UNIT - 2

Why DPCM? Convert analog signal into Digital signal in which analog signal is sampled and then difference between the actual value and its predicted value (predicted value is based on previous sample or samples) is quantized and then encoded forming a digital value.

Differential PCM

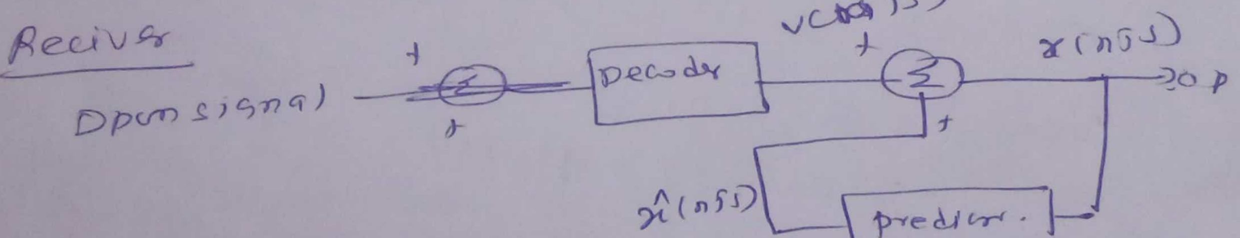
Why DPCM?

In PCM, successive samples

It represents a quantized version of $x(nT_s)$. Therefore, the quantized sample $u(nT_s)$ at the prediction filter input differs from the original I/P sample $x(nT_s)$ by quantization error $e(nT_s)$.

> **Note**: PCM scheme, the successive samples are carrying same information with small difference. When these samples are encoded by standard PCM, the resulting encoded signal contains redundant information (highly correlated it means similarity).
>
> Redundancy: For the signal which do not change rapidly from one sample to next sample.

Reciver



The quantized error signal $v(nT_s)$ is reconstructed from the incoming binary signal by a decoder block.

$$x(nT_s) = v(nT_s) + \hat{x}(nT_s)$$

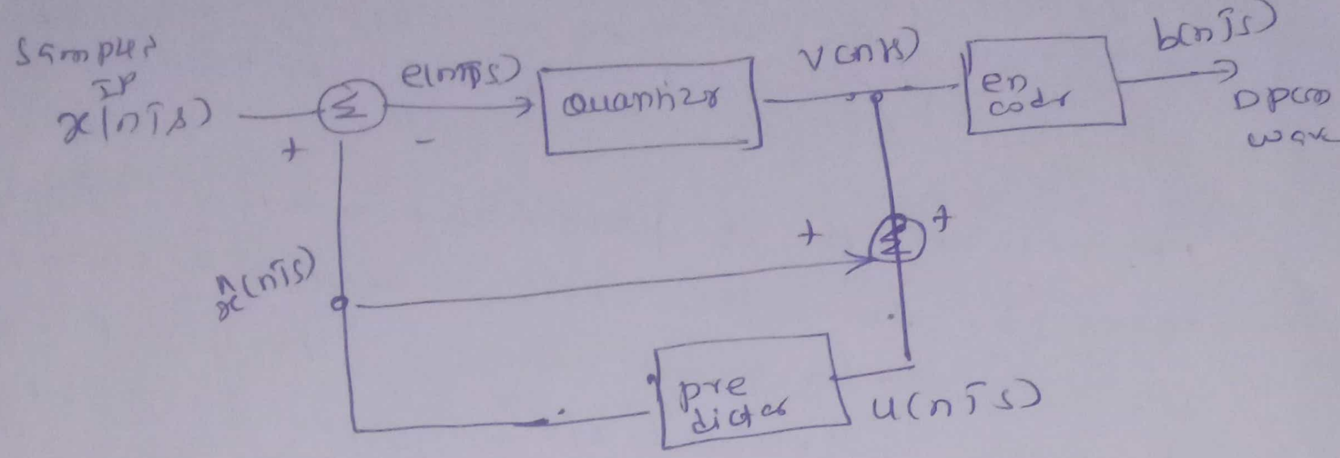
$$x(nT_s) = e(nT_s) + q(nT_s) + \hat{x}(nT_s) \quad (5)$$

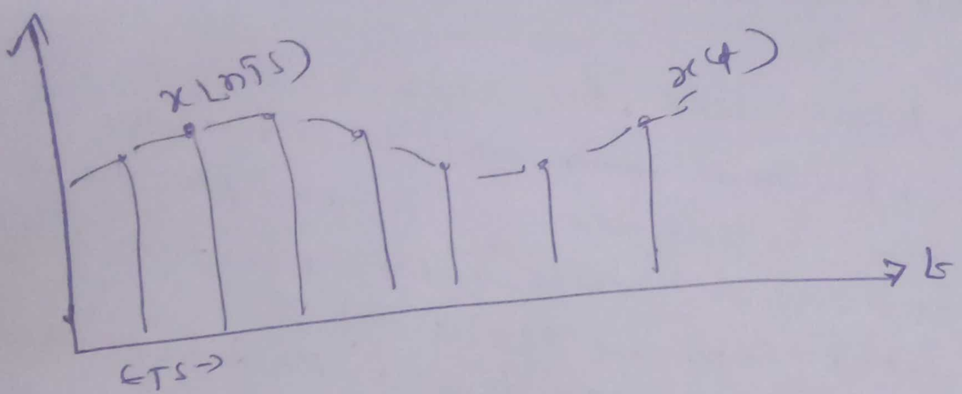
Using equation (1), (5) becomes

$$x(nT_s) = \hat{x}(nT_s) + e(nT_s)$$

The received signal not only contain original signal but also contains quantized error $q(nT_s)$.

DPCM: The difference in amplitude of two successive samples is transmitted rather than actual sample.





$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{—(1)}$$

$$V(nT_s) = Q[e(nT_s)] \quad \text{—(2)}$$
$$= e(nT_s) + q(nT_s)$$

$q(nT_s) \rightarrow$ Quantization error

The quantizer output $V(nT_s)$ is added to the predicted value $\hat{x}(nT_s)$ to produce the prediction - filter ~~output~~ input.

$$u(nT_s) = \hat{x}(nT_s) + V(nT_s) \quad \text{—(3)}$$

Sub (2) in (3)

$$u(nT_s) = \hat{x}(nT_s) + \underline{e(nT_s)} + q(nT_s)$$

Then, using equation (1)

$$x(nT_s) = \hat{x}(nT_s) + e(nT_s)$$

then $\boxed{u(nT_s) = x(nT_s) + q(nT_s)}$ —(4)

The output signal to quantization noise ratio of a signal coder is defined as

$$(SNR)_o = \frac{\sigma^2_x}{\sigma^2_Q}$$

Where σ^2_x is variance of the original signal $x(nTs)$ assumed to be zero mean and σ^2_Q is the variance of the quantization error $q(nTs)$, we may rewrite the equation as

$$(SNR)_o = \left(\frac{\sigma^2_x}{\sigma^2_E}\right)\left(\frac{\sigma^2_E}{\sigma^2_Q}\right)$$

$$= G_p \, (SNR)_p.$$

Where $(SNR)_p$ is the prediction error to quantization noise ratio, defined by

$$(SNR)_p = \frac{\sigma^2_E}{\sigma^2_Q}$$

G_p is the prediction gain produced by the differential quantization, defined by

$$G_p = \frac{\sigma^2_x}{\sigma^2_E}$$

The quantity G_p, when greater than unity, represents the gain in signal to noise ratio that is due to the dpcm scheme.

<u>Conclusion</u> For given base band signal, the variance σ^2_x is fixed. So that G_p is maximized by reducing σ^2_E of the prediction error $e(nTs)$. So that our goal to design a predictor to minimize the error is satisfied.

Advantages and Disadvantages of DPCM

Advantages:

1) only a small difference voltage is to be quantized and encoded. This will need less number of quantization levels and hence less number of bits to represent them.
2) The signaling rate and bandwidth of DPCM system will be less than PCM.

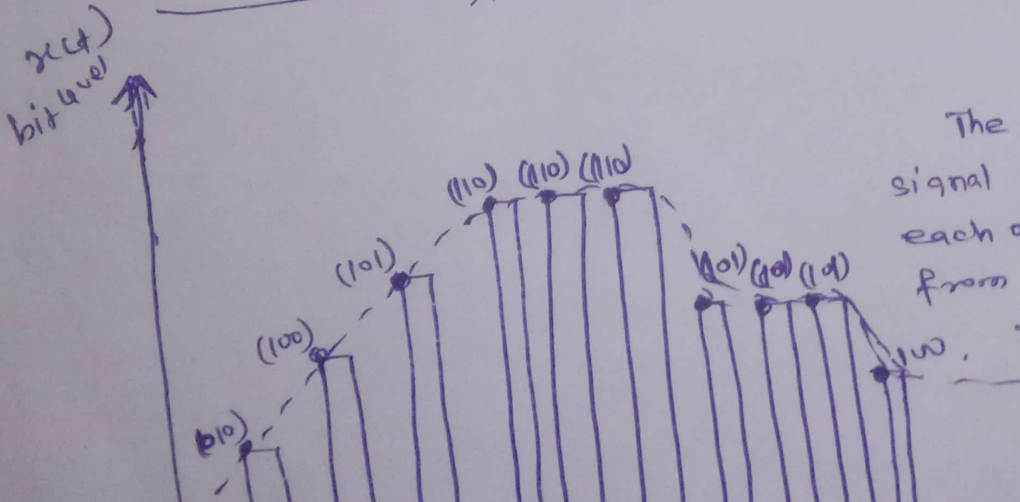
Disadvantages:

(i) Need predictor circuit to be used which is very complex.
(ii) practical usage is limited.

Applications:

Mainly used in image and Audio signal compression. The DPCM conducted on signals with the correlation between successive samples leads to good compression ratio.

———— X ———— X ———— X ———— X ————



The samples of a signal highly correlated with each other. The signal value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with a small difference.

When these samples are encoded by the std PCM, resulting encoded signal contain some redundant information of bits. The above figure 4TS, 5TS, 6TS encoded to same value of (110)

But threee samples (4Ts, 5Ts and 6Ts) are carrying the same information meas redunant

Let consider 9Ts and 10Ts

(101, 100) → the difference between the two samples only due to the last bit and first two bits are redunant. since do not change.

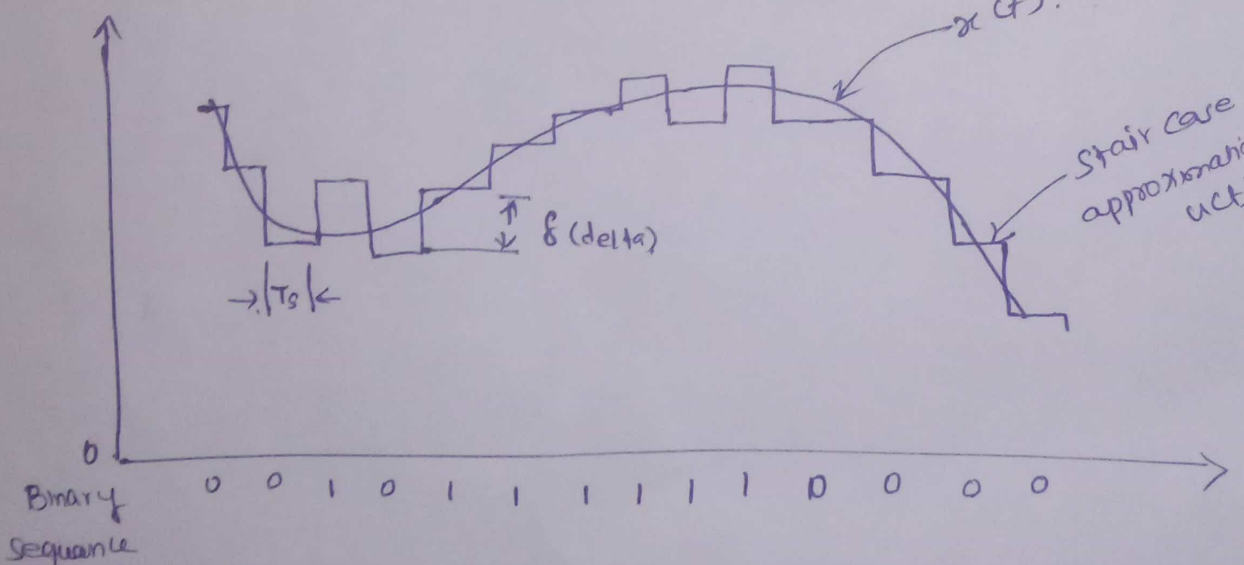
In order to process this redunant information and have a better Design and o/p to take a predicted sampled value.

Delta modulation

In pcm system, multiple bit codes are required to represent the sample values and all these bits have to transmit. Hence the signaling rate and transmission channel Bandwidth of pcm system is very high. To overcome this problem Delta modulation is used.

Definition: The delta modulation, is a one bit (or two level) version of DPCM. In this scheme only a single bit is transmitted which simply indicates whether the present sample is larger or smaller than the privious sample. This reduces ~~the~~ its signaling rate and bandwidth requirement to a great extent compare with pcm.

illustration of delata modulation

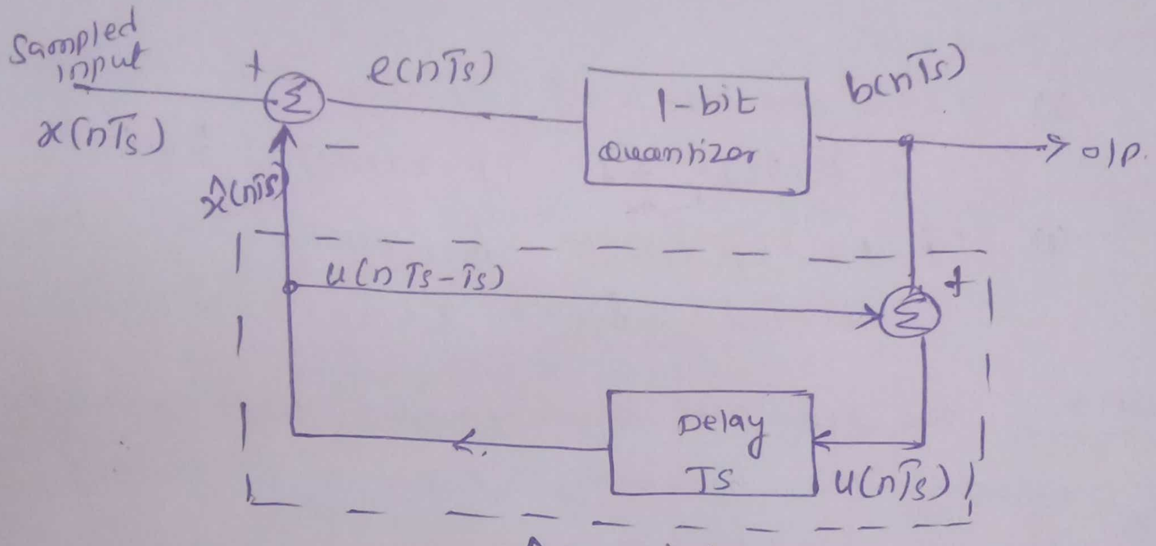


* Dm provides a staircase approximation to the over sampled version of an input base band signal. as shown in above figure.
* In Dm present sample is compared with previous sample value and indicates whether the amplitude is increased or decreased is transmitted.

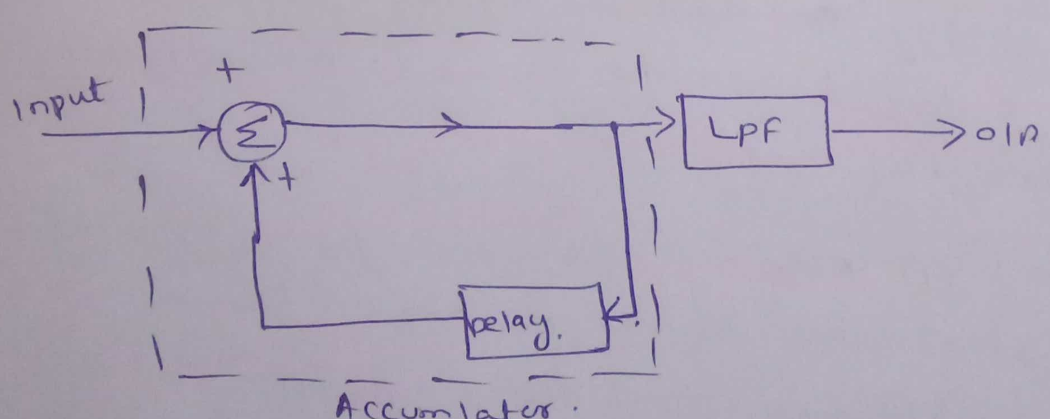
* As the step size is fixed, the levels $+\delta$ and $-\delta$ are used to represent the difference between the input signal $x(t)$ and the staircase

* If the difference is positive, then the approximated signal is increased by $+\delta$
* If the difference is Negative, then the approximated signal is decreased by $-\delta$.

Delta modulator:



Delta Demodulator



At Transmitter

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$
$$= x(nT_s) - u(nT_s - T_s) \quad \text{—— (1)}$$

$\hat{x}(nT_s) = u(nT_s - T_s)$

$$b(nT_s) = \delta \, sgn[e(nT_s)] \quad \text{--- (2)}$$

and

$$u(nT_s) = u(nT_s - T_s) + b(nT_s) \quad \text{--- (3)}$$

Where T_s is the sampling period

$e(nT_s)$ is a prediction error representing the difference between the present sample value $x(nT_s)$ of the input signal and the latest approximation to it namely $\hat{x}(nT_s) = u(nT_s - T_s)$

$$u(nT_s) = u[nT_s - T_s] + [\pm\delta] \quad \text{--- (4)}$$

From equation (4).

(1) If the step size is $+\delta$, binary '1' is transmitted.

$$b(nT_s) = +\delta : \text{ if } x(nT_s) \geq \hat{x}(nT_s)$$

(2) If the step size is $-\delta$, binary '0' is transmitted

$$b(nT_s) = -\delta, \text{ if } x(nT_s) < \hat{x}(nT_s)$$

At Reciver: The staircase approximation $u(t)$ is reconstructed by passing the incoming sequence of positive and Negative pulses through an accumlator.

The out-of-band quantization Noise in the high frequency staircase wave form $u(t)$ is rejected by LPF with a bandwidth equal to the original signal bandwidth.

Features of DM: (1) A one bit codeword for the output, which eliminates the need for word framing.

2mark

(2) Simplicity of design for both transmitter and Reciver

Advantages of DM over PCM

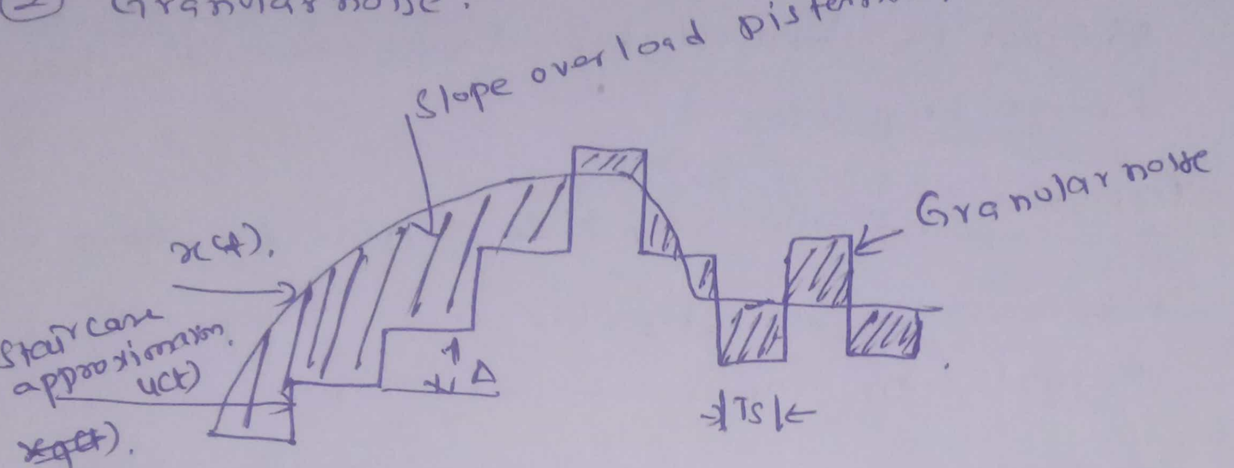
(1) Low signaling rate and low transmission Channel BW because of only one bit is transmitted per sample

(2) Simplicity in design for both transmitter and Receiver

DM.

Quantization noise (Distortions in DM system) or error

1. Slope overload distortion
2. Granular noise.



Slope overload distortion.

If the slope of the analog signal x(t) is much higher than that of its staircase approximation $x_q(t)$ over a long ~~distance~~ duration then $x_q(t)$ will not able to follow x(t). The difference between x(t) and $x_q(t)$ is called slope overload distortion.

$$x_q(t) = \frac{\Delta}{T_s} = \Delta f_s.$$

Elimination

1. Filtering the signal to limit the maximum rate of change
2. increasing step size Δ
3. increasing sampling rate(f_s)

However increasing Δ, the granular noise increases and if f_s is increased, signaling rate and BW requirement ~~is~~ are increased.

Best way: To reduce slope overload error is to detect the overload condition and increase the step size when overloading occurs ie detected.

Granular Noise :

It occurs when they very small variations in the input signal, the staircase approximated signal is changed by large-amount because of large step size.

To Reduce: Smaller step size required ~~where ...~~ whereas large step size are prepared for minizing the slope overload error.

Conclusion: We need to have a larger step size to accommodate a wide dynamic range, whereas a small step size is required for ~~accumulate~~ accurate representation of relatively lower level signals. it is clear that choice of the optimum step size that minimizes the mean square value of quantization error in a linear delta modulator will be the result of a compromise between slope overload distortion and Granular noise.

SNR

$$SNR_{o,max} = \frac{P_{max}}{WT_s(\delta^2/3)}$$

$$P_{max} = \frac{a_0^2}{2} = \frac{\delta^2}{8\pi^2 f_0^2 T_s^2}$$

$$\boxed{SNR_{o\,max} = \frac{3}{8\pi^2 W f_0^2 T_s^3}}$$

The above equation shows that, under the assumption of no slope-overload distortion, the maximum o/p signal to noise ratio of a delta modulator is proportional to the sampling rate cubed. This, therefore, indicates a 9dB improvement with doubling of the sampling rate

—x—

Adaptive Delta modulation

Why ADM (or) What is Adaptive Delta modulation?

The performance of a delta modulator can be improved significantly by making the step size of the modulator as a time varying form ie the step size of the modulator is increased, when the input varying rapidely and step size of the modulator decreased, when the input is varying slowely. Then, the system which uses the above said technique for reducing the quantization errors is called Adaptive Delta modulation system.

~~Block diagram~~:

In pratical implementation of system, the step size $\Delta(nT_s)$ or $2\delta(nT_s)$ is constrained to lie between minimum and maximum values. In particulare, we write

$$\boxed{\delta_{min} \le \delta(nT_s) \le \delta_{max}}$$

The upper limit, δ_{max}, controls the amount of slope-overload distortion. The lower limit, δ_{min}, controls the amount of idle channel noise.

Inside these limits the adaption rule for $\delta(nT_s)$ is expressed in the general form

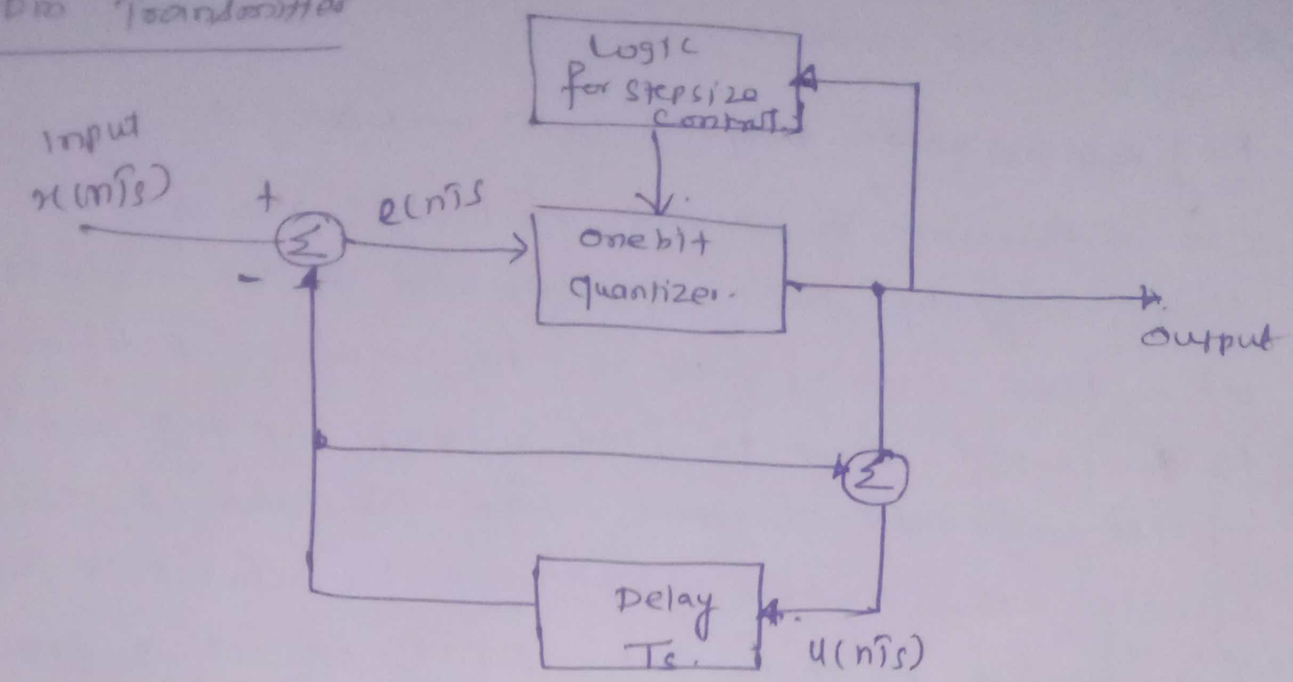
$$\boxed{\delta(nT_s) = g(nT_s)\,\delta(nT_s - T_s)}$$

Where $g(nT_s)$ depends on the present binary output $b(nT_s)$ of the Delta modulator and the 'M' previous values $b(nT_s - T_s) \ldots b(nT_s - MT_s)$. The algrothm is initiated with a starting step size $\delta_{start} = \delta_{min}$.

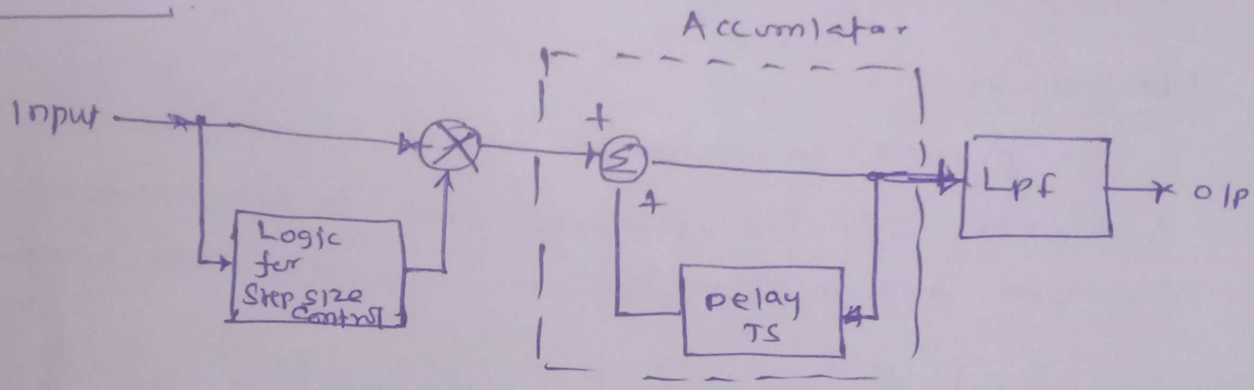
$$g(nT_s) = \begin{cases} K & \text{if } b(nT_s) = b(nT_s - T_s) \\ K^{-1} & \text{if } b(nT_s) \ne b(nT_s - T_s) \end{cases}$$

This adaption algrothim is called a constant factor ADM with 1 bit memory.

ADM Transmitter



ADM Reciver.



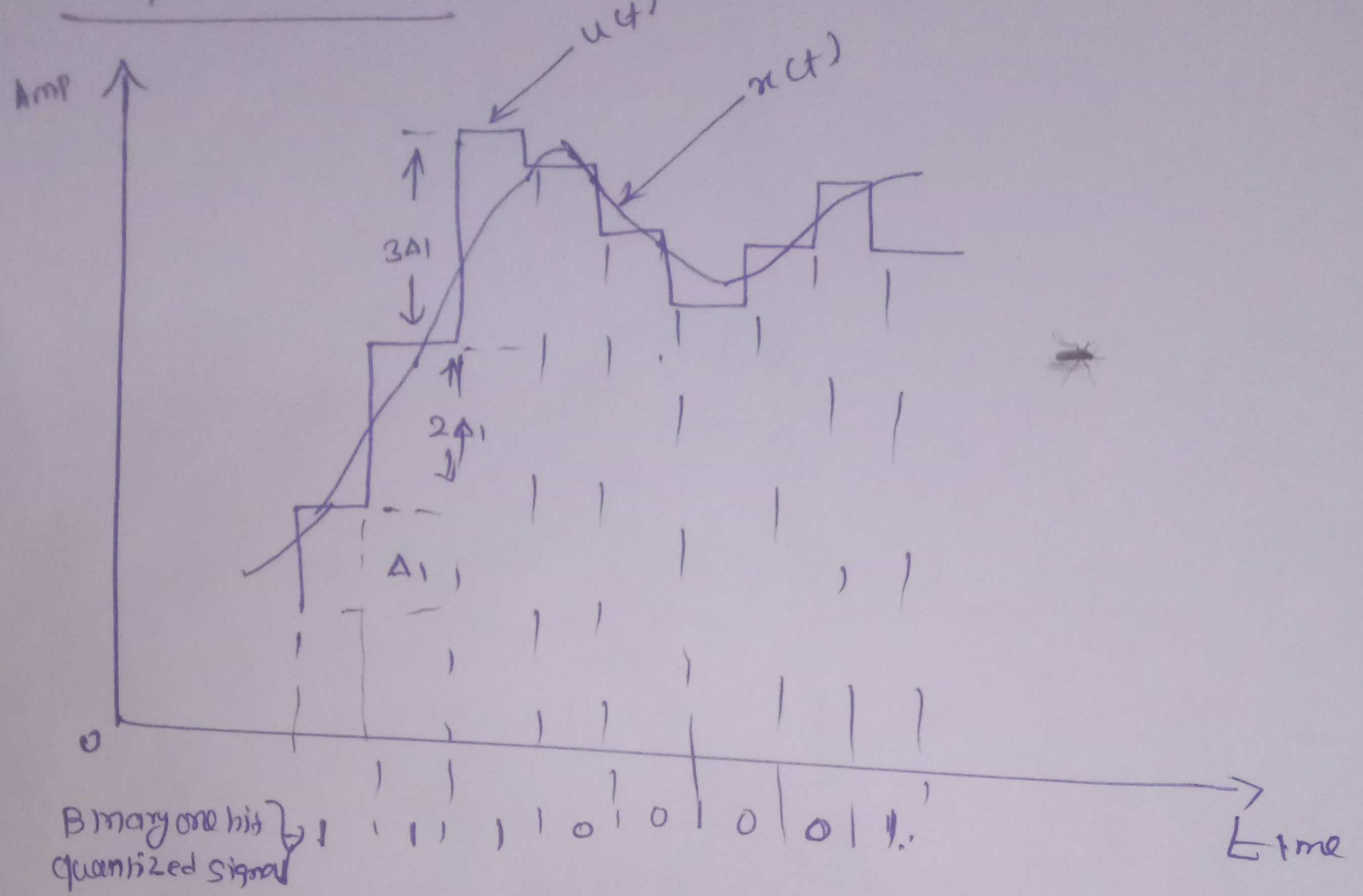
At Transmitter: Depending upon the output of one bit quantizer, the step size is increased or decreased according to a specified logic

* When the one bit quantizer output is High (ie 1), the step size may be increased.
* When the one bit quantizer output is Low (ie 0), the step size may be decreased.

At Reciver:

The Accumlator block generate a stair case waveform depending on the input applied it. Then the original signal is reconstructed from the stair case waveform using LPF which allows maximum frequency of the message signal remains high frequency will be attenuated.

Waveform of ADM



Advantages of ADM over DM.

1> The SNR is better than DM because of slope overload distortion and granular noise is reduced.

2) wide dynamic range due to variable step size

3> Better utilization of BW.

4) Low signaling Rate.

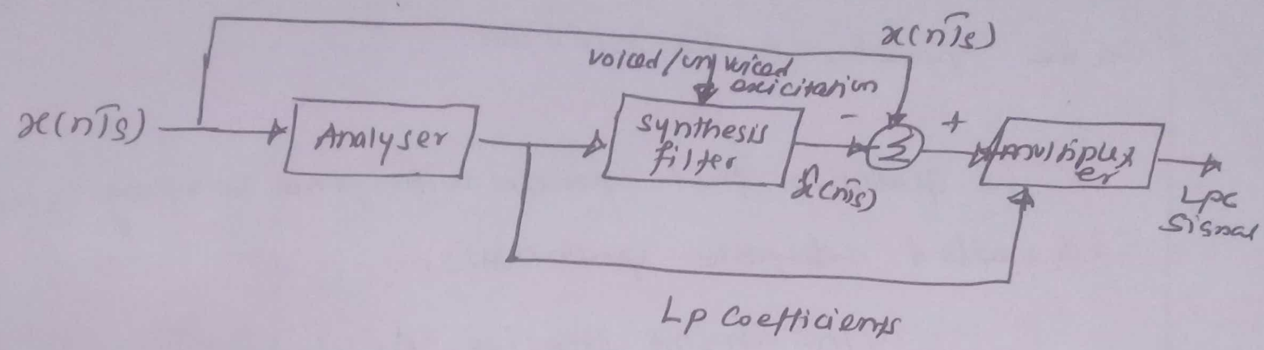
5> simplicity Design in Txer and Rxver.

6) Very good noise immunity.

Application: speech and image processing.

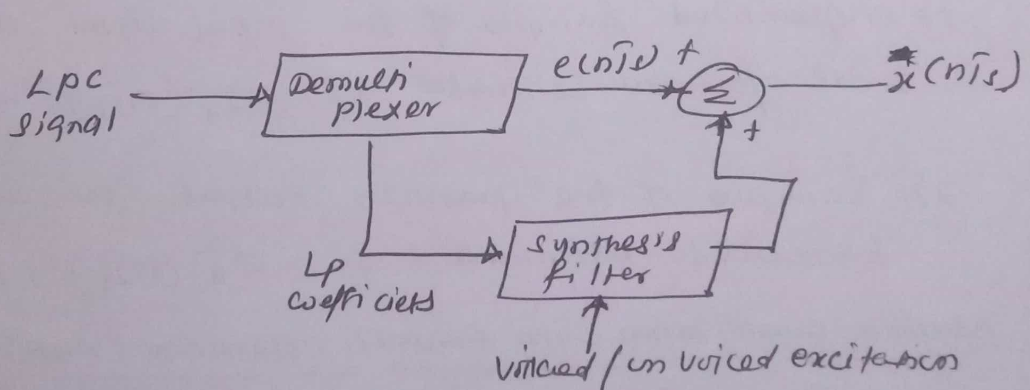
Linear predeetive Coding [LPC]

LPC Transmitter



$x(nT_s)$ is speech signal (digitized version). It is applied to the analyser. The analyser analyse the Lp coefficients for the synthesis filter. Based on these Lp coefficients, the synthesis filter reconstructs the speech signal $x(nT_s)$. The reconstructed signal $\hat{x}(nT_s)$ and the original signal $x(nT_s)$ are compared and the error $e(nT_s)$ is obtained. The Lp coefficients and error signal is multiplexed and transmitted.

Lpc Receiver (Decoder)



Lpc signal applied to the Demultiplexer. It seperates the filter parameters and error signal $e(nT_s)$. Lp coefficients are given to the synthesis filter. O/p of filter added to error signal and gives a $x(nT_s)$. This produce the bit rate of about 3 to 8 kbits/sec.

ADPCM: Adaptive Differential PCM

What is ADPCM? : A Digital coding sheme that uses both adaptive quantization and adaptive prediction is called adapative differential pcm. It is used to code speech at 32 k bits/sec..

The quantizer that operates with time varying step size $\Delta(nT_s)$ is called adaptive quantization.

Time varying stepsize $\Delta(nT_s)$

$T_s \rightarrow$ is the sampling period.

The step size of quantizer depends on variance of the input signal then we write.

$$\boxed{\underbrace{\Delta(nT_s)}_{\text{stepsize at instant } T_s} = \phi\, \underbrace{\hat{\sigma}_x(nT_s)}_{\text{estimate of the standard deviation } \sigma_x(nT_s)\ [\text{ie } \sigma_x^2]}}$$

To proceed with the ~~ab~~ application of above equation we may compute the estimate $\hat{\sigma}_x(nT_s)$ in one of two ways.

1) unquantized samples of the input signal are used to derive forward estimates of $\sigma_x(nT_s)$ in Fig (1)

2). Samples of the quantizer output are used to derive backward estimates of $\sigma_x(nT_s)$ in Fig(2)

Adaptive Quantization with Forward estimation (AQF)

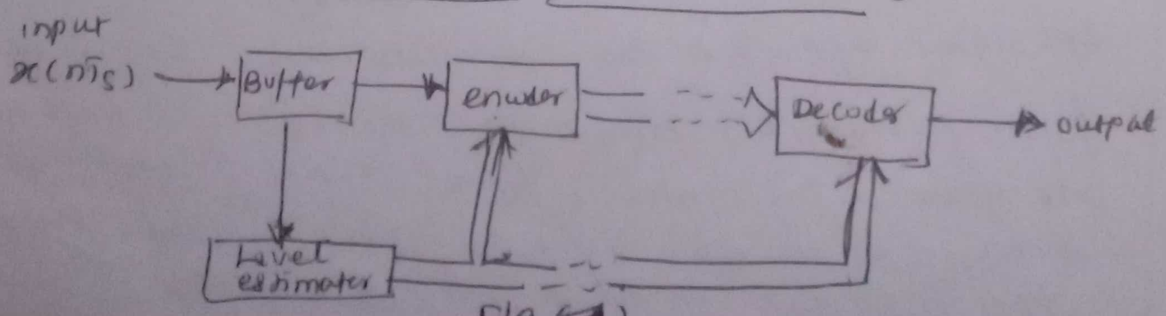


Fig (1)

Adapative quantization with backward estimation

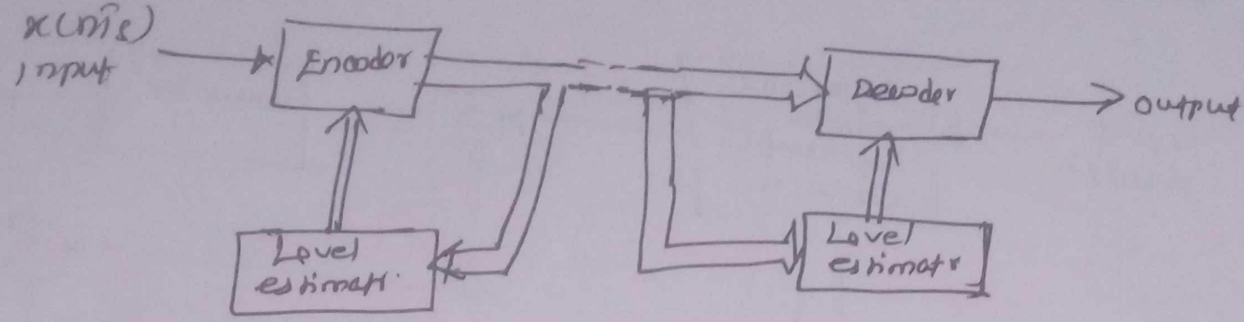


Fig (2)

From Fig (1) : The unquantized Samples are stored in the buffer. From these Samples an estimate $\hat{\sigma}_x(nT_s)$ of standard devation is obtained from $\hat{\sigma}(nT_s)$. This estimates is independent of quantizing noise. So we find that the step size $\Delta(nT_s)$ is obtained from AQF is more reliable than that from AQB.

problems encounted in AQF: 1) Level transmission [5 to 6 bits per step size sample required]

2). Buffering, processing delay.

The above said problems are avoide by AQB ie Fig (2)

From Fig (b): Recent History of the quantizer output to extract information for the Computation of the step size $\Delta(nT_s)$. So AQB usualy prepared than AQF practice. because of no Buffering delay.

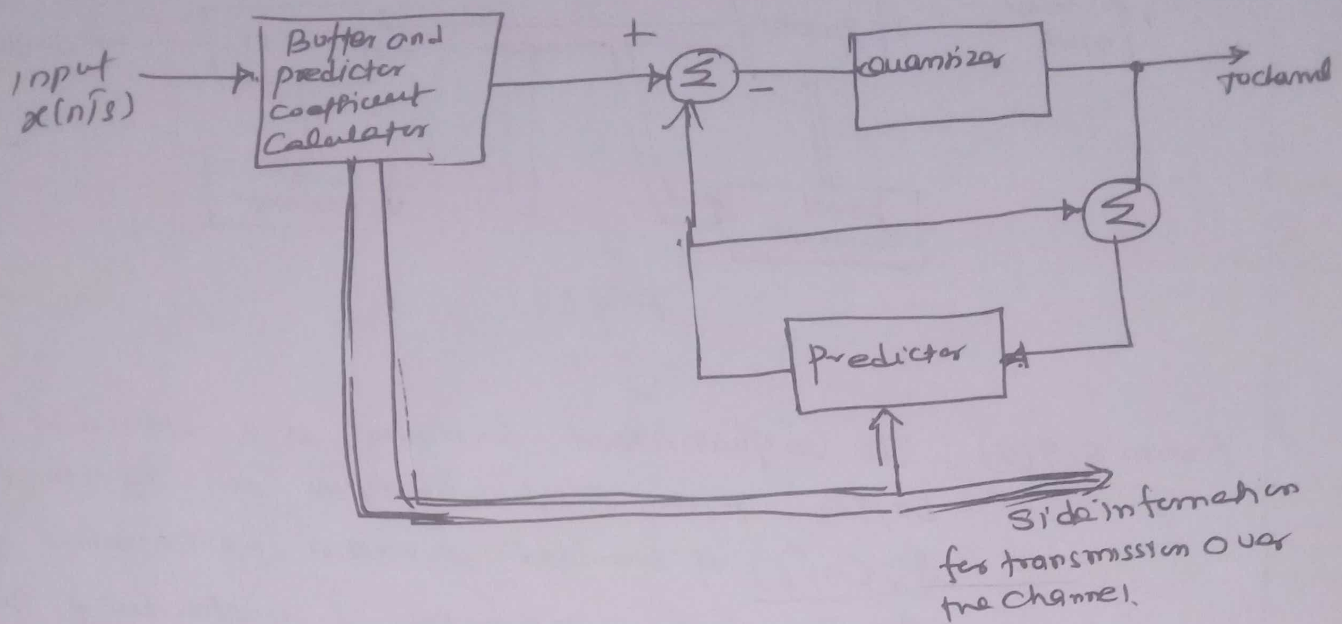
Adaptive prediction with forward estimation

As with adaptive quantization, there are two schemes for performing adaptive prediction.

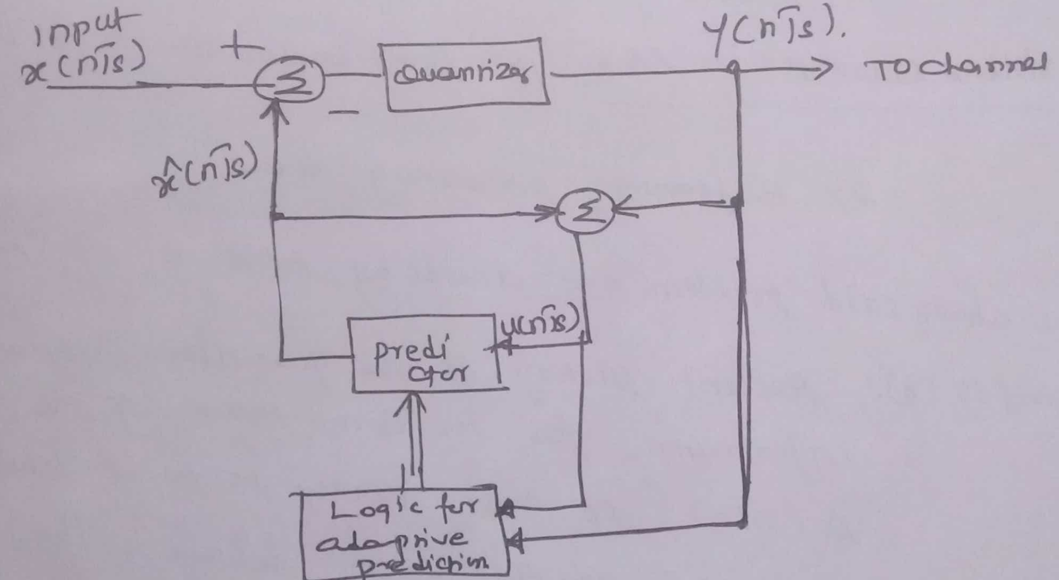
1). Adaptive prediction with forward estimation (APF), in which unquantized Samples of the input signal are used to derive estimates of the predictor coefficients

2) Adaptive prediction with backward estimation (APB), in which Samples of the quantizer output and the prediction error are used to derive estimates of the predictor Coefficients.

1) Adaptive prediction with forward estimation



2) Adaptive prediction with Backward estimation



$$u(nT_s) = \hat{x}(nT_s) + y(nT_s)$$

$\hat{x}(nT_s)$ is the prediction of the speech input sample $x(nT_s)$

$$y(nT_s) = u(nT_s) - \hat{x}(nT_s)$$

$u(nT_s)$ – representing a sample value of the predicter input

$\hat{x}(nT_s)$ – " " " predicter output

$y(nT_s) \rightarrow$ o/p.

— x —

Line Codes.

Let we know channel capacity of a Gussian channel is given as

$$C = B \log_2 \left[1 + S/N\right] \text{ bits/sec.}$$

from above equation channel capacity depends on two factors

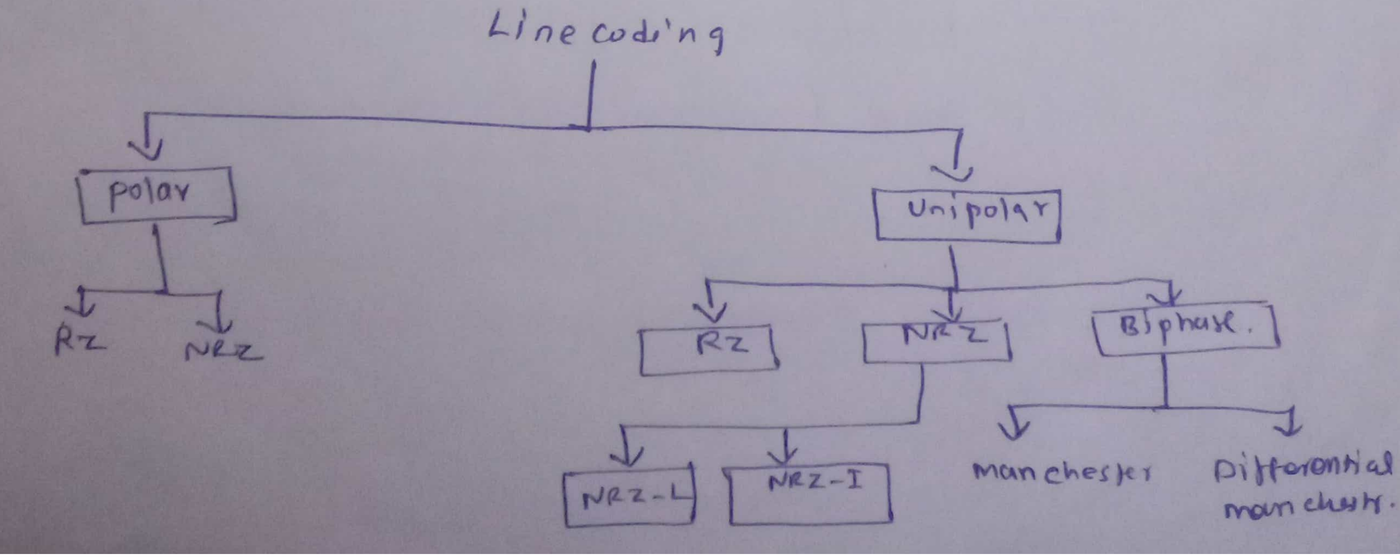
1) Bandwidth of the channel (B)
2) signal to noise ratio (S/N)

The encoding technique which follows the channel capacity theorem is called as Bandwidth (BW) -SNR trade off codes. This code is also called as line encoding or line coding.

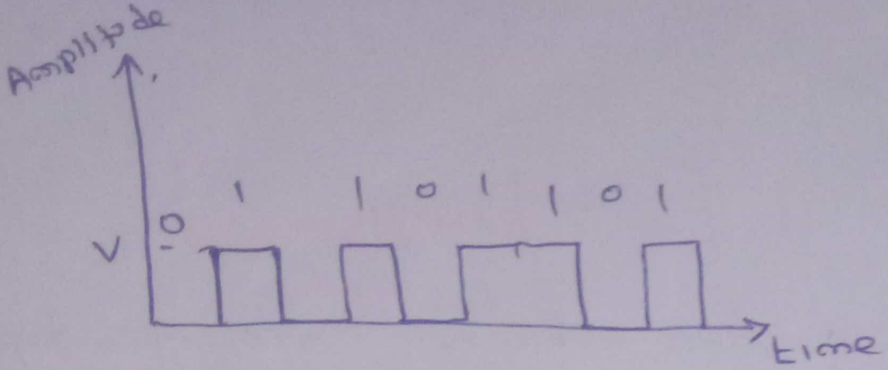
The line coding is used for converting a binary information sequence into a digital signal.

The wave form pattern of voltage or current used to represent the 1s and 0s of a digital signal on a transmission link is called line coding.

Types:



unipolar Representation



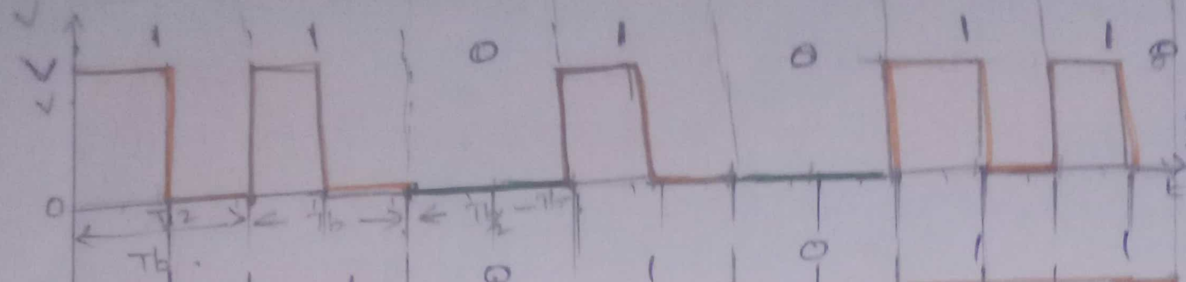
1s are encoded as positive voltage (eg. +5V) and 0s are encoded as the zero voltage (0V).

problems in unipolar

1). DC component
2). synchronization.

polar encoding

1). It eliminates the residual DC component but still a small DC residual present.
2) When the digital encoding is symmetrical around '0' volts, it is called a polar code.
3) It has two Levels (positive and negative) of amplitude. It reduces the power required to transmit the signal by one-half compared with unipolar.
4) polar line encoding has the same synchronization problem as unipolar line encode. If there is a long string of 1s or 0s the receiving oscillator may drift and become unsynchronized.

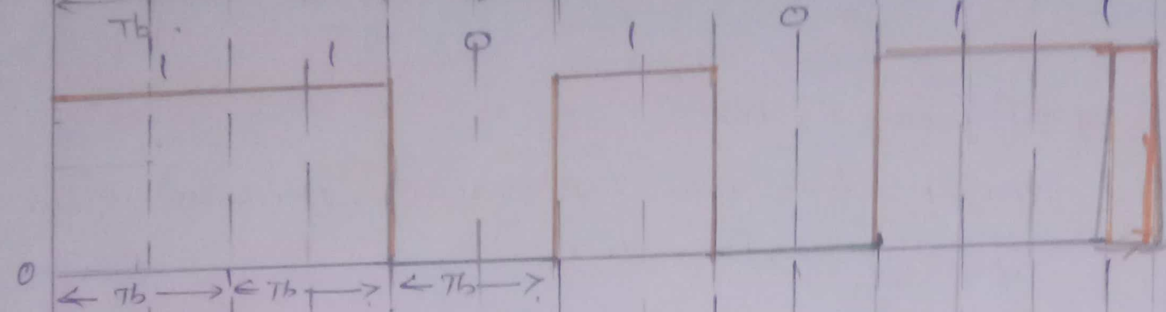


Unipolar RZ

$x(t) = \begin{cases} V & \text{for } 0 \le t \le T_b/2 \\ 0 & \text{for } T_b/2 \le t \le T_b \end{cases}$

For symbol '0'

$x(t) = 0$ for $0 \le t \le T_b$



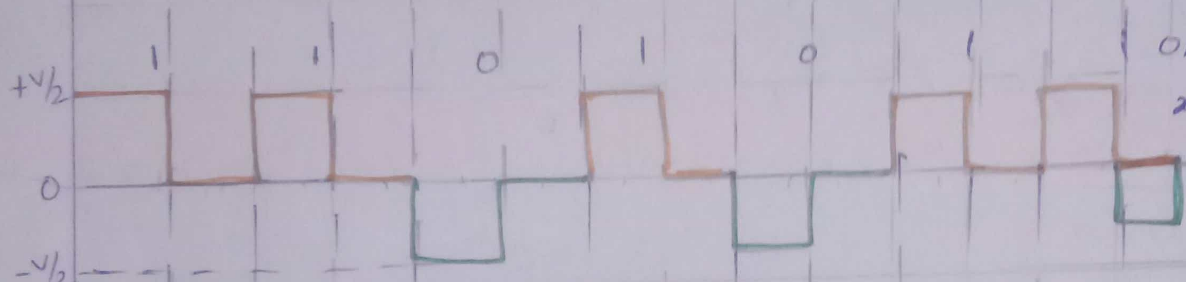
Unipolar NRZ

if symbol 1 is Txed

$x(t) = A$ for $0 \le t \le T_b$.

if '0' is Txed

$x(t) = 0$ for $0 \le t \le T_b$.



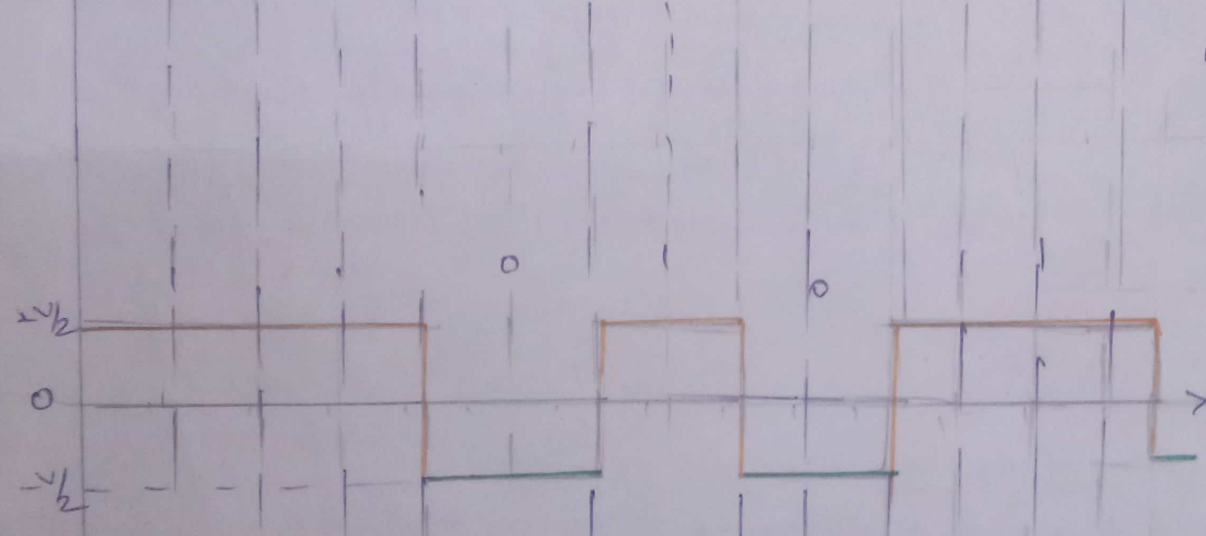
polar RZ:

if symbol '1' is Txed

$x(t) = \begin{cases} +\frac{V}{2} & \text{for } 0 \le t \le T_b/2 \\ 0 & \text{for } T_b/2 \le t < T_b \end{cases}$

if symbol '0' is Txed

$x(t) = \begin{cases} -V/2 & \text{for } 0 \le t \le T_b/2 \\ 0 & \text{for } T_b/2 \le t \le T_b \end{cases}$



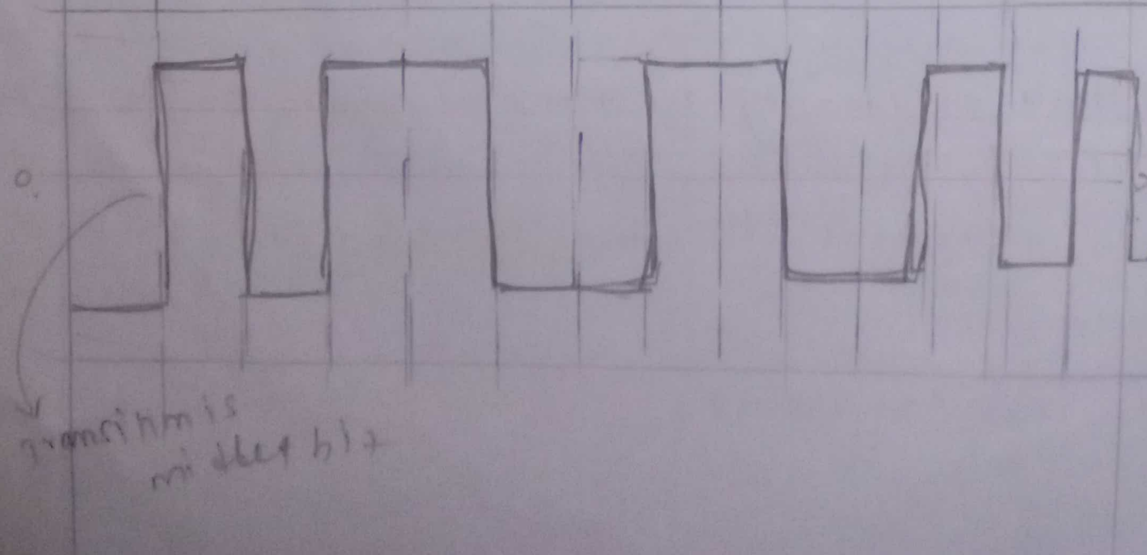
Polar NRZ

if symbol '1' is Txed

$x(t) = +V/2$ for $0 \le t$

if symbol '0' is Txed

$x(t) = -V/2$ for $0 \le t \le$



Manchester Encoding.

The transition at the middle of the bit is used.

> Negative to +ve rep '1'

positive to -ve rep '0'

Unipolar NRZ:

1) pulse does not return to zero on it's own. If the symbol '0' is to be transmitted, then pulse becomes zero.

2) Since there is no separation between the pulses, the receiver needs synchronization to det unipolar NRZ pulses.

3) Energy of the pulses is more

4) Unipolar format has some DC value, this DC value does not carry any information.

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polar RZ format: symbol 1 is represented by positive voltge polarity and symbol '0' represented by negative voltge polarity. — In RZ format, the pulse is Txed only for half Duration.

polar NRZ format: symbol 1 is represented by +ve polarity and symbol '0' is represented by Negative polarity. These polarities are maintained over the complete pulse Duration. Ie for polar NRZ

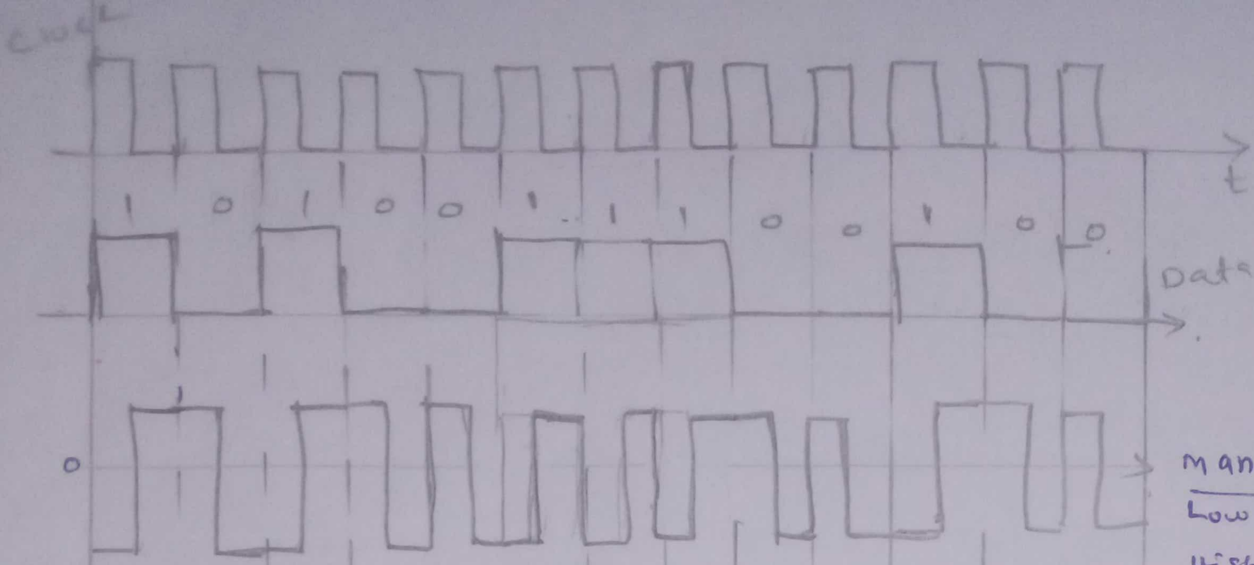
Bipolar NRZ-L:

1) Binary bit 1 is represented by positive voltage and bit 0 is represented by negative voltage level

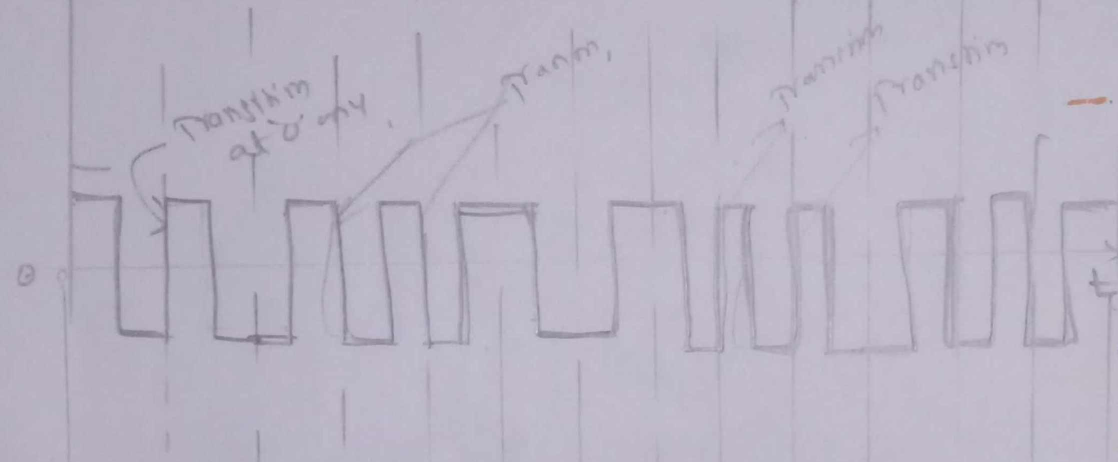
2) If the data bit stream contain continues 0's or 1's then receiver receives a continuous constant voltage

3) It is difficult to identify the total number of bits transmitted by the sender, clock synchronization is required to identify the total number of bits to be transmitted.

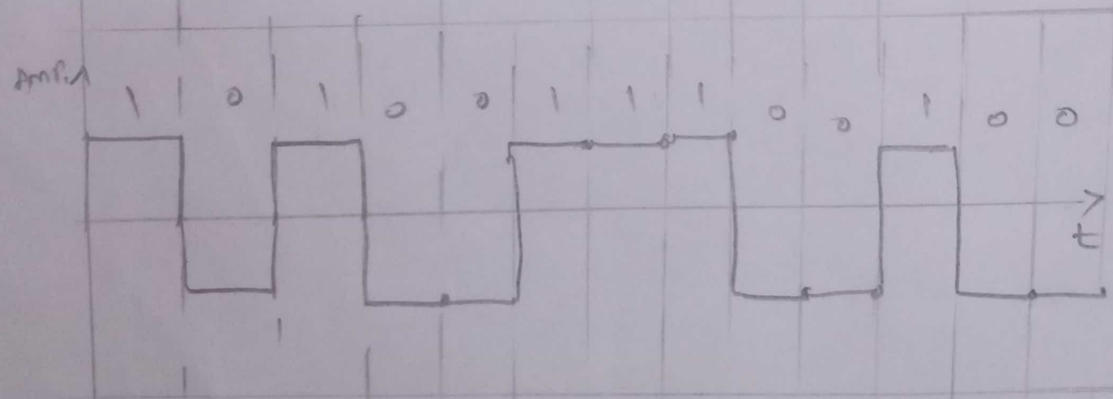
Solve for: Data (00110101)



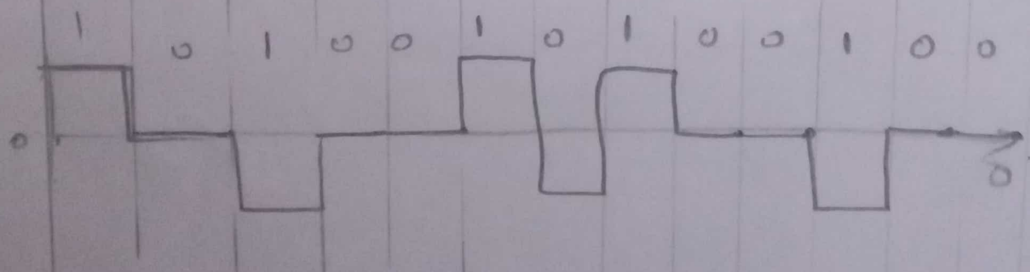
Manchester.
Low to High — 1
High to Low — 0



Differennce manchester.
Transition at middle of the bit
Data representation is mapped as per bit start time instand
No transistion at start of a bit period rep as 1
Transistion at start of a bit period rep as '0'



Bipolar NRZ-Level
Binary bit '1' is represented by postive Logic and bit '0' is represented by Negative Voltage Level



Bipolar - Alternate mark inversion (AMI)
A Neutral zero voltge represents binary 0, Binary 1 represented by alternating +ve and -ve voltge.

PSD OF NRZ UNIPOLAR LINE CODING

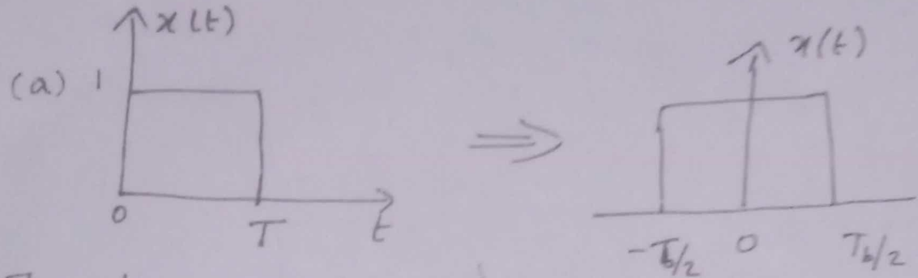
Step 1 : Find the fourier transform of NRZ pulse $X(f)$

Step 2 : Find the autocorrelation of unipolar $R_A(n)$

Step 3 : Calculate PSD based on $X(f)$ and $R_A(n)$

Unipolar NRZ

(a)



The basic formula for FT is

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

Applying fourier transform,

$$X(f) = \int_{-T_b/2}^{T_b/2} (1)\, e^{-j2\pi f t}\, dt$$

$$= \left[\frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-\frac{T_b}{2}}^{\frac{T_b}{2}}$$

$$= \frac{e^{-j\frac{2\pi f T_b}{2}} - e^{j\frac{2\pi f T_b}{2}}}{-j2\pi f}$$

$$= \frac{e^{j\pi f T_b} - e^{-j\pi f T_b}}{j2\pi f}$$

$$= \frac{\left[e^{j\pi f T_b} - e^{-j\pi f T_b}\right] T_b}{j2\pi f T_b}$$

$$= \frac{\sin \pi f T_b}{\pi f T_b} \times T_b$$

$$= T_b \ \text{sinc}(\cancel{\pi} f T_b)$$

$$\boxed{X(f) = T_b \ \text{sinc}(f T_b)}$$

Since

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\text{sinc}\,\theta = \frac{\sin \pi\theta}{\pi\theta}$$

$$\frac{\text{sinc function}}{\text{sinc}(\lambda)} = \frac{\sin(\pi\lambda)}{\pi\lambda}$$

Step 2

For auto correlation $R_A(n)$

W.K.T for unipolar, $A_L = \begin{cases} a & \text{for bit '1'} \\ 0 & \text{for bit '0'} \end{cases}$

Probability of A_L for large sequence

$$\boxed{P(A_L = 0) = P(A_L = 1) = 1/2}$$

Now auto correlation, (Average Auto Corrolation function)

$$R_A[A] = E[A_L \ A_{L-n}]$$

↳ estimation with current & shifted version.

i) for $n = 0$, we may write

$$R_A(n) = E[A_L \cdot A_{L-0}]$$

$$R_A(n) = E[A_L^2]$$

Estimation for

Continous fn, $E[x^2] = \int x^2 f(x)\,dx$

Discrete fn, $E[x^2] = \sum x^2 p(x)$

bit	A_L	A_L^2	$P(A_L^2)$
0	0	0	1/2
1	a	a^2	1/2

$(0^2)\,P(A_L = 0) + (a^2)\,P(A_L = a)$

$$\therefore E[x^2] = E[A_L^2] = 0^2(1/2) + a^2(1/2)$$

$$= 0 + \frac{a^2}{2}$$

$$R_A = E[x^2] = \frac{a^2}{2}$$

$$\boxed{R_A = \frac{a^2}{2}} \quad \text{for } n=0.$$

ii) for $n \neq 0$, The product A_L, A_{L-n} has four possible values such as 0, 0, 0 and a^2

bit	A_L	A_{L-n}	$A_L * A_{L-n}$	P
0 0	0	0	0	1/4
0 1	0	a	0	1/4
1 0	a	0	0	1/4
1 1	a	a	a^2	1/4

$$E[x^2] = E[A_L \cdot A_{L-n}]$$

$$R_{A_L A_{L-n}} = \sum A_L A_{L-n} \, P(x)$$

$$R_{A_L A_{L-n}} = 0(1/4) + 0(1/4) + 0(1/4) + a^2(1/4)$$

$$\therefore \boxed{R_{A_L A_{L-n}} = \frac{a^2}{4}} \quad n \neq 0.$$

$\therefore$ Autocorrelation function $R_A(n)$ can be expressed as

$$R_A(n) = \begin{cases} a^2/2 & , \text{for } n=0 \\ a^2/4 & , \text{for } n \neq 0 \end{cases}$$

Step 3

To find PSD, apply $X(f)$ and $R_A(n)$ to Wiener khintchine equation.

FT of the basic pulse set

power spectral density of discrete pam signal

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}$$

$$= \frac{1}{T_b} (T_b^2 \operatorname{sinc}^2(f T_b)) \left[\sum_{n=0}^{?} R_A(n) e^{-j2\pi f n T_b} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} R_A(n) e^{-j2\pi f n T_b} \right]$$

$$= \frac{T_b^2}{T_b} \operatorname{sinc}^2(fT_b) \left[\frac{a^2}{2} e^0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{a^2}{4} e^{-j2\pi f n T_b} \right]$$

$$= T_b \operatorname{sinc}^2(fT_b) \left[\frac{a^2}{2} + \frac{a^2}{4} \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-j2\pi f n T_b} \right]$$

$$= \frac{T_b a^2}{2} \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-j2\pi f n T_b}$$

$$= \frac{a^2}{2} T_b \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) \left[\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} - 1 \right]$$

(removing $n \neq 0$ term subtract 1)

$$= \frac{a^2}{2} T_b \operatorname{sinc}^2(fT_b) - \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b}$$

$$P(f) = \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} T_b \operatorname{sinc}^2(fT_b) \sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b}$$

Using Poissons formula,

i.e $$\sum_{n=-\infty}^{\infty} e^{-j2\pi f n T_b} = \frac{1}{T_b} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$

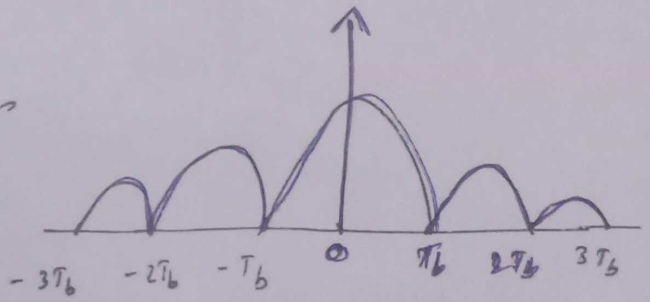
$\delta(f) \rightarrow$ Dirac Delta function at $f = 0$

so PSD will be

$$P(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2(f T_b) + \frac{a^2 T_b}{4} \left[\frac{\operatorname{sinc}^2(f T_b)}{T_b}\right] \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$

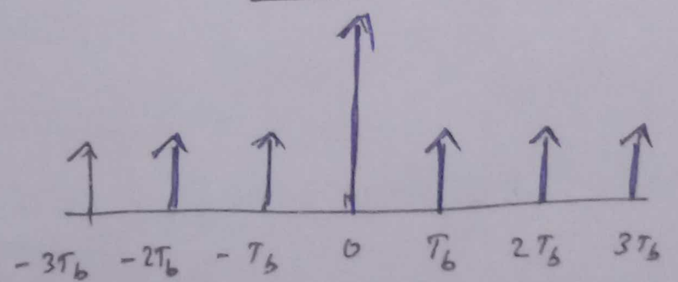
$$= \frac{a^2 T_b}{4} \operatorname{sinc}^2(f T_b) + \frac{a^2}{4} \operatorname{sinc}^2(f T_b) \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right)$$

Sinc fn

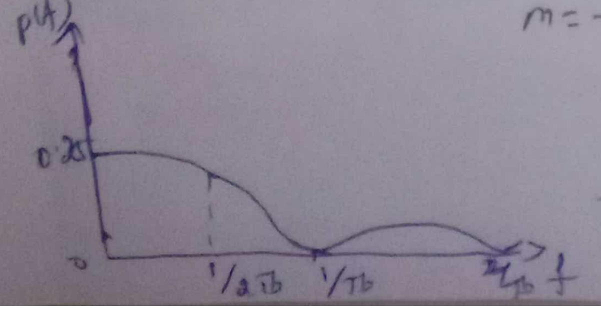


Multiplying both sinc and impulse function results in 0 at null terms.

Impulse



so $$\operatorname{sinc}^2(f T_b) \times \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right) = \delta(f)$$

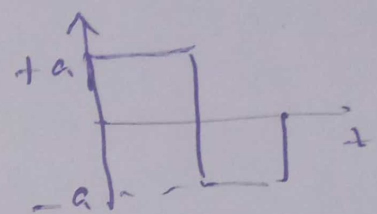


$$\boxed{P(f) = \frac{a^2 T_b}{4} \operatorname{sinc}^2(fT_b) + \frac{a^2}{4} \delta(f)}$$

The second term '$\frac{a^2}{4} \delta(f)$' is an DC component and it leads to distortion in signal.

C) Power spectral density of polar NRZ

In Polar NRZ | '0' → −a
1 → +a



$\parallel^{lr}$ to PSD of unipolar, FT & auto correlation has to be calculated.

w.k.T

$$\boxed{X(f) = T_b \operatorname{sinc}(fT_b)}$$ (from FT of Unipolar)

For polar,

$$A_L = \begin{cases} +a, & \text{for '1'} \\ -a, & \text{for '0'} \end{cases}$$

Auto correlation

$$R_A(n) = E[A_L \cdot A_{L-n}]$$

for $n = 0$,

$$R_A(0) = E[A_L \cdot A_L] = E[A_L^2]$$

	A_L	A_L^2	Prob
1	a	a^2	1/2
0	$-a$	a^2	1/2

$$R_A(0) = \sum A_L^2 \, p(x)$$

$$= a^2(1/2) + a^2(1/2)$$

$$\boxed{R_A(0) = a^2} \quad \text{for } n=0$$

ii) for $n \neq 0$

$$R_A(n) = E[A_L \cdot A_{L-n}]$$

A_L	A_{L-n}	$A_L A_{L-n}$	Prob	bit
$-a$	$-a$	$+a^2$	1/4	0 0
$-a$	a	$-a^2$	1/4	0 1
a	$-a$	$-a^2$	1/4	1 0
a	a	$+a^2$	1/4	1 1

$$R_A(n) = a^2(1/4) + (-a^2)(1/4) + (-a^2)(1/4) + a^2(1/4)$$

$$\boxed{R_A(n) = 0} \quad \text{for } n \neq 0.$$

$\therefore$ Auto correlation

$$R_A = \begin{cases} a^2, & n=0 \\ 0, & n \neq 0 \end{cases}$$

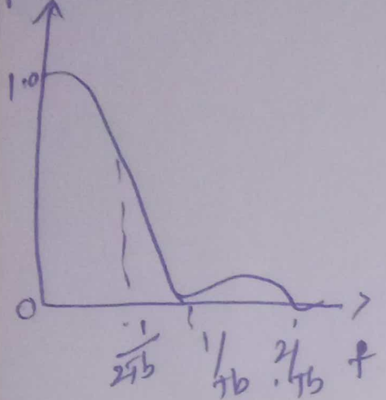
Now PSD

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) \, e^{-j2\pi f n T_b}$$

waveform has small DC value. It contains most of the power from DC to bit rate $\left(\frac{1}{Tb}\right)$

$$\frac{1}{T_b}\left[T_b \operatorname{sinc}(fT_b)\right]^2 \cdot \sum_{n=-\infty}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}$$

p(f)



$$= T_b \operatorname{sinc}^2(fT_b)\left\{\sum_{n=0} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} R_A(n)\, e^{-j2\pi f n T_b}\right\}$$

$$= T_b \operatorname{sinc}^2(fT_b)\left[\sum_{n=0} R_A(n)\, e^{-j2\pi f n T_b} + 0\right]$$

$$= T_b \operatorname{sinc}^2(fT_b)\left[a^2 \cdot e^0 + 0\right]$$

$$\boxed{P(f) = a^2 T_b \operatorname{sinc}^2(fT_b)}$$

(-) <u>PSD of NRZ bipolar format</u>

(<u>Pseudo trinary</u> code)

<u>Step 1</u>. FT of NRZ

$$X(f) = T_b \operatorname{sinc}(fT_b)$$

for bipolar format, NRZ

$$A_L = \begin{cases} 0 & \text{for symbol '0'} \\ +a & \\ -a & \text{for symbol '1'} \end{cases}$$

<u>Auto Correlation</u>

$$R_A(n) = E[A_L \cdot A_{L-n}]$$

i) for $n=0$, $R_A = E[A_L \cdot A_L]$

$$= E[A_L^2]$$

	A_L	A_L^2	Prob
for 0	0	0	1/2
for 1	a	a^2	1/4
	$-a$	a^2	1/4

$$R_A(0) = 0(1/2) + a^2(1/4) + a^2(1/4)$$

$$= \frac{a^2}{2}$$

(ii) for $n=1$,

	A_L	A_{L-1}	$A_L A_{L-1}$	Prob
00	0	0	0	1/4
01	0	$\pm a$	0	1/4
10	$\pm a$	0	0	1/4
11	$\pm a$	$\pm a$		
	$-a$	$+a$	$-a^2$	1/8
	$+a$	$-a$	$-a^2$	1/8

$$R_A = 0(1/4) + 0(1/4) + 0(1/4) - a^2(1/8) - a^2(1/8)$$

$$\boxed{R_A(1) = -\frac{a^2}{4}}$$

iii) for $n>1$

	A_L	A_{L-n}	$A_L A_{L-n}$	Prob
00	0	0	0	1/4
01	0	$\pm a$	0	1/4
10	$\pm a$	0	0	1/4

11	$+a$	$+a$	a^2	$1/16$
	$-a$	$+a$	$-a^2$	$1/16$
	$+a$	$-a$	$-a^2$	$1/16$
	$-a$	$-a$	a^2	$1/16$

$$R_A = 0(1/4) + 0(1/4) + 0(1/4) + a^2(1/16) - a^2(1/16) - a^2\left(\frac{1}{16}\right) + a^2\left(\frac{1}{16}\right)$$

$$\boxed{R_A = 0} \quad \text{for } n > 1$$

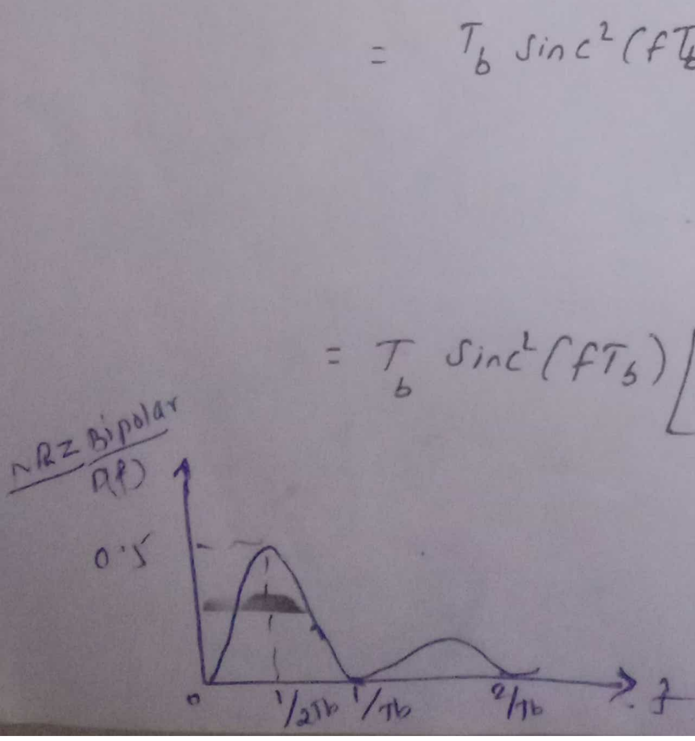
$$\therefore R_A = \begin{cases} a^2/2, & n = 0 \\ -a^2/4, & n = \pm 1 \\ 0, & \text{for } n \geq 2 \end{cases}$$

NOW PSD

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}$$

$$= T_b \operatorname{sinc}^2(fT_b) \Big[R_A(0) + R_A(1) e^{-j2\pi f T_b} + R_A(-1) e^{-j2\pi f T_b} + \underbrace{R_A(n \geq 2) e^{-j2\pi f n T_b}}_{0} \Big]$$

$$= T_b \operatorname{sinc}^2(fT_b) \left[R_A(0) + \left(\frac{-a^2}{4}\right)\left(e^{-j2\pi f T_b} + e^{-j2\pi f T_b}\right) \right]$$



peak power at $1/2T_b$

$$= T_b \, \text{Sinc}^2(f T_b)\left[\frac{a^2}{2} - \frac{a^2}{2}\left(\frac{e^{-j2\pi f T_b} + e^{+j2\pi f T_b}}{2}\right)\right]$$

$$= T_b \, \text{sinc}^2(f T_b)\left[\frac{a^2}{2}\right]\left[1 - \cos(2\pi f T_b)\right]$$

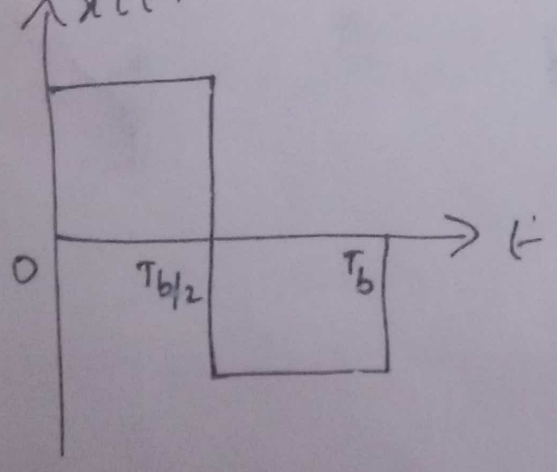
since $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$$= T_b \, \text{Sinc}^2(f T_b) \cdot \frac{a^2}{2}\left[1 - \cos(2\pi f T_b)\right]$$

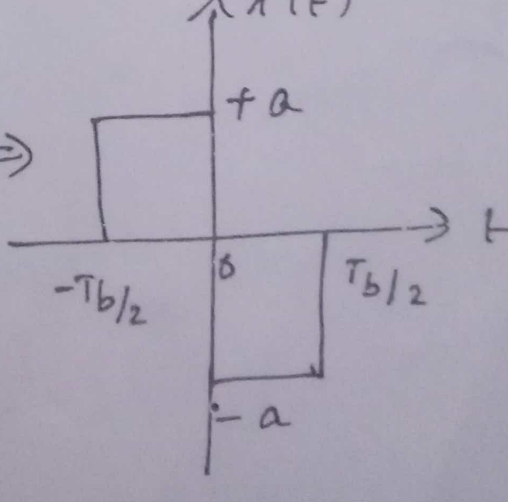
$$= \frac{a^2 T_b \, \text{Sinc}^2(f T_b)}{2}\left[2\sin^2(\pi f T_b)\right]$$

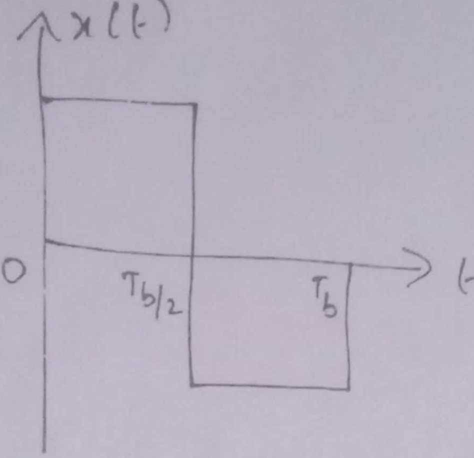
$$\boxed{P(f) = a^2 T_b \, \text{Sinc}^2(f T_b) . \sin^2(\pi f T_b)}$$

) PSD of manchester polar

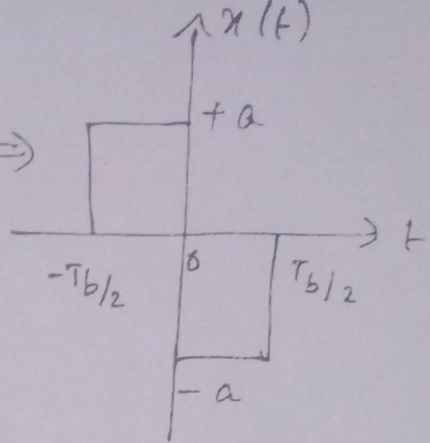


$\Rightarrow$





$\Rightarrow$



Step1

$$X(f) = \int_{-T_b/2}^{0} (1)\, e^{-j2\pi f t}\, dt + \int_{0}^{T_b/2} (-1)\, e^{-j2\pi f t}\, dt$$

$$= \left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-T_b/2}^{0} + \left[\frac{-e^{-j2\pi f t}}{-j2\pi f}\right]_{0}^{T_b/2}$$

$$= \frac{e^{0} - e^{-j2\pi f\left(-\frac{T_b}{2}\right)}}{-j2\pi f} + \frac{-e^{-j2\pi f\left(\frac{T_b}{2}\right)} + e^{0}}{-j2\pi f}$$

$$= \frac{1}{-j2\pi f}\left\{1 - e^{+j\pi f T_b} - e^{-j\pi f T_b} + 1\right\}$$

$$= \frac{1}{-j2\pi f}\left[2 - e^{+j\pi f T_b} - e^{-j\pi f T_b}\right]$$

$$= -\frac{1}{j\pi f}\left[1 - \left(\frac{e^{j\pi f T_b} + e^{-j\pi f T_b}}{2}\right)\right]$$

$$= -\frac{1}{j\pi f}\left[1 - \cos(\pi f T_b)\right]$$

$$= -\frac{2\sin^2\left(\frac{\pi f T_b}{2}\right)}{j\pi f}$$

$$= \frac{-(-j)2\sin^2\left(\frac{\pi f T_b}{2}\right)}{\pi f}$$

$$= \frac{jT_b \sin\frac{\pi f T_b}{2}}{\frac{\pi f}{2} T_b} \sin\frac{\pi f T_b}{2}$$

$$\boxed{X(f) = jT_b \, \mathrm{sinc}\left(\frac{f T_b}{2}\right) \cdot \sin\frac{\pi f T_b}{2}}$$

Scanned with CamScanner

Auto Correlation

$$R_A(n) = \begin{cases} a^2 & , n=0 \\ 0 & , n\neq 0 \end{cases}$$

PSD

$$P(f) = \frac{1}{T_b} |X(f)|^2 \sum_{n=-\infty}^{\infty} R_A(n) e^{-j2\pi f n T_b}$$

$$= \frac{1}{T_b} |X(f)|^2 \left[\sum_{n=0} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \right]$$

$$= \frac{1}{T_b} |X(f)|^2 \left[\sum_{n=0} R_A(n) e^{-j2\pi f n T_b} + 0 \right]$$

since $R_A(n)=0$ for $n\neq 0$

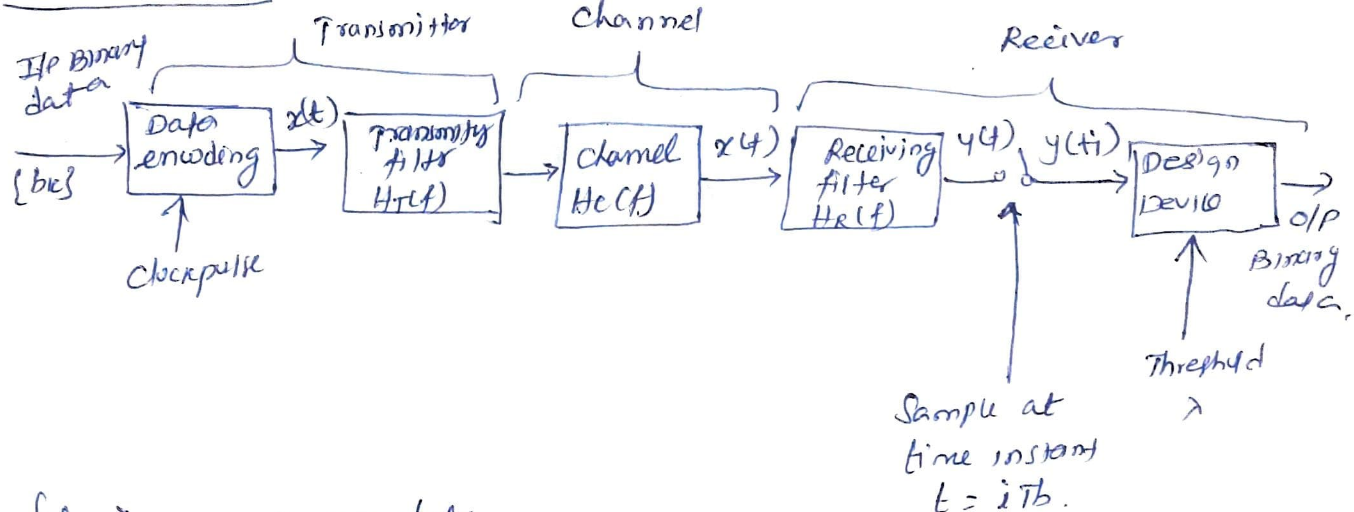
$$= \frac{1}{T_b} T_b^2 \, \text{sinc}^2\left(\frac{f T_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right) \left[a^2 \cdot e^0 \right]$$

$$\boxed{P(f) = T_b a^2 \, \text{sinc}^2\left(\frac{f T_b}{2}\right) \sin^2\left(\frac{\pi f T_b}{2}\right)}$$

Base band transmission

The signals are transmitted without any modulation over the channel in base band transmission.

Block Diagram :



$\{b_k\}$ - I/P Binary data

$x(t) \rightarrow$ o/p of encoder pulse wave form

$$x(t) = \sum_{k=-\infty}^{\infty} A_k\, g(t - kT_b) \quad \text{——— (1)}$$

T_b is the duration of each input binary bit

$g(t)$ is the shaping pulse.

and

$$A_k = \begin{cases} +a & \text{if } b_k = 1 \\ -a & \text{if } b_k = 0 \end{cases}$$

Let $H_T(f)$ is the combined transfer function of the Transmitting filter

$H_C(f)$ is the transfer function of the channel

$H_R(f)$ is the combined transfer function of the Receiving filter

$y(t)$ is the o/p of Receiving filter and it is noisy replica of the transmitted signal $x(t)$.

The signal $y(t)$ is sampled synchronously with the transmitter with sampling instants $\boxed{t = iT_b}$

The sampling instants are synchronous to the clock pulses at the transmitter.

The $y(t)$ is given to decision device and it compare the input signal with threshold 'λ'. o/p of decision device is

if $y(t_i) > \lambda$ select symbol '1' ——— (2)
If $y(t_i) < \lambda$ select symbol '0'.

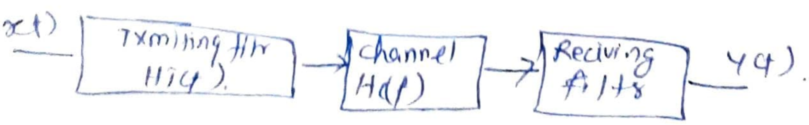
Intersymbol interference problem

from Receiving filter o/p, $y(t)$ can be given interms of A_k ~~g(t)~~ as

$$y(t) = \mu \sum_{K=-\infty}^{\infty} A_K\, p(t - kT_b) \quad \text{——— (3)}$$

μ - is the scalling factor
$p(t)$ is the shape different from that of $g(t)$,

from Block Diagram, we can observe that $A_k g(t)$ is the signal applied to the input of cascade of transmitting filter, channel and Receiving filter.



o/p of Cascade connection is

$\mu A_k p t$

Let Fourier Tranform of $g(t)$ be $G(f)$.

and $p(t)$ be $P(f)$.

Then in frequency domain we can write

$$\mu A_k P(f) = H(f)\, A_k\, G(f) \quad \text{——— (4)}$$

and

$$H(f) = H_T(f)\, H_C(f)\, H_R(f) \quad \text{——— (5)}$$

$H(f)$ is the Combined Transfer function of Transmitting filter, Chanel and Receiving filter [cascade connection].

Sub (5) in (4)

$$\mu \cancel{A_k} P(f) = H_T(f)\, H_C(f)\, H_R(f)\, \cancel{A_k}\, G(f)$$

$$\mu P(f) = H_T(f)\, H_C(f)\, H_R(f)\, G(f) \longrightarrow (6)$$

The Receiving filter output is sampled at $t_i = iT_b$.

from equation (3), at $t = iT_b$ we can write.

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} A_k\, p(iT_b - kT_b).$$

$$= \mu \sum_{k=-\infty}^{\infty} A_k\, p[(i-k)T_b] \quad \text{——— (7)}$$

Sub $k=i$ and $k \neq i$ in equation (7) and rearrange the above equation (7) and we get

$$y(t_i) = \underbrace{\mu A_i\, p(0)}_{\text{1st term}} + \underbrace{\mu \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} A_k\, p[(i-k)T_b]}_{\text{2nd term}} \quad \text{——— (8)}$$

First term represents the value of $y(t_i)$ when $i = k$.

$p(t)$ is normalized such that

$p(0) = 1$, Hence the above equation becomes

$$y(t_i) = \mu A_i + \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} A_k\, p[(i-k)T_b] \quad \text{——— (9)}$$

and $i = 0, \pm 1, \pm 2, \pm 3 \ldots$

Conclution :

* The first term in above equation (9) is μA_i.
It is the contribution of the ith transmitted bit

* The second term represents the residual effect of all other bits transmitted before and ~~aft~~ after sampling instant t_i

Defintion of ISI

The presence of outputs due to other bits (symbols) interfere with the output of required bit (symbol). This effect is called intersymbol interference.

NOTE: We have not considred the effect of channel noise, Actually, channel noise and ISI both interfere the transmitted signal

If the intersymbol interference is absent, then the second term will not be preset in equation (8)

$$\boxed{y(t_i) = \mu A_i} \quad \longrightarrow 10$$

At $t = iT_b$, the correct bit is A_i, observe that it is decoded correctly in absense of ISI. It is not possible to eliminate the second term (ISI) totally

So the ISI can be reduced by proper design of

pulse spectrum $G(f)$
transmitt filter $H_T(f)$
recive filter $H_R(f)$
channel $H_C(f)$.

Definition of ISI

The presence of outputs due to other bits (symbols) interfere with the output of required bit (symbol). This effect is called Intersymbol Interference (ISI). Here note that we have not considered the effect of channel noise. Actually, channel noise and ISI both interfere the transmitted signal.

- If the intersymbol interference is absent, then the second term will not be present in equation (3.2.7) i.e.,

$$y(t_i) = \mu A_i \qquad \dots (3.2.10)$$

At $t = i T_b$, the correct bit is A_i. Observe that it is decoded correctly in absence of ISI. It is not possible to eliminate the second term of equation (3.2.7) (and hence ISI) totally. The ISI can be reduced by proper design of pulse spectrum $G(f)$, transmit filter $H_T(f)$, receive filter $H_R(f)$ and the channel $H_C(f)$. We will discuss some of these issues in subsequent sections.

Review Questions

1. *What do you understand by Intersymbol Interference (ISI) ?*

   AU : Dec.-05, Marks 6, May-09, Marks 4

2. *Describe the baseband transmission system with a neat block diagram.* **AU : Dec.-09, Marks 16**
3. *Explain in detail M-ary base band system.* **AU : May-10, Marks 10**
4. *Explain intersymbol interference (ISI). How it is avoided ?*

3.3 Nyquist Criterion for Distortionless Transmission

AU : May-04, 05, 09, 11, 13, 14, Dec.-04, 05, 06

3.3.1 Nyquist Pulse Shaping Criterion

Time Domain Criterion

From equation (3.2.9) we know that the second term (summation) must be zero to eliminate effect of ISI. This is possible if the received pulse p(t) is controlled such that,

$$p[(i-k)T_b] = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases} \qquad \dots (3.3.1)$$

If p(t) satisfies the above condition, then we get a signal which is free from ISI. i.e.,

$$y(t_i) = \mu A_i \qquad \text{from equation (3.2.10)}$$

Hence equation (3.3.1) gives the condition for perfect reception in absence of noise. Equation (3.3.1) is the condition in time domain. This condition gives more useful criteria in frequency domain.

Criterion in Frequency Domain

- Let $p(nT_b)$ represent the impulses at which $p(t)$ is sampled for decision. These samples are taken at the rate of T_b. Fourier spectrum of these impulses is given as

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b) \qquad \text{... (3.3.2)}$$

This means the spectrums of $p(t)$ are periodic with period f_b. Here note that the sampling frequency (instants) is f_b. Here $P_\delta(f)$ represents the spectrum of $p(nT_b)$ and $P(f)$ is the spectrum of $p(t)$.

- We can think of $p(nT_b)$ as the infinite length of impulses with period T_b, which are weighted with amplitudes of $p(t)$. i.e.,

$$p_\delta(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\,\delta(t - nT_b) \qquad \text{... (3.3.3)}$$

- Fourier transform of $p_\delta(t)$ becomes,

$$P_\delta(f) = \int_{-\infty}^{\infty} p_\delta(t)\, e^{-j2\pi ft}\, dt = \int_{-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} p(nT_b)\,\delta(t - nT_b) \right] e^{-j2\pi ft}\, dt$$

- Let $\quad n = i - k$ in above equation,

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} p[(i-k)T_b]\,\delta[t - (i-k)T_b]\, e^{-j2\pi ft}\, dt$$

- Now let us apply the condition of equation (3.3.1) to above equation,

$$P_\delta(f) = \begin{cases} \displaystyle\int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft}\, dt & \text{for } i = k \\ \displaystyle\int_{-\infty}^{\infty} 0\ \delta(t)\, e^{-j2\pi ft}\, dt & \text{for } i \neq k \end{cases}$$

$$\therefore \quad P_\delta(f) = \int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft}\, dt \quad \text{for } i = k = p(0) \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt$$

An integration in above equation is the Fourier transform of $\delta(t)$, which is 1. Hence,

$$P_\delta(f) = p(0) \quad \text{for } i = k \qquad \text{... (3.3.4)}$$

$$= 1 \qquad \text{by normalization of } p(0).$$

- Hence equation (3.2.2) becomes (with $P_\delta(f) = 1$),

$$1 = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b)$$

or

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = \frac{1}{f_b} \qquad \text{... (3.3.5)}$$

Since $\frac{1}{f_b} = T_b$,

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = T_b \qquad \cdots (3.3.6)$$

This is the frequency domain condition for zero ISI. Above equation is called Nyquist pulse shaping criterion for baseband transmission.

3.3.2 Ideal Nyquist Channel (Ideal Solution to ISI)

- **Spectrum of $p(t)$:** We will now derive the function $p(t)$ which confirms to Nyquist criterion and eliminates ISI. Equation 3.3.6 represents the spectrum which repeats with period f_b and it has amplitude of T_b. This spectrum is shown in Fig. 3.3.1. $P(f)$ shows the spectrum of the expected signal $p(t)$. It can be represented using *rect* function as,

$$P(f) = \frac{1}{f_b} \text{rect}\left(\frac{f}{f_b}\right)$$

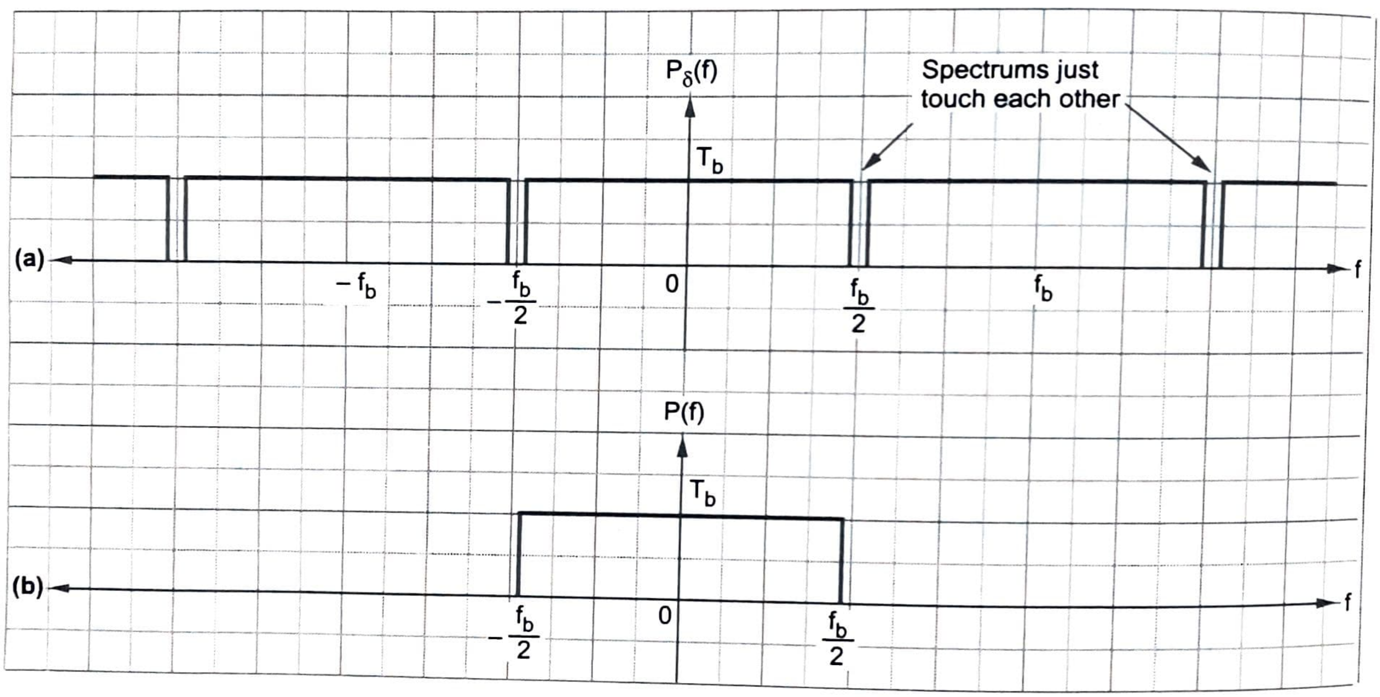


Fig. 3.3.1 (a) Spectrum of sampled signal

(b) Spectrum of p(t)

- **To obtain $p(t)$ from spectum :** Inverse fourier transform of above function can be obtained from standard fourier transform pairs as,

$$\text{sinc}\,(f_b t) \xleftrightarrow{FT} \frac{1}{f_b} \text{rect}\left(\frac{f}{f_b}\right)$$

i.e., $\quad p(t) = \text{sinc}\,(f_b t)$

The bandwidth of the pulse can be represented by $B_0 = \frac{f_b}{2}$. Then above equation becomes,

$$p(t) = \text{sinc}\,(2B_0 t) \qquad \text{... (3.3.7)}$$

The sinc function can also be represented as,

$$p(t) = \frac{\sin\,(2\pi B_0\, t)}{2\pi B_0\, t} \qquad \text{... (3.3.8)}$$

- **Nyquist bandwidth :** Here B_0 is called the Nyquist bandwidth. The Nyquist bandwidth is the minimum transmission bandwidth for zero ISI.
- Fig. 3.3.2 illustrates how sinc pulse produces zero ISI. Fig. 3.3.2 shows the three sinc pulses transmitted successively.

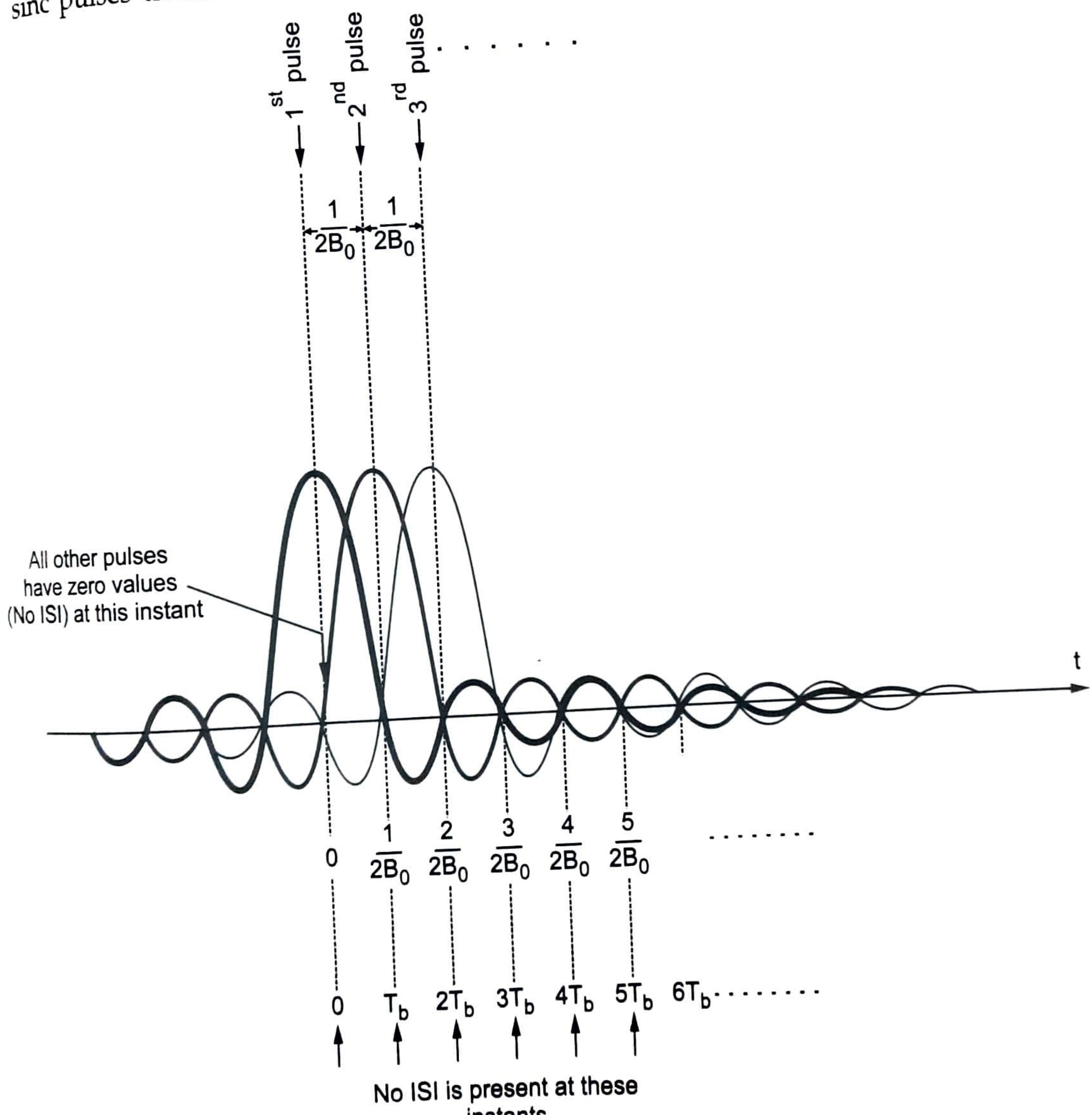


Fig. 3.3.2 Sinc pulse for baseband transmission produces zero ISI

One sinc pulse is transmitted for one bit. Hence the sinc pulses are transmitted at $t = 0, T_b, 2T_b, \ldots$ and so on. In the figure, observe that at $t = 0$, all the sinc pulses have zero values except the transmitted pulse. Hence ISI will be zero if we sample the first pulse at $t = 0$. Similarly at $t = T_b$, the second pulse have nonzero value and all other pulses have zero values. Hence ISI will be zero if we sample the 2^{nd} pulse at $t = T_b$. In general we can say that ISI will be zero if we sample the pulses at $t = 0, T_b, 2T_b, \ldots$ and so on.

From Fig. 3.3.2, the Nyquist bandwidth B_0 is related to bit period T_b as,

Bit period, $$T_b = \frac{1}{2B_0} \quad \ldots (3.3.9)$$

Or Nyquist bandwidth, $$B_0 = \frac{1}{2T_b} \quad \ldots (3.3.10)$$

Or $$B_0 = \frac{\text{Bit rate}}{2} \quad \ldots (3.3.11)$$

Thus Nyquist bandwidth is half the bit rate, $\frac{1}{T_b}$.

- **Ideal sinc pulse physically not possible :**

Just now we have seen that sinc pulse is ideal for zero ISI in baseband transmission. Now let use see whether transmission of exact sinc pulse is physically possible. From equation (3.3.7), the sinc pulse is given as,

$$p(t) = \text{sinc}(2B_0 t)$$

The Fourier transform of the sinc function is given as,

$$P(f) = \frac{1}{2B_0} \text{rect}\left(\frac{f}{2B_0}\right) \quad \ldots (3.3.12)$$

The 'rect' function indicates a rectangular pulse. This pulse is given as,

$$P(f) = \begin{cases} \frac{1}{2B_0} & \text{for } \quad -B_0 < f \leq B_0 \\ 0 & \text{elsewhere} \end{cases} \quad \ldots (3.3.13)$$

This pulse spectrum is shown in Fig. 3.3.3.

In the Fig. 3.3.3 observe that the frequency response of the sinc pulse is flat from $-B_0$ to $+B_0$ and zero elsewhere. There are abrupt transitions in the frequencies at $\pm B_0$. Such frequency spectrum is not physically

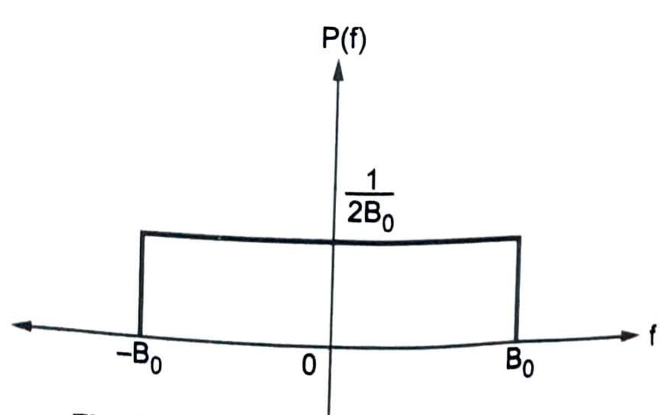


Fig. 3.3.3 Spectrum of the sinc pulse

realizable. Hence the ideal sinc pulse cannot be physically generated, even though it eliminates ISI completely.

- **Merits of Nyquist channel**

  i) It eliminates an ISI completely.

  ii) The method seems to be most easy.

- **Limitation of Nyquist channel**

  i) The transmission of exact sinc pulse is physically not possible.

3.3.3 Raised Cosine Channel

- **Raised cosine spectrum :** In the raised cosine spectrum, the frequency response $P(f)$ decreases towards zero gradually (That is there is no abrupt transition). The raised cosine spectrum is given as follows :

$$P(f) = \begin{cases} \dfrac{1}{2B_0} & \text{for } \quad -f_1 < f < f_1 \\ \dfrac{1}{4B_0}\left\{1+\cos\left[\dfrac{\pi(|f|-f_1)}{2B_0-2f_1}\right]\right\} & \text{for } \; f_1 < |f| < 2B_0 - f_1 \\ 0 & \text{elsewhere} \end{cases} \quad \ldots (3.3.14)$$

In the above equation, the middle term represents gradual roll off from f_1 to $2B_0 - f_1$. The frequency f_1 and Nyquist bandwidth B_0 are related by the roll off factor. i.e.,

$$\text{Roll off factor, } \alpha = 1 - \frac{f_1}{B_0} \quad \ldots (3.3.15)$$

- The frequency spectrum $P(f)$ is plotted in Fig. 3.3.4 for $\alpha = 0, 0.5$ and 1. This spectrum is normalized on both the scales. Observe that at $\alpha = 0$, there is abrupt transition and it becomes ideal case discussed in previous section. As 'α' increases the rolloff is slower.

- **Time domain pulse for raised cosine spectrum** : The inverse Fourier transform of the raised cosine spectrum of equation (3.3.14) gives time domain pulse $p(t)$. i.e.,

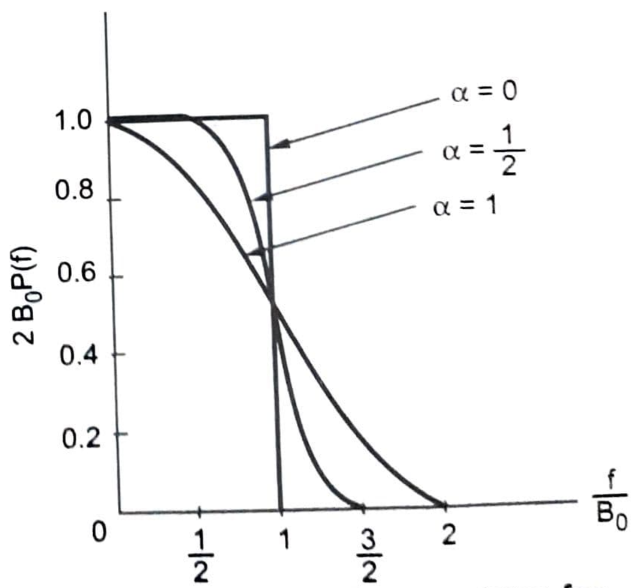


Fig. 3.3.4 Raised cosine spectrum for α = 0, 0.5 and 1

$$p(t) = \text{sinc}(2B_0 t) \frac{\cos(2\pi\alpha B_0 t)}{1-16\alpha^2 B_0^2 t^2} \quad \cdots (3.3.16)$$

Comments

i) The above pulse function is the product of two functions. The first, i.e. $sinc(2B_0 t)$ is the sinc pulse. The second factor decreases as $1/|t|^2$.

ii) Because of the second factor, the abrupt transitions in the frequency domain are avoided.

iii) The first factor, i.e. sinc function makes $p(t)$ zero at the sampling instants of $t = 0, \pm\frac{1}{2B_0}, \pm\frac{2}{2B_0}, \ldots\ldots$

That is $t = 0, \pm T_b, \pm 2T_b, \ldots$ and so on.

iv) Fig. 3.3.5 shows the pulse $p(t)$ for $\alpha = 0, 0.5$ and 1. This pulse is normalized on both the scales. Observe that at $\alpha = 0$, it becomes sinc pulse. Also observe that the pulse becomes zero (for all values of α) at the sampling instants of $\pm T_b, \pm 2T_b, \ldots$. Thus it is possible to eliminate ISI by using raised cosine spectrums.

v) From equation (3.3.14) observe that the spectrum has nonzero values upto $2B_0 - f_1$.

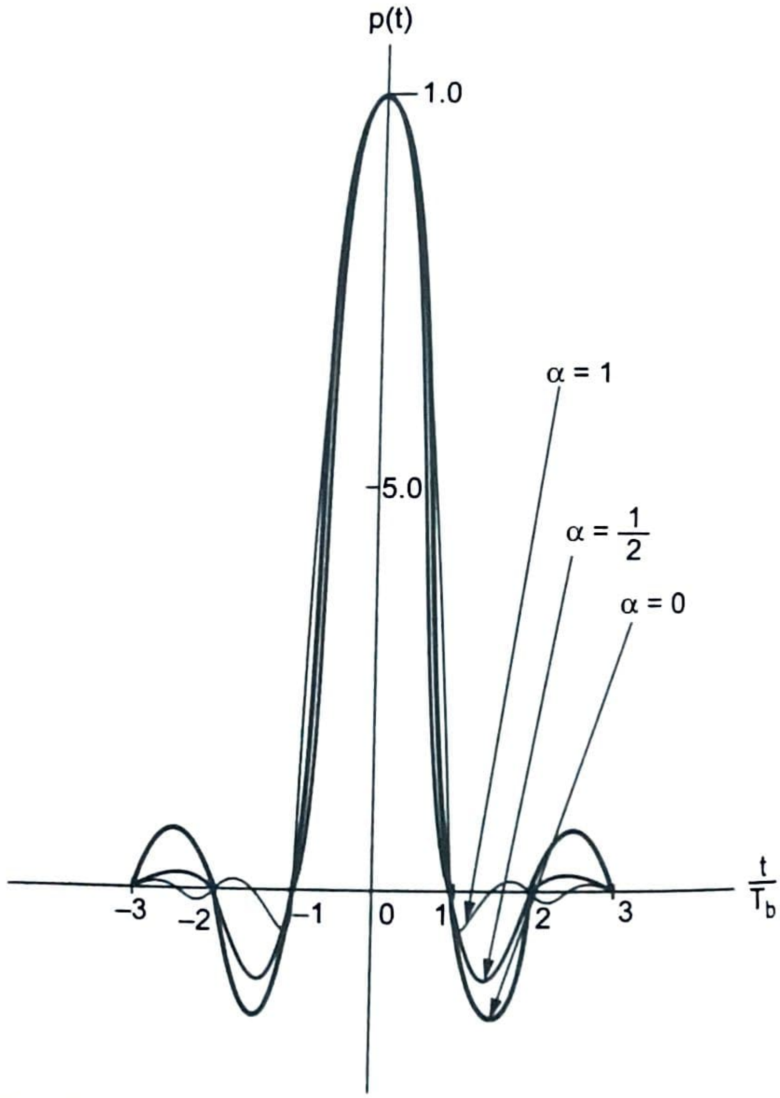


Fig. 3.3.5 Time response of raised cosine spectrum

- **Bandwidth required** : Hence the bandwidth required for raised cosine spectrum will be,

$$\boxed{B = 2B_0 - f_1} \quad \ldots (3.3.17)$$

Consider equation (3.3.15),

$$\alpha = 1 - \frac{f_1}{B_0}$$

$$f_1 = B_0 - B_0 \alpha$$

Putting for f_1 in equation (3.3.17) we get,

$$B = 2B_0 - B_0 + B_0 \alpha$$

$$\boxed{B = B_0 (1+\alpha)} \quad \text{... (3.3.18)}$$

For $\alpha = 0$, $B = B_0$, this is the case of ideal solution. As α increases, the bandwidth 'B' increases above Nyquist bandwidth. When $\alpha = 1$, $B = 2B_0$ i.e. the bandwidth required by the raised cosine spectrum is double of the Nyquist bandwidth.

Review Questions

1. *Explain any one method of ISI control.* **AU : May-14, Marks 12**
2. *Explain Nyquist first criterion to minimize ISI.* **AU : May-13, Marks 16**
3. *Sketch the time response and frequency response of signal with raised cosine pulse spectrum.* **AU : May-04, Marks 8; Dec.-09, Marks 16**
4. *What is meant by the ideal Nyquist channel ? What are its merits and limitations ?* **AU : Dec.-04, Marks 8**
5. *Obtain an expression for Nyquist criterion for distortionless baseband transmission for zero intersymbol interference.* **AU : May-05, 12, Marks 6, May-09, Marks 8, Dec.-05, May-10, Marks 10**
6. *With necessary expressions, explain the practical difficulties encountered in ideal Nyquist channel and how are they overcomed by raised cosine spectrum.* **AU : Dec.-06, Marks 5**
7. *Discuss on signal design for ISI elimination.* **AU : May-11, Marks 8**
8. *Explain how raised cosine spectrum reduces ISI.*

3.4 Correlative Coding to Eliminate ISI

AU : May-12, Dec.-12, 15

- In the previous section we observed that raised cosine spectrum eliminates ISI at the cost of increased transmission bandwidth. From equation 3.3.17 it is clear that bit rate or signaling rate is $2B_0$ in ideal baseband transmission system. But such system is not physically realizable.
- Correlative level coding allows the signaling rate of $2B_0$ in the channel of bandwidth B_0. This is made physically possible by allowing ISI in the transmitted signal in controlled manner. This ISI is known to the receiver. Hence effects of ISI are eliminated at the receiver.
- The correlative coding is implemented by duobinary signaling and modified duobinary signaling.

Correlative Coding to Eliminate ISI

Let we know that raised cosine spectrum eliminates ISI at the cost of increased transmission BW.

and $B = 2B_0 - f_1$. - [BW required for raise cosine spectrum]

$$\alpha = 1 - \frac{f_1}{B_0}$$

Signaling rate or bit rate is $2B_0$ in Ideal base band transmission system. But such system is not physically realizable.

Correlative Level Coding allows the signaling rate of $2B_0$ in the channel of BW B_0.

This is made physically possible by allowing ISI in the transmitted signal in controlled manner.

This ISI is known to the reciver. Hence effect of ISI is eliminated at the reciver.

The correlative coding is implemented by duobinary signaling and modified duobinary signaling.

Duobinary Encoding

Duo binary encoding reduces the maximum frequency of the base band signal.

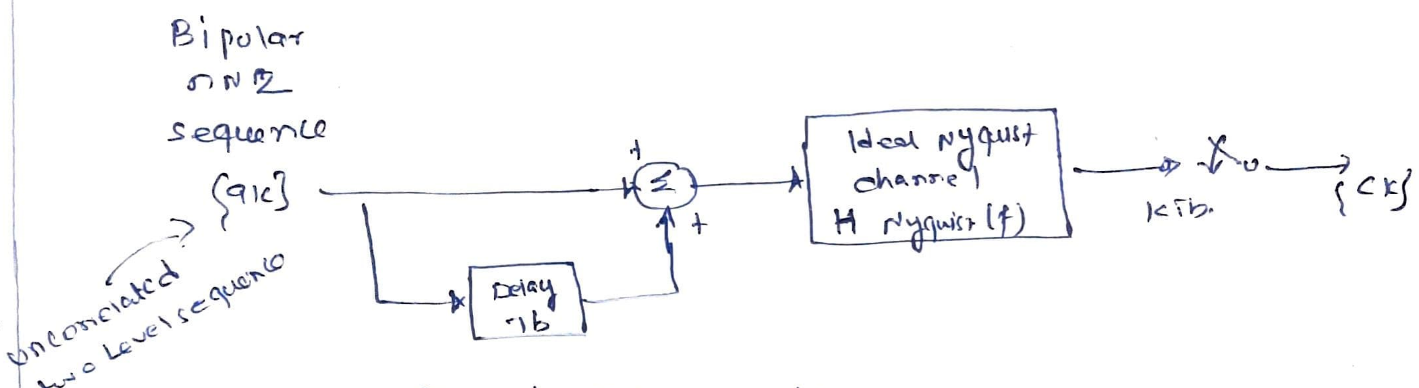
Duo => Double the transmission capacity of the binary system.

→ Consider the I/p sequence $\{b_k\}$, It Contain 1 and 0

→ Use of Level shifter this sequence is Converted into bipolar NRZ sequence $\{a_k\}$.

$$\left.\begin{array}{l} a_k = +1 \text{ if } b_k = 1 \\ a_k = -1 \text{ if } b_k = 0. \end{array}\right\}$$

Encoder :



Duo-binary encoder.

→ It accepts $\{a_k\}$ and Convert into three Level signals ie -2, 0 and +2.

The o/p is expressed as

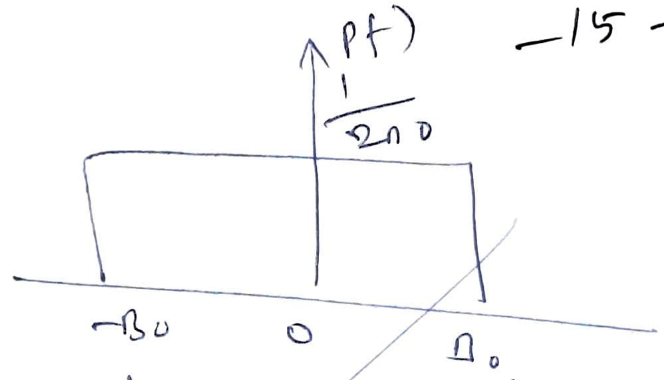
$$c_k = a_k + a_{k-1}$$

→ ie it Convert two level uncorrelated pulse into three level Correlated sequence. This introduce ISI in the signal in the artificial manner to reduce the BW.

Reconstruction

Let $\hat{a}_k$ represents the estimate of a_k

then $\hat{a}_k = c_k - \hat{a}_{k-1}$



response is for -B0 to B0 and zero elsewhere.

Phase is a abrupt form in the

---

this shows that if c_k is recived with error, then $\hat{a}_k$ will have error. The error will propagate in the o/p sequence.

Draw Back: Error propagation taking place in the decoder.

Frequency Response

Let delay element Tb. in Block diagram.

Frequency response of delay element is $e^{-j2\pi f T_b}$.

Hence frequency response of delay line filter will be

$1+e^{-j2\pi f T_b}$.

The delay line filter is connected in cascade with ideal Nyquist channel. Hence overall frequency response of the scheme will be.

$$H(f) = H_{Nyquist}(f)\left[1+e^{-j2\pi f T_b}\right].$$

Let rearrange the term in square bracket

$$H(f) = H_{Nyquist}(f)\left[e^{-j\pi f T_b} \cdot e^{j\pi f T_b} + e^{-j\pi f T_b} \cdot e^{-j\pi f T_b}\right]$$

$$= H_{Nyquist}(f)\left[e^{j\pi f T_b} + e^{-j\pi f T_b}\right] e^{-j\pi f T_b}.$$

$$= H_{Nyquist}(f)\, \{ \cdot\, 2\cos(\pi f T_b)\, e^{-j\pi T_b}. \quad \text{—— (1)}$$

Let we know that.

spectrum of the sinc pulse.

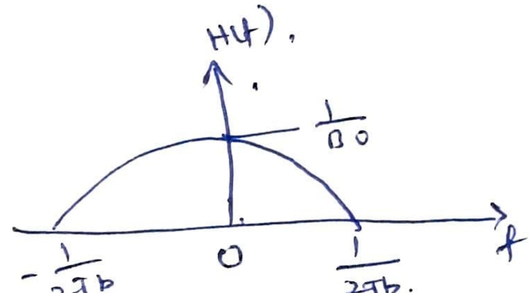
$$\underline{H_{Nyquist}(f)} = \begin{cases} \frac{1}{2B_0} & \text{for } -B_0 < f \le B_0 \\ 0 & \text{else.} \end{cases}$$

with above expression we can write above exp. (1)

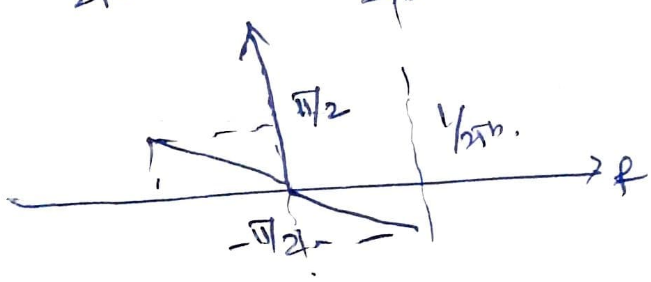
$$H(f) = \begin{cases} \frac{1}{B_0} \cos(\pi f T_b)\, e^{-j\pi f T_b} & |f| \le B_0 \\ 0 & \text{otherwise.} \end{cases}$$

Here $|H(f)| = \frac{1}{B_0}\cos(\pi f T_b)$ for $|f| \le B_0$

and $\angle H(f) = -\pi f T_b.$



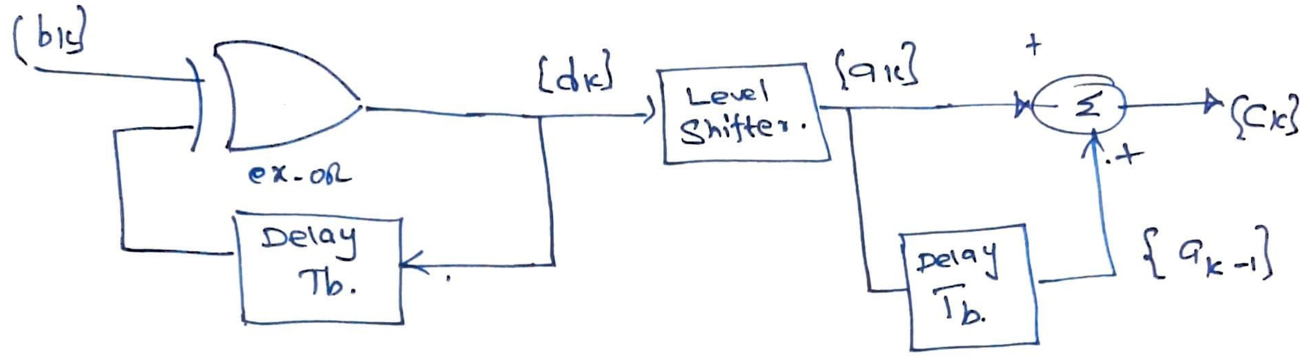
frequency Response.



Duo binary Encoder with precoder [Differential encoder].

precoder: Used to avoid error propagation to o/p.

Block Diagram



From above digram

$d_k = b_k \oplus d_{k-1}$ ——— (1)

ie $d_k = 1$ if either b_k or b_{k-1} is 1.

$d_k = 0$ otherwise.

The sequence $\{d_k\}$ is applied to the I/p of level shifter and we get output a_k ie bipolar signal, given by

if $d_k = 1$, $a_k = 1$
$d_k = 0$, $a_k = -1$ } ——— (2)

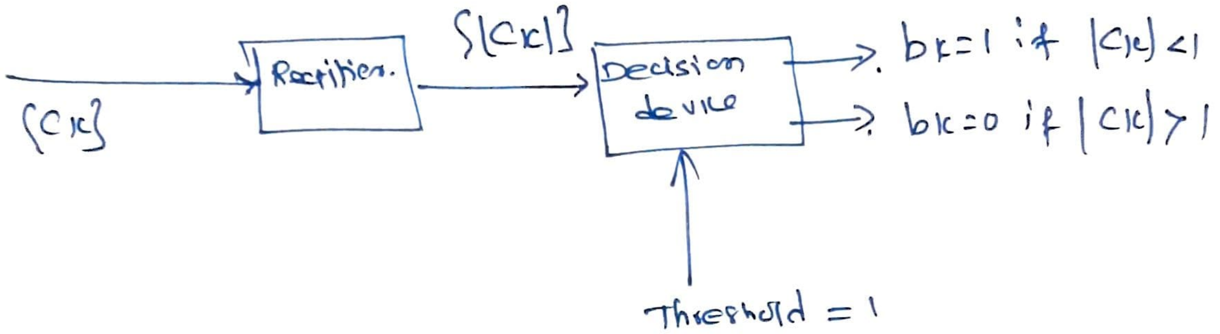
The sequence $\{a_k\}$ is then applied to duoBinary encoder. the output c_k is.

$C_k = a_k + a_{k-1}$ ——— (3)

the value of c_k will be

$$\left.\begin{aligned} c_k &= 0 \quad \text{if } b_k \text{ is } 1 \\ &= \pm 2 \quad \text{if } b_k \text{ is } 0. \end{aligned}\right\} \longrightarrow (4)$$

Decoder



In decoder magnitude of c_k is taken and compared with Threshold '1' to take decision of P.

if $|c_k| < 1$ take $b_k = 1$

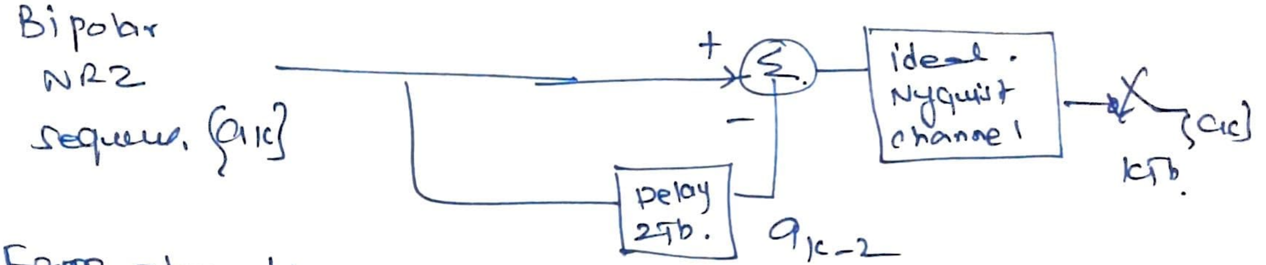
if $|c_k| > 1$ take $b_k = 0$.

Here 'b_k' depends on present value of 'c_k' previous value of the output not required. This shows the there is a no propagation of error in this system. compared to duo biary system

Modified duo Binary encoding

The correlation is over two binary digits in modified duobinary encoding.

Encoder:



From above diagram.

Let $c_k = a_k - a_{k-2}$ —— (1).

Due to Bipolar NRZ sequence. the a_k is ± 1

ie $a_k = \pm 1$,

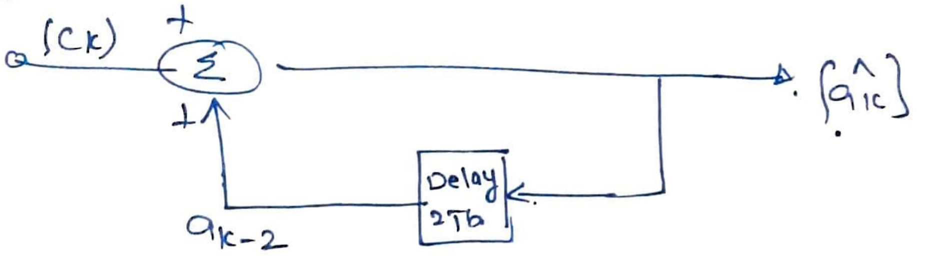
We get c_k as a Correlated seque of one of the three levels o/p s.

Let $\hat{a}_k$ is the estimate of a_k.

then we can obtain

$$\hat{a}_k = c_k + \hat{a}_{k-2} \quad \text{—— (2)}$$

Decoder:



From equation (2) if c_k is error, $\hat{a}_k$ also error ie error is propagated.

frequency Response and Impulse Response of modified duobinary encoder

Let Delay element '$2T_b$'

The Frequency response of delay element will be

$1 - e^{-j2\pi f T_b}$.

The delay line filter is connected in cascade with channel. Hence total frequency response will be

$$H(f) = H_{Nyquist}(f)\left[1 - e^{-j2\pi f 2T_b}\right] \quad (1)$$

Rearranging the terms inside square brackets

$$H(f) = H_{Ny}(f)\left[e^{j2\pi f T_b} \cdot e^{-j2\pi f T_b} - e^{-j2\pi f T_b} \cdot e^{-j2\pi f T_b}\right]$$

$$= H_{Ny}(f)\left[e^{j2\pi f T_b} - e^{-j2\pi f T_b}\right] \cdot e^{-j2\pi f T_b}.$$

$$H(f) = H_{Ny}(f) \cdot 2j \sin(2\pi f T_b) \cdot e^{-j2\pi f T_b}. \quad (2)$$

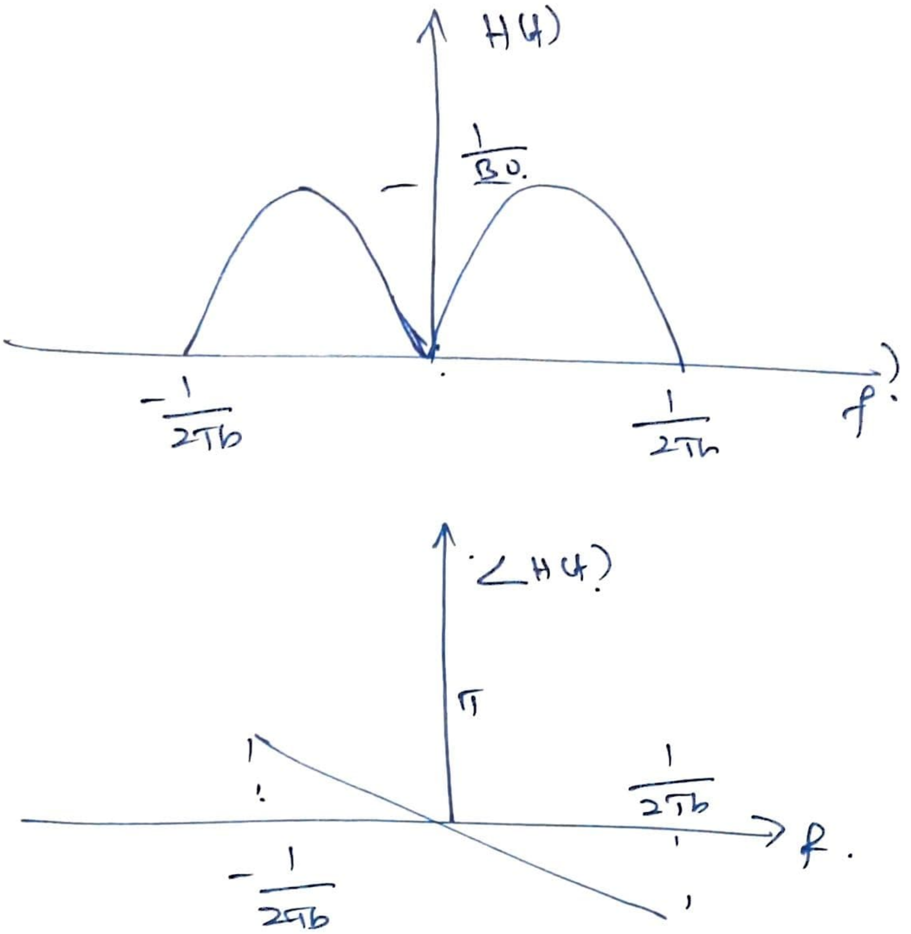
Here $H_{Ny}(f) = \begin{cases} \frac{1}{2B_0} & \text{for } -B_0 \le f \le B_0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$

$$\therefore H(f) = \begin{cases} \frac{1}{B_0} j\sin(2\pi f T_b) \cdot e^{-j2\pi f T_b} & \text{for } -B_0 \le f \le B_0 \\ 0 & \text{else} \end{cases} \quad (4)$$

or

$$|H(f)| = \frac{1}{B_0} \sin(2\pi f T_b) \quad \text{for } f \le |B_0|$$

and $\angle H(f) = -2\pi f T_b$



Taking Inverse F.T of equation no (4) we get impulse response.

$$\boxed{h(t) = \frac{2T_b^2 \sin(\pi t / T_b)}{\pi t(2T_b - t)}}$$

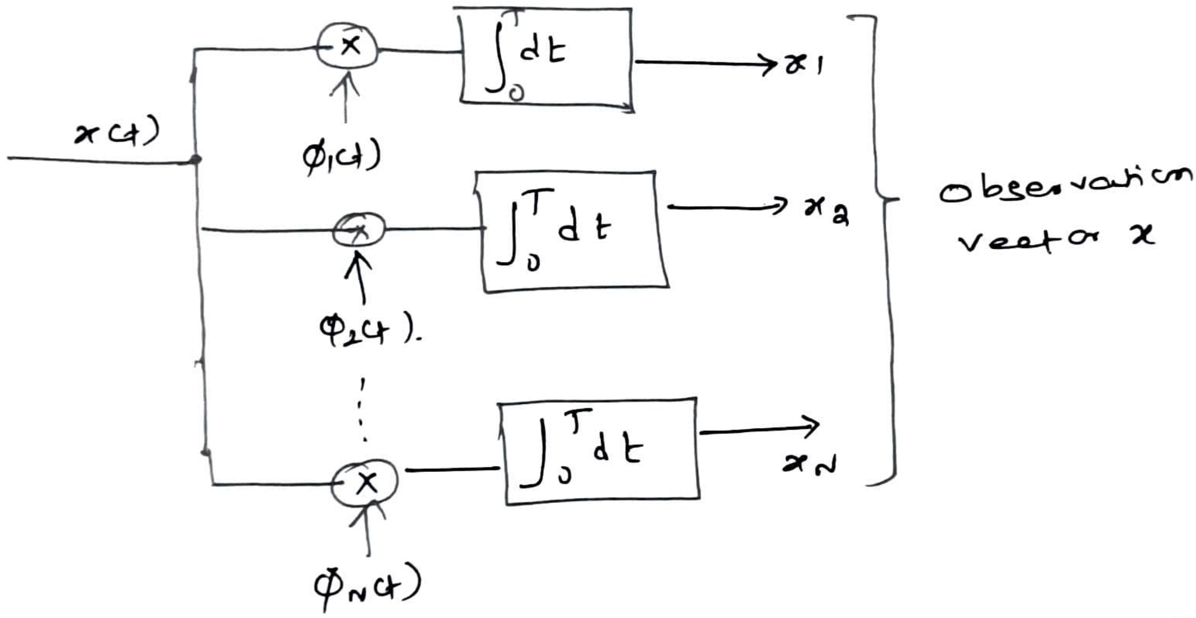
Correlation Reciver (Or) optimum Receiver.

What is optimum Receiver?

A Receiver is said to be optimum if it yields the minimum probability error. ie it designed to produce minimum probability error. called optimum Receiver

For an AWGN channel and transmitted signals $s_1(t), s_2(t) \ldots s_m(t)$ are equally likely, the optimum receiver consist of two subsystems.

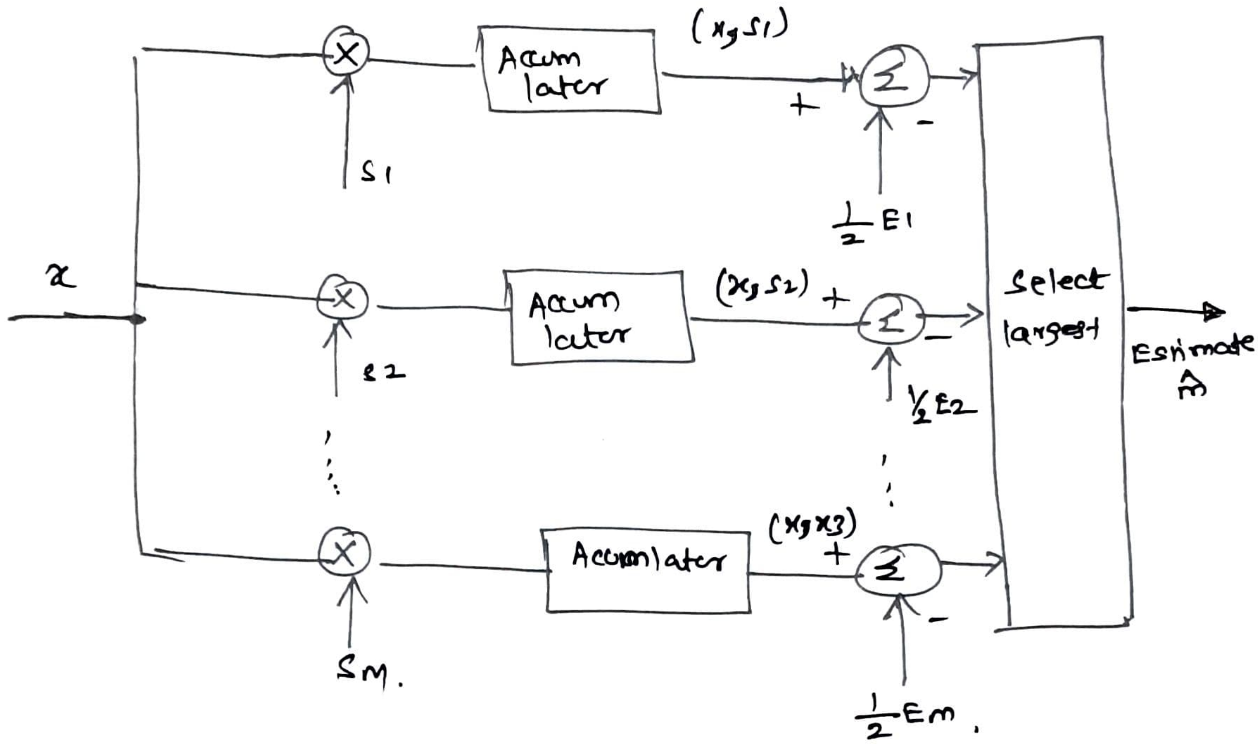
1. Detector part of Receiver



It consist of a bank of M product - integrators or correlators supplied with a Corresponding set of coherent referance signal (or) Correlators supplied with a corresponding set of coherent referance signals or orthogonal basis function $\phi_1(t), \phi_2(t) \ldots \phi_N(t)$ that are generated locally.

This bank of corrolators operate on recived signal $x(t)$, $0 \le t \le T$, produce the observation vector x.

2. Vector Receiver



It is implemented in the form of a maximum-likelihood detector.

I/p observation vector 'x' is applied to produce an estimate $\hat{m}$ of the transmitted symbol m_i, $i = 1, 2 \ldots M$, in a way that would minimize the average probability of symbol error.

Input observation vector x is multiplied with M signal vectors $s_1, s_2 \ldots s_m$. Then this two vectors summed in accumlater to produce set of inner products $\{(x, s_k)\}$, $k = 1, 2 \ldots M$.

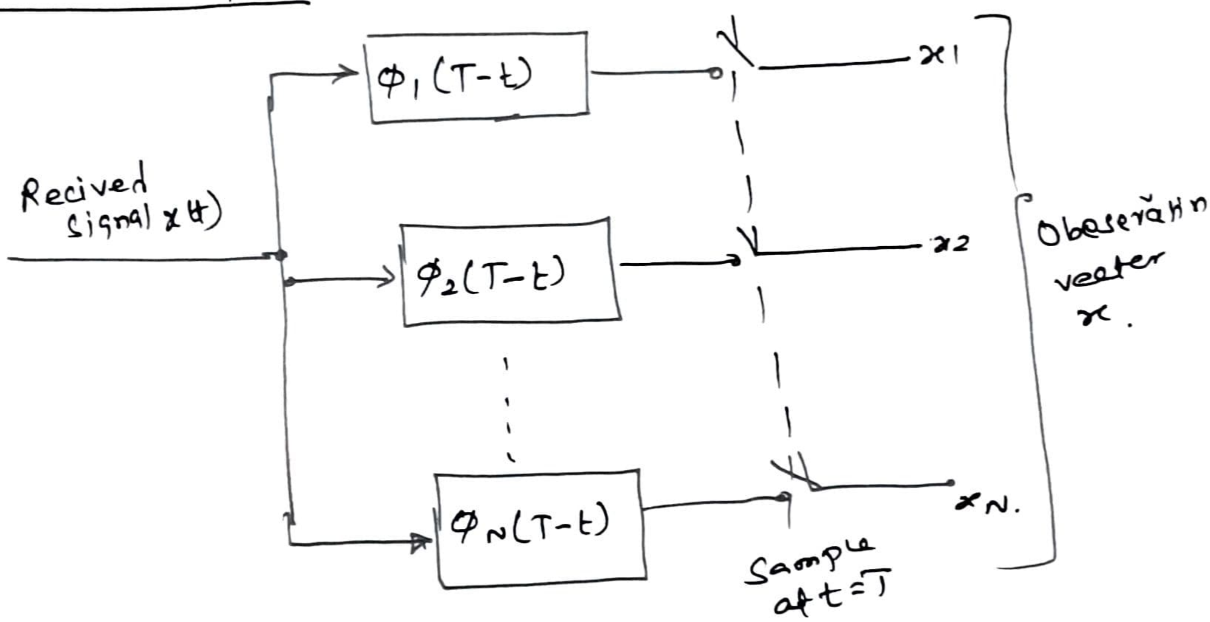
Then the inner product summed with signal energies.

Finaly, the largest in the resulting set of number is selected, and a corresponding decision on the transmitted message is made.

Matched filter Reciver

If a filter generates an output to maximize the output peak power ratio to mean noise power within its frequency response then it is called a matched filter. It is the optimum liner filter used to increase the SNR in the existence of additive noise.

Reciver (Detector part)



NOTE:
Here the orthogonal basis function $\phi_1(t), \phi_2(t) \cdots \phi_N(t)$ is assumed to zero out side the interval $0 \le t \le T$, the use of multipliers may be avoided.

Consider, a liner filter with impulse response $h_j(t)$.
$x(t)$ is a Recived signal used as a input to detector.
$y_j(t)$ is a output

The $y_j(t)$ is defined by the Convolution integral

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, h_j(t-\tau)\, d\tau \quad \text{---} (1)$$

Let we set impulse Response

$$h_j(t) = \phi_j(T-t) \quad \text{---} (2)$$

Then the Resulting filter output is

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(T - t + \tau)\, d\tau \quad \text{---} \ (3)$$

Sampling this output at time $t = T$, we get

$$y_j(T) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(\tau)\, d\tau \quad \text{---}$$

ie
$$y_j(T) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(T - T + \tau)\, d\tau$$

$$y_j(T) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(\tau)\, d\tau \quad \text{---} \ (4)$$

Since $\phi_j(T)$ is zero outside the interval $0 \leq t \leq T$, so we get

$$y_j(T) = \int_{0}^{T} x(\tau)\, \phi_j(\tau)\, d\tau \quad \text{---} \ (5).$$

A filter whose impulse Response is a time reversed and delayed version of some signal $\phi_j(t)$, as in equation (2), is said to be matched to $\phi_j(t)$. The crosponding, the optimum Reciver based on the detector is reffered to as the matched filter Reciver.

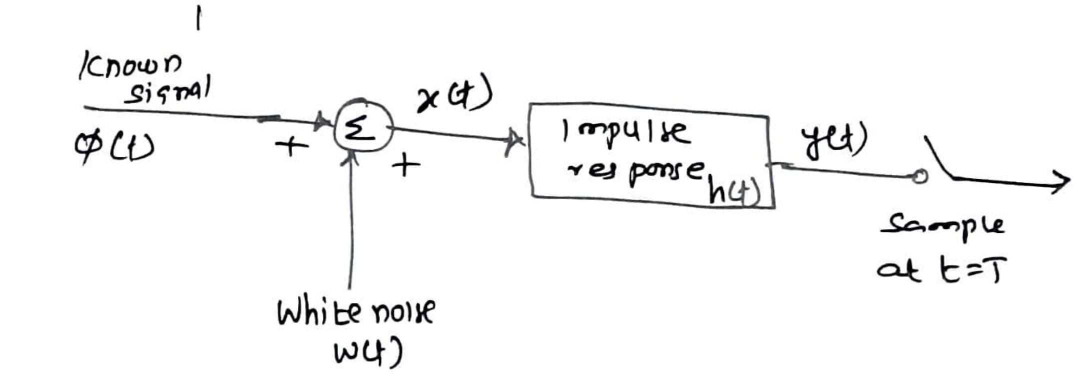
For a matched filter operating in real time to be physically realizable, it must be <u>causal</u>

ie impulse Response must be zero for negative time

$$\boxed{h_j(t) = 0 \quad t < 0}$$

from equation (2), the causality condition is satisfied provided that the signal $\phi_j(t)$ is zero outside the interval $0 \leq t \leq T$.

Maximization of output signal to noise Ratio. [matched filter]



Consider a linear filter with impulse Response $h(t)$

Known input signal $\phi(t)$

Additive noise component $w(t)$, Zero mean and PSD of $N_0/2$

then $x(t) = \phi(t) + w(t) \quad 0 \le 0 \le T$ ——— (6)

$$y(t) = \underbrace{\phi_0(t)}_{\text{Signal}} + \underbrace{n(t)}_{\text{Noise}} \qquad (7)$$

$\phi_0(t) > n(t)$ is to have the filter make the instantaneous power in the output signal $\phi_0(t)$, measure at $t=T$, as large as possible compared with the average power of o/p noise $n(t)$.

The equivalent term is defined to maximize the output signal to noise ratio as

$$(SNR_0) = \frac{|\phi_0(T)|^2}{E[n^2(t)]} \qquad (8)$$

Let $\Phi(f)$ is FT of known sinal $\phi(t)$

$H(f)$ is the transfer function of the filter

FT of o/p signal $\rightarrow \phi_0(t)$ equal to $H(f)\Phi(f)$. and $\phi_0(t)$ is itself given by the inverse FT

$$\phi_o(t) = \int_{-\infty}^{\infty} H(f)\, \phi(f)\, \exp(j2\pi f t)\, df$$

$t = T$ then, we may write

$$|\phi_o(t)|^2 = \left| \int_{-\infty}^{\infty} H(f)\, \phi(f)\, \exp(j2\pi f T)\, df \right|^2 \quad \text{——— (9)}$$

Effect of Noise

The noise signal $w(t)$ is present on the filter output. Then

$$\left.\begin{matrix}\text{PSD of} \\ \text{o/p noise } n(t)\end{matrix}\right\} = S_N(f) = \frac{N_0}{2} |H(f)|^2 \quad \text{——— (10)}$$

The average power of output noise is therefore

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)\, df \quad \text{——— (11)}$$

Sub (10) in (11)

$$E[n^2(t)] = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df \quad \text{——— (12)}$$

Sub equation (9) and (12) in equation (8)

$$(SNR_0) = \frac{\left| \int_{-\infty}^{\infty} H(f)\, \phi(f)\, \exp(j2\pi f T)\, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df} \quad \text{——— (13)}$$

Goal: Keeping $\phi(f)$ of the input signal fixed and using $H(f)$ we have to make $(SNR)_0$ is maximum. So we have to optimize.

To optimize, we take Schwarz's inequality to the numerator of equation (13), we may write

$$\left| \int_{-\infty}^{\infty} H(f)\, \phi(f)\, \exp(j2\pi f T)\, df \right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |\phi(f)|^2\, df \quad \text{——— (14)}$$

using the relation in equation (14), we may simplify the o/p signal to noise ratio as

$$(SNR)_0 \leq \frac{2}{N_0}\int_{-\infty}^{\infty} |\phi(f)|^2 df \longrightarrow (15)$$

The RHS of equation (15) is uniquely defined by

1. The signal energy given by

$$\int_{-\infty}^{\infty} |\phi(t)|^2 dt = \int_{-\infty}^{\infty} |\phi(f)|^2 dt$$

2. The noise power spectral density $N_0/2$

The RHS of equation (15) does not depend on $H(f)$. consequently, the output signal to noise ratio will be maximum when $H(f)$ is choosen so that the equality holds: that is

$$(SNR)_{0,max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |\phi(f)|^2 df \text{ ——— } (16)$$

For this condition, $H(f)$ assumes its optimum value denotes as $H_{opt}(f)$, and from schwarz inequality, except scaling factor,

$$\boxed{H_{opt}(f) = \phi^*(f)\exp(-j2\pi fT)} \text{ ——— } (17)$$

Conclusion: $\phi^*(f)$ is complex conjugate of the FT of the I/p signal $\phi(t)$. without time delay factor $\exp(-j2\pi fT)$

$$\boxed{H_{opt}(f) = \phi^*(f)}$$

equation (17) shows the matched filter in frequency domain

Time Domain representation

Take inverse FT of $H_{opt}(f)$, to obtain the impulse response of the matched filter as

$$h_{opt}(t) = \int_{-\infty}^{\infty} \phi^*(f) \exp[-j2\pi f(T-t)]\,df$$

$\phi^*(f) = \phi(-f)$ for Real valued signal.

$$h_{opt}(t) = \int_{-\infty}^{\infty} \phi(-f) \exp[-j2\pi f(T-t)]\,df$$

$$\boxed{h_{opt}(t) = \phi(T-t).} \quad (18)$$

equation (18) shows the impulse Response of the optimum filter is a time reversed and delayed version of the input signal $\phi(t)$: ie it matched to input signal.

From this we conclude, to maximize SNR is equivalent to minimization of the average probability of symbol error under two assumptions.

1. The additive white Gaussian noise at the receiver input is stationary with Gaussian statistics
2. The a periori probabilities of the transmitted signal are known.

Advantages :

1. This filter enhance the SNR by decreasing symbol errors.
2. This filter has optimal performance and its computational cost is low.

Disadvantages :

Properties of matched filter -

1. The spectrum of the output signal of a matched filter with the matched signal as input is except for a time delay factor, proportional to the energy spectral density of the input signal.

$$\phi_o(f) = H_{opt}(f)\,\phi(f)$$
$$= \phi^*(f)\,\phi(f)\exp(-j2\pi fT)$$
$$= |\phi(f)|^2 \exp(-j2\pi fT)$$

2. The output signal of a matched filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.

$$\phi_o(t) = R_\phi(t-T).$$
$$\phi_o(T) = R_\phi(0) = E$$

3. The output signal to noise ratio of a matched filter depends only on the ratio of the signal energy to the power spectral density of the white noise at the filter input.

4. Matched filter operation may be separated in two matching conditions

(1) Spectral phase matching. $\phi_o(t) \le \phi_o'(T) = \int_{-\infty}^{\infty} |\phi(f)||H(f)|\,df$

(2) Spectral Amplitude matching. $|H(f)| = |\phi(f)|$

Equalization Techniques

Why do we need equalization in baseband pulse transmission?
When the signal is passed through the channel, distortion is introduced in terms of i) Amplitude (ii) Delay. This distortion creates a problems of ISI. The detection of the signal also becomes difficult. This distortion can be compensated with the help of equalizers. Equalizer are basically filters which correct the channel distortion.

Types

1) Zero forcing equalizer
2) Adaptive equalizer.

Adapative Equalization

What is the necessity of Adapative equalition? Explain modes of operation and LMS algorithm.

Necessity: Most of the channels are made up of individual links and the distortion induced depends upon

1) Transmission characteristics of individual links
2) No of links in ~~the~~ connection.

So the transmission characteristics keep on changing Hence adapative equalization is required.

Basic principle:

The filters adapt themselves to the dispersive effect of the channel ie the filter coefficients are changed continuously according to the recived data and distortion in the data is reduced.

Types: prechannel equalization: used in Transmitting side it uses feed back to know about the amount distortion in the received data:

postchannel equalizer: It is used after the receiving filter, no feed back is required.

Structure of Adapative equalizer:

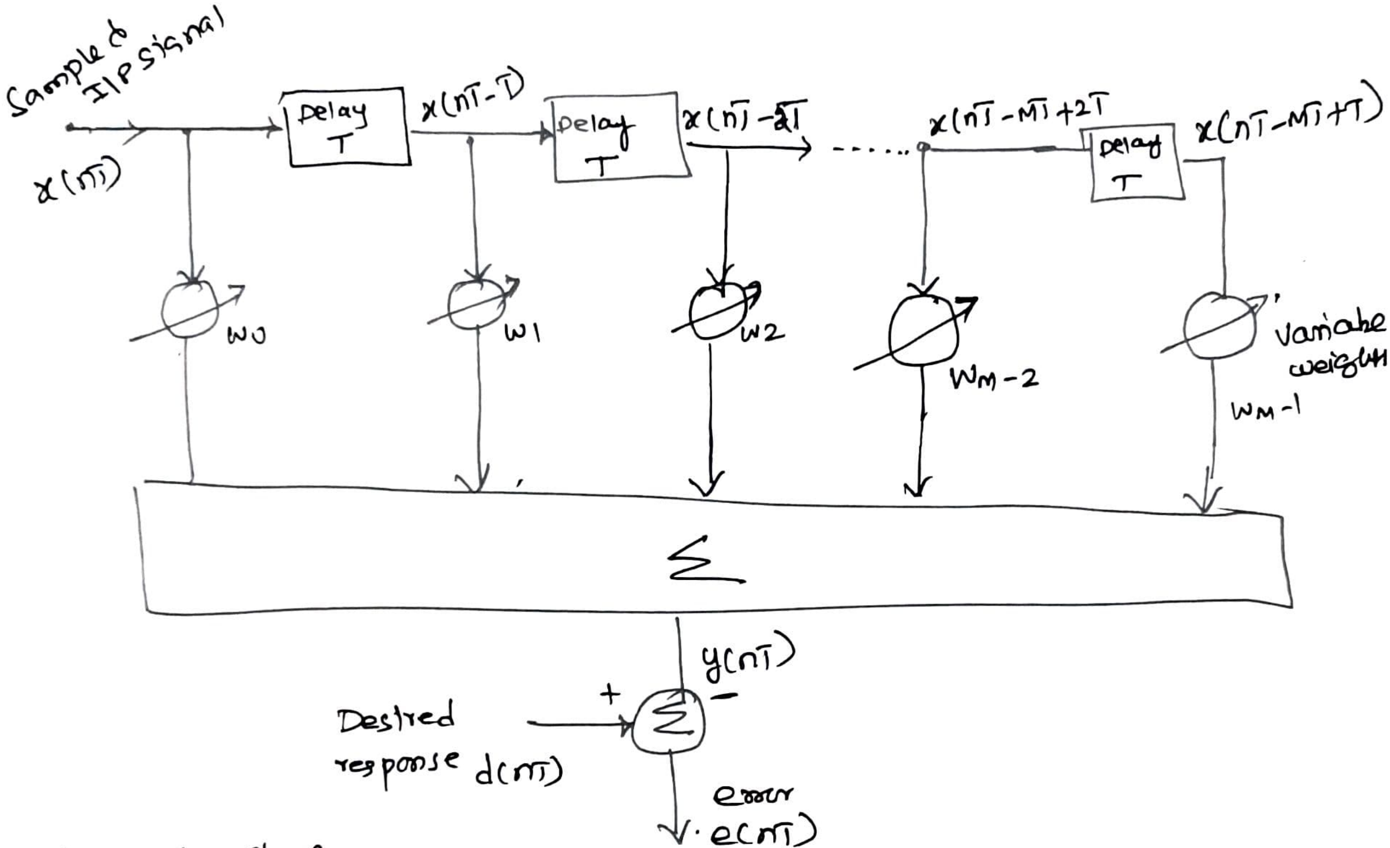


Figure above shown ~~It consist of~~ tapped delay line filter. It consist of set of delay elements and variable multiplies.

$x(nT)$ – input

$y(nT)$ is O/P.

$$y(nT) = \sum_{i=0}^{M} w_i \, x(nT - iT)$$

$w_i \rightarrow$ adaptive filter coefficients.

Let $\{d(nT)\}$ is known sequence transmitted first, and also known to receiver.

The response sequence $y(nT)$ is observed. and the error sequence between the two seque is calculated.

$$e(nT) = d(nT) - y(nT) \quad , n = 0, 1, \ldots N-1.$$

From equation, we know that there is no distortion in the channel.

example $n = 0$

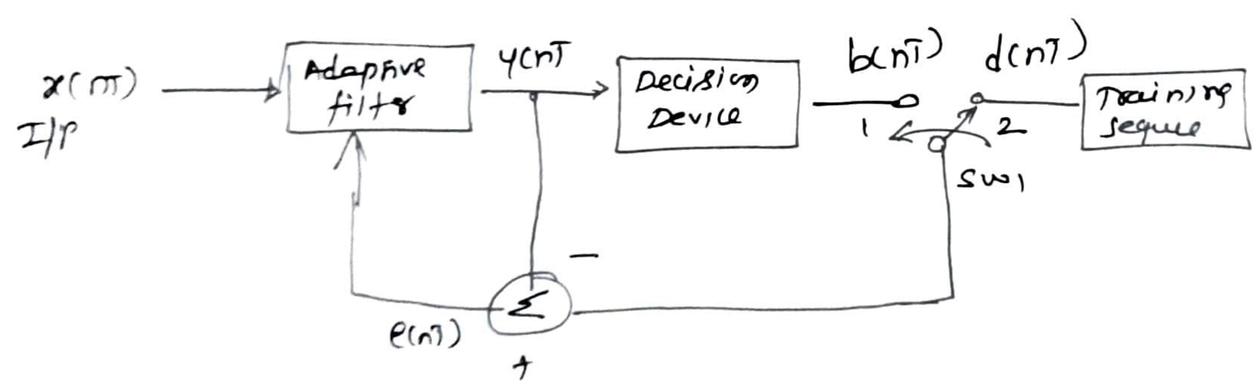
$$e(0T) = d(0T) - y(0T)$$

$$e(0T) = 0.$$

ie $d(nT)$ and $y(nT)$ will be same producing zero error sequence. Then the weights of filter w_i are changed recursivey such that error $e(nT)$ is minized. There are standard algrothims to change weights of the filter recursively.

operating modes

1) Training mode
2) Decision directed mode.



Training mode

When switch is in position 2, training sequence applied to equalizer [maximal length pseudo Noise sequence (PN). It generates the synchronized version in the receiver. The training mode sets the coefficient of adaptive equalizer.

2) Decision dereeted mode:

When switch in '2', it sets decision mode.

$$e(nT) = b(nT) - y(nT)$$

$b(nT)$ is the correct estimate of transmitted symbol.

$y(nT)$ is the output of adaptive equalizer.

Adepative filter coefficients are adjusted continuously such that error signal $e(nT)$ is minized.

Least means square [LMS] Algrothim.

Algrothim change the tap weights of the adaptive filtr recursively.

$$\hat{w}_i(nT+T) = \hat{w}(nT) + \mu e(nT)\, x(nT - iT).$$

$$i = 0, 1, \ldots M-1$$

$\hat{w}_i(nT)$ is the present estimate for tap 'i' at time nT

$\hat{w}_i(nT+T)$ is the updated estimate for tap 'i' at time nT

μ is the adaption constant

$x(nT - iT)$ is the filter input

$e(nT)$ is the error signal.

The parameter μ controls the amount of correction applied to the old estimate to produce updated estimate.

Ans. : In baseband binary PAM, symbols are transmitted one after another. These symbols are separated by sufficient time durations. The transmitter, channel and receiver acts as a filter to this baseband data. Because of the filtering characteristics, transmitted PAM pulses are spread in time. Let the transmitted waveform be represented as,

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \, g(t-kT_b)$$

Here A_k is the amplitude of k^{th} pulse.

And g(t) is shaping pulse.

The output pulse at $t = i\,T_b$ can be expressed as,

$$y(t_i) = \mu A_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k \, p[(i-k)T_b]$$

Here T_b is the bit duration and 't_i' indicates instant of i^{th} pulse. μA_i is the contribution of i^{th} transmitted bit. The second term in above equation occurs due to filtering nature of the transmitter receiver and channel. The second term represents the residual effect (time spread) of all other bits transmitted before and after t_i. This presence of outputs (second term) due to other bits (symbols) interfere with the output of required bit (symbol). This effect is called Intersymbol Interference (ISI).

Q.10 What are eye pattens ? **AU : Madras Univ., Nov.-97**

OR

Draw an eye pattern and represent the ways in which it could be used to evaluate the performance a baseband pulse transmission system. **AU : May-08**

Ans. : Eye pattern is used to study the effect of ISI in baseband transmission. Fig. 3.5.2 shows an interpretation of eye pattern.

i) Width of eye opening defines the interval overwhich the received wave can be sampled without error from ISI.

ii) The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied.

iii) Height of the eye opening at sampling time is called margin over noise.

Q.11 How is eye pattern obtained on the CRO ? **AU : May-04, 09**

Ans. : Eye pattern can be obtained on CRO by applying the signal to one of the input channels and giving an external trigger of $\frac{1}{T_b}$ Hz. This makes one sweep of beam equal to 'T_b' seconds.

Q.12 What is the condition for zero inter symbol interference ? AU : May-04, Dec.-07

Ans. : Zero ISI can be obtained if the transmitted pulse satisfies the following condition :

Time domain :
$$p[(i-k)T_b] = \begin{cases} 1 & for \quad i = k \\ 0 & for \quad i \neq k \end{cases}$$

Frequency domain
$$= \sum_{n=-\infty}^{\infty} p(f - nf_b) = T_b$$

Q.13 A TDM signal with bit time of 0.5 μs is to be transmitted using a channel with raised cosine roll off factor of 0.5. What is the bandwidth required ? AU : Dec.-04

Ans. : $T_b = 0.5\ \mu s, \quad \alpha = 0.5$

$$B_0 = \frac{f_b}{2} = \frac{1}{2T_b} = \frac{1}{2 \times 0.5 \times 10^{-6}} = 1 \times 10^6$$

$$\therefore \quad B = B_0(1+\alpha) = 1 \times 10^6 (1 + 0.5) = 1.5 \times 10^6$$

Q.14 From the eye pattern, how is the best time for sampling determined AU : Dec.-04

Ans. : It is preferable to sample the instant at which eye is open widest. At this instant, the chances of error are minimum.

Q.15 What is the purpose of using an eye pattern ? AU : Dec.-05, 12, 13, May-12

Ans. : Eye pattern can be used for :

i) To determine an interval over which the received wave can be sampled without error due to ISI.

ii) To determine the sensitivity of the system to timing error.

iii) The margin over the noise is determined from eye pattern.

Q.16 Why do you need adaptive equalization in a switched telephone n AU : Dec.-05, 14

Ans. : In switched telephone network the distortion depends upon

i) Transmission characteristics of individual links.

ii) Number of links in connection.

Hence fixed pair of transmit and receive filters will not serve the equalization problem. The transmission characteristics keep on changing. Therefore adaptive equalization is used.

Q.17 What is an ideal Nyquist channel ? AU : May-06, Dec.-09

Ans. : The ideal Nyquist channel uses sinc pulse for transmission . i.e.,

$$p(t) = \frac{\sin(2\pi B_0 t)}{2\pi B_0 t}$$

Such pulse have the spectrum of,

$$p(f) = \begin{cases} \frac{1}{2B_0} & \text{for } -B_0 < f \le B_0 \\ 0 & \text{elsewhere} \end{cases}$$

Q.18 What is the use of "eye pattern" ? **AU : Dec.-06**

Ans. : Refer section 3.5.

Q.19 Bring out the difference between carrier recovery and clock recovery. **AU : Dec.-06**

Ans. :

Sr. No.	Carrier recovery	Clock recovery
1.	Carrier is required for coherent detection at the receiver.	Clock is required to estimate correct bit timing at the receiver.
2.	M^{th} power loop, costas loop are used for carrier recovery.	Closed loop bit synchronizer, early-late synchronizer are used for clock recovery.

Q.20 Why do we need equalization in base band pulse transmission ? **AU : May-07, Dec.-08**

Ans. : When the signal is passed through the channel, distortion is introduced in terms of i) Amplitude and ii) Delay. This distortion creates the problems of ISI. The detection of the signal also becomes difficult. This distortion can be compensated with the help of equalizers. Equalizers are basically filters which correct the channel distortion.

Q.21 Give the Nyquist criterion for zero ISI. Plot the impulse response of an ideal Nyquist channel. **AU : May-08, Dec.-11**

Ans. : Refer answer of Q.12.

Impulse response is given as,

$$p(t) = \frac{\sin(2pB_0 t)}{2pB_0 t}$$

Fig. 3.8.5 shows the plot of above equation. (See Fig. 3.8.5 on next page)

Q.22 How does pulse shaping reduce inter symbol interference ? **AU : Dec.-10**

Ans. : 1) The shape of the pulse is selected such that at the instant of detection, the interference due to all other symbols is zero. Fig. 3.3.2 illustrates such situation.

2) The effect of ISI is totally eliminates if signal is sampled at $T_b, 2T_b, 3T_b, \ldots$ and so on.

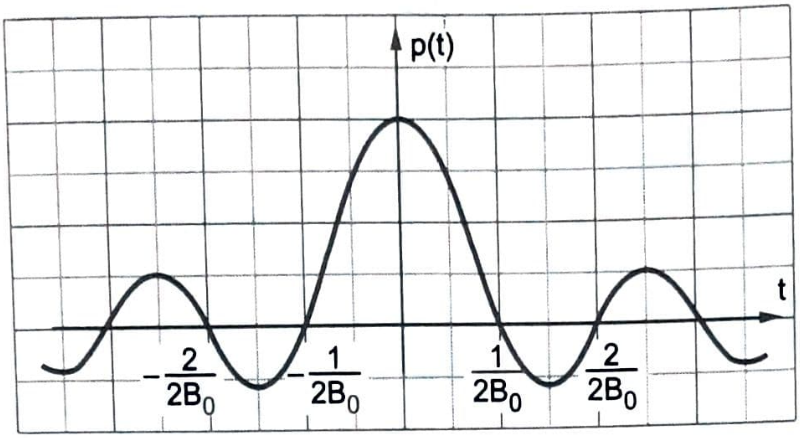


Fig. 3.8.5 Impulse response

Q.23 ISI cannot be avoided. Justify the statement. AU : May-13

Ans. : When the pulse $p(t)$ is transmitted across the channel, the output y(t) is given as,

$$y(t_i) = \underbrace{\mu A_i}_{i^{th}\ bit} + \underbrace{\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k p[(i-k)T_b]}_{\text{Residual effect}}$$

Here $\quad i = 0, \pm 1, \pm 2, \pm 3$

Here note that at any i^{th} instant, the residual effect of all other bits transmitted before and after the sampling instant t_i is present. This residual effect forms ISI and it cannot be totally avoided.

❑❑❑

UNIT 4 -1-

Digital modulation Techniques

Digital modulation provides more information capacity, high data security, quicker system availability with great quality communication. Hence, digital modulation Techniques have a grater demand, for their capacity to convey larger amounts of data than analog one

Modulation schemes are

1) ASK - Amplitude shift keying
2) FSK - Frequency shift keying
3) PSK - Phase shift keying
4) QAM = Quadrature amplitude modulation.

$$V(t) = V \sin(2\pi \cdot ft + \theta) \quad \text{--- (1)}$$

V → ASK

f → FSK

θ → PSK

ASK, PSK → QAM

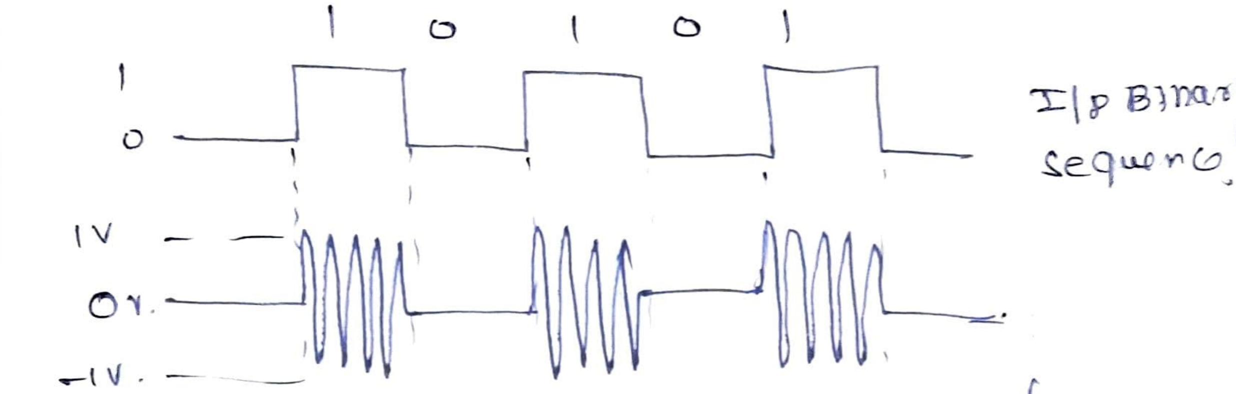
From equation (1) Amplitude of the carrier is varied proportional to the modulating signal is called ASK.

If frequency of the carrier is varied proportional to the modulating signal is called FSK.

If phase of the carrier is varied proportional to the information signal is called PSK

If both Amplitude and phase is varied proportional to the information signal is called QAM.

1) ASK : Amplitude Shift Keying.



ASK modulated o/p wave form

The input Binary signal is modulated, it gives zero value for Low input (ie '0') and ~~High~~ it gives the carrier output for High input (ie 1).

Mathamatically we can Represented by [general expression]

$$V_{(ASK)}(t) = [1 + V_m(t)] \left[\frac{A}{2} \cos(\omega_c t)\right]$$

$V_{(ASK)}(t)$ → Amp shift Keying wave.

$V_m(t)$ → Input signal

$\frac{A}{2}$ → Unmodulated Carrier Amplitude

$\omega_c t$ → Analog Carrier frequency.

Now $V_m(t) = 1$

$$V_{(ASK)}(t) = [1+1] \left[\frac{A}{2} \cos(\omega_c t)\right]$$

$$= \left[\frac{2}{2} A \cos(\omega_c t)\right]$$

$$V_{(ASK)}t = A \cos(\omega_c t)$$ ie It Contain only Carrier with amplitude 'A'

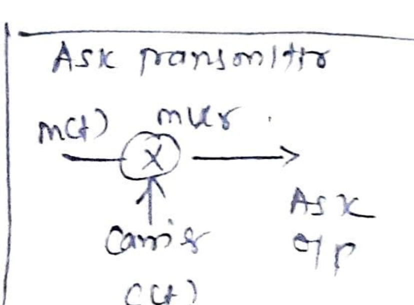
if $V_m(t) = 0$

$$V_{(ASK)}(t) = 0 \left[\frac{A}{2} \cos(\omega_c t)\right]$$

$$= 0.$$

ie o/p contain Zero value.

It also called as on-off Keying.



UNIT - 4 - DIGITAL MODULATION SCHEME

Geometric representation of signals. Generation, detection, PSD & BER of Coherent BPSK, BFSK, QPSK - QAM - Carrier Synchronization - structure of Non- Coherent receivers - Principle of DPSK.

Digital modulation:

→ The binary input is baseband modulated to get the waveform.

→ This baseband signal is used to modulate the txd carrier. Similar to analog modulation, the amplitude, frequency or phase of the carrier signal is varied as per the binary input.

(·) Signal space analysis:

Consider an msg source with M symbols $\{m_1, m_2, \dots m_M\}$ which emits one symbol for every T seconds.

The probability of occurrence $P_1, P_2, \dots P_M$ are known and assumed to be equiprobable.

$$P_i = \frac{1}{M} \quad \text{for } i = 1, 2, \dots M.$$

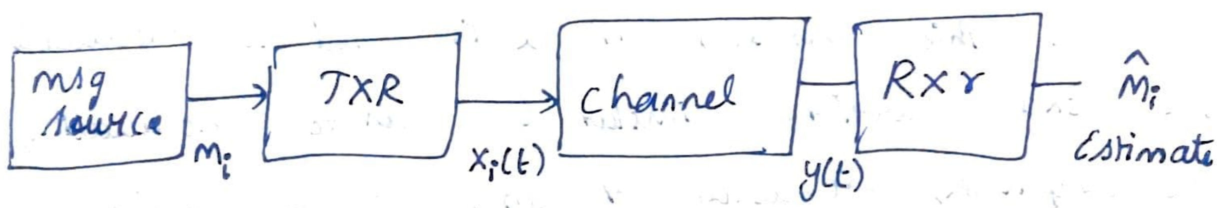
The txr codes each signal into an distinct signal $x_i(t)$.

$$\boxed{\text{Energy } (E_i) = \int_0^T x_i^{(2)}(t)\, dt}\ , \quad i = 1, 2, \dots M$$

The Channel is assumed to be

(i) Linear : The bandwidth of the channel is large enough to accomodate the txd signal $x_i(t)$ without any distortion.

(ii) AWGN noise : The Channel is said to have noise $n(t)$ as additive white gaussian Noise with zero mean $(\mu = 0)$ and variance $\left(\frac{N_0}{2}\right)$.



The rxd signal

$$y(t) = x_i(t) + n(t), \quad \begin{cases} 0 \le t \le T \\ i = 1, 2, 3, \dots M \end{cases}$$

Geometric representation of signals

→ A set of M energy signals can be represented as a linear combination of N orthonormal basis functions.

→ The no of orthonormal basis functions are always less than or equal to no of possible energy signals.

i.e $N \le M$.

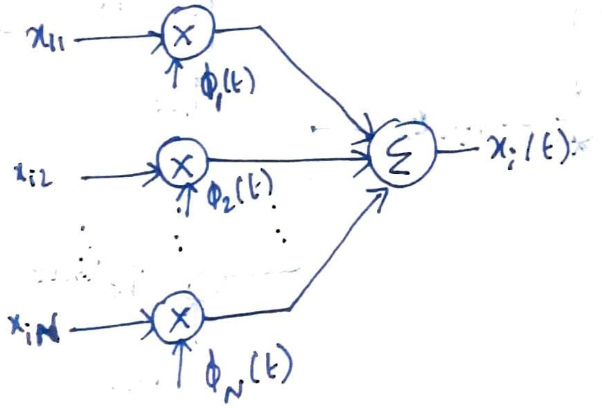
Consider a set of real valued signals $x_1(t), x_2(t), \ldots x_M(t)$ each of duration 'T' sec

Then the i^{th} signal can be represented as

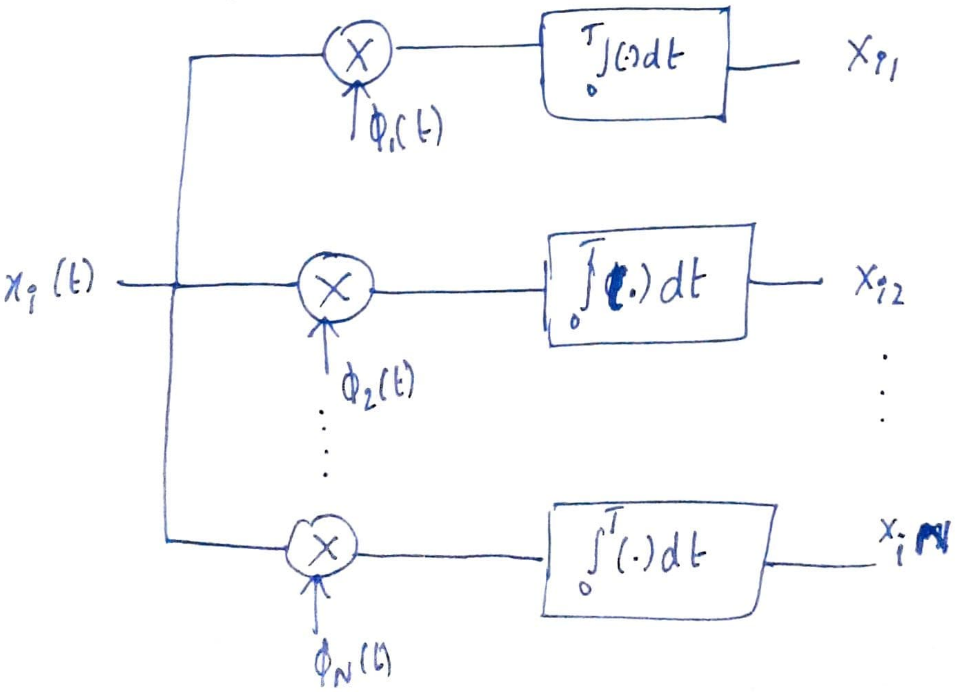
$$x_i(t) = \sum_{j=1}^{N} x_{ij}\, \phi_j(t) \qquad i = 1, 2, \ldots M$$

$$\text{where } x_{ij} = \int_0^T x_i(t)\, \phi_j(t)\, dt \qquad \begin{array}{l} i = 1, 2, \ldots M \\ j = 1, 2, \ldots M \end{array}$$

Based on these eqns the synthesizer & analyzer can be constructed.

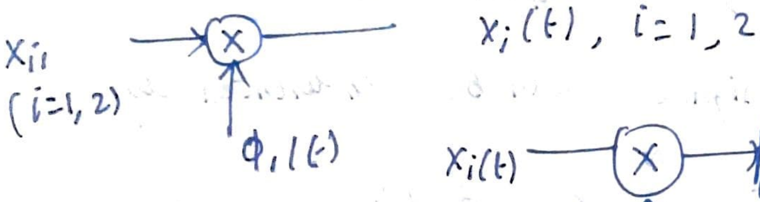


Synthesizer

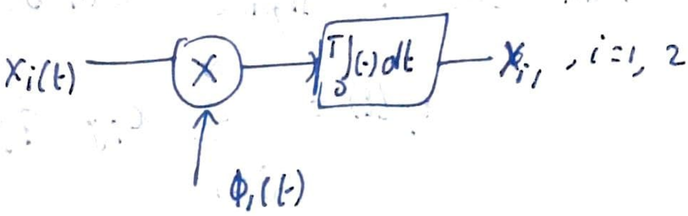


Analyzer

For BPSK modulation, $M = 2$, $N = 1$

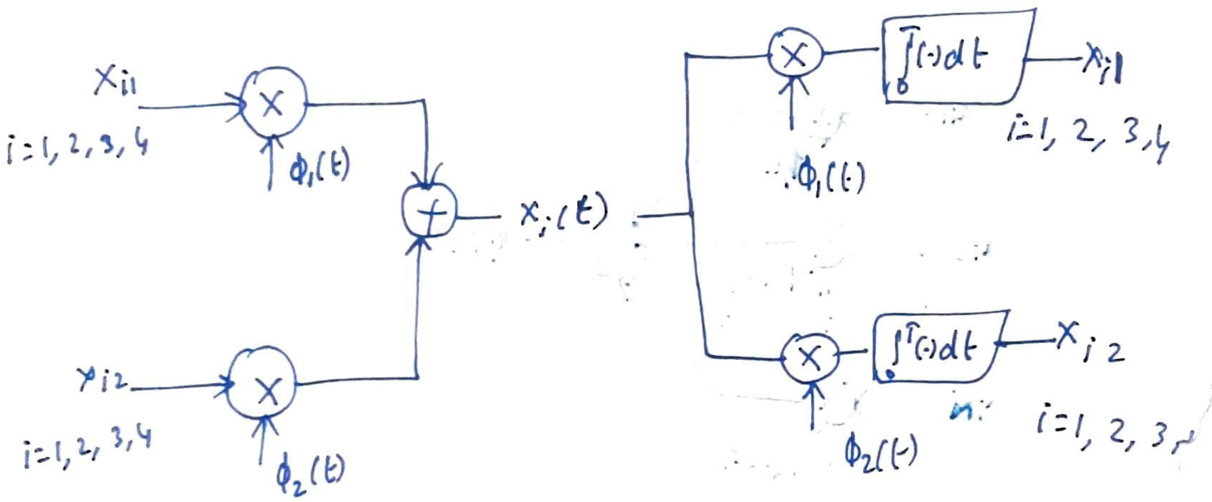


TXR



RXR

For QPSK, $N = 2$, $M = 4$



TXR RXR

GRAM - SCHMIDT ORTHOGONALIZATION PROCEDURE

Consider a set of M energy signals
$x_1(t), x_2(t), \ldots x_M(t)$.

The first basis function

$$\phi_1(t) = \frac{x_1(t)}{\sqrt{E_1}} \longrightarrow (1)$$

$E_1 \rightarrow$ energy of the signal $x_1(t)$

so, $$x_1(t) = \sqrt{E_1}\, \phi_1(t) \longrightarrow (2)$$

W.K.T $$x_i(t) = \sum_{j=1}^{N} x_{ij}\, \phi_j(t)$$

Consider $N = 1$,

$$x_1(t) = x_{11}\, \phi_1(t) \longrightarrow (3)$$

Comparing (2) & (3),

$x_{11} = \sqrt{E_1}$ is the co-efficient

To define other basis fn, an new intermediate function is introduced.

$$g_i(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij}\, \phi_j(t) \longrightarrow (4)$$

where $$x_{ij} = \int_0^T x_i(t)\, \phi_j(t)\, dt \quad , j = 1, 2, \ldots i-1$$

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\, dt}} \quad i = 2 \ldots N$$

Second basis function,

$$g_2(t) = x_2(t) - \sum_{j=1}^{1} x_{2j}\, \phi_j(t)$$

$$= x_2(t) - x_{21}\, \phi_1(t)$$

So the basis fn; $\phi_2(t) = \dfrac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}}$

$$\int_0^T g_2^2(t)\,dt = \int_0^T [x_2(t) - x_{21}\phi_1(t)]^2\,dt$$

$$= \int_0^T x_2^2(t)\,dt + \int_0^T x_{21}^2\,\phi_1^2(t)\,dt$$

$$- 2\int_0^T x_2(t)\, x_{21}\, \phi_1(t)\,dt$$

$$= E_2 + x_{21}^2 \int_0^T \phi_1^2(t)\,dt - 2x_{21}\int_0^T x_2(t)\,\phi_1(t)\,dt$$

$$= E_2 + x_{21}^2 - 2x_{21}^2$$

(since $x_{ij} = \int_0^T x_i(t)\, \phi_j(t)\, dt$)

So $\int_0^T g_2^2(t)\,dt = E_2 - x_{21}^2$

Now $\phi_2(t) = \dfrac{g_2(t)}{\sqrt{E_2 - x_{21}^2}}$

$$= \frac{x_2(t) - x_{21}\,\phi_1(t)}{\sqrt{E_2 - x_{21}^2}}$$

$$\phi_2^2(t) = \frac{g_2^2(t)}{\int_0^T g_2^2(t)\,dt}$$

$$\int_0^T \phi_2^2(t)\,dt = \frac{\int_0^T g_2^2(t)\,dt}{\int_0^T g_2^2(t)\,dt} = 1$$

So $\phi_1(t)$ & $\phi_2(t)$ are orthogonal.

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = \int_0^T \frac{x_2(t) - x_{21}\,\phi_1(t)}{\sqrt{E_2 - x_{21}^2}}\,\phi_1(t)\,dt$$

$$= \frac{1}{\sqrt{E_2 - x_{21}^2}} \int_0^T x_2(t)\,\phi_1(t) - \int_0^T x_{21}\,\phi_1^2(t)\,dt$$

$$= \frac{1}{\sqrt{E_2 - x_{21}^2}} \left[x_{21} - x_{21}(1) \right]$$

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0$$

Hence $\phi_1(t)$ & $\phi_2(t)$ are orthogonal.

DIGITAL MODULATION - Types

Coherent : In this modulation, the local carrier generated at rxr is phase locked with the txr. This carrier phase information at the ~~coherent~~ rxr is used by matched filters to detect the data.

Non-coherent : The demodulators operate without knowledge of the phase information of the txr.

1. Binary shift Keying (BSK)
   (a) BPSK
   (b) BASK or OOK
   (c) BFSK
2. Quadrature phase shift Keying (QPSK)
3. QAM (Quadrature Amp. mod)
4. DPSK (Differential ~~pulse~~ phase shift Keying)

(5)

(·) COHERENT BINARY PHASE SHIFT KEYING

→ In the BPSK modulation the input digital data 0 or 1 is directly converted to the phase of 0 & π respectively.

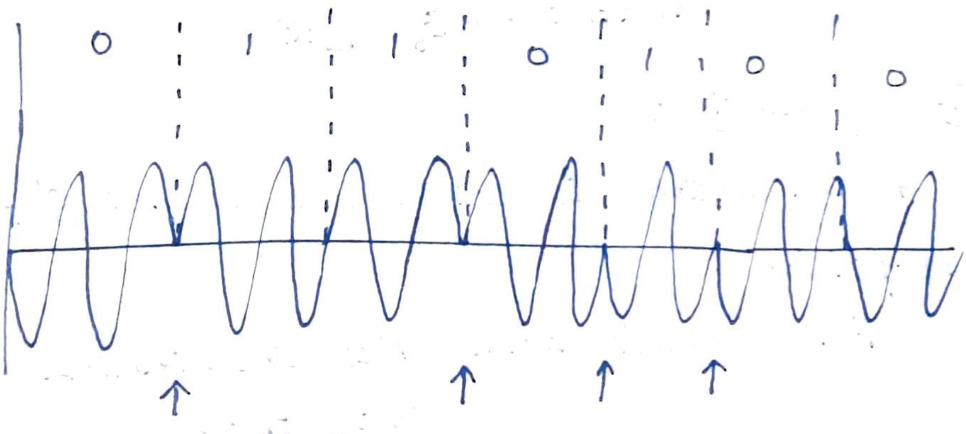
→ The pair of signals $x_1(t)$ and $x_2(t)$ are used to represent binary symbols 1 and 0. They are defined by

$$x_1(t) = A \cos 2\pi f_c t \qquad 0 \le t \le T_b \text{ for '1'}$$

$$x_2(t) = A \cos(2\pi f_c t + \pi) \qquad 0 \le t \le T_b$$

$$= -A \cos 2\pi f_c t \qquad \text{for '0'}$$

where A is the max. amplitude of the txd signal
f_c is the carrier frequency.



BPSK waveform

Generally the amplitudes are represented as energy signals.

$$\text{Energy per bit } (E_b) = \int_0^{T_b} x_1^2(t)\, dt$$

$$= A^2 \int_0^{T_b} \cos^2 2\pi f_c t\, dt$$

$$= \frac{A^2}{2} \int_0^{T_b} (1 + \cos 4\pi f_c t)\, dt$$

$\because \cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$E_b = \frac{A^2 T_b}{2} \Rightarrow A = \sqrt{\frac{2E_b}{T_b}}$$

$T_b \rightarrow$ bit duration.

Now $$x_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$$x_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

It is clear that BPSK uses only one basis fn:

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

Representing the BPSK signals in terms of basic fn gives,

$$x_1(t) = \sqrt{E_b}\, \phi_1(t)$$

$$x_2(t) = -\sqrt{E_b}\, \phi_2(t)$$

BPSK Constellation:

→ Constellation is nothing but the set of possible msg points.

For Coherent BPSK, $N=1$ (one dimensional) and $M=2$ (two msg points).

→ To identify the possible msg points, the following operation is performed.

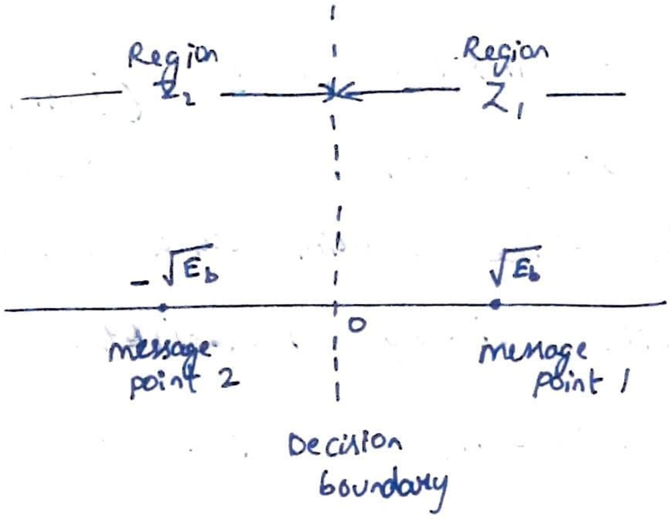
$$x_{11} = \int_0^{T_b} x_1(t)\, \phi_1(t)\, dt$$

$$= \sqrt{E_b} \int_0^{T_b} \phi_1^2(t)\, dt$$

$$= \sqrt{E_b}$$

$$x_{21} = \int_0^{T_b} x_2(t)\, \phi_1(t)\, dt = -\sqrt{E_b} \int_0^{T_b} \phi_1^2(t)\, dt$$

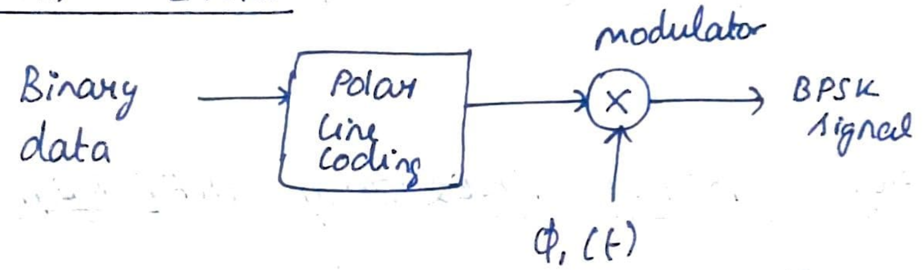
$$= -\sqrt{E_b}.$$



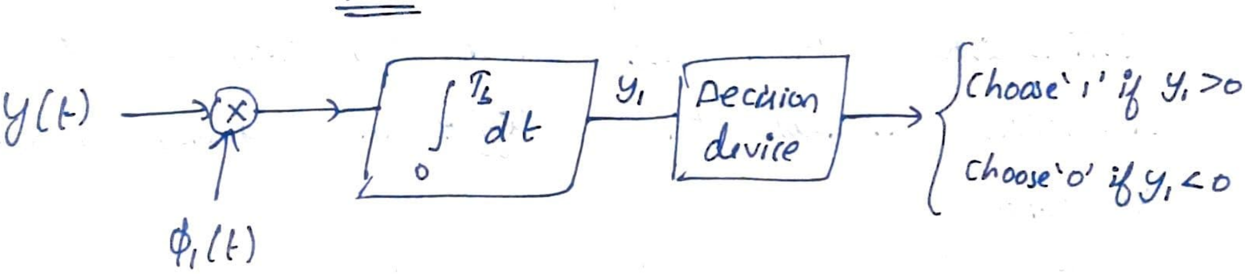
Signal space diagram

Since there are only 2 msg points, the decision boundary is the midpoint of the line joining 2 msg points.

BPSK TXR & RXR



TXR



→ Convert the binary data to polar signal

Bit '1' → $\sqrt{Eb}$; Bit '0' → $-\sqrt{E_b}$

→ Multiply the polar signal with basis function to get BPSK signal as

$x_1(t) = \sqrt{E_b}\ \phi_1(t)$; $x_2(t) = -\sqrt{E_b}\ \phi_2(t)$

→ The rxd signal is passed through the correlator to obtain the observation.

$$y_1 = \int_0^{T_b} y(t) \cdot \phi_1(t)\, dt$$

The observation is compared with threshold to decide rxd bits

Probability of Bit Error analysis

①. BPSK

For BER analysis, Consider AWGN Channel. i.e Zero mean & $\sigma^2 = N_0/2$

The rxd sgl is given by

$$y(t) = \begin{cases} x_1(t) + n(t) & \text{for symbol '1'} \\ x_2(t) + n(t) & \text{for symbol '0'.} \end{cases} \quad \text{--- ①}$$

Observation Scalar → (rxd signal × basis fn) sent into Integrator

$$y_1(t) = \int_0^{T_b} y(t) \cdot \phi_1(t)\, dt \quad \text{--- ②}$$

$$y_1 = \begin{cases} \sqrt{E_b} + n & \text{for '1'} \\ -\sqrt{E_b} + n & \text{for '0'} \end{cases} \quad \text{--- ③}$$

Mean of observation vector,

$$E(y_1) = \begin{bmatrix} \sqrt{E_b} & \text{for '1'} \\ -\sqrt{E_b} & \text{for '0'} \end{bmatrix} \quad \text{--- ④}$$

since $E[n] = 0$.

Variance of observation vector

$$E\left[\{y_1 - E(y_1)\}^2\right] = \sigma^2 = \frac{N_0}{2} \quad \text{--- ⑤}$$

for both symbols '0' & '1'.

The Probability density function (pdf) of Gaussian random variable is given by

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad \text{--- (6)}$$

The likelihood function when symbol '0' is txd is

$$f_{y_1}(y_1/0) = \frac{1}{\sqrt{2\pi \frac{N_0}{2}}} \exp\left[\frac{-(y_1+\sqrt{E_b})^2}{2\frac{N_0}{2}}\right] \quad \text{--- (7)}$$

$\mu = -\sqrt{E_b}$
$x = y_1$

Substituting $\sigma^2 = \frac{N_0}{2}$, $E(y_1) = -\sqrt{E_b}$

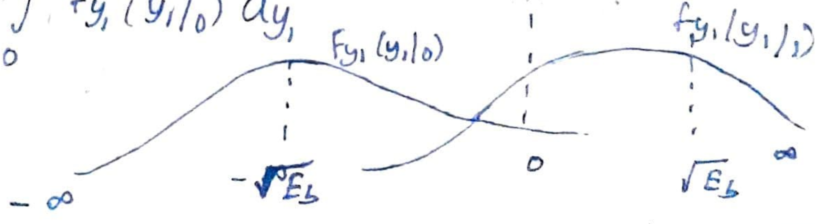
$$f_{y_1}(y_1/0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(y_1+\sqrt{E_b})^2}{N_0}\right] \quad \text{--- (8)}$$

Similarly likelihood for symbol '1' is

$$f_{y_1}(y_1/1) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(y_1-\sqrt{E_b})^2}{N_0}\right] \quad \text{--- (9)}$$

The first type of error occurs when the observation value falls between $0 < y_1 < \infty$ for symbol '0'.

$$P_e(0) = \int_0^\infty f_{y_1}(y_1/0)\, dy_1 \quad \text{--- (10)}$$



Sub ④ in ⑩

$$\therefore P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(y_1+\sqrt{E_b})^2\right] dy_1 \quad \text{---(11)}$$

Consider $\frac{1}{\sqrt{N_0}}(y_1+\sqrt{E_b}) = z$,

then $dz = \frac{1}{\sqrt{N_0}} dy_1$

$\Rightarrow \therefore dy_1 = \sqrt{N_0} \cdot dz.$

$y_1 = 0,$
$\Rightarrow z = \sqrt{\frac{E_b}{N_0}}$
$y_1 \to \infty,\ z \Rightarrow \infty$

Sub z value and dy_1 value in equation (11) and change the limit value.

Now $$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} \exp\left[-z^2\right](\sqrt{N_0}\, dz)$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} \exp(-z^2)\, dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\, dz \quad \text{---(12)}$$

The probability of error fn is represented in terms of complimentary error function

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-z^2)\, dz \quad \text{---(13)}$$

Rewrite the equation (12), we get $P_e(0)$

$x = \sqrt{\frac{E_b}{N_0}}$

So $$\boxed{P_e(0) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

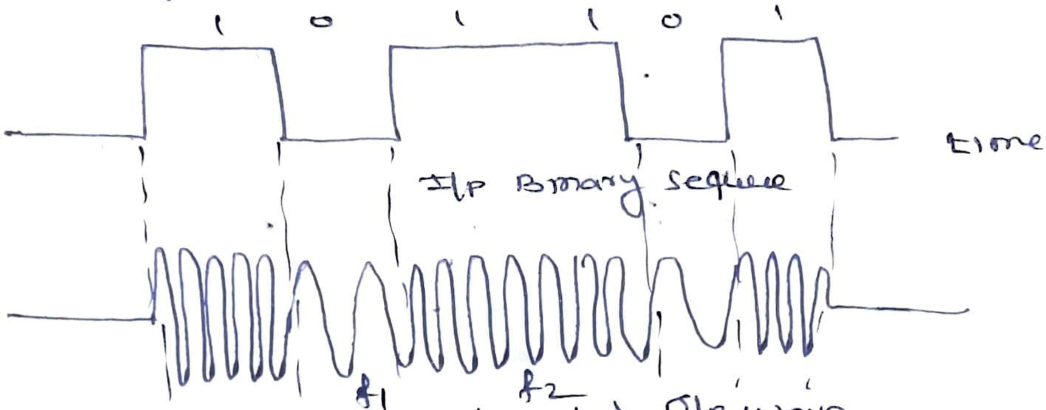
Illy

$$P_e(1) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

So $$P_e = \frac{P_e(0)+P_e(1)}{2} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Binary FSK. [BFSK]

FSK is the digital modulation technique in which the frequency of the carrier signal varies according to the modulating input ie input binary sequence.



FSK modulated O/p wave

The output of a FSK modulated wave is in high frequency for a binary High input and is Low in frequency for a binary Low input. The binary 1's and 0's are called mark and space frequencies.

The general expression for FSK is

$$V_{fsk}(t) = V_c \cos\left[2\pi\left[f_c + V_m(t)\,\Delta f\right]t\right] \quad \text{—} \; (1)$$

Carrier Amplitude   Analog carrier frequency.   Binary I/p.   peak shift in the carrier frequency.

Here modulating signal is a normalized Binary waveform

So we take   logic 1 = +1

logic 0 = −1

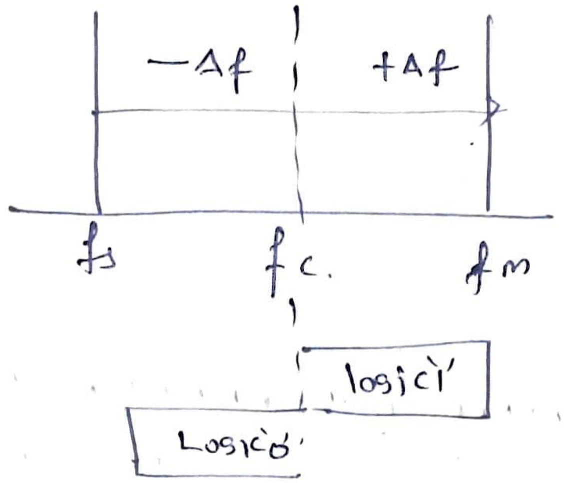
if $V_m(t) = +1$

$$V_{fsk}(t) = V_c \cos\left\{2\pi\left(f_c + \Delta f\right]t\right\}. \quad \text{—} \; (2)$$

if $V_m(t) = -1$

$$V_{fsk}(t) = V_c \cos\left\{2\pi\left(f_c - \Delta f\right]t\right\}. \quad \text{—} \; (3)$$

From above equation ② and ③, the centre frequency f_c is shifted ie deviated up and down in the frequency domain by the binary input signal is below



FSK in frequency Domain

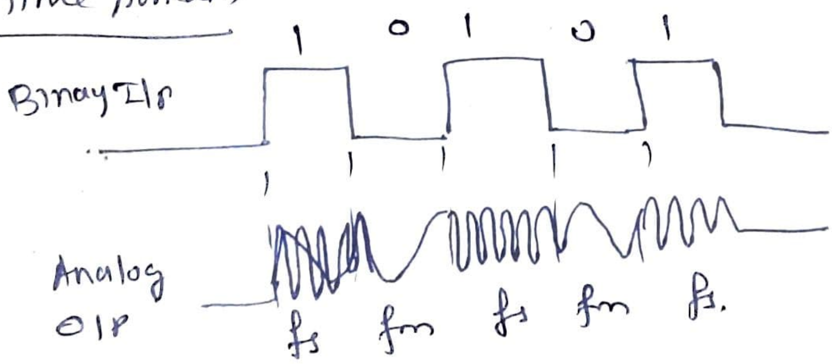
frequency Deviation is expressed mathematically as

$$\boxed{f = |f_m - f_s| / 2}$$

f = frequency deviation (Hz)

$|f_m - f_s|$ → difference between the mark and space frequencies (Hz)

FSK in Time Domain



f_s → space frequency
f_m → mark frequency.

Minimum BW of FSK $\quad B = |(f_s - f_m)| + 2f_b$.

$f_s - f_m$ → frequency Deviation in Hz
f_b = Input bit rate (bps)

$B = 2f + 2f_b$

Minimum Nyquist BW = $\boxed{B = 2(f + f_b)}$

Determine the (a) peak frequency Deviation (b) minimum Bandwidth, and (c) baud for a binary FSK signal with a mark frequency of 49 KHZ, a space frequency of 51 KHZ, and an input bit rate of 2 kbps.

(a) peak frequency deviation is determined by.

$$f = \frac{|f_m - f_s|}{2}$$

$$= \frac{|49 - 51|}{2} = 1 \text{ KHZ.}$$

(b) minimum BW.

$$B = 2(f + f_b)$$

$$= 2(1000 + 2000) = 6 \text{ KHZ}$$

(c) For FSK, N = 1, and the baud is determin by

baud $2000/1 = 2000$

— x —

BINARY AMPLITUDE SHIFT KEYING

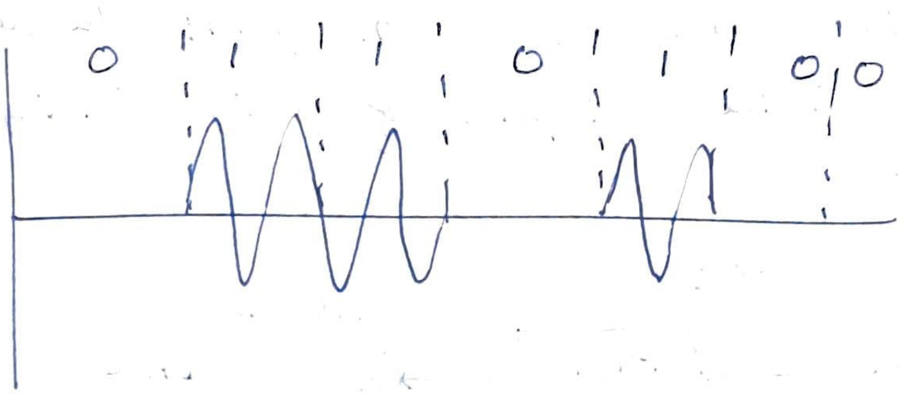
(just for 2 mrks)

(ON-OFF Keying) (OOK)

The amplitude of the carrier takes 2 values 'A' and '0' based on the binary symbol 1 & 0.

$x_1(t) = A\cos 2\pi f_c t$ for '1'

$x_2(t) = 0$ for '0'



COHERENT BINARY FREQUENCY SHIFT KEYING (BFSK)

→ In BFSK, the symbols '0' and '1' are distinguished from each other by txng one of two sinusoidal signals that differ in frequency by a fixed amount.

→ The txd BFSK signal is given by

$$x_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_i t, \quad 0 \le t \le T_b$$

When $M=2$, $i=1,2$

$$f_i = \frac{W_c + i}{T_b}$$

Representing the BFSK signal in terms of basis function,

$$x_1(t) = \sqrt{E_b}\sqrt{\frac{2}{T_b}} \cos 2\pi f_i t \quad \text{(For Symbol 1)}$$

$$= \sqrt{E_b}\, \phi_1(t)$$

where $\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_i t$ is the basis fn

|||ly $x_2(t) = \sqrt{E_b}\sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$ for symbol 0

$x_2(t) = \sqrt{E_b}\ \phi_2(t)$

$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$ is the second basis $\underline{Fn}$

BFSK Constellation : Here $N=2,\ M=2.$

$$x_{ij} = \int_0^{T_b} x_i(t)\ \phi_j(t)\,dt \qquad i=1,\ j=1,2$$

$\underline{i=1,\ j=1}$

$$x_{11} = \int_0^{T_b} x_1(t)\ \phi_1(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\ \phi_1^2(t)\,dt = \sqrt{E_b}$$

$\int_0^{T_b} \sqrt{E_B}\cdot\frac{2}{T_b}\cos^2(2\pi f_1 t)$

$\frac{\sqrt{E_B}}{T_b}\int 1-\cos 2(2\pi f_1 t)$

$\frac{\sqrt{E_b}}{T_b}\int_0^{T_b} 1-\cos 4\pi f_1 t$

$\frac{\sqrt{E_B}}{T_b}\left[t-\frac{\sin(4\pi f_1 t)}{4\pi f_1}\right]_0^{T_b}$

$\frac{\sqrt{E_b}}{T_b}\left[T_b-\frac{\sin(4\pi f_1 T_b)}{4\pi f_1}\right] - (0-0)$

$=\sqrt{E_b}$

$f_1=\frac{1}{T_b} \Rightarrow f_1 T_b = 1$

$\underline{i=1,\ j=2}$

$$x_{12} = \int_0^{T_b} x_1(t)\,\phi_2(t)\,dt = \int_0^{T_b} \sqrt{E_b}\ \phi_2(t)\phi_1(t)\,dt$$

$\cos^2 x = \frac{1-\cos 2x}{2}$

$$= \sqrt{E_b}\int_0^{T_b} \phi_1(t)\ \phi_2(t)\,dt$$

$= 0.$ since $\phi_1(t)\phi_2(t) = 0.$

$\underline{i=2,\ j=1}$

$$x_{21} = \int_0^{T_b} x_2(t)\ \phi_1(t)\,dt = \int_0^{T_b} \sqrt{E_b}\ \phi_2(t)\,\phi_1(t)\,dt$$

$= 0$

$\underline{i=2,\ j=2}$

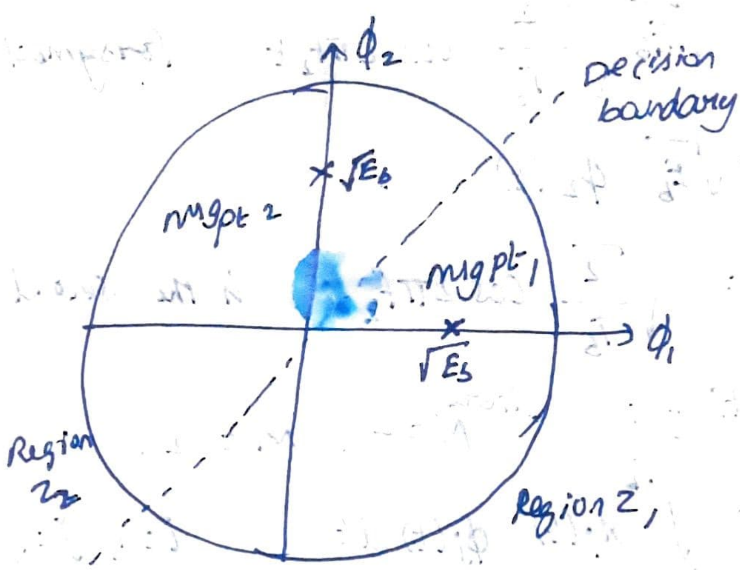
$$x_{22} = \int_0^{T_b} x_2(t)\,\phi_2(t)\,dt = \int_0^{T_b} \sqrt{E_b}\ \phi_2^2(t)\,dt$$

$= \sqrt{E_b}.$

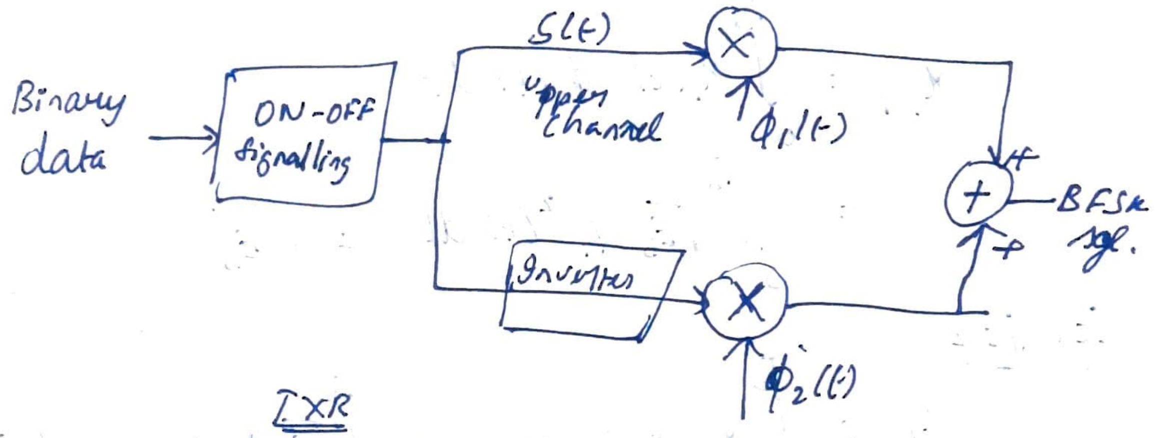
The msg points are given by vectors $x_1 = \begin{bmatrix}\sqrt{E_b}\\0\end{bmatrix}$

$x_2 = \begin{bmatrix}0\\\sqrt{E_b}\end{bmatrix}$

BFSK Constellation



BFSK Generation



TXR

→ The binary data is line coded using unipolar

$1 \rightarrow \sqrt{E_b}$

$0 \rightarrow 0.$

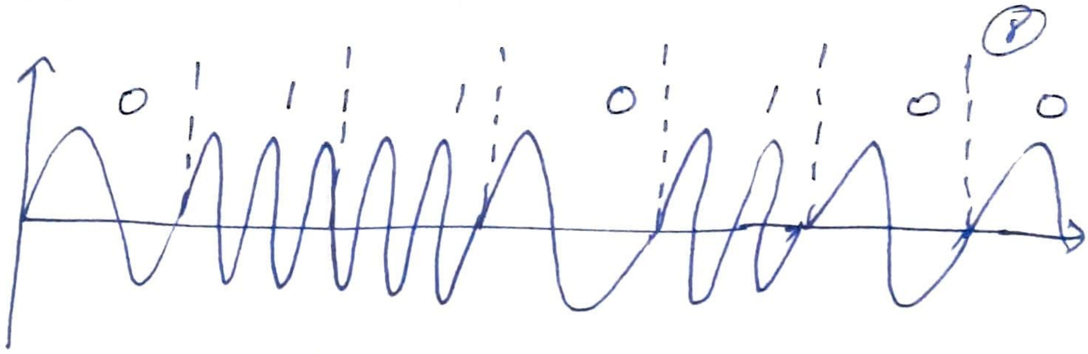
→ Multiply $s(t)$ with 1st basis function $\phi_1(t)$ in upper channel.

→ $s(t)$ is inverted in lower channel & multiplied with second basis fn.

→ Adding upper and lower channel will produce BFSK signal.

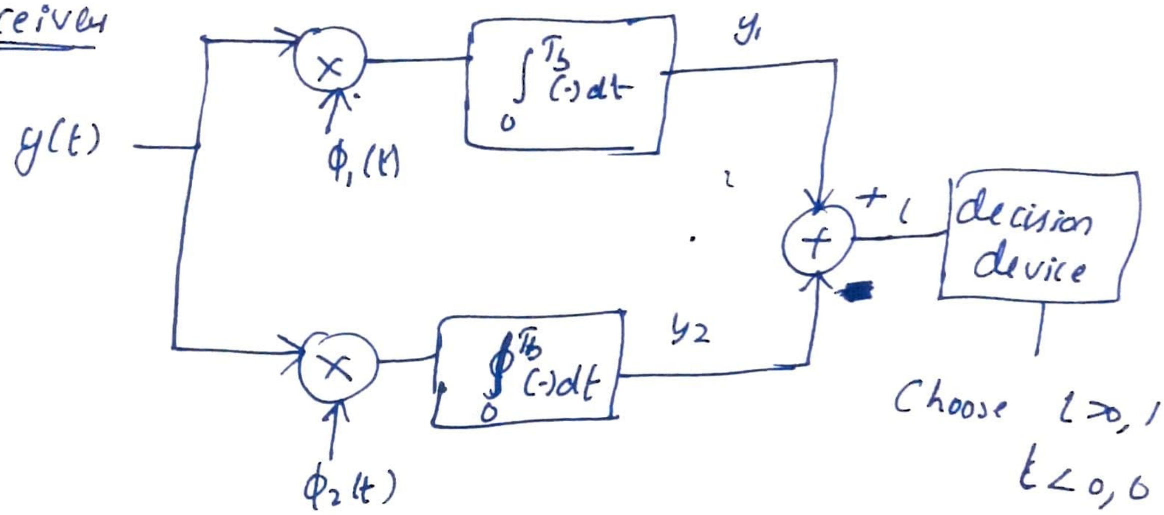
→ The rxd signal is given as i/p to upper & lower correlators. The difference l is obtained and if $l>0$, bit '1' else bit '0' is obtained.

(8)



Waveform - BFSK

Receiver



→

Differential Phase Shift Keying (DPSK)

→ Non-coherent version of PSK.

i.e It doesnot require any reference signal at the rxr from txr.

→ This is achieved by combining 2 operations at the txr namely

1. Differential encoding of input binary signal
2. Phase shift keying.

So Differential encoding + PSK → DPSK.

→ For txning symbol '1', we leave the phase of the current signal waveform unchanged and to txd '0', we advance the phase by 180°.

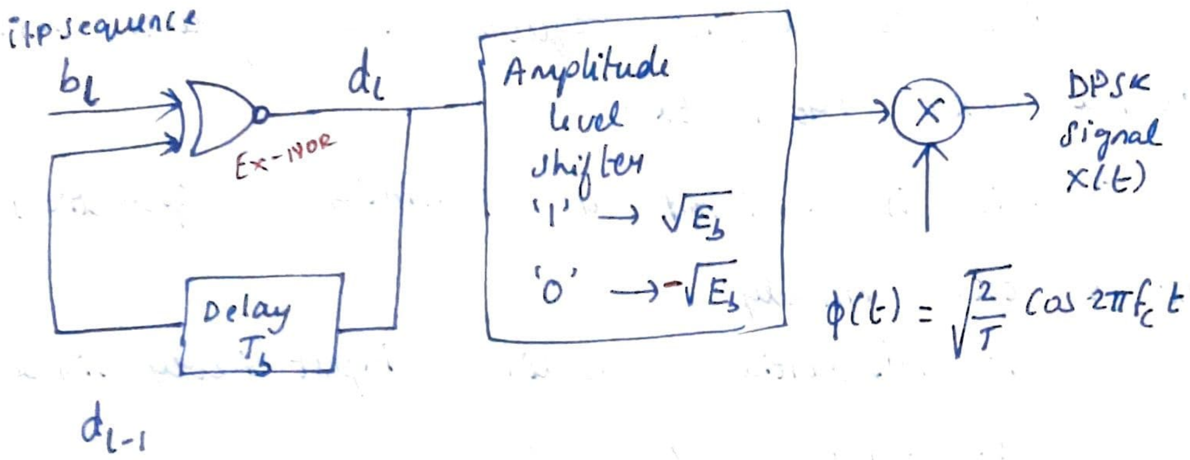
'1' → phase unchanged

'0' → Phase advanced by 180°.

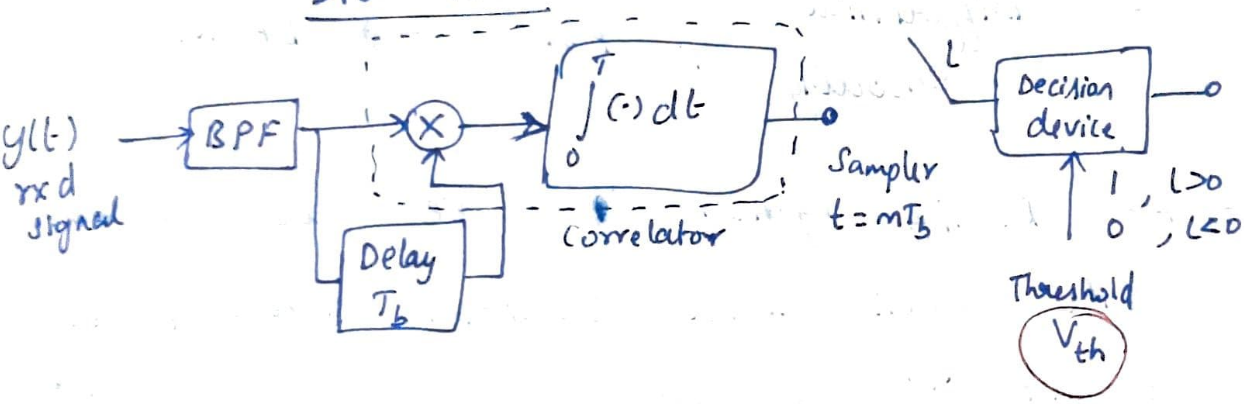
→ The rxr is enabled with storage capacity, so it can measure the relative phase difference between the waveform rxd. during 2 successive bit intervals.

→ The unknown phase θ in the rxd signal is slowly varied so that the phase difference b/w the signals rxd in two successive bit intervals will be independent of θ.

DPSK txr



DPSK Rxr



→ The i/p bits are differentially encoded using Ex-NOR operation.

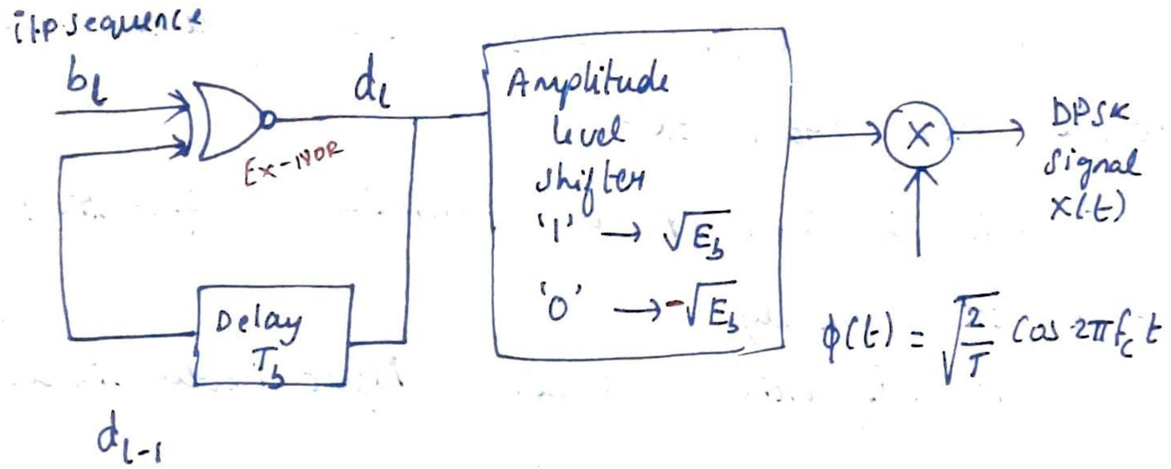
differentially encoded output is

$$d_l = \overline{b_l \oplus d_{l-1}}$$

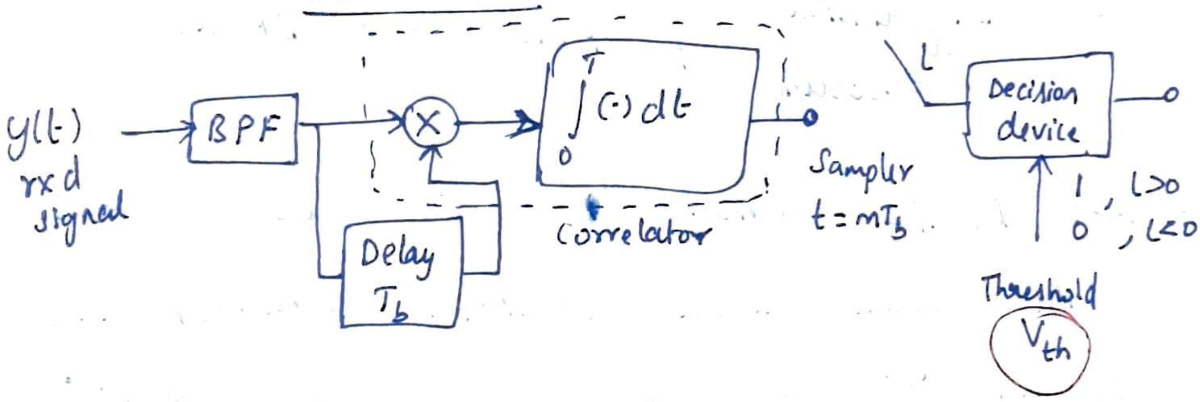
These bits are used to modulate the carrier with phase angles 0 & π radians.

the signals rxd in two successive bit intervals will be independent of θ.

DPSK txr



DPSK Rxr



→ The i/p bits are differentially encoded using Ex-NOR operation.

differentially encoded output is

$$d_l = \overline{b_l \oplus d_{l-1}}$$

These bits are used to modulate the carrier with phase angles 0 & π radians.

DPSK Signal generation

b_l		1	0	0	1	0	0	1
d_{l-1}		1	1	0	1	1	0	1
d_l	1 (reference)	1	0	1	1	0	1	1
Txd phase (ϕ)	0	0	π	0	0	π	0	0

→ The rxd signal is band pass filtered to limit the noise components.

→ If the consecutive bits are same ($d_l = d_{l-1}$) then the inputs to product modulator are inphase

$$\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t+\theta)\cdot\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t+\theta)$$

$$= \frac{2E_b}{T_b}\cos^2(2\pi f_c+\theta)$$

The integrator o/p at $t = T_b$ is

$$\int_0^{T_b}\frac{2E_b}{T_b}\cos^2(2\pi f_c+\theta)\,dt$$

$$= \frac{2E_b}{T_b}\int_0^{T_b}\frac{1+\cos 2(2\pi f_c+\theta)}{2}\,dt$$

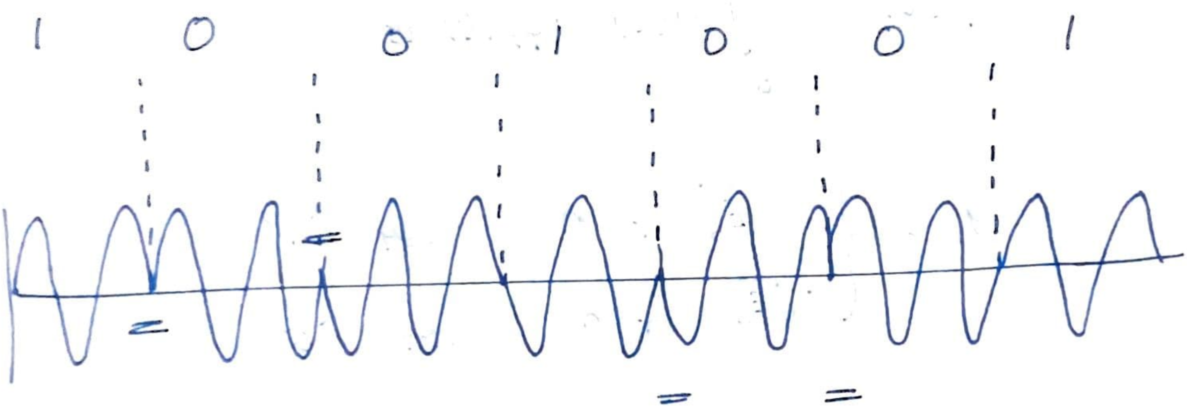
$\boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$

The detected sequence is obtained by

Rxd phase	0	0	π	0	0	π	0	0
	S (same)		D (Different)	D	S	D	D	S
Detected bits	1		0	0	1	0	0	1

DPSK waveform

Undergoes a phase change when '0' is sent.



Avg Probability of error,

$$P_e = \frac{P_e(0) + P_e(1)}{2}$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)}$$

③ BFSK BER analysis

Assuming to be AWGN channel with $\mu = 0$, $\sigma^2 = \frac{N_0}{2}$, the rxd signal is

$$y(t) = \begin{cases} x_1(t) + n(t) & \text{for symbol } 1 \\ x_2(t) + n(t) & \text{for symbol } 0 \end{cases}$$

Observation vector

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad \text{where} \quad y_1 = \int_0^{T_b} y(t)\,\phi_1(t)\,dt$$

$$y_2 = \int_0^{T_b} y(t)\,\phi_2(t)\,dt$$

When symbol 1 is txd, observation vector is

$$y_1 = \int_0^{T_b} [x_1(t) + n(t)]\,\phi_1(t).dt$$

$$= \int_0^{T_b} x_1(t)\,\phi_1(t)\,dt + \int_0^{T_b} n(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_1^2(t)\,dt + n,$$

$$= \sqrt{E_b} + n$$

$$y_2 = \int_0^{T_b} [x_1(t) + n(t)]\,\phi_2(t)\,dt$$

$$= \int_0^{T_b} x_1(t)\,\phi_2(t)\,dt + \int_0^{T_b} n(t)\,\phi_2(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\, \phi_1(t)\, \phi_2(t) + n$$

$$y_2 = 0 + n = n$$

$$\therefore y = \begin{bmatrix} \sqrt{E_b} + n \\ n \end{bmatrix}$$

$\| ^{ly}$ we can prove the observation scalar for '0'

as $y = \begin{bmatrix} n \\ \sqrt{E_b} + n \end{bmatrix}$

In the BFSK rxr, the rxd signal is given as i/p to both upper & lower correlators and the difference (l) is obtained. If it is greater than 0, txd bit is '1' and if not '0'

So $L = y_1 - y_2$

$$E[L/1] = E[(y_1/1) - (y_2/1)]$$

$$E[L/1] = E\left[\sqrt{E_b} + n - n\right] = \sqrt{E_b}$$

The conditional mean of the random variable L under the condition that symbol 0 was txd is given by

$$E[L/0] = E[(y_1/0) - (y_2/0)]$$

$$= E[n - (\sqrt{E_b} + n)]$$

$$\boxed{E[L/0] = -\sqrt{E_b}}$$

(15)

The random variables Y_1 & Y_2 are statistically independent with a variances equal to $\frac{N_0}{2}$. Then the variance of random variable L is

$$Var[L] = Var[Y_1] + Var[Y_2] = \frac{N_0}{2} + \frac{N_0}{2}$$

$$= N_0.$$

When symbol '0' is txd, cpdf random variable L is given by

$$f_L(l/0) = \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{(l+\sqrt{E_b})^2}{2N_0}}$$

Prob. of error, $P_e(0) = P(l>0)$ when symbol '0' was sent

$$= \int_0^\infty f_L(l/0)\,dl$$

$$= \int_0^\infty \frac{1}{\sqrt{2\pi N_0}} \exp\left(-\frac{(l+\sqrt{E_b})^2}{2N_0}\right) dl$$

Let we Take $\frac{l+\sqrt{E_b}}{\sqrt{2N_0}} = z$,

$l=0 \rightarrow z = \sqrt{\frac{E_b}{2N_0}}$

$l=\infty \Rightarrow z=\infty$

$dz = \frac{dl}{\sqrt{2N_0}}$

$dl = \sqrt{2N_0}\cdot dz$

$$\therefore P_e(0) = \frac{1}{\sqrt{2\pi N_0}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-z^2} \cdot \sqrt{2N_0}\cdot dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-z^2} dz$$

This is the Complimentary error fn,

$$P_e(0) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

IIIly

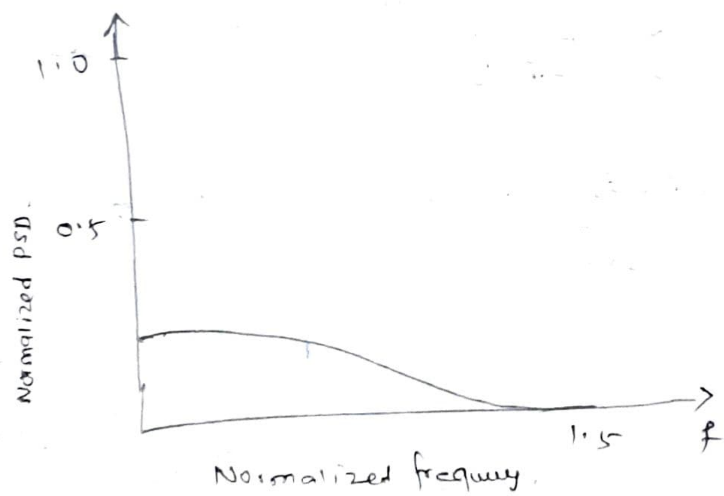
$$P_e(1) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Avg. prob of error is

$$P_e = \frac{P_e(0) + P_e(1)}{2}$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

PSD of FSK



(-) Coherent Quadrature phase shift Keying (QPSK)

In QPSK, the phase of transmitted carrier takes one of the four equally spaced values such as $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$.

The txd QPSK signal is given by

$$x_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right]$$

$0 \le t \le T,\ i = 1, 2, 3, 4$

$T \rightarrow$ Symbol duration

$E \rightarrow$ Symbol Energy

sub

$$x_1(t) = \sqrt{\frac{2E}{T}} \cos[2\pi f_c t + \pi/4]$$

$$x_2(t) = \sqrt{\frac{2E}{T}} \cos[2\pi f_c t + 3\pi/4]$$

$$x_3(t) = \sqrt{\frac{2E}{T}} \cos[2\pi f_c t + 5\pi/4]$$

$$x_4(t) = \sqrt{\frac{2E}{T}} \cos[2\pi f_c t + 7\pi/4]$$

$$x_i(t) = \sqrt{\frac{2E}{T}} \cos 2\pi f_c t \cos(2i-1)\frac{\pi}{4} - \sqrt{\frac{2E}{T}} \sin 2\pi f_c t \sin(2i-1)\frac{\pi}{4}$$

$\boxed{\cos A \cos B - \sin A \sin B = \cos(A+B)}$

$0 \le t \le T$

$i = 1, 2, 3, 4$

$$= \sqrt{E}\cos(2i-1)\pi/4 \cdot \sqrt{\frac{2}{T}}\cos 2\pi f_c t - \sqrt{E}\sin(2i-1)\pi/4 \sqrt{\frac{2}{T}}\sin 2\pi f_c t$$

$$= x_i(t) = \sqrt{E}\cos(2i-1)\pi/4\ \phi_1(t) - \sqrt{E}\sin(2i-1)\pi/4 \cdot \phi_2(t).$$

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$$ are the 2 basis fns.

Constellation

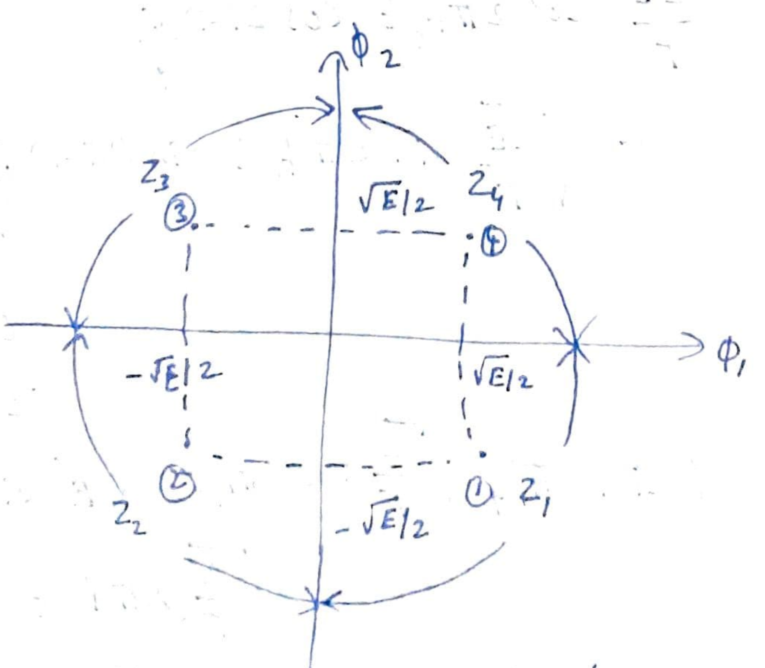
There are 4 msg points and the corresponding signal vectors are

$$X_i = \begin{bmatrix} \sqrt{E} \cos (2i-1) \pi/4 \\ -\sqrt{E} \sin (2i-1) \pi/4 \end{bmatrix}, \quad i = 1, 2, 3, 4$$

$\cos \pi/4 = \frac{1}{\sqrt{2}}$

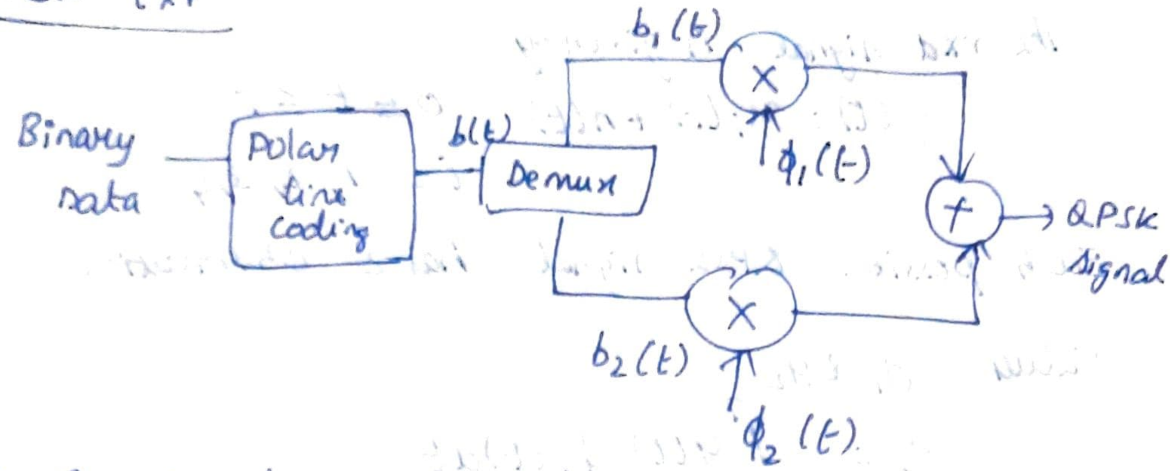
By substituting values $i = 1, 2, 3, 4$

di Bits	Carrier	Msg co-ordinates X_{i1}	X_{i2}
1 0	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
0 0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
0 1	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
1 1	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$



QPSK space signal dgm.

QPSK txr



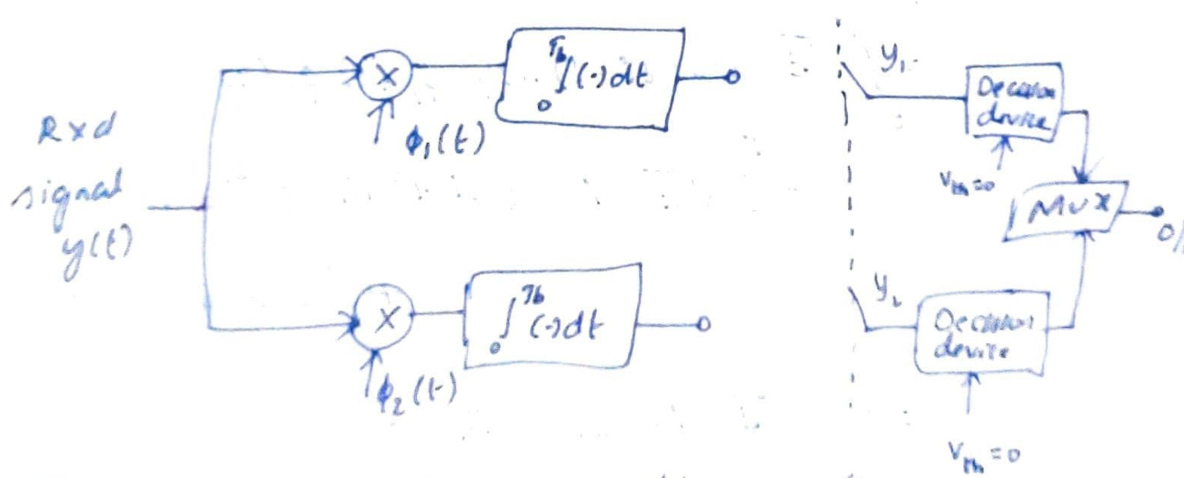
→ Input binary data is polar line coded to get b(t)

→ Using Demux, polar signal is divided into 2 waves one with odd bits and the other one with even bits.

→ These two waves are seperately BPSK modulated using corresponding basis functions

→ Addition of two BPSK results in QPSK wave.

QPSK Rxr



→ The rxd signal is passed through the correlators in upper and lower channels. The correlator outputs are sampled and the samples are compared with the threshold which gives inphase & quadrature bits. These bits are combined which gives the txd bit sequence back.

BER analysis (QPSK)

The rxd signal is given by

$$y(t) = x_i(t) + n(t) \qquad 0 \le t \le T$$
$$i = 1, 2, 3, 4$$

Each possible QPSK signal has 2 observation values y_1 & y_2

$$y_1 = \int_0^T y(t)\,\phi_1(t)\,dt$$
$$= \int_0^T [x_i(t) + n(t)]\,\phi_1(t)\,dt$$
$$= \int_0^T x_i(t)\,\phi_1(t)\,dt + \int_0^T n(t)\,\phi_1(t)\,dt$$
$$= \int_0^T \sqrt{E}\cos(2i-1)\pi/4\;\phi_1^2(t)\,dt - \int_0^T \sqrt{E}\sin(2i-1)\pi/4\;\phi_1(t)\,\phi_2(t)\,dt + n_1$$

$$y_1 = \sqrt{E}\cos(2i-1)\pi/4 + n$$

$$y_2 = \int_0^T y(t)\,\phi_2(t)\,dt = \int_0^T [(x_i(t) + n(t))]\cdot\phi_2(t)\,dt$$

$$y_2 = \int_0^T \sqrt{E}\cos(2i-1)\pi/4\;\phi_1(t)\,\phi_2(t)\,dt$$
$$- \int_0^T \sqrt{E}\sin(2i-1)\pi/4\;\phi_2^2(t)\,dt$$
$$+ \int_0^T n(t)\,\phi_2(t)\,dt$$

$$= 0 - \sqrt{E}\sin(2i-1)\pi/4 + n$$

$$y_2 = -\sqrt{E}\sin(2i-1)\pi/4 + n.$$

Observation vector is

$$y = \begin{bmatrix} \sqrt{E}\cos(2i-1)\pi/4 + n \\ -\sqrt{E}\sin(2i-1)\pi/4 + n \end{bmatrix} \quad i = 1, 2, 3, 4$$

Mean

$$E(y_1) = \sqrt{E}\cos(2i-1)\pi/4 + E(n)$$

$$= \sqrt{E}\cos(2i-1)\pi/4$$ $\boxed{E(n)=0}$

$$E(y_2) = -\sqrt{E}\sin(2i-1)\pi/4 + E(n)$$

$$= -\sqrt{E}\sin(2i-1)\pi/4, \quad i = 1,2,3,4$$

Variance of y is N_0

Assume $x_4(t)$ is dibit 11. For correct decision, observation values y_1 and y_2 should be greater than 0.

Prob. of correct decision $= P(y_1>0) \cdot P(y_2>0)$

$$P_c = \int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(y_1-\sqrt{E/2})^2}{N_0}\right] dy_1$$

$$\int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-(y_2-\sqrt{E/2})^2}{N_0}\right] dy_2$$

Let $\dfrac{y_1-\sqrt{E/2}}{\sqrt{N_0}} = \dfrac{y_2-\sqrt{E/2}}{\sqrt{N_0}} = z$

Changing variables of integration from y_1 & y_2 to z

$y_1 = 0 \rightarrow z = -\sqrt{E/2N_0}$

$y_1 = \infty \rightarrow z = \infty$

$\dfrac{dy_1}{\sqrt{N_0}} = dz \Rightarrow dy_1 = \sqrt{N_0}\, dz$

$$P_c = \underbrace{\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-z^2/\sqrt{N_0}}\, dz}_{\mathrm{I}} \underbrace{\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-z^2/\sqrt{N_0}}\, dz}_{\mathrm{II}}$$

consider term I,

$$\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = \frac{1}{2}\frac{2}{\sqrt{\pi}} \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} e^{-z^2} dz$$

$$= \frac{1}{2} \operatorname{erfc}\left(-\sqrt{\frac{E}{2N_0}}\right)$$

$$= \frac{1}{2}\left[2 - \operatorname{erfc}\sqrt{\frac{E}{2N_0}}\right]$$

$\operatorname{erfc}(-x) = 2 - \operatorname{erfc}(x)$

$$P_c = \left(1 - \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E}{2N_0}}\right)^2$$

Prob. of error is

$$P_e = 1 - P_c$$

$$= 1 - \left(1 - \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E}{2N_0}}\right)^2$$

$$= 1 - \left(1 - \frac{1}{4}\operatorname{erfc}^2\sqrt{\frac{E}{2N_0}} - \operatorname{erfc}\sqrt{\frac{E}{2N_0}}\right)$$

$$= 1 - 1 - \frac{1}{4}\operatorname{erfc}^2\sqrt{\frac{E}{2N_0}} + \operatorname{erfc}\sqrt{\frac{E}{2N_0}}$$

$$P_e = \operatorname{erfc}\sqrt{\frac{E}{2N_0}} - \frac{1}{4}\operatorname{erfc}^2\sqrt{\frac{E}{2N_0}}$$

approx, $P_e \simeq \operatorname{erfc}\sqrt{\frac{E}{2N_0}}$

$$E = 2E_b$$

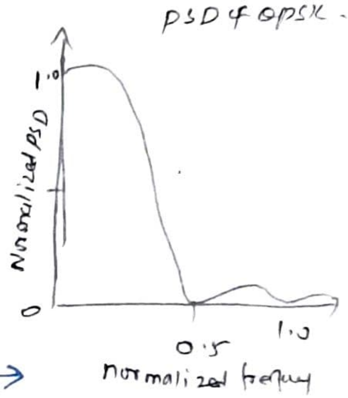
$$P_e \simeq \text{erfc}\sqrt{\frac{2E_b}{2N_0}}$$

$$P_e^{QPSK} \simeq \text{erfc}\sqrt{\frac{E_b}{N_0}}$$

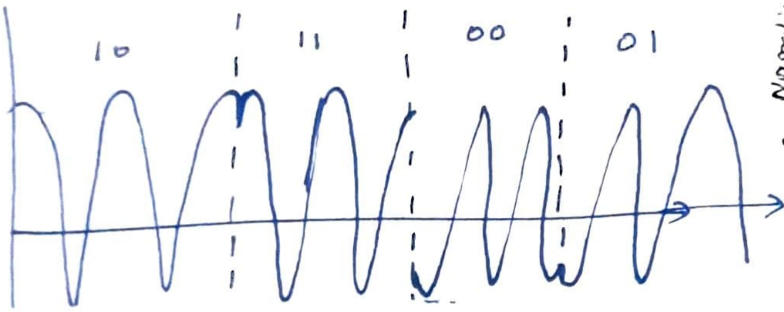
With Gray coding used, bit error rate of QPSK signal is

$$BER = \frac{P_e^{QPSK}}{2} = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E_b}{N_0}}$$

PSD of QPSK.



QPSK waveform



QUADRATURE AMPLITUDE MODULATION (QAM)

→ QAM is a form of modulation, used for modulating data signals into a carrier used for radio communication.

→ It is the combination of (ASK & PSK).

→ QAM uses 2 carriers for modulation, which are seperated by 90° degrees. The resultant signals are summed together and it consist of both amplitude and phase variations. Hence it is considered as a combination of amplitude & phase modulation.

Generation of QAM:

In a QAM signal, it uses two carriers each having the frequency and different phase by 90^o.

One signal is called as I-inphase which is a sine wave and the other is called as Q - quadrature which is a consine wave.

The block diagram used to generate the QAM signal is shown in the figure.

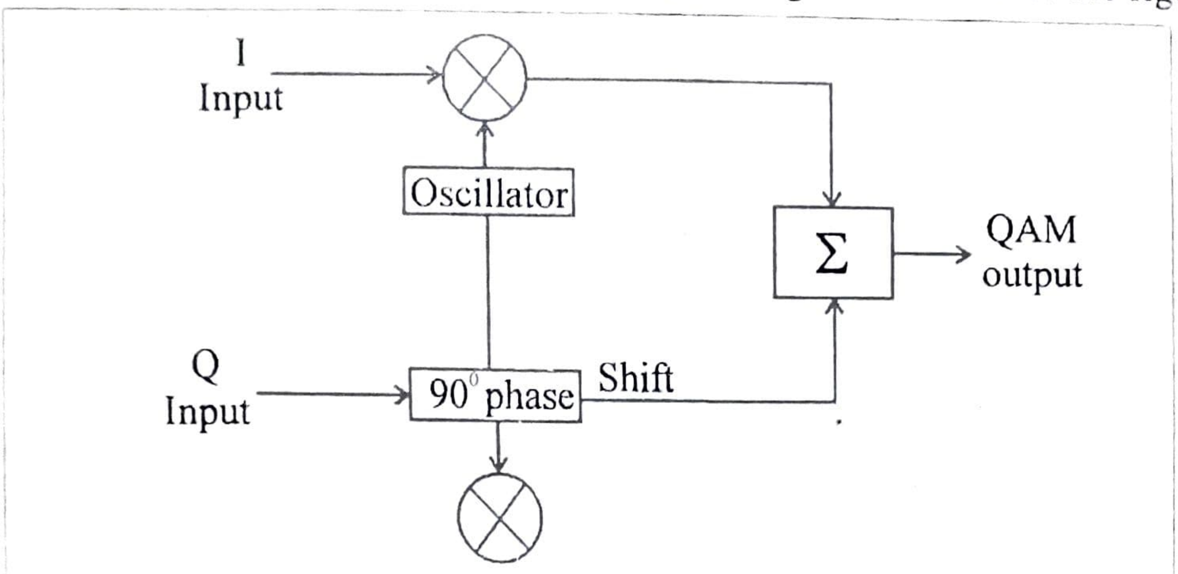


Fig. 4.28 Basic QAM Modulator

There are two carrier signals used with a 90^o phase shift between them. the two input message signals are given at the one of the input to the mixer. The other input to the mixer is the carrier signal. First mixer receives the carrier with 0^o phase shift and the second mixer receives the carrier with 90^o phase shift.

The modulated signals by the two mixers are given to a summer. The two resultant signals are summed and then processed for transmission.

$I = A.\cos\theta$ (Inphase signal)

$Q = A.\sin\theta$ (Quadrature signal).

Detection of QAM:

The detection of the QAM signal at the receiver is shown in the figure.

The transmitted signal are given as a input to the splitter. This will split the signal into two and are applied to the mixers.

One half of the signal is applied with inphase local oscillator and demodulated.

The other half is applied with quadrature oscillator signal and demodulated

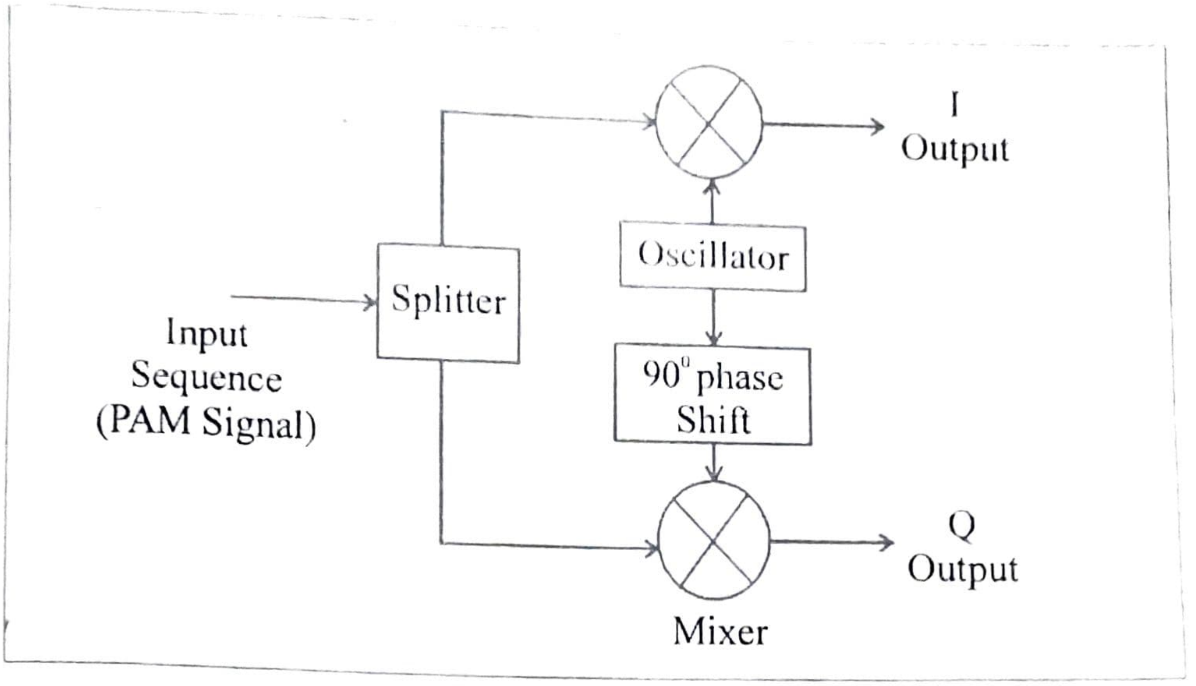


Fig. 4.29 QAM Demodulation

The demodulation circuitary includes the circuitary for carrier recovery. For this purpose usually phase locked loop is used.

Signal space diagram of QAM :

The QAM is considered as the two pulse amplitude signals on quadrature carriers. The first rectangular QAM is usually 16 - QAM and it is represented in the figure.

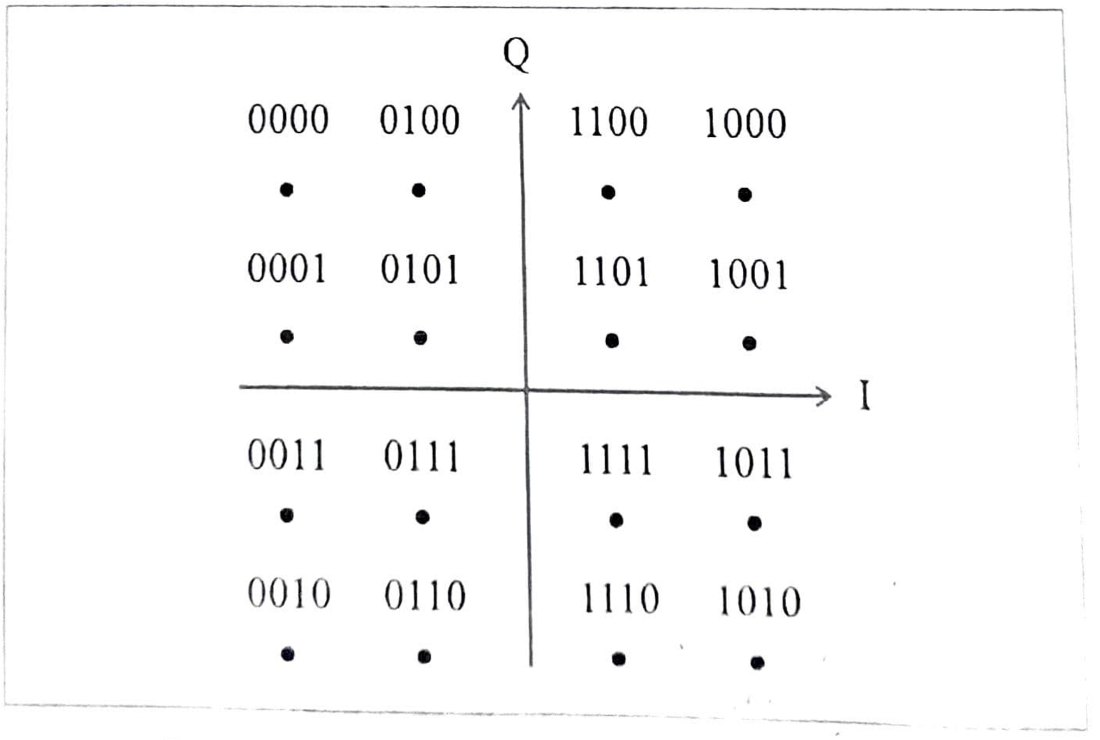


Fig. 4.30 Signal Space Diagram of QAM

Formula summary for the Bit error rate of different digital modulation schemes.

S.No	modulation type	BER
1.	Coherent BPSK	$\frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$
2.	Coherent BFSK	$\frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_o}}\right)$
3.	DPSK	$\frac{1}{2}\exp\left(\frac{-E_b}{N_o}\right)$
4.	Non coherent BFSK	$\frac{1}{2}\exp(-E_b/2N_o)$
5.	Coherent QPSK ε coherent MSK	$\frac{1}{2} erfc\sqrt{\frac{E_b}{N_o}}$

4.10 SYNCHRONIZATION :

If any two event occur simultaneously then the two sequence of events are said to be 'synchronous'. The process of making any situation synchronous and maintaining it in this condition is called 'synchronization'.

In digital modulation techniques, there are two modes of synchronization.

1. Carrier synchronization
2. Symbol synchronization

4.10.1 Carrier Synchronization:

During the coherent type of detection the knowledge about frequency and the phase of the carrier is important. The estimation of both the frequency and phase of the carrier is called carrier synchronization.

The carrier synchronization at the receiver can be achieved by the way of modulating the message signal onto the carrier in a way that the power spectrum contains a discrete component at the carrier frequency.

Phase locked loop (PLL) can be used at the receiver to track this component.

A phase locked loop consist of a voltage controlled oscillator (VCO), filter, and a multiplier connected by the way of negative feedback system.

Some time there is also possibility of absence of the DC component in the power spectrum of the message signal. In that case the receiver use a technique called "suppressed carrier-tracking loop" to provide the carrier reference.

The following figure is the example or method of carrier synchronization with the consideration of the impulse is available at the carrier frequency.

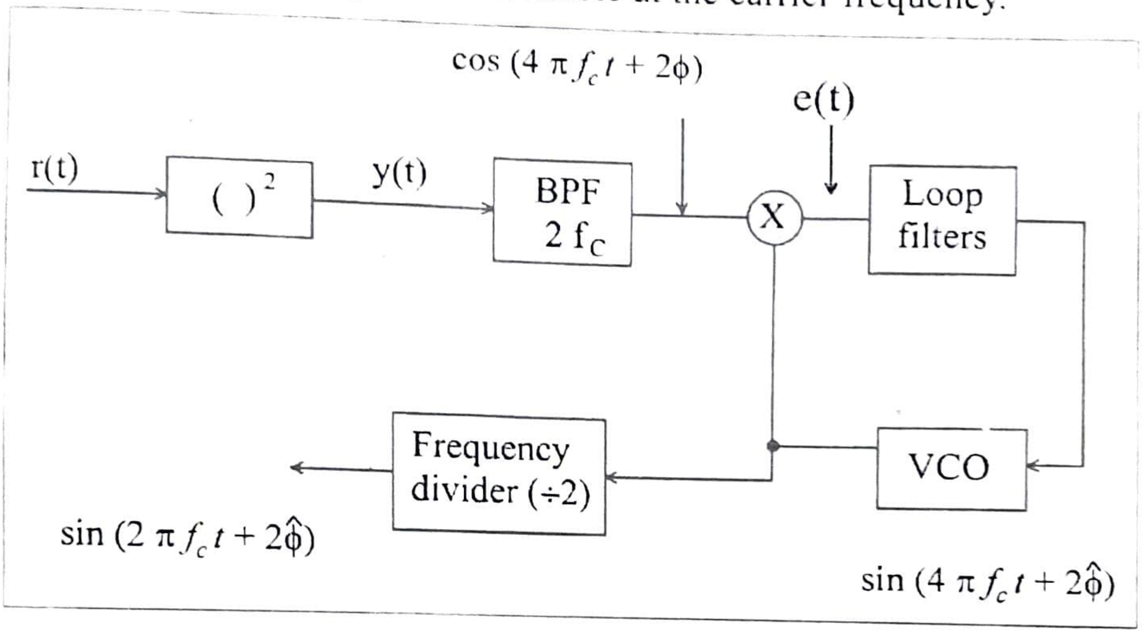


Fig. 4.31 Carrier Recovery

The impulse present in the carrier signal can be recovered with the help of phase – locked – loop (PLL).

In the modern way of digital communication system the presence of carrier is considered as wastage of energy. Therefore the suppressed carrier systems are employed. In these type systems two ways are adopted for the carrier recovery at the receiver.

1. Data aided synchronization
2. Non – data aided synchronization

1. Data aided synchronization :-

In this method along with the input information a special signal is also sent which is called as pilot signal. This pilot signal is nothing but the unmodolated carrier.

The disadvantages of the systems are,

1. As the pilot signal is transmitted in each frame, the throughput of the system will be reduced.

2. Because of the transmission of power signals the power efficiency is also decreases.

2. Non data – aided synchronisation:-

In this method pilot signal is not used. The receiver recreates the carrier by extracting necessary information from the received signal itself.

The main disadvantage in this system is,

1. Increase in time taken for synchronization of the carrier.

At the receiver side two different approaches are used for the recreation of carrier,

a) Forward acting carrier recovery

b) Decision directed carrier recovery.

The forward acting carrier recovery method is presented here in detail,

The noise contained received waveform is assumed as $r(t)$.

$$r(t) = s(t) + n(t)$$

where,

$s(t)$ – signal

$n(t)$ – noise

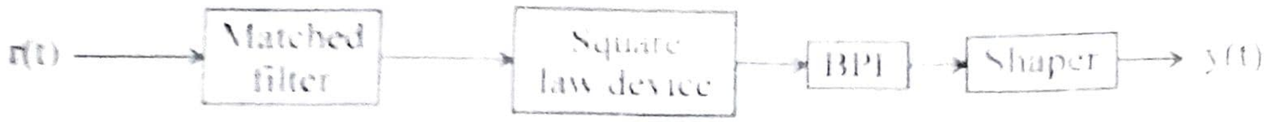
the s (t) can be written as,

$$s(t) = A(t)\cdot\cos(2\pi f_c t + \phi) \quad \text{... (4.102)}$$

If we passed the received signal directly onto the low pass (or) band pass, it will average the signal and it doesn't contain any phase (or) frequency component. So, it is sent to a squaring device before filtering. The square of the signal component is,

$$s^2(t) = \frac{1}{2}A^2(t) + \frac{1}{2}A^2(t)\cdot\cos(4\pi f_c t + 2\phi) \quad \text{... (4.103)}$$

One of the example of such type of synchronizer is shown in the following Figure.



In the above example the matched filter output is a triangular waveform and it is given to a square law device.

The square law device rectifies the signal and the resulting signal has the frequency that is equal to the transmitted signal frequency.

The Fourier component can be separated by a BPF tuned to the frequency $f_s = R_s$. This is further shaped by a saturating amplifier. Now the receiver can able to recover the clock which is used in the transmitter.

2. Closed loop symbol synchronization:

In this method, the clock can be generated on comparative measurements.

This method is more accurate but it is costly and complex in design.

The following figure shows the method of early late gate synchronizer.

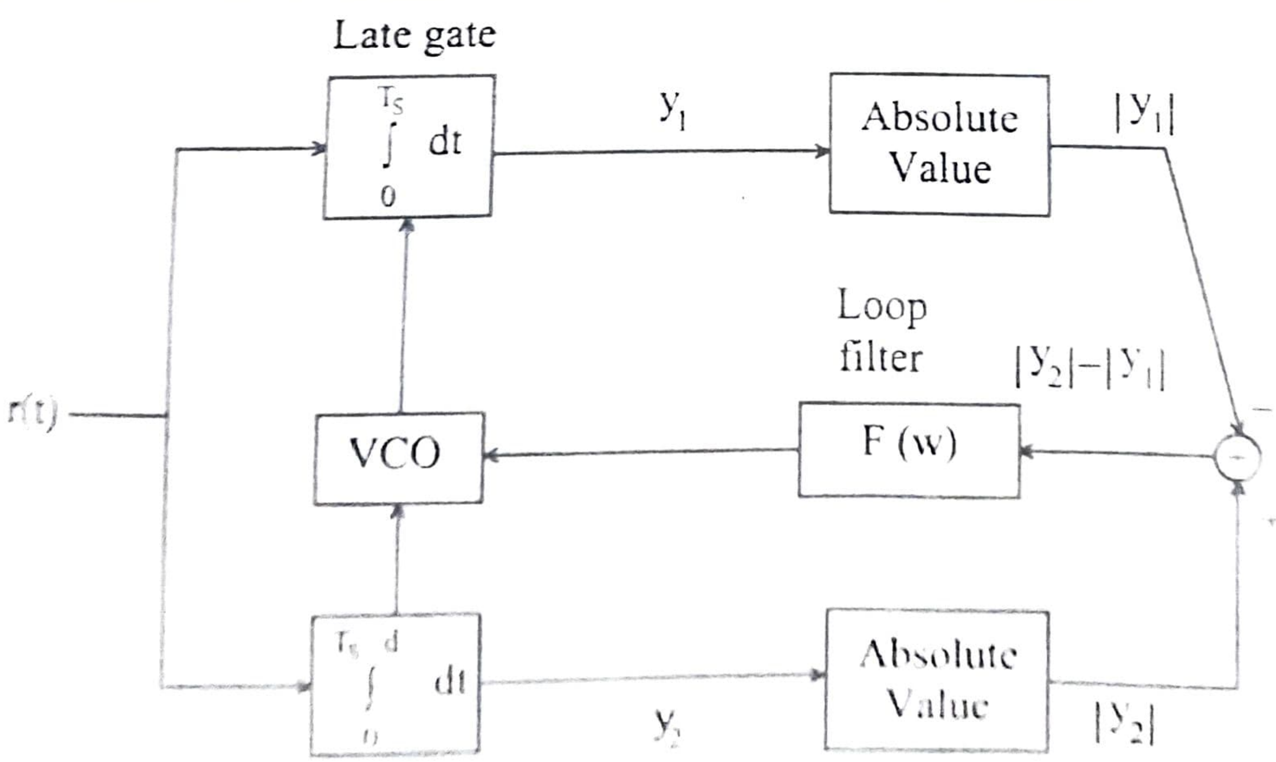


Fig. 32 Early Late Gate Synchroniser

This is given to a band pass filter. The BPF produce the average (or) expected value of the signal

$$E[s^2(t)] = \frac{1}{2} E[A^2(t)] + \frac{1}{2} E \frac{1}{2} E[A^2(t)] \cdot \cos(4\pi f_c t + 2\phi) \quad \text{... (4.104)}$$

The spectral component at $2 f_c$ has sufficient power, so the band pass filter is tuned to $2 f_c$.

Then it is filtered by loop filter and the output is given to the VCO. The feedback circuit locks the phase of the carrier. The output of the VCO is frequency divided to generate the phase locked carrier. This is the carrier used for the demodulation of the received signal $r(t)$.

4.10.2 Symbol Synchronization:

"During the demodulation it is also necessary for the receiver to know about the instant of modulation time. That is, the starting and finishing times of the symbols. The estimation of these times is called clock recovery (or) symbol synchronization".

The symbol synchronization can be processed along side of the carrier recovery or it can be processed first. There are many approaches to accomplish the clock recovery.

Case (i): A clock can be transmitted along with the message signal in a multiplexed form. At the receiver, the clock is extracted by some filtering methods.

Case (ii): First extracting the clock by using non coherent detector. Then the carrier is recovered by the output in each clocked interval.

A symbol synchronizer can able to generate a square wave of frequency, which is equal to the transmitted frequency and the zero crossing coincide with the signals transitions.

This can be done by two ways

1. Open loop synchronization
2. Closed loop synchronization

1. **Open loop symbol synchronization:-**

It performs direct operations on the incoming data.

This type of synchrosers generate a frequency component at the symbol rate, with the help of a filter & non linear devices.

This type of synchronisation can be performed on the received signal by two separate integration over two different portions of the symbol interval.

The first integration (early gate) begins for ($Ts - d$) time where d is the delay between two integrators. Usually d is less than Ts.

The second integrator (Late gate) starts its integration 'd' times after the first integration.

The difference in the absolute value of these two value is the measure of symbol timing error.

It is then feed back to the loops timing reference for correction. The operation can be best understood by the following timing diagram.

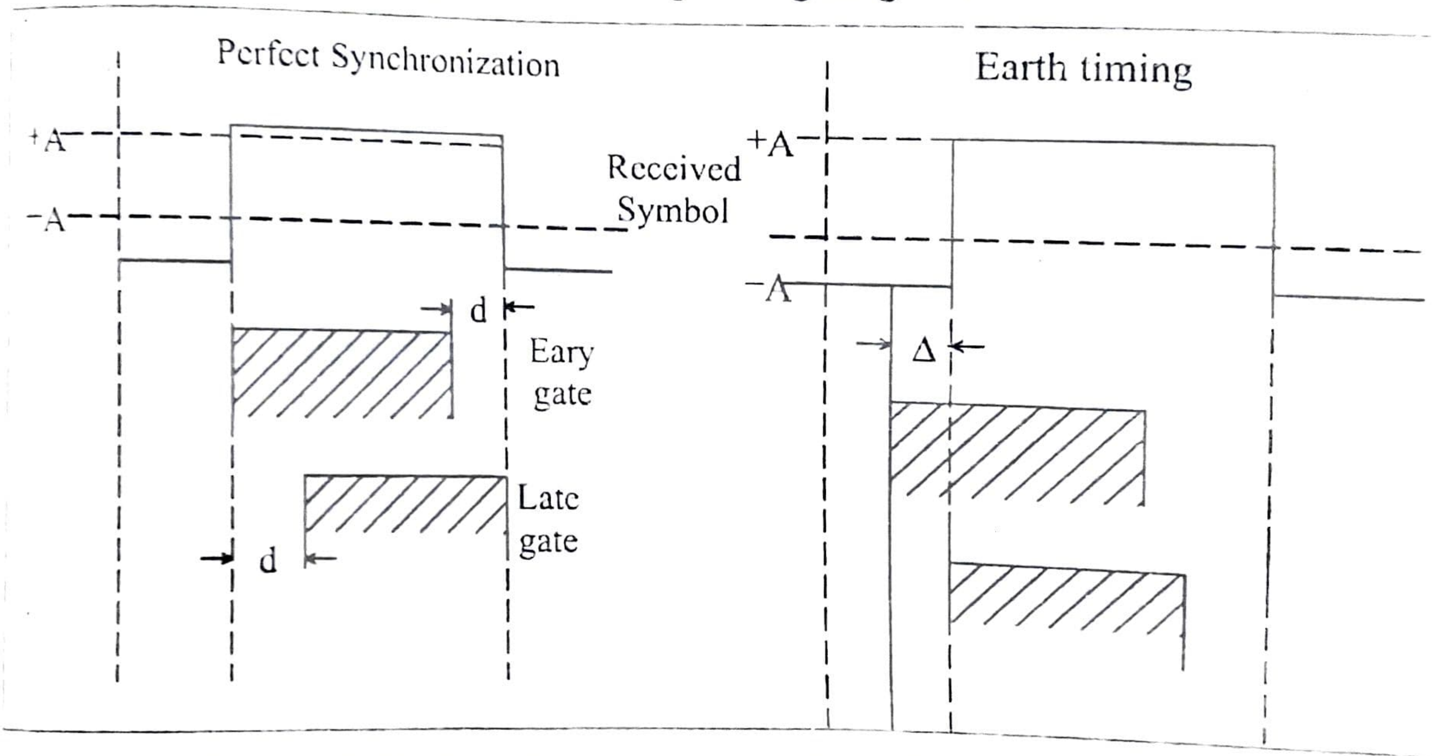


Fig. 4.33 Timing Diagram – Early Late Gate Synchronizer

During the perfect synchronisation both the gates are within the symbol interval. Therefore the receiver is in time synchrorisation with the transmitted signals.

During the early clock timing with the interval Δ, the early gate falls on the previous bit interval and the difference is $-2\Delta A$. A is the amplitude of the signal.

This error signal lower the VCO voltage and as a consequence VCO output frequency is also lowered. So that the clock timing falls in line with the incoming signal.

If the receiver timing been late, then the error will be positive and VCO frequency increased towards that of the incoming signal.

In practical circuits the two gates are not identical there will be a presence of some offset. This is not considered in normal symbol sequence. But for long symbol it may arise problems.

The solution is incorporating transparency in the line codes.

Comparison of the digital modulation schemes by Error Performance.

S.No	Type of Modulation	Transmission Bandwidth	Coherent detection	Non coherent detection
1	BASK	R	$\frac{1}{2} erfc \sqrt{\frac{E}{4N_0}}$	–
2	BPSK	R	$\frac{1}{2} erfc \sqrt{\frac{E}{N_0}}$	–
3	BFSK	$2\Delta F+R$ $(2\Delta f = f_2 - f_1)$	$\frac{1}{2} erfc \sqrt{\frac{E_b}{2N_0}}$	$\frac{1}{2} e^{-(1/2)(E_b/N_0)}$
4	DPSK	R	–	$\frac{1}{2} e^{(-E_b/N_0)}$
5	QPSK	1.5 R	$\frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$	–

Problems with Solutions :

1. *In a BFSK system a bit rate of 3Kbps in used. If the lower frequency signal is 10 KHZ, find higher frequency signal of minimum if minimum separation is used between the two signals.*

Solution:

$f_b = 3$ Kbps $f_L = 10$ KHZ

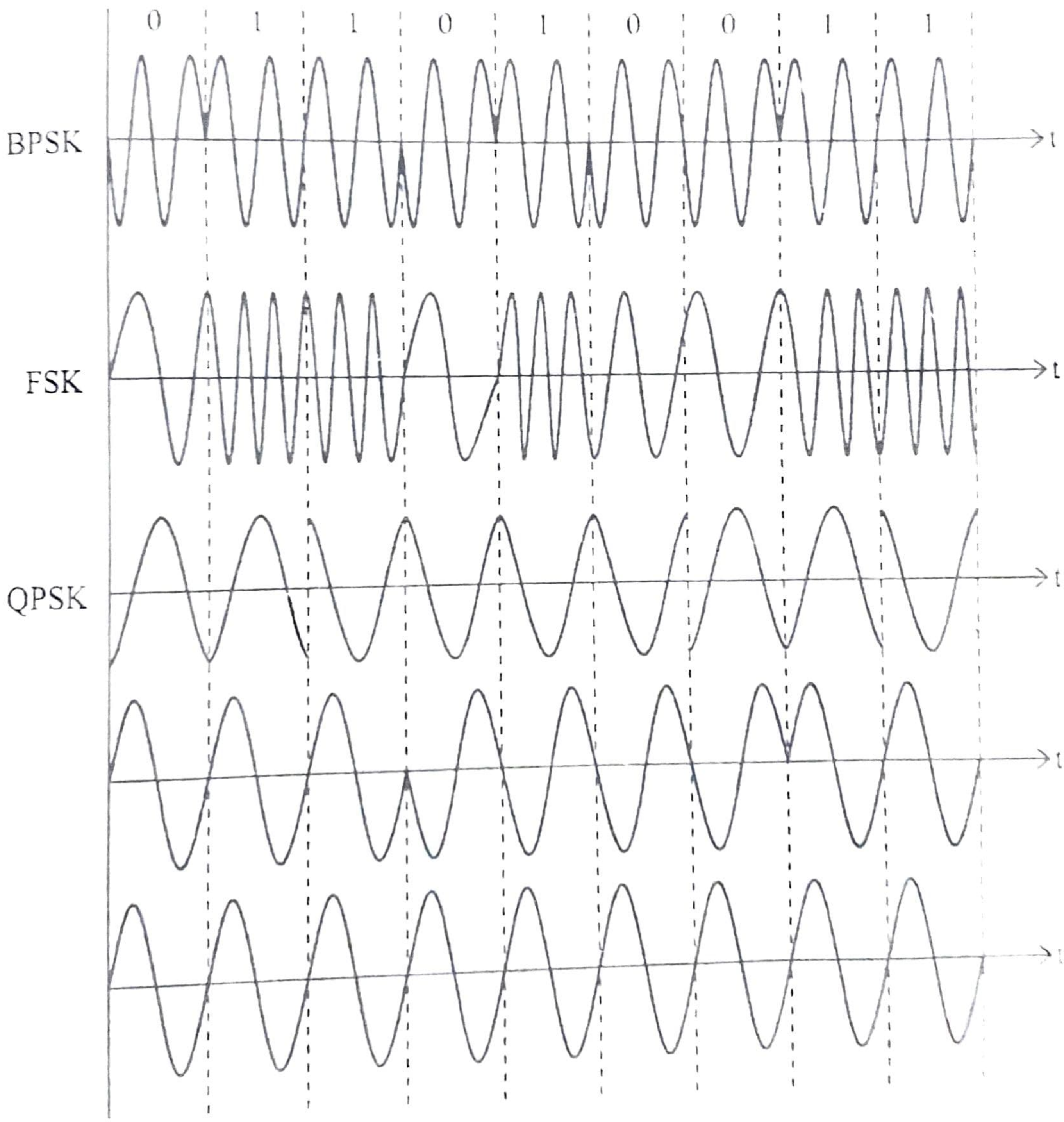
Minimum separation between f_L and f_H is $2f_b$

ie.., $f_H - f_L = 2f_b$

$f_H = 2f_b + f_L$

$= 2(\text{Kbps}) + 10\text{KHZ} \quad 16\text{KHZ}$

2. *For the bit stream of 0 1 1 0 0 0 1 1 plot the waveforms of i) BPSK ii) FSK ($f_H = 2f_b, f_L = f_b$) iii) QPSK*



3. *Given that amplitude of input at coherent optional receiver is 10 mv and frequency 1 MHZ. The signal is corrupted with white noise of PSD* 10^{-9} W/HZ. *The data rate is* 10^4 bits/sec. *Find probability of error.*

$[erfc(1.01) = 0.1531\ erfc(1.11) = 0.1164, erfc(1.22) = 0.0844]$

Solution:

$$A = 10 \times 10^{-3}\ V$$

$$r_b = 10^4\ \text{bits/sec} \qquad T_b = \frac{1}{r_b} = \frac{1}{10^4}$$

$$\frac{N_o}{2} = 10^{-9} \qquad N_o = 2 \times 10^{-9}$$

$$E = \frac{A^2}{2} \cdot T_b = \frac{(10 \times 10^{-3})^2}{2} \times \frac{1}{10^4} = 5 \times 10^{-9}\ J$$

For binary orthogonal coherent FSK,

$$p_e = \frac{1}{2} erfc\sqrt{\frac{0.5E}{N_o}} = \frac{1}{2}\ erfc\sqrt{\frac{0.5 \times 5 \times 10^{-9}}{2 \times 10^{-9}}}$$

$$p_e = \frac{1}{2} erfc\,(1.25) = \frac{1}{2} \times 0.0844 = 0.0422$$

For binary orthogonal coherent FSK,

4. *Binary data is transmitted over a microwave link at a rate of* 10^6 bits/sec *and the PSD of noise at the receiver input is* 0^{-10} watts/HZ. *Find the average carrier power required to maintain an average probability of error* $p_e \leq 10^4$ *for coherent Binary FSK. What is the required channel Bandwidth?*

$[erfc(2.8) = 0.9998\ erfc(2.5) = 0.9959]$

Energy of one bit.

$$E = pT_b$$

$$p = \frac{E}{T_b} = \frac{3 \times 10^{-9}}{1/10^6} = 3 \times 10^{-9} \times 10^6$$

$$= 3mW$$

Channel Bandwidth $B_T = \frac{1}{T_b} = 10^6 \text{ bits/sec} = 1\text{MHZ}$

5. ***Determine the minimum bandwidth for a BPSK modulator with a carrier frequency of 40 MHz and an input bit rate of 500 kbps.***

Solution :

$$f_b = 500 \text{ kbps} = 500 \text{ KHz}$$

$$BW = 2 f_b = 2 \times 500 \text{ KHz} = 1\text{MHz}.$$

6. ***For a BPSM modulator with a carrier frequency of 70 MHz and an input bit rate of 10 Mbps, draw the spectrum of output signal and determine the minimum Nyquist bandwidth.***

Solution :

$$f_b = 10 \ Mbps = 10 \times 10^6 \text{ Hz}$$

$$\text{Minimum Nyquist Bandwidth} = 2 f_b = 2 \times 10 \times 10^6$$

$$= 20 \text{ MHz}$$

7. ***Binary data is transmitted using PSK at a rate 2 Mbps over RF link having bandwidth of 2 MHz. Find signal power required at receiver input so that error probability is less than or equal to 10^{-4} Assume noise PSD to be 10^{-10} watt/Hz.[Q (3.71) = 10^{-4}]***

Solution :

$$f_b = 2 \times 10^6 \qquad T_b = \frac{1}{f_b} = \frac{1}{2 \times 10^6}$$

$$P_e = 10^{-4}, \qquad \frac{N_0}{2} = 10^{-10} \text{ W/Hz}$$

$$Q(3.71) = 10^{-4}$$

Solution:

$$\text{Data Rate} = \frac{1}{T_b} = 10^6 \text{ bits / sec}$$

$$[\text{PSD}]_{\text{Noise}} = \frac{N_o}{2} = 10^{-10} \text{ watts / HZ}$$

$$p_e \leq 10^{-4}$$

Error probability of coherent binary FSK.

$$p_e = \frac{1}{2} erfc \sqrt{\frac{0.5E}{N_o}}$$

$$10^4 = \frac{1}{2} erfc \sqrt{\frac{0.5E}{N_o}}$$

$$erfc \sqrt{\frac{0.5E}{N_o}} = 2 \times 10^{-4}$$

$$1 - erfc \sqrt{\frac{0.5E}{N_o}} = 1 - 2 \times 10^{-4} = 0.9998$$

$$\sqrt{\frac{0.5E}{N_o}} = 2.8$$

$$\frac{0.5E}{N_o} = (2.8)^2 = 7.84$$

$$\frac{N_o}{2} = 10^{-10}$$

$$N_o = 2 \times 10^{-10}$$

$$\frac{0.5E}{2 \times 10^{-10}} = 7.84$$

$$E = \frac{7.84 \times 2 \times 10^{-10}}{0.5} = 3 \times 10^{-9} \times 10^6$$

For BPSK error probability is given as,

$$P_e = Q\sqrt{\frac{2E}{N_o}}$$

$$10^{-4} = Q\sqrt{\frac{2E}{N_o}}$$

$$\text{Since } Q(3.71) = 10^{-4}, \sqrt{\frac{2E}{N_o}} = 3.71$$

$$\frac{2E}{N_o} = (3.71)^2$$

$$E = (3.71)^2 \times \frac{N_o}{2} = (3.71)^2 \times 10^{-10}$$

$$\therefore pT_b = (3.71)^2 \times 10^{-10} \quad E = pT_b$$

$$p = \frac{(3.71)^2 \times 10^{-10}}{T_b}$$

$$= \frac{(3.71)^2 \times 10^{-10}}{T_b} = (3.71)^2 \times 10^{-10} \times 2 \times 10^6$$

$$= 2.753\, mW$$

8. ***Find the bit error probability for a BPSK system with bit rate of 1 Mbps. The received waveforms $s_1(t) = -A\cos\omega_0 t$ and $s_2(t) = -A\cos\omega_0 t$ are coherently detected with a matched filter. The value of A is 10 mV. Assume that noise power spectral density $N_o = 10^{-11}$ W/Hz and that signal power and energy per bit are normalized relative to 1Ω load $[Q(3.16) = 0.0008]$***

Solution:

$$r_b = 1 \times 10^6 \quad T_b = \frac{1}{r_b} = \frac{1}{1 \times 10^6}$$

$$A = 10\, mV = 10 \times 10^{-3}\, V$$

$$N_o = 10^{-10}\, W/Hz$$

$$E_b = \frac{1}{2}A^2 T_b = \frac{1}{2}\times(0.01)^2 \times \frac{1}{1\times 10^6}$$

$$= 5\times 10^{-11} J$$

$$\frac{E_b}{N_o} = \frac{5\times 10^{-11}}{10^{-11}} = 5$$

$$p_e = Q\sqrt{\frac{2E_b}{N_o}} = Q\sqrt{2\times 5} = Q(3.16)$$

$$= 0.0008 = 8\times 10^{-4}$$

9. ***In a digital CW communication system, the bit rare of NRZ data stream is 1 Mbps and carrier frequency of 100 MHz complete the symbol rate of transmission and bandwidth requirement of channel for i) BPSK system ii) QPSK system iii) Comment in result***

Given:

$$f_b = 1\text{Mbps},\ f_c = 100\text{MHz}$$

i) BPSK:

$$\text{symbol Rate} = \frac{1}{T_s} = \frac{1}{T_b} = f_b = 1\text{Mbps}$$

$$\text{Bandwidth} = 2\times f_b = 2\times 1\text{Mbps}$$

$$= 2\text{MHz}$$

(ii) QPSK

$$\text{For QPSK symbol Rate} = \frac{1}{T_s} = \frac{1}{2T_b} = \frac{f_b}{2}$$

$$= \frac{1\text{Mbps}}{2}$$

$$= 500\text{ Kbps}$$

$$\text{Bandwidth BW} = f_b = 1\text{MHz}$$

(iii) comment on result:

Bandwidth required by QPSK is half compared to BPSK.

Symbol rate of QPSK is also half compared to BPSK

10. *A QPSK signal is received at the input of a coherent optimal receiver with amplitude 1 mV and frequency 2 MHz. The signal is corrupted with white noise of PSD* 10^{-11} W/Hz. *If the data rate* 10^4 bits/sec *is find* $p_e \cdot [Q(2.23) = 0.0129]$

Solution:

$$A = 1mV = 1 \times 10^{-3}\, V$$

$$\frac{N_o}{2} = 10^{-11}\, W/Hz \Rightarrow r_b = \frac{1}{T_b} = 10^4\, bps$$

$$E_b = \frac{1}{2} A^2 T_b = \frac{1}{2}[1 \times 10^{-3}]^2 \times \frac{1}{10^4} = 5 \times 10^{-11}$$

$$\frac{E_b}{N_o} = \frac{5 \times 10^{-11}}{2 \times 10^{-11}} = 2.5$$

i) QPSK

$$p_e = 2Q\sqrt{\frac{2E_b}{N_o}} = 2Q\sqrt{2 \times 2.5} = 2Q(2.23)$$

$$= 2 \times 0.0129$$

$$= 0.0258$$

11. *A binary frequency shift keying system employs two signaling frequencies* f_1 *and* f_2. *The lower frequency* f_1 *is 1200 Hz and signaling rate is 500 baud calculate* f_2.

For BFSK,

$$f_b = 500\, Hz$$

FM modulation index $\dfrac{|f_m - f_s|}{f_b} = h = 1$

$$|f_m - f_s| = f_b$$

Since

$$f_s = f_1 = 1200\, Hz$$

$$f_m - 1200 = 500$$

$$f_m = 1700\, Hz$$

4.9.2015 UNIT-V - ERROR CONTROL CODING

The physical medium through the messages are transmitted are called channels. The channels are pronto different kind of noises such as lightening, human error, equipment defects, etc. In digital communication system the information transmitted from one end to another depends on,

* Transmitted signal power (E_b)
* Channel Bandwidth

In digital communication systems, there is a limit to value of E_b/N_0, where N_0 is the noise spectral density. Error control coding is needed to achieve good data quality at the receiver with a limit on E_b/N_0 (SNR). They are useful for accurate transfer of information.

Some of ECC (Error control codes) add redundancy in the form of extra symbols to a message prior to transmission.

Advantages of ECC:

* Reduces the required E_b/N_0 for a fixed bit error rate
* Reduces the transmitted power where E_b/N_0 is low.
* Reduces the size of the antenna for radio communication

* Reduces the hardware cost.

Disadvantages of ECC:

* Addition of redundancy increases the transmission Bandwidth.
* Increases the complexity in implentation of decoder.

Channel Coding:

ECC is used to achieve error performance which include considerations of bandwidth & system complexity. Since these ECC's are used to overcome the effects of noise in the channel, the encoding procedure is called channel coding.

Objectives of ECC:

* Capacity to rectify more errors.
* Fast & efficient encoding of message.
* Fast & efficient decoding of message.
* Maximum information rate.

ECC
- Block codes
  - Linear Block codes
  - Cyclic codes
- Convolution codes

Block codes:

The channel encoder accepts information from the source encoder in successive k-bit blocks. For each block, n-k redundant bits are added which are algebrically related in the k^{th} message bit, thereby, producing an overall ~~bit~~ encoded block of 'n'. The n-bit code is called as code word. n - length of the code word. The channel encoder produces the bits at the rate of R_0,

$$R_0 = \left(\frac{n}{k}\right) R_s$$

R_s = bit rate of the source.

Code rate, $r = \frac{k}{n}$

Algorithm for Encoding:

Step1: Write the message vector $m_{1 \times k}$ (written in rows & columns ie matrix)

Step2: Compute the parity matrix $P_{k \times n-k}$

Step3: Compute the generator matrix.

$$G = \left[P_{k \times n-k} \, . \, I_{k \times k} \right]$$

$I_{k \times k} \rightarrow$ Identity matrix.

Step4: Compute the code word,

$$C = mG$$

Algorithm for decoding:

Step1: Compute Parity check matrix.

$$H_{n-k \times n} = \left[I_{n-k \times (n-k)} \;\vdots\; P^T_{(n-k) \times n} \right]$$

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Step2: Let the received data be 'r'.

Step3: Compute the Syndrome, $S = r H^T$

Step4: Construct the decoding table.

Step5: Find the error pattern 'e'.

Step6: Correct the error by adding the error pattern with the received vector.

1. Construct a (7,4) Linear Block code having the parity information as, $b_0 = m_0 + m_1 + m_2$; $b_1 = m_0 + m_2 + m_3$; $b_2 = m_1 + m_2 + m_3$. Find the code of word for the message vector, m = 1110 & correct the error that has occurred in C_5 position.

→ length of code word (n)
→ message bits (k)
= (n, k)

Soln:

The no. of message bits = k = 4.

The length of the code word, n = 7

The no. of parity bits = n - k
= 7 - 4
= 3

Bits that we add are called parity

Encoding:

1) $m_{1\times k} = [1\ 1\ 1\ 0]_{1\times 4}$

2) Parity matrix:

$$P_{k\times n-k} = P_{4\times 3} = \begin{array}{c} \\ m_0 \\ m_1 \\ m_2 \\ m_2 \end{array} \begin{array}{c} \begin{array}{ccc} b_0 & b_1 & b_2 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{4\times 3} \end{array}$$

3) Generator matrix:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 7}$$

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7) $C = mG$

$$= [1\ 1\ 1\ 0]_{1\times4} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{4\times7}$$

$$= [1\oplus1\oplus1\oplus0 \quad 1\oplus0\oplus1\oplus0 \quad 0\oplus1\oplus1\oplus0 \quad 1\oplus0\oplus0\oplus0$$
$$0\oplus1\oplus0\oplus0 \quad 0\oplus0\oplus1\oplus0 \quad 0\oplus0\oplus0\oplus0]$$

$$C = \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\ \end{array}$$
$$C = [\underbrace{1\quad 0\quad 0}_{\text{Parity bits}}\quad \underbrace{1\quad 1\quad 1\quad 0}_{\text{Message bits}}]$$

Due to the channel noise, the error has occurred in C_5^{th} position. $\therefore$ The received vector,

$$r = [1\ 0\ 0\ 1\ 1\ \underline{\underline{0}}\ 0]$$

[error: 1 is changed to 0]

Decoding: 1) $r = [1\ 0\ 0\ 1\ 1\ 0\ 0]$

$$H = [I_{3\times3} \mid P^T_{3\times4}]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$S = rH^T$

$$S = [1\ 0\ 0\ 1\ 1\ 0\ 0]_{1\times7} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{7\times3}$$

$S = [1 \oplus 0 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \quad 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \quad 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0]$

$S = [1 \quad 1 \quad 1]_{1\times 3}$

Decoding table:

Syndrome	Error pattern
0 0 0	0 0 0 0 0 0 0
1 0 0	1 0 0 0 0 0 0
0 1 0	0 1 0 0 0 0 0
0 0 1	0 0 1 0 0 0 0
1 1 0	0 0 0 1 0 0 0
1 0 1	0 0 0 0 1 0 0
S ← $\boxed{1\ 1\ 1}$	$\boxed{0\ 0\ 0\ 0\ 0\ 1\ 0}$
0 1 1	0 0 0 0 0 0 1

Syndrome = HT

$\Rightarrow e = 0000010$

5) The error pattern,

$e = 0000010$

6) $c = r + e$

$r = 100\,1100$

$e = 0000010$

$$c = \begin{array}{r} 100\ 1100 \\ (+)\ 000\ 0010 \\ \hline 100\ 1110 \end{array} \rightarrow c$$

∴ We have received the original data transmitted

$c = 1001110$

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Hamming distance:

It is defined as no. of locations in which the elements of two code vectors differ. It is represented by $d(m, q)$

$m = [1 0 1 0 0 1 0]$

$q = [0 \underbrace{0 1}_{\text{same bit}} 1 \underbrace{0 1 0}_{\text{same bit}}]$ $\therefore d(m, q) = 2$

Hamming weight:

It is defined as no. of non-zero elements in a code vector. It is denoted as w_m.

Minimum hamming distance, "d_{min}" is the smallest hamming weight of all the non-zero code vectors.

The total no. of 1's.

$m = [1 0 1 0 0 1 0]$

$\Rightarrow$ Hamming weight $w_m = 3$.

$q = [0 0 1 1 0 1 0]$

$w_m = 3$

Find the minimum hamming weight among the following.

$m = [1 1 1 0 0 1 1] = 5$

$q = [0 0 1 1 0 1 1] = 4$

$v = [1 1 0 0 0 1 0] = 3$

$c = [0 0 0 0 0 0 0] = 0$

For finding the minimum haming weight the non-zero code vector, which contain non-zero element should only we consider,

$d_{min} = 3$

Syndrome & its properties.

A syndrome 'S' is defined as a $1 \times n-k$ for a (n,k) linear block code. It contains the information above the error pattern and is used for error detection & correction. It is calculated by the formula,

$$S = r H^T$$

where, r - received vector.

H^T - parity check matrix

If $S \neq 0$, then $r \neq c$ and says that error has occurred.

$S = 0$, $r = c$, and says that no error has occurred.

Property 1: The syndrome depends only on the error pattern and not on the transmitted code word.

we know that,

$$c = mG$$

$$r = c + e$$

$$S = r H^T$$

$$S = (c+e) H^T$$

$$S = cH^T + eH^T$$

$$S = mG H^T + eH^T$$

As per linear block code technique (LBC), $GH^T = 0$

$$\therefore S = eH^T$$

Property 2:

All the error patterns that differ by a code word have the same syndrome.

c_i = new error pattern

Proof: Let $e_i = e + c_i$

we know that, $S = e_i H^T$ from property - I.

$$S = (e + c_i)\, H^T$$

$$= eH^T + c_i H^T \qquad \because c_i = m_i G$$

$$= eH^T + m_i G H^T$$

By LBC, $GH^T = 0$

$$\therefore S = eH^T$$

1) All the linear block codes satisfy the condition, $G^T H = H^T G = 0$

2) Closure Property:

States that sum of any two code words is a code word.

Consider a (6,3) LBC, with a parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

Find the generator matrix and all possible code words

Solu:

The length of code word, $n = 6$

The no: of message bits, $K = 3$

The no: of parity bits, $n - k = 6 - 3$
$= 3$

As the no: of message bits is '3', the possible message vectors are, $m = 2^3 = 8$

$m =$ 000
001
010
011
100
101
110
111

$$H_{3\times 7} = \left[\, I_{3\times 3} \;\vdots\; P^T_{3\times 4} \,\right]$$

$$\begin{array}{c} \quad P \qquad\qquad I \\ H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

To find Generator matrix:

$$G = \left[\, P_{k\times n-k} \;\vdots\; I_{k\times k} \,\right]$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Note:

From the parity check matrix, H, it is clear that, P^T & P are the same ∴ The parity check matrix G, will be same

If message contains all zeros then the code words are also all zeros.

To find code words:

$C = mG$

$$c_1 = [0\ 0\ 1] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 1\ 1\ 0\ 0\ 1]$$

$$c_2 = [0\ 1\ 0] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [0\ 1\ 1\ 0\ 1\ 0]$$

$$c_3 = [0\ 1\ 1] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 0\ 0\ 0\ 1\ 1]$$

$$c_4 = [1\ 0\ 0] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 0\ 1\ 1\ 0\ 0]$$

0 0 1
0 1 1
0 1 1
0 0 0
0 1 0
0 0 1

1 0 0
0 0 0
1 0 0
1 0 0
0 0 0
0 0 0
0 0 0

$$c_5 = [1\,0\,1]\begin{bmatrix}1&0&1&1&0&0\\0&1&1&0&1&0\\1&1&1&0&0&1\end{bmatrix}$$

$$= [0\;1\;0\;1\;0\;1]$$

$$c_6 = [1\,1\,0]\begin{bmatrix}1&0&1&1&0&0\\0&1&1&0&1&0\\1&1&1&0&0&1\end{bmatrix}$$

$$= [1\;1\;0\;1\;1\;0]$$

$$c_7 = [1\,1\,1]\begin{bmatrix}1&0&1&1&0&0\\0&1&1&0&1&0\\1&1&1&0&0&1\end{bmatrix}$$

$$= [0\;0\;1\;1\;1\;1]$$

To find syndrome:

Consider, c_7 & c_5 bit in it,

$c_7 = [0\,0\,1\,1\,1\,1]$

c_5^{th} bit of c_7

$\Rightarrow r = [0\,0\,1\,1\,1\,\underline{\underline{0}}]$

$$S = rH^T$$

$$= [0\,0\,1\,1\,1\,0]\begin{bmatrix}1&0&1\\0&1&1\\1&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$S = [0\;\;0\;\;1]_{1\times 3}$$

Decoding table:

Syndrome	Error Pattern
0 0 0	0 0 0 0 0 0
1 0 1	1 0 0 0 0 0
0 1 1	0 1 0 0 0 0
1 1 1	0 0 1 0 0 0
1 0 0	0 0 0 1 0 0
0 1 0	0 0 0 0 1 0
0 0 1	0 0 0 0 0 1

$e = 000001$

$c = r + e$

$$\begin{array}{rl} c = \quad 001110 & \rightarrow r \\ 000001 & \rightarrow e \\ \hline 001111 & = c_7 \end{array}$$

$\therefore c_7 = 001111$

$\therefore$ we received the original signal transmitted.

Cyclic Codes:

It is a code in which cyclic shift of a code word produces another code word.

Advantages of cyclic codes:

Easy to encode

They possess a well-defined mathematical structure, thereby, providing a efficient decoding scheme.

Properties:

Linear Property:

The sum of any two code words is also a code word.

Cyclic property:

The cyclic shift of a code word is also a code word (ie)

Consider a (n, k) block code having a code word $\{c_0, c_1, \ldots c_{n-1}\}$

It is said to be cyclic iff $\{c_{n-1}, c_0, c_1, \ldots c_{n-2}\}$, $\{c_{n-2}, c_{n-1}, \ldots c_{n-3}\}$ is also a code word.

Representation of a codeword as a polynomial:

$$c_x = c_0 + c_1 x + \ldots\ldots + c_{n-1} x^{n-1}$$

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Advantages of representing a code word as polynomial:

1. These are algebric codes, hence the algebric operations such as addition, subtraction, multiplication & division are very simple.

2. The position of the bits are represented with the help of powers of x in a polynomial (ie) $x^{n-1} \rightarrow MSB$; $x^0 \rightarrow LSB$.

Message Bit as polynomial:

$$m(x) = m_0 + m_1 x + m_2 x^2 + \dots + m_{k-1} x^{k-1}$$

Parity bit as a polynomial:

$$b(x) = b_0 + b_1 x + \dots + b_{n-k-1}$$

$$\therefore C(x) = b(x) + x^{n-k} m(x)$$

Generator Polynomial: [G(x)]

It is a polynomial of degree (n-k), which is a factor of $x^n + 1$

$$G(x) = 1 + \sum_{i=1}^{n-k-1} g_i x^i + x^{n-k}$$

$$g_i = 0 \text{ or } 1$$

Generator matrix: $[G_{k \times n}]$

The generator matrix is constructed using 'k' polynomials (ie), $g(x), xg(x), \dots x^{k-1} g(x)$

$$G_{k \times n} = \begin{bmatrix} g(x) \\ x g(x) \\ \vdots \\ x^{k-1} g(x) \end{bmatrix}$$

Parity check polynomial:

$$h(x) = 1 + \sum_{i=1}^{k-1} h_i x^i + x^k$$

$h_i = 0$ or 1

Parity check matrix:

$H_{n-k \times n}$

This matrix is constructed using $(n-k)$ polynomials where, the polynomials are,

$$x^k h(x^{-1}),\ x^{k+1} h(x^{-1}), \ldots\ x^{n-1} h(x^{-1})$$

These are used as rows in matrix represented as shown,

$$\begin{bmatrix} x^k h(x^{-1}) \\ \vdots \\ x^{n-1} h(x^{-1}) \end{bmatrix}$$

Encoding Algorithm for Cyclic Codes:

Step1: Factorize x^n+1 into irreducible polynomials of degree 1.

Step2: Find the primitive polynomial of degree m. It satisfies the condition, $n = 2^m - 1$

Step3: Assign, one of the primitive polynomial, as generator polynomial, $g(x)$.

Step4: Assign the other remaining polynomials as parity check polynomial as, $h(x)$.

Step5: Write the message polynomial, $m(x)$

Step6: Find the product of $m(x) \cdot x^{n-k}$

Step7: Divide $m(x) \cdot x^{(n-k)}$ by $g(x)$.

$$m(x)\, x^{(n-k)} / g(x)$$

Step8: Assign the remainder as $b(x)$

Step 4: Find the code polynomial,

$$C(x) = b(x) + x^{n-k} \cdot m(x)$$

1. Find the code word for the message 1001 by constructing a (7,4) cyclic code. (Do this for generator poly)

Soln:

$n = 7$

$k = 4$

1) $x^n + 1$

$\Rightarrow x^7 + 1$

$m=1$ $\quad$ $m=3$ $\quad$ $m=3 \to$ degree

$= (1+x)(1+x+x^3)(1+x^2+x^3)$

2) $n = 2^m - 1$

for, $m=1$;

$n = 2^1 - 1 = 2-1 = 1 \neq 7 \neq n$

$\therefore (1+x)$ is not primitive,

for $m=3$, $n = 2^3 - 1 = 8-1 = 7 = n$

$\therefore 1+x+x^3$ is primitive polynomial.

Similarly,

$1+x^2+x^3$, is also a primitive polynomial.

as, $2^m - 1 = 2^3 - 1 = 7 = n$

3) $g(x) = 1+x+x^3$ (Select the one with lower order)

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4) $h(x) = (1+x)(1+x^2+x^3)$

$= 1+x^2+x^3+x+x^3+x^4$

$= 1+x+x^2+2x^3+x^4$

Neglect $2x^3$, since the coefficients of x should be either '1' or 0.

$\therefore h(x) = 1+x+x^2+x^4.$

5) $m = 1001$

$\therefore m = 1\times x^0 + 0\times x^1 + 0\times x^2 + 1\times x^3$

$= 1+x^3$

6) $m(x) \cdot x^{n-k}$

$= (1+x^3)\cdot x^{(7-4)} = (1+x^3)(x^3)$

$= x^3+x^6$

7) $m(x)(x^{n-k})/g(x)$

$$= \frac{x^3+x^6}{1+x+x^3}$$

$$1+x+x^3 \;\big)\; x^3+x^6 \;\big(\; x^3-x$$

$$\frac{\overset{(-)}{x^3} + \overset{(-)}{x^6} + \overset{(-)}{x^4}}{-x^4}$$

$$\frac{\overset{(+)}{-x^4} - \overset{(+)}{x^2} - \overset{(+)}{x}}{x^2+x}$$

8) $b(x) = x^2+x$

9) $c(x) = b(x) + m(x)(x^{n-k})$

$= x+x^2+x^3+x^6$

$\therefore c(x) = (0\,1\,1\,1\,0\,0\,1)$

Find the generator matrix and parity check matrix for the above problem.

Solu:

Generator matrix:

$$\begin{bmatrix} g(x) \\ x(g(x)) \\ x^2\, g(x) \\ x^3\, g(x) \\ \vdots \\ x^{k-1}(g(x)) \end{bmatrix} = \begin{bmatrix} 1+x+x^3 \\ x+x^2+x^4 \\ x^2+x^3+x^5 \\ x^3+x^4+x^6 \end{bmatrix} \begin{bmatrix} g(x) \\ x\, g(x) \\ x^2\, g(x) \\ x^3\, g(x) \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc|cccc} & P & & & I & & \end{array} \\ G = & \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \end{array}$$

Since we did not get the identity matrix

∴ The generator matrix is not systematic.

Decoding of Cyclic codes:

The code word $c_0, c_1, \ldots c_{n-1}$ is transmitted over a noisy channel which results in a received word $r_0, r_1, \ldots r_{n-1}$

This received word can be represented as a polynomial

$$r(x) = r_0 + r_1 x + \ldots + r_{n-1} x^{n-1}$$

$$\frac{r(x)}{g(x)} = q(x) + \frac{s(x)}{g(x)}$$

$$r(x) = q(x)\, g(x) + s(x)$$

where, $s(x) \rightarrow$ remainder of Syndrome.

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1. Find the parity check matrix for the above problem.

Solu:

$$H(x) = \begin{bmatrix} x^k h(x^{-1}) \\ x^{k+1} h(x^{-1}) \\ \vdots \\ x^{n-1} h(x^{-1}) \end{bmatrix}$$

$[n=7$
$k=3]$
$[n-k=4]$

we know that,

$h(x) = 1+x+x^2+x^4$ (from previous problem)

$$h(x^{-1}) = \left(1+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^4}\right)$$

$$= \frac{x^4+x^3+x^2+1}{x^4}$$

$$H(x) = \begin{bmatrix} x^4\left(\frac{x^4+x^3+x^2+1}{x^4}\right) \\ x^5\left(\frac{x^4+x^3+x^2+1}{x^4}\right) \\ x^6\left(\frac{x^4+x^3+x^2+1}{x^4}\right) \end{bmatrix}$$

$$= \begin{bmatrix} x^4+x^3+x^2+1 \\ x^5+x^4+x^3+x \\ x^6+x^5+x^4+x^2 \end{bmatrix}$$

$$\therefore H(x) = \left[\begin{array}{ccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{array}\right] \Rightarrow H(x) = [\underbrace{I_{3\times3}}_{I_{n-k}} \mid P]$$

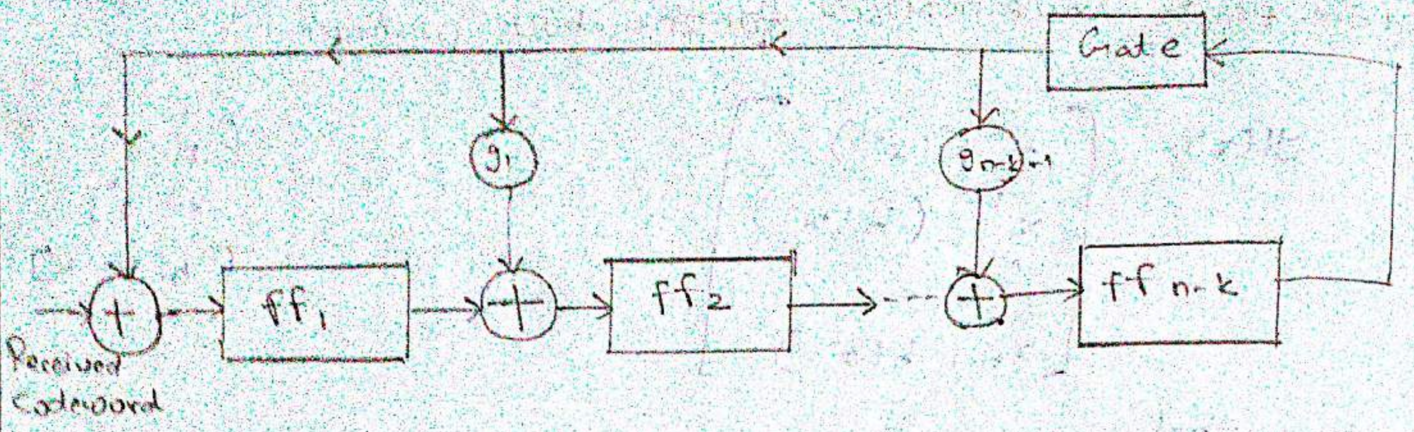
$I \qquad P$

$\therefore H(x)$ is not systematic

Structure of decoder:

No. of flip flops $= n-k$

no. of modulo-2 $= n-k-1$



1. Find the syndrome using syndrome calculator & decode the cyclic code (7, 4)

Modulo-2 adders- Adders performing Ex-OR operation

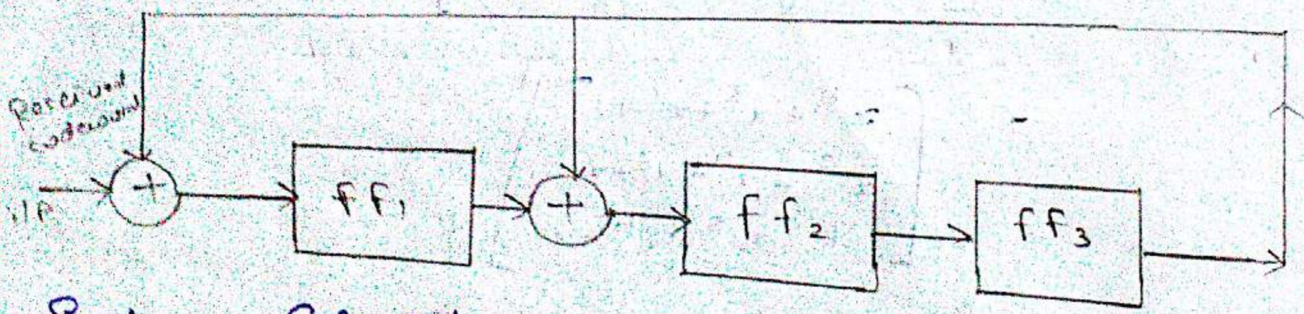
Solu:

for (7, 4)

no: of flip flops = 7 − 4 = 3

no: of adders = 7 − 4 − 1 = 2

1) adder
2) flipflop
3) last flip flop o/p should be to the 1st adder.



Syndrome Calculator:

I/P	I/P to ff₁ o/p of ff₃ + I/P	I/P to ff₂ o/p of ff₃ + o/p of ff₁	I/P to ff₃ o/p of ff₂
0	0	0	0
1	1 ⊕ 0 = 1	0 ⊕ 0 = 0	0
0	0 ⊕ 0 = 0	1 ⊕ 0 = 1	0
0	0 ⊕ 0 = 0	0 ⊕ 0 = 0	1
0	0 ⊕ 1 = 1	1 ⊕ 0 = 1	0
1	0 ⊕ 1 = 1	0 ⊕ 1 = 1	1
1	1 ⊕ 1 = 0	1 ⊕ 1 = 0	1
0	1 ⊕ 0 = 1	1 ⊕ 0 = 1	0

1st assume from LSB of the code word

o/p → previous o/p → add with previous o/p

Code word: 0111001 LSB

If C_3 is error,

r = 0110001

$S = [1\ 1\ 0]$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Symbol	Error pattern
000	0 000000
101	1000000
010	0100000
101	0010000
110	0001000
111	0000100
011	0000010
001	0000001

$$\begin{aligned} r &= 0110001 \\ (+)\ e &= 0001000 \\ \hline c &= 0111001 \end{aligned}$$

Properties of syndrome in cyclic codes:

1) The syndrome of the received word polynomial is also the syndrome of the corresponding error polynomial.

2) The syndrome of the received word polynomial xr, is the syndrome for cyclic shifted received word polynomial.

3) The syndrome polynomial is identical to the error polynomial assuming that the errors are confined to n-k parity bits.

Convolution codes:

By default assume G_1 for the antenna 1 to generator sequence is given

Given the generator sequence $\underbrace{G_1 = 1 + x^2, \; G_2 = 1 + x + x^2}$. And the message bits at 10101. Find the code word with the help of encoder diagram.

$G_1 = 1x^0 + 0x^1 + 1x^2 = \{1\,0\,1\}$

$G_2 = 1x^0 + 1x^1 + 1x^2 = \{1\,1\,1\}$

$k = 3$

Solu:

Convert the generator sequence G_1 & G_2 into bit format. (write the co-efficients of the polynomial given above)

$G_1 = \{1\,0\,1\}$

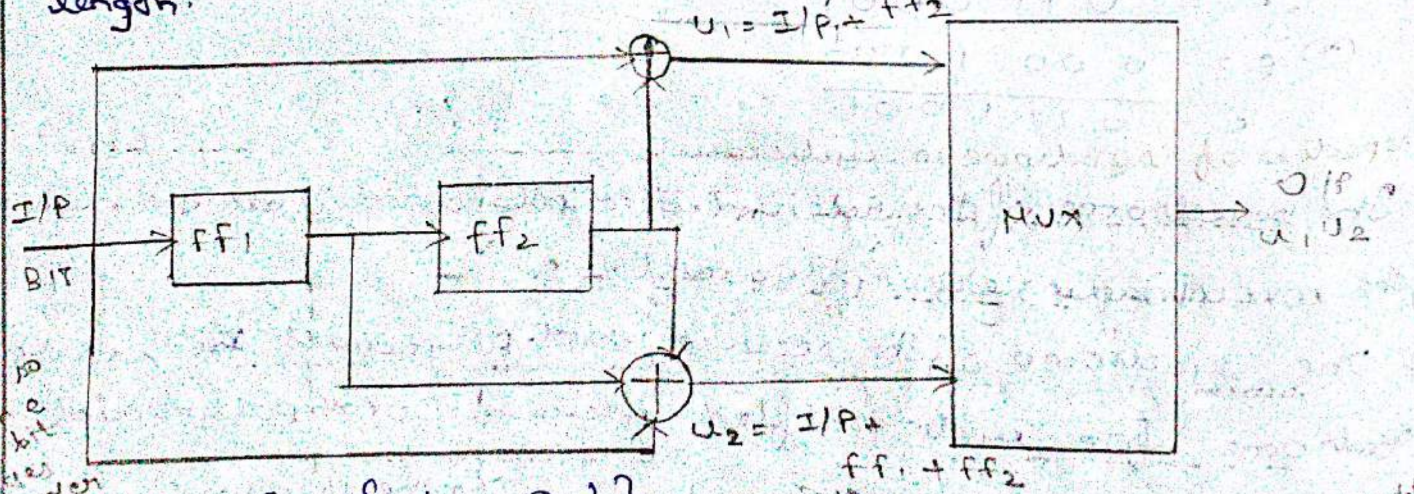
$G_2 = \{1\,1\,1\}$

No: of adders: $k-1$

Constraint Length $= 3 = k$

To draw the encoder diagram flip flop, modulo-2 adder & multiplexer is required. The no: of flip flop & the no: of modulo-2 adders is equal to $k-1$, where 'k' is the constraint length.

→ Encoder Diagram



$G_1 = \{\underset{I/P}{1} \;\; \underset{ff_1}{0} \;\; \underset{ff_2}{1}\}$ → Previous value

State Table:

Message bits (i/p)	State ff₁	State ff₂	u_1: I/P + FF₂	u_2: I/P + FF₁ + FF₂	Mux O/P u_1	Mux O/P u_2
Initial	0	0				
1	1	0	$1 \oplus 0 = 1$	$1 \oplus 0 \oplus 0 = 1$	1	1
0	0	1	$0 \oplus 0 = 0$	$0 \oplus 1 \oplus 0 = 1$	0	1
1	1	0	$1 \oplus 1 = 0$	$1 \oplus 0 \oplus 1 = 0$	0	0
0	0	1	$0 \oplus 0 = 0$	$0 \oplus 1 \oplus 0 = 1$	0	1
	1	0	$1 \oplus 1 = 0$	$1 \oplus 0 \oplus 1 = 0$	0	0

General representation

$G_1 = \{\underset{I/P}{1} \;\; \underset{ff_1}{0} \;\; \underset{ff_2}{1}\}$

$\Rightarrow u_1 = I/P + ff_2$

$G_2 = \{\underset{I/P}{1} \;\; \underset{ff_1}{1} \;\; \underset{ff_2}{1}\}$

$\Rightarrow u_2 = I/P + ff_1 + ff_2$

2

∴ Codeword = {11, 01, 00, 01, 00}

In order to find the code word using a "code tree or Trellis" the state table that defines all the possibilities of the i/p & the states should be calculated.

2. Find the codeword for message 10101 having $G_1 = 1 + x^2$ & $G_2 = 1 + x + x^2$ using code tree & trellis.

Soln:

Since the convolutional code for a single bit is a dibit there are four possibilities for a single bit. (00, 01, 10, 11). Each of these states are assigned a variable as,

a → 00
b → 01
c → 10
d → 11

State Table:

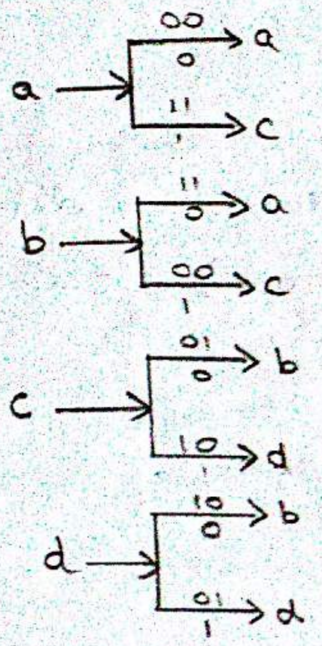
Message bit	Present state			Next state		output	
						i/p + ff2	i/p + ff1 + ff2
	ff1	ff2		ff1	ff2	u_1	u_2
0	0	0	a	0	0 → a	0	0
Ex-OR 1	0	0		1	0 – c	1	1
Ex-OR 0	0	1	b	0	0 – a	1	1
1	0	1		1	0 – c	0	0
0	1	0	c	0	1 – b	0	1
1	1	0		1	1 – d	1	0
0	1	1	d	0	1 – b	1	0
1	1	1		1	1 – d	0	1

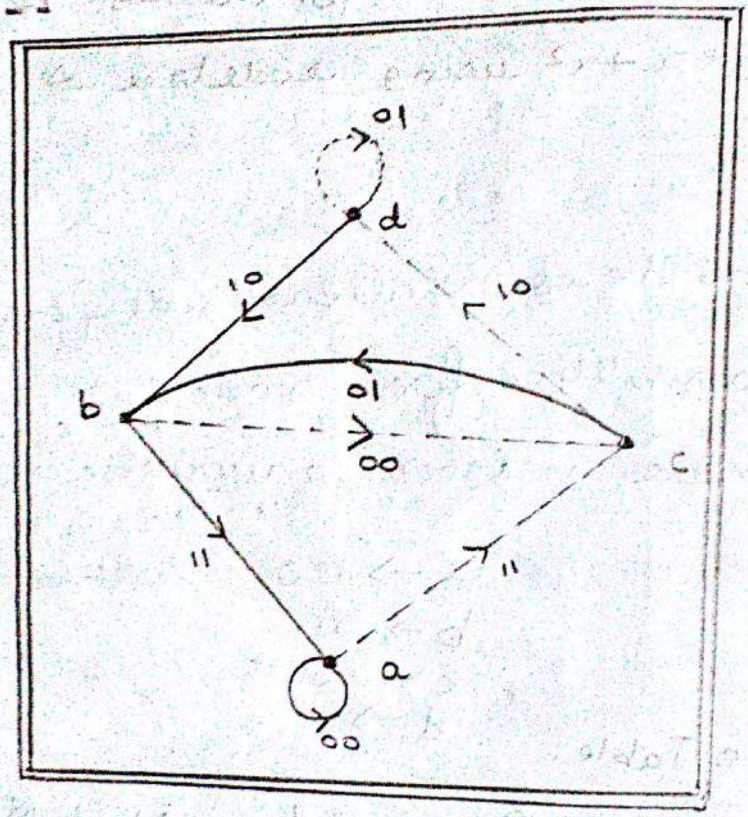
State Diagram:

Consider 'a' to be as your initial state. If your initial bit is zero then draw a bold line to next state if the initial bit is one draw a dotted line to next state.

Message bit is 0 ———

Message bit is 1 - - - -





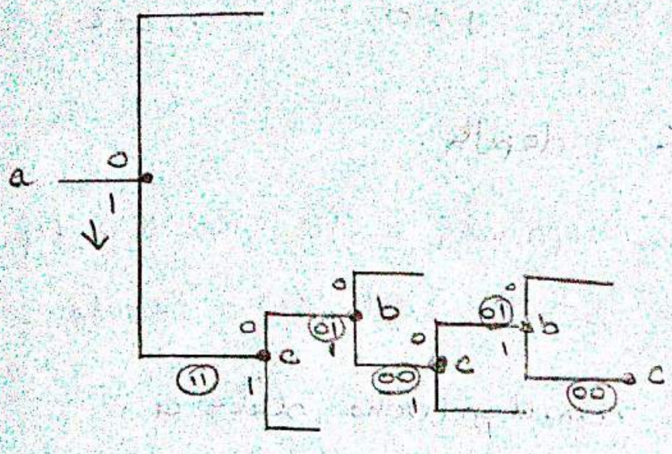
Code tree:

Start with a initial state 00 & the two possibilities of message 0's & 1's and representing using bifocation.

i.e) when message (MSB) is '0' the upper part of the tree is only drawn, similarly when the message (MSB) is '1' the lower part of the tree is only drawn. And the o/p for each transition of state is written on the line connecting the 2 states.

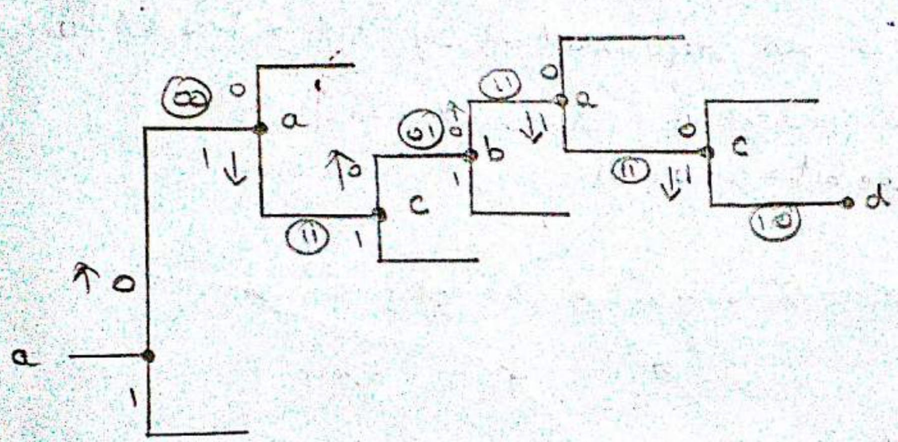
For 16-Mark complete this Code tree.

Message = 10101



∴ Codeword = {11, 01, 00, 01, 00}

3. Draw a code tree for message 010011



∴ codeword = {00, 11, 01, 11, 11, 10}

Trellis:

It is another method to find the code word for a convolutional code. A trellis is a tree like structure with remerging branches. A code branch produced by i/p message '0' is drawn as a solid line. A code branch produced by i/p message '1' is drawn as dashed line.

A trellis is a more instructive structure than a code tree. It brings out explicitly that the convolutional encoder is a finite state machine. The trellis contains 'L+K' levels where 'L' is the length of the incoming message and 'K' – constraint length of code.

The levels of trellis are, j = 0, 1, ... , L+K-1

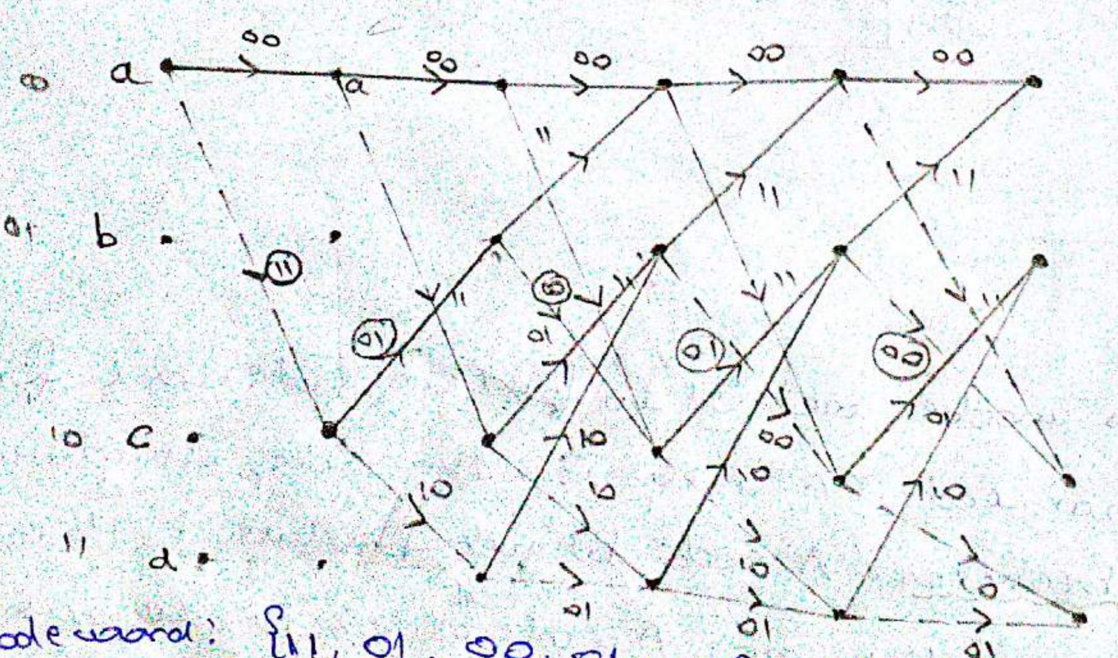
The level 'j' is referred as depth.

The first 'K-1' levels corresponds to the encoders departure from the initial stage 'a' & the last 'K-1' levels, corresponds to the encoders written to the state 'a'.

All the states of the encoder are labelled as a, b, c, d.

The left nodes represents the current state & right node represents next state. A transition from one state to another for i/p '0' is represented by "solid branch" & for a i/p '1' it is represented as "dashed line"

Message bit = 10101



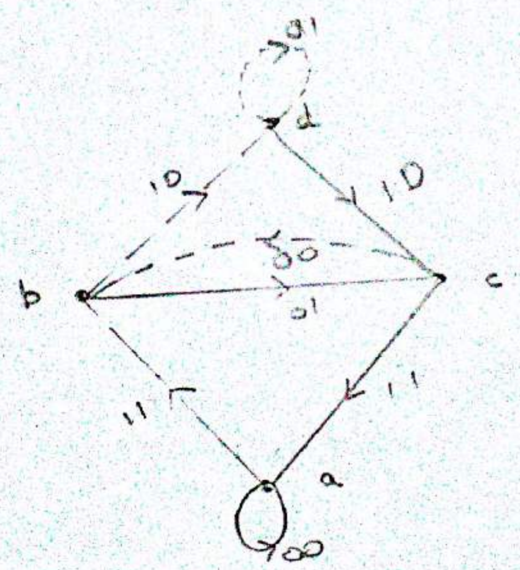
Codeword: {11, 01, 00, 01, 00} → Message = 10101

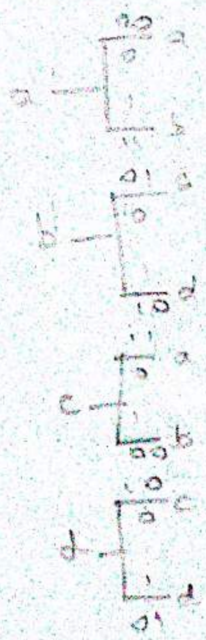
Codeword: {11, 10, 10, 00, 01} → Message = 11010

Construct a state table, state diagram, code tree, trellis for a → 00, b → 10, c → 01, d → 11. Find codeword for message 1011.

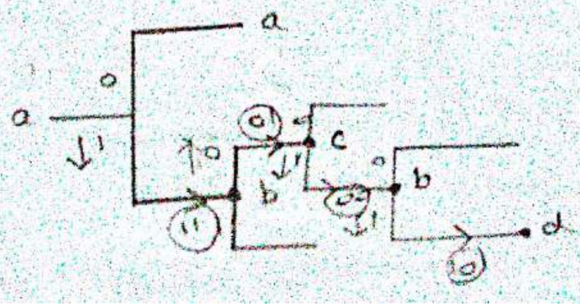
Message bits	Present state ff1	Present state ff2		Next state ff1	Next state ff2	Output u_1 i/p + ff2	Output u_2 i/p+ff1+ff2
0	0	0	a	0	0 - a	0	0
1	0	0		1	0 - b	1	1
0	1	0	b	0	1 - c	0	1
1	1	0		1	1 - d	1	0
0	0	1	c	0	0 - a	1	1
1	0	1		1	0 - b	0	0
0	1	1	d	0	1 - c	1	0
1	1	1		1	1 - d	0	1

State Diagram:



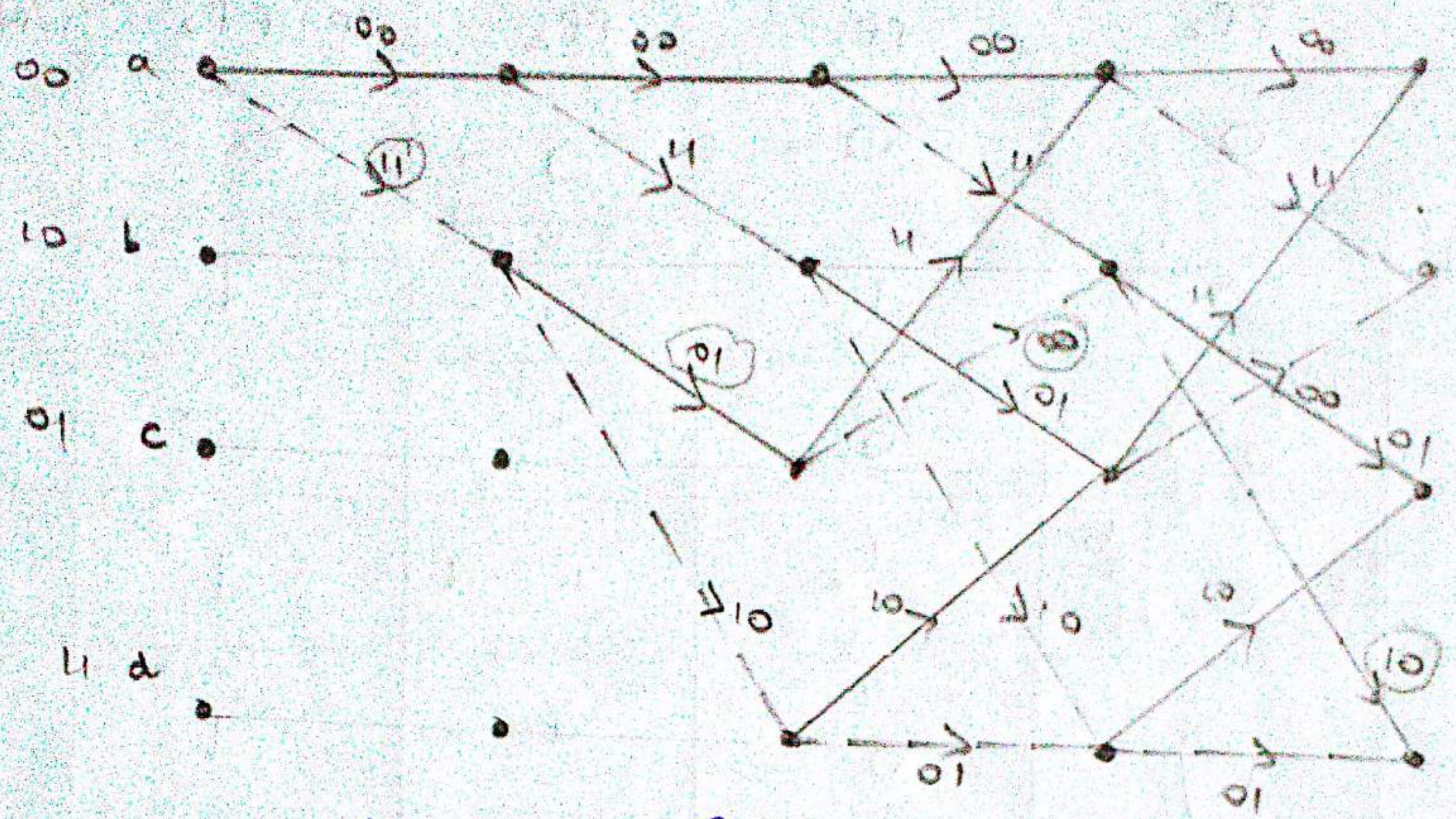


Code tree:- Message : 1011



∴ Codeword = {11, 01, 00, 10}

Trellis:



∴ Codeword = {11, 01, 00, 10}