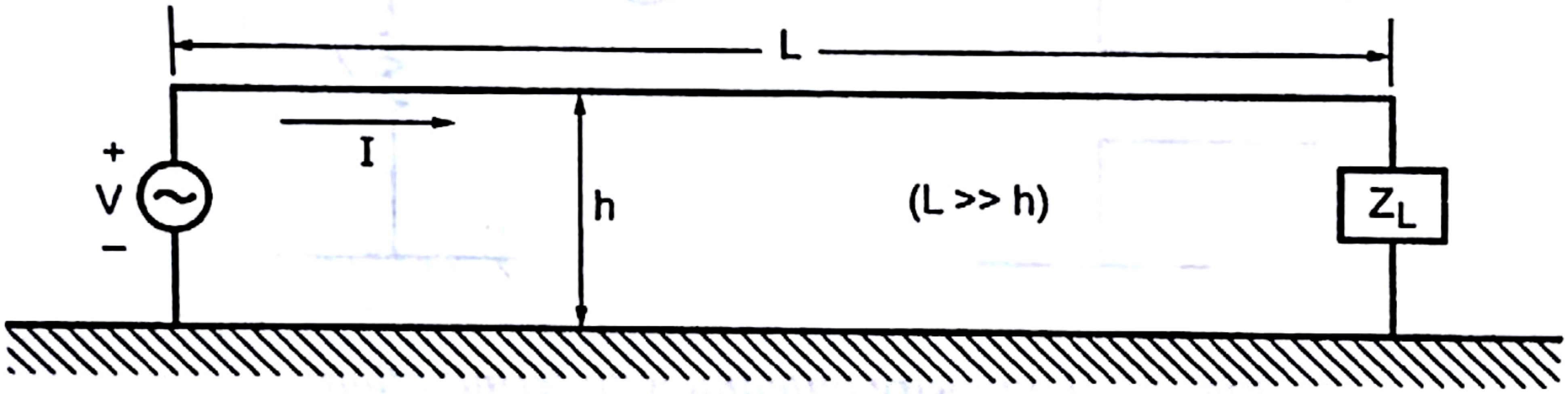
$$\frac{S_O}{N_O} = \frac{S_i}{kB[\eta_{rad} T_b + (1 - \eta_{rad}) T_P + (L_T - 1) T_P + L_T T_{REC}]}$$

UNIT II
RADIATION MECHANISMS AND DESIGN ASPECTS

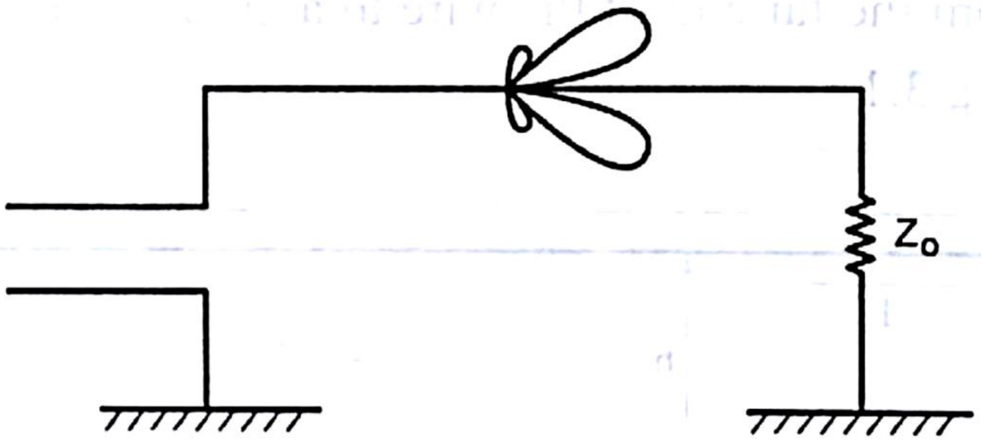
Radiation Mechanisms of Linear Wire and Loop antennas, Aperture antennas, Reflector antennas, Microstrip antennas and Frequency independent antennas, Design considerations and applications

Radiations from Linear Wire Antenna

A single wire antenna is typically a straight copper wire, between one and two wavelength long, running parallel to the earth's surface.

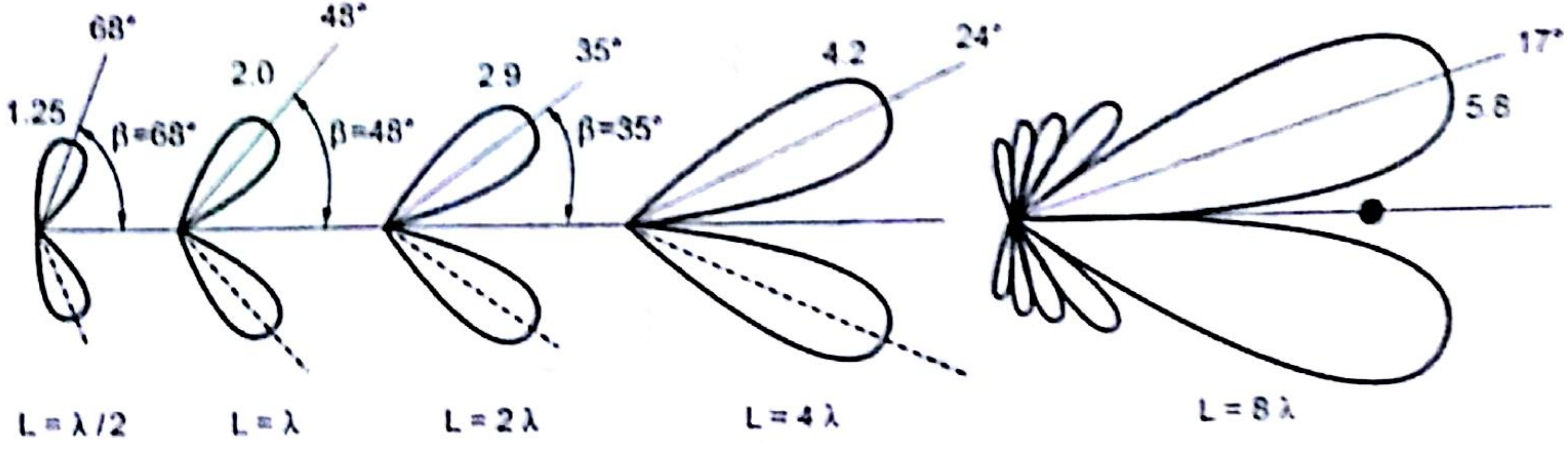


Long Wire Antenna



Long wire antenna with termination

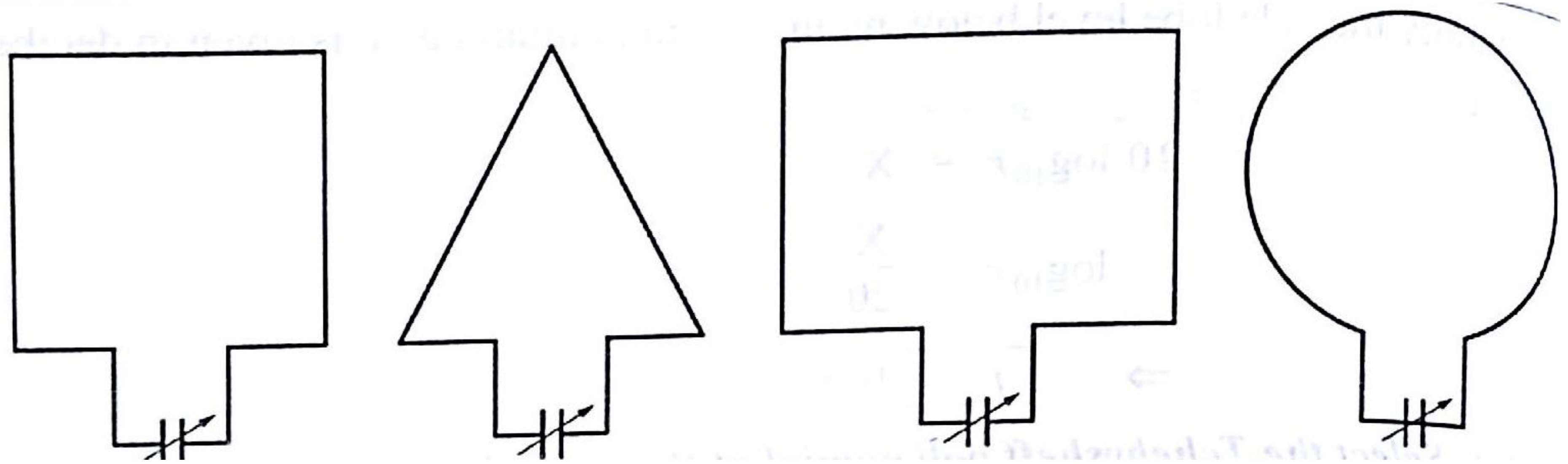
A beverage antenna is called as a travelling wave radiator, when it is terminated with a characteristic impedance. They are called as non resonant type.



Radiation pattern of a long wire antenna

Loop Antenna

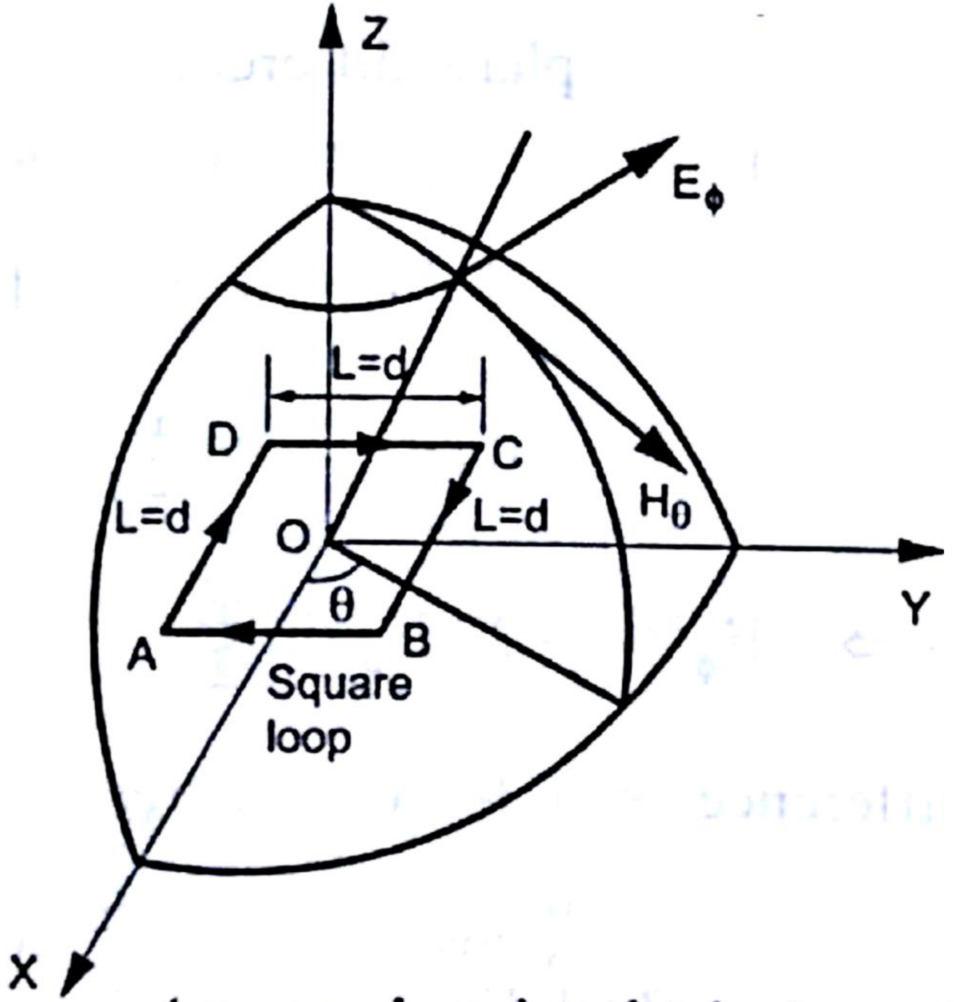
- The loop antenna is a radiating coil of any convenient cross section of one or more turns carrying radio frequency current
- A loop of more than one turn is called as a frame
- Loop is designed that its dimensions are small in comparison to wavelength



Square, Triangle, Rectangular and Circular loops

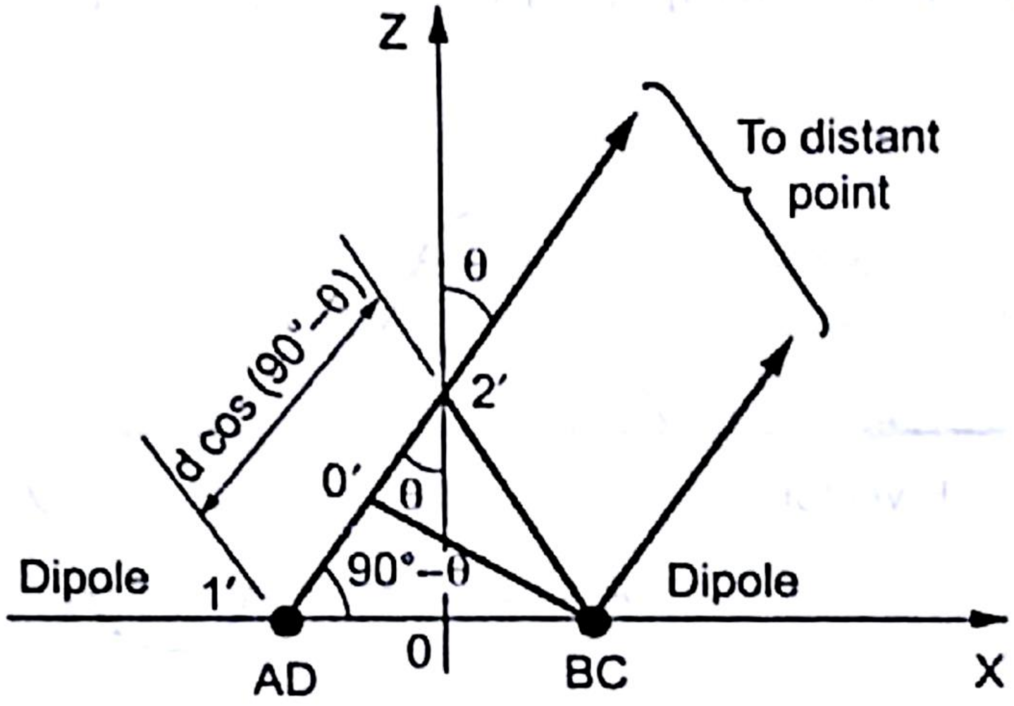
Radiations from small Loop

- The radiation pattern pf the loop is independent of the exact shape of the loop. If the loop is small compared with λ and is similar to the radiation pattern of an elementary dipole



A square loop is spherical coordinate system

In order to find out the far field radiation pattern, consideration of two short dipole AD and BC will do, rather than all the four. Since the sides AD and BC of the loop are being treated as short dipole their radiation pattern will be as shown in Fig.2.42.



A square loop as two short dipoles AD and BC

The individual dipoles AD and BC will behave like two isotropic point sources. Fields due to dipoles AB and CD is negligible and hence neglected. Now the far field radiation pattern due to isotropic sources AD and BC with reference to center point O, we have,

$$E_\phi = \text{Field component due to AD} + \text{Field component due to BC}$$

Field component due to AD, $E_{AD} = -E_o e^{j\psi/2}$

Field component due to BC, $E_{BC} = E_o e^{-j\psi/2}$

Where, E_o = amplitude of electric field

ψ = phase difference

$$\therefore \quad E_\phi = -E_o e^{j\psi/2} + E_o e^{-j\psi/2}$$

$$= -E_o [e^{j\psi/2} - e^{-j\psi/2}]$$

$$= -E_o \cdot 2j \cdot \sin\frac{\psi}{2} \qquad \because \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$E_\phi = -2j\, E_o \sin\frac{\psi}{2}$$

But path difference $= 1'0 + 02'$ metres

$$= \frac{d}{2}\cos(90-\theta) + \frac{d}{2}\cos(90-\theta)$$

$$= d\sin\theta$$

$\therefore$ Phase difference $\psi = 2\pi \cdot$ path difference

$$\psi = 2\pi \cdot d\sin\theta \text{ metres}$$

Now ψ in terms of wavelength, $\psi = \dfrac{2\pi}{\lambda} d\sin\theta$ wave length

$$E_{\phi} = -2\, E_o\, j \cdot \sin\left[\frac{\pi d}{\lambda} \sin\theta\right]$$

The term j indicates that total field E_{ϕ} is in phase quadrature with the individual dipole field E_o. But for a short dipole,

$$E_o = \frac{I_m L\, e^{j\omega(t - \frac{r}{c})}}{4\pi\varepsilon_0} \cdot \frac{j\omega}{C^2 r}$$

Substituting $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = f\lambda$ and $\omega = 2\pi f$, we get,

$$E_o = \frac{I_m L^{j\omega(t - \frac{r}{c})}}{4\pi\varepsilon_0} \cdot \frac{j\, 2\pi f}{\frac{1}{\sqrt{\mu_0\varepsilon_0}} \cdot f\lambda \cdot r}$$

$$= j \frac{I_m L e^{j\omega(t - \frac{r}{c})} \sqrt{\mu_0 \varepsilon_0}}{2 \lambda r \sqrt{\varepsilon_0} \sqrt{\varepsilon_0}} \qquad \because \varepsilon_0 = \sqrt{\varepsilon_0} \sqrt{\varepsilon_0}$$

$$= j \frac{I_m L e^{j\omega(t - \frac{r}{c})}}{2 \lambda r} \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= j \frac{I_m L e^{j\omega(t - \frac{r}{c})}}{2 \lambda r} \times 120 \pi \qquad \because \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120 \pi$$

$$\therefore E_0 = j \frac{60\pi}{r} I_m e^{j\omega(t - \frac{r}{c})} \cdot \frac{L}{\lambda}$$

Substituting E_o in equation

$$E_\phi = -2 \cdot j \frac{60\pi}{r} I_m e^{j\omega(t - \frac{r}{c})} \cdot \frac{L}{\lambda} \cdot j \sin\left(\frac{\pi d}{\lambda} \sin\theta\right)$$

$$= \frac{120\pi}{r} I_m e^{j\omega(t - \frac{r}{c})} \cdot \frac{L}{\lambda} \cdot \frac{\pi d}{\lambda} \sin\theta$$

since for small angles $\sin\theta = \theta$.

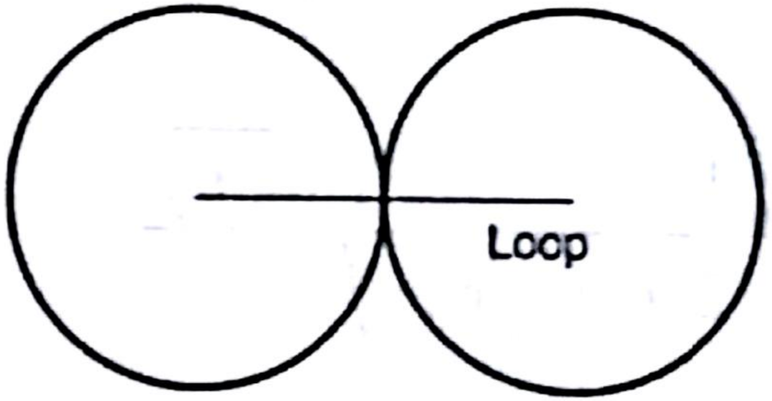
$$\boxed{E_\phi = \frac{120\pi^2}{r\lambda^2} I_m e^{j\omega(t - \frac{r}{c})} \cdot \sin\theta \cdot A}$$

We know that, $\dfrac{E_\phi}{H_\theta} = \eta_0 = 120\pi$

$$\Rightarrow H_\theta = \frac{E_\phi}{120\pi}$$

$$H_\theta = \frac{120\pi^2 I_m e^{j\omega(t - \frac{r}{c})} \sin\theta \cdot A}{r\lambda^2 \cdot 120\pi}$$

$$\boxed{H_\theta = \frac{\pi I_m e^{j\omega(t - \frac{r}{c})} \sin\theta \cdot A}{r\lambda^2}}$$



Radiation pattern of loop antenna

Aperture Antennas

The term aperture refers to an opening in a closed surface.

The aperture antennas are most common at microwave frequency band. It must have an aperture length and width of atleast several wavelengths in order to have a high gain.

Typical antennas that fall in this category are the slot, horn, reflector, and lens antennas.

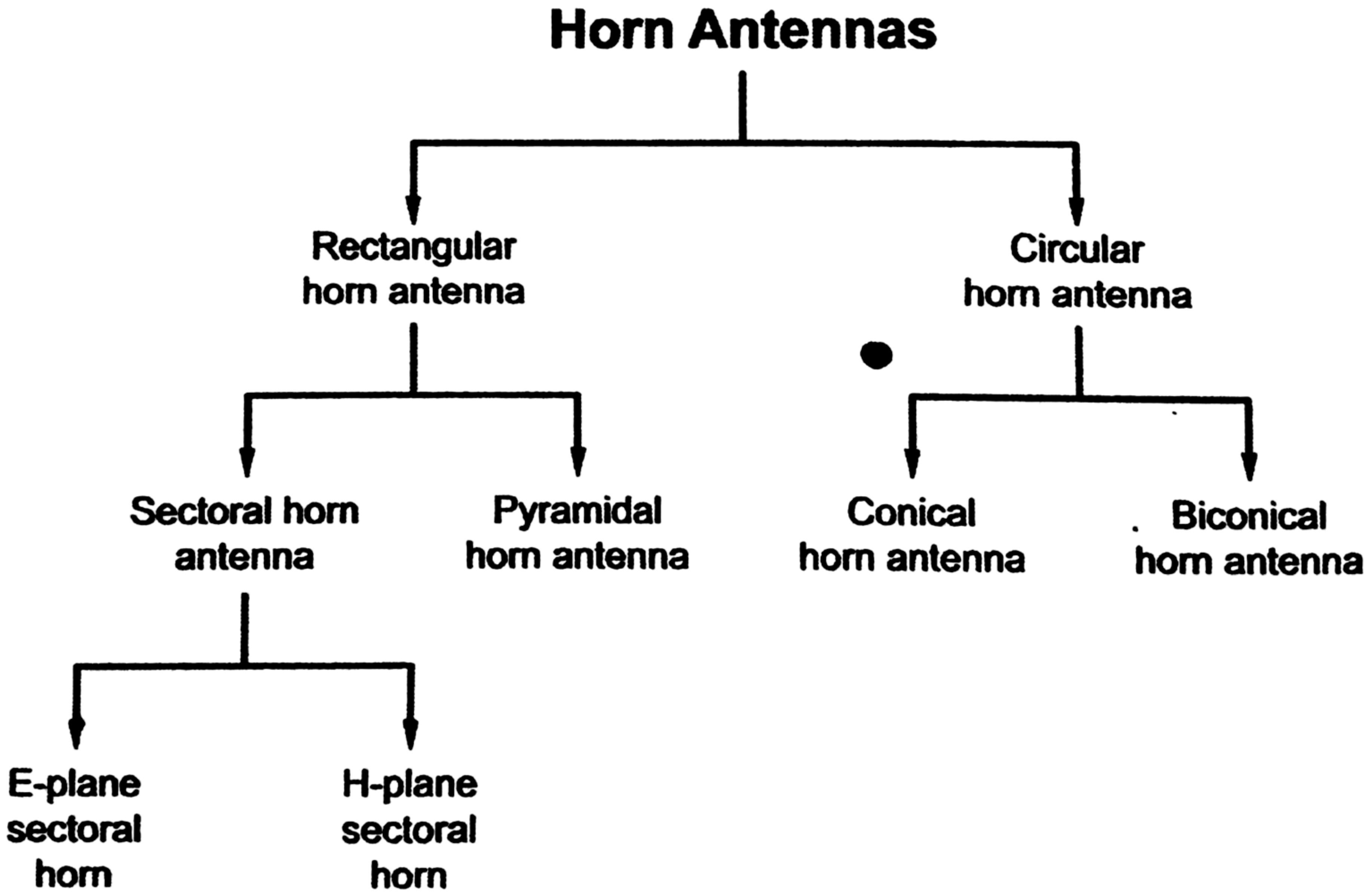
HORN ANTENNA

One of the simplest and probably the most widely used microwave antenna is the horn and may be considered as an aperture antenna.

A horn antenna may be regarded as a ***flared out*** or ***opened out waveguide***. When one end of the waveguide is excited and the other end is kept open, it radiates in open space in all directions.

Types:

1. Rectangular horn antenna
2. Circular horn antenna

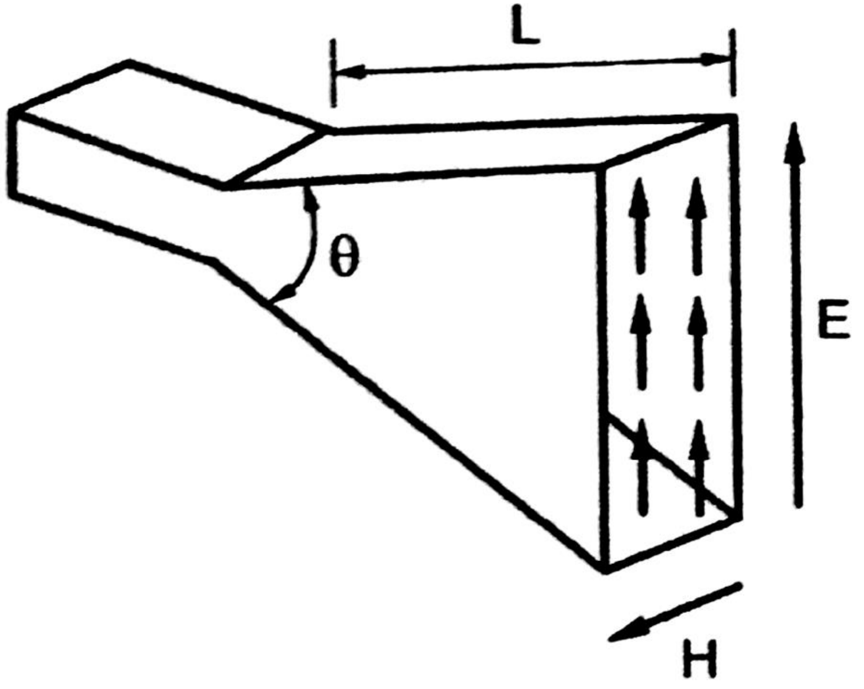

Horn Antennas
Rectangular horn antenna
Circular horn antenna
Sectoral horn antenna
Pyramidal horn antenna
Conical horn antenna
Biconical horn antenna
E-plane sectoral horn
H-plane sectoral horn

1. Rectangular horn antenna

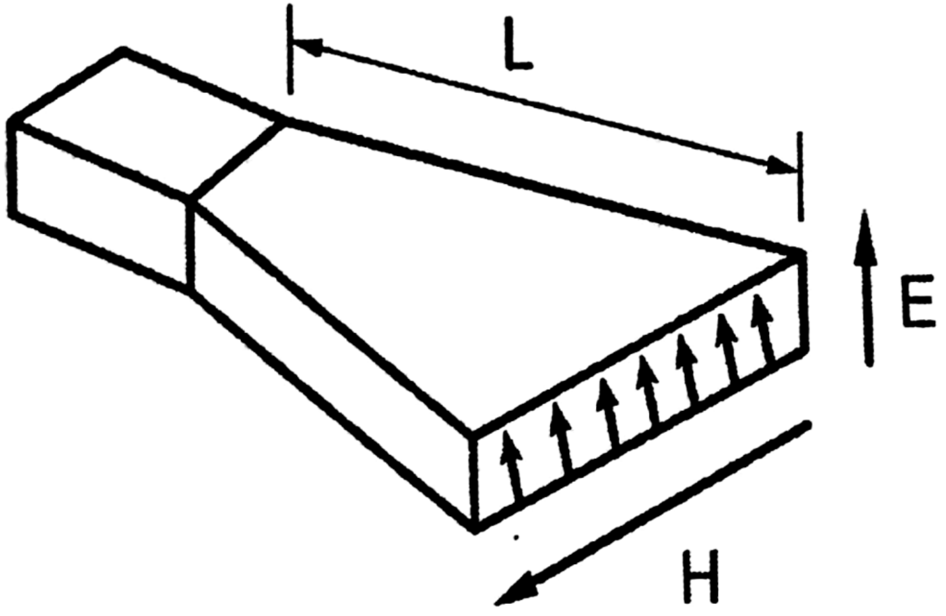
    1. Sectoral horn antenna

    1. E – plane sectoral horn – Flaring is done in the direction of the electric field vector

    2. H – plane sectoral horn - Flaring is done in the direction of the magnetic field vector

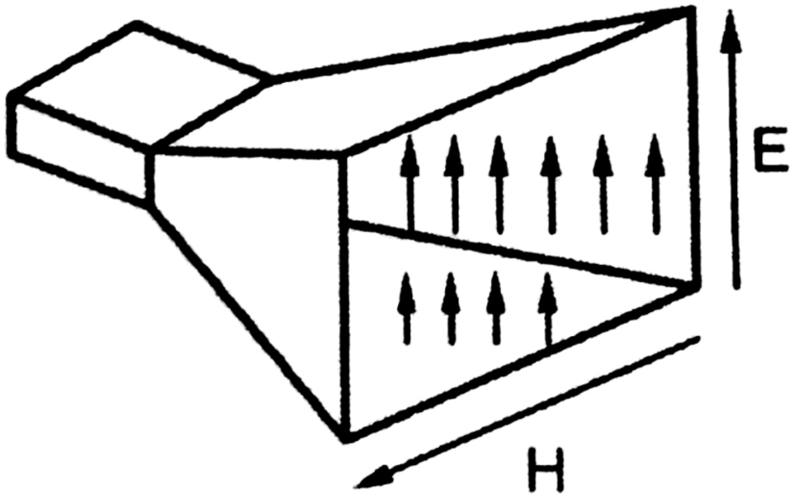


E-plane sectoral horn



H-plane sectoral horn

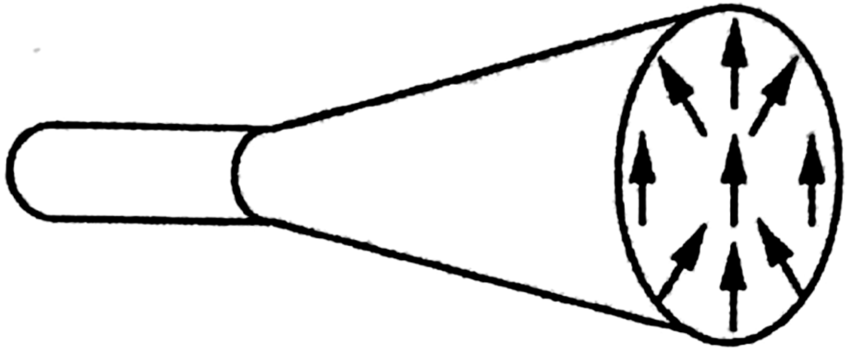
Pyramidal horn antenna - Flaring is done in the direction of both the electric field and magnetic field



Pyramidal horn antenna

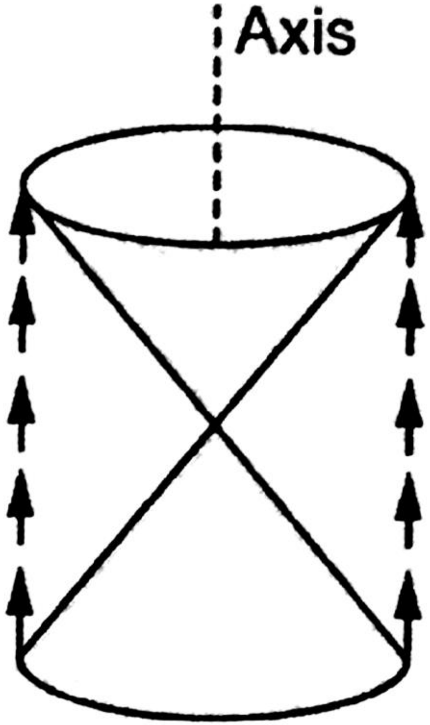
Circular horn antenna

1. Conical horn antenna



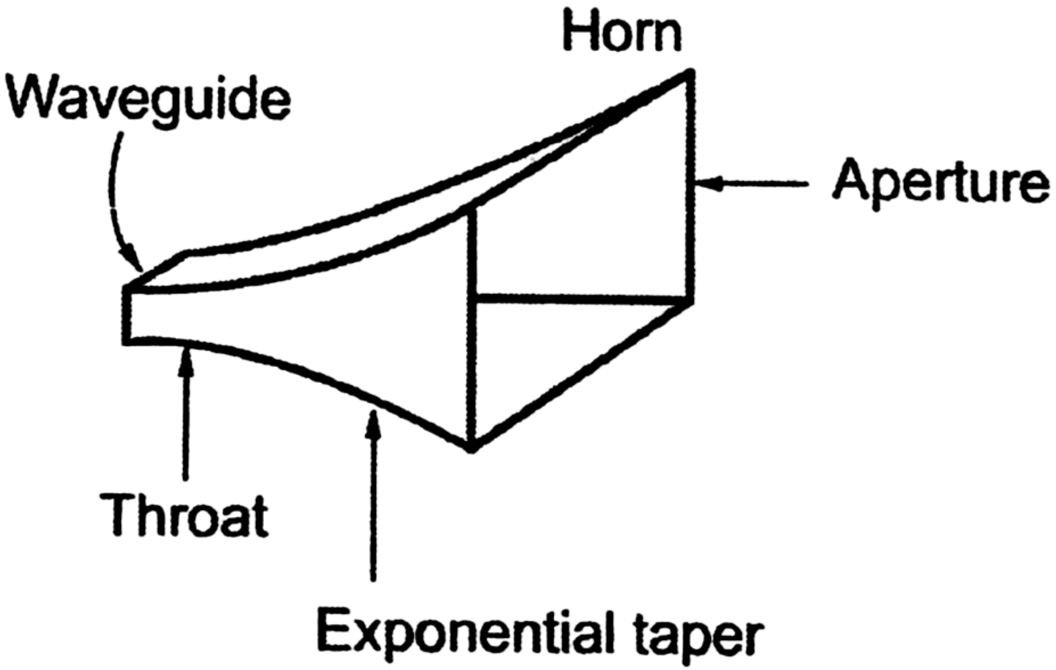
Conical horn

2. Biconical horn antenna

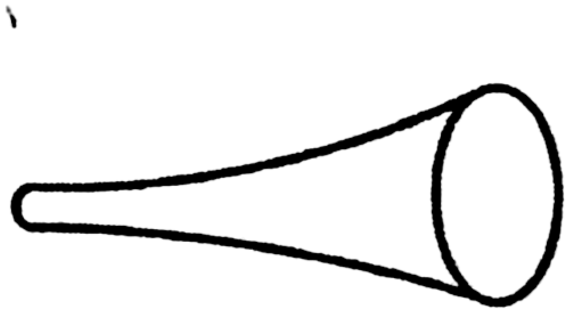


Biconical horn

Exponentially Tapered Horn Antenna



Exponentially tapered pyramidal



Exponentially tapered conical

Principle of Horn Antenna

Huygene's principle says that, each point on a primary wave front can be considered to be a new source of a secondary spherical wave and the secondary wave front can be constructed as the envelope of these secondary spherical waves.

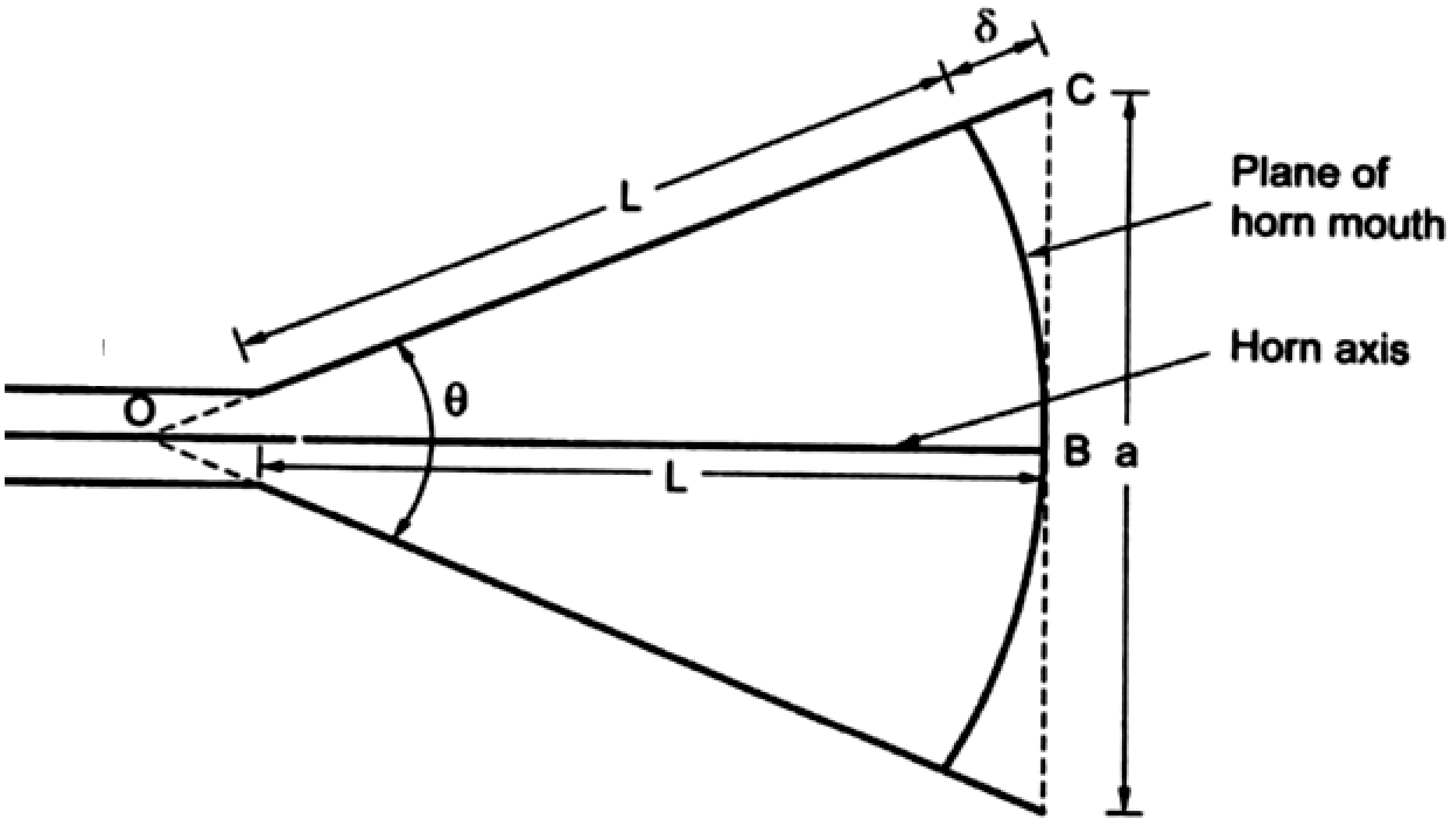
Design of Horn Antenna

Consider a pyramidal horn of length 'L' and aperture height '*a*' with flaring along 'θ' as shown in Fig

From the geometry ΔOBC

$$\cos\frac{\theta}{2} = \frac{OB}{OC} = \frac{L}{L+\delta} \qquad \ldots\ldots(1)$$

$$\tan\frac{\theta}{2} = \frac{BC}{OB} = \frac{a/2}{L} = \frac{a}{2L} \qquad \ldots\ldots(2)$$


δ
C
L
Plane of
horn mouth
Horn axis
O
θ
B
a
L

where, θ – Flare angle (θ_E *for E plane*, θ_H *for H plane*) in degree

a – Aperture (a_E *for E plane*, a_H *for H plane*) in m,

L – Length of horn in m, and

δ – Path length difference in m.

The flare angle θ can be expressed from equations (1) and (2) as,

$$\boxed{\theta = 2 \tan^{-1} \frac{a}{2L} = 2 \cos^{-1}\left(\frac{L}{L+\delta}\right)} \quad \ldots\ldots (3)$$

In the E plane of the horn, δ is usually $0.25\,\lambda$ *or* less and in the H plane, it can be larger or about $0.4\,\lambda$

From triangle OBC,

$$(L+\delta)^2 = L^2 + \left(\frac{a}{2}\right)^2$$

$$L^2 + \delta^2 + 2 L \delta = L^2 + \frac{a^2}{4}$$

If 'δ' is small, then δ^2 can be neglected.

$$\therefore \quad 2 L \delta = \frac{a^2}{4}$$

$$\boxed{L = \frac{a^2}{8\delta}} \quad \ldots\ldots (4)$$

Equations (3) and (4) are the ***design equations of the horn antenna***.

For an optimum flare horn, the ***half power beam width*** can be approximated as,

$$\theta_H = \frac{67^\circ \lambda}{a_H} = \frac{67 \lambda}{w} \deg ree \quad \ldots\ldots(5)$$

$$\theta_E = \frac{56° \lambda}{a_E} = \frac{56 \lambda}{a} \text{ degree} \qquad \ldots\ldots(6)$$

Assume that there is no loss, the ***directivity*** is given in terms of the effective aperture of the horn as,

$$D = \frac{4 \pi A_e}{\lambda^2} = \frac{4\pi \varepsilon_{ap} A_p}{\lambda^2} \qquad \ldots\ldots(7)$$

where, A_e = Effective aperture in m^2

A_p = Physical aperture in m^2 = Area of horn mouth opening, and

$$\varepsilon_{ap} = \frac{A_e}{A_p} = \text{Aperture efficiency}$$

For a pyramidal rectangular horn,

$$A_p = a_E \cdot a_H = a \times w \qquad \text{........(8)}$$

where, a = *Height of the aperture* $= a_E =$ E-plane aperture in m

w = *Height of the aperture* $= a_H =$ H-plane aperture in m

Similarly for a conical horn,

$$A_p = \pi r^2 \qquad \text{.........(9)}$$

where, r = Radius of aperture in metre

For example if $a_E = a_H = \lambda = 1$ m and $\varepsilon_{ap} \approx 0.6$, then the directivity of the rectangular horn is given by

$$D = \frac{4\pi(0.6) A_p}{\lambda^2} \approx \frac{7.5 A_p}{\lambda^2} \qquad \text{.........(10)}$$

$$D(dB) \approx 10 \log_{10} \frac{7.5 A_p}{\lambda^2} \qquad \text{......... (11)}$$

Reflector Antenna

Reflector type of antennas or reflectors are widely used to modify the radiation pattern of a radiating element

Reflector antenna means a reflector of suitable size and shape, which may produce a direct radiation(energy) in a desired direction

The antenna which is a radiating source in the reflector antenna is called ***primary antenna or feed***, while the reflector antenna is called the ***secondary antenna***. The most common feeds are dipole, horn and slot.

Types of Reflector antennas

(*i*) Plane reflector *or* flat sheet reflector,

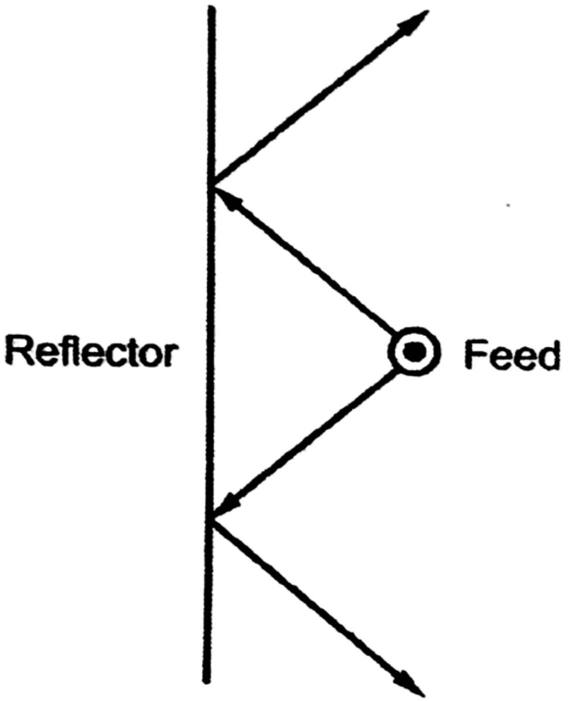
(*ii*) Corner reflector,

(*iii*) Parabolic reflector,

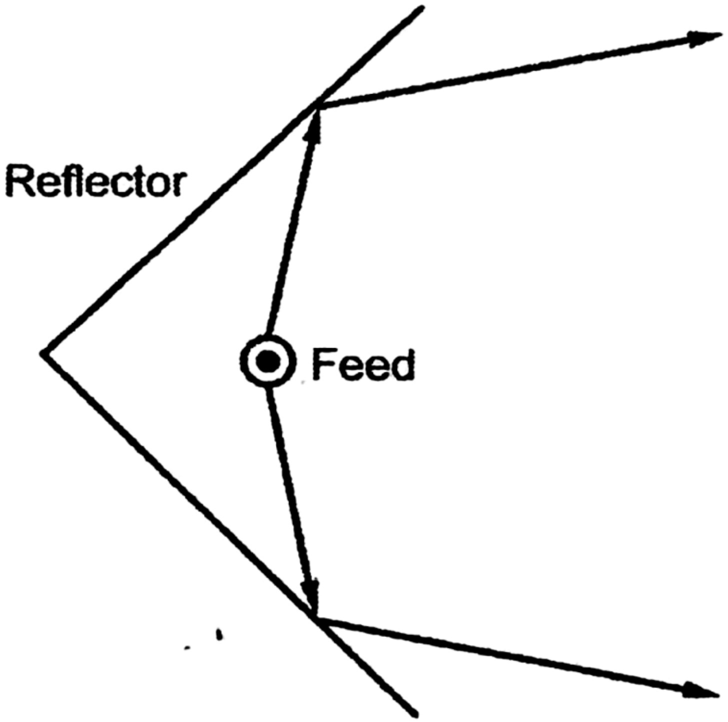
(*iv*) Hyperbolic reflector,

(*v*) Elliptical reflector, and

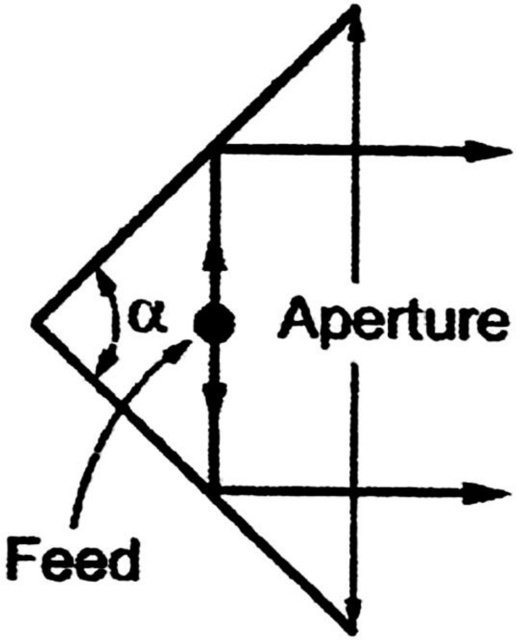
(*vi*) Circular reflector.



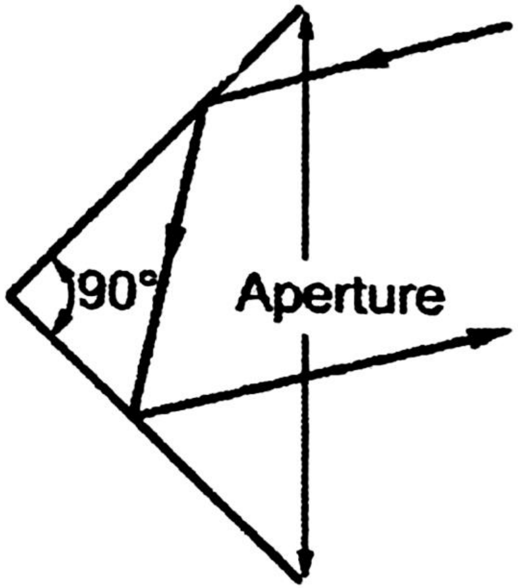
(a) Plane reflector



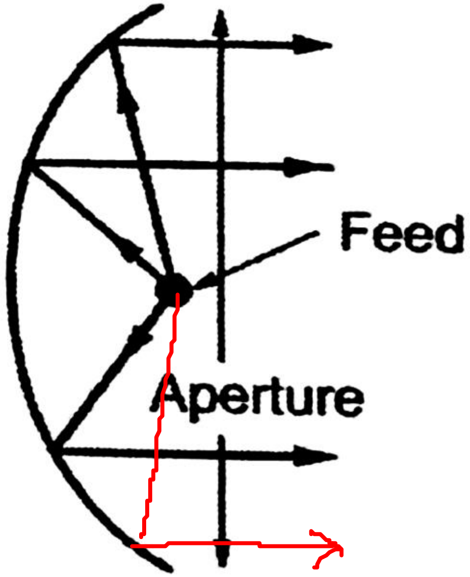
(b) Corner reflector



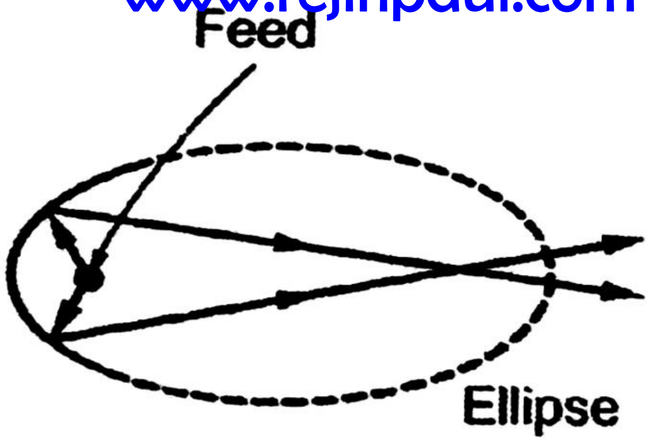
(C). Active corner reflector



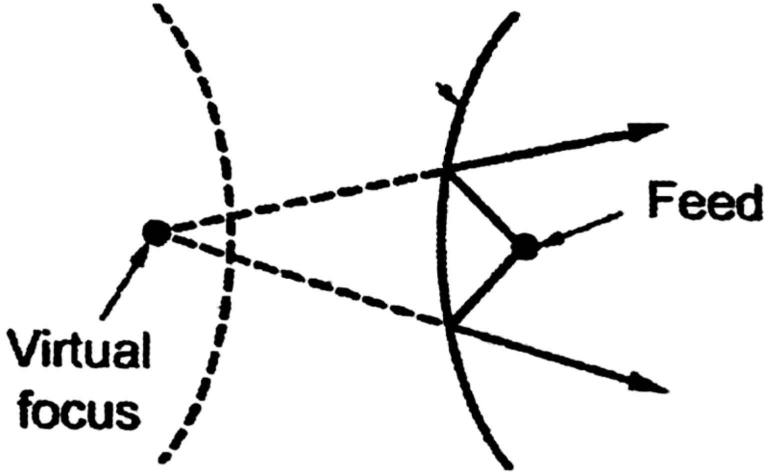
(d). Passive corner reflector



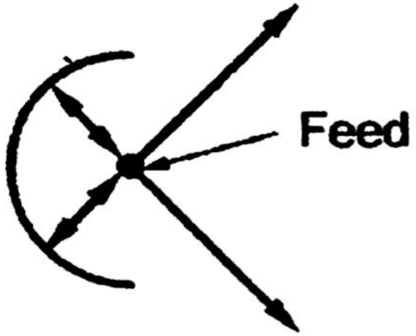
(d). Parabolic reflector



(e).Elliptical reflector



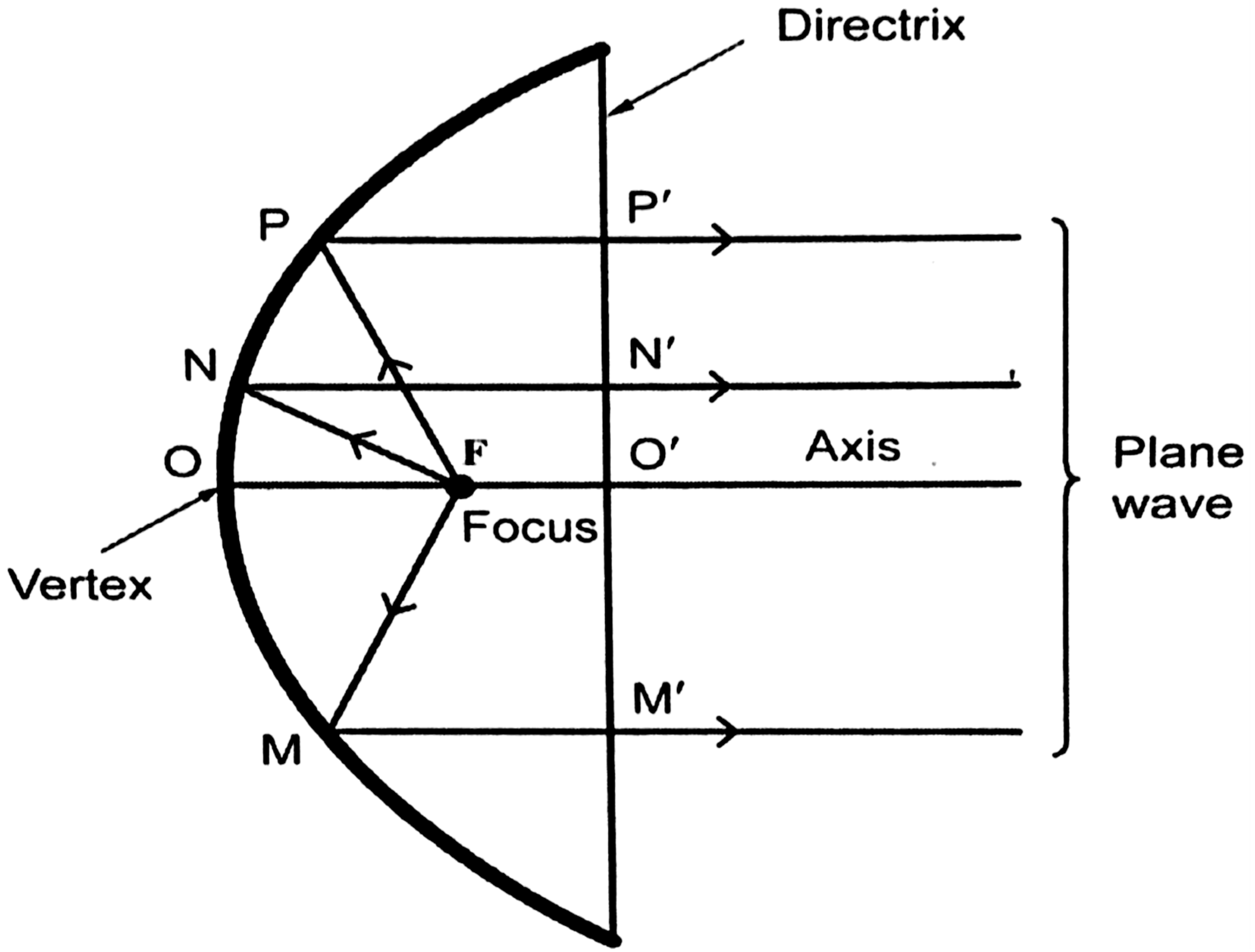
(f) Hyperbolic reflector



(g) Circular reflector

Parabolic Reflector

*The **parabolic structure** is used to improve the overall radiation characteristics such as antenna pattern, antenna efficiency, polarization etc of the reflector antenna.*



$$FN + NN' = FP + PP' = FM + MM'$$

The parabola is a two dimensional plane curve.

Where, OF = Focal length

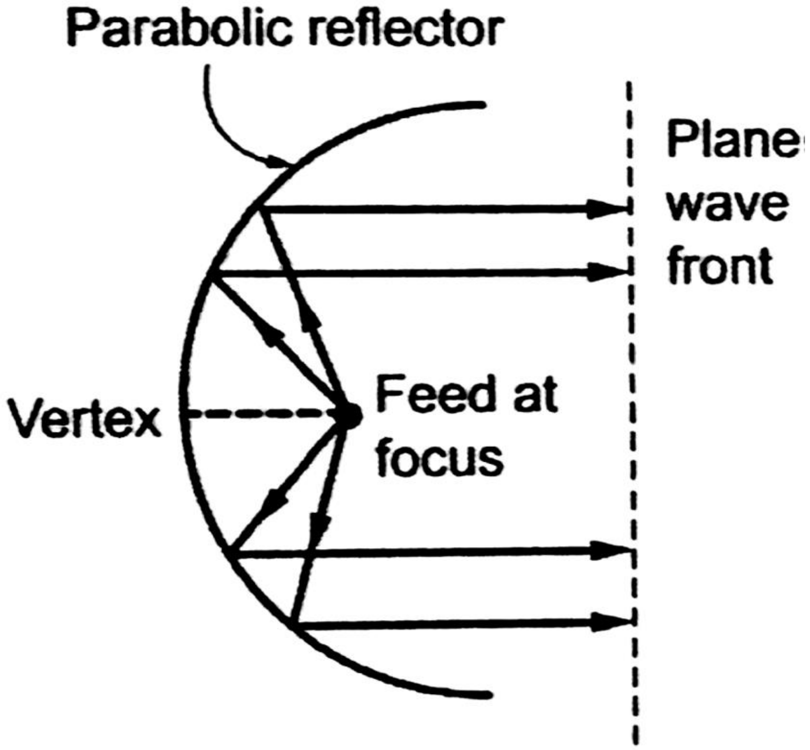
F = Focus

O = Vertex

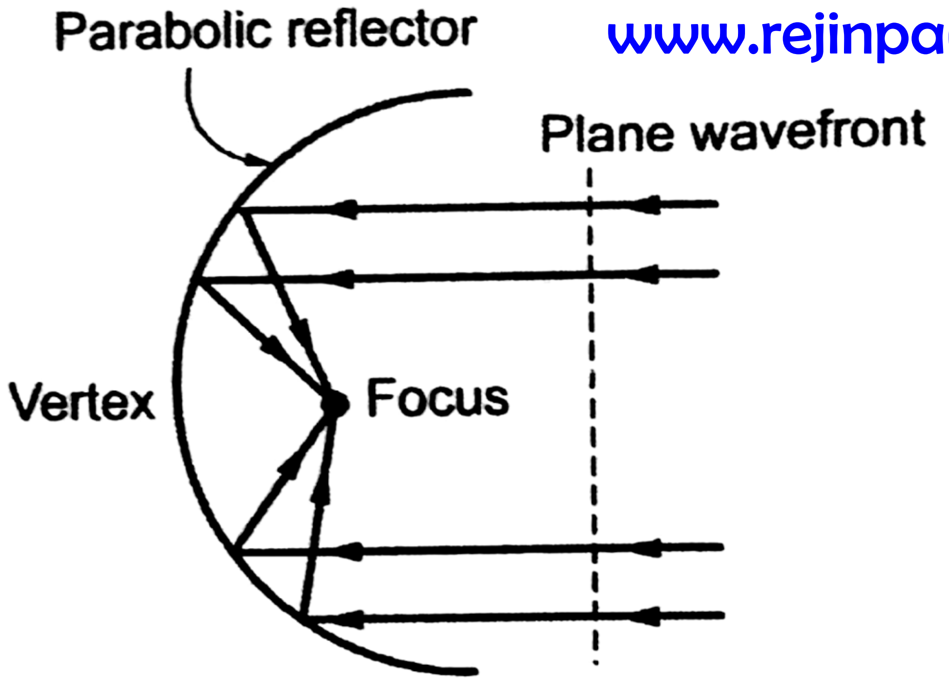
OO' = Axis of parabola

By the geometrical optics, when the point source is placed at the focal point, then the rays reflected by the parabolic reflector form a parallel wave front. This principle is normally used in the ***transmitting antenna****.*

Similarly at the receiving antenna, when the beam of parallel rays is incident on a parabolic reflector, then the radiations focus at a focal point.



(a) Parabolic reflector at transmitting end

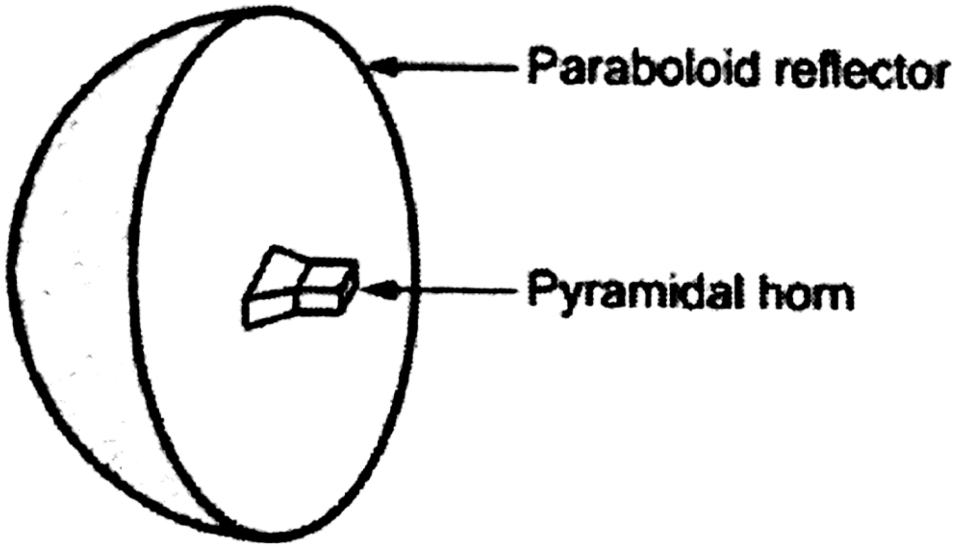


(b) Parabolic reflector at receiving end

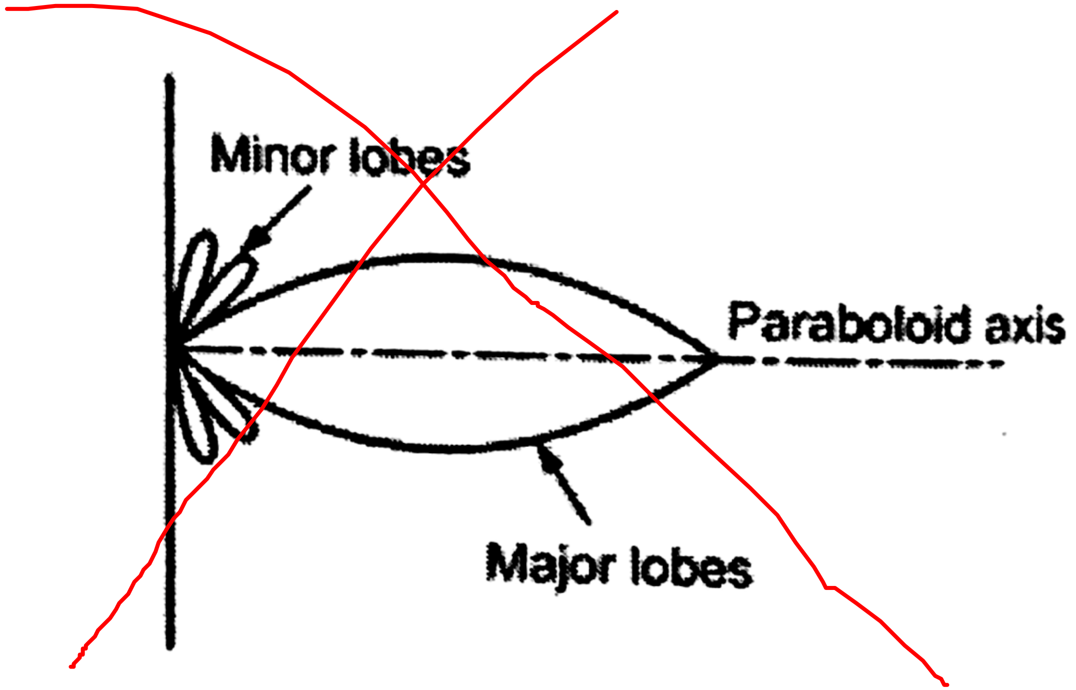
The open mouth (D) of the parabola is known as the aperture. The ratio of focal length to aperture (*i.e.*, f/D) is known as ***"f over D ratio"*** and it is an important characteristics of parabolic reflector (f/D varies from 0.25 to 0.50).

PARABOLOID (OR) PARABOLOIDAL REFLECTOR (OR) MICROWAVE DISH

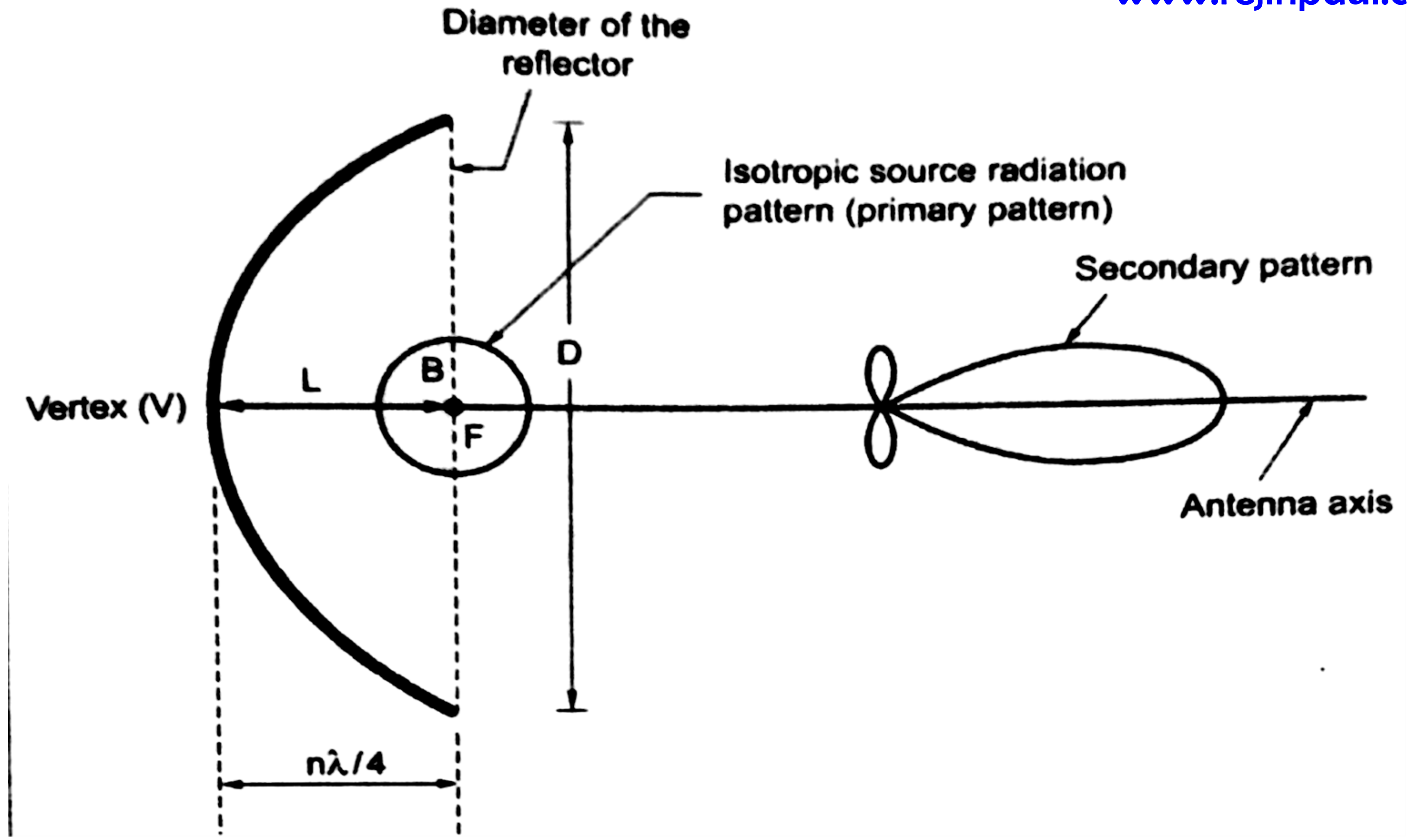
A parabola is a two dimensional plane curve. In practical applications, a three dimensional structure of the parabolic reflector is used.



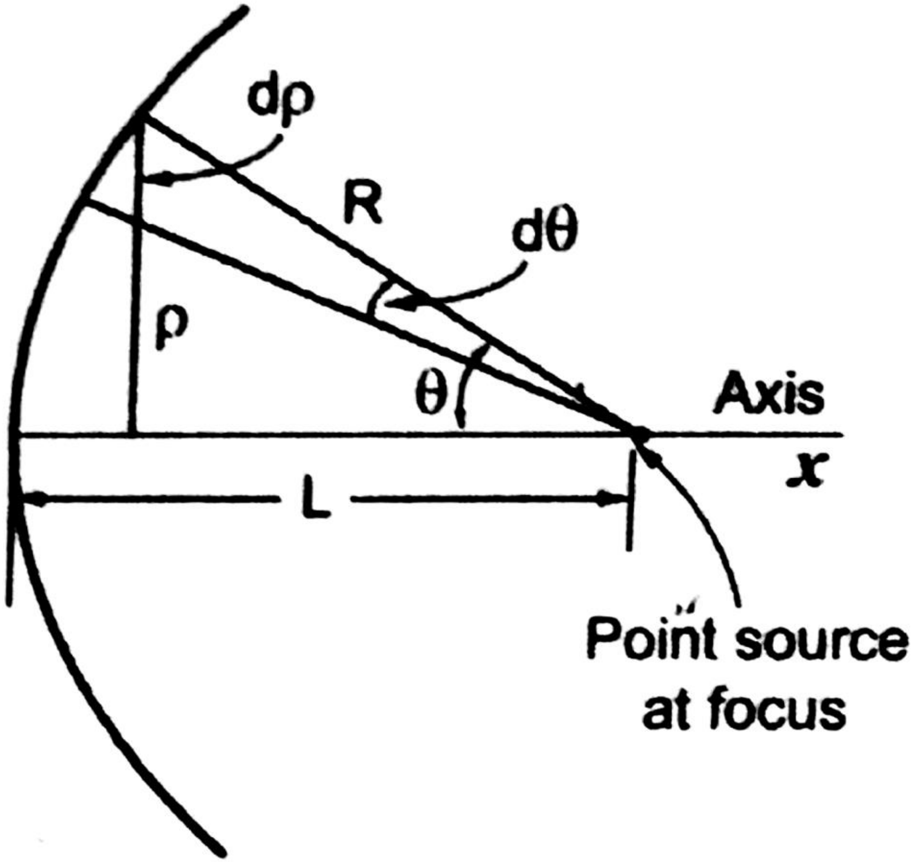
(a) Paraboloid



(b) Radiation pattern


Diameter of the reflector
Isotropic source radiation pattern (primary pattern)
Secondary pattern
Vertex (V)
L
B
F
D
Antenna axis
nλ/4

Field Distribution



Cross sections of paraboloid

Consider a paraboloid with an isotropic source used as a line source as given in Fig.5.21(b). The total power 'P' from distance of 'ρ' from the axis and strip of width '$\partial\rho$' is expressed as,

$$P = 2\pi\,\rho\,d\rho\,S_\rho$$

Where, S_ρ is the power density at a distance ρ from the axis, W/m^2

This power must be equal to the power radiated by the isotropic source over the solid angle $2\pi\sin\theta\,d\theta$.

$$P = 2\pi\sin\theta\,d\theta\,U$$

Where, U is the radiation intensity, W/sr

$$2\pi\rho\,d\rho\,S_\rho = 2\pi\sin\theta\,d\theta\,U$$

$$\frac{S_\rho}{U} = \frac{\sin\theta}{\rho(d\rho/d\theta)}$$

Where, $\rho = R\sin\theta = \dfrac{2L\sin\theta}{1+\cos\theta}$ $\quad \Big|\therefore\ R = \dfrac{2L}{1+\cos\theta}$

$$\boxed{S_\rho = \frac{(1+\cos\theta)^2}{4L^2}U}$$

The ratio of the power density $\dfrac{S_\theta}{S_0} = \dfrac{(1+\cos\theta)^2}{4}$

The field-intensity ratio $\dfrac{E_\theta}{E_0} = \dfrac{1+\cos\theta}{2}$

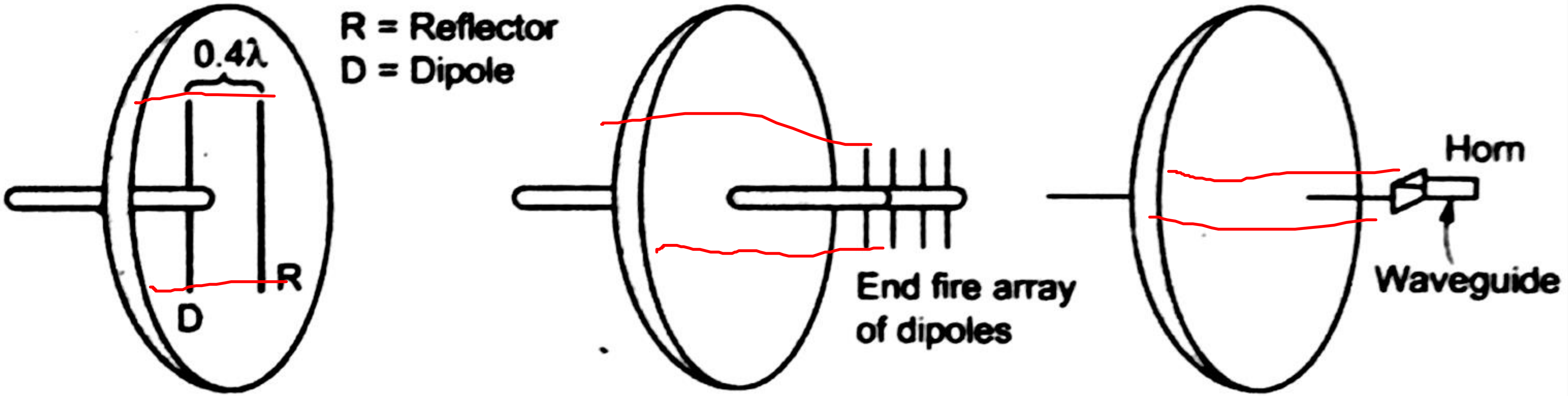
Feeding systems or structures:

Parabolic reflector antenna consists of two basic parts

1. A source of radiation placed at the focus called primary radiator or feed
2. The reflector called secondary radiator

The various feed used in reflectors are

1. Dipole antenna
2. Horn antenna
3. End fire antenna
4. Cassegrain feed
5. Offset feed



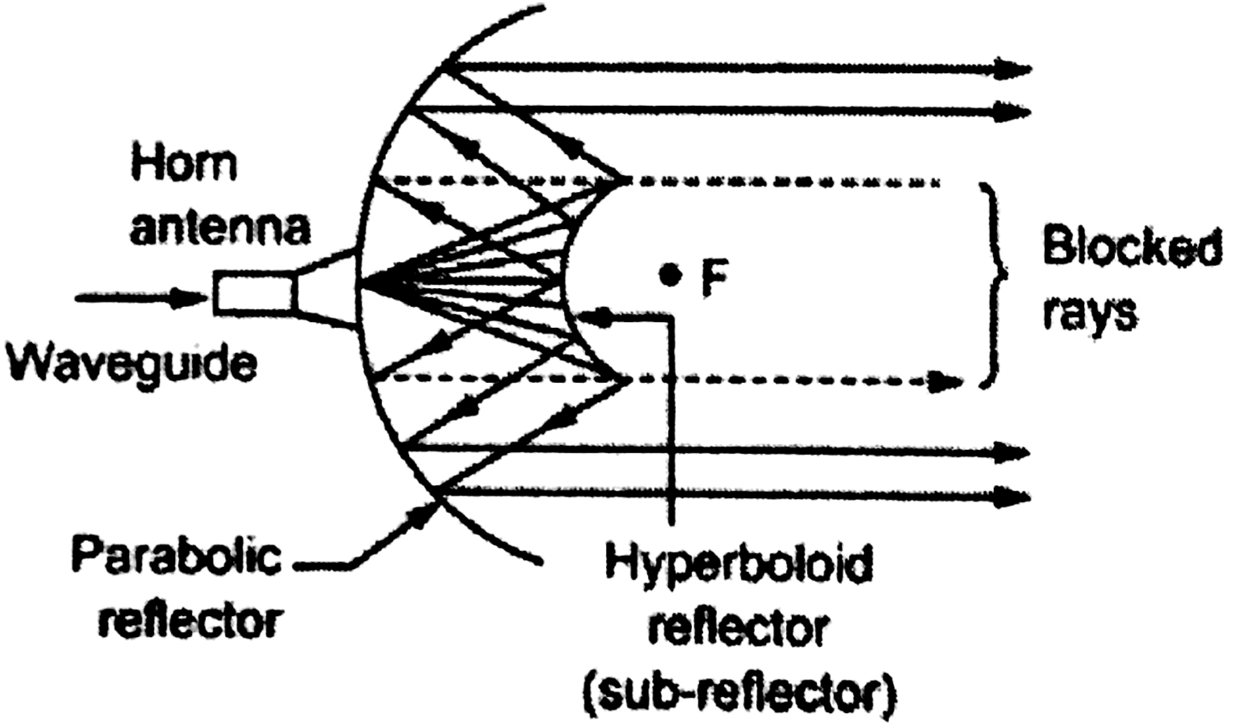
(a) Dipole with plane reflector

(b) End fire array of dipole

(c) Horn with waveguide

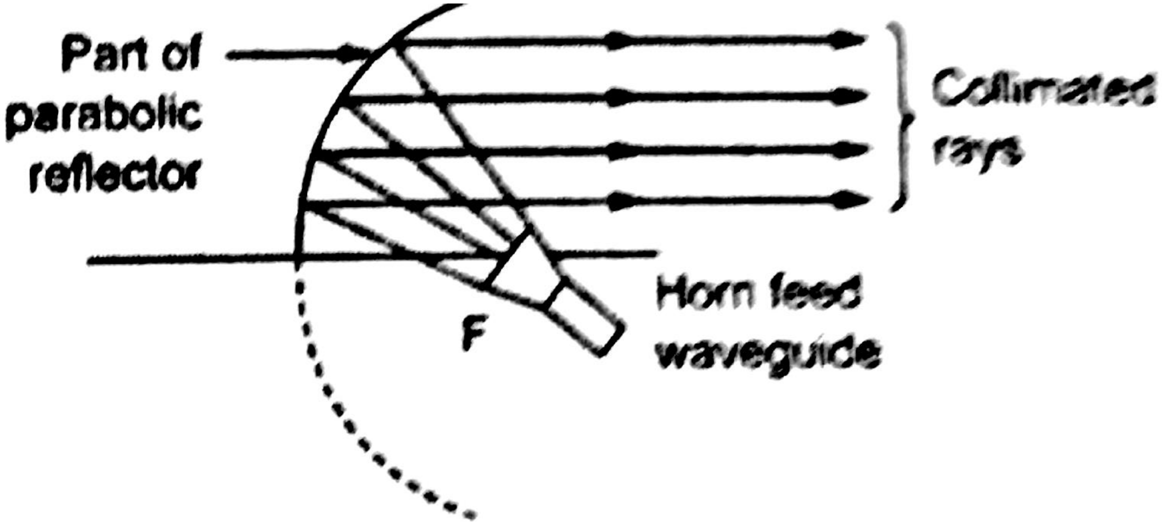
Different types of Feed system

Cassegrain feed



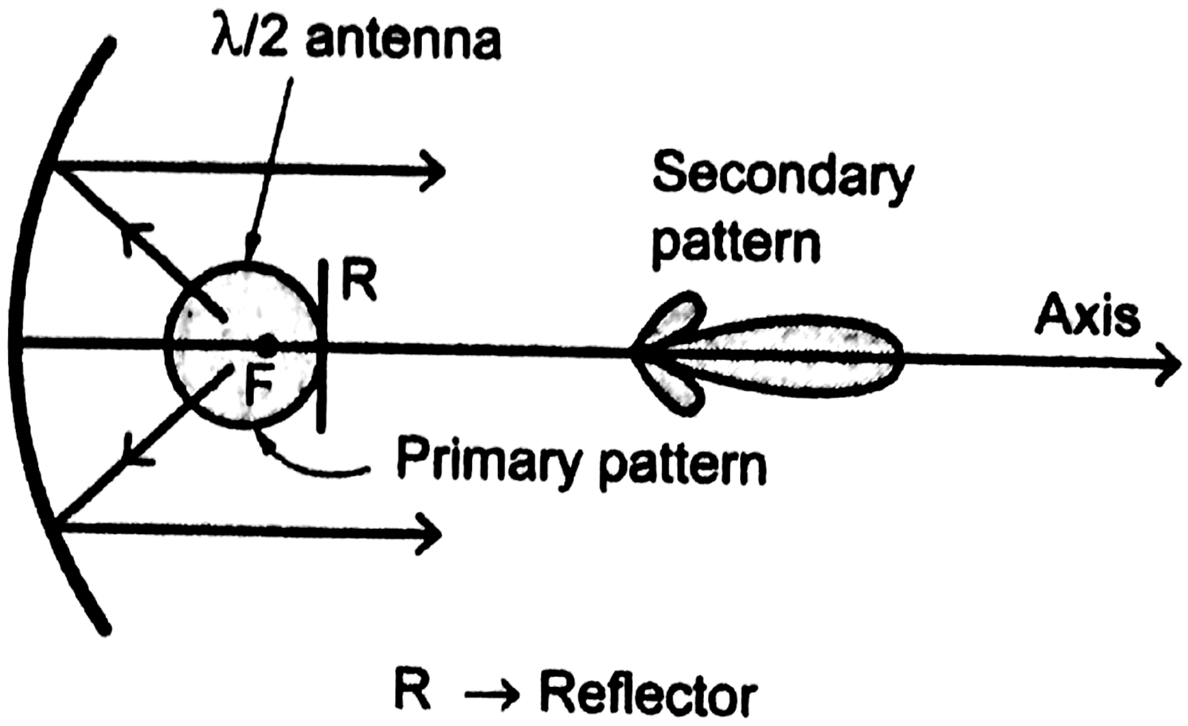
F = Focus of parabolic reflector and hyperboloid

Cassegrain feed system



Offset feed system

Aperture Blockage



Full parabolic reflector using $\frac{\lambda}{2}$ ***antenna***

Slot Antennas

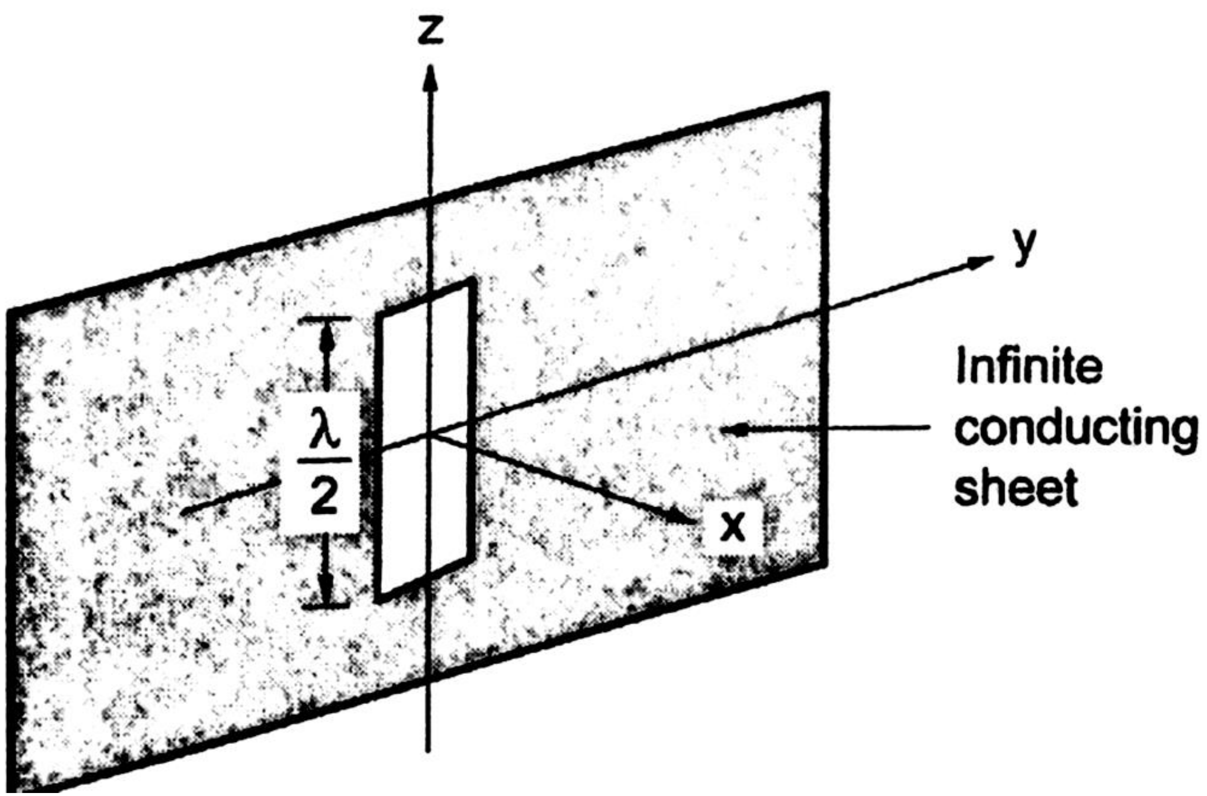
The slot antenna is an opening (slot) cut in a sheet of conductor which is energized through a co-axial cable or wave guide

It is the best suitable radiator at frequencies above 300MHz

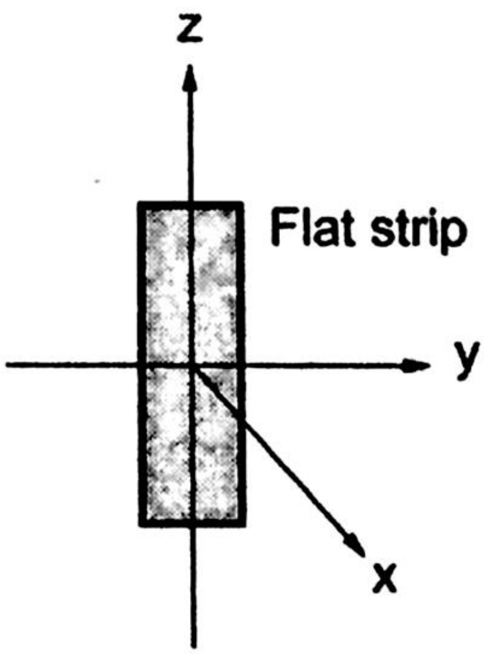
The shape, size and operating frequency of the slot determines the radiation pattern

Whenever a high frequency field exists across a very narrow slot in an infinite conducting sheet, the energy is radiated through that slot

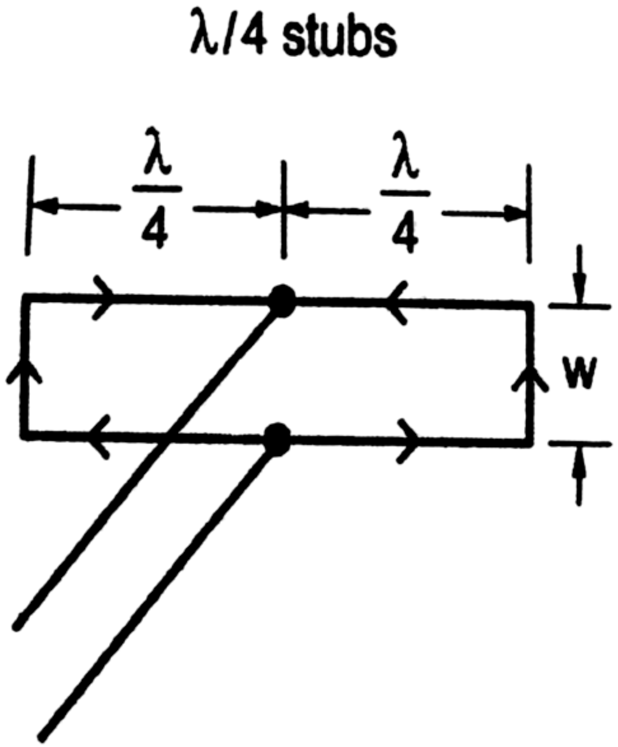
Construction



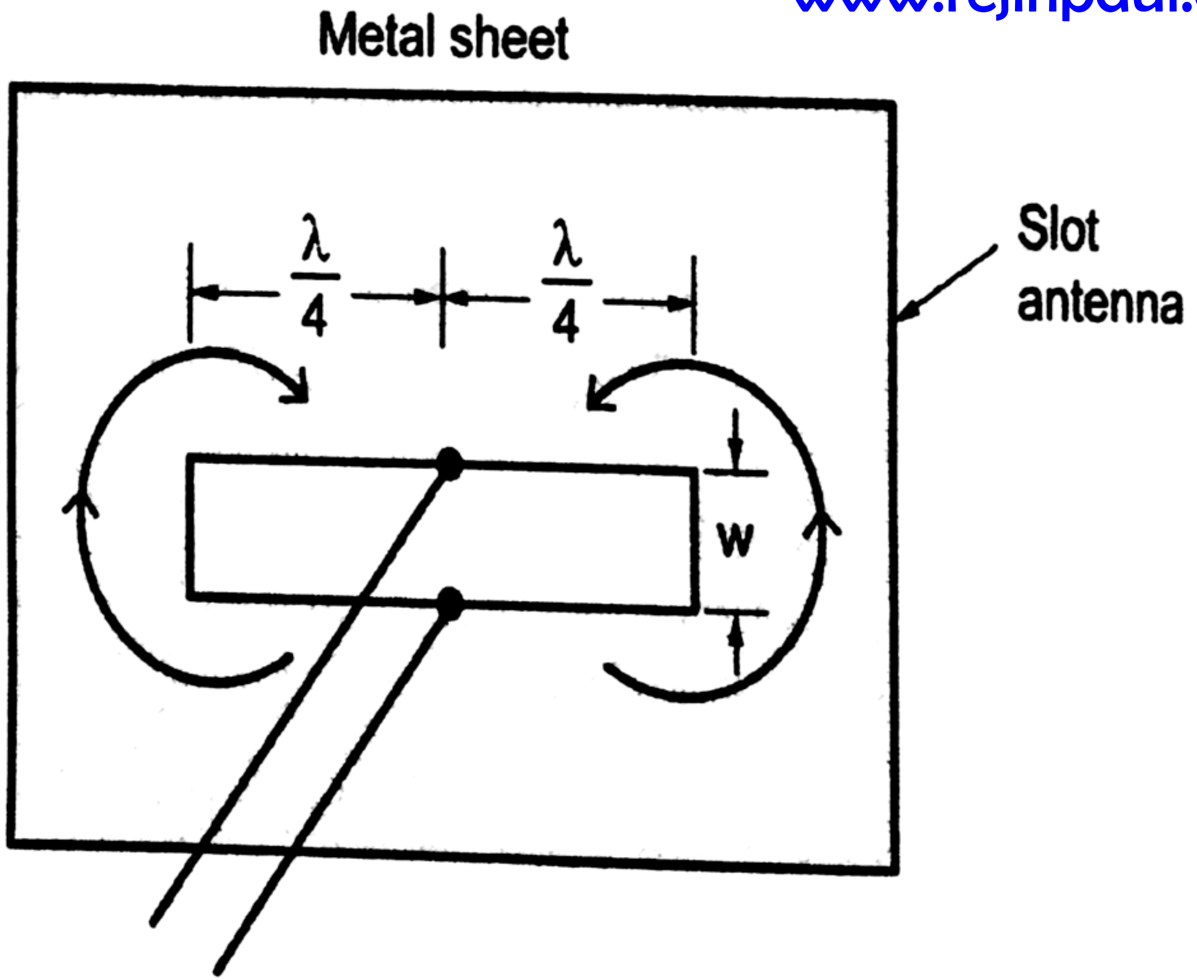
Metallic conducting sheet (slot antenna)



complementary flat strip

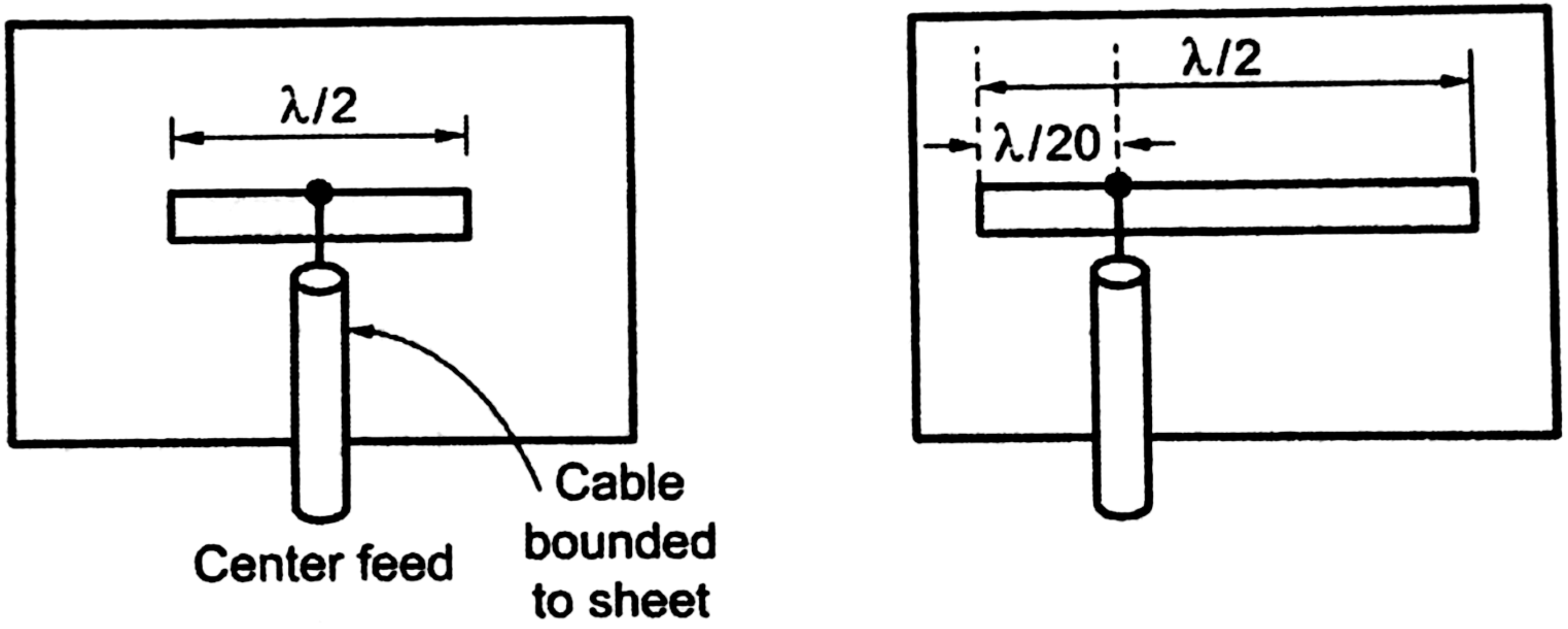


Radiator using stubs



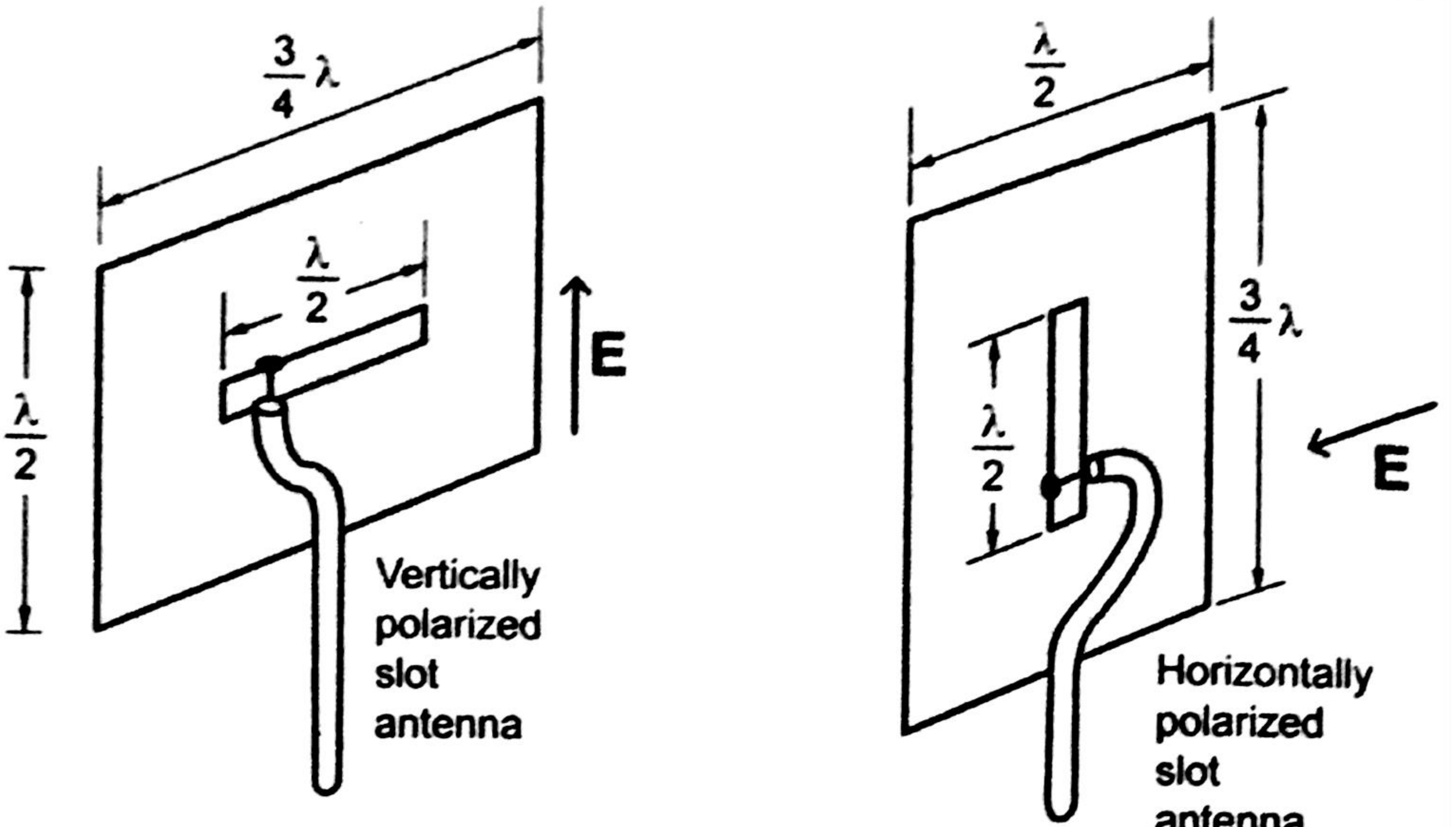
slot antenna

Method of feeding for Slot Antenna

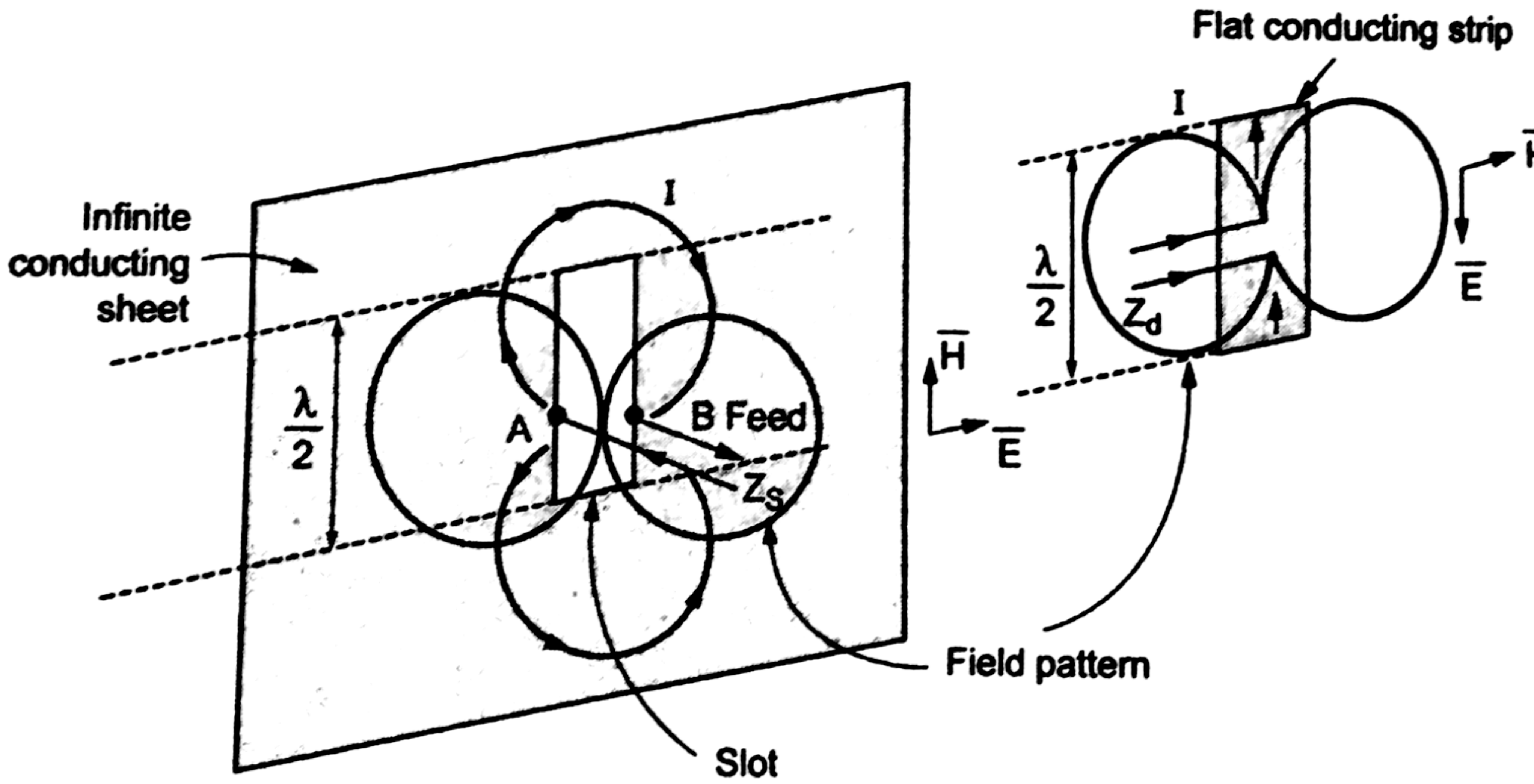


(a) Center feed *(b) Off-center feed*

Types of Slot Antenna



Working Principle: Pattern of the Slot Antenna



Slot and complementary dipole antenna

If, $Z_S \rightarrow$ Terminal impedance of the slot, and

$Z_d \rightarrow$ Terminal impedance of the dipole,

Then, Z_S and Z_d are related to each other in terms of intrinsic impedance of the free space η_0 and it is expressed as,

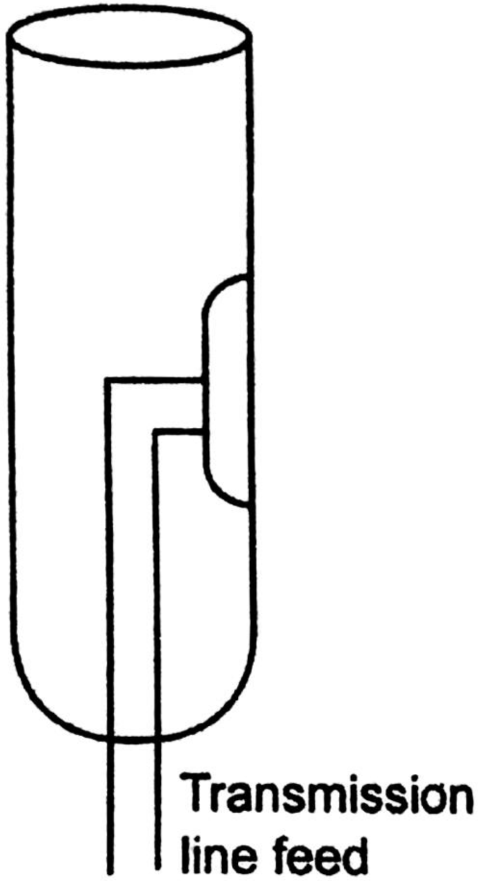
$$Z_d \cdot Z_S = \frac{\eta_0^2}{4} = \frac{(376.7)^2}{4} \approx 35,476 \qquad (\because \eta_0 = 120\,\pi \text{ ohms})$$

Hence the terminal impedance of the slot antenna is given as,

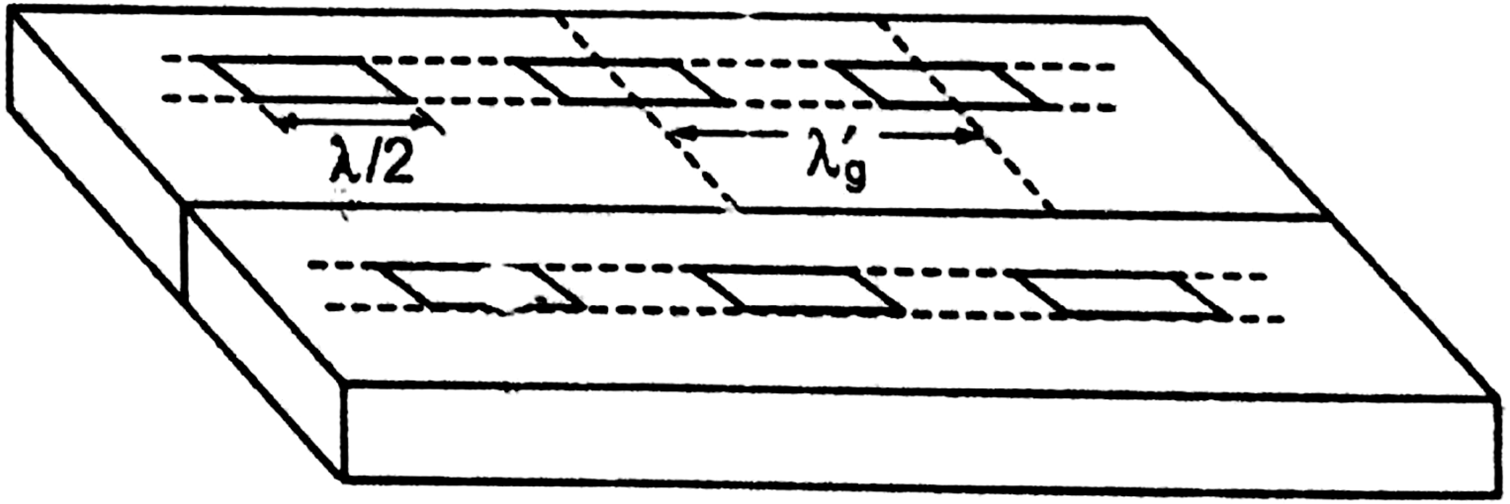
$$\boxed{Z_S = \frac{35,476}{Z_d}} \text{ or } \boxed{Z_S = 35,476\, Y_d} \qquad \ldots\ldots(1)$$

where, $Z_d = 73 + j\,42.5$ ohms

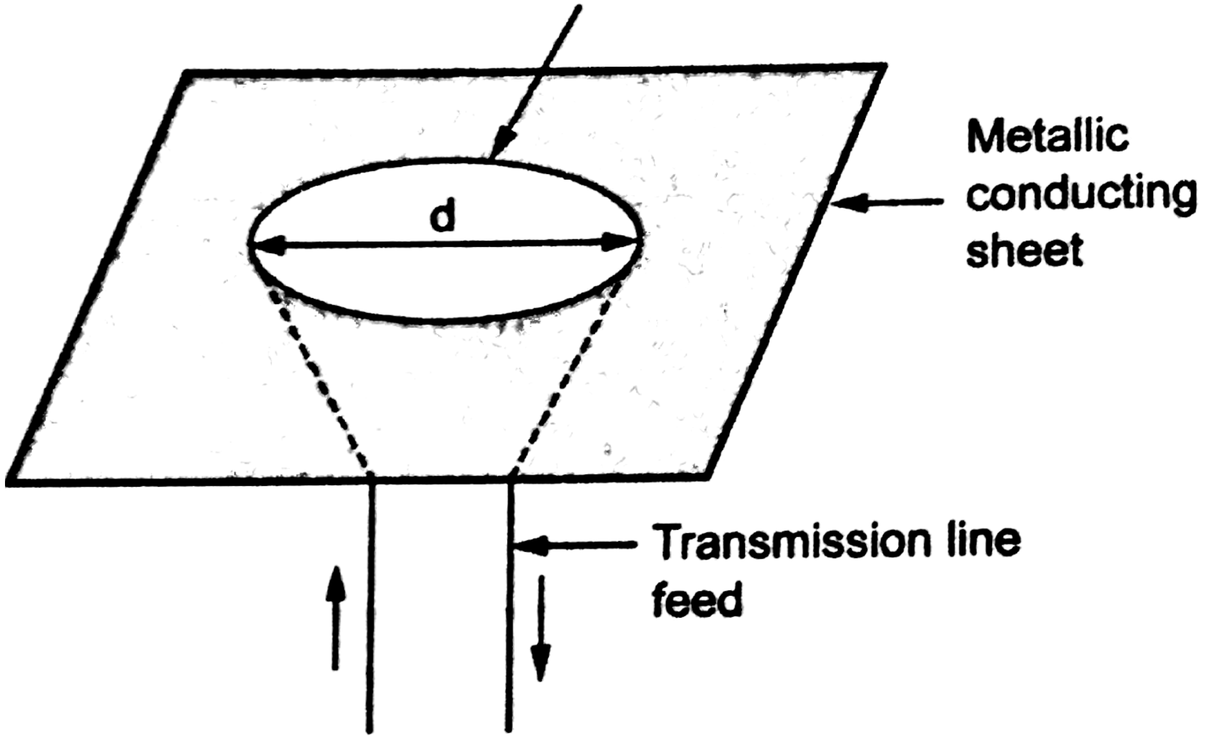
Various shapes of slot antenna



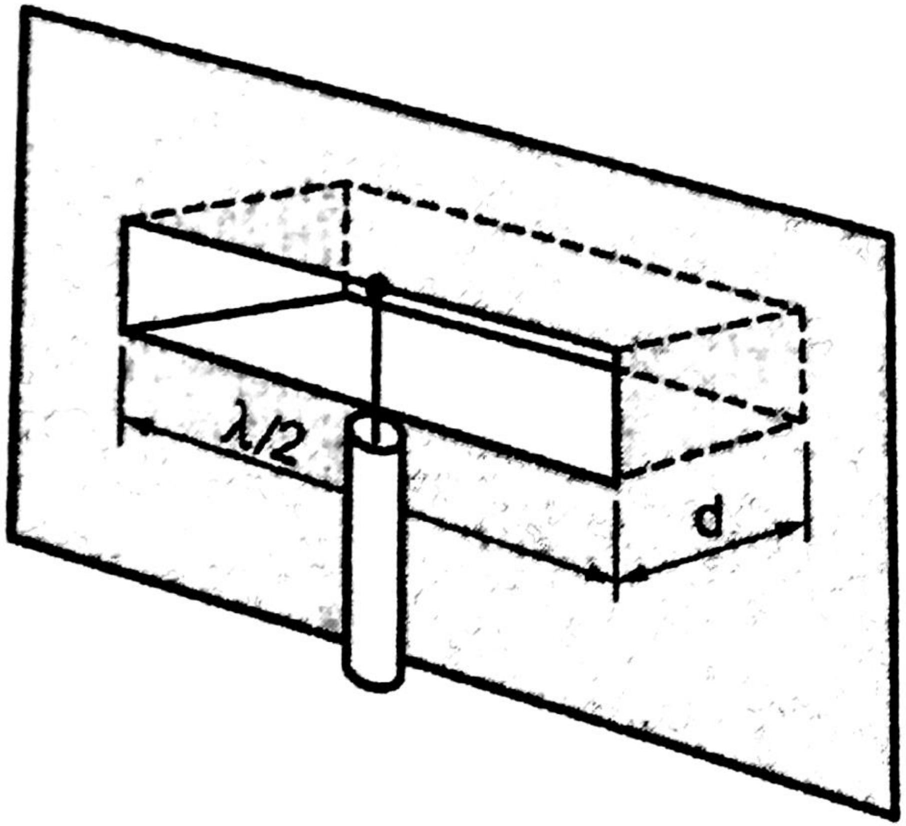
Slotted cylinder antenna



Planar array of slot antenna



Annular slot antenna



Boxed-in slot antenna

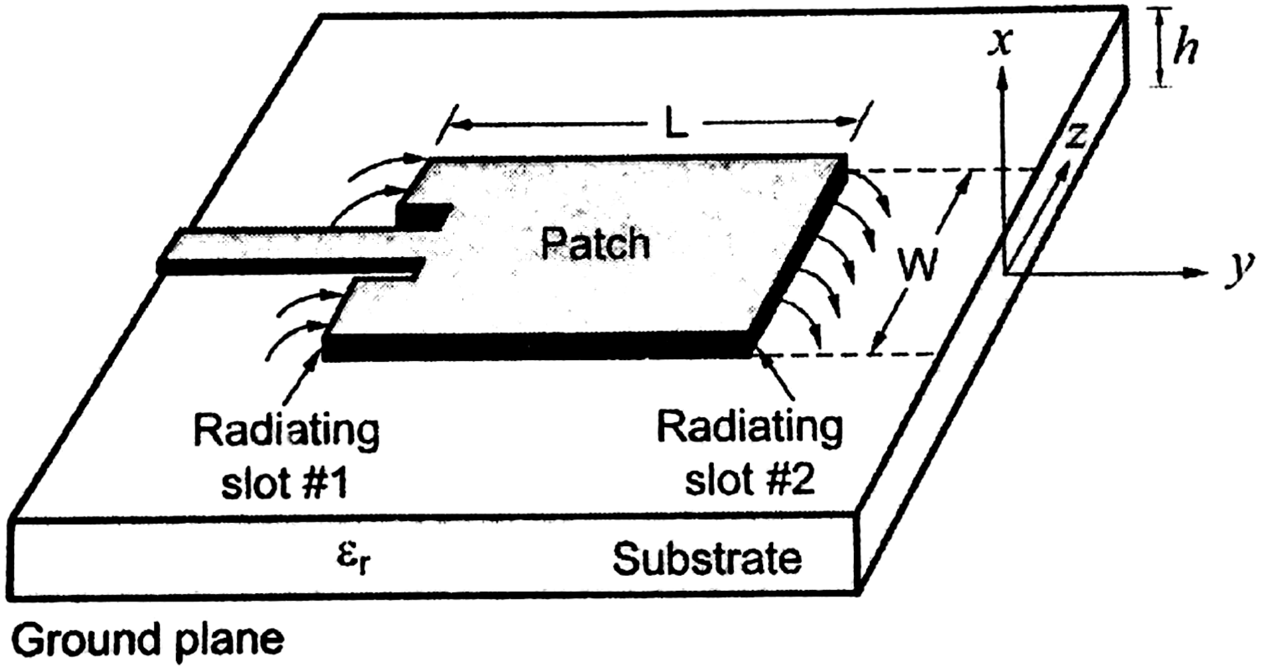
Microstrip Antennas (MAS) or Patch Antenna

*The Antenna which is made up of metal patches placed on dielectric and fed by microstrip or coplanar transmission line is called **microstrip antenna**. It is also called as **patch antenna** or **microstrip patch antenna**.*

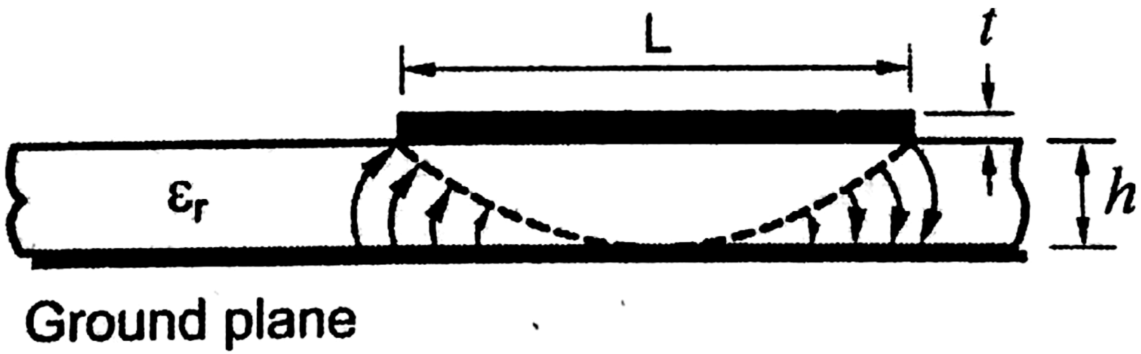
The simplest patch antenna uses a half-wavelength long patch with a larger ground plane to give better performance but at the cost of larger antenna size.

As the MSA are directly printed on to the circuit boards, so it is also called as ***printed antenna.*** The micro strip antenna is constructed on a thin dielectric sheet which uses a printed circuit board and etching techniques.

construction



Microstrip antenna



Side view

Types of patch in Microstrip Antenna

The following features are common for all MSA

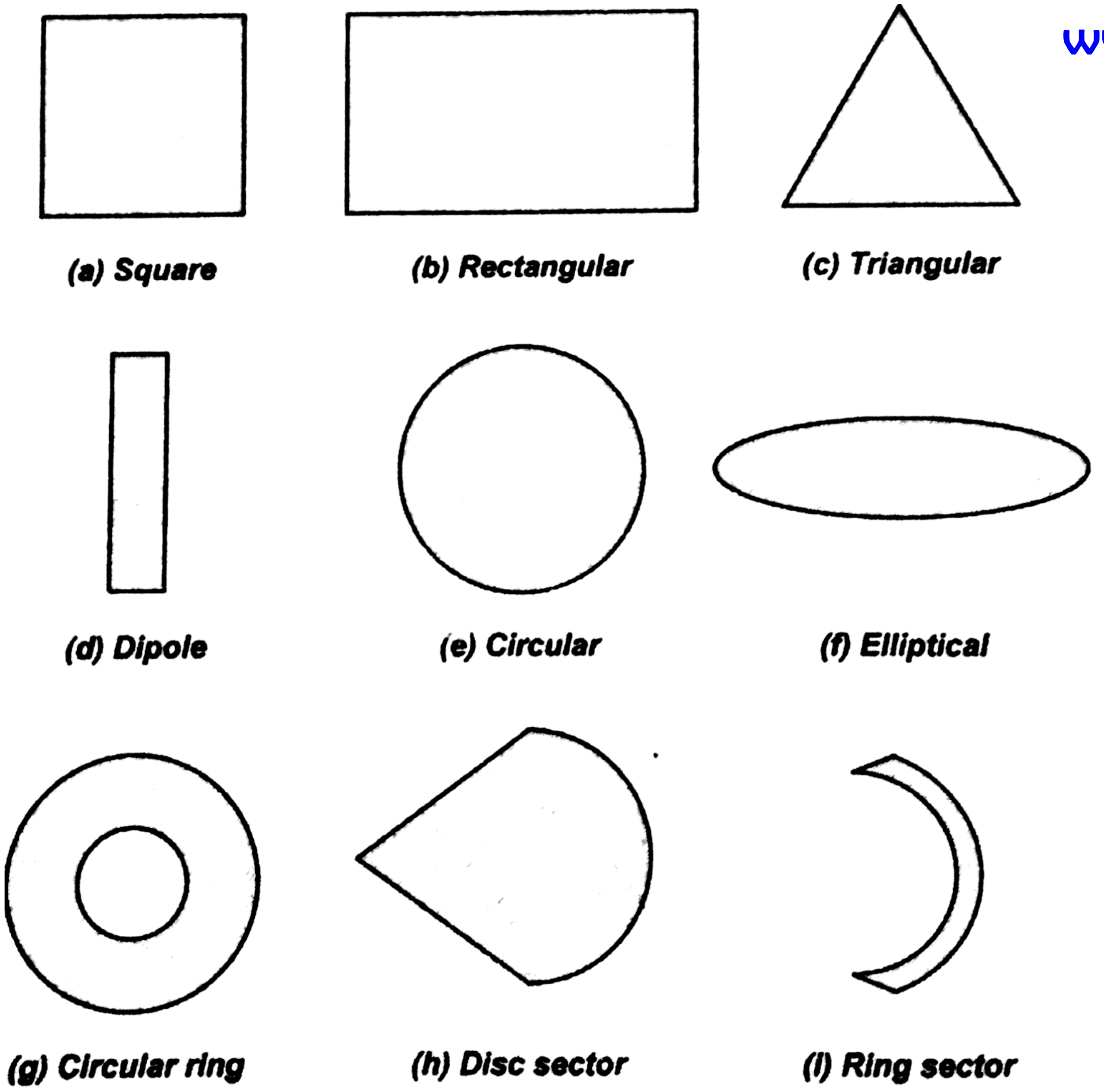
(i) A thin, flat metallic region which is commonly called patch

(ii) A dielectric substrate

(iii) A ground plane which is much larger than patch considering dimensions

(iv) A feed network which supplied power to antenna elements

In microstrip antenna, the radiating element and the feed lines are generally photo etched on the dielectric substrate



Different shapes of patch in microstrip antenna

Feed methods of Microstrip Antenna

1. Contacting feed
2. Non-contacting feed

(a) Microstrip feed

(i) Center feed

(ii) Offset feed

(iii) Inset feed

(iv) Quarter wave line feed

(b) Co-axial feed

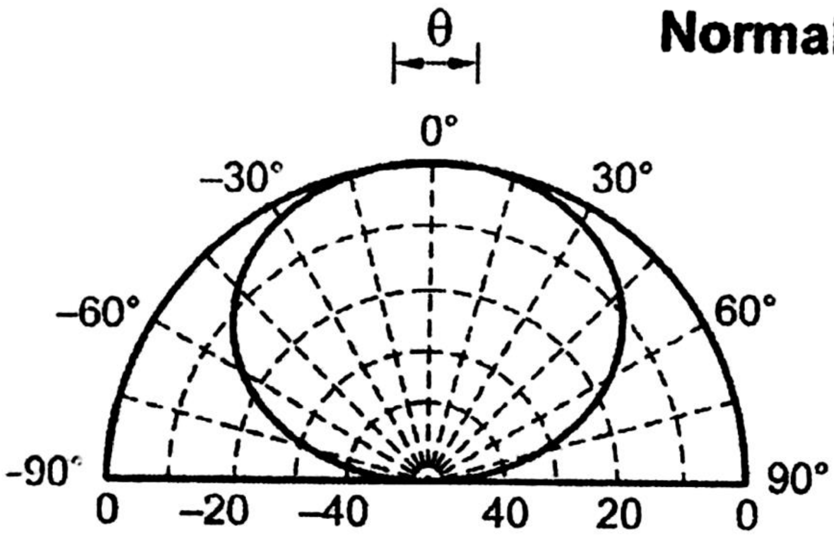
(c) Aperture coupled feed

(d) Proximit coupled feed

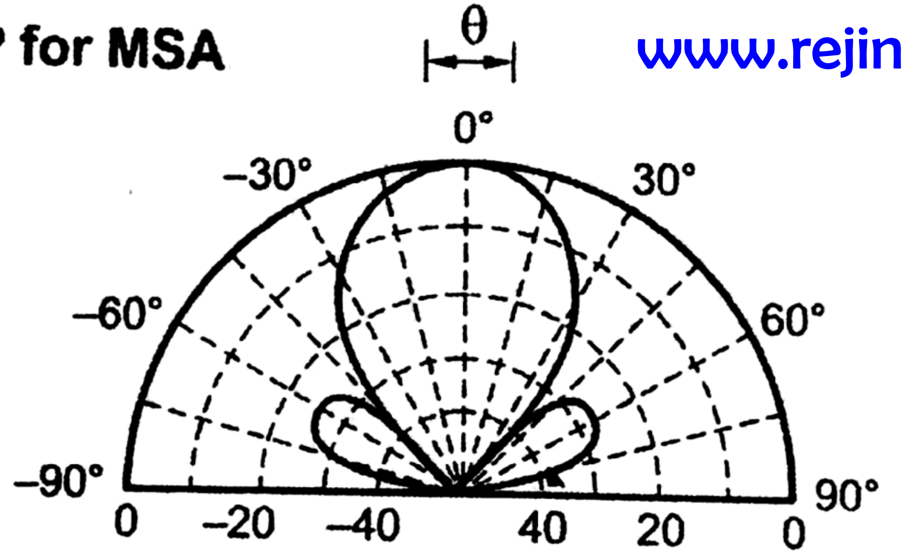
Applications

(i) Mobile and satellite communication application
(ii) Radio frequency identification
(iii) Worldwide interoperability for Microwave access (WiMax)
(iv) Radar application
(v) Telemedicine application
(vi) Medicinal applications of patch
(vii) Military applications
(viii) Space applications

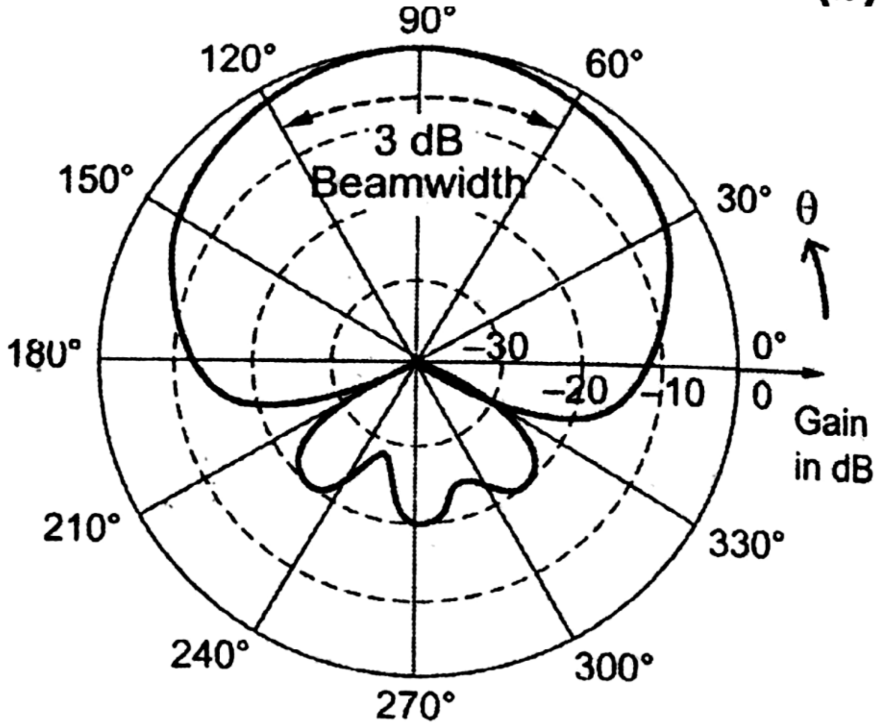
Normalized RP for MSA



(a) $\phi = 0°$



(b) $\phi = 90°$



RP for linearly polarized MSA

Numerical tool for Antenna Synthesis

Computer Aided design (CAD) software

The main advantages of CAD tool are:

(i) CAD relations are independent of specific feeding mechanism with the exception of input resistance.

(ii) It requires less computational time.

(iii) Implementation is easy.

(iv) It does not require rigorous mathematical steps

(v) Accuracy is more

(vi) Results are closer to the experimental results.

Two of the commercially available CAD packages are listed as:

PCAAD 3.0

ENSEMBLE 2.0

CYLINDRICAL+

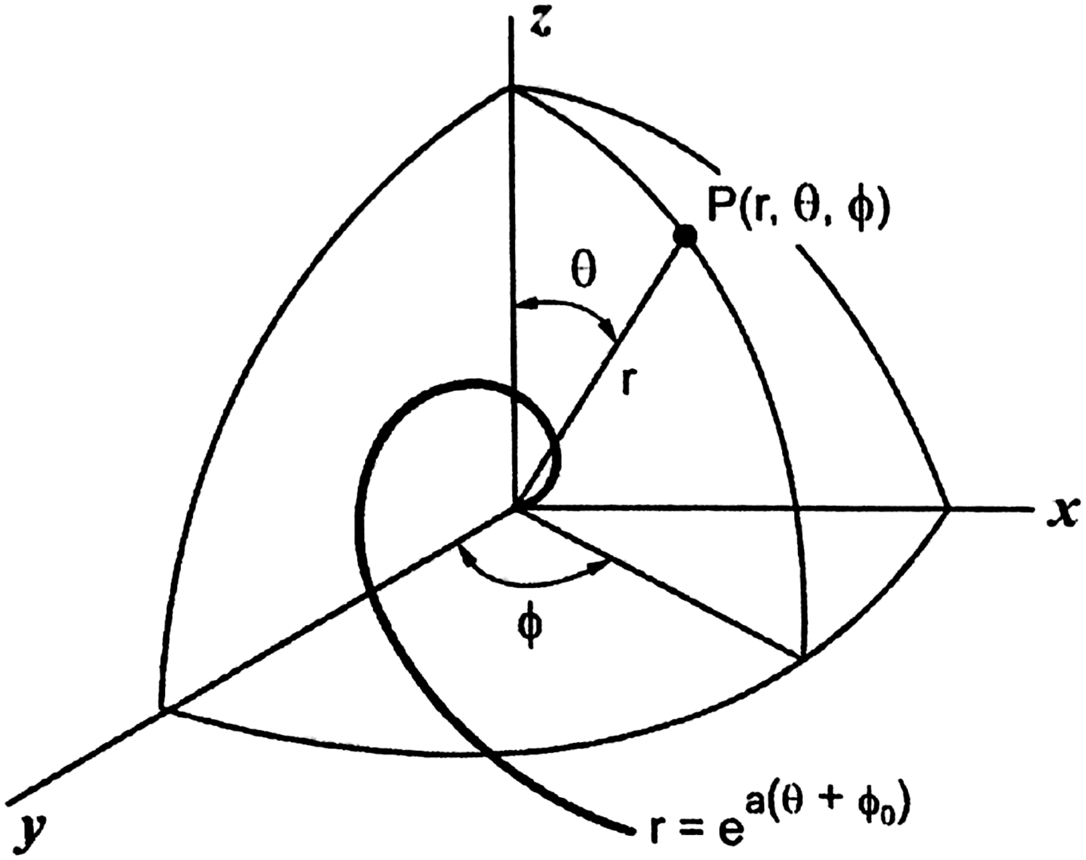
Principle of frequency independent antennas

A frequency independent antenna is physically fixed in size and operates on an over a wide bandwidth (entire frequency band) with relatively constant impedance, pattern, polarization and gain

These antennas are broadband antennas which are using 10 to 10,000 MHz

RUMSEY'S PRINCIPLE

"The performance that is, the impedance and pattern properties of a lossless antenna is independent of frequency if the dimensions of the antenna are specified in terms of angles such that they remain constant in terms of wavelength"



Spherical co-ordinate system for equiangular spiral antenna

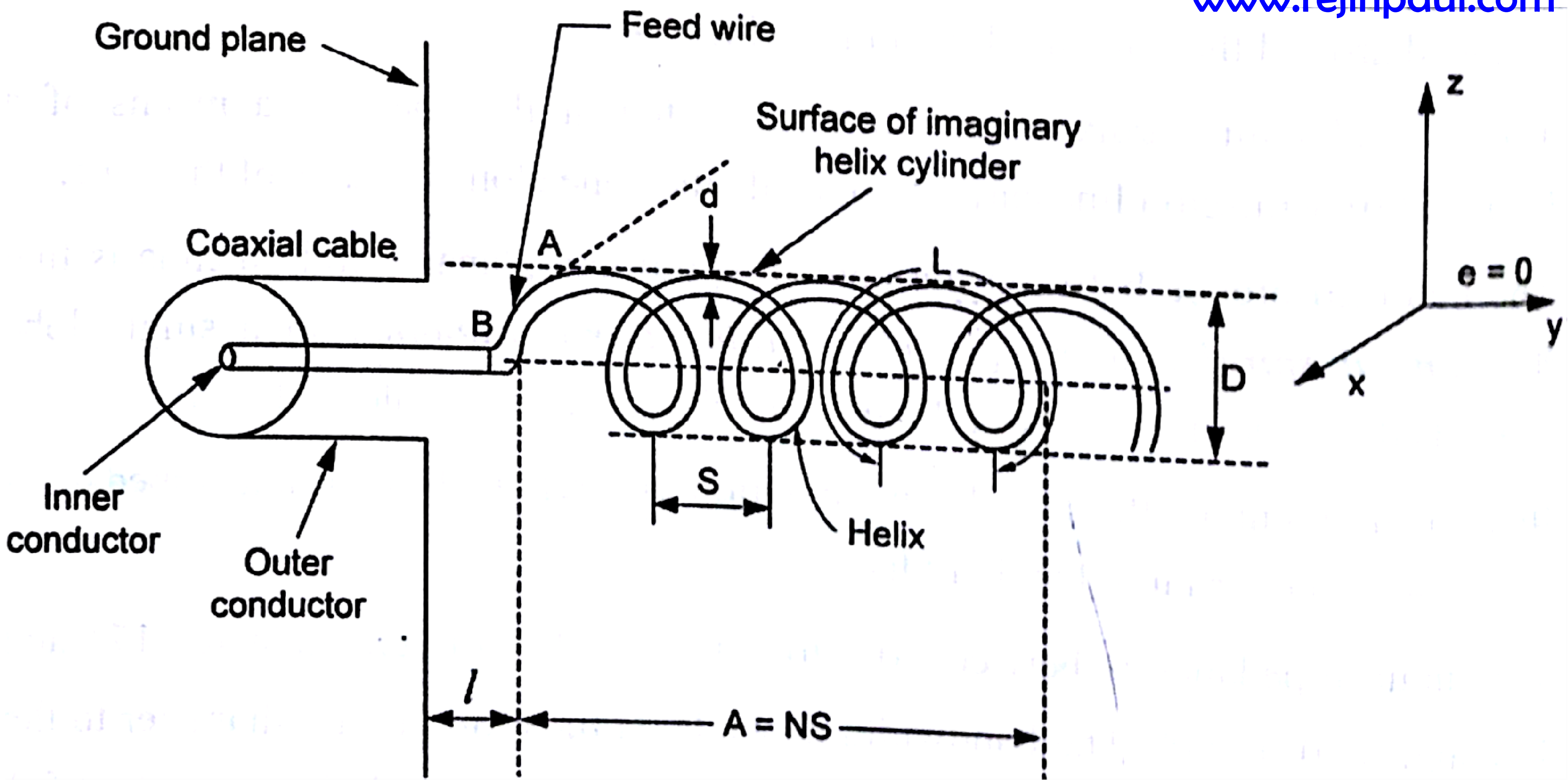
Helical Antenna

Helical antenna is a simplest type of antenna (radiator) which provides circularly polarized waves; it is used in extra terrestrial communications where satellite relays are involved.

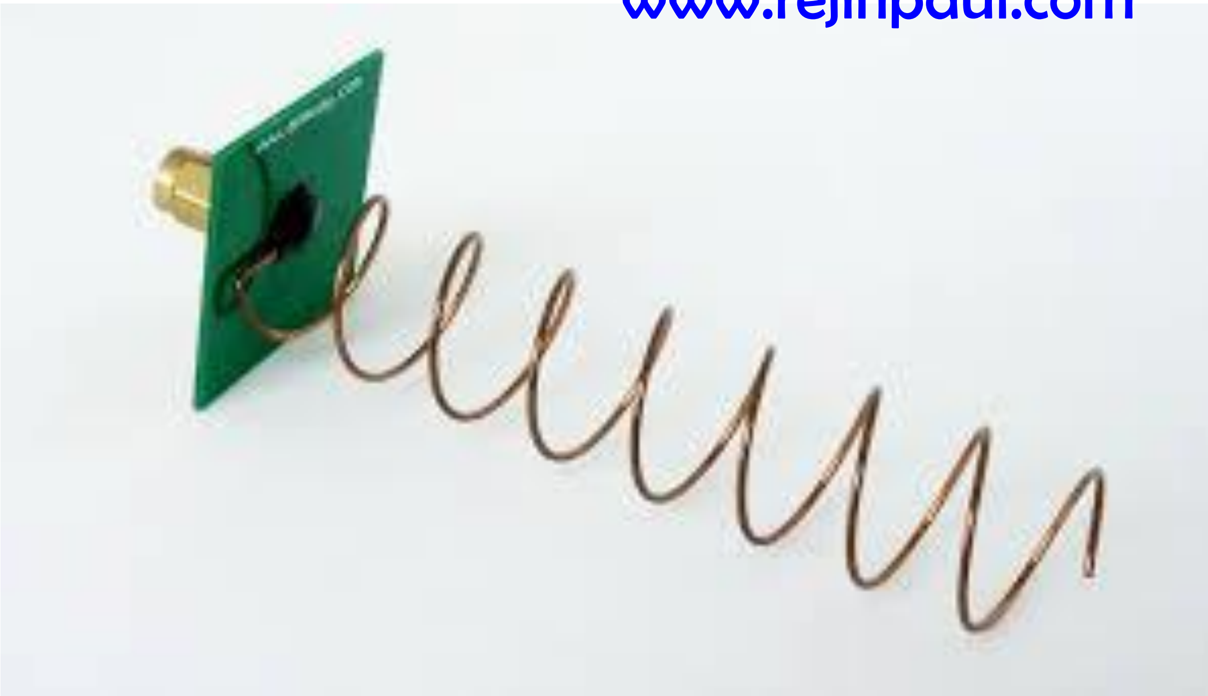
The helical antenna is a broadband VHF and UHF antenna to provide circular polarization characteristics.

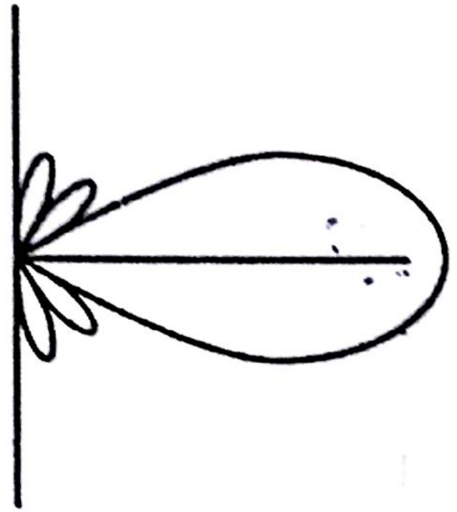
Construction

Helical antenna consists of a helix of thick copper wire or tubing wound in the shape of a screw thread and used with a flat metal called a ***ground plane*** or ***ground plate***



Helical Antenna





Radiation pattern of helical antenna (axial mode)

The following symbols are used to describe a helix

C = Circumference of helix = π D

d = Diameter of helix conductor

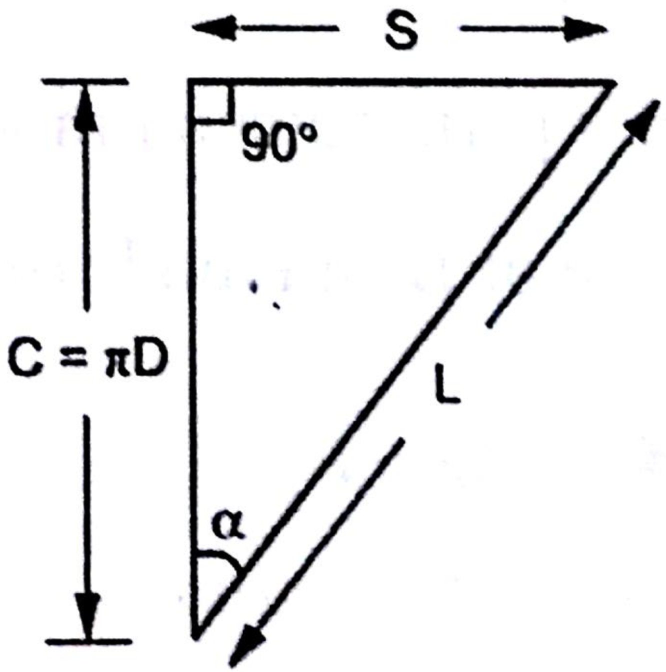
A = Axial length = NS

N = Number of turns

L = Length of one turn

l = Spacing of helix from ground plane

α = Pitch angle



Inter-relation between circumference, spacing, turn length and pitch angle

For N turn of helix, the total length of antenna is equal to NS

If one turn of helix is unrolled, then circumference (πD), spacing S, turn length "L" and pitch angle α are related by the triangle as shown in fig.

Then the length of one turn is expressed as

$$L = \sqrt{S^2 + C^2} = \sqrt{S^2 + (\pi D)^2} \qquad \text{......(1)}$$

Pitch angle (α) is the angle between a line tangent to the helix wire and the plane normal to the helix axis.

$$\tan\alpha = \frac{S}{C} = \frac{S}{\pi D}$$

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) \qquad \text{......(2)}$$

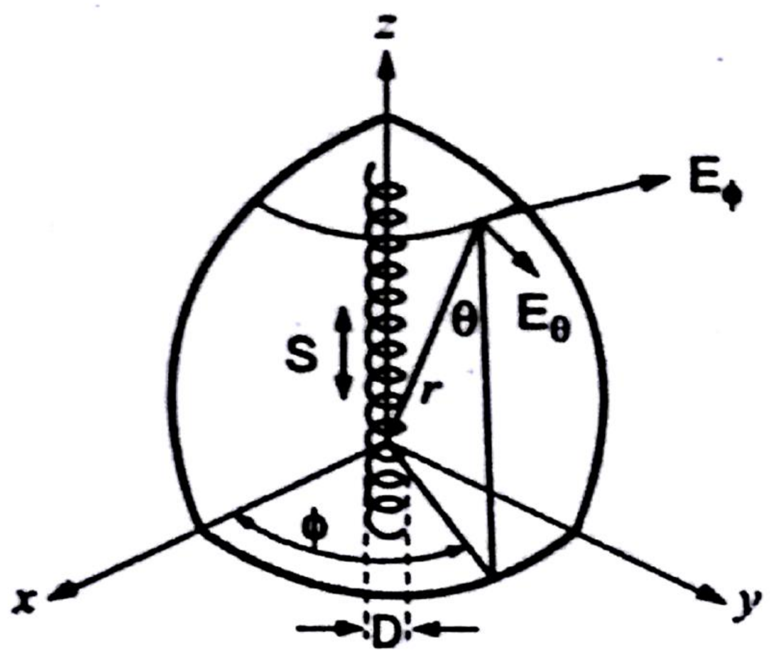
MODES OF RADIATION

In general, a helical antenna can radiate in many modes. But the most important modes of radiation are as follows:

(i) Normal mode or perpendicular mode.

(ii) Axial or End fire or Beam mode of radiation.

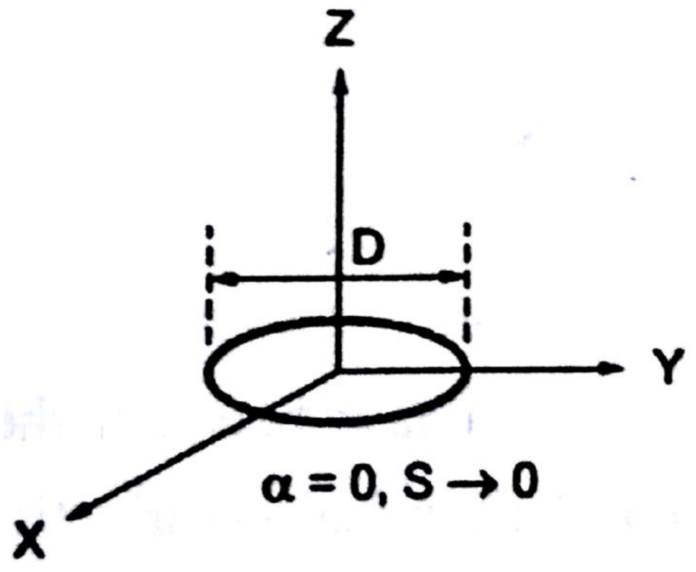
Normal Mode of Radiation



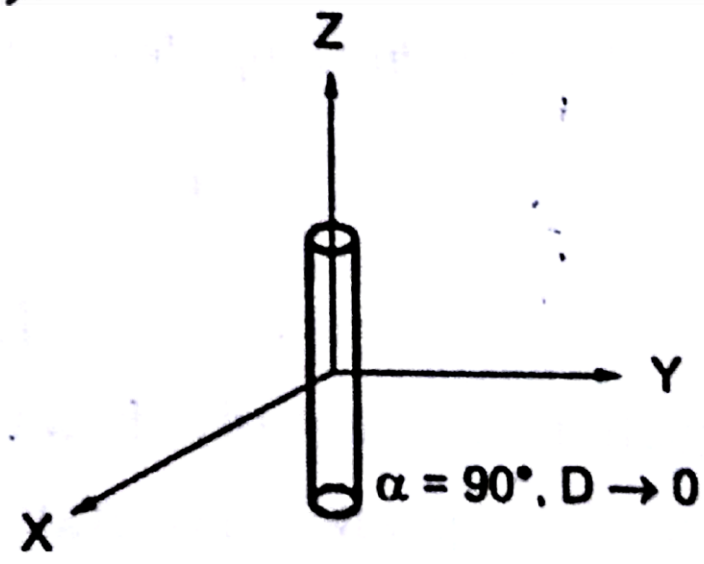
Helix in 3-dimensional spherical coordinate

When $\alpha = 0°$ helix corresponds to a loop and $\alpha = 90°$ the helix becomes a linear dipole as shown in Figure

If S = 0, helix collapse to a loop and if S = constant and D = 0, the helix straightens into a linear conductor (short dipole).



(a) Loop



(b) Short dipole

Limiting conditions on helix

Axial Ratio (AR)

The far field of the ***small loop*** is given by,

$$E_\phi = \frac{120\,\pi^2\,[I]\sin\theta}{r}\cdot\frac{A}{\lambda^2} \qquad \text{......(3)}$$

where, [I] - Retarded current

r - Distance

A - Area of loop $= \dfrac{\pi D^2}{4}$

The far field of a ***short dipole*** is given by,

$$E_\theta = \frac{j\,60\,\pi\,[I]\sin\theta}{r}\cdot\frac{S}{\lambda} \qquad \text{......(4)}$$

where, S = L = Length of dipole

The Equations (3) and (4) shows that there is 90^0 phase between them due to presence of 'j' operator. The Axial Ratio (AR) of Elliptical polarization is given by

$$AR = \frac{E_\theta}{E_\phi} = \frac{\left| \frac{j\, 60\, \pi\, [I] \sin\theta \cdot S}{\lambda\, r} \right|}{\left| \frac{120\, \pi^2\, [I] \sin\theta \cdot A}{r\, \lambda^2} \right|}$$

$$= \frac{S\,\lambda}{2\,\pi\, A} = \frac{2\, S\, \lambda}{\pi^2\, D^2} \qquad \text{where, } A = \frac{\pi\, D^2}{4}$$

$$AR = \frac{2\, S\, \lambda}{\pi^2\, D^2} = \text{Axial ratio}$$

For circular polarization, $AR = 1 = \frac{E_\theta}{E_\phi}$

$$|E_\theta| = |E_\phi|$$

$$\therefore \quad |2 S \lambda| = |\pi^2 D^2|$$

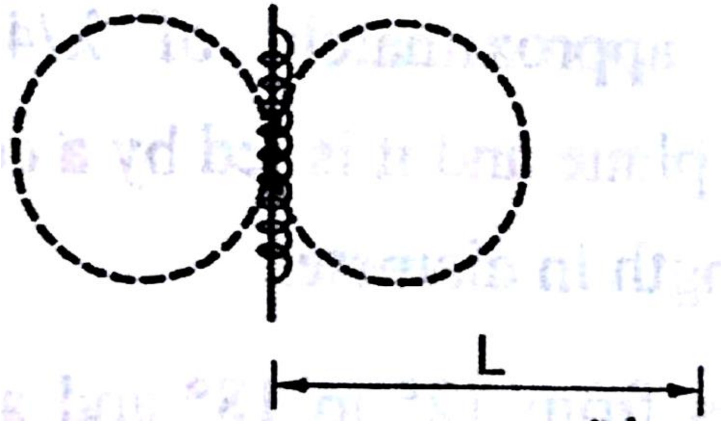
$$S = \frac{\pi^2 D^2}{2 \lambda} = \frac{C^2}{2 \lambda}, \text{ where } C = \pi D \qquad \ldots\ldots (6)$$

By substituting the equation (6) on equation (2), we get

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) = \tan^{-1} \frac{\frac{\pi^2 \cdot D^2}{2 \lambda}}{\pi D}$$

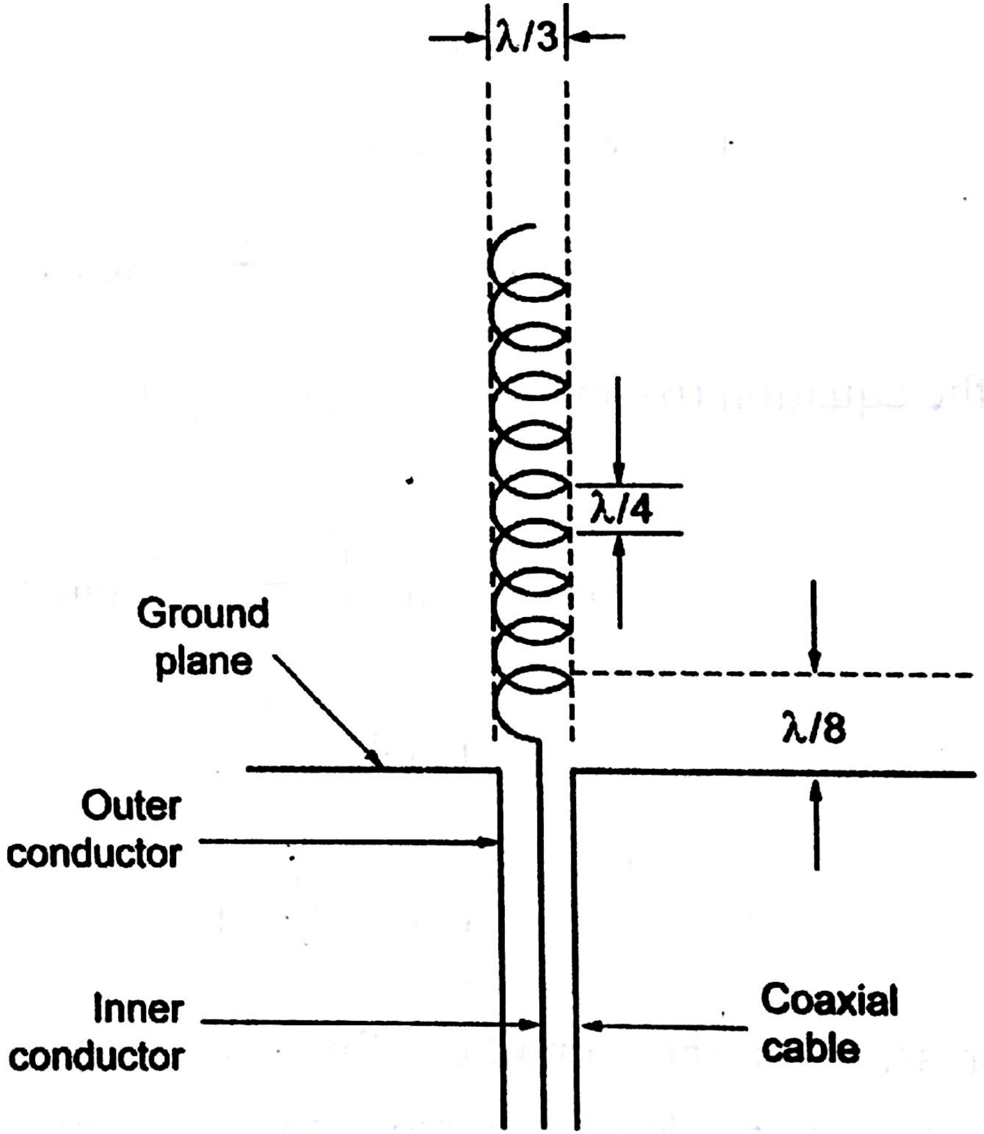
$$\alpha = \tan^{-1}\left(\frac{\pi D}{2 \lambda}\right) = \tan^{-1}\left(\frac{C}{2 \lambda}\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{C}{2 \lambda}\right)} \qquad \ldots\ldots (7)$$



Normal mode of radiation

Axial (OR) Beam Mode of Radiation



Arrangement for generating axial mode

In general, the terminal impedance of helical antenna lies between 100 Ω to 200 Ω pure resistive. Within 20% approximation, the ***terminal impedance*** is given by

$$\boxed{R = \frac{140\,C}{\lambda}\ \text{ohms}} \qquad \ldots\ldots(8)$$

The HPBW (Beamwidth between half power points) is given by,

$$\text{HPBW} = \frac{52}{C}\sqrt{\frac{\lambda^3}{N\,S}}\ \text{degrees} \qquad \ldots\ldots(9)$$

where, λ = free space wave length

S = Spacing

The beamwidth between first nulls is given by

$$\text{BWFN} = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}\ \text{degree} \qquad \ldots\ldots(10)$$

The maximum ***directive gain (directivity)*** for axial mode is given by

$$D = \frac{15\,N\,S\,C^2}{\lambda^3} \quad \text{......(11)}$$

$$\text{Axial Ratio (AR)} = 1 + \frac{1}{2N} \quad \text{......(12)}$$

The normalized far field pattern is given as,

$$E = \sin\left(\frac{\pi}{2N}\right)\cos\theta \cdot \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \quad \text{......(13a)}$$

$$\psi = 2\pi\left[\frac{S}{\lambda}(1-\cos\theta) + \frac{1}{2N}\right] \quad \text{......(13b)}$$

where, α =12° to 15°, $N \geq 3$, $NS \leq 10$ and $C = \frac{3}{4}\lambda$ to $\frac{4}{3}\lambda$

Log Periodic Antenna

A log periodic antenna is a broadband narrow beam antenna. It is a frequency independent antenna.

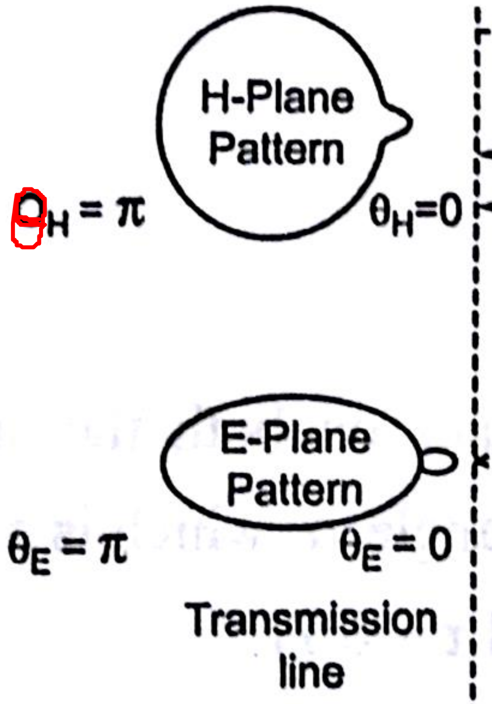
This frequency independent concept can be obtained by adjusting the antenna structure (either expanded or contracted) in proportion to the wavelength. If it is not possible to adjust the antenna mechanically, then the size of ***active or radiating region*** should be proportional to the wavelength.

Log-Periodic Concept

Here, the geometry of the antenna structure is adjusted such that all the electrical properties of the antenna must repeat periodically with the logarithm of the frequency

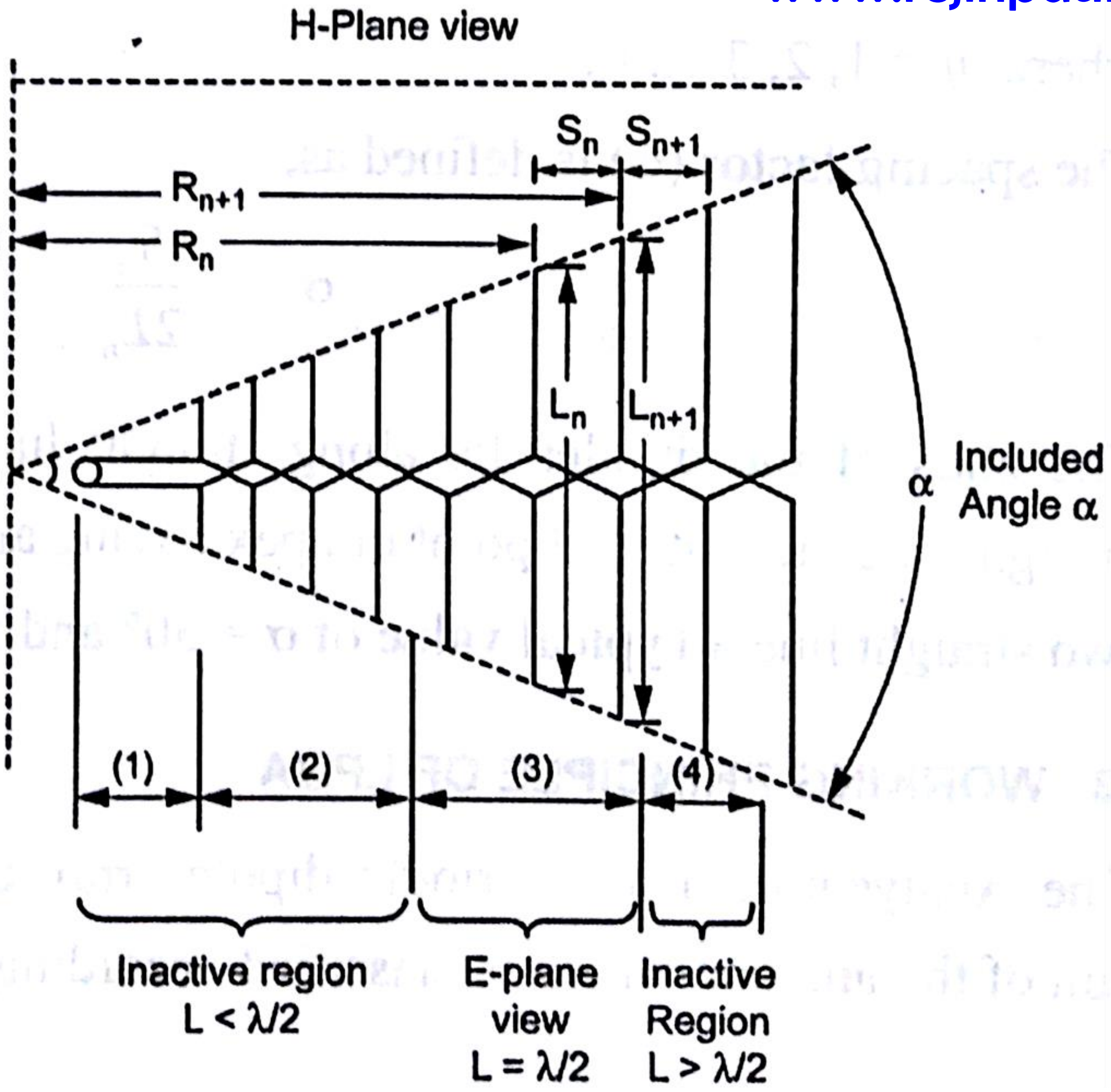

VHF Log Periodic Antenna

CONSTRUCTION OF LPDA



(1) Included transmission line region
(2) Loaded transmission line region
(3) Active region
(4) Reflective region

Radiation pattern of a LPDA in E-plane and H-plane



A log periodic dipole array with main region of operation

The relationship between spacings S and lengths L of adjacent elements are scaled as,

$$\frac{S_n}{S_{n+1}} = \frac{L_n}{L_{n+1}} = \tau \qquad \text{.....(1)}$$

τ is also called ***periodicity factor*** which is always less than 1. The above expression can be written in terms of constant k with the radii of the arm as

$$\frac{R_{n+1}}{R_n} = \frac{S_{n+1}}{S_n} = \frac{L_{n+1}}{L_n} = \frac{1}{\tau} = k\,; \quad k > 1 \qquad \text{.....(2)}$$

where $n = 1, 2, 3 \ldots. n$

The spacing factor (σ) is defined as,

$$\sigma = \frac{S_n}{2L_n} \qquad \text{......(3)}$$

WORKING PRINCIPLE OF LPDA

(i) Inactive transmission - line region ($L < \lambda/2$)

(ii) Active region $L \approx \lambda/2$

(iii) Inactive reflective region ($L > \lambda/2$)

DESIGN OF LOG PERIODIC DIPOLE ARRAY

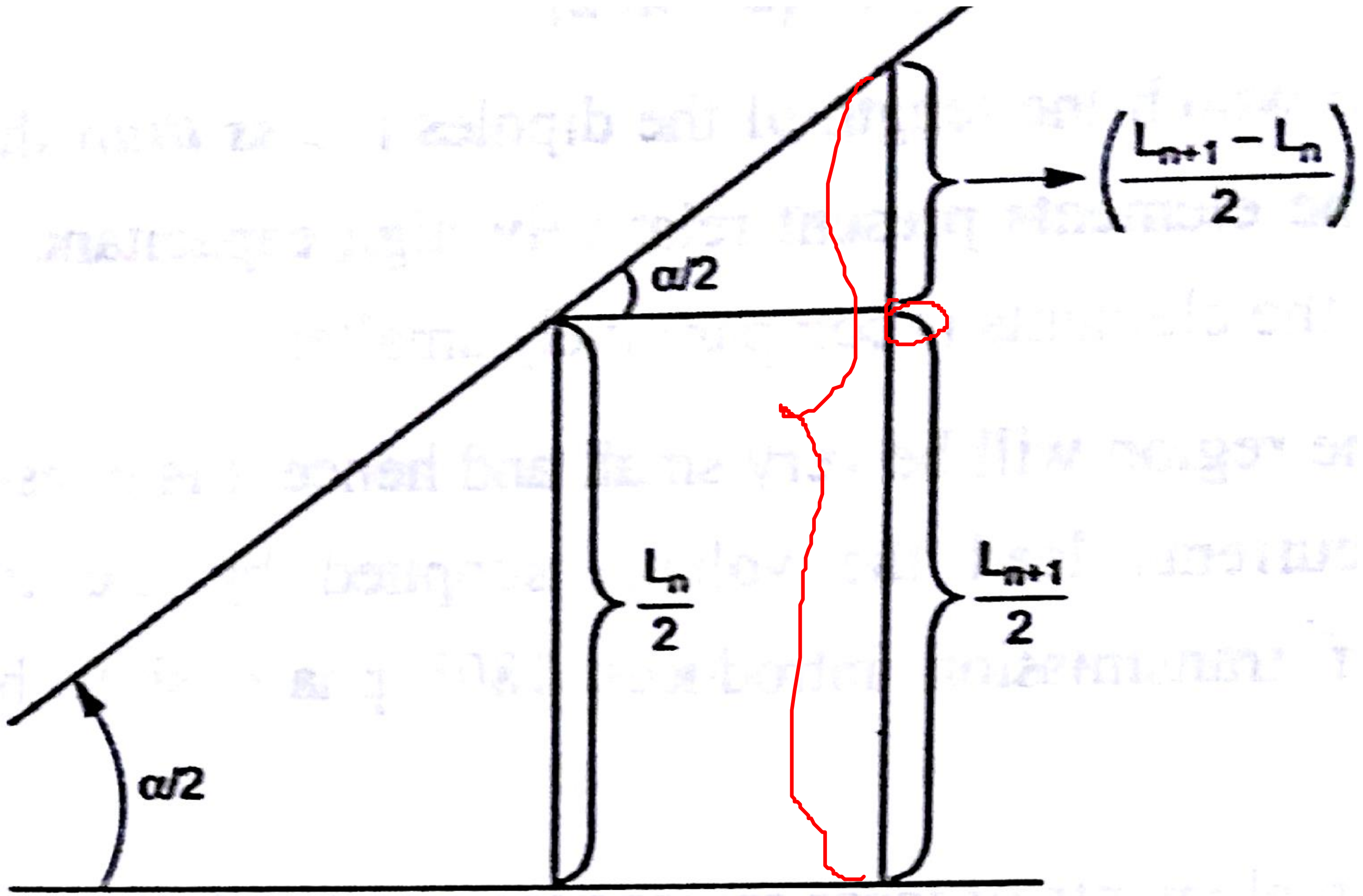
The performance of a log periodic dipole array depends on the following parameters.

(*i*) Apex angle (α)

(*ii*) Design ratio (τ)

(*iii*) Spacing factor (σ)

Consider a part of a log periodic array as shown in the Fig.



Geometry of log-periodic array

From Fig.

$$\tan(\alpha/2) = \frac{\frac{L_{n+1} - L_n}{2}}{S} \qquad \ldots\ldots(4)$$

$$\tan(\alpha/2) = \frac{L_{n+1} - L_n}{2S}$$

$$= \frac{L_{n+1}\left[1 - \frac{L_n}{L_{n+1}}\right]}{2S} \qquad \ldots\ldots(5)$$

But

$$\frac{L_{n+1}}{L_n} = k$$

$$\frac{L_n}{L_{n+1}} = \frac{1}{k} \qquad \ldots\ldots(6)$$

By substituting the equation (6) in equation (5), we get

$$\tan(\alpha/2) = \frac{\left(1-\frac{1}{k}\right)L_{n+1}}{2S} \qquad \text{......(6)}$$

For active region $L_{n+1} = \lambda/2$ ......(7)

By substituting the equation (7) in equation (6), we get

$$\tan(\alpha/2) = \frac{\left(1-\frac{1}{k}\right)\lambda/2}{2S} = \frac{\left(1-\frac{1}{k}\right)}{4\left(\frac{S}{\lambda}\right)}$$

$$\tan(\alpha/2) = \frac{\left(1-\frac{1}{k}\right)}{4\sigma} \qquad \text{......(8)}$$

where, $\sigma = \frac{S}{\lambda} =$ Spacing factor

α Apex angle

$k =$ Scale factor

But $\tau = \frac{1}{k}$

$$\boxed{\tan(\alpha/2) = \frac{1-\tau}{4\sigma}} \quad \ldots\ldots (9)$$

From equation (9), σ can be obtained as

$$\boxed{\sigma = \frac{1-\tau}{4 \tan \alpha/2}} \quad \ldots\ldots (10)$$

$$\tan(\alpha/2) = \frac{1-\tau}{4\sigma}$$

$$\alpha/2 = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right)$$

$$\boxed{\alpha = 2\tan^{-1}\frac{1-\tau}{4\sigma}} \qquad \text{......(11)}$$

The number of elements in an array(n) can be obtained from the upper frequency (f_U) and lower frequency(f_L) and it is given as,

$$\log(f_U)-\log(f_L) = (n-1)\log\left(\frac{1}{\tau}\right) \qquad \text{......(12)}$$

UNIT III
ANTENNA ARRAYS AND APPLICATIONS

Two-element array, Array factor, Pattern multiplication, Uniformly spaced arrays with uniform and non-uniform excitation amplitudes, Smart antennas.

Antenna Arrays

Several antennas of similar type are arranged in a system to radiate more in desired direction with high gain

This can be achieved by combining the individual antenna radiations in desired direction and canceling the radiation in undesired direction

Such system is called an antenna array

An antenna array is a system of similar antennas oriented similarly to get greater directivity in a desired direction

The antenna array is said to be linear if the elements of the antenna array are equally spaced along a straight line

The linear antenna array is said to be uniform linear array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line

Various forms of Antenna Arrays

Practically various forms of the antenna array are used as radiating systems. Some of the practically used forms are as follows

(i) Broadside array

(ii) End fire array

(iii) Collinear array

(iv) Parasitic array

Array of 2 Point Sources

Point source is nothing but an isotropic radiator occupying zero volume

A number of similar point source is arranged in the form of array

The simplest condition of number of point sources in the array is two

The array of 2 point sources can be analyzed in 3 different ways

(i) Two point sources of equal magnitude and same phase

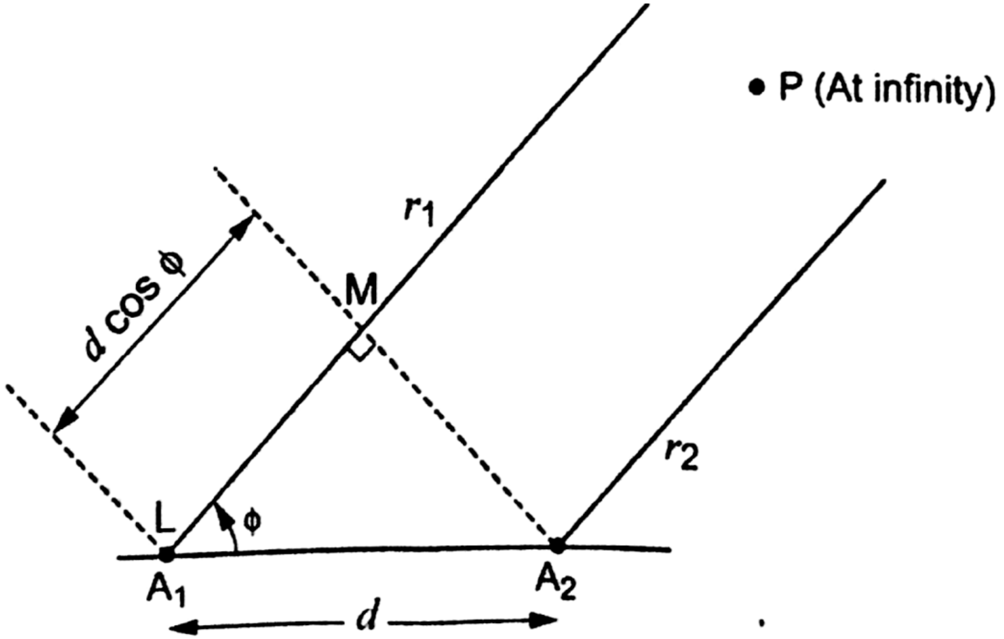
(ii) Two point sources with currents of equal magnitude and opposite phase

(iii) Two point sources with currents of unequal magnitudes and any phase

Two point sources with currents equal in magnitude and phase

Consider two point sources A_1 and A_2 separated by distance 'd' as shown in Fig.2.21. Let both the point sources are supplied with currents equal in magnitude and phase.

Consider a distant point 'p' far away from the array. Let the distance between point sources A_1 and A_2 and point 'p' be r_1 and r_2 respectively. As these radial distances are extremely large as compared with 'd' (distance between 2 point sources). We can assume $r_1 = r_2 = r$.



$$\text{the path difference} = d \cos \phi$$

In terms of wavelength,

$$\text{Path difference} = \frac{d \cos \phi}{\lambda}$$

$$\text{Phase angle } \psi = 2\pi \times \text{path difference}$$

$$\text{Phase angle } \psi = 2\pi \left(\frac{d \cos \phi}{\lambda} \right)$$

$$\boxed{\psi = \beta\, d \cos \phi \quad \text{radian}} \qquad \left(\because \beta = \frac{2\pi}{\lambda} \right)$$

Let $E_1 \rightarrow$ Far field at a distant point 'p' due to point source A_1

$$E_1 = E_0\, e^{-j\psi/2}$$

Similarly $E_2 \rightarrow$ Far field at point 'p' due to point source A_2

$$E_2 = E_0\, e^{j\psi/2}$$

where $E_0 \rightarrow$ Amplitude of both the field components

The total field (E_T) at point '*p*' is given by

$$E_T = E_1 + E_2 = E_0\, e^{-j\psi/2} + E_0 \cdot e^{j\psi/2}$$

$$E_T = E_0\left(e^{j\psi/2} + e^{-j\psi/2}\right)$$

$$E_T = 2\, E_0 \cos(\psi/2) \qquad \left[\because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}\right]$$

Substituting the value of ψ from equation

$$\boxed{E_T = 2\, E_0 \cos\left(\frac{\beta\, d \cos\phi}{2}\right)}$$

Array Factor

It is ratio of the magnitude of the resultant field to the magnitude of the maximum field.

$$\text{Array factor} = \frac{|E_T|}{|E_{max}|}$$

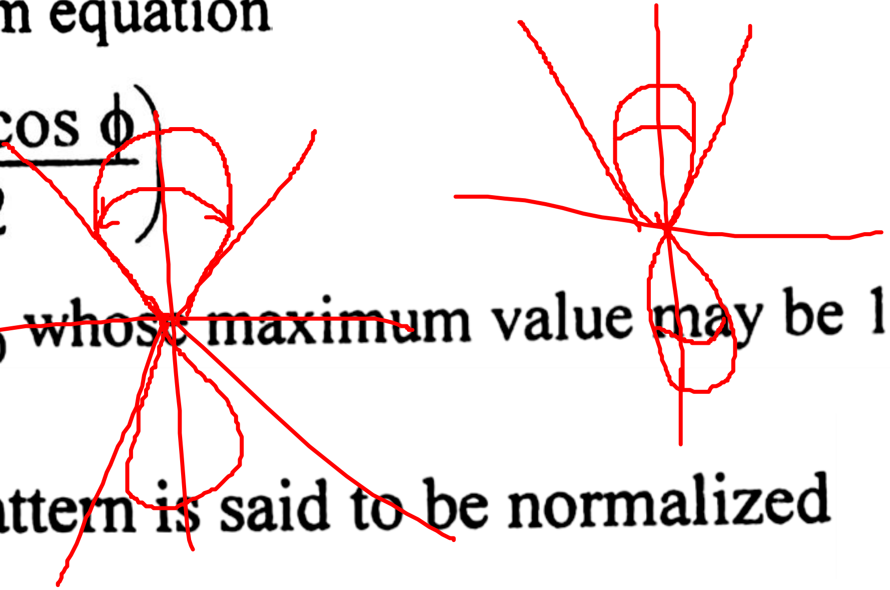
But maximum field is $E_{max} = 2\,E_0$

$$\text{Array factor} = \frac{|E_T|}{|2\,E_0|} = \cos\left(\frac{\beta\, d \cos\phi}{2}\right)$$

Field Pattern

To draw the field pattern, the directions of maxima, minima and half power points must be known which can be calculated from equation

$$E_T = 2\,E_0 \cos\left(\frac{\beta\, d \cos\phi}{2}\right)$$



Here the amplitude of the total field is $2\,E_0$ whose maximum value may be 1

$\therefore$ By putting $2\,E_0 = 1$ or $E_0 = \frac{1}{2}$, the pattern is said to be normalized

$$E = \cos\left(\beta d \frac{\cos\phi}{2}\right)$$

Let $d = \lambda/2$ and $\beta = \frac{2\pi}{\lambda}$

$$E = \cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \frac{\cos\phi}{2}\right)$$

$$\boxed{E = \cos\left(\frac{\pi}{2}\cos\phi\right)}$$

Maxima Direction

The direction through which maximum radiation occurs is called as maxima direction or maxima. It is obvious that the electric field is maximum at maxima direction.

The total field strength 'E' is maximum when $\cos\left(\frac{\pi}{2}\cos\phi\right)$ is maximum and its maximum value is ± 1.

$$E = \cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\frac{\pi}{2}\cos\phi_{max} = \cos^{-1}(\pm 1) = \pm n\pi \qquad \text{where } n = 0, 1, 2\ldots\ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{max} = 0$$

$$\cos\phi_{max} = 0$$

$$\boxed{\phi_{max} = 90° \text{ or } 270°}$$

Minima Direction

The total field strength 'E' is minimum when $E = \cos\left(\frac{\pi}{2}\cos\theta\right)$ is minimum and its minimum value is zero.

$$\therefore \quad E = \cos\left(\frac{\pi}{2}\cos\phi\right) = 0$$

$$\frac{\pi}{2}\cos\phi_{min} = \cos^{-1}(0) = \pm(2n+1)\frac{\pi}{2} \qquad \text{where } n = 0, 1, 2\ldots$$

If $n = 0$, then $\frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$

$$\cos\phi_{min} = \pm 1$$

$$\boxed{\phi_{min} = 0° \text{ or } 180°}$$

Half power point directions

At half power points, power is $\frac{1}{2}$ (or) voltage and current is $\frac{1}{\sqrt{2}}$ times the maximum value.

$\therefore$ At half power point direction, the electric field is $\pm\frac{1}{\sqrt{2}}$

i.e.,

$$E = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \quad E = \cos\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = \pm(2n+1)\frac{\pi}{4}, \qquad \text{where } n = 0, 1, 2\ldots$$

If $n = 0$, then

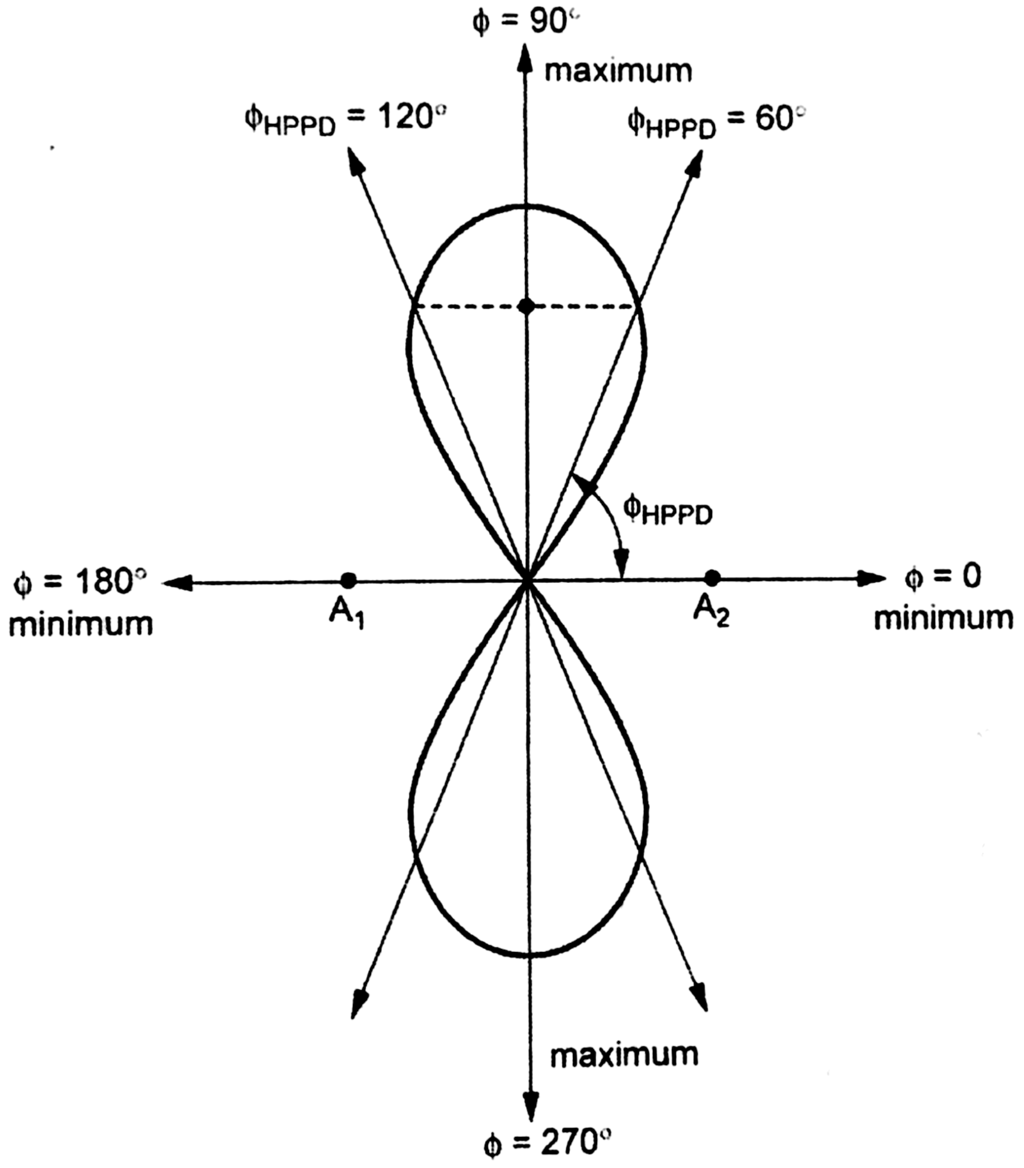
$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$\boxed{\phi_{HPPD} = 60^\circ \text{ (or) } 120^\circ}$$

Now the field pattern with E against ϕ is drawn for $d = \lambda/2$



Two point sources with currents equal in magnitudes but opposite in phase

Consider 2 point sources separated by distance '*d*' and supplied with currents ***equal in magnitude but opposite in phase***. It is similar to the previous case except that source A_1 has current out of phase (180°) (or) opposite phase to source A_2. *i.e.*, when there is maximum in source A_1 at one particular instant, then there is minimum in source A_2 at that instant and vice-versa.

Total far field at distant point '*p*' is given by

$$E_T = -E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

Let $E_1 = E_2 = E_0$

$$\therefore \quad E_T = E_0 \left(e^{j\psi/2} - e^{-j\psi/2} \right)$$

$$E_T = E_0 \cdot 2j \sin\frac{\psi}{2}$$

$$\left[\because \frac{e^{j\theta/2} - e^{-j\theta/2}}{2j} = \sin\theta/2 \right]$$

$$\underline{E_T = 2j E_0 \sin\left(\frac{\beta d \cos\phi}{2}\right)}$$

Field Pattern

To draw the field pattern, the directions of maxima, minima and half power points must be known which can be calculated from equation

$$E_T = 2\,j\,E_0 \sin\left(\frac{\beta\, d \cos\phi}{2}\right)$$

Here the amplitude of the total field is $2\,E_0$ whose maximum value may be 1

By putting $|2\,E_0| = 1$, the pattern is said to be normalized

$$\therefore \quad E = \sin\left(\frac{\beta\, d \cos\phi}{2}\right)$$

$$\text{Let} \quad d = \frac{\lambda}{2} \quad \text{and} \quad \beta = \frac{2\pi}{\lambda}$$

$$E = \sin\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \frac{\cos\phi}{2}\right)$$

$$\boxed{E = \sin\left(\frac{\pi}{2}\cos\phi\right)}$$

Maxima Directions

The direction through which maximum radiation occurs is called as maxima direction or maxima. It is obvious that the electric field is maximum at maxima direction.

$$E = \pm 1$$

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\frac{\pi}{2}\cos\phi_{max} = \sin^{-1}(\pm 1) = \pm(2n+1)\frac{\pi}{2} \text{ where } n = 0, 1, 2\ldots\ldots$$

If $n = 0$, then $\frac{\pi}{2}\cos\phi_{max} = \pm\frac{\pi}{2}$

$$\cos\phi_{max} = \pm 1$$

$$\boxed{\phi_{max} = 0° \text{ and } 180°}$$

Minima Direction

The total field strength 'E' is minimum when $E = \sin\left(\frac{\pi}{2}\cos\phi\right)$ is minimum

i.e., zero.

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right) = 0$$

$$\frac{\pi}{2}\cos\phi_{min} = \sin^{-1}(0) = \pm n\pi \qquad \text{where } n = 0, 1, 2, \ldots\ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{min} = 0$$

$$\cos\phi_{min} = 0$$

$$\boxed{\phi_{min} = \pm 90°}$$

Half Power Point Direction (HPPD)

At half power points, power is $\frac{1}{2}$ (or) voltage and current is $\frac{1}{\sqrt{2}}$ times the maximum value.

$\therefore$ At half power points direction, the electric field is $\pm \frac{1}{\sqrt{2}}$

$$E = \pm \frac{1}{\sqrt{2}}$$

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$= \pm(2n+1)\frac{\pi}{4} \qquad \text{where } n = 0, 1, 2, \ldots\ldots$$

If $n = 0$, then

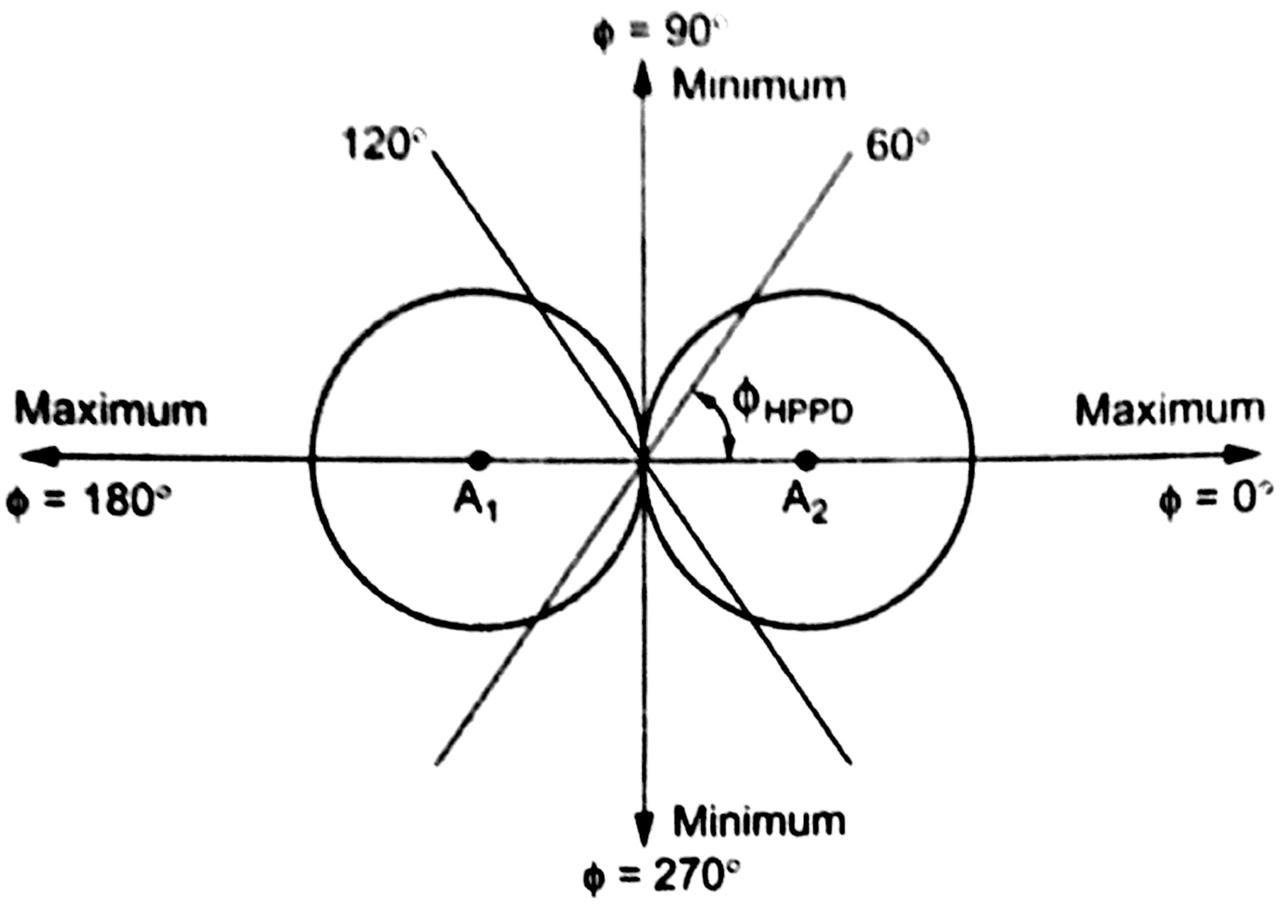
$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

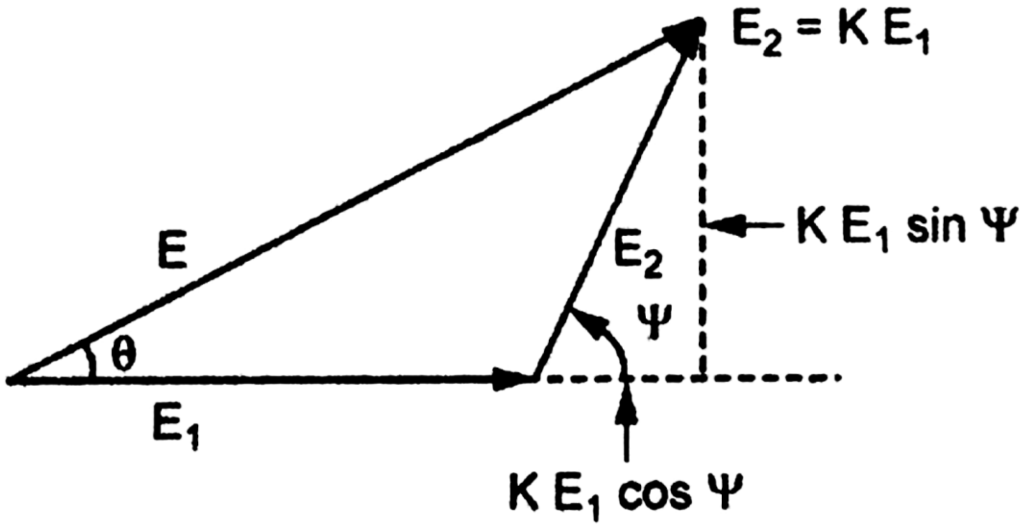
$$= 60° \text{ and } 120°$$

$$\therefore \phi_{HPPD} = 60° \text{ and } 120°$$



φ = 90°
Minimum
120°
60°
Maximum
φHPPD
Maximum
φ = 180°
A1
A2
φ = 0°
Minimum
φ = 270°

Two point sources with currents of unequal magnitudes and any phase



Vector diagram of fields E_1 and E_2

Now the total phase difference between the radiations by the 2 point sources at any far point '*p*' is given by

$$\psi = \frac{2\pi}{\lambda}\cos\phi + \alpha$$

Assume the value of 'α' as $0 < \alpha < 180°$, then the resultant field at point 'p' is given by

$$E_T = E_1 e^{j0} + E_2 e^{j\psi}$$

(Source 1 is assumed to be reference, hence phase angle is '0')

$$E_T = E_1 + E_2 e^{j\psi}$$

$$E_T = E_1 \left(1 + \frac{E_2}{E_1} e^{j\psi} \right)$$

Let

$$\frac{E_2}{E_1} = k$$

Since $E_1 > E_2$, the value of k is less than unity. ($0 \leq k \leq 1$)

$$E_T = E_1 (1 + k e^{j\psi})$$

$$\boxed{\therefore E_T = E_1 [1 + k (\cos \psi + j \sin \psi)}$$

∴ The magnitude of the resultant field at point '*p*' is given by

$$|E_T| = \{E_1 [1 + k \cos \psi + j\, k \sin \psi]\}$$

$$\boxed{|E_T| = E_1 \sqrt{(1 + k \cos \psi)^2 + (k \sin \psi)^2}}$$

The phase angle between 2 fields at the far point '*p*' is given by

$$\boxed{\theta = \tan^{-1} \frac{k \sin \psi}{1 + k \cos \psi}}$$

N Element uniform linear array

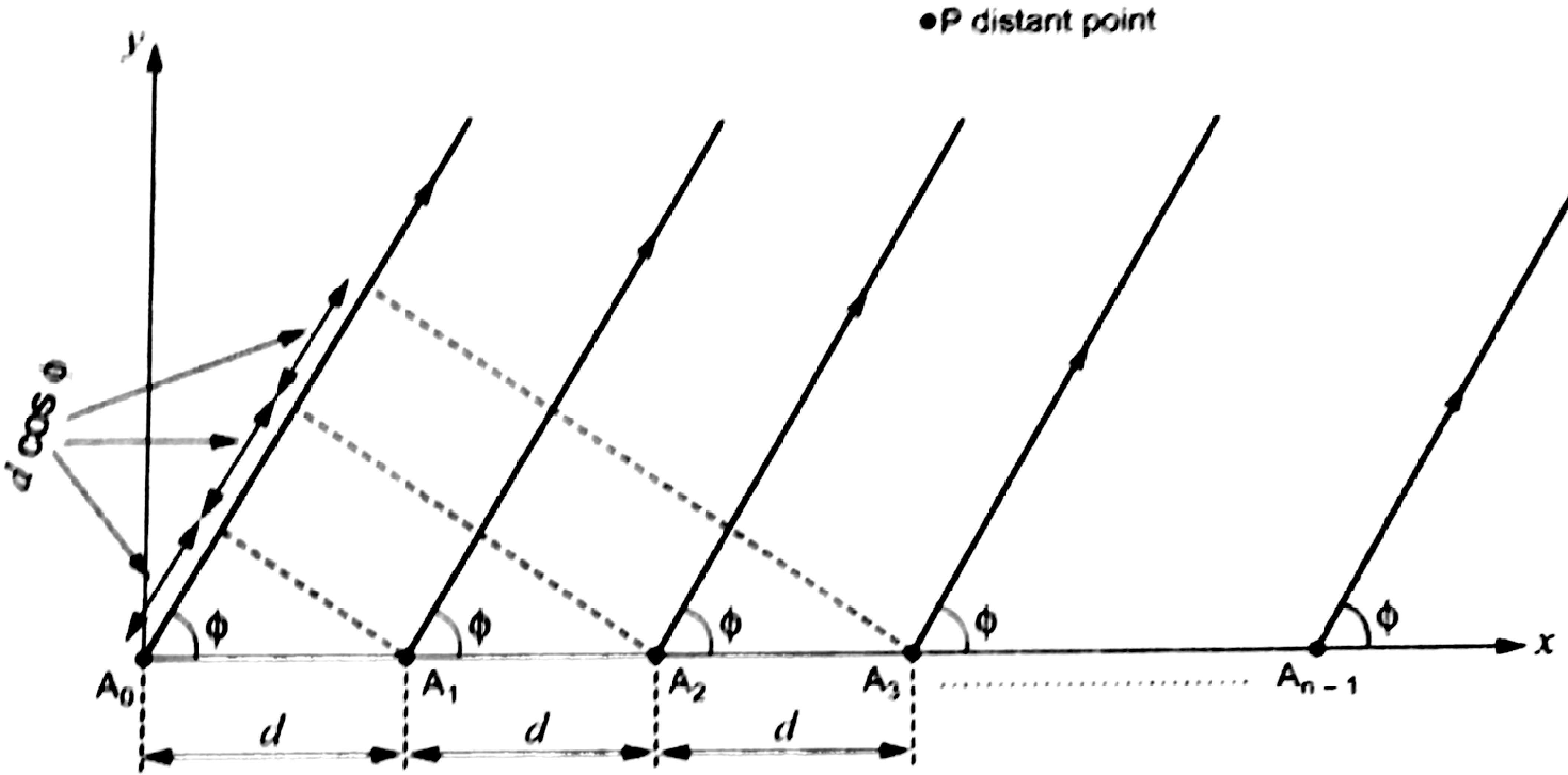
At higher frequencies, for point to point communications, it is necessary to have a pattern with single beam radiation. Such highly directive single beam pattern can be obtained by increasing the point sources in the array from 2 to '*n*' number of sources.

Linear Array

The antenna array is said to be linear if the elements of the antenna array are equally spaced along a straight line

Uniform Linear Array

The linear antenna array is said to be uniform linear array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line



Uniform linear array of 'n' elements

Consider a general 'n' element uniform linear array as shown in Fig.

Here point sources are equally spaced and fed with a current of equal amplitude and phase shift is uniform progressive phase shift.

Total field at a distant point 'p' is obtained by adding the fields due to 'n' individual sources vectorically.

$$E_t = E_0 e^{0j\psi} + E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{j3\psi} \ldots\ldots\ldots\ldots E_0 e^{j(n-1)\psi}$$

$$E_T = E_0 (1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \ldots\ldots\ldots\ldots e^{j(n-1)\psi}) \qquad \ldots\ldots(1)$$

ψ is the total phase difference of the fields at distant point 'P' from adjacent sources and it is expressed as,

$$\psi = \beta d \cos \theta + \alpha \quad radian \qquad \ldots\ldots(2)$$

where,

α is the phase difference in adjacent point sources.

$\beta\, d \cos\theta$ is the phase difference due to path difference, and

$$\text{Propagation constant } \beta = \frac{2\pi}{\lambda}$$

Multiplying equation (1) by $e^{j\psi}$ becomes,

$$E_T e^{j\psi} = E_0 \left(e^{j\psi} + e^{j2\psi} + e^{j3\psi} + e^{j4\psi} + \ldots\ldots\ldots\ldots e^{nj\psi}\right) \quad \ldots\ldots (3)$$

By subtracting equation (3) from equation (1), we get

$$E_T - E_T e^{j\psi} = E_O \left\{\left[1 + e^{j\psi} + e^{j2\psi} + \ldots\ldots\ldots + e^{j(n-1)\psi}\right] - \left[e^{j\psi} + e^{j2\psi} + \ldots\ldots + e^{jn\psi}\right]\right\}$$

$$E_T(1 - e^{j\psi}) = E_0 \left(1 - e^{jn\psi}\right)$$

$$\boxed{E_T = E_0 \left(\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right)} \quad \ldots\ldots(4)$$

Equation (4) may be written as

$$E_T = E_0 \frac{(1 - e^{jn\psi/2} \cdot e^{jn\psi/2})}{(1 - e^{j\psi/2} \cdot e^{j\psi/2})}$$

$$= E_0 \frac{(e^{jn\psi/2} \cdot e^{-jn\psi/2} - e^{jn\psi/2} \cdot e^{jn\psi/2})}{e^{j\psi/2} \cdot e^{-j\psi/2} - e^{j\psi/2} \cdot e^{j\psi/2}}$$

$$= E_0 \left[\frac{e^{j\frac{n\psi}{2}} \left(e^{-j\frac{n\psi}{2}} - e^{j\frac{n\psi}{2}} \right)}{e^{j\frac{\psi}{2}} \left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}} \right)} \right]$$

According to trigonometric identity,

$$e^{-j\theta} - e^{j\theta} = -2j\sin\theta \qquad \ldots\ldots (5)$$

Using the equation (5), then the resultant field in equation (4) becomes,

$$E_T = E_O\left[\frac{\left(-2j\sin\frac{n\psi}{2}\right)e^{j\frac{n\psi}{2}}}{\left(-2j\sin\frac{n\psi}{2}\right)e^{j\frac{\psi}{2}}}\right]$$

$$E_T = E_0 \cdot e^{j\left(\frac{n-1}{2}\right)\psi}\,\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} \qquad \ldots\ldots (6)$$

The phase angle of the resultant field at point P is given as

$$\phi = \frac{(n-1)}{2}\psi = \left(\frac{n-1}{2}\right)\beta d\cos\theta + \alpha \text{ (from equation 2)} \qquad .....(7)$$

Then, the equation (6) becomes,

$$E_T = E_0\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right]e^{j\phi} = E_0\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right](\cos\phi + j\sin\phi)$$

$$E_T = E_0\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right]\angle\phi \qquad ......(8)$$

This equation (8) indicates the resultant field due to 'n' element linear array at distant point P. The magnitude of the resultant field is given as

$$E_T = E_0 \left[\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] \qquad \text{......(9)}$$

maximum value of E_T is 'n' times the field from a single source

$$E_{T(Max)} = E_0 n$$

$$E_{Nor} = \frac{E_T}{E_{T(Max)}} = \frac{E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}}{E_0 n}$$

$$\boxed{E_{Nor} = \frac{\sin \frac{n\psi}{2}}{n \sin \frac{\psi}{2}} = (Array\ Factor)_n}$$

Pattern Multiplication

“The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source patterns and the pattern of array of isotropic point sources each located at the phase center of individual source and having the relative amplitude and phase, whereas the total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources”

The total field pattern of an array of non-isotropic but similar sources may be expressed as

$$Total\ Field\,(E_T) = (Multiplication\ of\ field\ pattern) \times (Addition\ of\ phase\ pattern)$$

$$E_T = \{E_i(\theta, \phi) \times E_a(\theta, \phi)\} \times \{E_{pi}(\theta, \phi) + E_{pa}(\theta, \phi)\}$$

where,

$E_i(\theta, \phi)$ = Field pattern of individual source,

$E_a(\theta, \phi)$ = Field pattern of array of isotropic point sources,

$E_{pi}(\theta, \phi)$ = Phase pattern of individual source,

$E_{pa}(\theta, \phi)$ = Phase pattern of array of isotropic point sources,

θ – Polar angles, and

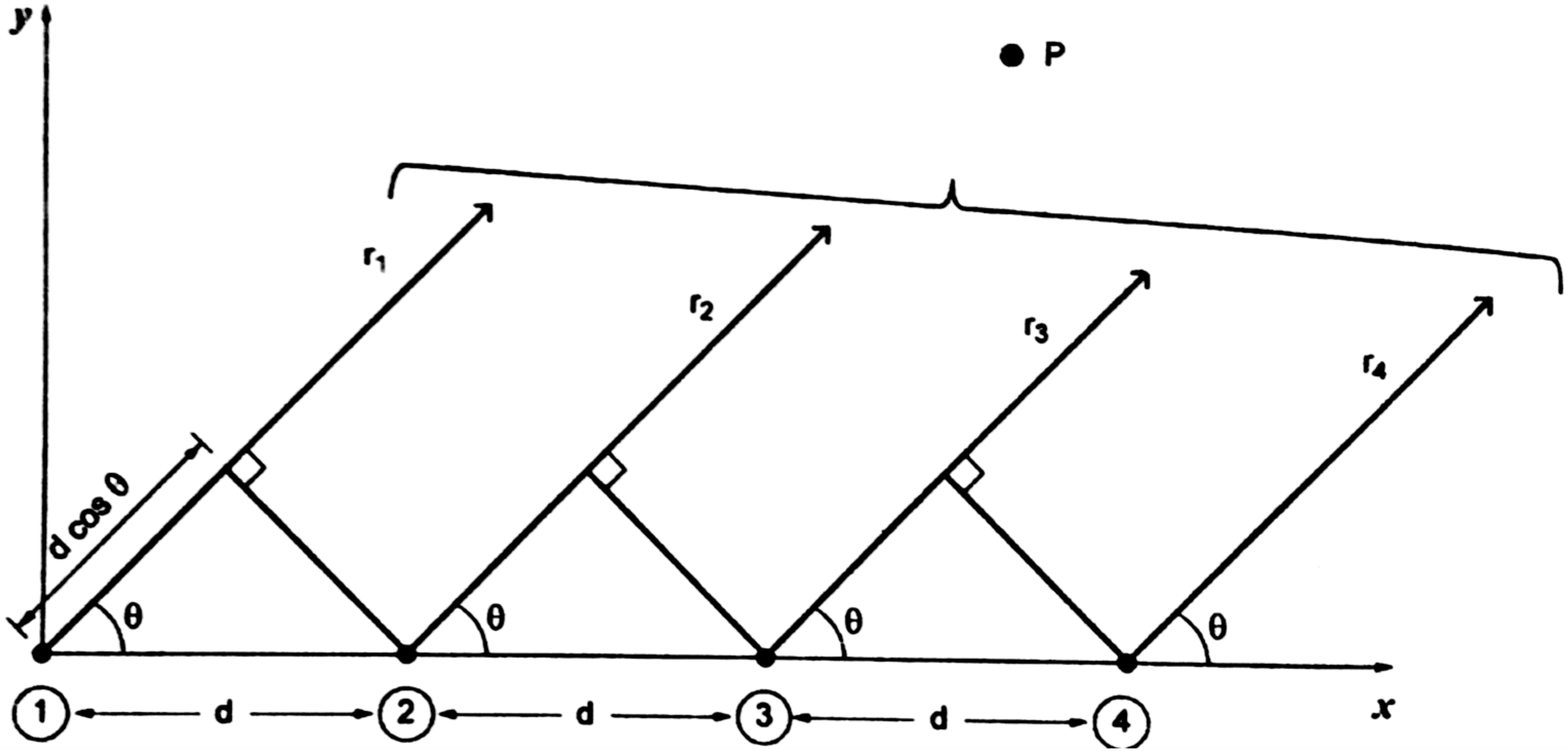
ϕ – Azimuth angles.

Advantages of pattern multiplication

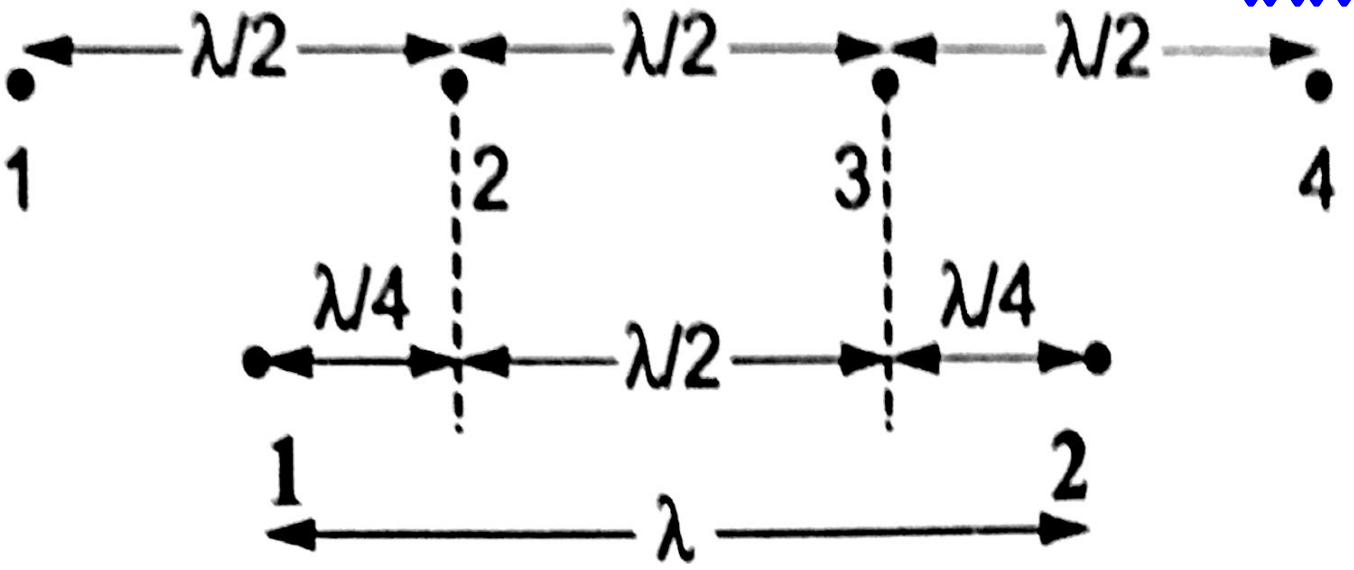
(*i*) It is a speedy method for sketching the pattern of complicated arrays just by inspection, and

(*ii*) It is a useful tool in the design of antenna arrays.

RADIATION PATTERN OF 4-ISOTROPIC ELEMENTS FED IN PHASE, SPACED $\frac{\lambda}{2}$ APART



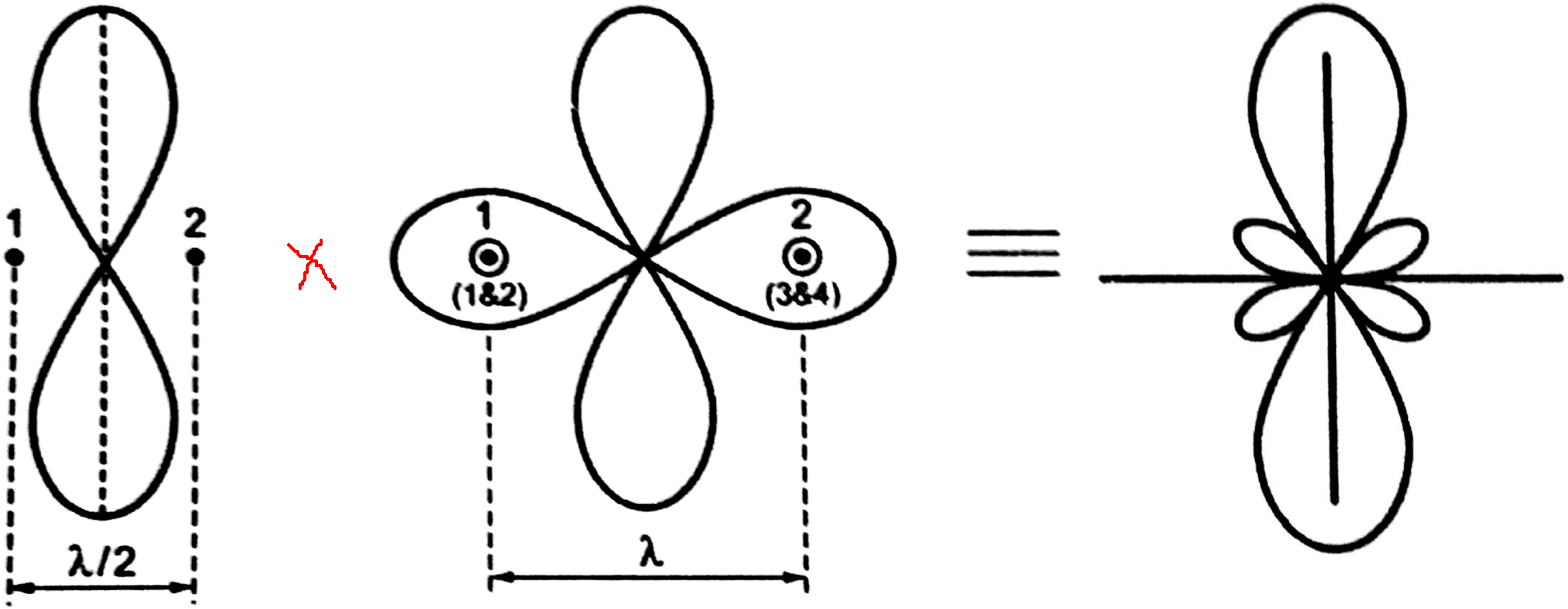
Linear array of 4 isotropic elements spaced $\frac{\lambda}{2}$ apart, fed in phase



Two units array spaced at λ

Two isotropic point source spaced λ/2 apart fed in phase provides a ***bidirectional pattern***. According to pattern multiplication, the radiation pattern of 4 elements is obtained as,

$$\left\{\begin{array}{c}\therefore \text{ Resultant radiation} \\ \text{pattern of 4 elements}\end{array}\right\} = \left\{\begin{array}{c}\text{Radiation pattern} \\ \text{of individual} \\ \text{elements}\end{array}\right\} \times \left\{\begin{array}{c}\text{Array of} \\ \text{two units} \\ \text{spaced '}\lambda\text{'}\end{array}\right\}$$



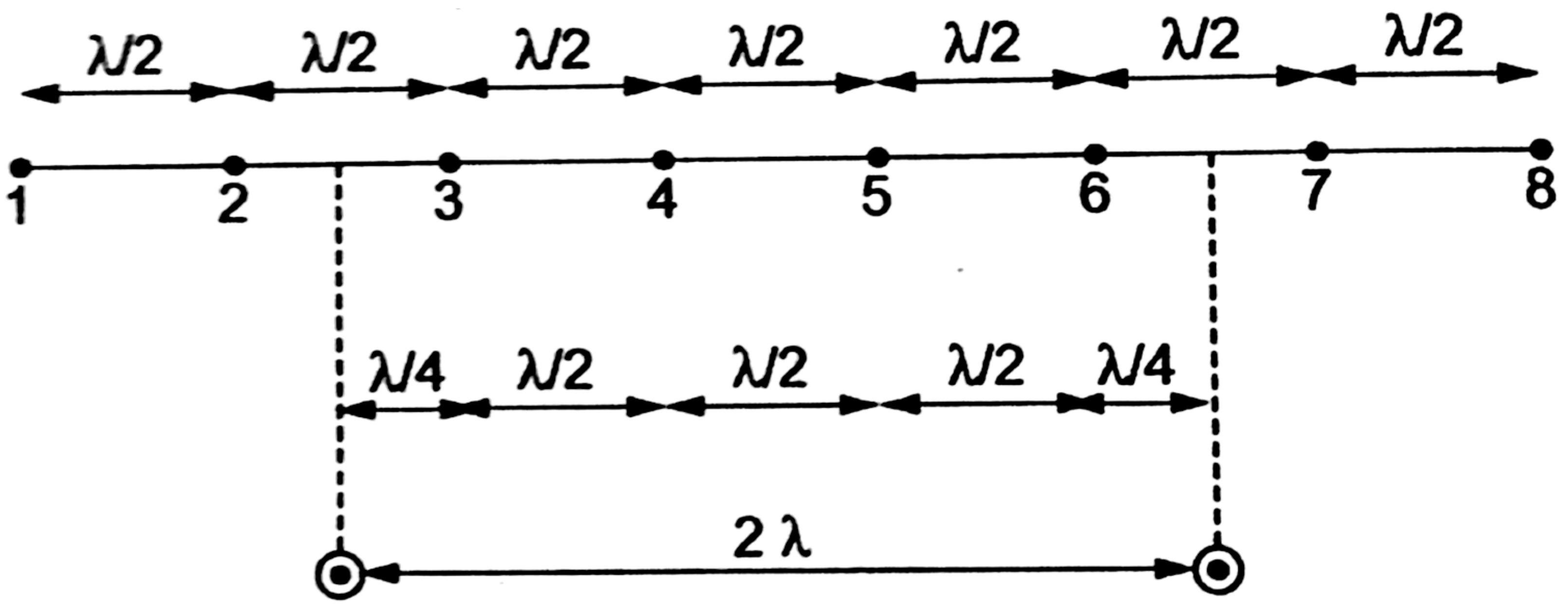
Individual (unit pattern) pattern due to 2 individual elements

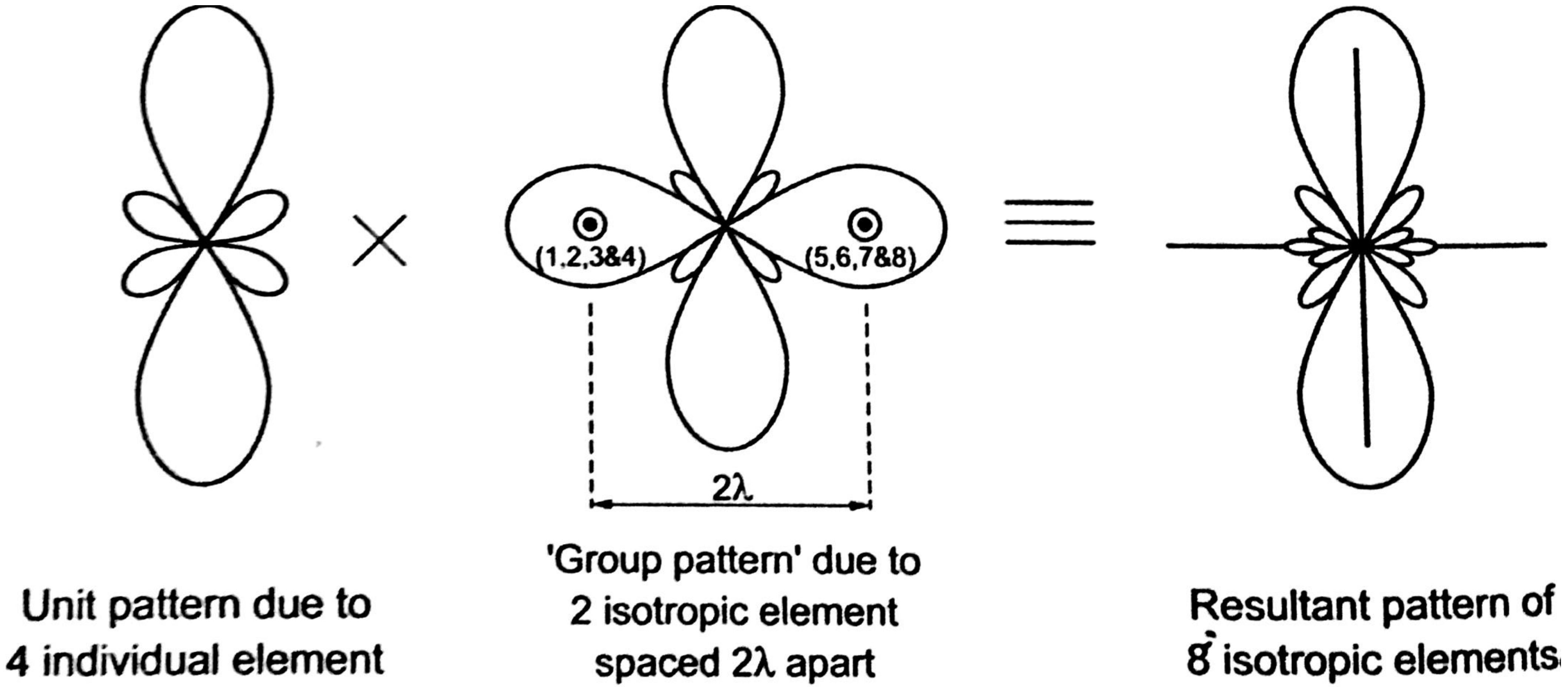
Group pattern due to array of two isotropic separated by λ

Resultant pattern of 4 isotropic elements

Resultant radiation pattern of 4 isotropic elements by pattern multiplication

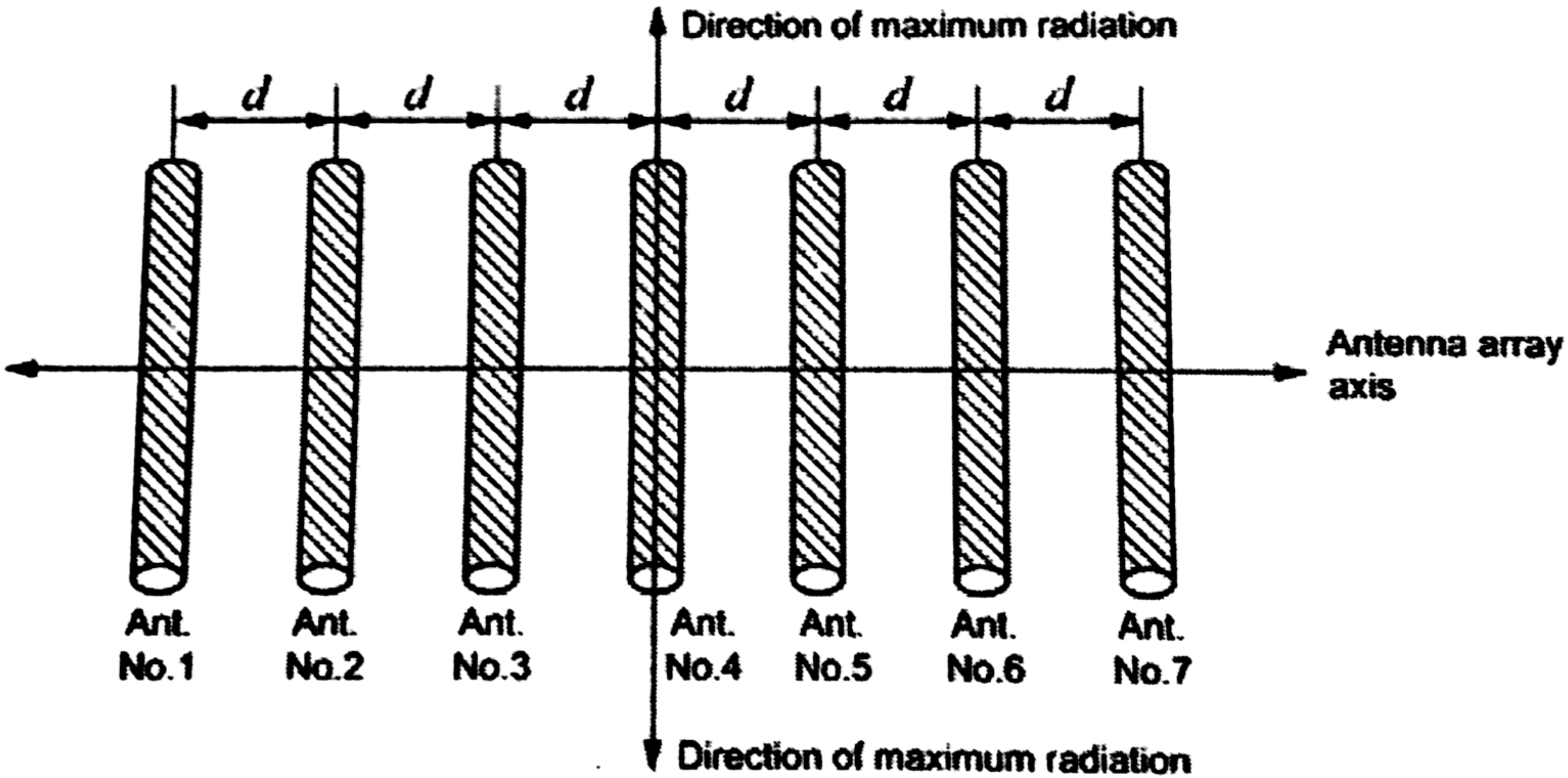
RADIATION PATTERN OF 8-ISOTROPIC ELEMENTS FED IN PHASE, AND $\frac{\lambda}{2}$ APART





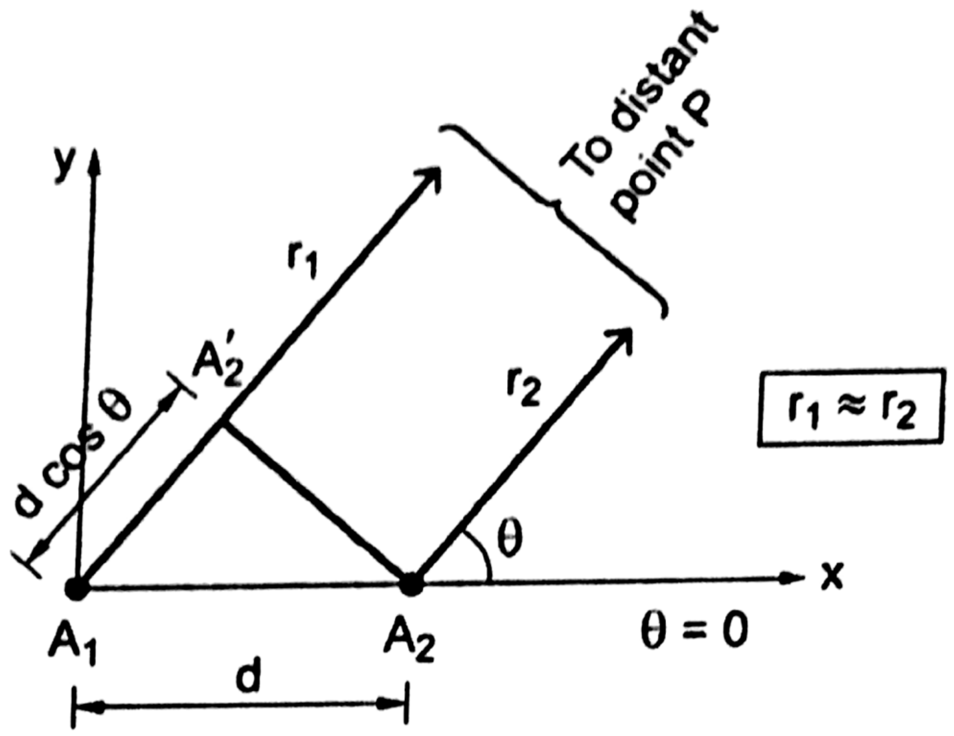
Resultant radiation pattern of 8-isotropic elements by pattern multiplication

Broadside Array



Broadside array of antennas

Array of 'n' isotropic sources of equal amplitude and spacing - Broadside Array



Two sources of equal amplitude and phase, separated by a distance 'd'

path difference $(A_1 A_2^1) = d \cos \theta$ meter

In terms of wavelength,

$$\text{Path difference} = \frac{d \cos\theta}{\lambda} \qquad \ldots\ldots(1)$$

$$\text{Phase angle } \psi = 2\pi \times \text{path difference}$$

$$= 2\pi\left(\frac{d \cos\theta}{\lambda}\right)$$

$$\psi = \frac{2\pi}{\lambda} d \cos\theta \quad \text{radians} \qquad \ldots\ldots(2)$$

$$\boxed{\psi = \beta d \cos\theta \quad \text{radians}} \qquad \ldots\ldots(3)$$

$$\psi = \beta d \cos\theta + \alpha \qquad \ldots\ldots(4)$$

1. Maxima Direction for Major Lobe

An array is said to be broadside array, if the phase angle makes maximum radiation perpendicular to the line of array. i.e. ***90° and 270°***. In the broad side array, all sources are in phase. i.e. $\alpha = 0$ *and* $\psi = 0$.

$$\psi = \beta d \cos\theta + \alpha = 0 \quad \ldots\ldots(5)$$

$$\beta d \cos \theta_{Max} = 0$$

$$\cos\theta_{Max} = 0$$

$$\boxed{\theta_{Max} = 90° \text{ or } 270°}$$

The major lobes maxima occurs in these directions

2. Maxima Direction for Minor Lobes

The minor lobe maxima occurs between first nulls and higher order nulls. The ***nulls*** are the directions through which an array ***radiate zero power***.

The total far field strength for array of '*n*' isotropic point sources of equal amplitude and spacing is expressed as,

$$E_T = E_0 \left[\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] \qquad \ldots\ldots(6)$$

In the above expression, E_T is maximum, when numerator is maximum. *i.e.*

$\sin \frac{n\psi}{2}$ is maximum provided $\sin \frac{\psi}{2} \neq 0$.

$$\therefore \qquad \sin\frac{n\,\psi}{2} = 1$$

$$\frac{n\,\psi}{2} = \pm(2\,N+1)\frac{\pi}{2} \qquad \text{where, } N = 1, 2, 3, 4\ldots\ldots\ldots$$

N is a constant and N = 0 corresponds to major lobe maxima where, '*n*' indicates the number of isotropic elements.

$$\frac{\psi}{2} = \pm(2\,N+1)\frac{\pi}{2\,n}$$

$$\psi = \pm(2\,N+1)\frac{\pi}{n} \qquad \ldots\ldots(7)$$

Equating equation (5) and equation (7), we get

$$\beta\,d\cos(\theta_{Max})_{minor} + \alpha = \pm(2\,N+1)\frac{\pi}{n}$$

$$\beta\,d\cos(\theta_{Max})_{minor} = \pm(2\,N+1)\frac{\pi}{n} - \alpha$$

$$\cos(\theta_{Max})_{minor} = \pm \frac{(2N+1)\frac{\pi}{n} - \alpha}{\beta d}$$

$$(\theta_{Max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{(2N+1)\pi}{n} - \alpha\right]\right\} \qquad \ldots\ldots(8)$$

For a broadside array $\alpha = 0$, then equation (8) becomes

$$(\theta_{Max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{(2N+1)\pi}{n}\right]\right\}$$

By substituting the propagation constant $\beta = \frac{2\pi}{\lambda}$ in the above expression, we get

$$\boxed{(\theta_{Max})_{minor} = \cos^{-1}\left\{\pm \frac{(2N+1)\lambda}{2nd}\right\}} \qquad \ldots\ldots(9)$$

where, $(\theta_{Max})_{min\,or}$ = Maxima direction of minor lobes

Consider n = 4, d = λ/2, N = 1 then equation (9) becomes

$$(\theta_{Max})_{minor} = \cos^{-1}\left\{\pm \frac{(2+1)}{2 \times 4 \times \frac{\lambda}{2}} \cdot \lambda\right\}$$

$$= \cos^{-1}\left(\pm \frac{3}{4}\right)$$

$$\boxed{(\theta_{Max})_{minor} = \pm 41.4^\circ \text{ or } \pm 138.6^\circ}$$

∴ Thus $+41.4^\circ$, $+138.6^\circ$, -41.4° and $+138.6^\circ$ are the 4 minor lobe maxima of the array of 4 isotropic sources fed in phase and spaced $\frac{\lambda}{2}$ apart. No other maxima exist for $N \geq 2$, because for N = 2, $\cos(\theta_{max})_{minor} = \pm 5/4$ which is >> 1, whereas cosine value is always << 1.

3. Minima Directions for Minor Lobes

Minima is the direction through which an array radiate zero power. It is otherwise called as ***null direction*** and the electric field intensity is zero along the null direction.

The direction of minima of minor lobes is the array of 'n' isotropic sources of equal amplitude and phase is given as,

$$E_T = E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} = 0$$

Minima occurs, when $\sin \frac{n\psi}{2} = 0$

$$\frac{n\,\psi}{2} = \pm N\pi \qquad \text{where, } N = 1, 2, 3 \ldots\ldots\ldots$$

$$\psi = \pm \frac{2N\pi}{n} \qquad \ldots\ldots(10)$$

But, $\psi = \beta\, d \cos\phi + \alpha$, in equation (10), we get

$$\beta\, d\, (\cos\theta_{Min})_{minor} + \alpha = \pm \frac{2N\pi}{n}$$

For broad side array, $\alpha = 0$, then

$$\beta\, d\, (\cos\theta_{Min})_{minor} = \pm \frac{2N\pi}{n}$$

$$\cos(\theta_{Min})_{minor} = \frac{1}{\beta\, d}\left\{\pm \frac{2N\pi}{n}\right\}$$

$$(\theta_{Min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n}\right\}\right] \qquad \ldots\ldots(11)$$

By substituting the propagation constant, $\beta = \frac{2\pi}{\lambda}$ in equation (11), we get

$$= \cos^{-1}\pm\left[\frac{1}{\frac{2\pi}{\lambda}d}\left\{\pm\frac{2N\pi}{n}\right\}\right]$$

$$\boxed{(\theta_{Min})_{minor} = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]} \qquad \ldots\ldots(12)$$

where, $(\theta_{Min})_{min\,or}$ = Direction of minor lobe minima

For example,

(i) If $N = 1, n = 4$ and $d = \lambda/2$

$$(\theta_{Min})_{minor} = \cos^{-1} \pm \frac{1 \cdot \lambda}{4 \cdot \frac{\lambda}{2}} = \cos^{-1}\left[\pm \frac{1}{2}\right]$$

$$(\theta_{Min})_{minor} = \pm 60, \pm 120°$$

(ii) If $N = 2$,

$$(\theta_{Min})_{minor} = \cos^{-1}\left[\pm \frac{2 \cdot \lambda}{4 \cdot \frac{\lambda}{2}}\right] = \cos^{-1}[\pm 1]$$

$$= \pm 0°, \pm 180° = 0°, 180°$$

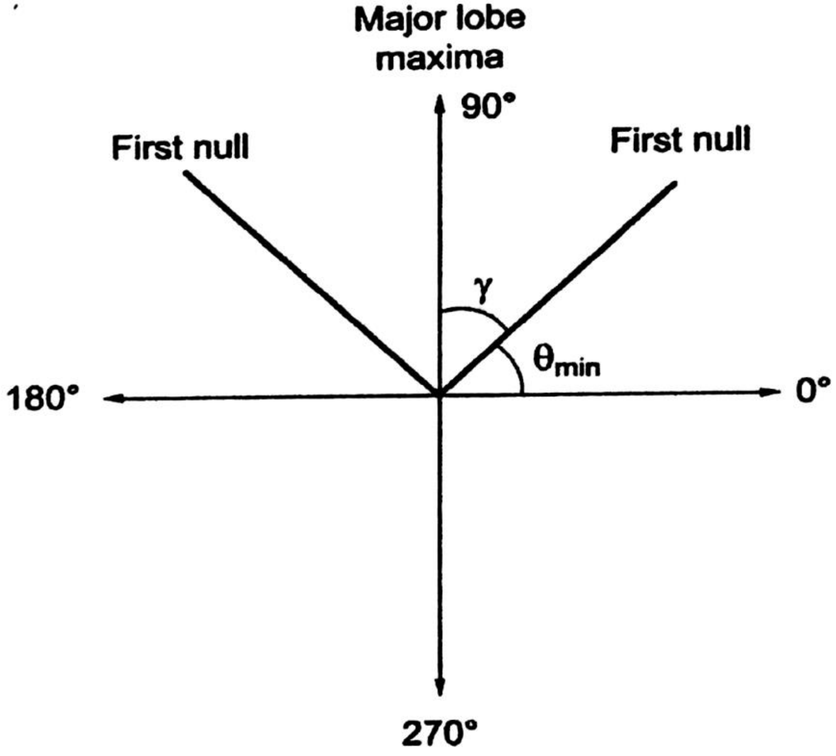
Thus 0°, 60°, 120°, 180°, – 60°, – 120° are the six minor lobe minima of the array of 4 isotropic sources spaced $\frac{\lambda}{2}$ apart. No other minima exist for which cosine functions becomes more than one which is not possible.

4. Beam Width of Major Lobe

(i) Beam width between First Null (BWFN)

BWFN is defined as,

The angle between first nulls (2γ) or double the angle between first null and major lobe in the maxima directions.



From Fig. $\gamma = 90 - \theta_{min} \Rightarrow \theta_{min} = 90° - \gamma$ ...... (13)

$$\text{Beam width (BW)} = 2 \times \left\{ \begin{array}{c} \text{Angle between first null and} \\ \text{maximum of major lobe} \end{array} \right\}$$

$$\boxed{BW = 2 \times \gamma} \qquad \text{...... (14)}$$

By substituting equation (13) in equation (12), we get

$$90° - \gamma = \cos^{-1}\left\{ \pm \frac{N\lambda}{n\,d} \right\}$$

$$\cos(90° - \gamma) = \pm \frac{N\lambda}{n\,d}$$

$$\sin\gamma = \pm \frac{N\lambda}{n\,d} \qquad [\sin\gamma = \gamma \text{ when '}\gamma\text{' is very small}]$$

$$\boxed{\gamma = \pm \frac{N\lambda}{n\,d}} \qquad \text{......(15)}$$

First null occurs, when N = 1

$$\gamma_1 = \pm \frac{\lambda}{n\,d} \qquad \text{......(16)}$$

From equation (14),

$$\text{BWFN} = 2 \times \gamma_1 = \frac{2\,\lambda}{n\,d} \qquad \text{......(17)}$$

Let L = Total length of the array in meters.

$$L = (n-1)\,d \approx n\,d \qquad (\text{if } n \text{ is large}) \qquad \text{......(18)}$$

By substituting equation (18) in equation (16), we get

$$\therefore \quad 2\,\gamma_1 = \frac{2\,\lambda}{L} = \frac{2}{L/\lambda} \text{ radian}$$

$$= \frac{2}{L/\lambda} \times 57.3 \text{ degree} = \frac{114.6^\circ}{L/\lambda}$$

$$\boxed{\text{BWFN} = \frac{114.6^\circ}{L/\lambda}} \qquad \text{......(19)}$$

(ii) Half Power Beam Width (HPBW)

$$HPBW = \frac{1}{2} BWFN \qquad \text{......(20)}$$

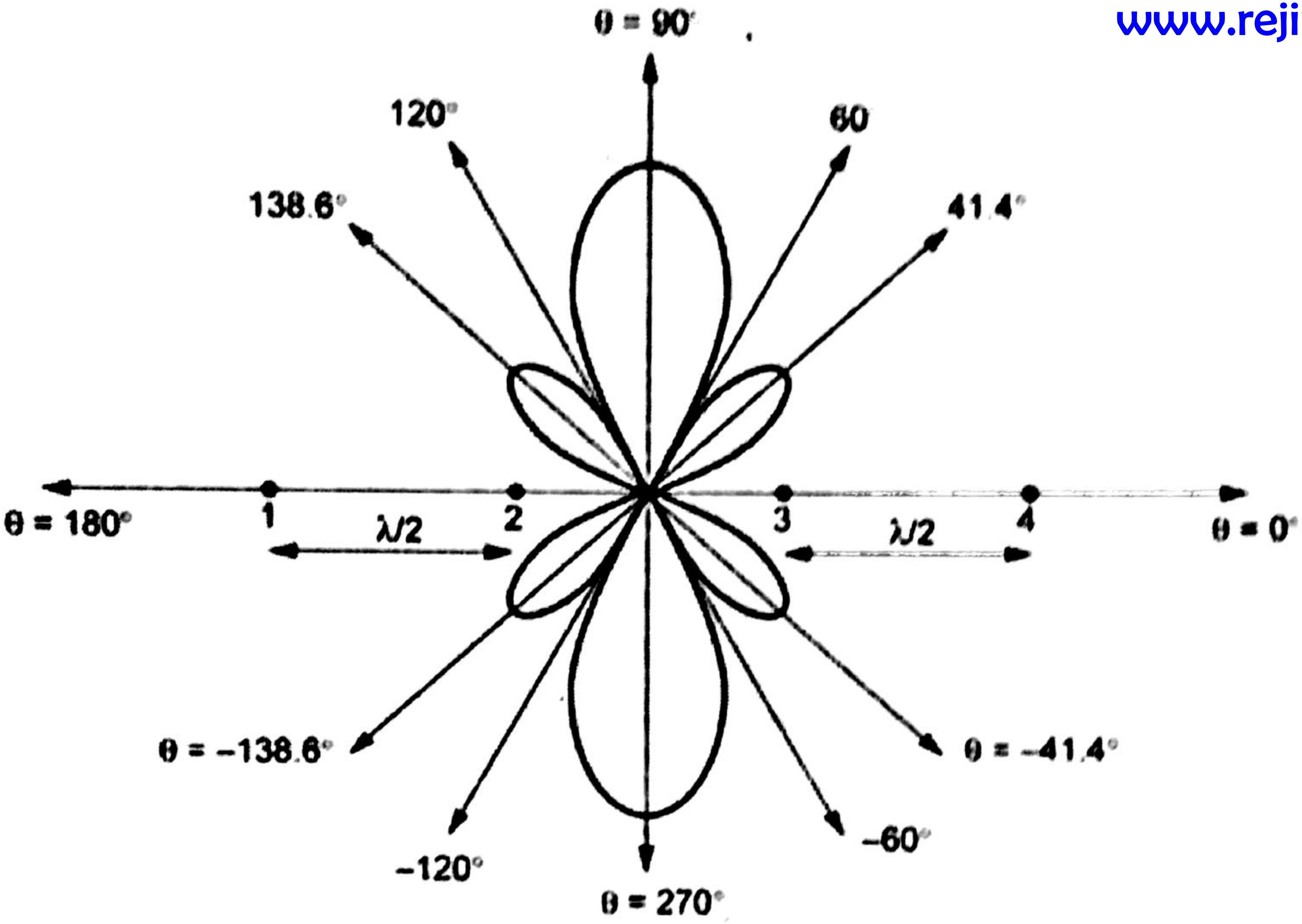
By substituting equation (19) in equation (20), we get

$$= \frac{57.3°}{L/\lambda}$$

$$\boxed{HPBW = \frac{57.3°}{L/\lambda}} \qquad \text{......(21)}$$

5. Directivity

$$D = 2n\left(\frac{d}{\lambda}\right) = 2\left(\frac{L}{\lambda}\right) \qquad \text{......(22)}$$

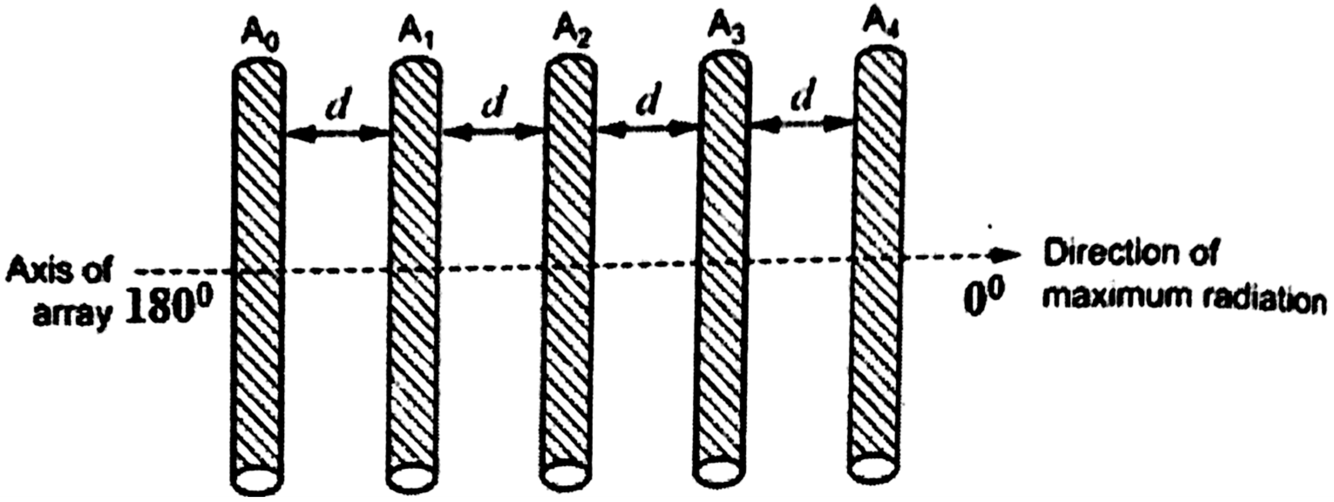


Field pattern of broadside array consisting of four isotropic sources of equal amplitude and in phase

End Fire Array

An array is said to be end fire, if the direction of maximum radiation coincides with the array axis to get unidirectional radiation.

In the end fire array, number of identical antennas are spaced equally along a line. All the antennas are fed individually with currents of equal magnitudes but their phases vary progressively along the line to make the entire arrangement to get unidirectional radiation along the axis of the array.



End fire array

For an array to be end fire, the phase angle is such that it makes the maximum radiation in the line of array. *i.e.*, $\theta = 0°$ or $180°$. The total phase difference is expressed as

$$\psi = \beta d \cos\theta + \alpha \qquad \text{......(1)}$$

For end fire array $\psi = 0$ and $\theta = 0°$ (or) $180°$, then the equation (1) becomes,

$$\beta d \cos 0° = -\alpha$$

$$\boxed{\alpha = -\beta d = \frac{-2\pi}{\lambda} d} \qquad \text{......(2)}$$

For an example, if spacing between 2 sources is $\frac{\lambda}{2}$ (or) $\frac{\lambda}{4}$, then the phase angle by which source 2 lags behind source 1 is

$$\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ (or) } \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \text{ radians}$$

1. Maxima Direction for Minor Lobe

The total field strength of 'n' element uniform linear array should be maximum for these directions.

$$E_T = E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}$$

E_T will be maximum, when

$$\sin \frac{n\psi}{2} = 1 \quad \text{if } \sin \frac{\psi}{2} \neq 0$$

$$\frac{n\psi}{2} = \sin^{-1}(1) = \pm (2N+1)\frac{\pi}{2}$$

where, N = 1, 2, 3.........and N = 0 corresponds to major lobe maxima.

$$\psi = \pm \frac{(2N+1)\pi}{n} \qquad ......(3)$$

By substituting equation (2) in equation (1),

$$\psi = \beta d \cos\theta - \beta d$$

$$\psi = \beta d (\cos\theta - 1) \qquad \ldots\ldots(4)$$

Equating equation (3) and equation (4), we get

$$\beta d (\cos\theta - 1) = \pm \frac{(2N+1)\pi}{n}$$

$$\cos\theta - 1 = \pm \frac{(2N+1)\pi}{n \beta d}$$

$$\cos\theta = 1 \pm \frac{(2N+1)\pi}{n \beta d}$$

$$(\theta_{Max})_{minor} = \cos^{-1}\left[1 \pm \frac{(2N+1)\pi}{n \beta d}\right] \qquad \ldots\ldots(5)$$

By substituting the propagation constant, $\beta = \frac{2\pi}{\lambda}$ in equation (5)

$$(\theta_{Max})_{minor} = \cos^{-1}\left[1 \pm \frac{(2N+1)\lambda}{2nd}\right] \quad \ldots\ldots(6)$$

For example, $n = 4$, and $d = \frac{\lambda}{2}$

$$(\theta_{Max})_{minor} = \cos^{-1}\left[1 \pm \frac{(2N+1)\lambda}{2.4.\frac{\lambda}{2}}\right]$$

$$(\theta_{Max})_{minor} = \cos^{-1}\left(1 \pm \frac{(2N+1)}{4}\right) \quad \ldots\ldots(7)$$

If $N = 1$,

$$(\theta_{Max})_{minor} = \cos^{-1}\left(1 \pm \frac{3}{4}\right)$$

$$= \cos^{-1}\left(\frac{1}{4}\right) \text{ [or] } \cos^{-1}\left(\frac{7}{4}\right) \Rightarrow \text{is invalid}$$

$$\boxed{(\theta_{Max})_{minor} = \cos^{-1}\left(\frac{1}{4}\right) = 75.5^\circ}$$

If $N = 2$,

$$(\theta_{Max})_{minor} = \cos^{-1}\left(1 \pm \frac{5}{4}\right)$$

$$= \cos^{-1}\left(\frac{-1}{4}\right) \text{ [or] } \cos^{-1}\left(\frac{9}{4}\right) \Rightarrow \text{is invalid}$$

$$= \cos^{-1}\left(\frac{-1}{4}\right)$$

$$\boxed{\therefore (\theta_{Max})_{minor} = -75.5^\circ}$$

2. Minima Direction for Minor Lobe

Minima is the direction through which the array radiate zero power, which is also called as null direction. The electric field intensity is zero along the null direction.

$$E_T = E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} = 0$$

E_T is zero, when $$\sin \frac{n\psi}{2} = 0$$

$$\frac{n\psi}{2} = \sin^{-1}(0) = \pm N\pi$$

Where N = 1, 2, 3......... and N = 0 corresponds to major lobe

$$\boxed{\psi = \pm \frac{2N\pi}{n}} \qquad \text{......(8)}$$

Equating the equations (4) and (8), we get

$$\beta\, d\, [\cos(\theta_{Min})_{minor} - 1] = \pm \frac{2N\pi}{n}$$

$$\cos(\theta_{Min})_{minor} - 1 = \pm \frac{2N\pi}{\beta\, n\, d}$$

$$= \pm \frac{2N\pi}{\frac{2\pi}{\lambda} \times d \times n}$$

$$\cos(\theta_{Min})_{minor} - 1 = \pm \frac{N\lambda}{nd} \qquad \text{......(9)}$$

$$1 - 2\sin^2 \frac{(\theta_{Min})_{minor}}{2} - 1 = \pm \frac{N\lambda}{n\, d} \qquad [\because \cos\theta = 1 - 2\sin^2 \frac{\theta}{2}]$$

$$-2\sin^2 \frac{(\theta_{Min})_{minor}}{2} = \pm \frac{N\lambda}{n\, d}$$

$$2\sin^2\frac{(\theta_{Min})_{min\,or}}{2} = \pm\frac{N\lambda}{nd}$$

$$\sin\frac{(\theta_{Min})_{min\,or}}{2} = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\frac{(\theta_{Min})_{min\,or}}{2} = \sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$(\theta_{Min})_{min\,or} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right) \qquad \ldots\ldots(10)$$

For example; if $n = 4$ and $d = \lambda/2$

(i) $N = 1$, $$(\theta_{Min})_1 = 2\sin^{-1}\left(\pm\sqrt{\frac{1\times\lambda}{2\times4\times\frac{\lambda}{2}}}\right)$$

$$= 2\sin^{-1}\left(\pm\sqrt{\frac{1}{4}}\right) = 2\sin^{-1}\left(\pm\frac{1}{2}\right)$$

$$= 2 \times (\pm 30^\circ)$$

$$(\theta_{Min})_1 = \pm 60^\circ$$

(ii) $N = 2$,

$$(\theta_{Min})_2 = 2\sin^{-1}\left(\pm\sqrt{\frac{2\times\lambda}{2\times 4\times\frac{\lambda}{2}}}\right)$$

$$= 2\sin^{-1}\left(\pm\sqrt{\frac{2}{4}}\right) = 2\sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$= 2 \times (\pm 45^\circ)$$

$$(\theta_{Min})_2 = \pm 90^\circ$$

(iii) N = 3,

$$(\theta_{Min})_3 = 2\sin^{-1}\left(\pm\sqrt{\frac{3\times\lambda}{2\times4\times\frac{\lambda}{2}}}\right)$$

$$= 2\sin^{-1}\left(\pm\sqrt{\frac{3}{4}}\right) = 2\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

$$= 2\times(\pm 60^\circ)$$

$$(\theta_{Min})_3 = \pm 120^\circ$$

(iv) N = 4,

$$(\theta_{Min})_4 = 2\sin^{-1}\left(\pm\sqrt{\frac{4\times\lambda}{2\times4\times\frac{\lambda}{2}}}\right) = 2\sin^{-1}\left(\pm\sqrt{\frac{4}{4}}\right) = 2\sin^{-1}(\pm 1)$$

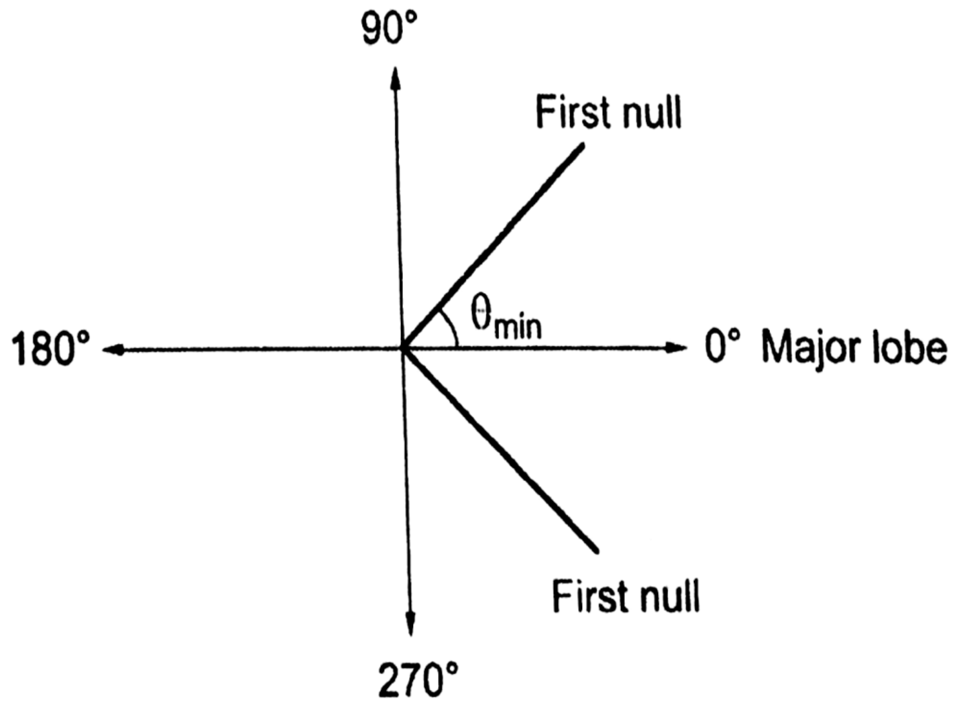
$$= 2\times(\pm 90^\circ)$$

$$(\theta_{Min})_4 = \pm 180^\circ$$

Therefore for an end fire array of 4 isotropic sources spaced $\lambda/2$ apart, there are six minor lobe maxima along the directions $\pm 60°$, $\pm 90°$, $\pm 120°$ and $\pm 180°$.

3. Beam Width for Major Lobe

(i) BWFN



Beam width of end fire array

$$\text{BWFN} = 2 \times \left\{ \begin{array}{c} \text{Angle between first nulls and the} \\ \text{maximum of major lobes} \end{array} \right\}$$

$$= 2 \times \theta_{min} \quad \ldots\ldots(11)$$

From equation (10),

$$\theta_{\min} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$\sin\theta_{\min} = 2\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right) \quad \ldots\ldots(12)$$

For small angles $\sin\theta_{\min} \approx \theta_{\min}$, then the equation (12) becomes

$$\theta_{\min} = \pm\sqrt{\frac{2N\lambda}{nd}} \quad \ldots\ldots(13)$$

Let L = Total length of the array in meters.

$$L = (n-1)\,d \approx n\,d \text{ (if } n \text{ is large)} \quad \ldots\ldots(14)$$

Therefore equation (13) becomes,

$$\theta_{min} = \pm\sqrt{\frac{2Nd}{L}} \qquad \ldots\ldots(15)$$

By substitute equation (15) in equation (11), we get

$$BWFN = 2\times\theta_{min} = 2\times\left(\pm\sqrt{\frac{2N\lambda}{L}}\right) \qquad \ldots\ldots\ldots(16)$$

(a) If $N = 1$,

$$BWFN = \pm 2\sqrt{\frac{2}{L/\lambda}} = \pm\frac{2\sqrt{2}}{\sqrt{\frac{L}{\lambda}}} \text{ radians}$$

$$= \pm\frac{2\sqrt{2}}{\sqrt{\frac{L}{\lambda}}}\times 57.3 \text{ degrees}$$

$$\text{BWFN} = \pm 114.6\sqrt{\frac{2}{L/\lambda}} \text{ degrees} \qquad \text{......(17)}$$

(ii) HPBW

$$\text{HPBW} = \frac{\text{BWFN}}{2}$$

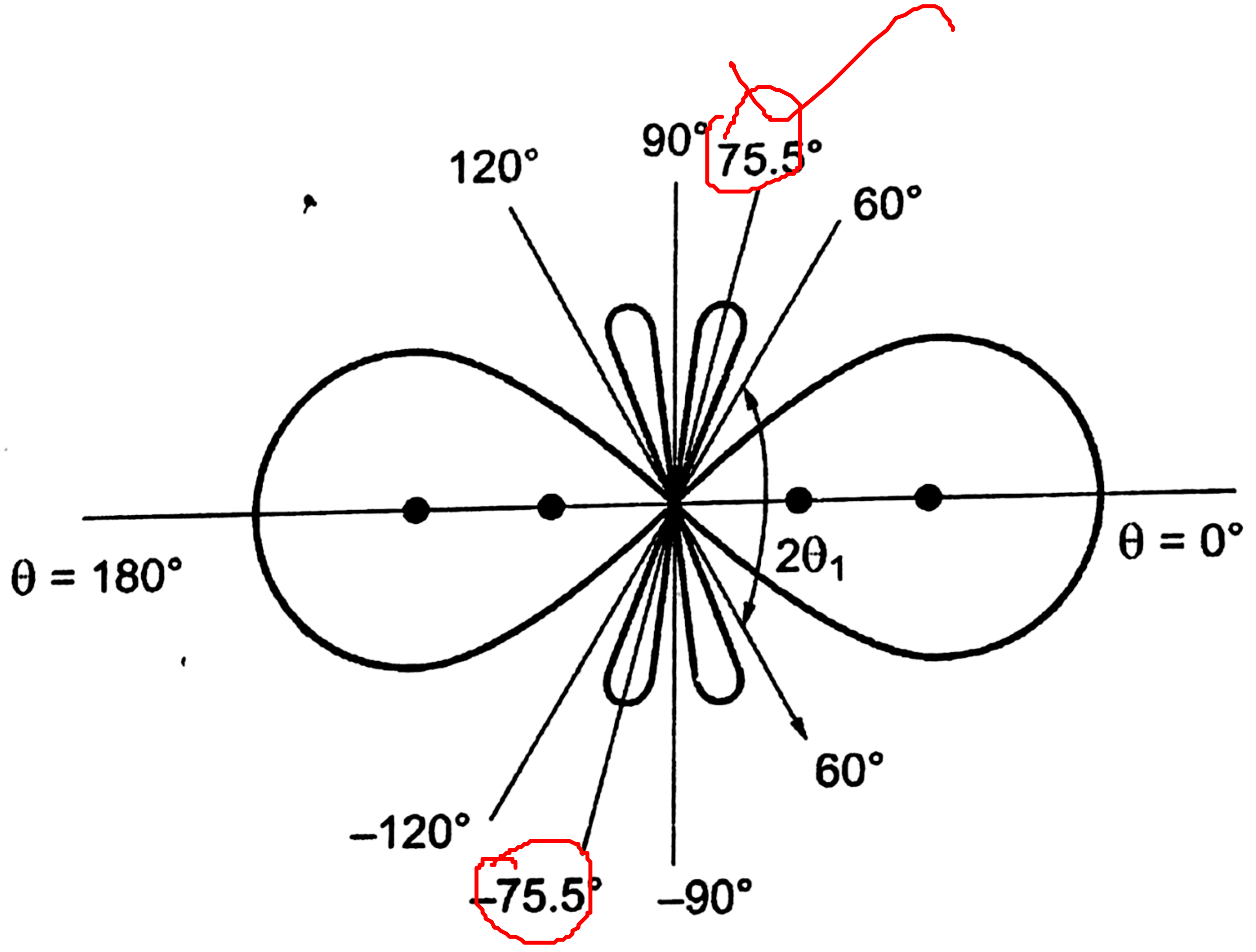
$$\text{HPBW} = \pm 57.3\sqrt{\frac{2}{L/\lambda}} \qquad \text{......(18)}$$

4. Directivity

$$D = 4n\left(\frac{d}{\lambda}\right) = 4\left(\frac{L}{\lambda}\right) \qquad \text{......(19)}$$

For an increased directivity,

$$D = 1.789\left[4n\left(\frac{d}{\lambda}\right)\right] = 1.789\left[4\left(\frac{L}{\lambda}\right)\right] \qquad \text{......(20)}$$



Field pattern of an end fire array

Phased Arrays

***Phased array** means an array of many elements and the phase of each element being a variable that provides **control of the beam direction**, that is, maximum radiation in any desired direction and pattern shape including the side lobes.*

Some of the specialized phased arrays are:

(i) Frequency scanning array,

(ii) Retrodirective array, and

(iii) Adaptive array.

*In the **frequency scanning array** or **scanning array**, phase change is accomplished by varying the frequency. It is one of the simplest phased arrays since no phase control is required at each element.*

*A **retrodirective** or **self-focusing** array is an array that will receive a signal from any direction in space and automatically reflects the signal back toward its source, usually after suitable modulation and amplification.*

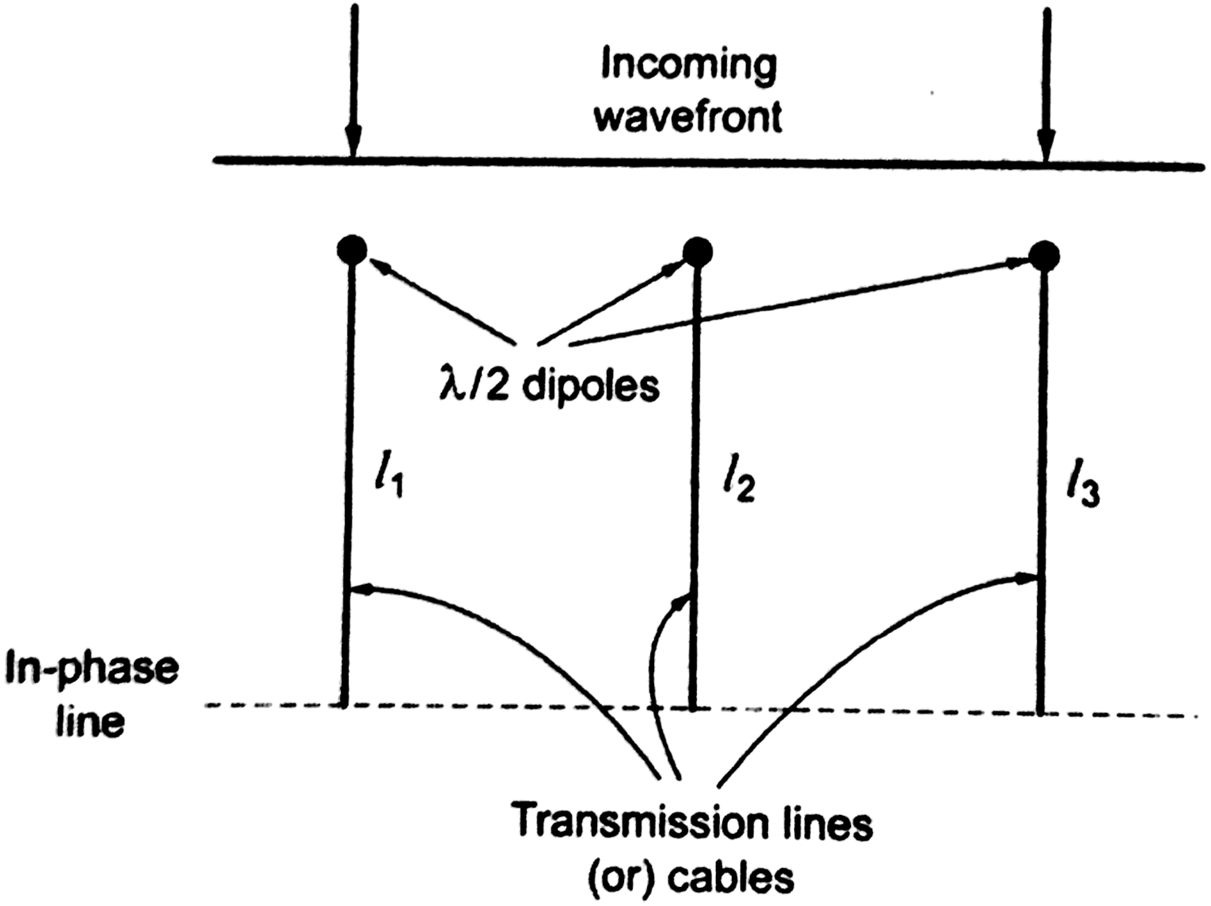
Phased array designs

The objectives of the phased array are:

(i) A phased array has to accomplish a beam steering without the mechanical and inertial problems of rotating the entire array, and

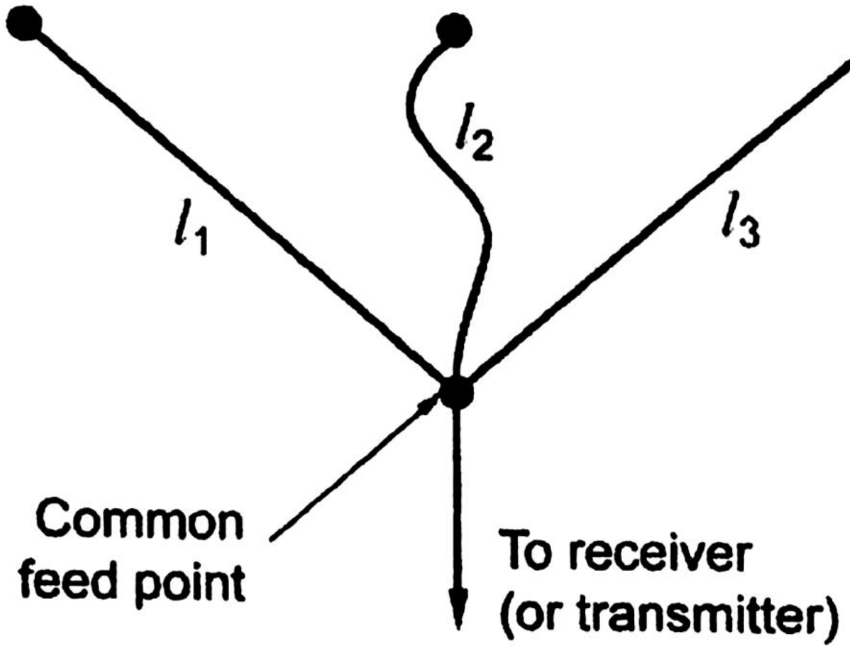
(ii) It has to provide beam control at a fixed frequency (or) at any number of frequencies within a certain bandwidth in a ***frequency-independent manner***.

In the simplest form of a phased array, beam steering can be done by mechanical switching. Consider a basic 3-elements array and each element be a $\frac{\lambda}{2}$ dipole antenna

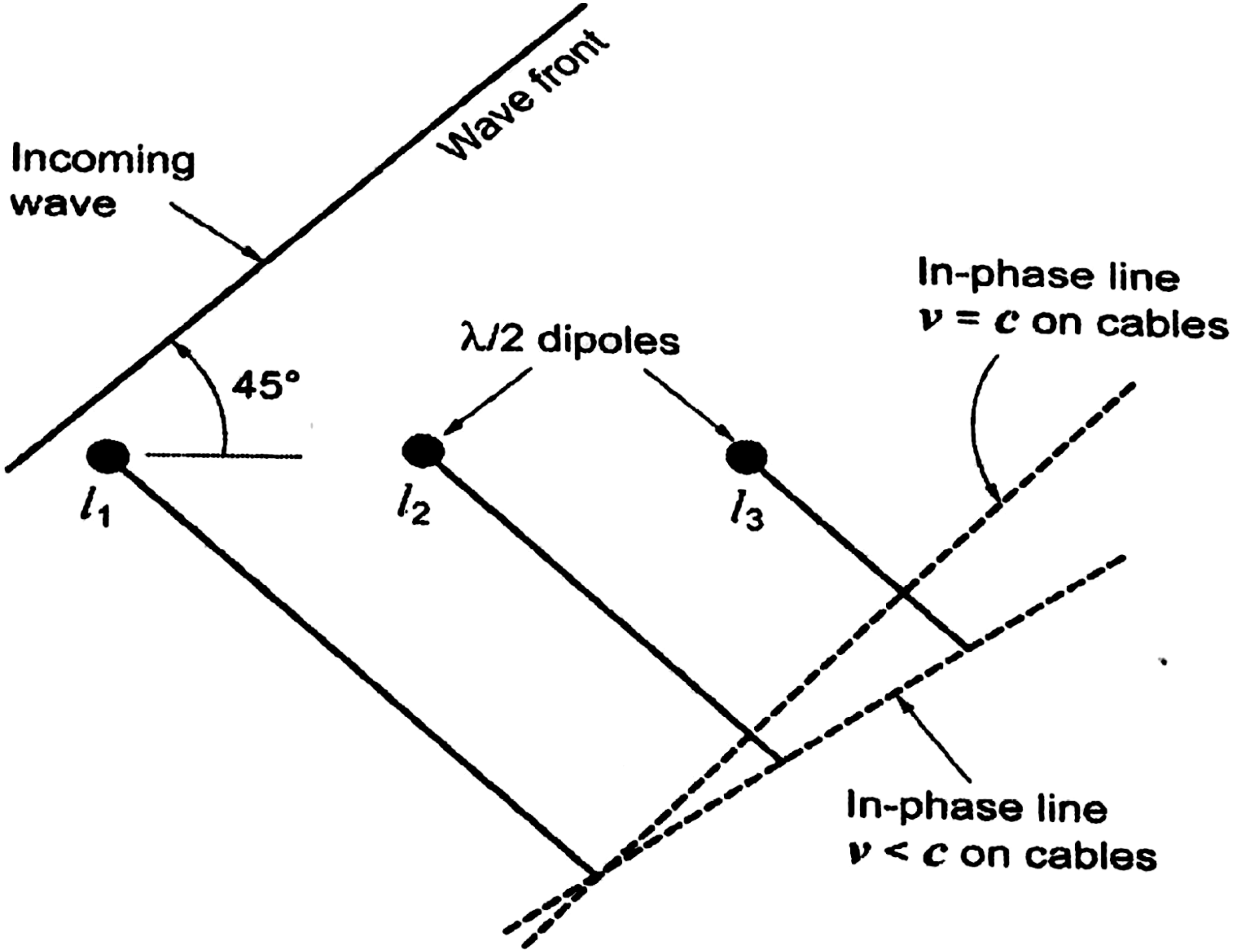


A simple phased array of three $\frac{\lambda}{2}$ dipoles

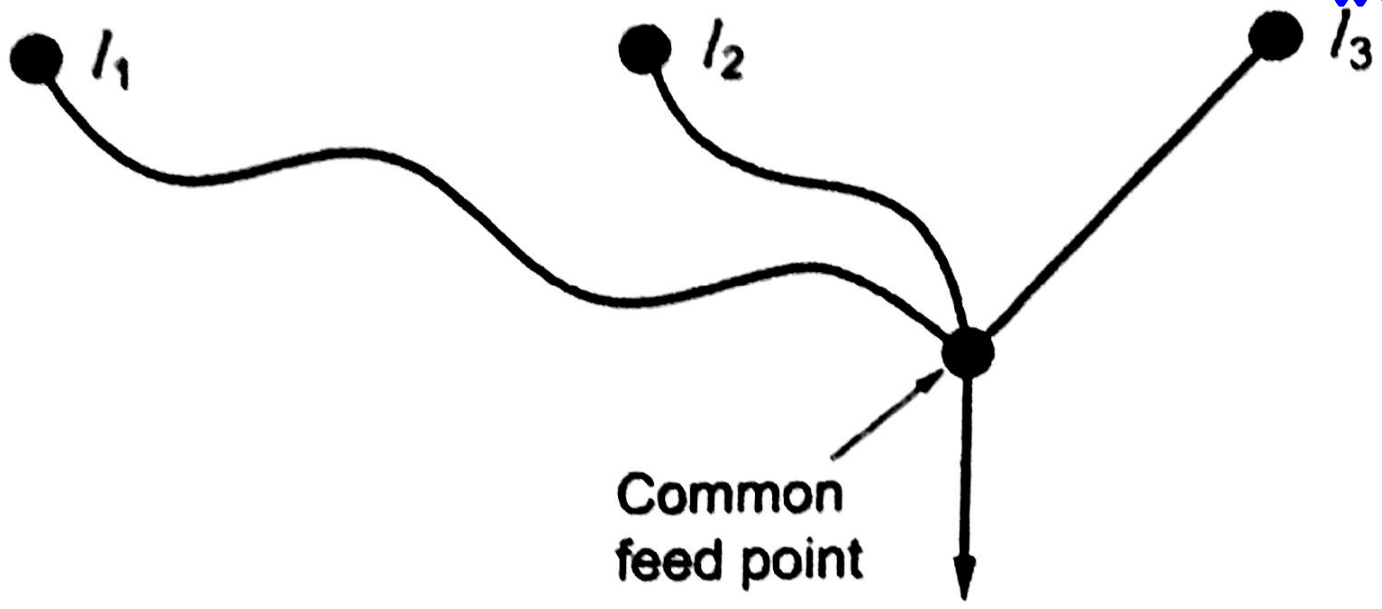
All three transmission cables are joined as a common point and this 3-element array will operate as a ***broadside array***.



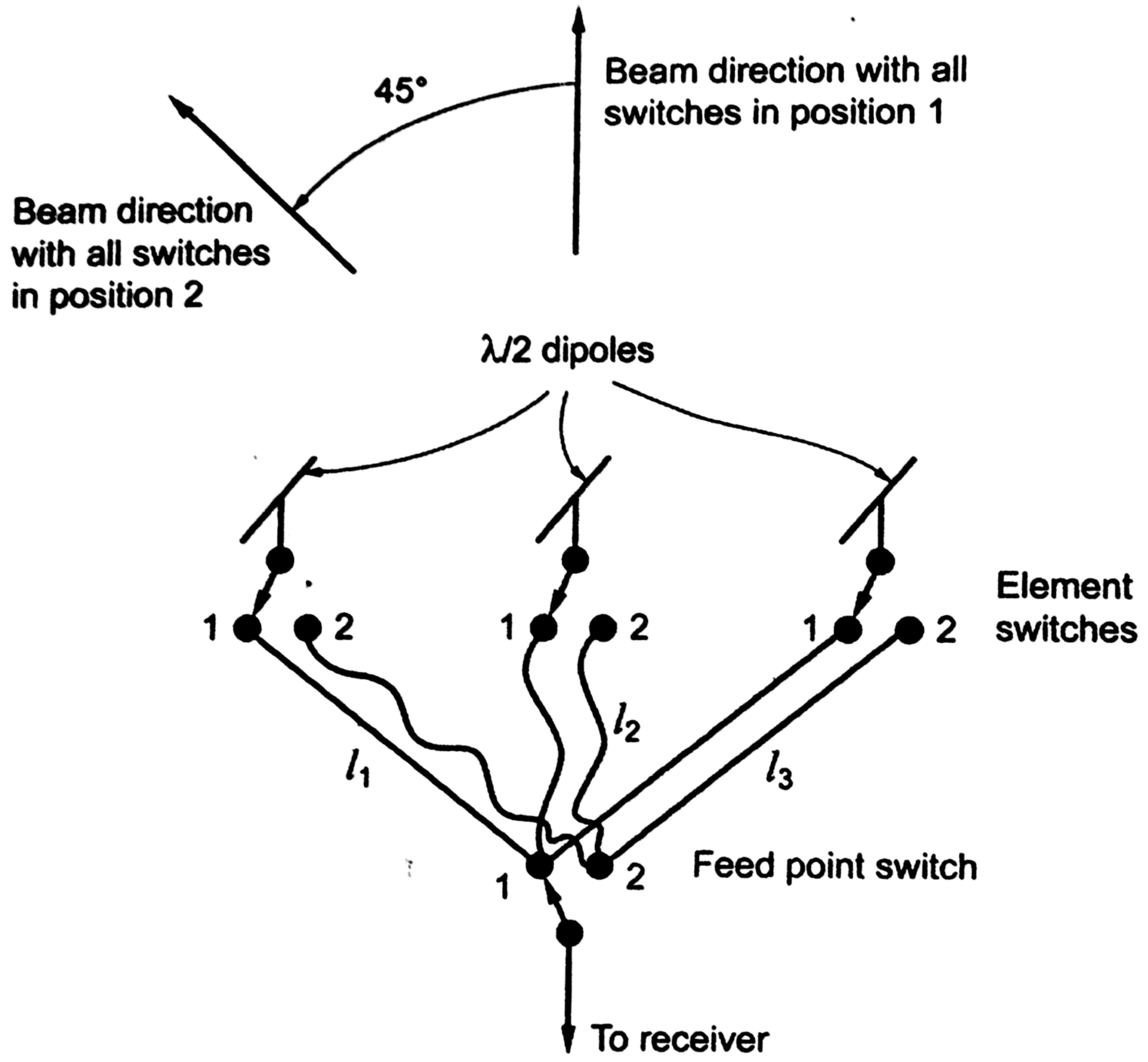
Equal length cables joined in common point



Incoming wave at 45° from broad side array



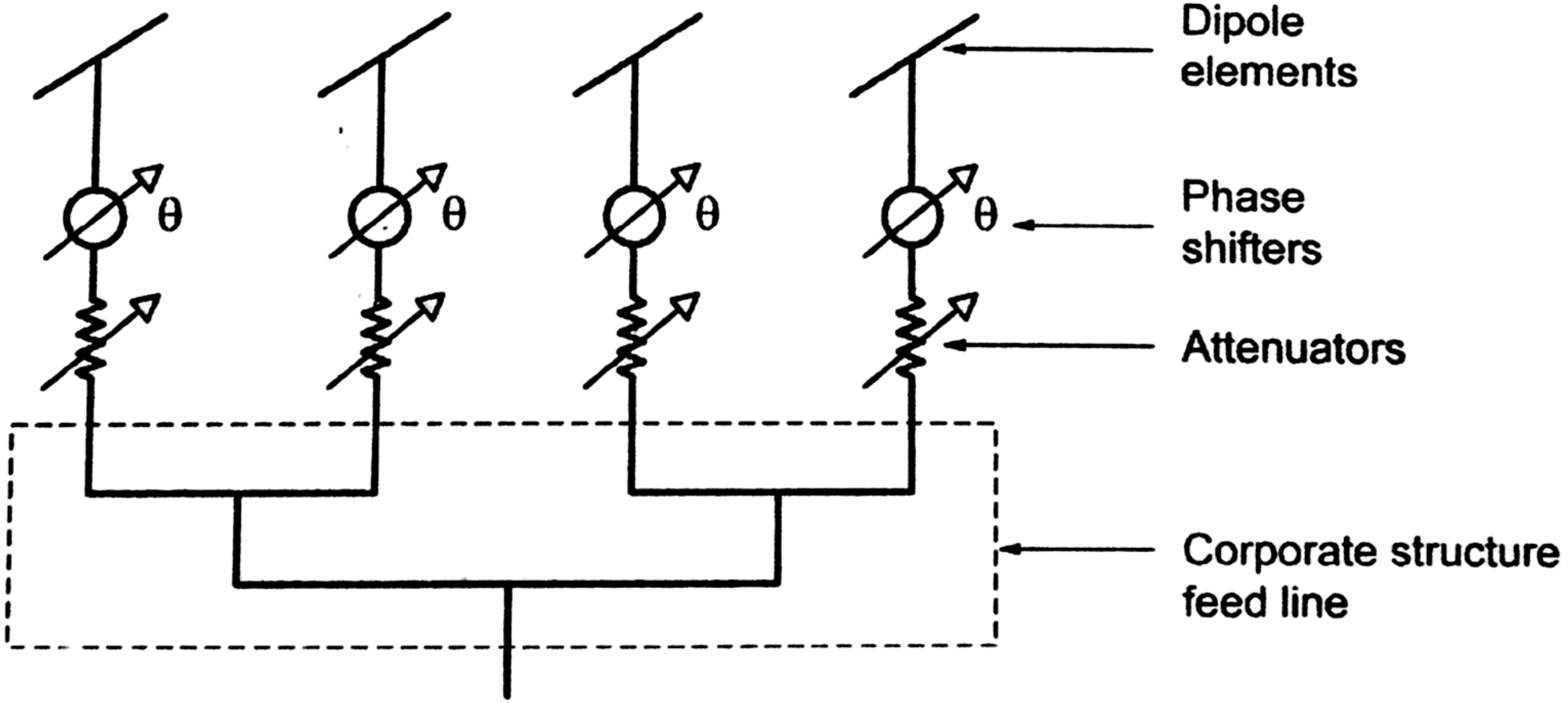
Joining on all cables in common point



Switches for shifting from broadside to 45° reception

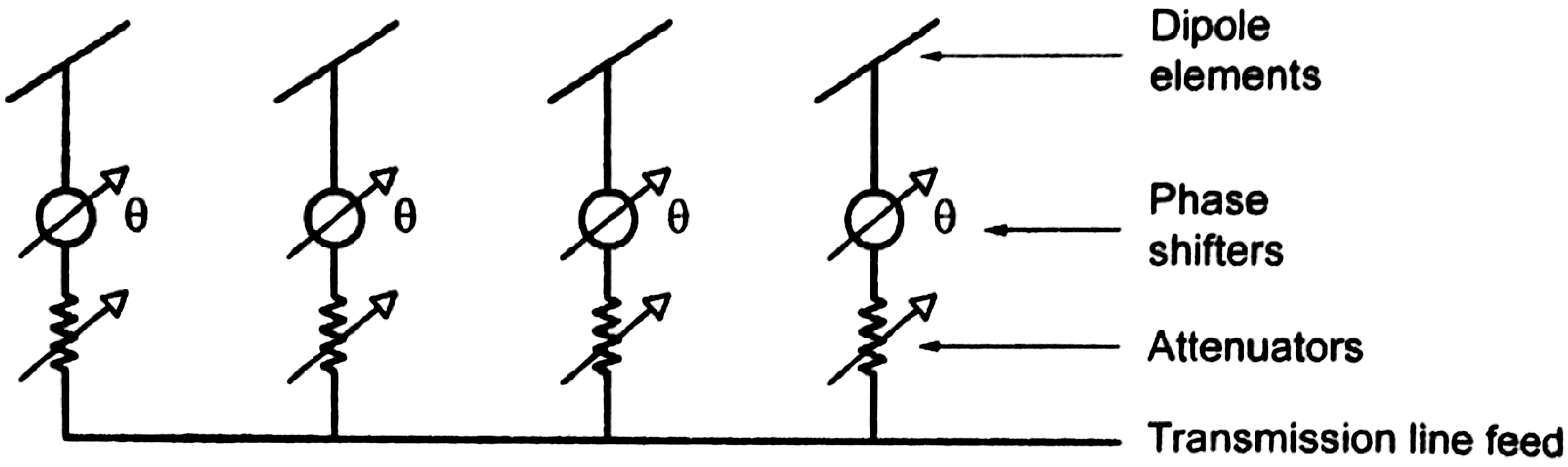
Different types of fed used in phased array

1. Corporate Structure:

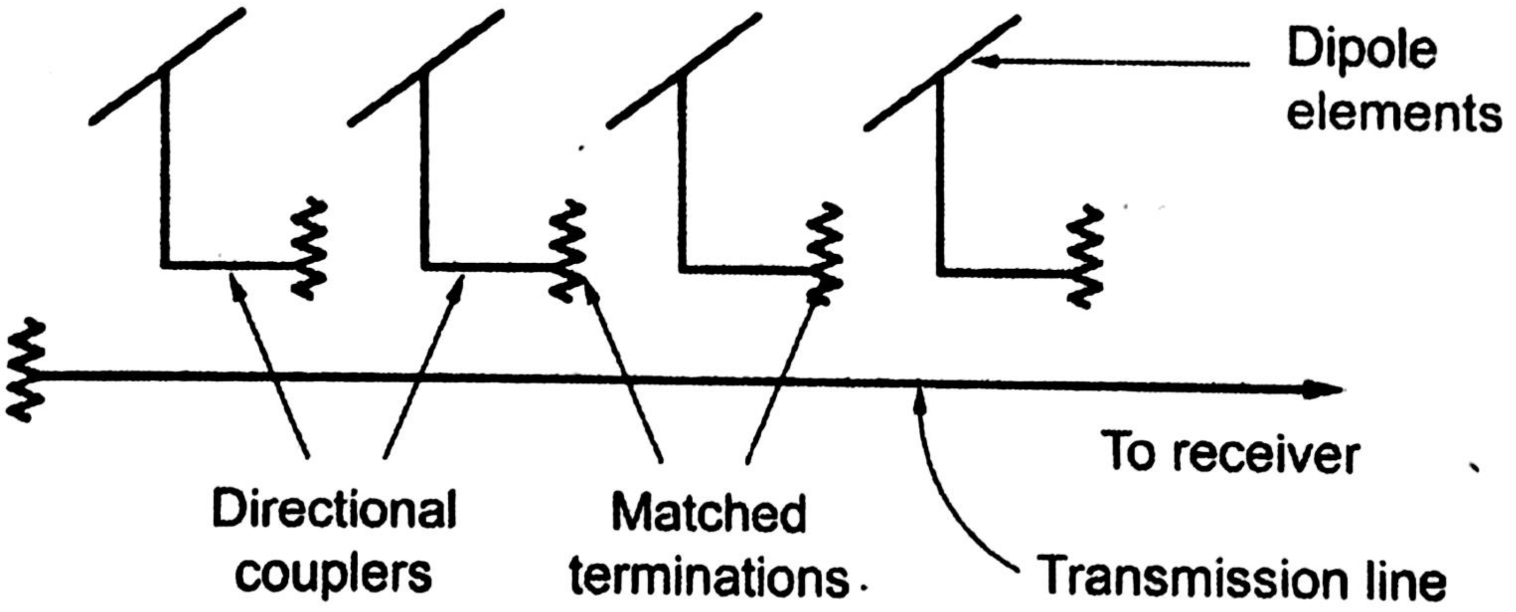


Schematic of phased array fed by corporate structure

2. End – Fed phased array



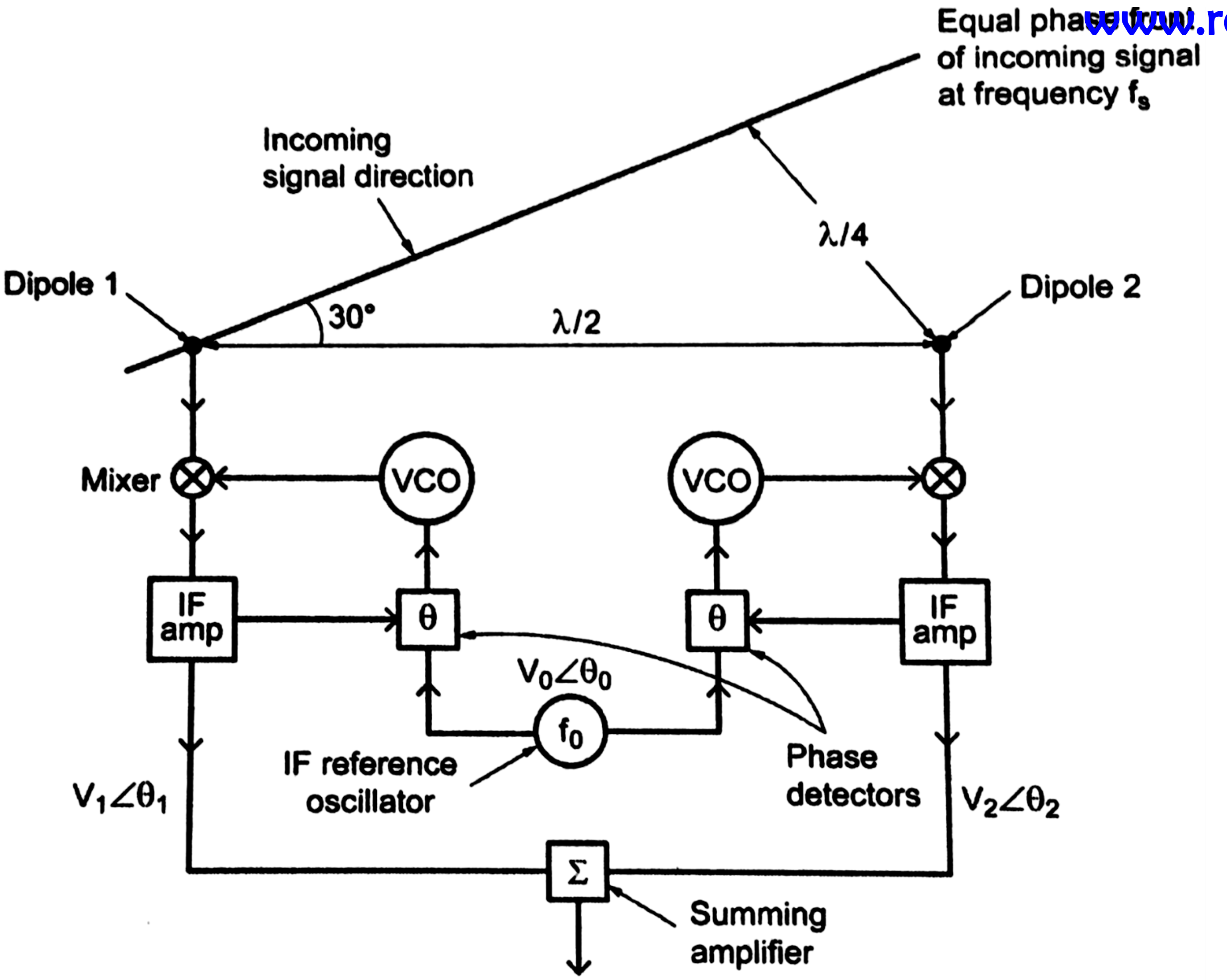
End-fed phase array with transmission line feed



End-fed phase array with directional coupler feed

Adaptive array

Adaptive arrays are arrays that can automatically self-adapt to various incoming signals conditions so as to maximize the signal from a particular source or to null out interfering signals.



Two- element adaptive array with signal-processing circuitry

Antenna Synthesis

Antenna analysis is the process of determining the radiation pattern for a given input distribution. Antenna synthesis is the inverse of antenna analysis.

Hence, antenna array synthesis is the process of determining input or source distribution for a specified radiation pattern.

In other words, antenna synthesis is the problem of determining the parameters of an antenna system that will produce a radiation pattern which accurately approximates some desired pattern.

The **various array synthesis techniques are as follows:**

(i) Fourier transformed method,

(ii) Dolph- Tchebyscheff method ,

(iii) Taylor's method,

(iv) Laplace transform method, and

(v) Binomial arrays

DOLPH-TCHEBYSCHEFF (D-T) OPTIMUM DISTRIBUTION [OR] CHEBYSHEV ARRAYS [OR] LINEAR ARRAY WITH NONUNIFORM AMPLITUDE DISTRIBUTIONS

While designing antenna arrays, it is necessary to determine the current ratios which results in smallest side lobe level for a specified beam width.

FUNDAMENTAL OF TCHEBYCHEFF POLYNOMIALS

The Tchebyscheff polynomial of *m*th degree with variable '*x*' is denoted by $T_m(x)$ and it is defined by

$$\left.\begin{aligned} T_m(x) &= \cos(m \cos^{-1} x), & -1 < x < +1 \\ &= \cos h(m \cos h^{-1} x), & |x| > 1 \end{aligned}\right\} \quad \ldots\ldots (1)$$

where '*m*' is an integer constant range from 0 to ∞

In general equation (1) can be written as,

$$T_m(x) = \cos\left(m\cos^{-1}x\right) = \cos(m\delta) = \cos\left(m\frac{\psi}{2}\right)$$

$$T_m(x) = \cos\left(m\frac{\psi}{2}\right) \qquad \text{......(2)}$$

where, $\delta = \cos^{-1}x \Rightarrow x = \cos\delta = \cos\dfrac{\psi}{2}$

Let us now obtain Tchebyscheff polynomials for different values of 'm'

If $m = 0$, $\quad T_0(x) = \cos(m\delta) = \cos(0.\delta)$

$$\boxed{T_0(x) = 1}$$

If $m = 1$, $T_1(x) = \cos(m\delta) = \cos(1.\delta) = x$

$$\boxed{T_1(x) = x}$$

If $m = 2$, $T_2(x) = \cos(m\delta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$= \cos(2\delta)$

$= 2\cos^2\delta - 1$

$$\boxed{\therefore T_2(x) = 2x^2 - 1}$$

If $m = 3$, $T_3(x) = \cos 3\delta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$= 4\cos^3\delta - 3\cos\delta$

$$\boxed{T_3(x) = 4x^3 - 3x}$$

If $m = 4$, $T_4(x) = \cos 4\delta$ $\cos 4\theta = 2\cos^2 2\theta - 1$

$$= \cos 2(2\delta) = 2\cos^2(2\delta) - 1$$

$$= 2[2\cos^2\delta - 1]^2 - 1$$

$$= 2[4\cos^4\delta - 4\cos^2\delta + 1] - 1$$

$$= 8\cos^4\delta - 8\cos^2\delta + 1$$

$$\boxed{T_4(x) = 8x^4 - 8x^2 + 1}$$

Further the polynomials with higher values of m can be obtained using recursive formula given by

$$T_{m+1}(x) = 2x\,T_m(x) - T_{m-1}(x)$$

For the particular values of 'm' the first ten Tchebyscheff polynomials are given as,

$m = 0$	$T_0(x) = 1$
$m = 1$	$T_1(x) = x$
$m = 2$	$T_2(x) = 2x^2 - 1$
$m = 3$	$T_3(x) = 4x^3 - 3x$
$m = 4$	$T_4(x) = 8x^4 - 8x^2 + 1$
$m = 5$	$T_5(x) = 16x^5 - 20x^3 + 5x$
$m = 6$	$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$
$m = 7$	$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$
$m = 8$	$T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$
$m = 9$	$T_9(x) = 256x^9 - 576x^7 - 432x^5 - 120x^3 + 9x$

Binomial Array

$$(a+b)^{n-1} = a^{n-1} + \frac{n-1}{1!}a^{n-2}\cdot b + \frac{(n-1)(n-2)}{2!}a^{n-3}b^2 + \frac{(n-1)(n-2)(n-3)}{3!}a^{n-4}b^3 \quad \ldots\ldots (1)$$

where, n - Number of radiating sources in the array

CONCEPTS OF BINOMIAL ARRAY

If the array is arranged in such a way that radiating sources are in the centre of the broad side array radiates more strongly than the radiating sources at the edges, minor lobes can be eliminated.

The secondary lobes can be eliminated entirely, when the following two conditions are satisfied:

(*i*) The space between the 2 consecutive radiating sources does not exceed $\frac{\lambda}{2}$:

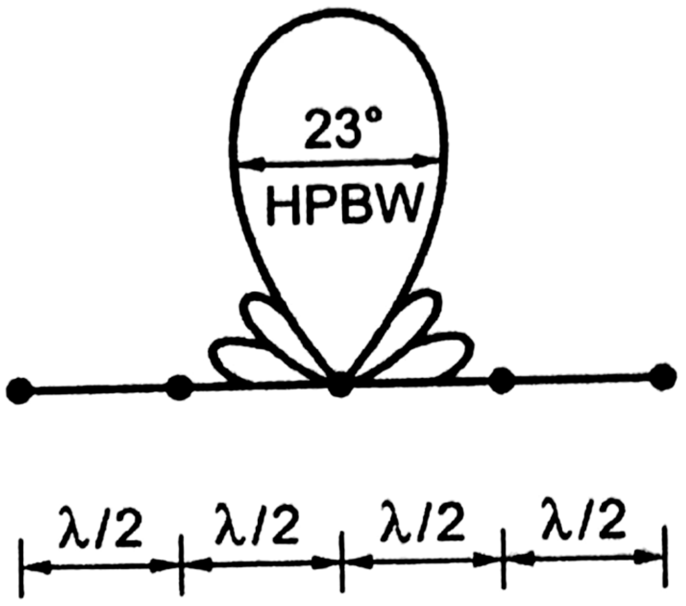
(*ii*) The current amplitudes in radiating sources (from outer towards centre source) are proportional to the coefficients of the successive terms of the binomial series.

For example, the relative amplitudes for the arrays of 1 to 10 radiating sources are as follows:

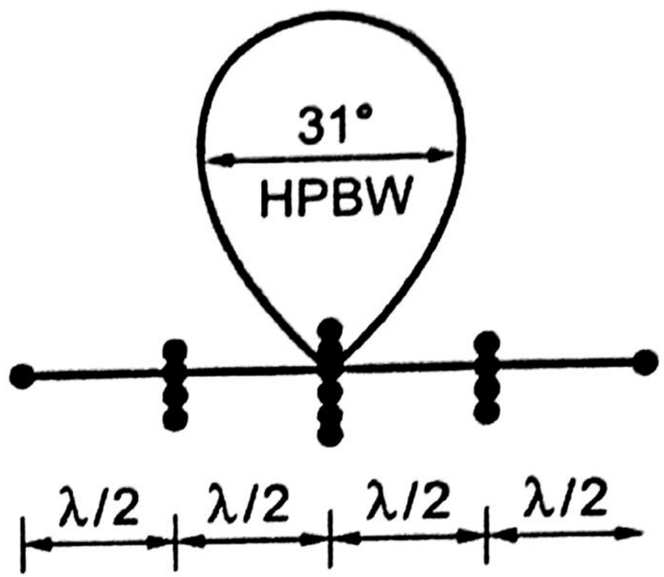
Number of sources	Relative Amplitude
$n = 1$	1
$n = 2$	1, 1
$n = 3$	1, 2, 1
$n = 4$	1, 3, 3, 1
$n = 5$	1, 4, 6, 4, 1
$n = 6$	1, 5, 10, 10, 5, 1
$n = 7$	1, 6, 15, 20, 15, 6, 1
$n = 8$	1, 7, 21, 35, 35, 21, 7, 1
$n = 9$	1, 8, 28, 56, 70, 56, 28, 8, 1
$n = 10$	1, 9, 36, 84, 126, 126, 84, 36, 9, 1

consider $n = 5$, $d = \frac{\lambda}{2}$, HPBW of binomial array is 31° and HPBW

of an uniform array is 23°



(a) Uniform array



(b) Binomial array with amplitude ratio 1 : 4 : 6 : 4 : 1

Disadvantages of Binomial Arrays

(i) HPBW increases and hence the directivity decreases.

(ii) For the design of a large array, the larger amplitude ratio of sources is required.

UNIT – IV
PASSIVE AND ACTIVE MICROWAVE DEVICES

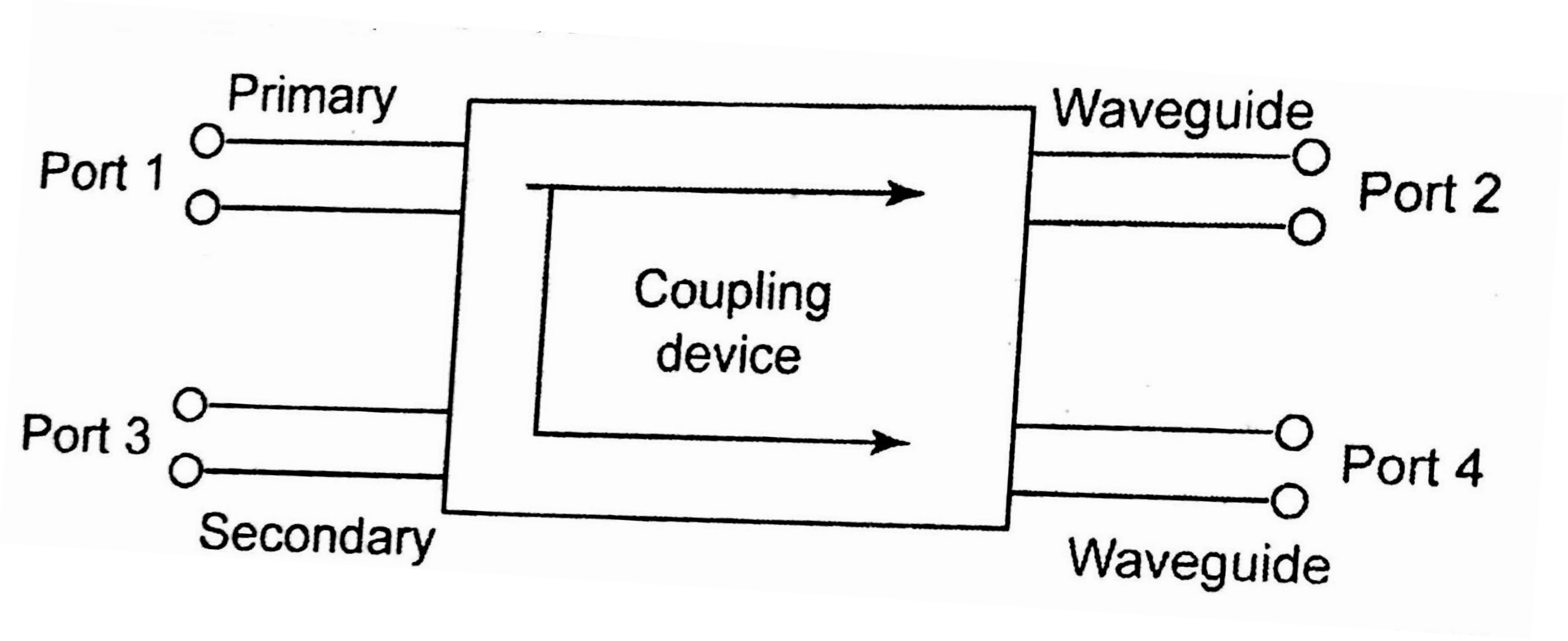
Microwave Passive components: Directional Coupler, Power Divider, Magic Tee, attenuator, resonator, Principles of Microwave Semiconductor Devices: Gunn Diodes, IMPATT diodes, Schottky Barrier diodes, PIN diodes, Microwave tubes: Klystron, TWT, Magnetron

Directional Couplers

A directional coupler is a four port passive device commonly used for coupling a known fraction of the microwave power to a port (coupled port) in the auxiliary line while flowing from input port to output port in the main line. The remaining port is ideally isolated port and matched terminated.

They can be designed to measure ***incident*** and/or ***reflected power, SWR (Standing Wave Ratio)*** values, provide a signal path to a receiver or perform other desirable operations.

They can be unidirectional (measuring only incident power) or bi – directional (measuring both incident and reflected) powers



Directional coupler

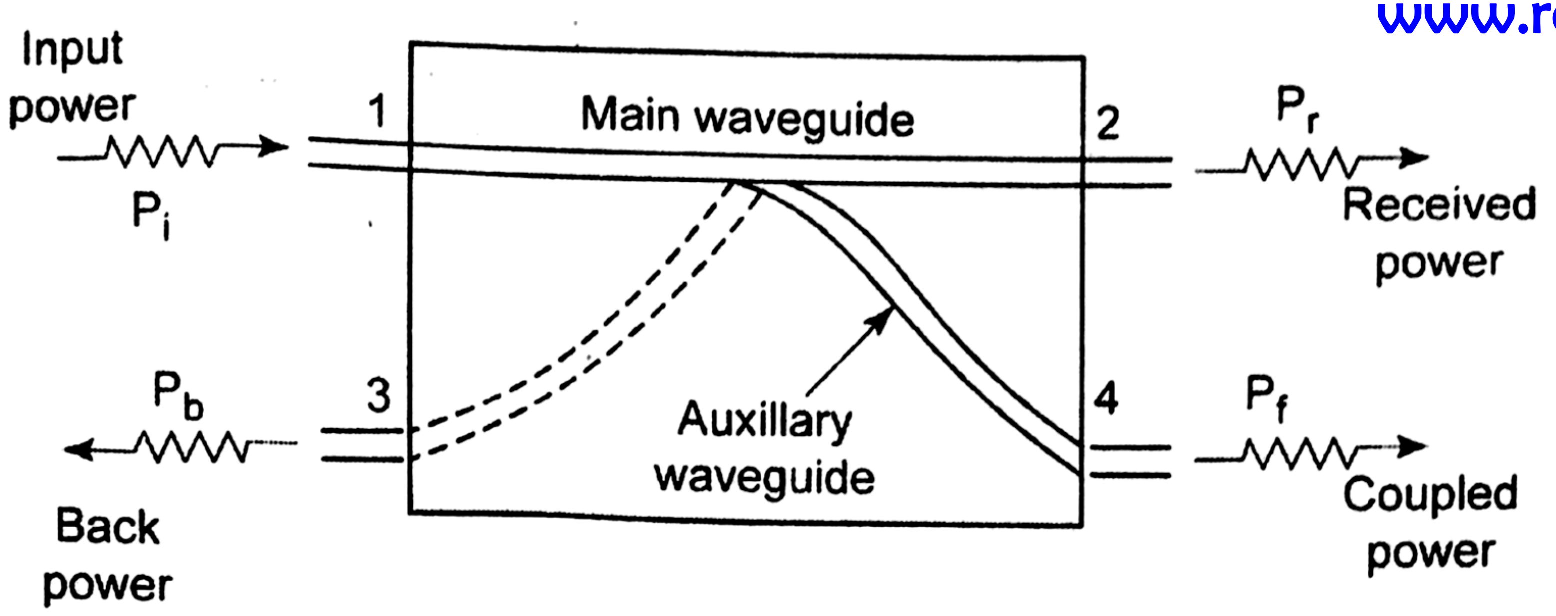
Properties of Directional Coupler:

With matched terminations at all its ports, the ***properties of an ideal directional coupler*** can be summarized as follows.

(i) A portion of power traveling from port 1 to port 2 is coupled to port 4 but not to port 3.

(ii) A portion of power traveling from port 2 to port 1 is coupled to port 3 but not to port 4.

(iii) A portion of power incident on port 3 is coupled to port 2 but not to port 1 and a portion of the power incident on port 4 is coupled to port 1 but not to port 2. Also ports 1 and 3 are decoupled as are ports 2 and 4.



Directional coupler indicating powers.

P_i - Incident power at port 1

P_r - Received power at port 2

P_f - Forward coupled power at port 4

P_b - Back power at port 3

Performance of a directional coupler is described by following terms:

1. Coupling Factor (C)
2. Directivity (D)
3. Isolation

Coupling Factor (C):

The coupling factor of a directional coupler is defined as ***the ratio of the incident power 'P_i" to the forward power 'P_f' measured in dB.***

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4} \text{ (or)}$$

$$\boxed{C(dB) = 10 \log_{10} \frac{P_i}{P_f}}$$

Directivity (D):

The directivity of a directional coupler is defined as ***the ratio of forward power 'P_f' to the back power 'P_b' expressed in dB.***

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3} \quad \text{(or)}$$

$$D(dB) = 10 \log_{10} \frac{P_f}{P_b}$$

The coupling factor ***is a measure of how much of the incident power is being sampled.***

Directivity ***is a measure of how well the directional coupler distinguishes between the forward and reverse traveling powers.***

Isolation:

The term isolation is sometimes used to ***describe the directive properties*** of a coupler.

It is defined as the ***ratio of the incident power*** ‘P_i’ to the ***back power*** ‘P_b’ expressed in dB.

$$\text{Isolation}\,(\text{dB}) = 10\log_{10}\frac{P_i}{P_b}$$

Isolation (dB) equals ***coupling plus directivity.***

Types of Directional Couplers:

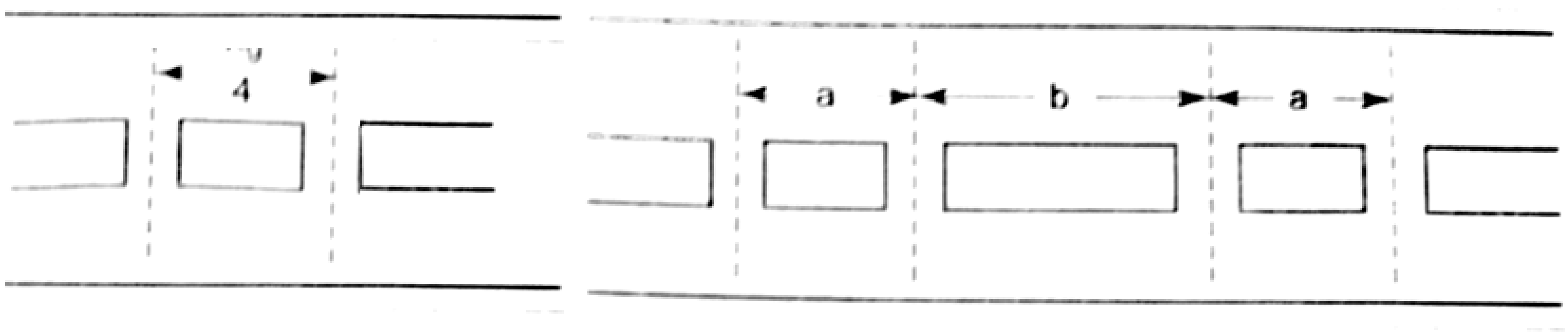
Several types of directional couplers exists, such as a

(i) ***Two – hole directional coupler,***

(ii) ***Four – hole directional coupler,***

(iii) ***Reverse – coupling directional coupler*** (Schwinger coupler), and

(iv) ***Bethe – hole directional coupler.***



(a) Two – hole

(b) Four hole

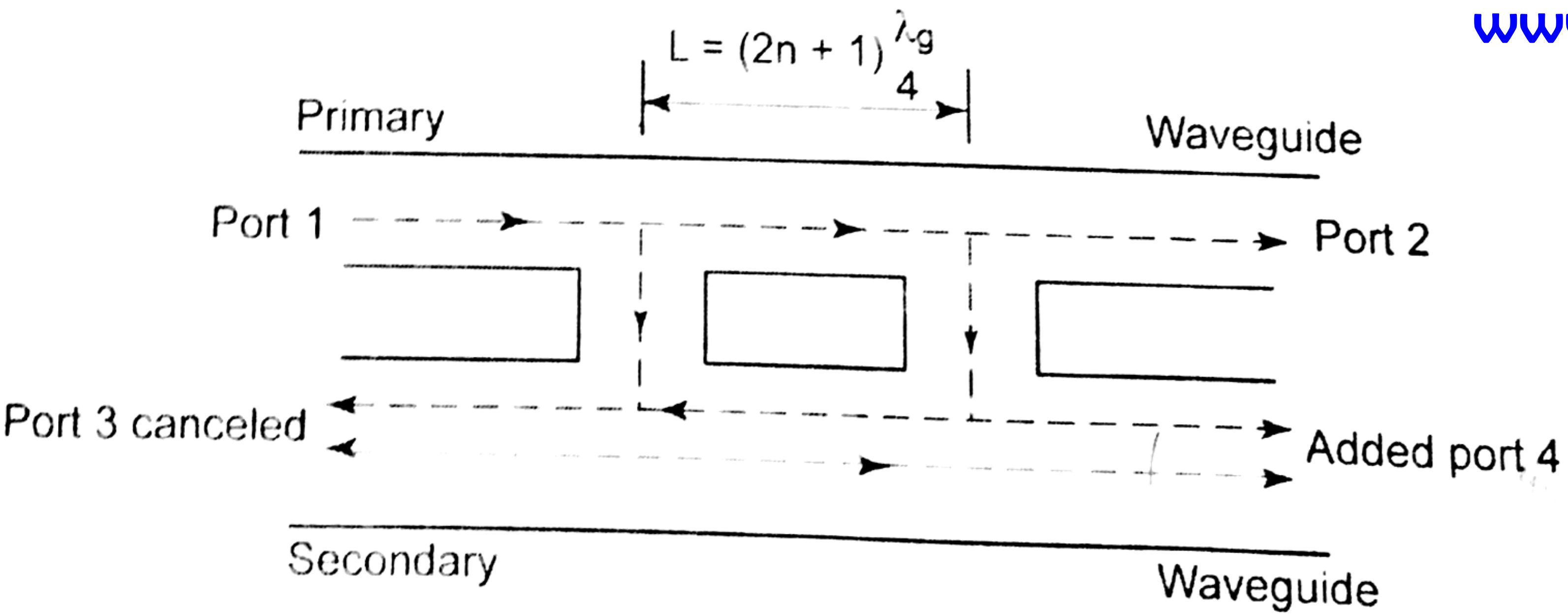
Two Hole Directional Coupler:

A two – hole directional coupler consists of two waveguides are, the primary, and the secondary with two tiny holes common between them.

The number of holes can be one (as in ***Bethe cross guide coupler***) or more than two (as in a ***multi hole coupler***).

The ***degree of coupling*** is determined by size and location of the holes in the waveguide walls.

The two holes are at a distance of $\frac{\lambda_g}{4}$ where λ_g is the guide wavelength.



Two – hole directional coupler

The spacing between the centers of two holes must be,

$$L = (2n+1)\frac{\lambda_g}{4}$$

Where, n is any positive integer & λ_g is the guide wavelength.

A fraction of the wave entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as ***slot antennas***.

The forward waves in the secondary guide are in the same phase, regardless of the hole space, and are added at port 4.

The coupling is given by,

$$C = -20 \log 2 |B_f|$$

Where, B_f – Amplitude in the forward direction

The backward waves in the secondary guide (waves are progressing from right to left) are out of phase by 180° at the position of the 1st hole and are canceled at port 3.

Scattering Matrix of a Directional Coupler

Directional coupler is a ***four port network.*** Hence [S] is a 4 × 4 matrix.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \ldots (1)$$

In a directional coupler all ***four ports*** are ***perfectly matched*** to the junction. Hence the diagonal elements are zero.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \quad \ldots (2)$$

From symmetric property, $S_{ij} = S_{ji}$

$$S_{12} = S_{21};\ S_{23} = S_{32};\ S_{13} = S_{31};\ S_{24} = S_{42};\ S_{34} = S_{43};\ S_{41} = S_{14} \qquad \ldots (3)$$

There is ***no coupling*** between port 1 and port 3

$$S_{13} = S_{31} = 0 \qquad \ldots (4)$$

Also there is no coupling between port 2 and port 4

$$S_{24} = S_{42} = 0 \qquad \ldots (5)$$

Substituting in equation (1), the values of scattering parameters as per equations (2) to (5) we get

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \qquad \ldots (6)$$

Since $[S]\,[S^*] = I$ we get

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $$|S_{12}|^2 + |S_{14}|^2 = 1 \quad \ldots(7)$$

R_2C_2: $$|S_{12}|^2 + |S_{23}|^2 = 1 \quad \ldots(8)$$

R_3C_3: $$|S_{23}|^2 + |S_{34}|^2 = 1 \quad \ldots(9)$$

R_1C_3: $$S_{12}S_{23}^{*} + S_{14}S_{34}^{*} = 0 \quad \ldots(10)$$

Comparing equation (7) and (8)

$$|S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2$$

$$S_{14} = S_{23} \quad \ldots(11)$$

Comparing equation (8) and (9)

$$|S_{12}|^2 + |S_{23}|^2 = |S_{23}|^2 + |S_{34}|^2$$

$$S_{12} = S_{34} \quad \ldots(12)$$

Let us assume that S_{12} is real and positive = 'P'

$$S_{12} = S_{34} = P = S_{34}^{*} \quad \dots (13)$$

Substitute equation (13) in equation (10)

$$S_{12} S_{23}^{*} + S_{14} S_{34}^{*} = 0 \qquad \because \left| S_{14} = S_{23} \right|$$

$$P\left(S_{23} + S_{23}^{*}\right) = 0$$

$$S_{23} + S_{23}^{*} = 0 \quad \dots (14)$$

$$S_{23} = -S^{*}{}_{23}$$

i.e., S_{23} must be ***imaginary***

$$S_{23} = jq$$

$$S_{23}^{*} = -jq$$

Let $S_{23} = jq = S_{14}$

$$S_{12} = S_{34} = P \qquad \ldots (15)$$

$$S_{23} = S_{14} = jq \qquad \ldots (15a)$$

Substitute equations (15) and (15a) in equation (7) then we get,

$$\boxed{P^2 + q^2 = 1}$$

Substituting these values in equation (6), [S] matrix of a directional coupler is reduced to

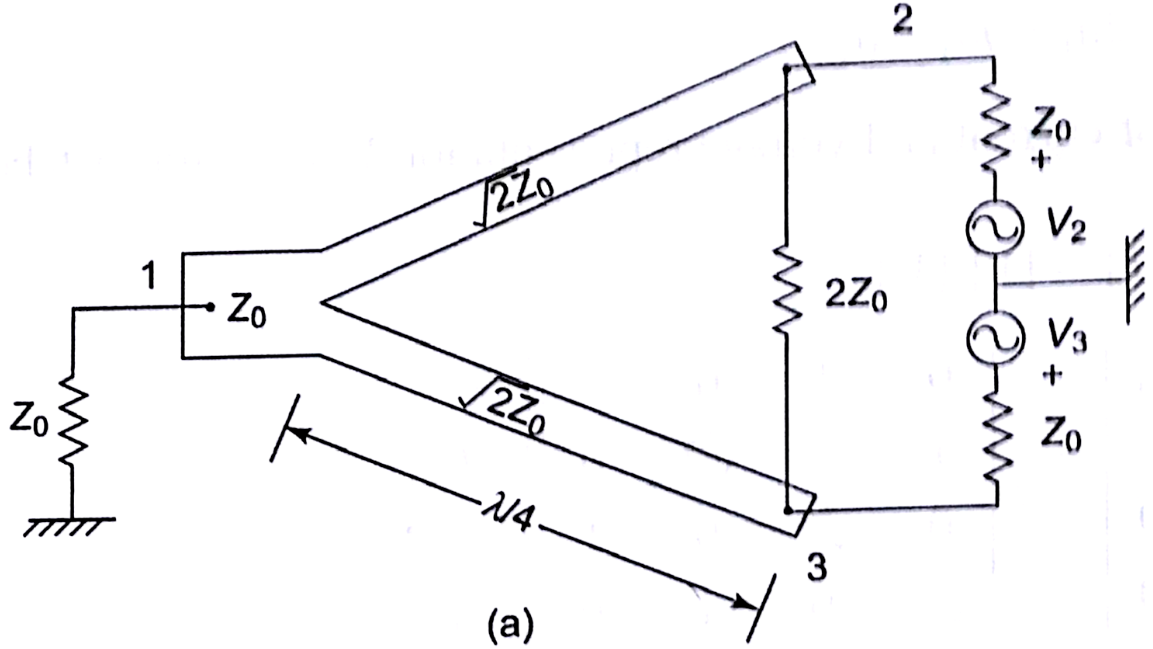
$$[S] = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix} \qquad \ldots (17)$$

Power Divider

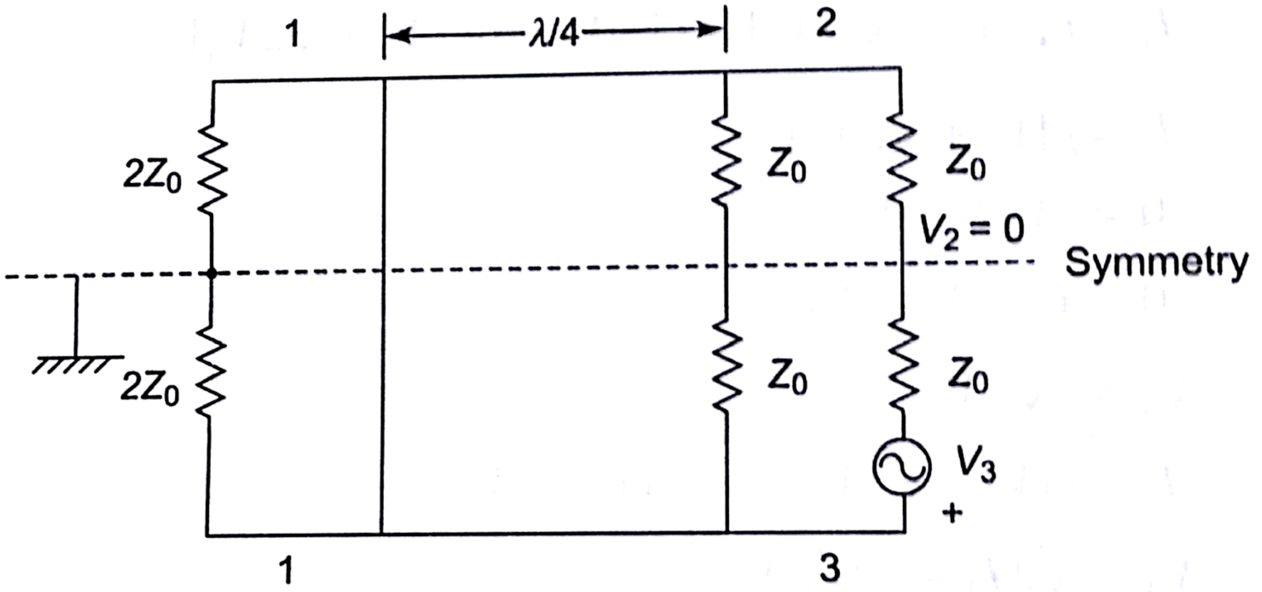
A power divider is a device to split the input power into a number of smaller amounts of power at multiple ports(N) to feed N number of branching circuits with isolation between the output ports

A two-way equal power divider is shown in figure which is a lossless three-port junction

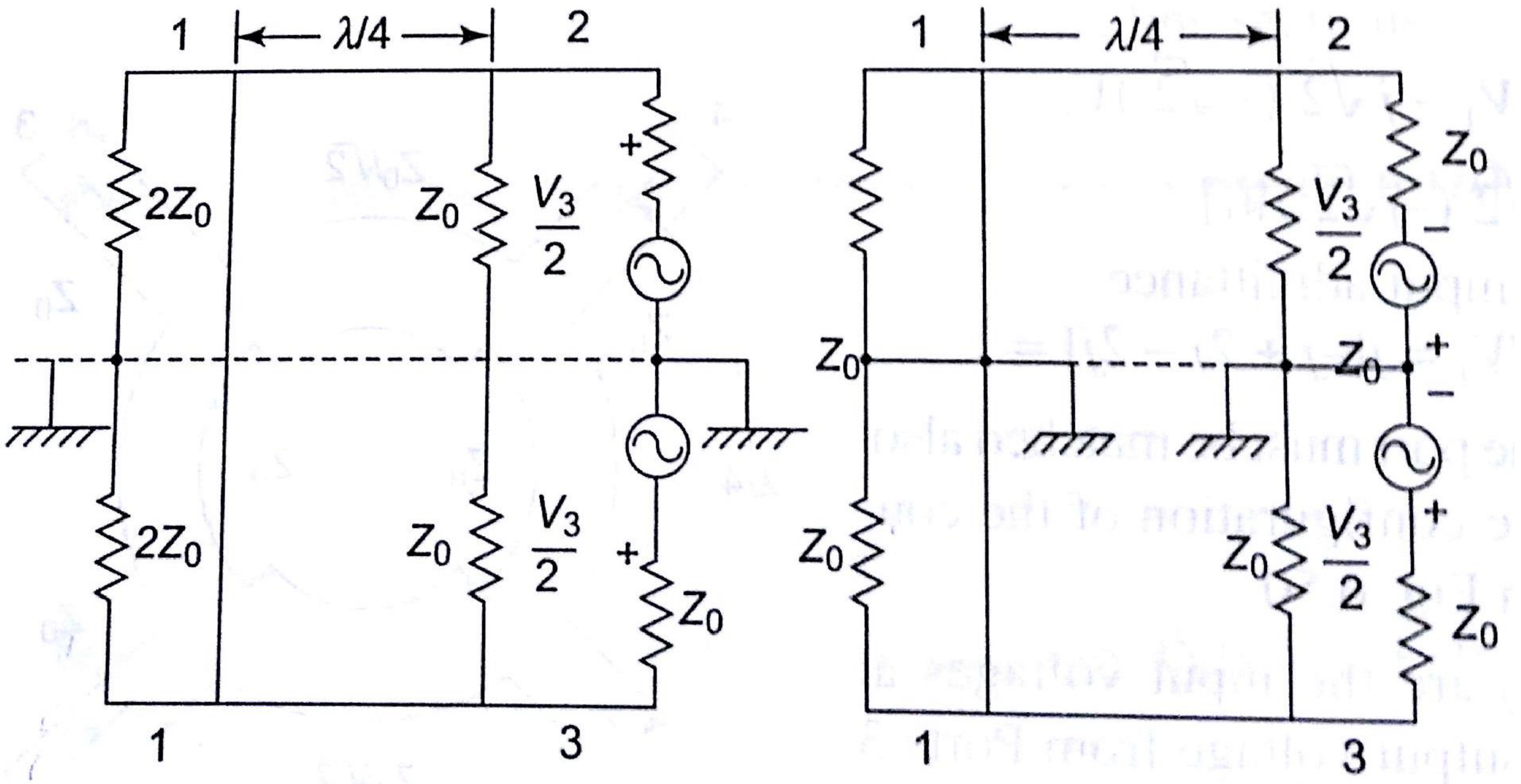
For equal power division, the device consists of two quarter wave sections with characteristic impedance Zo connected in parallel with input line


2
Z_0
+
V_2
1
Z_0
$\sqrt{2}Z_0$
$2Z_0$
V_3
+
Z_0
$\sqrt{2}Z_0$
Z_0
λ/4
3

(a)


1
λ/4
2
$2Z_0$
Z_0
Z_0
$V_2 = 0$
Symmetry
$2Z_0$
Z_0
Z_0
V_3
+
1
3

(b)



(c)

Two-way power divider: (a) microstrip configuration (b) equivalent circuit (c) even and odd modes symmetries

Hybrid Junctions

- A hybrid junction is a four – port network in which a signal incident on any one of the ports divides between two output ports with the remaining port being isolated

Magic Tee:

Here rectangular slots are cut both along the width and breadth of a long waveguide and side arms are attached as shown in fig.

A magic *tee is a* ***combination of the E – plane tee*** *and* ***H – plane tee.***

Ports 1 and 2 are collinear arms, port 3 is the H – arm, and port 4 is the E – arm.

Characteristics of Magic Tee:

The magic – T has the following ***characteristics*** when all the ports are terminated with matched load.

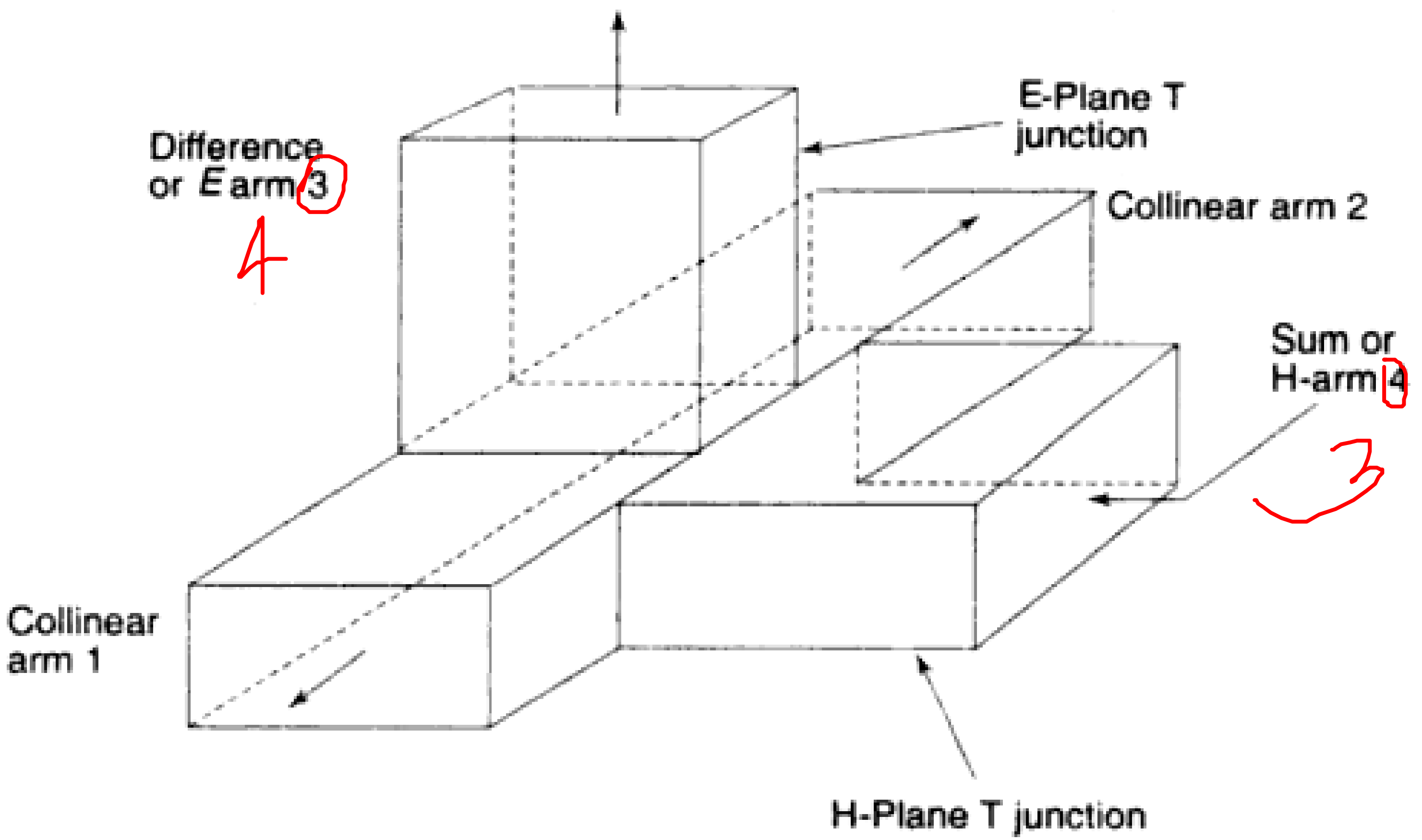
(i) If two in phase waves of equal magnitude are fed into ports 1 and 2, the output at port 4 is **subtractive** and hence zero and total output will appear additively at port 3. Hence port 4 is called the **difference (or) E – arm** and port 3 the **sum (or) H – arm.**

(ii) A wave incident at port 4 (E – arm) divides equally between ports 1 and 2 but opposite in phase with **no coupling** to port 3 (H – arm).

(iii) A wave incident at port 3 (H – arm) divides equally between ports 1 and 2 in phase with no coupling to port 4 (E – arm).

i.e.,

$$\boxed{S_{43} = S_{34} = 0} \quad \dots (1)$$



Difference
or E arm 3
4
E-Plane T
junction
Collinear arm 2
Sum or
H-arm 4
3
Collinear
arm 1
H-Plane T junction

Fig. *Magic-T*

(iv) A wave fed into one collinear port 1 or 2 will not appear in the other collinear port 2 or 1. Hence two collinear ports 1 and 2 are isolated from each other.

$$\boxed{S_{12} = S_{21} = 0} \quad \ldots (2)$$

A magic – T can be matched by putting screws suitably in the E and H arms without destroying the symmetry of the junction.

For an ideal loss less magic – T matched at ports 3 and 4.

$$\boxed{S_{33} = S_{44} = 0}$$

S-matrix for Magic Tee

Using the properties of E – H plane Tee, its scattering matrix can be obtained as follows:

[S] is a ***4 × 4 matrix*** since there are 4 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad \ldots (4)$$

From symmetric property, $S_{ij} = S_{ji}$

$$\boxed{S_{12} = S_{21}, S_{13} = S_{31}, S_{14} = S_{41}, S_{23} = S_{32}, S_{24} = S_{42}, S_{34} = S_{43}} \quad \ldots (5)$$

Port 3 has H-plane tee section

$$S_{23} = S_{13} \quad \ldots (6)$$

Similarly port 4 has E-plane tee section

$$S_{24} = -S_{14} \quad \ldots (7)$$

Substitute equations (1), (3), (5),(6) and (7) in equation (4). the S – matrix for a magic – T, matched at ports 3 and 4 is given by

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \qquad \ldots (8)$$

From unitary property [S] . [S]* = [I]

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the unitary property applied to rows 1 and 2, we get

R_1C_1: $$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \dots (9)$$

R_2C_2: $$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad \dots (10)$$

R_3C_3: $$|S_{13}|^2 + |S_{13}|^2 = 1 \quad \dots (11)$$

R_4C_4: $$|S_{14}|^2 + |S_{14}|^2 = 1 \quad \dots (12)$$

Equating equations (9) and (10), we get

$$|S_{11}|^2 - |S_{22}|^2 = 0$$

$$\boxed{|S_{11}| = |S_{22}|} \quad \dots (13)$$

From equation (11),

$$|S_{13}|^2 + |S_{13}|^2 = 1$$

$$2|S_{13}|^2 = 1$$

$$|S_{13}|^2 = \frac{1}{2}$$

$$\boxed{|S_{13}| = \frac{1}{\sqrt{2}}} \quad \dots (14)$$

From equation (12).

$$|S_{14}|^2 + |S_{14}|^2 = 1$$

$$2|S_{14}|^2 = 1$$

$$\boxed{|S_{14}| = \frac{1}{\sqrt{2}}} \quad \ldots (15)$$

Substituting equations (14) and (15) in equation (9)

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

Which is valid if

$$\boxed{S_{11} = S_{12} = 0} \quad \ldots (16)$$

From Equations (13) and (16)

$$\boxed{S_{22} = 0} \quad \dots (17)$$

The [S] of magic tee is obtained by substituting the scattering parameters from equations (13) to (17) in equation (8).

$$[S] = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad \dots (18)$$

Where $|S_{13}| = \frac{1}{\sqrt{2}} = |S_{14}|$

The scattering matrix for an ideal hybrid tee may be stated in the following form.

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

Applications of magic tee:

A magic tee has several applications

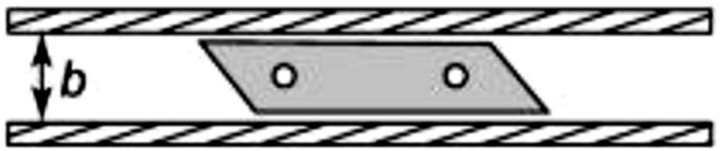
(i) Measurement of impedance
(ii) As duplexer
(iii) As mixer and
(iv) As an isolator

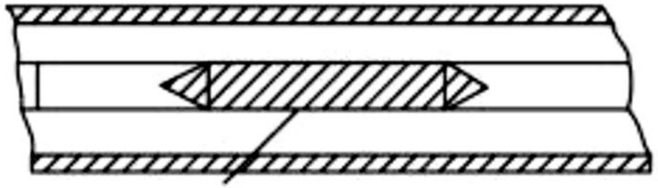
Attenuators

- Attenuators are passive devices used to control power levels in a microwave system by partially absorbing the transmitted signal wave
- Both fixed and variable attenuators are designed using resistive films
- Types:
  1. Coaxial line fixed attenuator
  2. waveguide attenuators (Variable type)

Coaxial line fixed attenuator:

- A coaxial fixed attenuator uses a film with losses on the center conductor to absorb some of the power as shown in fig
- The fixed waveguide type consists of a thin dielectric strip coated with resistive film and placed at the center of the waveguide parallel to the maximum E field



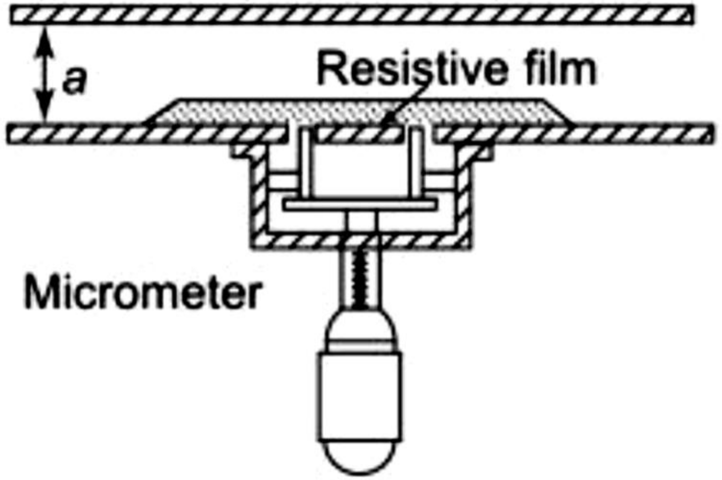


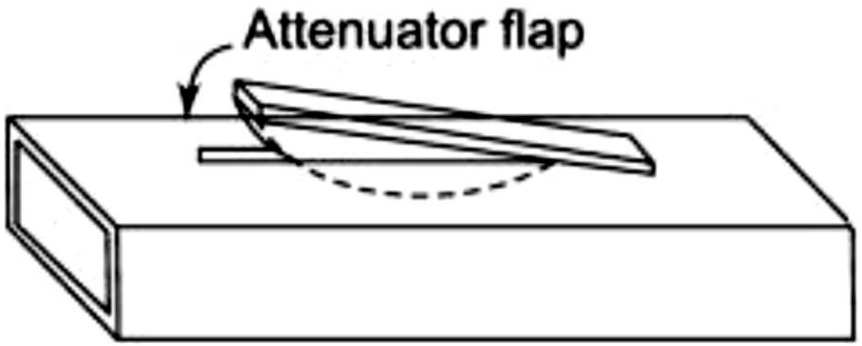
Lossy material on centre conductor

- Induced current on the resistive film due to the incident wave results in power dissipation, leading to attenuation of microwave energy
- The dielectric strip is tapered at both ends up to a length of more than half wavelength to reduce reflections
- The resistive van is supported by two dielectric rods separated by an odd multiple of quarter wave length and perpendicular to the electric filed

Variable type attenuator:

- A variable-type attenuator can be constructed by moving the resistive vane by means of micrometer screw from one side of the narrow wall to the center where the E – filed is maximum
- A maximum of 90 dB attenuation is possible with VSWR of 1.05





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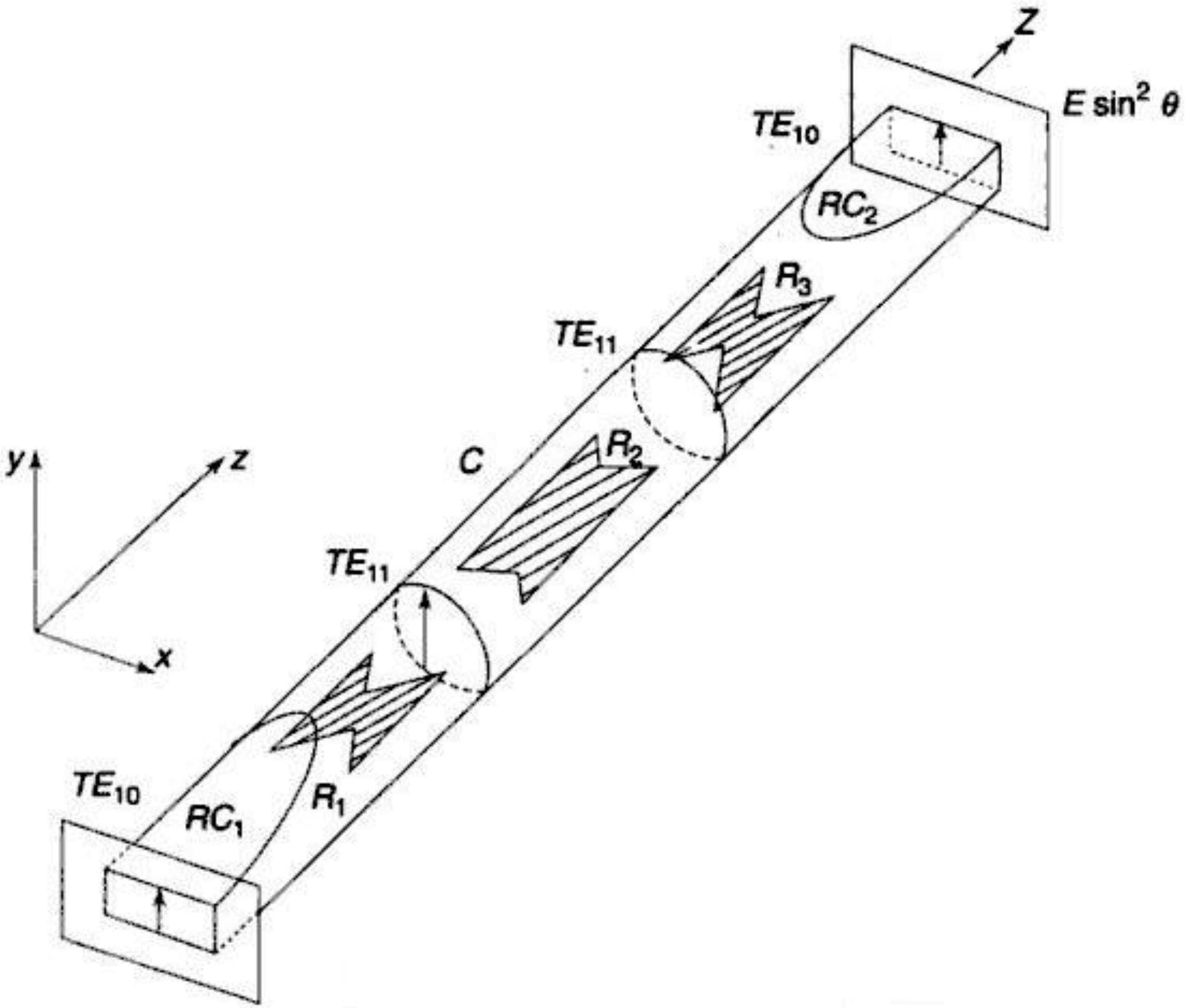
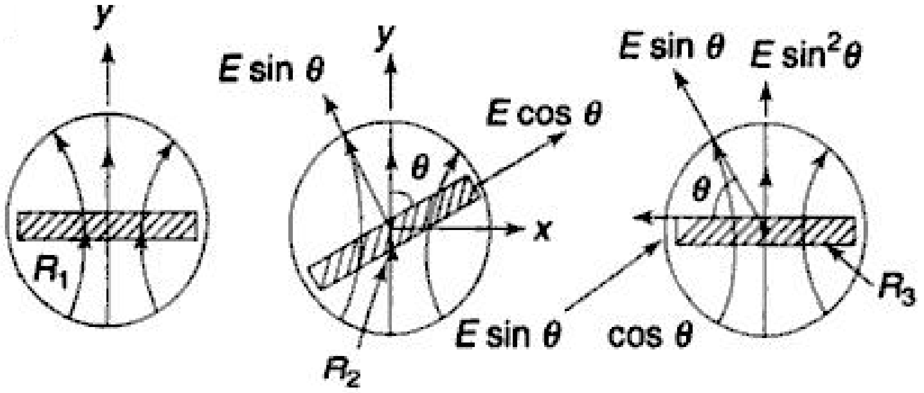
Precision type variable attenuator:

- A precision type variable attenuator makes use of a circular waveguide section(C), containing a very thin tapered resistive card (R2), to both sides of which are connected axisymmetric sections of circular to rectangular waveguide tapered transitions (RC1 and RC2) as shown in fig
- The center circular section with the resistive card can be precisely rotated by 360 degree with respect to the two fixed sections of circular to rectangular waveguide transitions
- The induced current on the resistive card R2 due to the incident signal is dissipated as heat producing attenuation of the transmitted signal
- The incident TE10 dominant wave in the rectangular wave guide is converted into a dominant TE11 mode in the circular waveguide





R_1, R_2, R_3 – Tapered resistive cards
RC_1 & RC_2 – Rectangular-to-circular waveguide transitions
C – Circular Waveguide Section

Precision type variable attenuator

- The attenuation of the transmitted wave is

$$\alpha = \frac{E}{E\sin^2\theta} = \frac{1}{\sin^2\theta} = \frac{1}{|S_{21}|}$$

$$\alpha\,(\text{dB}) = -40\log(\sin\theta) = -20\log|S_{21}|$$

Attenuators are normally matched reciprocal devices $|S_{21}| = |S_{12}|$

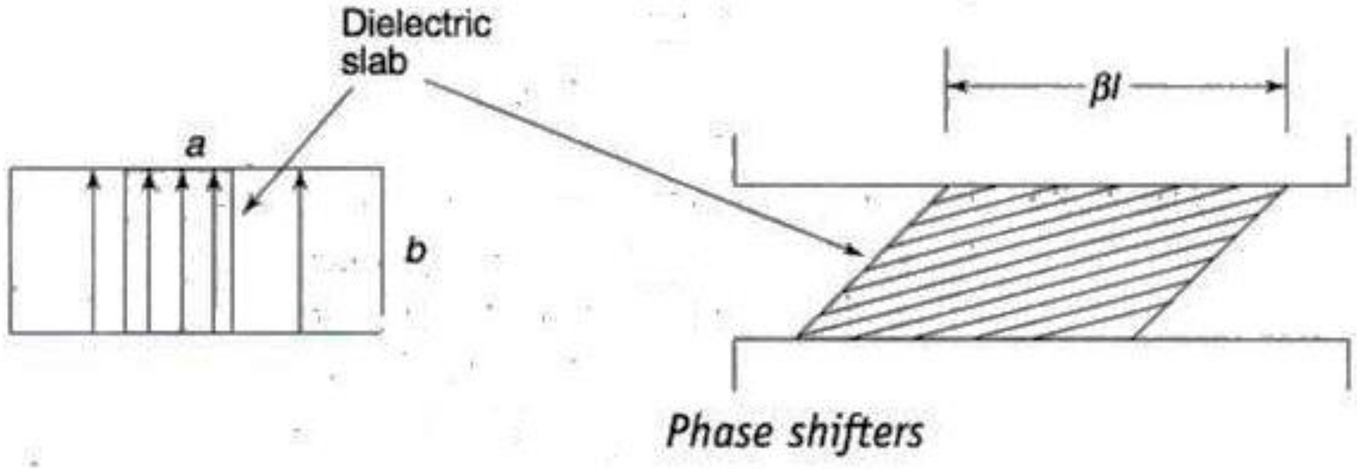
$$|S_{11}| \text{ or } |S_{22}| = \frac{\text{VSWR}-1}{\text{VSWR}+1} << 0.1$$

where the VSWR is measured at the port concerned. The S-matrix of an ideal precision rotary attenuator is

$$[S] = \begin{bmatrix} 0 & \sin^2\theta \\ \sin^2\theta & 0 \end{bmatrix}$$

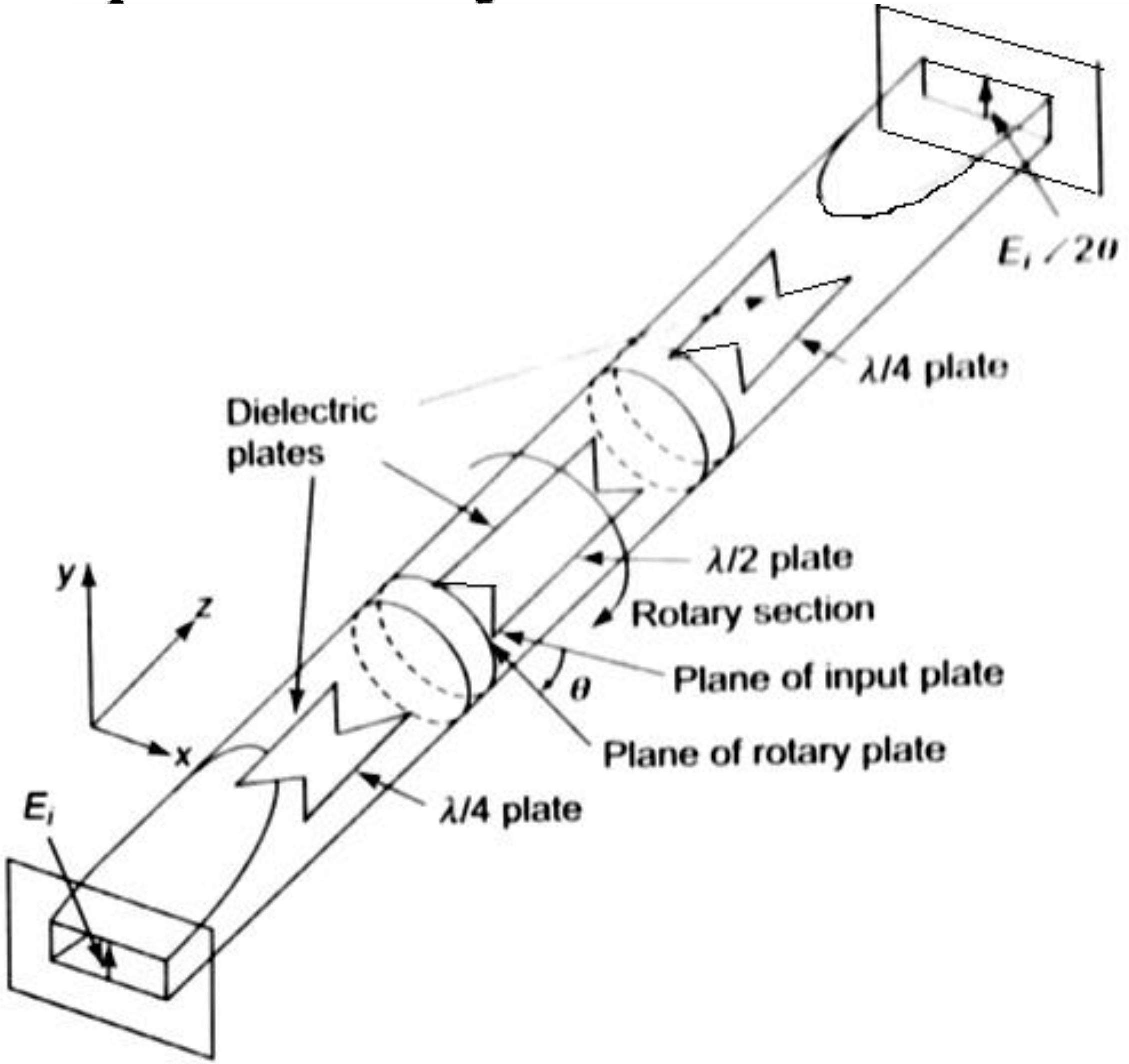
Phase Shifter

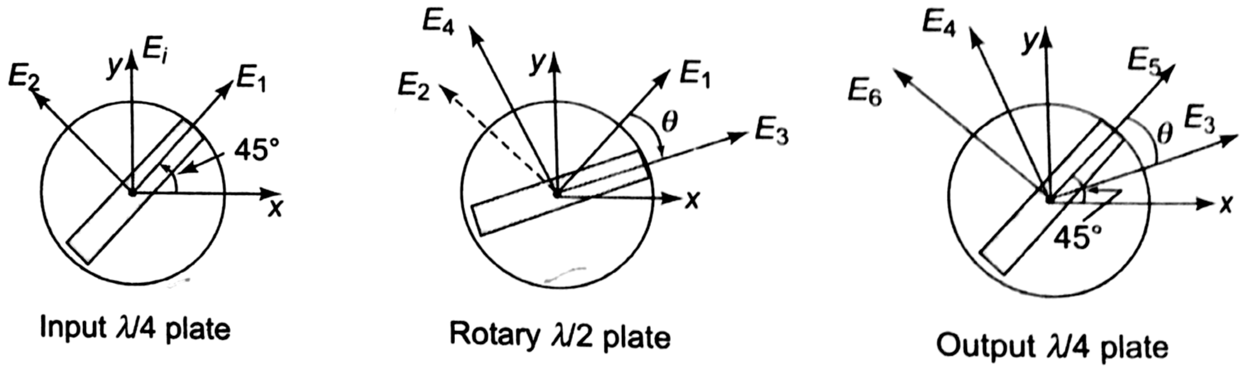
- A phase shifter is a two-port passive device that produces variable change in phase of the wave transmitted through it
- A phase shifter can be realized by placing a lossless dielectric slab within a waveguide parallel to and at the position of maximum E-field
- A differential phase change is produced due to the change of wave pared to that thro



Precision Rotary Phase Shifter:

The important parts of rotary phase shifter are three circular waveguide sections, two fixed sections and one rotatable. The fixed sections are quarter wave plates and rotatable sections are half wave plate.





The rotary phase shifter is suitable for low power applications typically few watts only.

A circularly polarized field is a field with x and y components of electric field that are equal in magnitude but 90° apart in time phase. A quarterwave plate is a device that produces a circularly polarized wave when a linearly polarized wave is incident upon it.

When TE_{11} mode is polarized parallel to the slab, the propagation constant β_1 is greater than when mode is polarized perpendicular to the slab. i.e. $\beta_1 > \beta_2$.

The length of quarterwave is chosen so that differential phase change $(\beta_1 > \beta_2)l = 90°$.

Let E_i be the maximum electric field strength of this mode TE_{11}

$$E_o = \frac{E_i}{\sqrt{2}} \quad ; \quad E_i = E_o\, e^{-j\beta_1 l}$$

The resultant electric field strength at the output is

$$E_{out} = E_i \cdot e^{-j(2\theta + 4\beta_1 l)}$$

The polarization of E_{out} and E_i are same except for the phase change

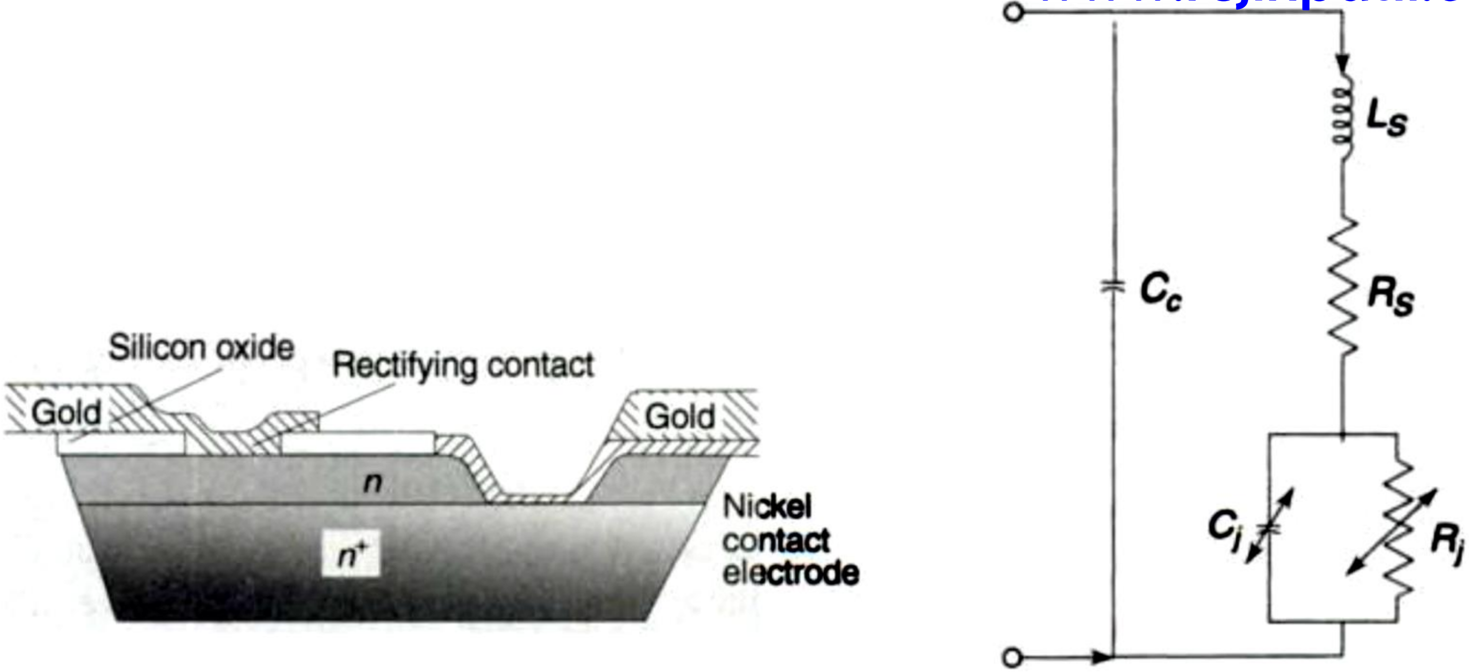
$$\Delta\phi = 2\theta + 4\beta_1 l \text{ radians}$$

At a given frequency, $4\beta_1 l$ remains constant and change in phase is achieved by varying angle θ of the half wave plate with respect to quarter wave plate.

Schottky Diode

Schottky diodes are metal-semiconductor barrier diodes as shown in Fig. 10.2. The diode is constructed on a thin silicon (n^+-type) substrate by growing epitaxially on n-type active layer of about 2 micron thickness. A thin SiO_2 layer is grown thermally over this active layer. Metal-semiconductor junction is formed by depositing metal over SiO_2. Schottky diodes also exhibit a square-law characteristic and have a higher burn out rating, lower $1/f$ noise and better reliability than point contact diodes. When the device is forward biased, the major carriers (electrons) can be easily injected from the highly doped n-semiconductor material into the metal. When it is reverse-biased, the barrier height becomes too high for the electrons to cross and no conduction takes place.

RF power flow in the device is limited by power dissipation in R_s and is shorted across C_j . C_c and L_s produce RF-mismatch and can be matched by external circuit.



Schottky diode and its equivalent circuit

R_j = resistance of metallic junction

C_j = barrier capacitance (0.3–0.5 pF)

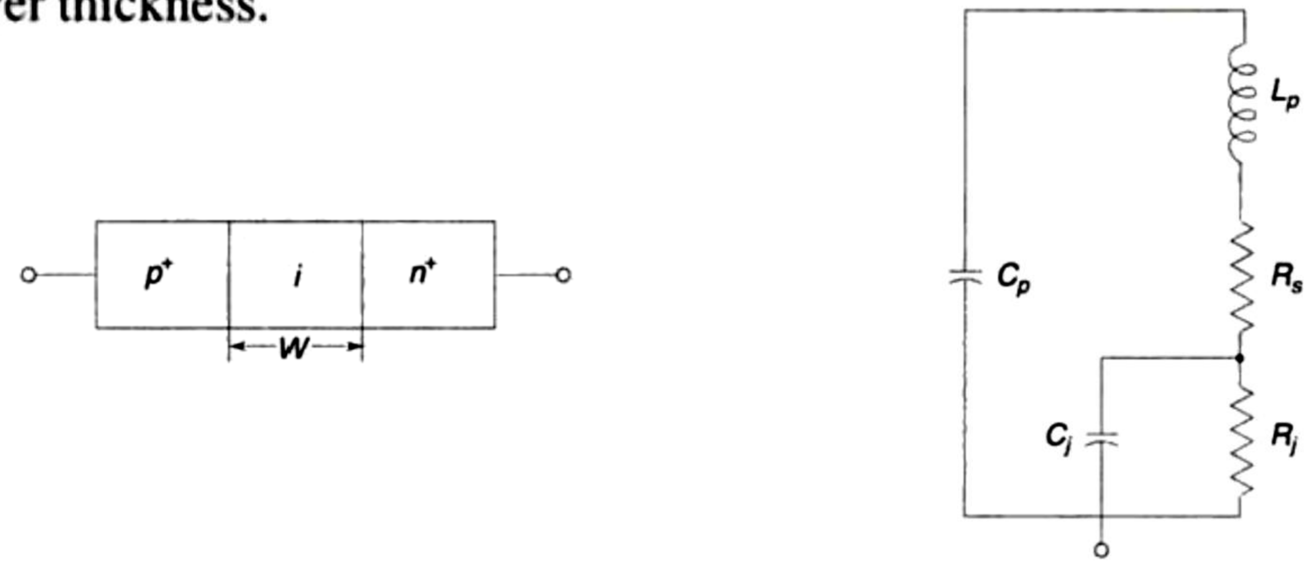
R_s = bulk resistance of heavily doped Si substrate (4–6 ohm)

L_s = inductance of gold whisker wire (0.4–0.9 nH)

C_c = Case capacitance

PIN Diode

A PIN diode consists of a high-resistivity intrinsic semiconductor layer between two highly doped p^+ and n^+ Si layers as shown in Fig. 10.8 along with its equivalent circuit. The device acts as electrically variable resistor related to the *i* layer thickness.



PIN diode and equivalent circuit

R_j, C_j = Junction resistance, capacitance of *i* layer

R_s = Bulk semiconductor (p^+ and n^+) layer and contact resistance

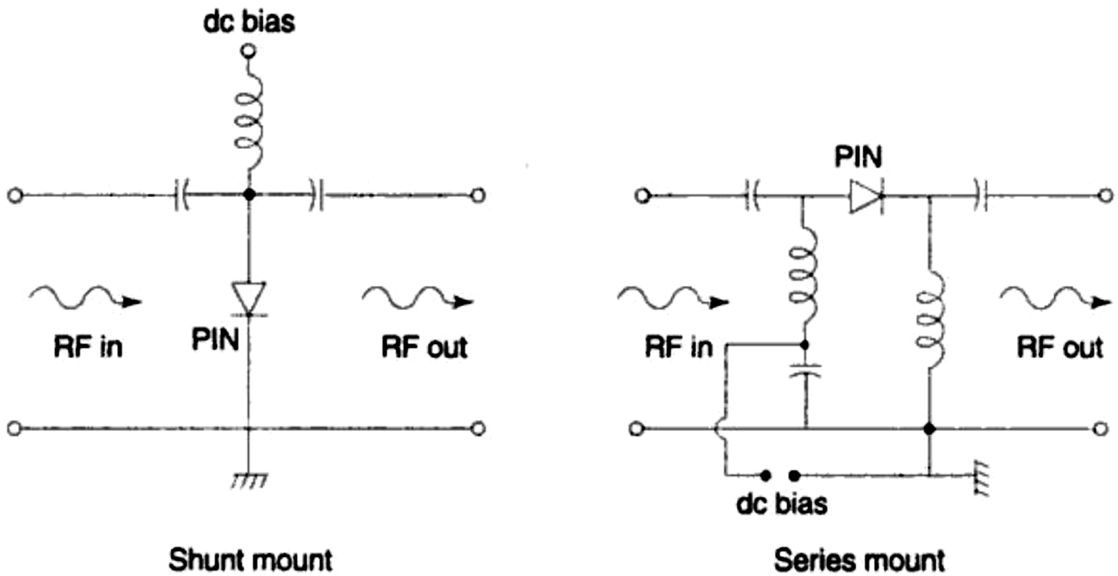
L_p, C_p = package inductance, capacitance

PIN Switch:

Types:

1. Single Switch
2. Double Switch

Single Switch



Single PIN switch

DC blocking inductor and Capacitor used

For Shunt configuration

Reverse Bias - Transmission ON

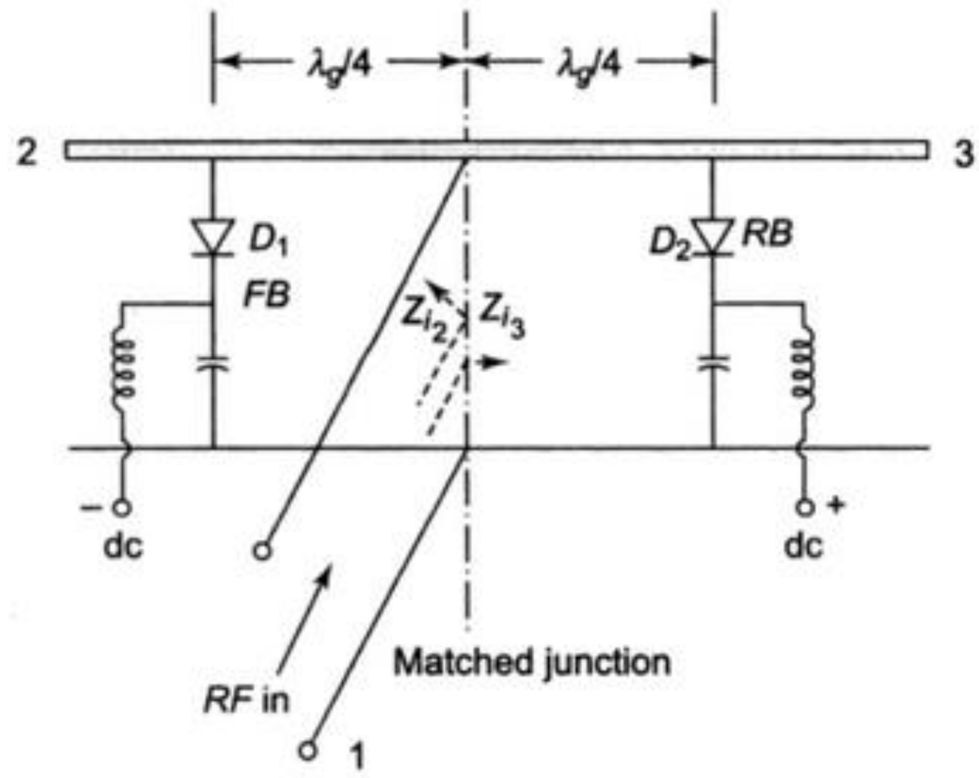
Forward Bias – Transmission OFF

For Series Configuration

Forward Bias – Transmission ON

Reverse Bias - Transmission OFF

Double Switch:



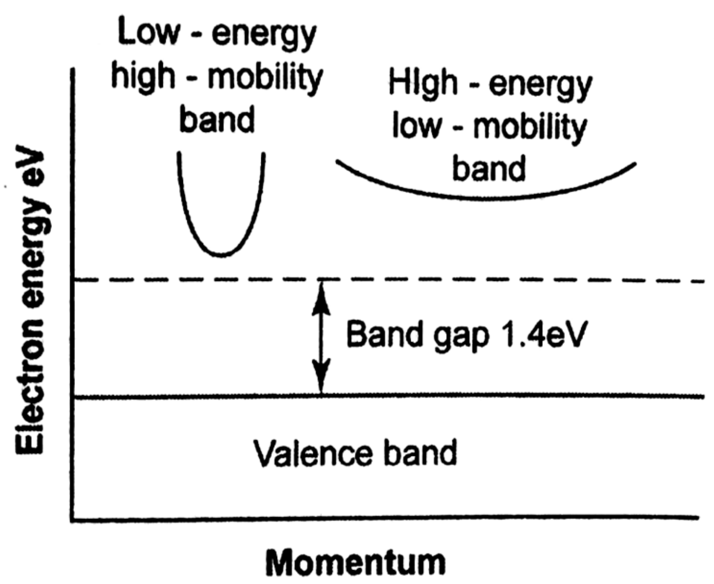
SPDT double switch

(i) When D_1 is forward biased, Z_{i2} = infinite
(ii) When D_2 is reverse biased, $Z_{i3} = 0$,

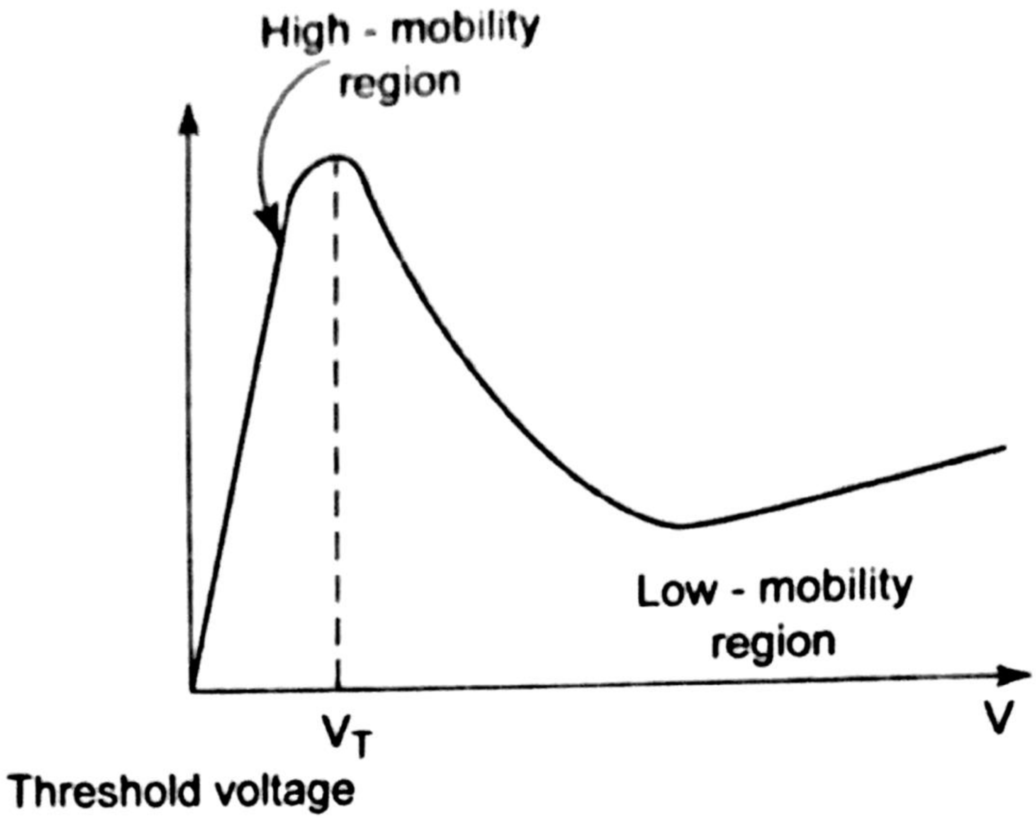
Transferred Electron Devices

Transferred Electron Effect:

Some materials like GaAs exhibit a negative differential mobility (i.e., a decrease in the carrier velocity with an increase in the electric field) when biased above a threshold value of the electric field. The electrons in the lower – energy band will be transferred into the higher – energy band. The behaviour is called ***transferred elecıron effect*** *and the device is also called* ***transferred electron device (TED) or Gunn diode.***



Energy conduction band for a Gunn material such as GaAs



Current – Voltage characteristics for a Gunn device

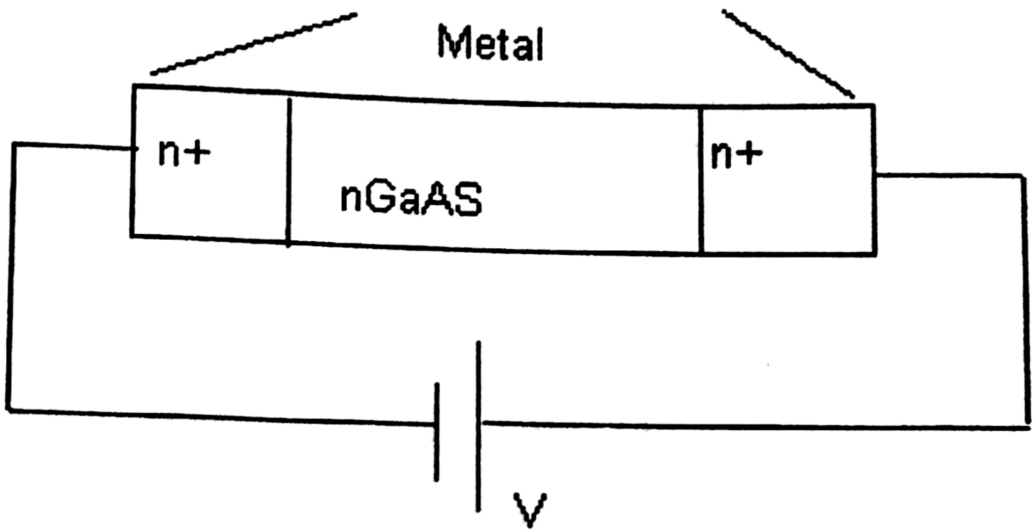
In the high – energy band the effective electron mass is larger and hence the electron mobility is lower than low – energy band.

Gunn diodes are negative resistance devices which are normally used as low power oscillator at microwave frequencies in transmitter and also as local oscillator in receiver front ends.

TEDs are fabricated from compound semiconductors, such as gallium arsenide (GaAs), indium phosphide (InP), or Cadmium telluride (CdTe).

The ***positive resistances absorb power*** (passive devices), whereas ***negative resistances generate power*** (active devices).

GUNN Diode – GaAs Diode



A simple Gunn Oscillator

The basic structure of a Gunn diode, which consists of n – type GaAs semiconductor with regions of high doping (n^+).

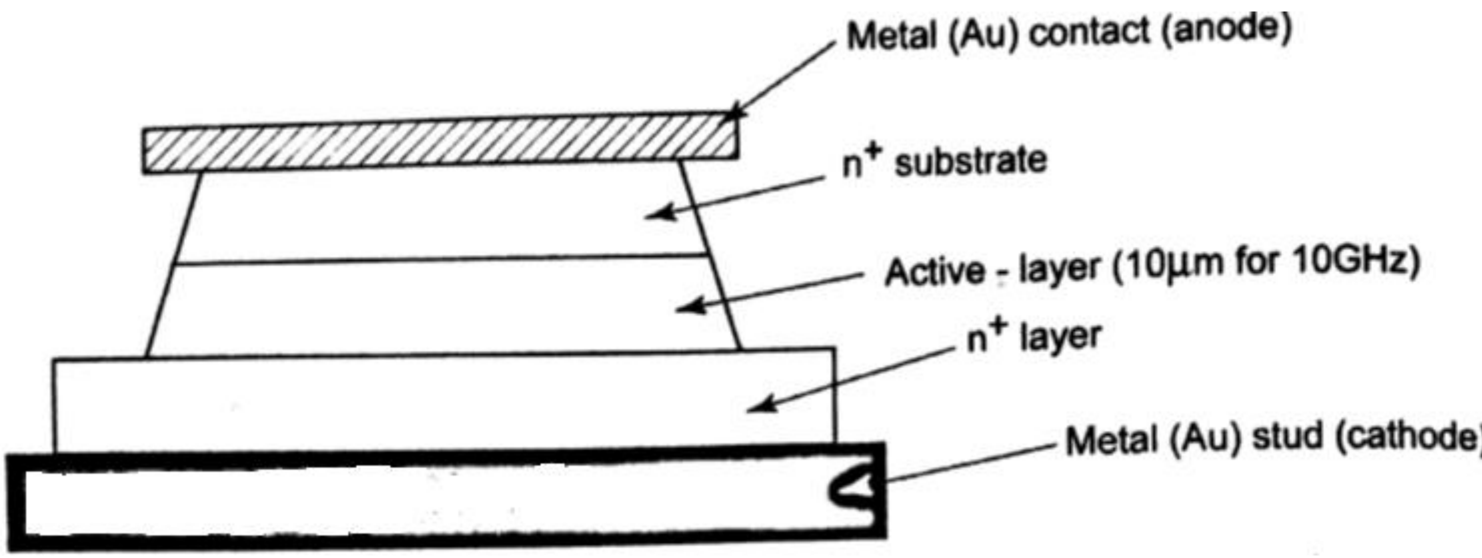
Even though there is no junction this is called a diode with reference to the positive end (anode) and negative end (cathode) of the dc voltage applied across the device.

If a dc (or) diode voltage or an electric field at low level is applied to the GaAs, an electric field is established across it. Initially the current will increase with a rise in the voltage.

At low E-field in the material, most of the electrons will be located in the lower energy band

When the diode voltage exceeds a certain threshold value, V_{th}, a high electric field (3.2kV/m for GaAs) is produced across the active regions and ***electrons*** are excited from their ***initial lower valley*** to the ***higher valley*** where they become virtually immobile.

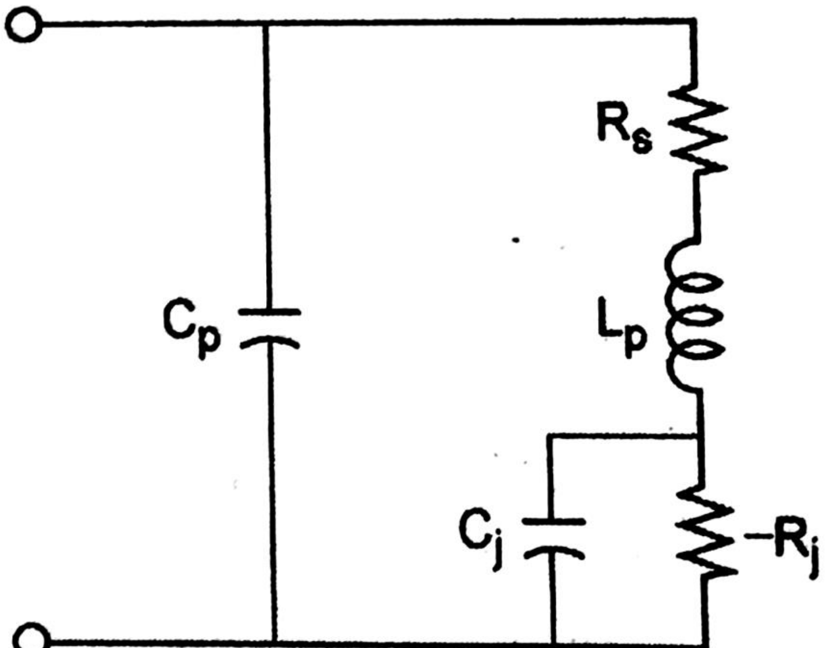
If the rate at which electrons are transferred is very high, the current will decrease with increase in voltage, resulting in equivalent negative resistance effect.



Construction of Gunn diode

Negative Resistance:

The carrier drift velocity is linearly increased from zero to a maximum when the electric field is varied from zero to a threshold value. When the electric field is beyond the threshold value of 3000V/cm, the drift velocity is decreased and the diode exhibits negative resistance.



C_j – Diode capacitance
$-R_j$ – Diode resistance
R_s – Total resistance of leads ohmic contact bulk resistance of diode
L_p – Package inductance and
C_p – Package capacitance.

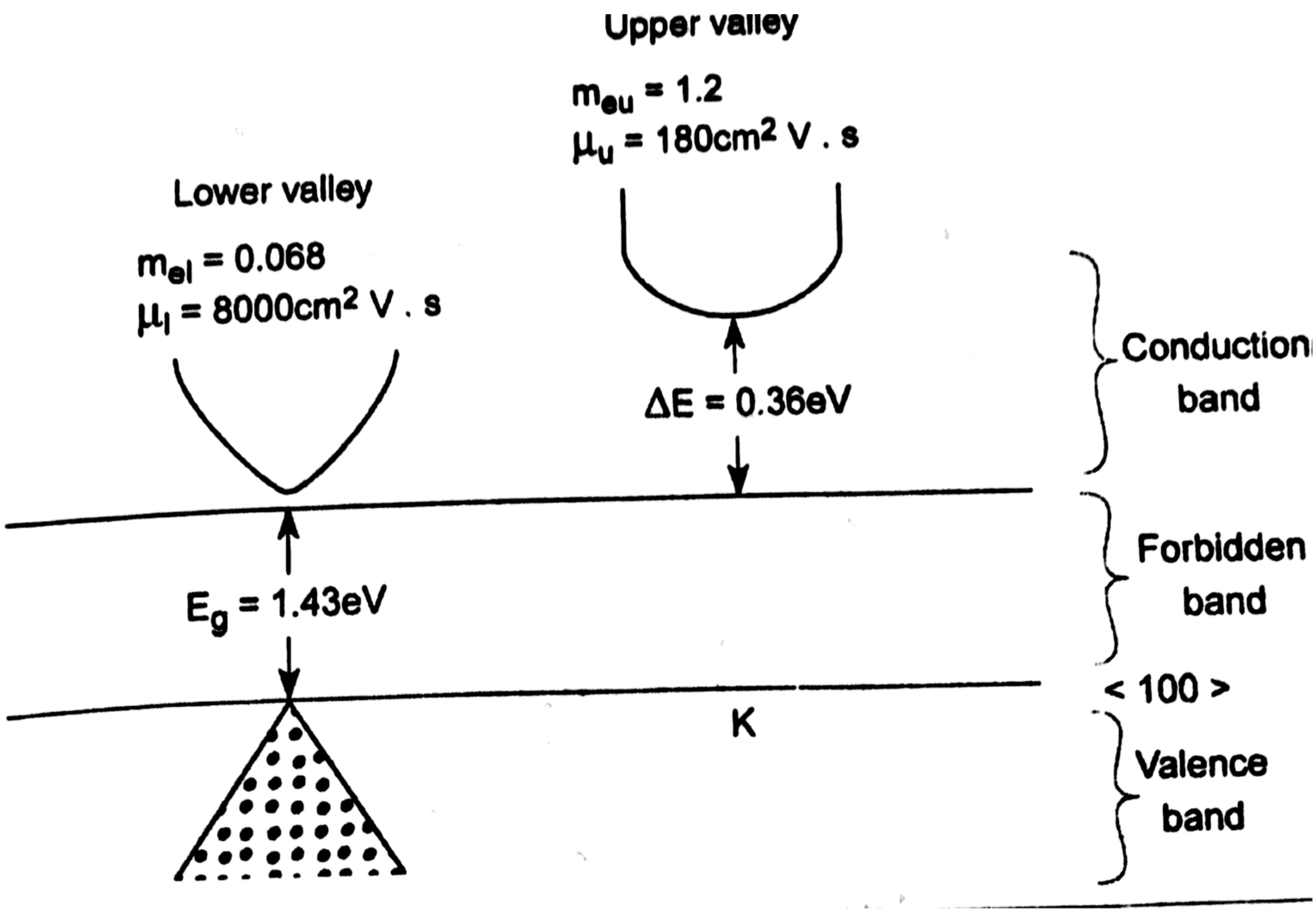
Equivalent circuit of a Gunn diode

Two valley model theory:

According to the energy band theory of the n – type GaAs, a high – mobility lower valley is separated by energy of ***0.36eV*** from a low – mobility upper valley.

Data for two valleys in GaAs

Valley	Effective Mass M_e	Mobility μ	Separation ΔE
Lower	$M_{el} = 0.068$	$\mu_l = 8000 cm^2 / V\text{-}sec$	$\Delta E = 0.36eV$
Upper	$M_{eu} = 1.2$	$\mu_u = 180\ cm^2 / V\text{-}sec$	$\Delta E = 0.36eV$



Two – valley model for n – type GaAs.

Transfer of electron densities:

Electron densities in the lower and upper valleys remain the same under an equilibrium condition.

(i) When the applied electric field is ***lower than*** the electric field of the lower valley ***($E < E_l$)***. ***No electrons*** will ***transfer*** to the ***upper valley***.

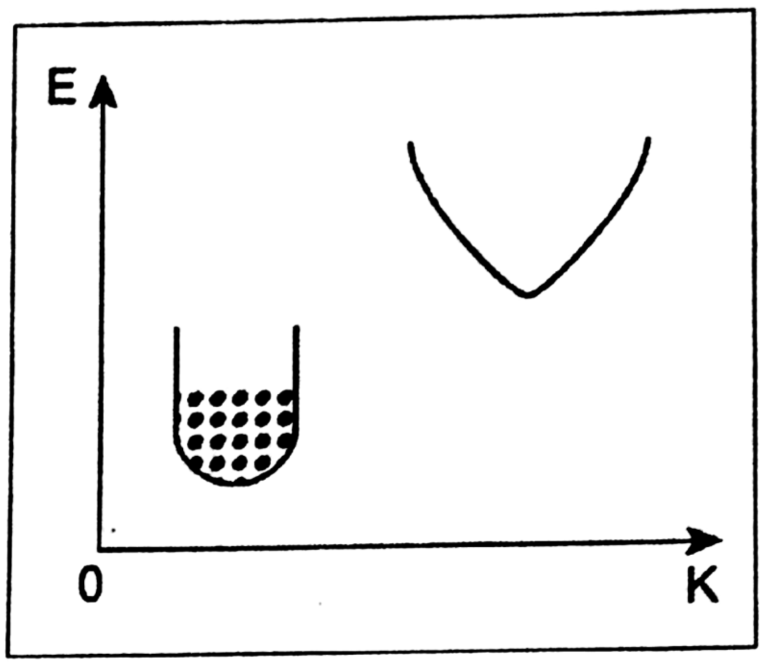


Fig. 9.10. $E < E_l$

(ii) When the applied electric field is higher than that of the lower valley and lower than that of the upper valley ($E_l < E < E_u$). ***Electrons*** will begin to ***transfer*** to the ***upper valley***.

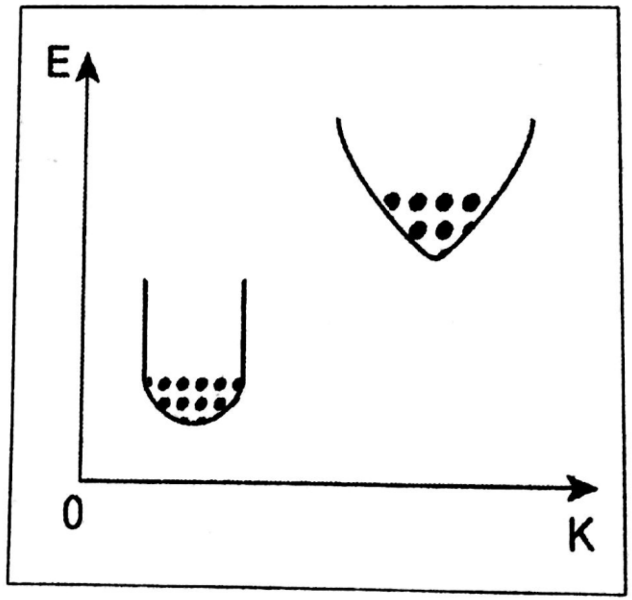


Fig. 9.11. $E_l < E < E_u$

(iii) When the applied electric field is higher than that of the upper valley ***($E_u < E$), all electrons*** will ***transfer*** to the ***upper valley.***

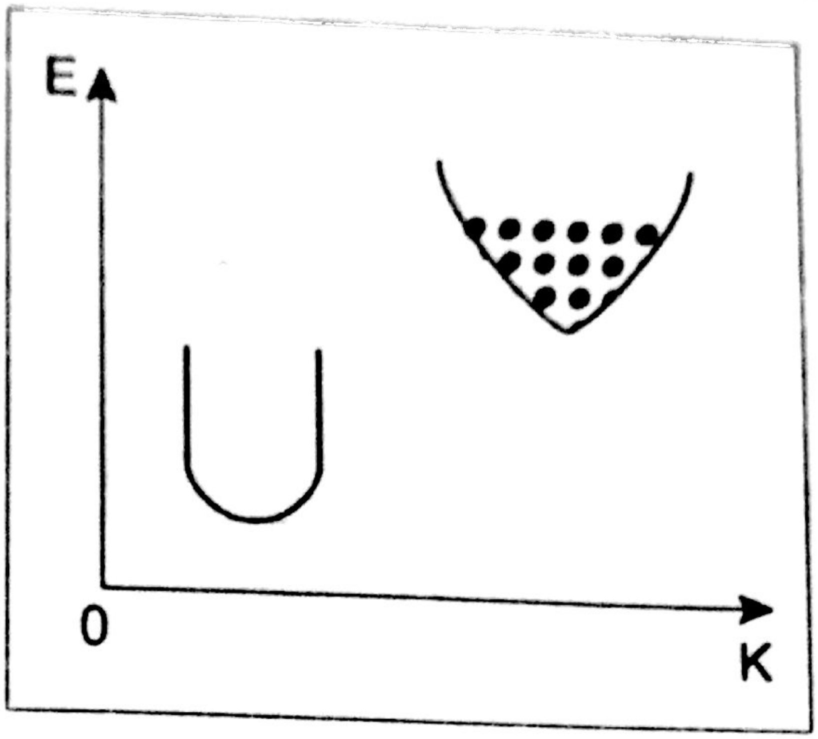
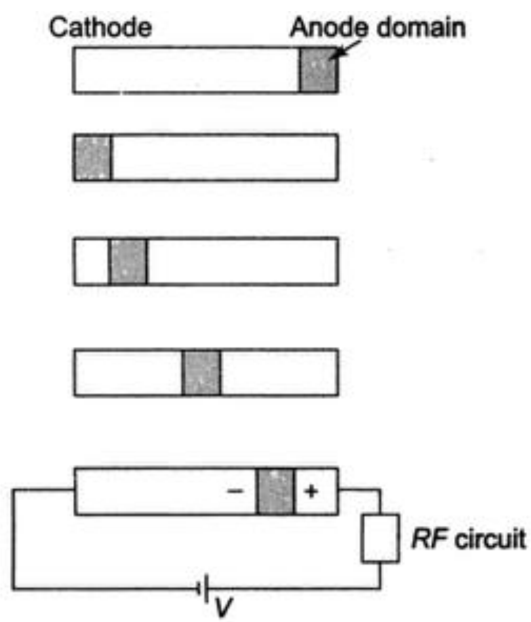


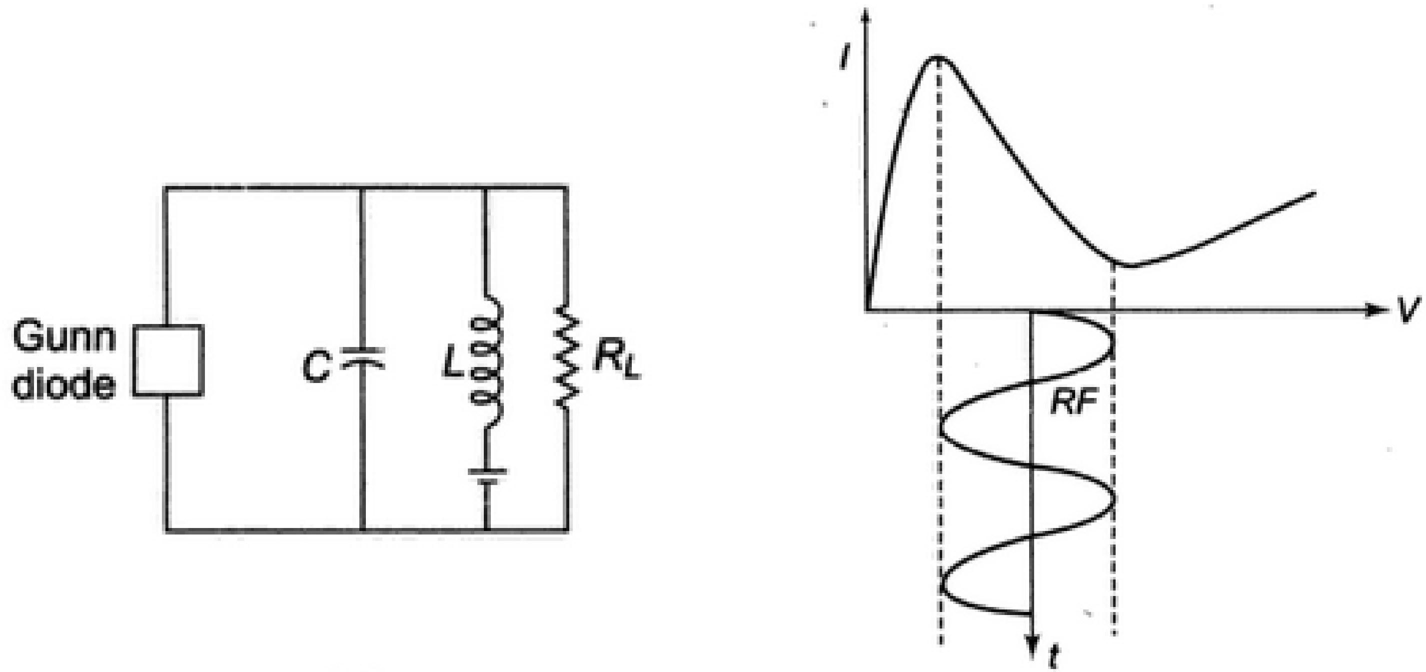
***Fig. 9.12.* $E_u < E$**

Operating Modes:

1. Gun or TT mode
2. LSA mode
3. Quenched Domain mode
4. Delayed mode



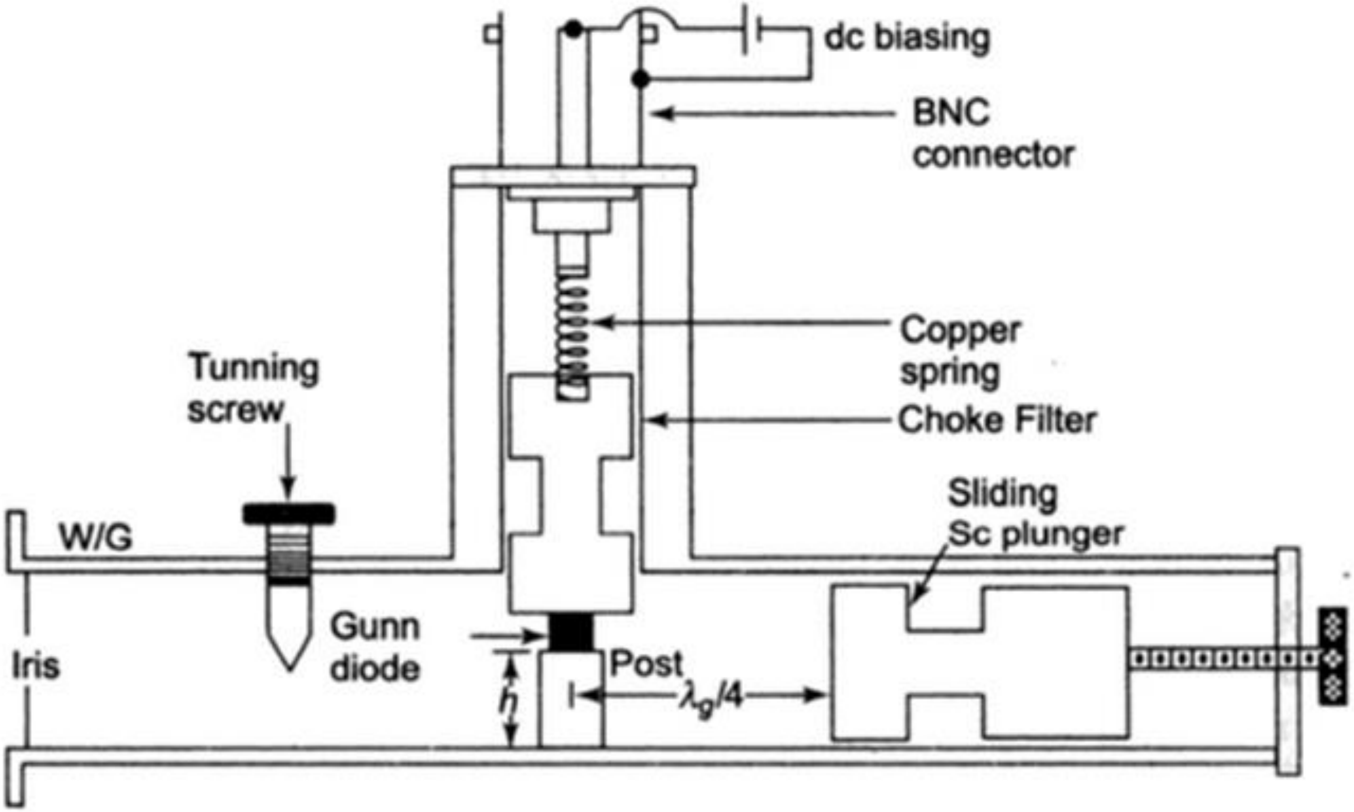
High-field domain movement



(a) (b)

Gunn oscillator operating in LSA mode and RF oscillating voltage

Gunn Diode Oscillator



Gunn diode oscillator circuit

Gunn diode oscillators are commonly used in radars as *LO* and also as signal source in the laboratory. A Gunn diode oscillator can be designed by mounting the diode inside a waveguide cavity formed by a short circuit termination at one end and by an iris at other end as shown in Fig. 10.24. The diode is mounted at the centre perpendicular to the broadwall where the electric field component is maximum under the dominant TE_{10} mode. The intrinsic frequency f_0 of oscillation depends on the electron drift velocity V_d due to high field domain through the effective length l.

$$f_0 = V_d/l$$

The power output of the Gunn diode oscillator is in the range of a few watts for CW operation at biasing values 10 V and 1A at 30–40 GHz. A frequency tuning range of nearly 2% can be achieved. For pulsed operation, peak powers are typically 100–200 W.

The power output of the Gunn diode is limited by the difficulty of heat dissipation from the small chip. The advantages are small size, ruggedness, and low cost.

Introduction:

Microwave tubes are constructed to overcome the limitations of conventional electronic vacuum tubes such as triodes, tetrodes and pentodes. These conventional electronic vacuum tubes fail to operate above 1 GHz.

Three important parameters of ordinary vacuum tubes become increasingly important as frequency rises

1. Inter electrode capacitance
2. Lead inductance
3. Electron Transit Time
4. Gain bandwidth product limitation

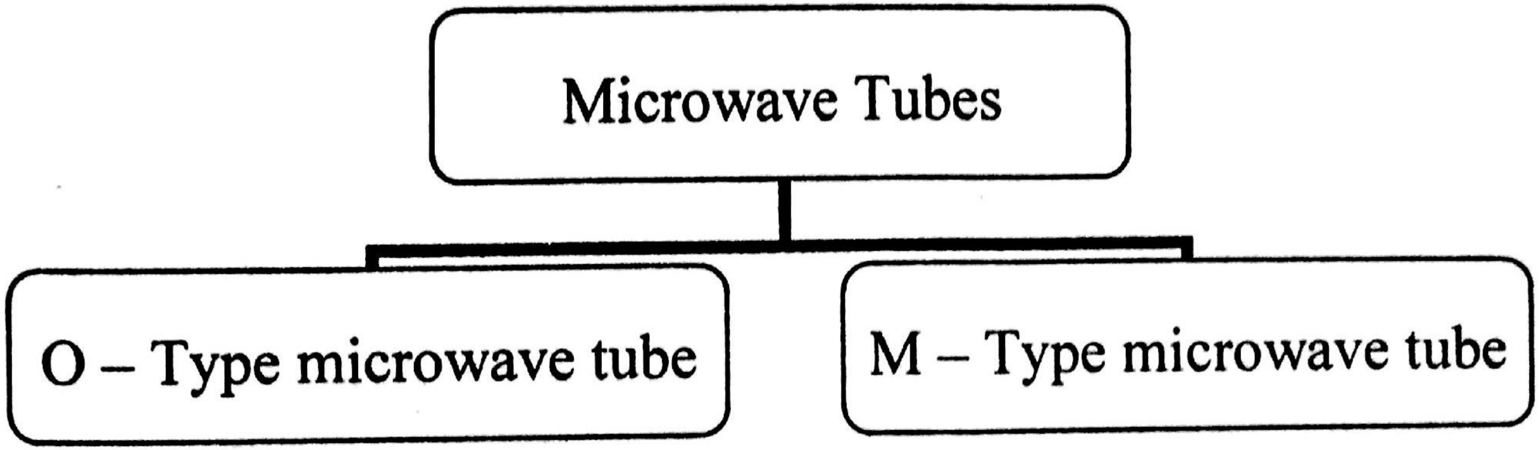
In microwave tubes the ***electron transit*** time is utilized for microwave ***oscillation*** or ***amplification***.

Transit Time:

Transit time is the ***time taken*** *for the* ***electron*** *to* ***travel*** *from* ***cathode to anode.***

The principle uses an electron beam on which ***space – charge waves*** interact with electromagnetic fields in the microwave cavities to transfer energy to the output circuit of the cavity (klystrons and magnetrons) or interact with the electromagnetic fields in a ***slow – wave structure*** to give amplification through transfer of energy (traveling wave tubes).

Classification of Microwave Tubes

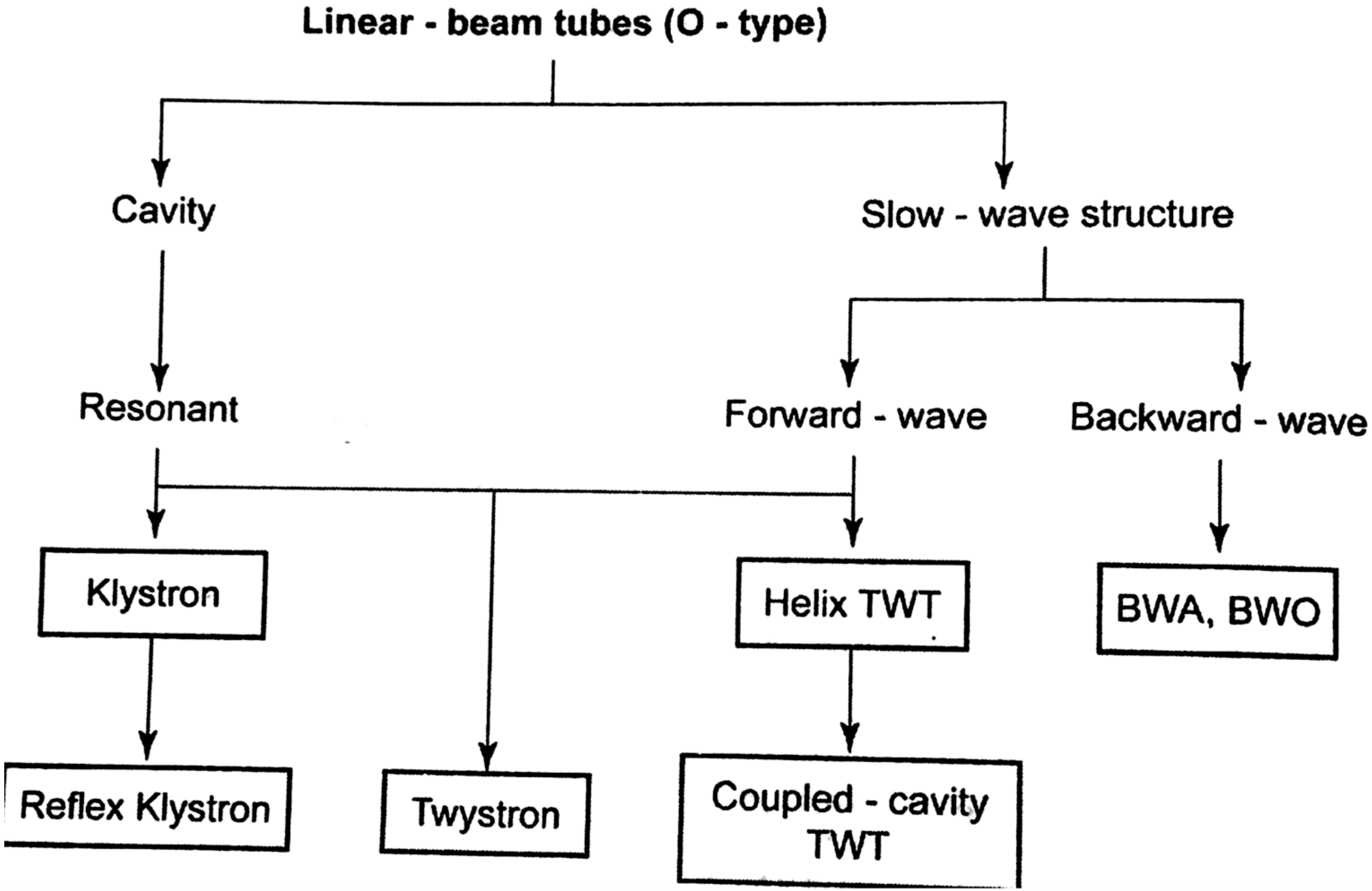


M – Type Microwave Tube

Magnetrons are crossed field devices (M – type) where the static magnetic field is ***perpendicular*** to the electric field. In this tube, the ***electrons travel*** in a ***curved path.***

O – Type Microwave Tube

The most important microwave tubes are ***linear beam*** or 'O' – type tubes in ***recognition of the straight*** path taken by the electron beam.



Klystron

A klystron is a vacuum tube that can be used either as a ***generator*** or as an ***amplifier*** of power at microwave frequencies operated by the principles of ***velocity*** and ***current modulation.***

There are ***two basic configurations*** of Klystron tubes.

(i) **Reflex Klystron** – It is used as low power ***microwave oscillator,*** and

(ii) **Two cavity** (or) **Multi cavity Klystron** – It is used as low power ***microwave amplifier.***

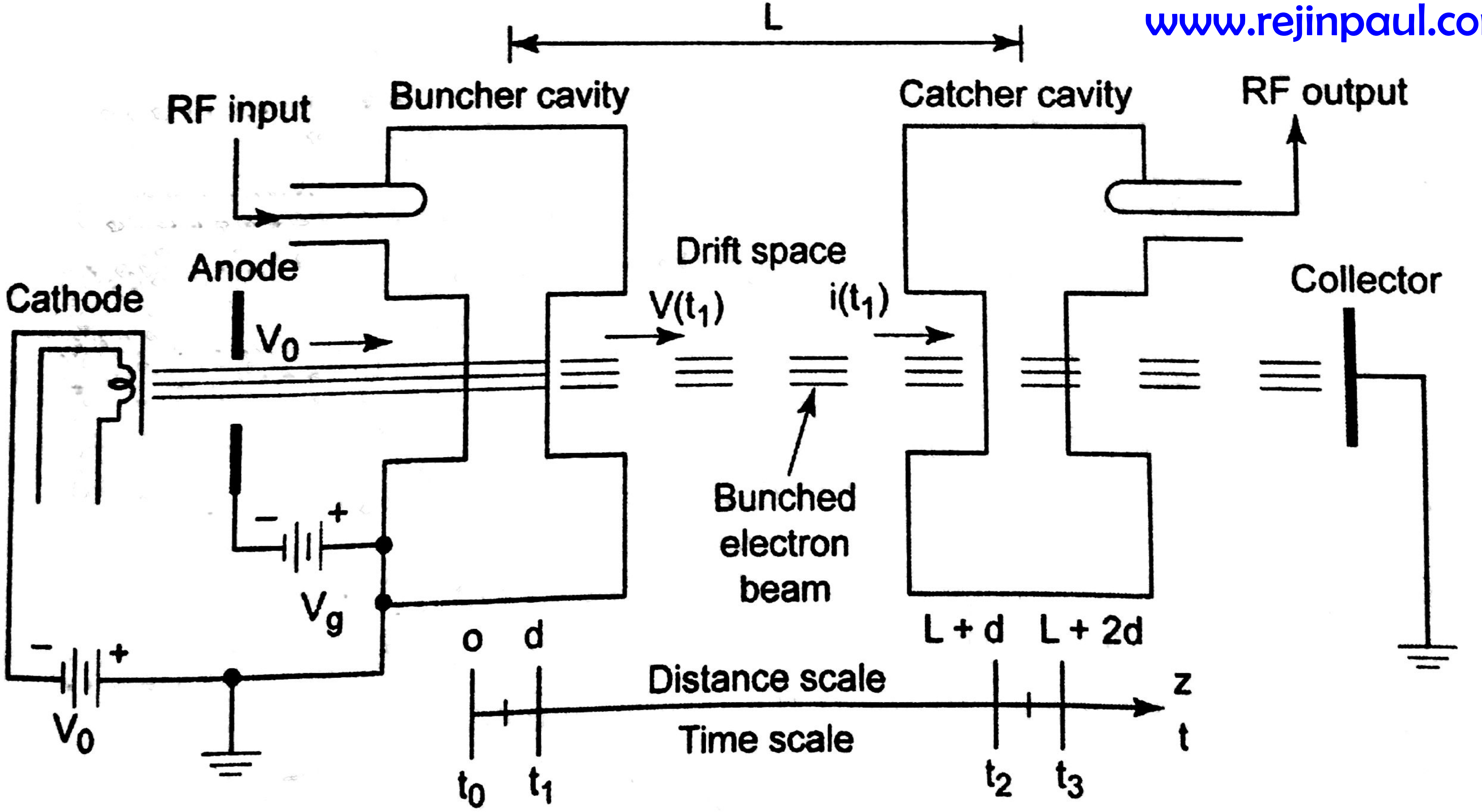
Two Cavity Klystron amplifier

A two-cavity klystron amplifier is a ***velocity modulated tube*** in which velocity modulation process produces ***density modulated stream of electrons.***

It consists of two cavities, ***buncher (input) cavity*** and ***catcher(output) cavity***

Drift Space:

The separation between buncher and catcher grids is called as ***drift space.***



Two – cavity Klystron amplifier.

Operation:

- ✓ Cathode emits the electrons beam. This electrons beam first reach the anode. The accelerating anode produces a high velocity electrons beam.
- ✓ The input RF signal to be amplified excites the buncher cavity with a coupling loop.
- ✓ The electrons beam passing the buncher cavity gap at zeros of the gap voltage (Voltage between buncher grids) passes through with unchanged velocity.
- ✓ The ***electrons beam*** passing through the ***positive half cycles*** of the ***gap voltage*** undergo an ***increase*** in ***velocity***, those passing through the ***negative swings*** of the gap voltage undergo a ***decrease*** in ***velocity***. As a result of these actions, the electrons ***gradually bunch*** together as they travel down the ***drift space***.

✓ The ***first cavity*** acts as the ***buncher*** and ***velocity – modulates*** the beam. Thus the electron beam is velocity modulated to form bunches or under goes density modulation in accordance with the input RF signal cycle.

Velocity – Modulation:

The variation in electron velocity in the drift space is known as velocity modulation

When this density modulated electron beam passing through the catcher cavity grid, it induces RF current (ac current) and thereby excite the RF field in the output cavity at input signal cycle.

The ac current on the beam is such that the level of excitation of the second cavity is much greater than that in the buncher cavity, and hence amplification takes place.

If desired, a portion of the amplified output can be fed back to the buncher cavity in a regenerative manner to obtain ***self – sustained oscillations.***

The maximum bunching should occur approximately midway between the second cavity grids during its retarding phase, thus the ***kinetic energy*** is ***transferred*** from the ***electrons*** to the ***field*** of the ***second cavity***.

The electrons then emerge from the second cavity with reduced velocity and terminate at the collector.

Catcher Cavity:

*The **output cavity** catches **energy** from the **bunched electron beam**. Therefore, it also called as **catcher cavity**.*

Velocity – Modulation Process

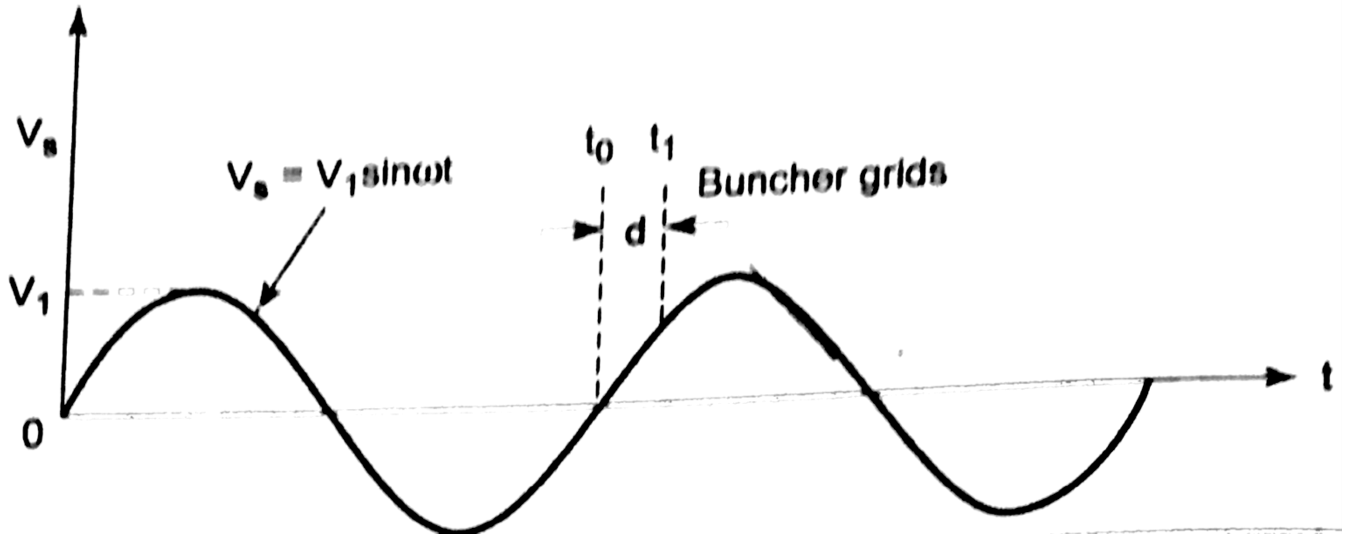
When electrons are ***first accelerated*** by the high dc beam voltage V_0 before entering the buncher grids, their velocity (v_o) is uniform

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593\times10^6 \sqrt{V_0} \text{ m/s} \quad \ldots (1)$$

When the microwave signal is applied to the input terminal of the buncher cavity, the gap voltage between the buncher grids can be written as

$$V_s = V_1 \sin(\omega t) \quad \ldots (2)$$

Where, V_1 is the amplitude of the signal and assume ($V_1 << V_0$)



Signal voltage in buncher gap

Average transit time through the buncher cavity grids gap distance d is

$$\tau \approx \frac{d}{v_0} = t_1 - t_0 \qquad \dots (3)$$

The ***average gap transit angle***

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \qquad \dots (4)$$

The ***average microwave voltage*** in the buncher gap can be written as

$$\langle V_s \rangle = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t) dt$$

$$= -\frac{V_1}{\omega \tau} [\cos(\omega t_1) - \cos(\omega t_0)] \quad \ldots (5)$$

By using trigonometric relation

$$\cos(A-B) - \cos(A+B) = 2 \sin A \ \sin B$$

$$\langle V_s \rangle = \frac{V_1}{\omega \tau} 2 \sin\left[\frac{\omega d}{2 v_0}\right] \sin\left(\omega t_0 + \frac{\omega d}{2 v_0}\right)$$

Substitute $\tau = \dfrac{d}{\nu_0}$

$$= \frac{V_1 \sin\left(\dfrac{\omega d}{2\nu_0}\right)}{\dfrac{\omega d}{2\nu_0}} \sin\left(\omega t_0 + \frac{\omega d}{2\nu_0}\right)$$

$$\boxed{\langle V_s \rangle = V_1 \frac{\sin\left(\dfrac{\theta_g}{2}\right)}{\dfrac{\theta_g}{2}} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)}$$

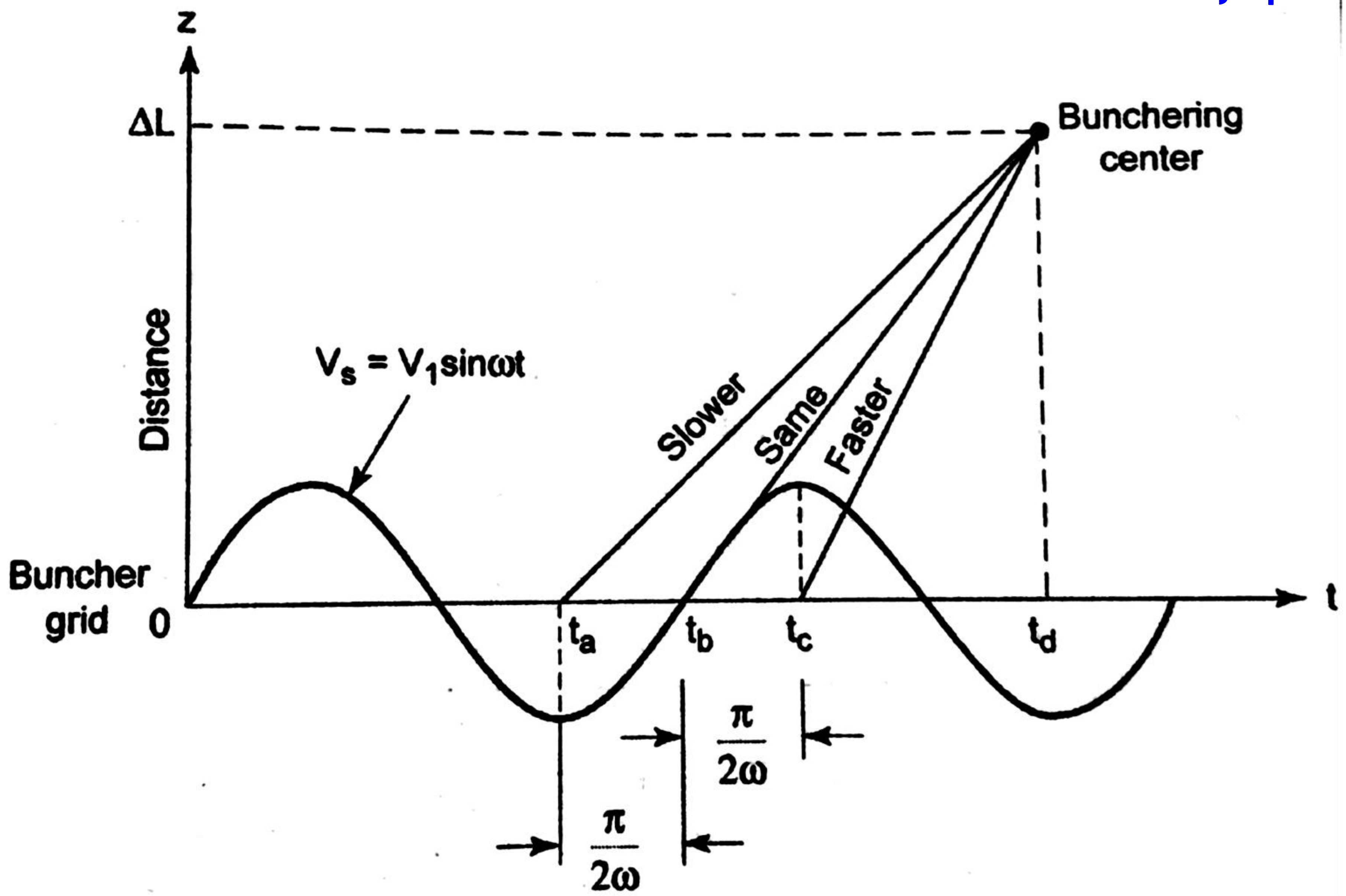
Bunching Process

The ***effect*** of ***velocity modulation*** produces bunching of the ***electron beam*** or ***current modulation***.

The electrons that pass the buncher at $V_s = 0$ travel through with unchanged velocity v_0.

The electrons that pass the buncher cavity during the ***positive half cycles*** of microwave input voltage V_s ***travel faster*** than the electrons that passed the gap when $V_s = 0$.

The electron beams that pass the buncher cavity during the ***negative half cycles*** of the voltage V_s travel ***slower than*** the electrons that passed the gap when $V_s = 0$.



Bunching distance

The distance from the buncher grid to the location of dense electron bunching for the electron at t_b is

$$\Delta L = v_0 (t_d - t_b) \qquad \ldots (11)$$

Where,

$$t_c = t_b + \frac{\pi}{2\omega}$$

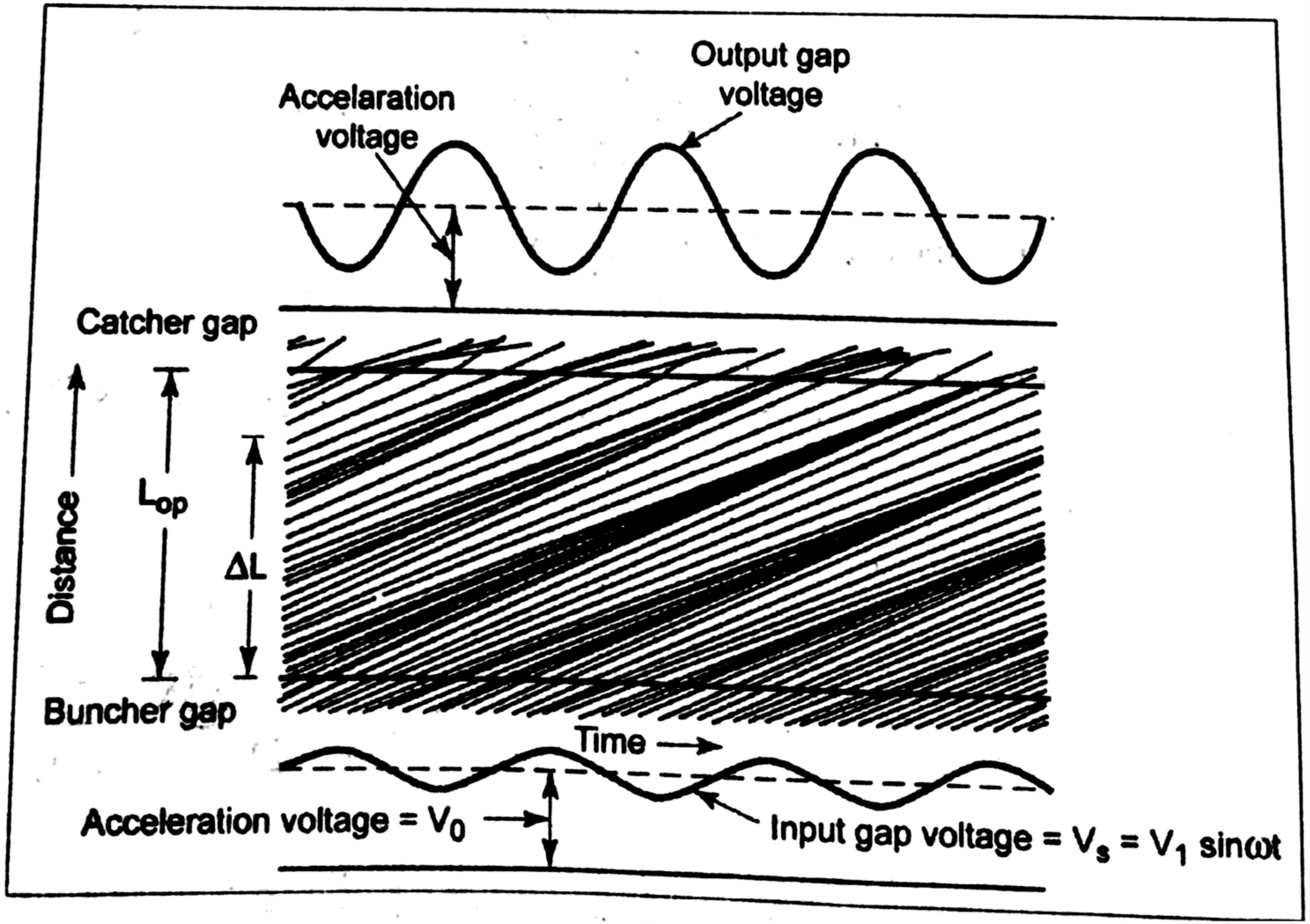
$$t_b = t_a + \frac{\pi}{2\omega}$$

$$t_a = t_b + \frac{\pi}{2\omega}$$

$$\Delta L = v_0 (t_d - t_b) + \left[v_0 \frac{\pi}{2\omega} - \frac{v_0 \beta_i V_1}{2V_0} (t_d - t_b) - \frac{v_0 \beta_i V_1}{2V_0} \frac{\pi}{2\omega} \right]$$

Applegate Diagram:

The applegate diagram represents the internal operation of two cavity klystron by ***distance–time plot***. It includes velocity modulation process, bunching process, energy transfer etc..



Bunching Parameter and DC Transit Angle:

$$\omega T = \omega t_2 - \omega t_1$$

$$= \left[\omega T_0 - \left(\frac{\omega T_0 \beta_i V_1}{2V_0}\right)\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

$$= \theta_0 - X\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)$$

dc transit angle between cavities $\theta_0 = \dfrac{\omega L}{v_0} = 2\pi N$ ...

Where, ***N*** is the ***number of electron transit cycles*** in the drift space.

The *bunching parameter* of a klystron

$$X = \frac{\beta_i V_1}{2V_0}\theta_0$$

Beam Current in Catcher Cavity:

✓ The bunched ***beam current*** at the catcher cavity is a periodic waveform of period $\frac{2\pi}{\omega}$ about ***dc current***.

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nX)\cos\left[n\omega(t_2 - \tau - T_0)\right]$$

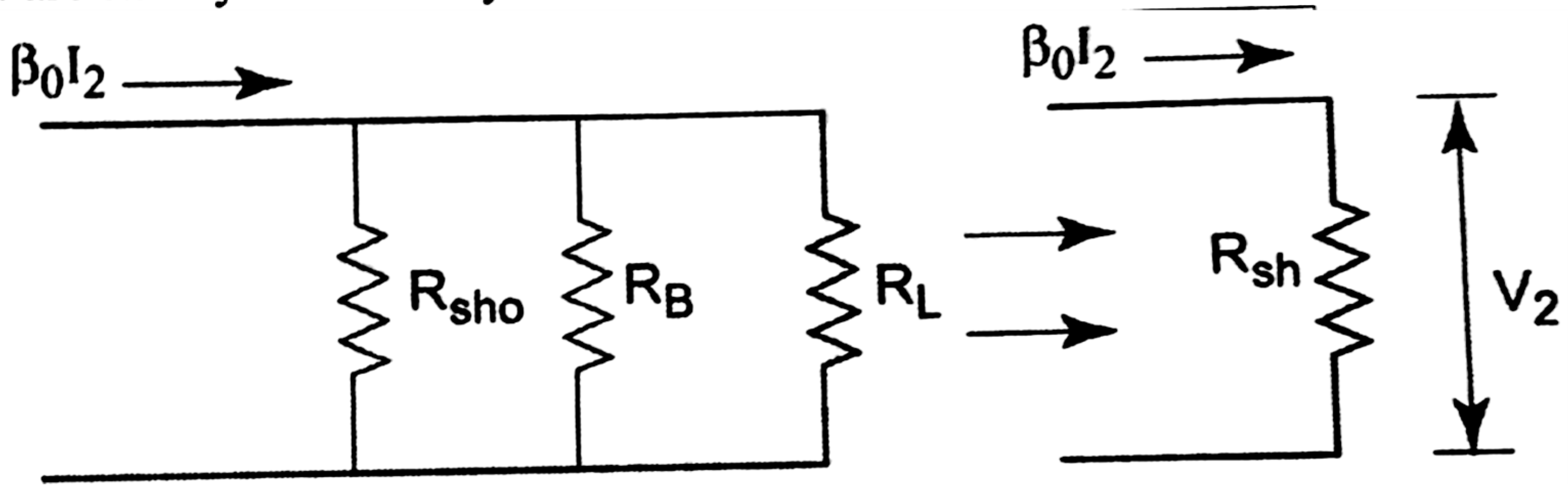
Where, I_0 - ***dc beam current in buncher cavity***

$$L_{opt} = \frac{3.682 V_0 v_0}{\omega \beta_i V_1}$$

Output Power

The maximum bunching should occur approximately midway between the catcher grids.

When the ***electrons*** emerge from the ***catcher grids,*** they have ***reduced velocity*** and are finally ***collected*** by the ***collector.***



Equivalent circuit of output cavity

The output cavity can be represented by an equivalent circuit.

Where

R_{sho} - Wall resistance of catcher cavity,

R_B - Beam loading resistance,

R_L - External load resistance, and

R_{sh} - Total equivalent shunt resistance of the catcher circuit, including the load.

$$P_{out} = \frac{\beta_0 I_2 V_2}{2}$$

Efficiency of Klystron

The electronic efficiency η of the two-cavity klystron amplifier is defined as the "***ratio of the output power to the input power*** (or) the **ratio of RF output power to the dc beam power**".

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{ac}}{P_{dc}}$$

The dc power supplied by the beam voltage, $P_{in} = V_0 I_0$

$$\eta = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$$

Maximum Efficiency:

$$\eta = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$$

$$= \frac{\beta_0 2 I_0 J_1(X) V_2}{2 I_0 V_0}$$

The *efficiency* becomes *maximum*, when $J_1(X) = 0.582$ at $X = 1.841$ and the output voltage V_2 is equal to V_0 ($V_2 = V_0$)

$$= \beta_0 J_1(X)$$

$$= 0.582\ \beta_0$$

If the *coupling is perfect* $\beta_0 = 1$, then

$$\boxed{\eta_{max} = 58.2\ \%}$$

Voltage Gain

The input voltage V1 can be expressed in terms of the bunching parameter X as

$$V_1 = \frac{2V_0}{\beta_0 \theta_0} X$$

Already we know,

$$R_{sh} = \frac{V_2}{\beta_0 I_2}$$

$$V_2 = \beta I_2 R_{sh}$$

The ***voltage gain*** of a ***klystron amplifier*** is defined as

$$A_v = \left|\frac{V_2}{V_1}\right| = \frac{\beta_0 I_2 R_{sh}}{V_1}$$

$$= \frac{\beta_0 \; 2 I_0 \; J_1(X)}{\dfrac{2 V_0 X}{\beta_0 \theta_0}} R_{sh}$$

$$= \frac{\beta_0^2 \theta_0 I_0 J_1(X)}{V_0 X} R_{sh}$$

$$A_v = \frac{\beta_0^2 \; \theta_0}{R_0} \; \frac{J_1(X)}{X} \; R_{sh}$$

$$A_v = G_m \; R_{sh}$$

Characteristics:

(i) ***Efficiency:*** ***≈40%.***

(ii) ***Power output:***

(a) Continuous wave average power ***≈500 KW***

(b) Pulsed ***power*** ***30 MW*** at ***10 GHz.***

(iii) ***Power gain:*** ***≈30 dB.***

Applications:

(i) Used in Troposphere scatter transmitters.

(ii) Satellite communication ground stations.

(iii) Used in UHF TV transmitters.

(iv) Radar transmitters.

Problem No. 1 (8)

A two cavity klystron amplifier has the following parameters:

Beam voltage, $V_0 = 1000$ V, Beam current $I_0 = 25mA$;

Frequency $f = 3GHz$, $R_0 = 40\ k\Omega$

Gap spacing in either cavity, $d = 1mm$

Spacing between the two cavities, $L = 4cm$

Effective shunt impedance, $R_{sh} = 30\ k\Omega$

Calculate input gap voltage, voltage gain and efficiency.

Solution:

(a) For maximum output voltage V_2, $J_1(X)$ must be maximum. This means $J_1(X) = 0.582$ at $X = 1.841$. The electron velocity just leaving the cathode is

$$v_0 = \left(0.593 \times 10^6\right)\sqrt{V_0}$$

$$= \left(0.593 \times 10^6\right)\sqrt{10^3}$$

$$= \left(0.593 \times 10^6\right) \times 31.62$$

$$= 0.593 \times 31.62 \times 10^6$$

$$\boxed{v_0 = 1.88 \times 10^7 \ m/s}$$

(b) The d.c electron transit time across the gap,

0.0562 ns

$$\tau = \frac{d}{v_0} = \frac{1 \times 10^{-3}}{1.779 \times 10^7} = 0.056 \text{ ns}$$

(c) Input voltage for maximum output voltage:

The gap transit angle is $\theta_g = \omega \frac{d}{v_0}$ $\quad (\because \omega = 2\pi f)$

$$= \frac{2\pi\left(3 \times 10^9\right) \times 10^{-3}}{1.88 \times 10^7}$$

$$= \frac{18.284 \times 10^6}{1.88 \times 10^7}$$

$$\boxed{\theta_g = 1 rad}$$

The beam coupling coefficient is

$$\beta_i = \beta_o = \frac{\sin\left(\frac{\theta_g}{2}\right)}{\frac{\theta_g}{2}} = \frac{\sin\left(\frac{1}{2}\right)}{\frac{1}{2}}$$

$$= \frac{0.479}{0.5} = 0.958 \qquad \textbf{(use radian mode)}$$

The dc transit angle between the cavities is

$$\theta_0 = \omega T_0 = \omega \frac{L}{\nu_0}$$

$$= 2\pi \left(3\times10^9\right)\frac{4\times10^{-2}}{1.88\times10^7}$$

$$= 6.28\times3\times2.128$$

$$\boxed{\theta_0 = 40\ rad}$$

The maximum input voltage V_1 is then given by

$$V_{1max} = \frac{2V_0X}{\beta_i\theta_0}$$

$$= \frac{2(1000)(1.841)}{(0.952)(40)}$$

$$= \frac{3682}{38.08}$$

$$\boxed{V_{1max} = 96.5V}$$

96.04

(b) The voltage gain

$$A_v = \frac{\beta_0^2 \ \theta_0}{R_0} \frac{J_1(X)}{X} R_{sh}$$

$$= \frac{(0.9599)^2 (40)(0.582)(30\times10^3)}{4\times10^4 \times1.841}$$

$$= \frac{0.92\times23.28\times30}{7.364}$$

$$= \frac{64.253}{7.364}$$

$$A_v = 8.595$$

8.704

(c) Efficiency
$$\eta = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$$

Where,

$$I_2 = 2\beta_o I_0 J_1(X) = 2\times25\times10^{-3}\times0.582 \quad (\beta_0 = 1)$$

$$I_2 = 29.1\times10^{-3}\ A$$

$$V_2 = \beta_0\ I_2 R_{sh}$$

$$= (0.959)(29.1\times10^{-3})(30\times10^{3})$$

$$V_2 = 831V$$

$$\text{Efficiency} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$$

$$= \frac{(0.959)(29.1\times10^{-3})(831)}{2\times(25\times10^{-3})(10^3)}$$

$$= \frac{23190.634}{50\times10^3}$$

$$\boxed{\eta = 46.38\ \%}$$

46.63%

Problem No. 2:

(8)

A pulsed cylindrical magnetron is operated with the following parameters:

Anode voltage = 25kV

Beam current = 25A

Magnetic density = $0.35\ Wb/m^2$

Radius of cathode cylinder = 4cm

Radius of anode cylinder = 8cm

Calculate *a) The angular frequency*

b) The cutoff voltage

c) The cutoff magnetic flux density.

Solution:

a) Angular frequency $\omega_c = \frac{e}{m} B_0$

$$= 1.759 \times 10^{11} \times 0.34$$

$$= 0.62 \times 10^{11} \text{ radian}$$

0.615×10^{11}

b) The cutoff voltage $V_{OC} = \frac{e}{8m} B_o^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2$

$$= \frac{1}{8} \times 1.759 \times 10^{11} \times (0.35)^2 \times (8 \times 10^{-2})^2 \left(1 - \frac{4^2}{8^2}\right)^2$$

$$= 0.125 \times 1.759 \times 0.1225 \times 64 \times 10^{-4} \times 10^{11} \times \left(\frac{64-16}{64}\right)^2$$

$$= 0.22 \times 7.84 \times 10^7 \times 0.5625$$

$$= 1.725 \times 10^7 \times 0.5625$$

$$= 9.7 MV$$

c) The cutoff magnetic flux density

$$B_{OC} = \frac{\left(8V_0 \frac{m}{e}\right)^{\frac{1}{2}}}{b\left(1-\frac{a^2}{b^2}\right)}$$

$$= \frac{\left(\frac{8\times 25\times 10^3 \times 1}{1.759\times 10^{11}}\right)^{\frac{1}{2}}}{\left[8\times 10^{-2}\left(1-\frac{4^2}{8^2}\right)\right]}$$

$$= \frac{\left(\dfrac{200}{1.759\times 10^{8}}\right)^{\frac{1}{2}}}{\left(8\times 10^{-2}\times 0.75\right)}$$

$$= \frac{\left(113.7\times 10^{-8}\right)^{\frac{1}{2}}}{6\times 10^{-2}}$$

$$= \frac{\left(1.13\times 10^{-6}\right)^{\frac{1}{2}}}{0.06}$$

$$= 17.7\, mWb/m^{2}$$

A reflex klystron operates under the following conditions:

$V_0 = 500V$ $\quad$ $L = 1mm;$ $R_{sh} = 10k\Omega$ $\quad$ $f_r = 8GHz$

$$\frac{e}{m} = 1.759 \times 10^{11} \text{ (MKS system)}$$

The tube is oscillating at f_r at the peak of the $n = 2$ mode or $1\frac{3}{4}$ mode. Assume that the transit time through the gap and beam loading can be neglected.

(a) *Find the value of the repeller voltage V_r.*

(b) *Find the direct current necessary to give a microwave gap voltage of 200 V.*

(c) *What is the electronic efficiency under this condition?*

Solution:

$$\frac{V_0}{(V_r + V_0)^2} = \left(\frac{e}{m}\right) \frac{\left(2\pi n - \frac{\pi}{2}\right)^2}{8\omega^2 L^2}$$

$$= \left(1.759 \times 10^{11}\right) \frac{\left(2\pi \times 2 - \frac{\pi}{2}\right)^2}{8\left(2\pi \times 8 \times 10^9\right)^2 \left(10^{-3}\right)^2}$$

$$= \left(1.759 \times 10^{11}\right) \frac{(12.57 - 1.57)^2}{8\left(50.27 \times 10^9\right)^2 \left(10^{-6}\right)}$$

$$= 1.759\times10^{11}\,\frac{(11)^2}{8(2527.1)\times10^{18}\times10^{-6}}$$

$$= 1.759\times10^{11}\,\frac{121}{20216.8\times10^{12}}$$

$$= \frac{212.839}{202168}$$

$$\boxed{\frac{V_0}{(V_r+V_0)^2} = 1.05\times10^{-3}}$$

$$(V_r + V_0)^2 = \frac{V_0}{1.05 \times 10^{-3}}$$

$$= \frac{500}{1.05 \times 10^{-3}} = 0.476 \times 10^6$$

$$(V_r + V_0) = 689.92$$

$$V_r = 689.92 - V_0$$

$$= 689.92 - 500$$

$$\boxed{V_r = 189.92 \text{ V}}$$

(b) Assume that, $\beta_0 = 1$

$$V_2 = I_2 R_{sh}$$

$$= 2 I_0 J_1(X') R_{sh}$$

The direct current,

$$I_0 = \frac{V_2}{2 J_1 (X') R_{sh}}$$

$$= \frac{200}{2 \times (0.582)(10 \times 10^3)}$$

$$= \frac{200}{1.164 \times 10^4} = 17.18 \times 10^{-3} \text{ A}$$

$$\boxed{I_0 = 17.18 \text{ mA}}$$

(c) Electronic efficiency

$$= \frac{2X' J_1(X')}{2\pi n - \frac{\pi}{2}}$$

$$= \frac{2(1.841)(0.582)}{2\pi(2) - \frac{\pi}{2}}$$

$$= \frac{2.143}{12.57 - 1.57}$$

$$\boxed{\eta = 19.48\%}$$

An X – band pulsed conventional magnetron has the following operating parameters:

Anode voltage, $V_0 = 5.5kV$

Beam current, $I_0 = 4.5A$

Operating frequency, $f = 9 \times 10^9 Hz$

Resonator conductance, $G_r = 2 \times 10^{-4}$ mho

Loaded conductance, $G_l = 2.5 \times 10^{-5}$ mho

Vane capacitance, $C = 2.5pF$

Duty cycle, $DC = 0.002$

Power loss, $P_{loss} = 18.5kW$

Compute:

a) The angular resonant frequency.

b) The unloaded quality factor.

c) The loaded quality factor.

d) The external quality factor.

e) The circuit efficiency.

f) The electronic efficiency.

Solution:

(a) The angular resonant frequency is,

$$\omega_0 = 2\pi f = 2\times 3.14\times 9\times 10^9$$

$$= 56.55\times 10^9 \text{ rad}$$

(b) The unloaded quality factor is,

$$Q_{un} = \frac{\omega_0 C}{G_r} = \frac{56.55\times 10^9 \times 2.5\times 10^{-12}}{2\times 10^{-4}}$$

$$= \frac{141.375\times 10^{-3}}{2\times 10^{-4}} = 707$$

(c) The loaded quality factor is,

$$Q_l = \frac{\omega_0 C}{G_r + G_l} = \frac{56.55 \times 10^{9} \times 2.5 \times 10^{-12}}{2 \times 10^{-4} + 2.5 \times 10^{-5}}$$

$$= \frac{141.375 \times 10^{-3}}{2.25 \times 10^{-4}}$$

$$\boxed{Q_l = 628.3}$$

(d) The external quality factor is,

$$Q_{ex} = \frac{\omega_0 C}{G_l} = \frac{56.55\times10^{9}\times2.5\times10^{-12}}{2.5\times10^{-5}}$$

$$= \frac{141.375\times10^{-3}}{2.5\times10^{-5}}$$

$$Q_{ex} = 5655$$

(e) The circuit efficiency is,

$$\eta_C = \frac{1}{1+\frac{Q_{ex}}{Q_{un}}} = \frac{1}{1+\frac{5655}{707}} = \frac{1}{1+8}$$

$$\boxed{\eta_C = 11.11\%}$$

(f) The electronic efficiency is,

$$\eta_e = \frac{P_{gen}}{P_{dc}} = \frac{V_0 I_0 - P_{lost}}{V_0 I_0}$$

$$= \frac{5.5\times10^3 \times 4.5 - 18.5\times10^3}{5.5\times10^3 \times 4.5}$$

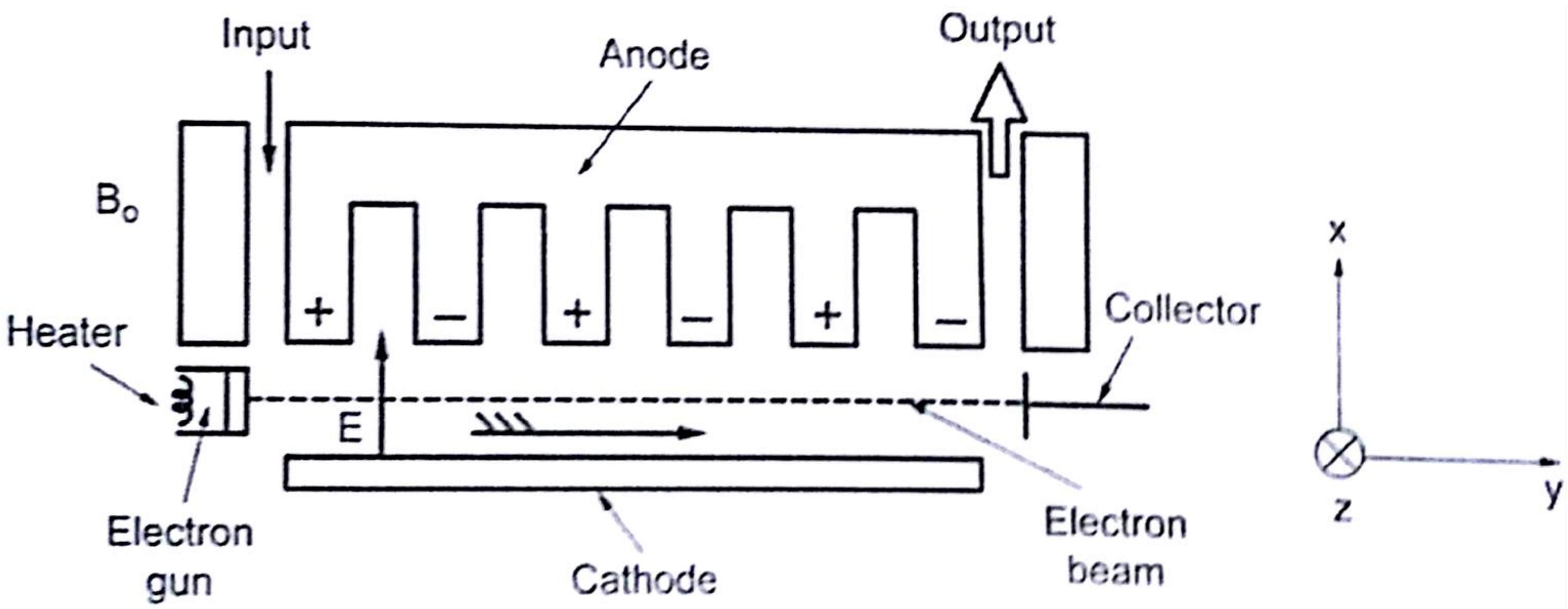
$$= \frac{24.75 - 18.5}{24.75}$$

$$\boxed{\eta_e = 25.25\%}$$

Types of Magnetron

- Cylindrical Magnetron
- Linear Magnetron
- Coaxial Magnetron
- Voltage Tunable Magnetron

Linear Magnetron



Schematic diagram of a linear magnetron

Hull Cutoff Equations

The ***Hull cutoff voltage*** for a linear magnetron is given by,

$$V_{0C} = \frac{1}{2}\frac{e}{m}B_0^2 d^2$$

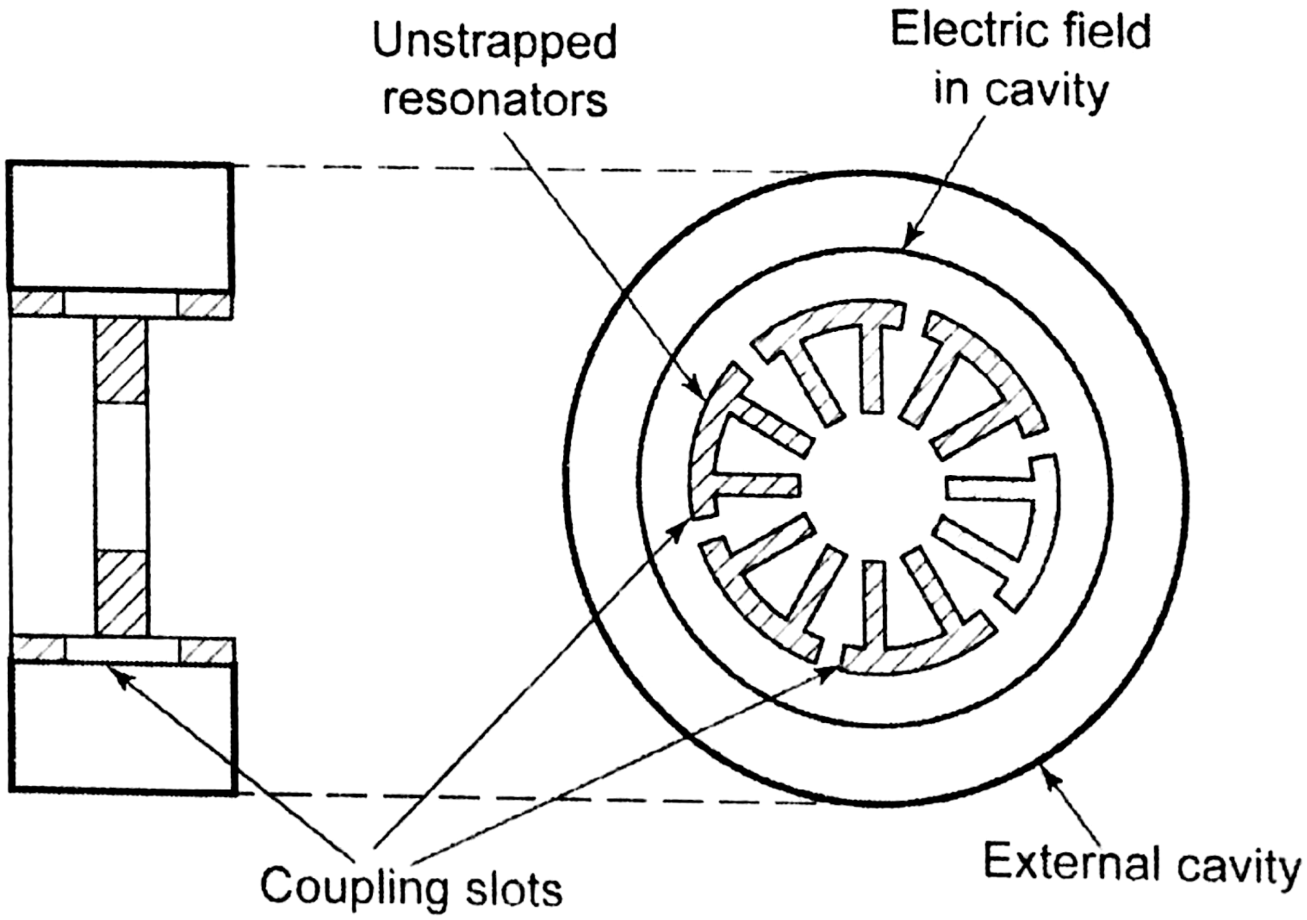
where, d is the anode-cathode distance.

$B_0 = B_z$ is the magnetic flux density in the positive *z* direction.

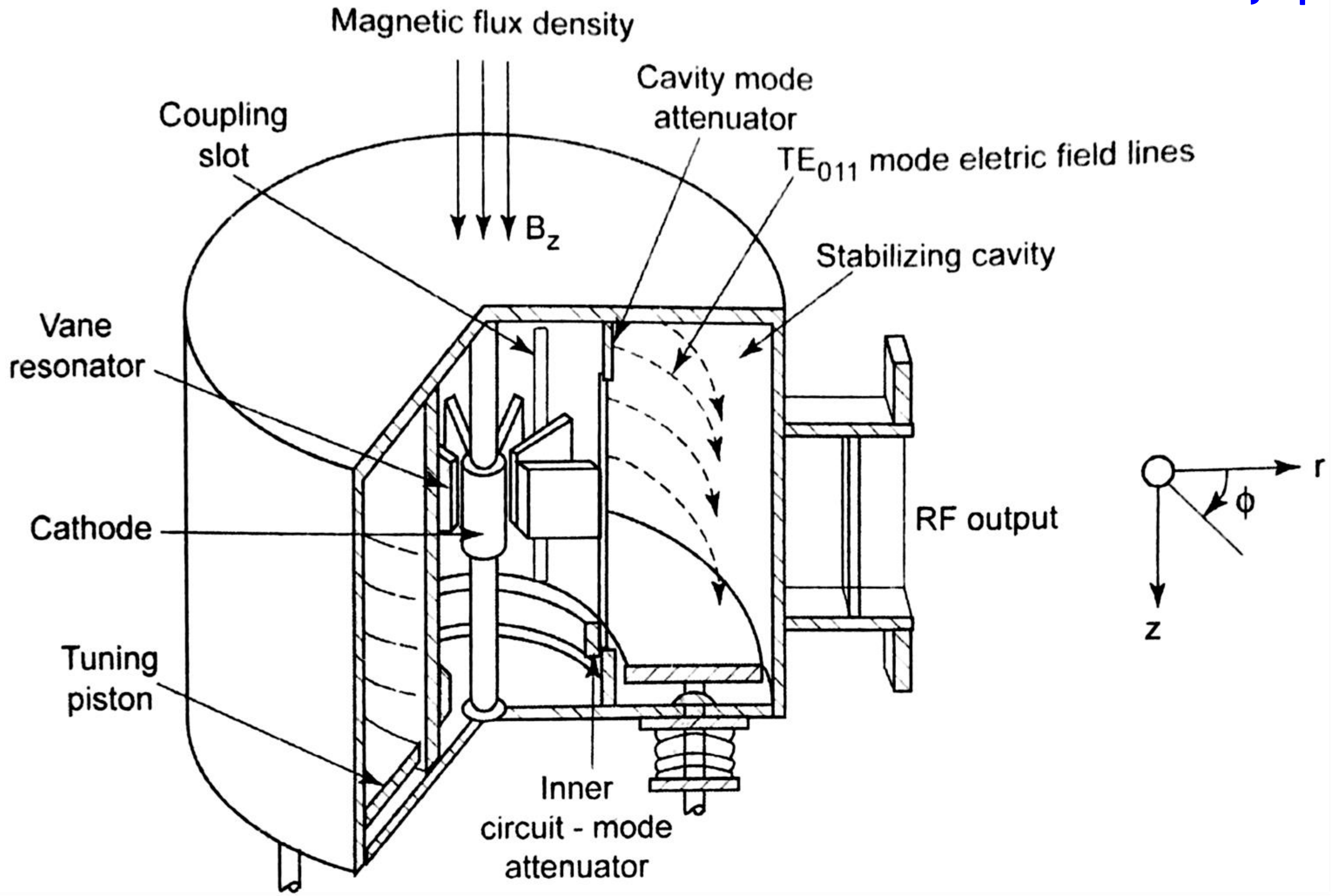
Hull cutoff magnetic flux for a linear magnetron is expressed as,

$$B_{0C} = \frac{1}{d}\sqrt{2\frac{m}{e}V_0}$$

Coaxial Magnetron



Cross section



Cutaway view

Characteristics:

The performance characteristics of coaxial magnetron are,

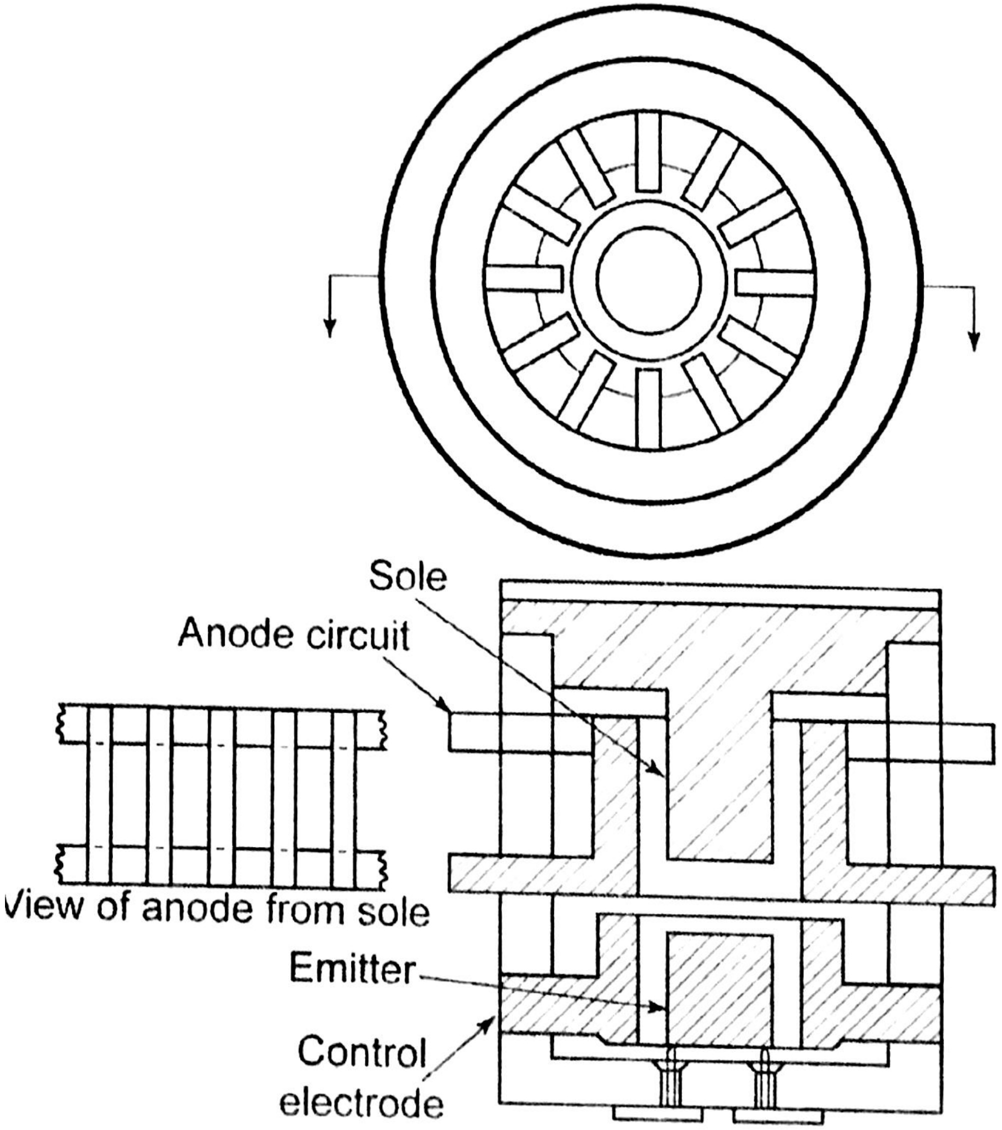
(i) Minimum peak power of 400 kW at a frequency range from 8.9 to 9.6GHz.

(ii) Its duty cycle is 0.0013.

(iii) Nominal anode voltage is 32kV.

(iv) Peak anode current is 32A.

Voltage – Tunable Magnetron



Cross section view of a voltage – tunable magnetron