MOHAMED SATHAK
A.J. COLLEGE OF ENGINEERING

Engineering Mechanics

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AP-Mechanical

Engineering Mechanics

Rigid-body Mechanics

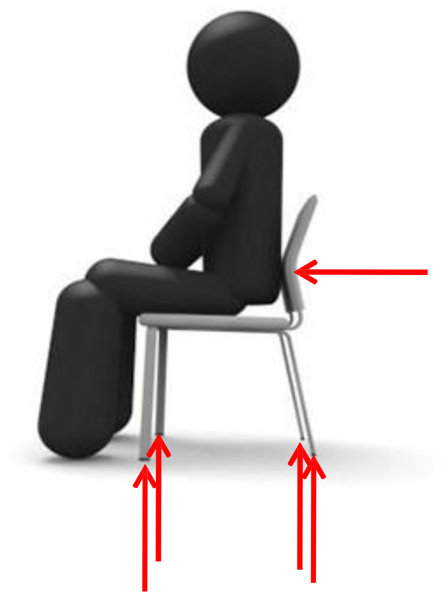
- a basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids (advanced courses).
- essential for the design and analysis of many types of structural members, mechanical components, electrical devices, etc, encountered in engineering.

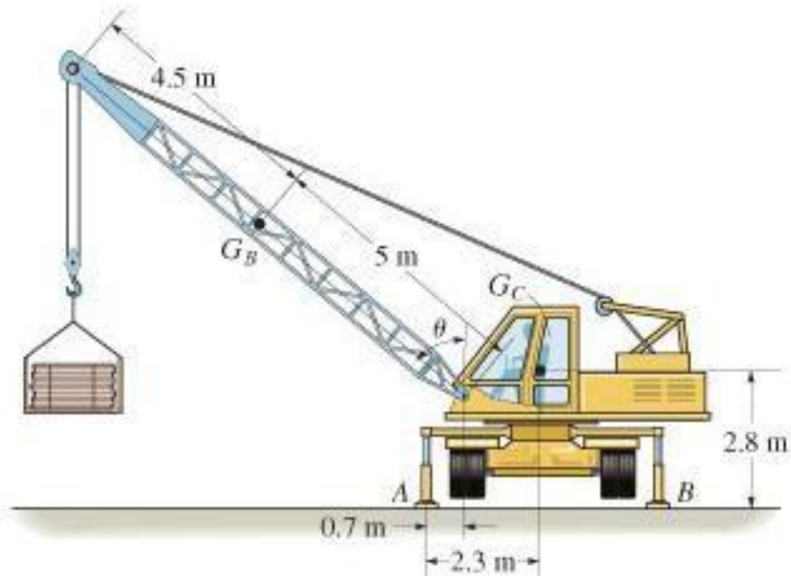
A rigid body does not deform under load!

Engineering Mechanics

Rigid-body Mechanics

Statics: deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).

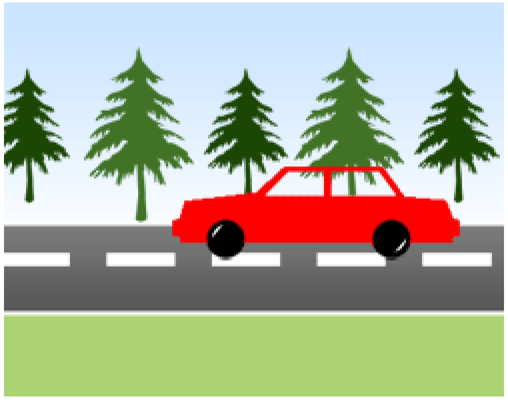


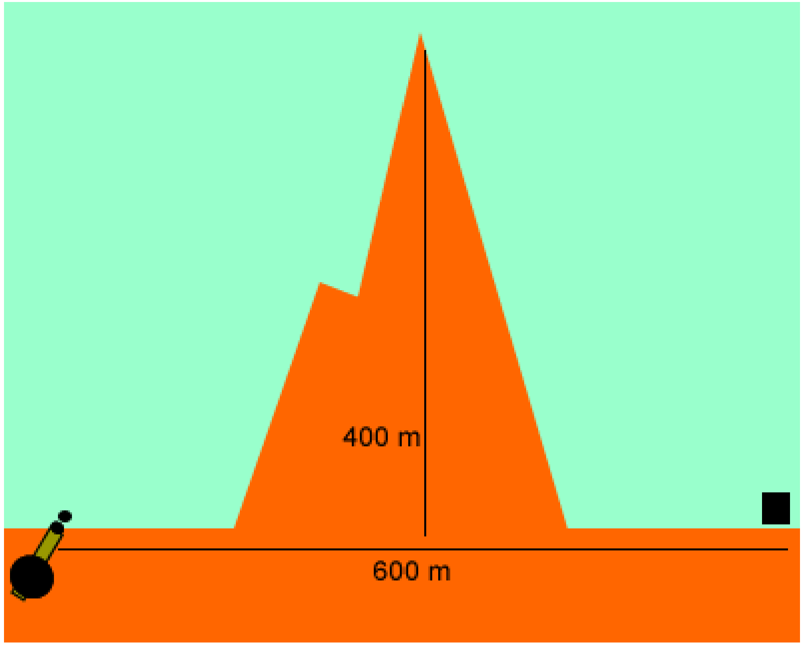


Engineering Mechanics

<u>Rigid-body Mechanics</u>

- Dynamics: deals with motion of bodies (accelerated motion)





Mechanics: Fundamental Concepts

Length (Space): needed to locate position of a point in space, & describe size of the physical system € Distances, Geometric Properties

Time: measure of succession of events € basic quantity in Dynamics

Mass: quantity of matter in a body € measure of inertia of a body (its resistance to change in velocity)

Force: represents the action of one body on another € characterized by its magnitude, direction of its action, and its point of application

€ Force is a Vector quantity.

Mechanics: Fundamental Concepts

Newtonian Mechanics

Length, Time, and Mass are absolute concepts
independent of each other

Force is a derived concept
not independent of the other fundamental concepts.
Force acting on a body is related to the mass of the body
and the variation of its velocity with time.

Force can also occur between bodies that are physically separated (Ex: gravitational, electrical, and magnetic forces)

Mechanics: Fundamental Concepts

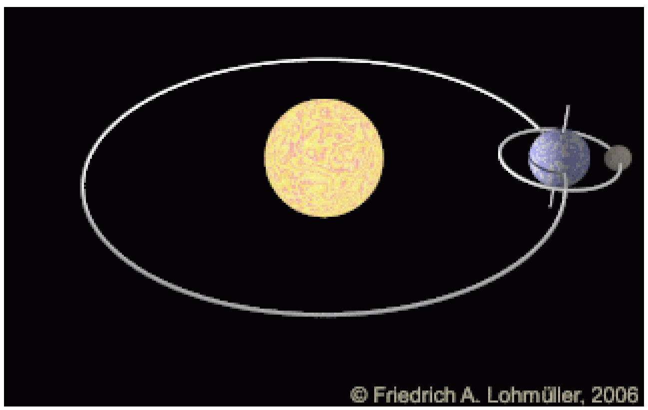
Remember:

- Mass is a property of matter that does not change from one location to another.
- Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located
- Weight of a body is the gravitational force acting on it.

Mechanics: Idealizations

To simplify application of the theory

Particle: A body with mass but with dimensions that can be neglected



Size of earth is insignificant compared to the size of its orbit. Earth can be modeled as a particle when studying its orbital motion

Mechanics: Idealizations

Rigid Body: A combination of large number of particles in which all particles remain at a fixed distance (practically) from one another before and after applying a load.

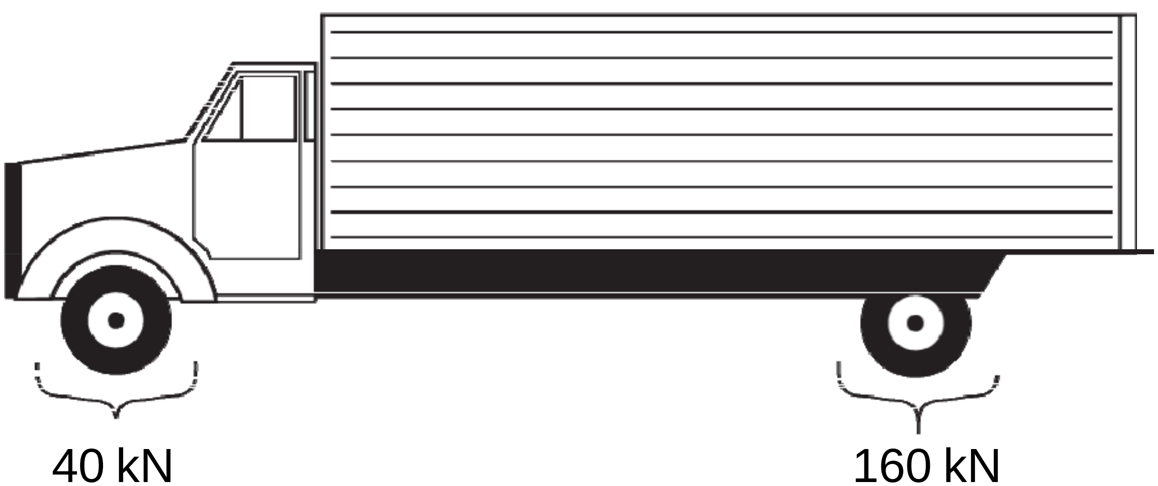
Material properties of a rigid body are not required to be considered when analyzing the forces acting on the body.

In most cases, actual deformations occurring in structures, machines, mechanisms, etc. are relatively small, and rigid body assumption is suitable for analysis

Mechanics: Idealizations

Concentrated Force: Effect of a loading which is assumed to act at a point (CG) on a body.

- Provided the area over which the load is applied is very small compared to the overall size of the body.



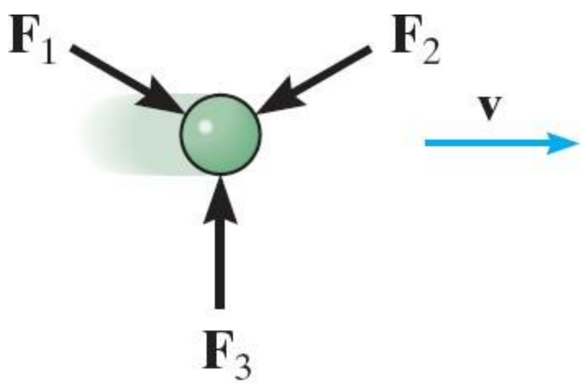
Ex: Contact Force between a wheel and ground.

Mechanics: Newton’s Three Laws of Motion

Basis of formulation of rigid body mechanics.

First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

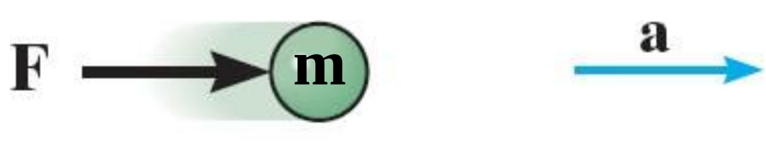
First law contains the principle of the equilibrium of forces € main topic of concern in Statics



Equilibrium

Mechanics: Newton's Three Laws of Motion

Second Law: A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force.



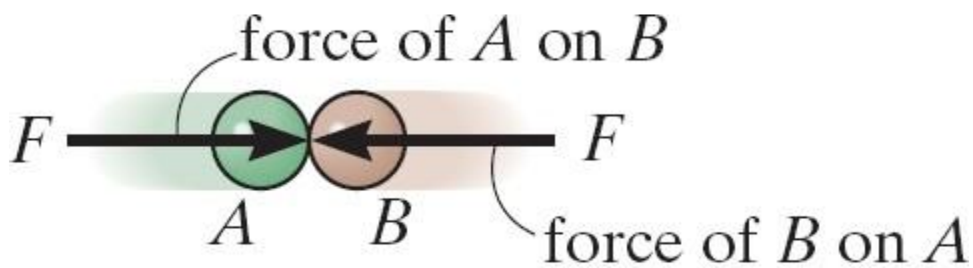
Accelerated motion

$F = ma$

Second Law forms the basis for most of the analysis in Dynamics

Mechanics: Newton's Three Laws of Motion

Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.



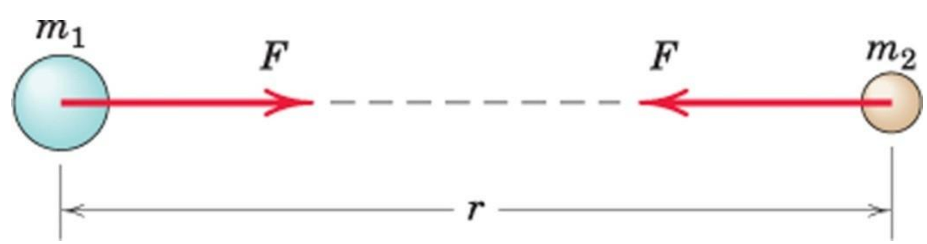
Action – reaction

Third law is basic to our understanding of Force € Forces always occur in pairs of equal and opposite forces.

Mechanics: Newton's Law of Gravitational Attraction

Weight of a body (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics.
This law governs the gravitational attraction between any two particles.

$$F = G\frac{m_1 m_2}{r^2}$$



$\boldsymbol{F}$ = mutual force of attraction between two particles

$\boldsymbol{G}$ = universal constant of gravitation

Experiments € G = 6.673x10^{-11} $m^3/(kg.s^2)$

Rotation of Earth is not taken into account

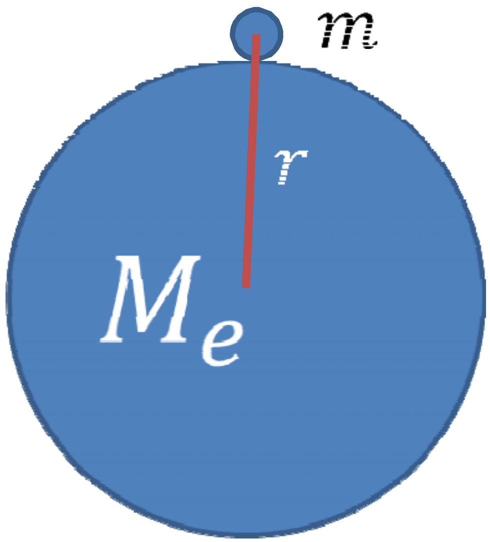
$\boldsymbol{m_1}$, $\boldsymbol{m_2}$ = masses of two particles

$\boldsymbol{r}$ = distance between two particles

Gravitational Attraction of the Earth

Weight of a Body: If a particle is located at or near the surface of the earth, the only significant gravitational force is that between the earth and the particle

Weight of a particle having mass $m_1 = m$:



Assuming earth to be a non-rotating sphere of constant density and having mass $m_2 = M_e$

$$W = G\frac{mM_e}{r^2}$$

r = distance between the earth's center and the particle

$$W = mg$$

Let $g = G\, M_e/r^2$ = acceleration due to gravity (9.81m/s^2)

Mechanics: Units

Four Fundamental Quantities

Quantity	Dimensional Symbol	SI UNIT	
		Unit	Symbol
Mass	M	Kilogram	Kg
Length	L	Meter	M
Time	T	Second	s
Force	F	Newton	N

Basic Unit

$\boxed{F = ma}$ € N = kg.m/s²

$\boxed{W = mg}$ € N = kg.m/s²

1 Newton is the force required to give a mass of 1 kg an acceleration of 1 m/s²

Mechanics: Units Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

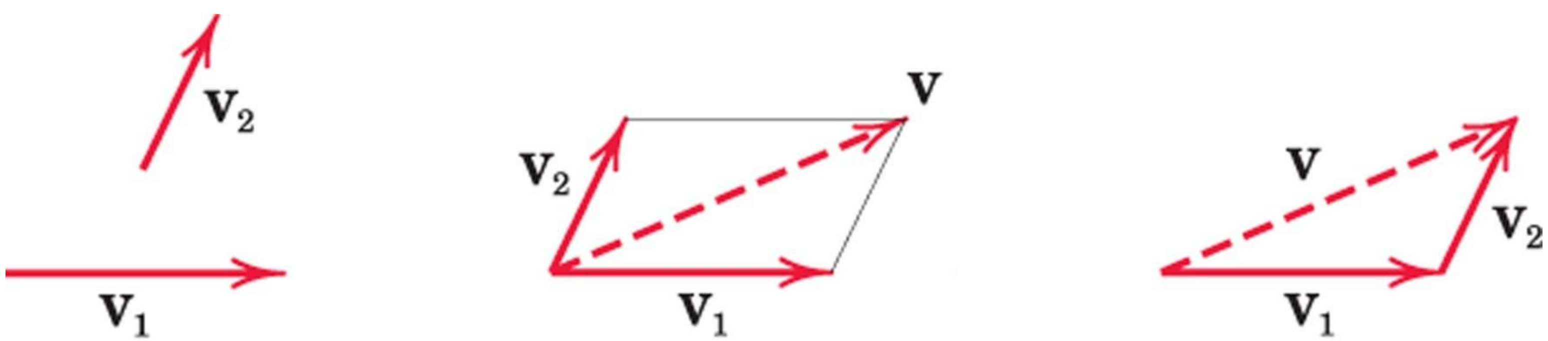
Scalars and Vectors

Scalars: only magnitude is associated.
Ex: time, volume, density, <u>speed</u>, energy, mass

Vectors: possess direction as well as magnitude, and must obey the parallelogram law of addition (and the triangle law).

Ex: displacement, <u>velocity</u>, acceleration,
force, moment, momentum

Equivalent Vector: $V = V_1 + V_2$ (Vector Sum)



Speed is the magnitude of velocity.

Vectors

A Vector V can be written as: V = *V*n

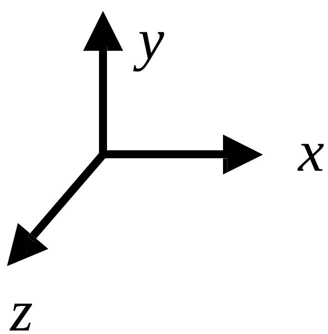
V = magnitude of V

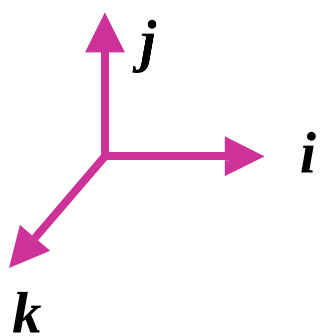
n = unit vector whose magnitude is one and whose direction coincides with that of V

Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

Vectors represented by Bold and Non-Italic letters (V)

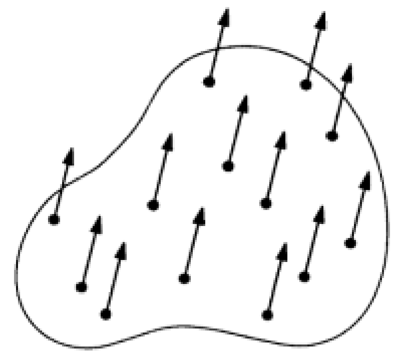
Magnitude of vectors represented by Non-Bold, Italic letters (*V*)



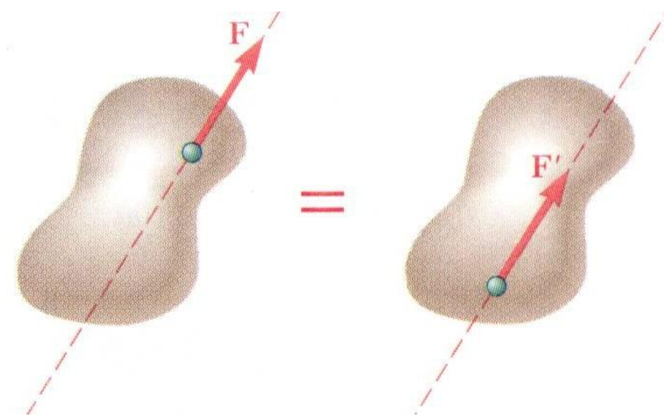


i, ***j***, ***k*** – unit vectors

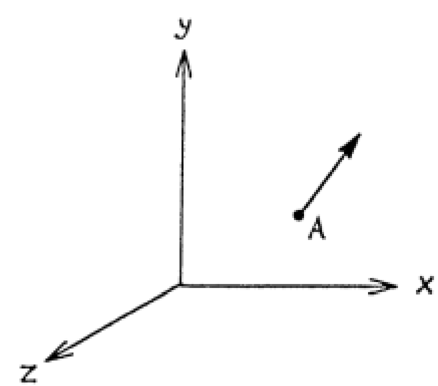
Vectors



Free Vector: whose action is not confined to or associated with a unique line in space
Ex: Movement of a body without rotation.

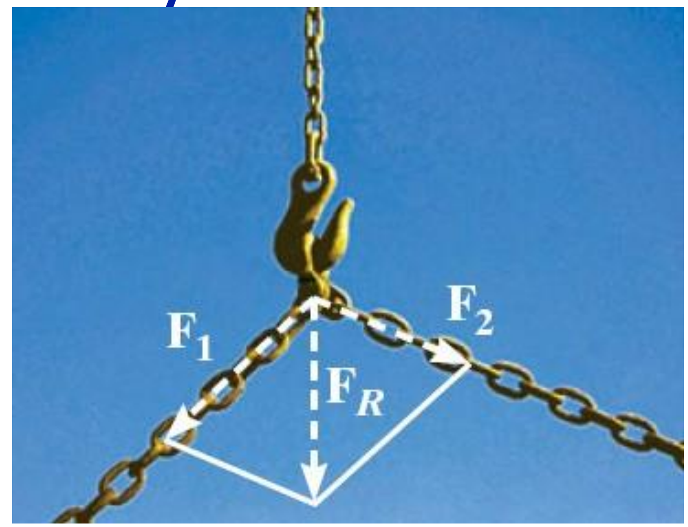


Sliding Vector: has a unique line of action in space but not a unique point of application
Ex: External force on a rigid body
€ Principle of Transmissibility
€ Imp in Rigid Body Mechanics



Fixed Vector: for which a unique point of application is specified
Ex: Action of a force on deformable body

Vector Addition: Procedure for Analysis



Parallelogram Law (Graphical)

Resultant Force (diagonal)
Components (sides of parallelogram)

Cosine law:

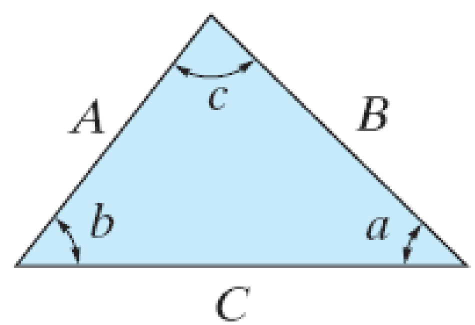
$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Algebraic Solution

Using the coordinate system



Trigonometry (Geometry)

Resultant Force and Components from Law of Cosines and Law of Sines

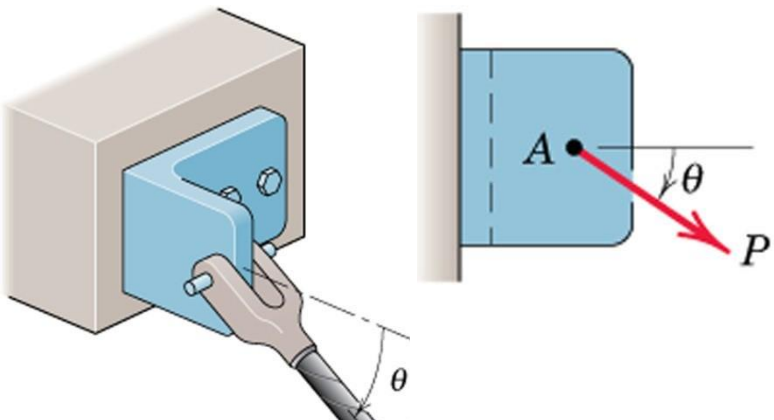
Force Systems

Force: Magnitude (P), direction (arrow) and point of application (point A) is important

Change in any of the three specifications will alter the effect on the bracket.

Force is a Fixed Vector

In case of rigid bodies, line of action of force is important (not its point of application if we are interested in only the resultant external effects of the force), we will treat most forces as



Cable Tension P

External effect: Forces applied (applied force); Forces exerted by bracket, bolts, Foundation (reactive force)

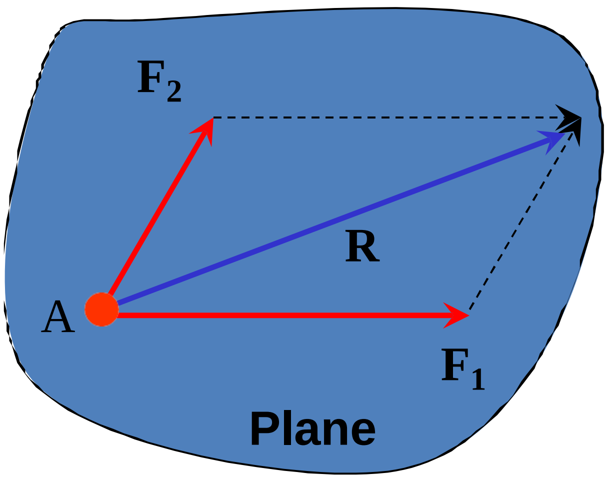
Internal effect: Deformation, strain pattern – permanent strain; depends on material properties of bracket, bolts, etc.

Force Systems

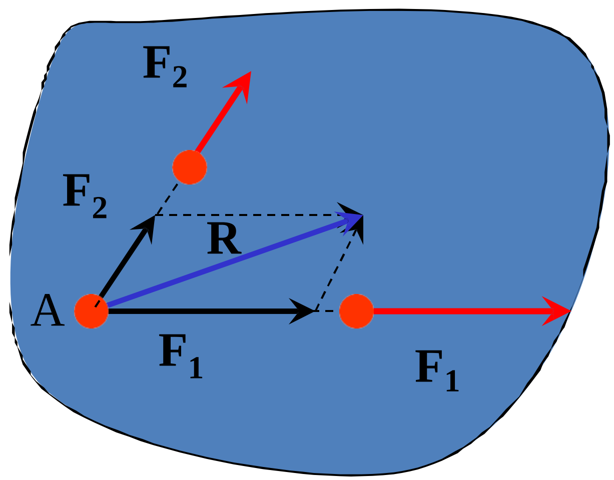
Concurrent force:

Forces are said to be concurrent at a point if their lines of action intersect at that point

F_1, F_2 are concurrent forces; R will be on same plane; $R = F_1 + F_2$

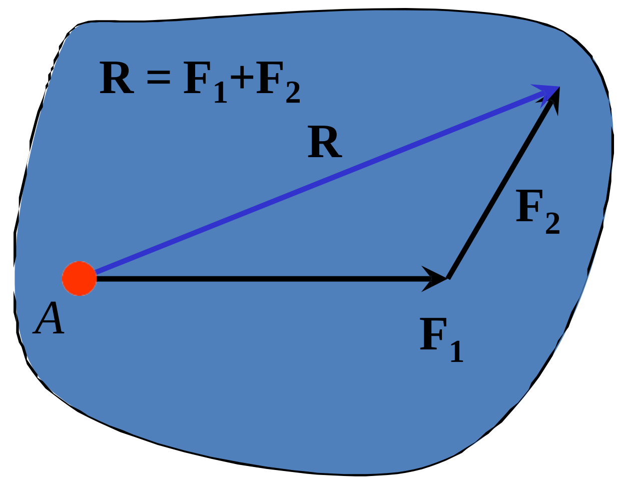


Forces act at same point



Forces act at different point

(Apply Principle of Transmissibility)

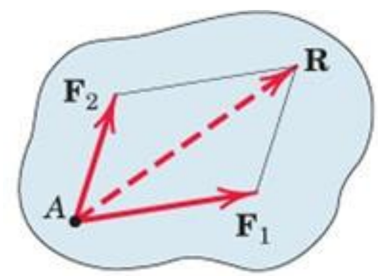


Triangle Law

Components and Projections of Force

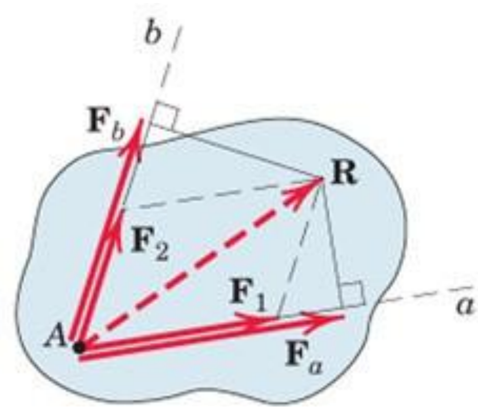
Components of a Force are not necessarily equal to the Projections of the Force unless the axes on which the forces are projected are orthogonal (perpendicular to each other).

F_1 and F_2 are components of R. $R = F_1 + F_2$



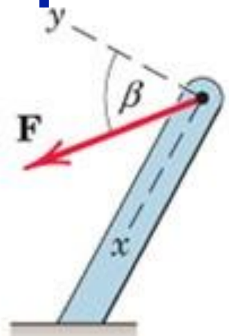
F_a and F_b are perpendicular projections on axes a and b, respectively.

$R \neq F_a + F_b$ unless a and b are perpendicular to each other

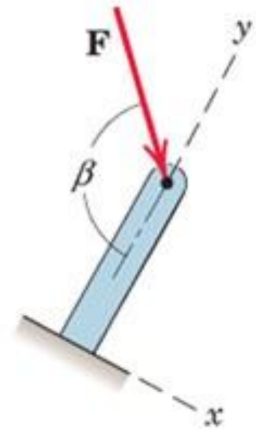


Components of Force

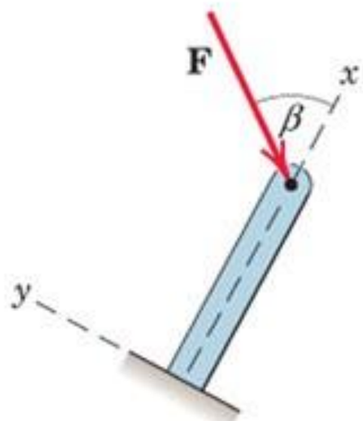
Examples



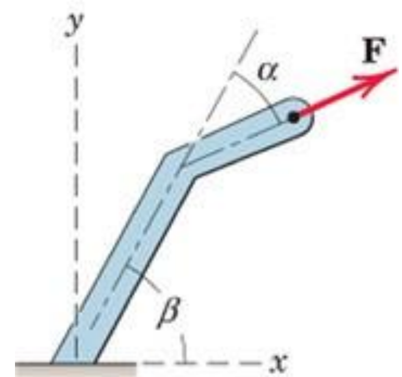
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



$$F_x = F \sin(\pi - \beta)$$
$$F_y = -F \cos(\pi - \beta)$$

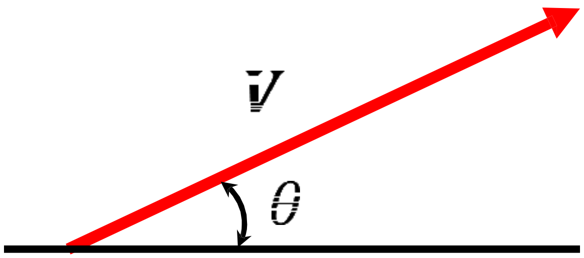


$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

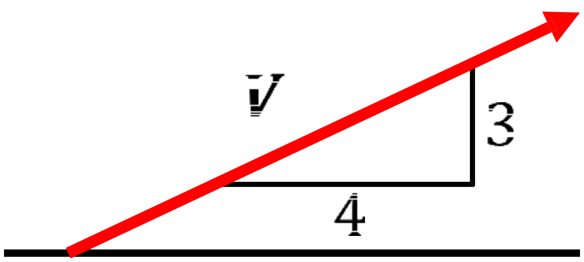


$$F_x = F \cos(\beta - \alpha)$$
$$F_y = F \sin(\beta - \alpha)$$

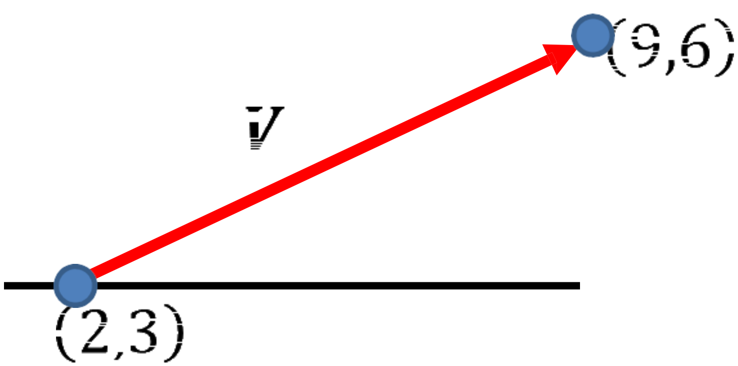
Vector



$$\boldsymbol{V} = V(cos\theta\boldsymbol{i} + sin\theta\boldsymbol{j})$$



$$\boldsymbol{V} = V\left(\frac{4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{4^2 + 3^2}}\right)$$

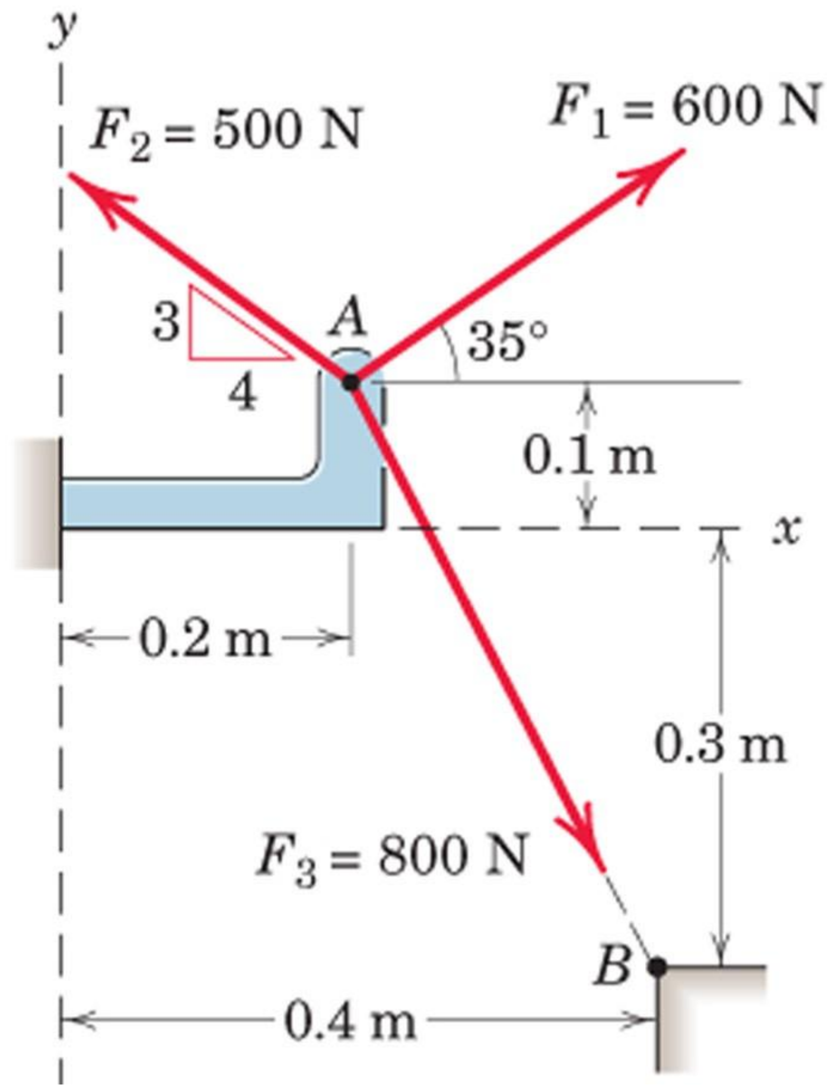


$$\boldsymbol{V} = V\left(\frac{(9-2)\boldsymbol{i} + (6-3)\boldsymbol{j}}{\sqrt{(9-2)^2 + (6-3)^2}}\right)$$

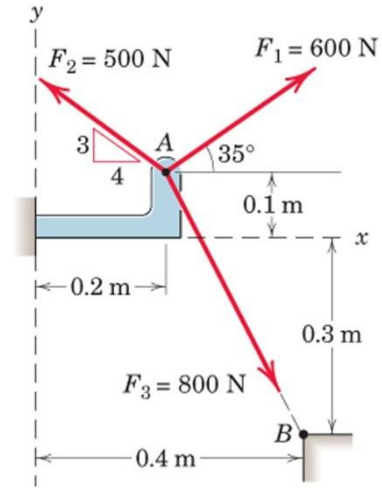
Components of Force

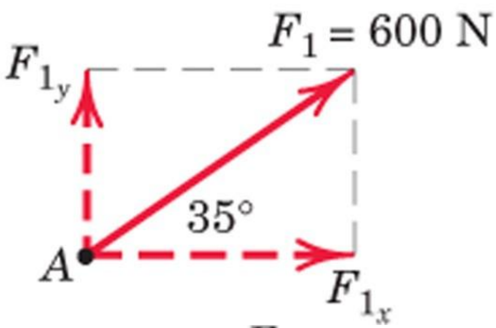
Example 1:

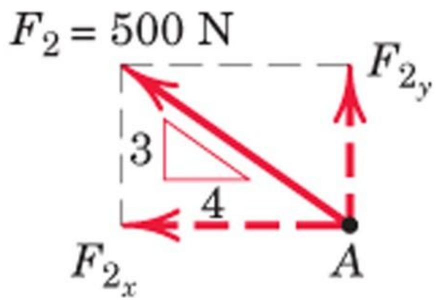
Determine the x and y scalar components of F_1, F_2, and F_3 acting at point A of the bracket

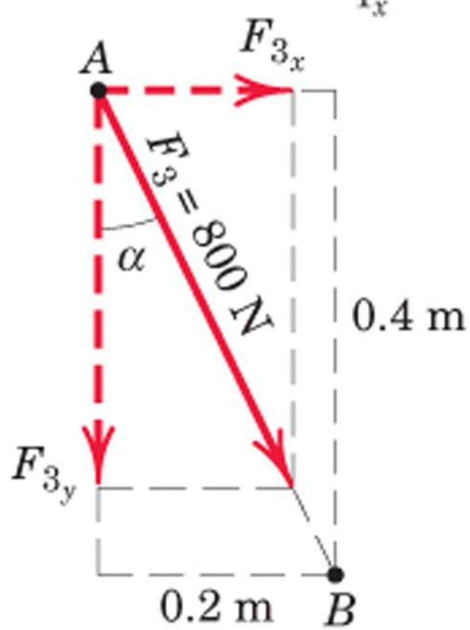


Components of Force









$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

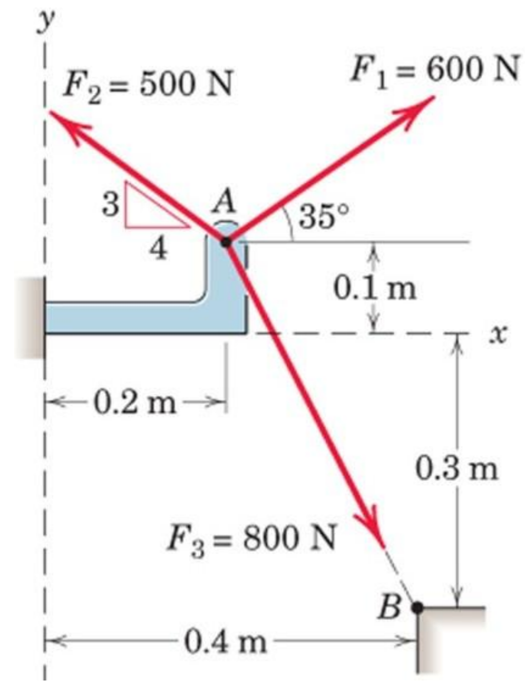
$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

Components of Force

Alternative Solution



$$\boldsymbol{F}_1 = F_1\boldsymbol{n}_1 = F_1\frac{cos(35^o)\boldsymbol{i} + sin(35^o)\boldsymbol{j}}{\sqrt{\left(cos(35^o)\right)^2 + \left(sin(35^o)\right)^2}}$$

$$= 600[0.819\boldsymbol{i} - 0.5735\boldsymbol{j}]$$

$$= 491\boldsymbol{i} - 344\boldsymbol{j}$$

$F_{1x} = 491\ N$ $\quad\quad$ $F_{1y} = 344\ N$

$$\boldsymbol{F}_2 = F_2\boldsymbol{n}_2 = F_2\frac{-4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{(-4)^2 + (3)^2}}$$

$$= 500[-0.8\boldsymbol{i} + 0.6\boldsymbol{j}] \quad = 400\boldsymbol{i} + 300\boldsymbol{j}$$

$F_{2x} = 400\ N$ $\quad\quad$ $F_{2y} = 300\ N$

Components of Force

Alternative Solution

$$\overrightarrow{AB} = 0.2\boldsymbol{i} - 0.4\boldsymbol{j}$$

$$\overline{AB} = \sqrt{(0.2)^2 + (-0.4)^2}$$

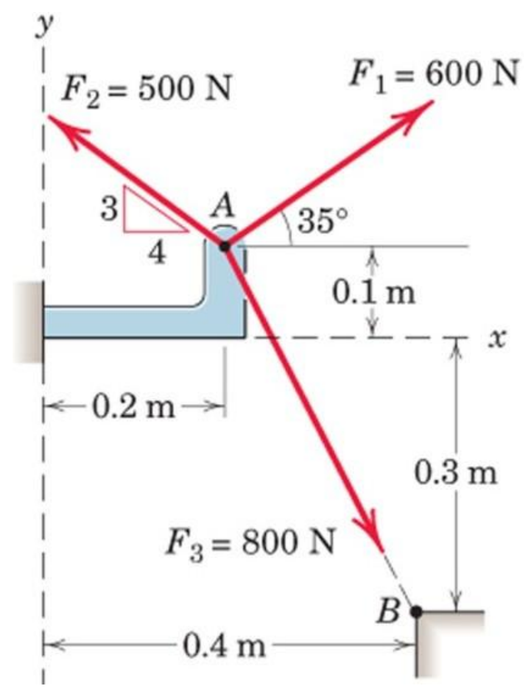
$$\boldsymbol{F_3} = F_3\boldsymbol{n}_3 = F_3\frac{\overrightarrow{AB}}{\overline{AB}}$$

$$= 800\frac{0.2\boldsymbol{i} - 0.4\boldsymbol{j}}{\sqrt{(0.2)^2 - (-0.4)^2}}$$

$$= 800[0.447\boldsymbol{i} - 0.894\boldsymbol{j}]$$

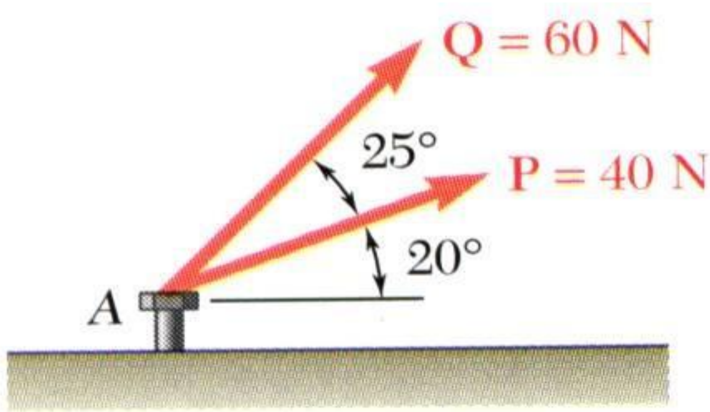
$$= 358\boldsymbol{i} - 716\boldsymbol{j}$$

$F_{3x} = 358\ N$ $\qquad$ $F_{3y} = 716\ N$



Components of Force

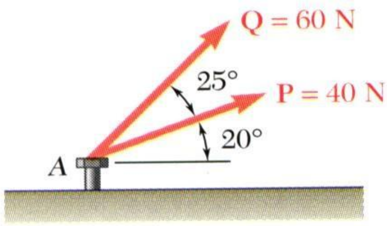
Example 2: The two forces act on a bolt at A. Determine their resultant.



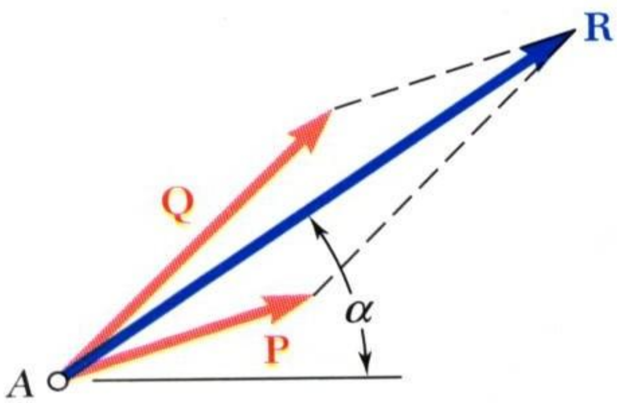
Graphical solution - construct a parallelogram with sides in the same direction as P and Q and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

Components of Force

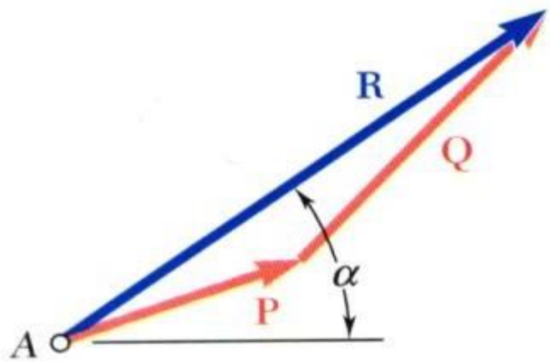


Solution:



- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

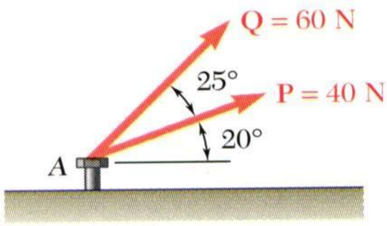
$$\boxed{\mathbf{R} = 98\ \text{N} \quad \alpha = 35^\circ}$$



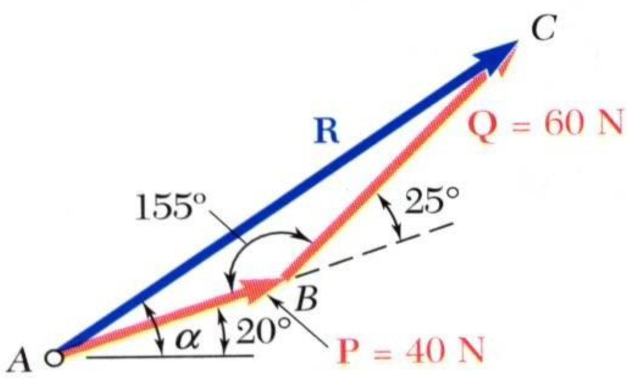
- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\ \text{N} \quad \alpha = 35^\circ}$$

Components of Force



Trigonometric Solution: Apply the triangle rule.



From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

$$= (40\,\mathrm{N})^2 + (60\,\mathrm{N})^2 - 2(40\,\mathrm{N})(60\,\mathrm{N})\cos 155°$$

$$\boxed{R = 97.73\ \mathrm{N}}$$

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B\,\frac{Q}{R}$$

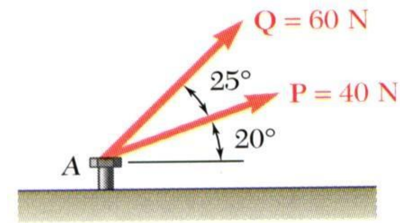
$$= \sin 155°\,\frac{60\ \mathrm{N}}{97.73\mathrm{N}}$$

$$A = 15.04°$$

$$\alpha = 20° + A$$

$$\boxed{\alpha = 35.04°}$$

Components of Force



$$\boldsymbol{R} = \boldsymbol{P} - \boldsymbol{Q}$$

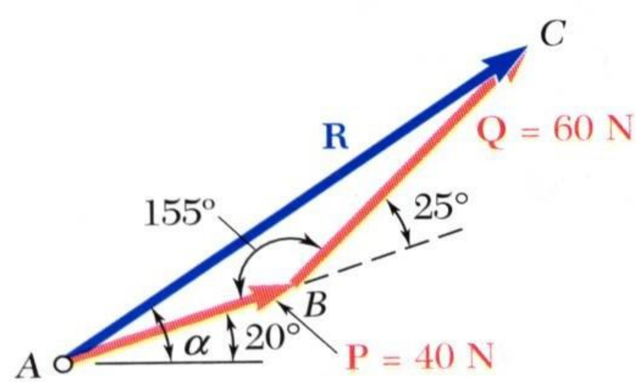
$$\boldsymbol{P} = 40[\cos(20)\boldsymbol{i} + \sin(20)\boldsymbol{j}]$$
$$= 37.58\mathbf{i} + 13.68\mathbf{j}$$

$$\boldsymbol{Q} = 60[\cos(45)\,\boldsymbol{i} + \sin(45)\boldsymbol{j}]$$
$$= 42.43\mathbf{i} + 42.43\mathbf{j}$$

$$\boldsymbol{R} = 80.01\boldsymbol{i} - 56.10\boldsymbol{j}$$

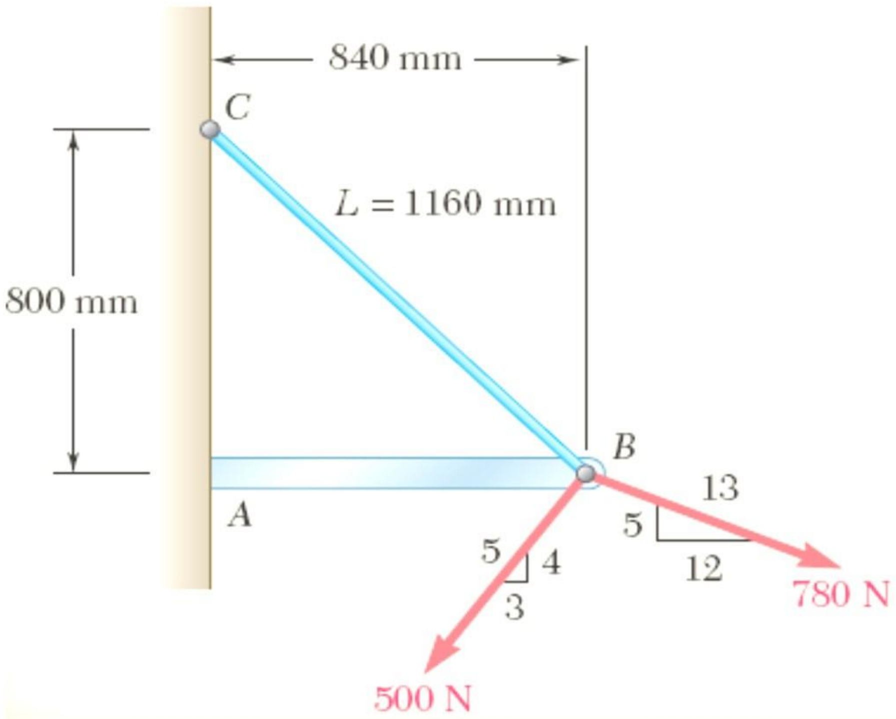
$$R = 97.72$$

$$\alpha = 35.03^o$$



Components of Force

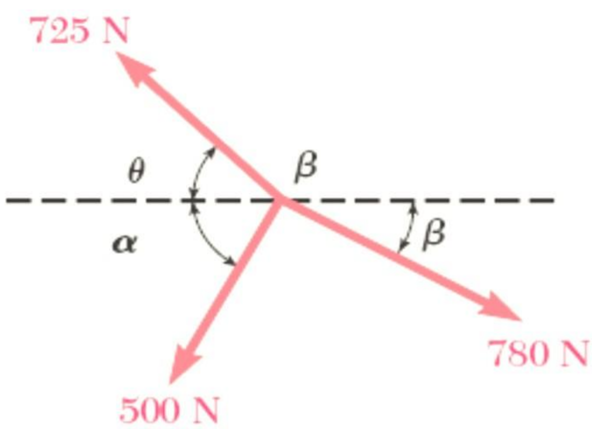
Example 3:Tension in cable *BC* is 725-N, determine the resultant of the three forces exerted at point *B* of beam *AB*.



Solution:

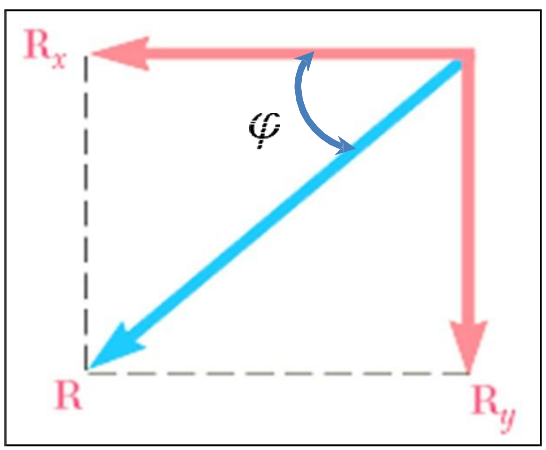
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Components of Force



Resolve each force into rectangular components

Magnitude (N)	X-component (N)	Y-component (N)
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$



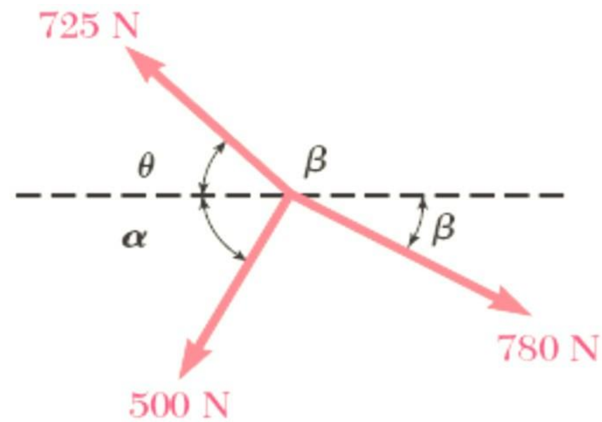
$\mathbf{R} = R_x i + R_y j$ $\qquad$ $\mathbf{R} = (-105)i + (-200)j$

Calculate the magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62\ 3^o$$

$$\mathbf{R} = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Components of Force



Alternate solution

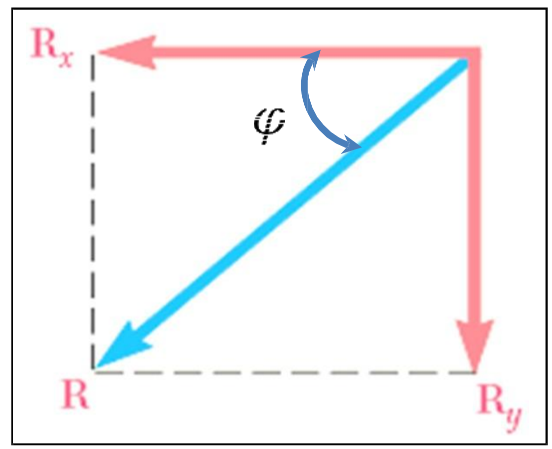
$$R = F_1 + F_2 + F_3$$

$$F_1 = 725[-0.724i + 0.689j$$

$$F_2 = 500[-0.6i - 0.8j$$

$$F_2 = 780[0.923i - 0.384j$$

$$R = -105i - 200j$$

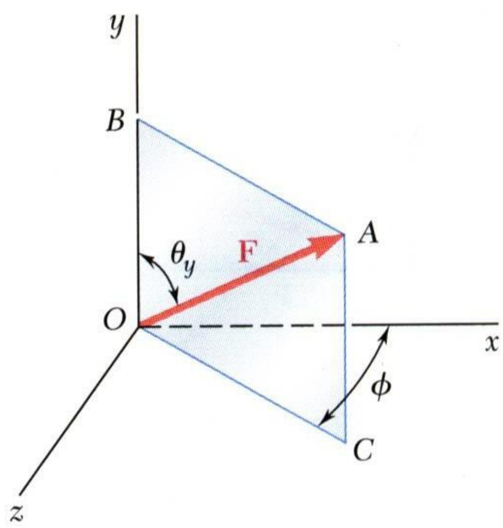


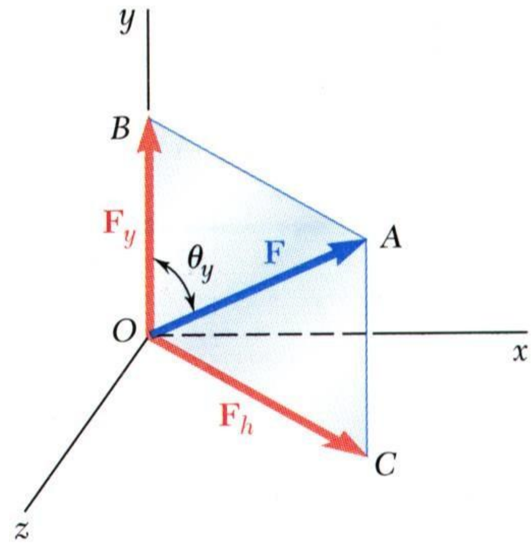
Calculate the magnitude and direction

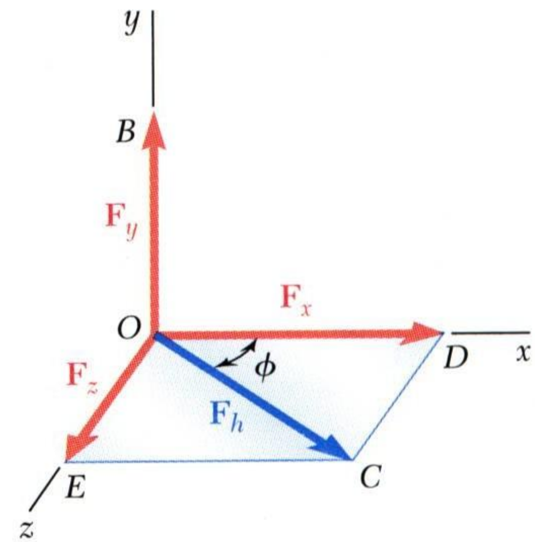
$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62\ 3^o$$

$$R = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Rectangular Components in Space







- The vector F is contained in the plane *OBAC*.

- Resolve F into horizontal and vertical components.

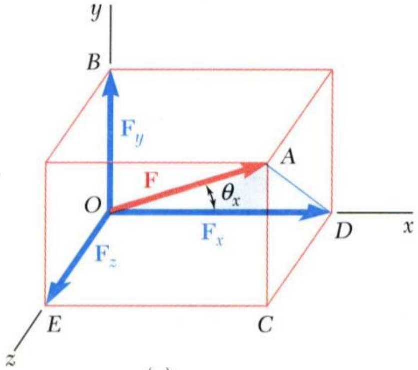
$$F_y = F\cos\theta_y$$

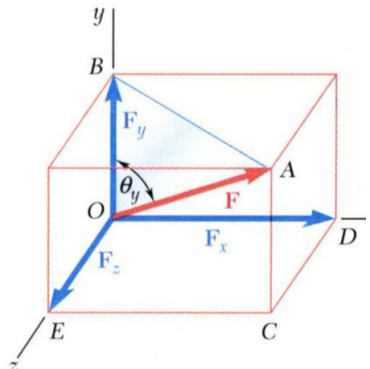
$$F_h = F\sin\theta_y$$

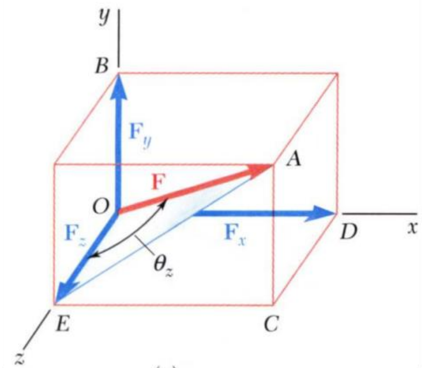
- Resolve F_h into rectangular components

$$F_x = F_h\cos\phi$$
$$= F\sin\theta_y\cos\phi$$
$$F_z = F_h\sin\phi$$
$$= F\sin\theta_y\sin\phi$$

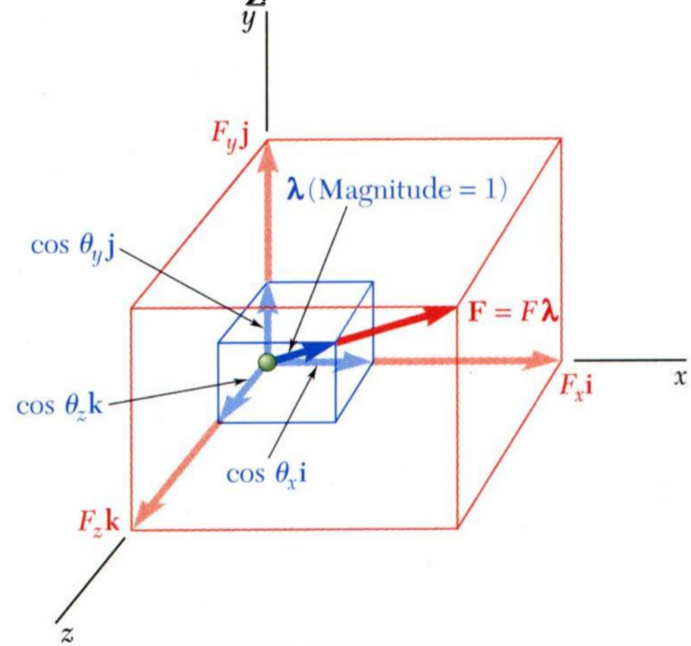
Rectangular Components in Space







$F_x = F cos\theta_x$ $\quad$ $F_y = F cos\theta_y$ $\quad$ $F_z = F cos\theta_z$



$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{F} = Fcos\theta_x\boldsymbol{i} + Fcos\theta_y\boldsymbol{j} + Fcos\theta_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\left(cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}\right)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

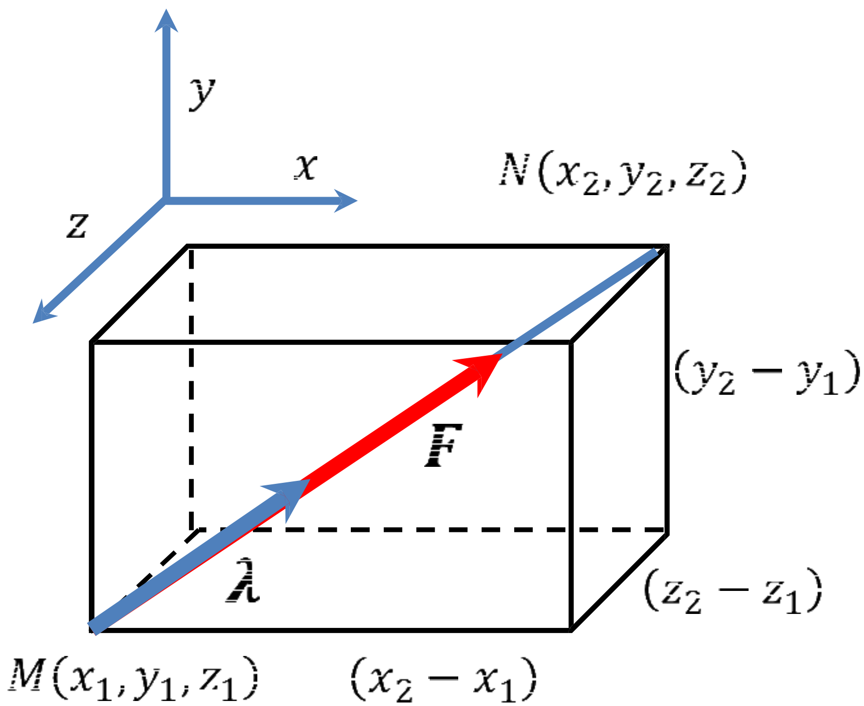
Where $\boldsymbol{\lambda} = cos\theta_x\boldsymbol{i} + cos\theta_y\boldsymbol{j} + cos\theta_z\boldsymbol{k}$

$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $cos\theta_x$, $cos\theta_y$ and $cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} - d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

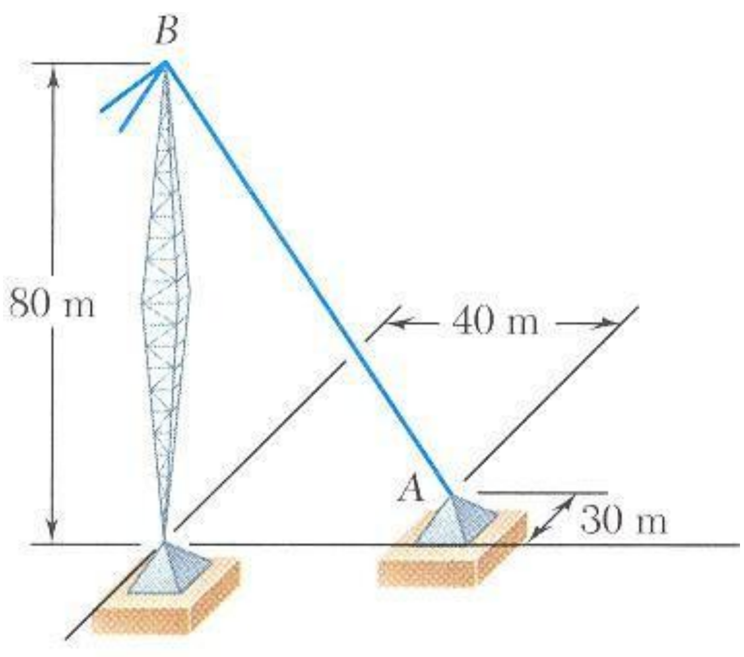
$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Rectangular Components in Space

Example: The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

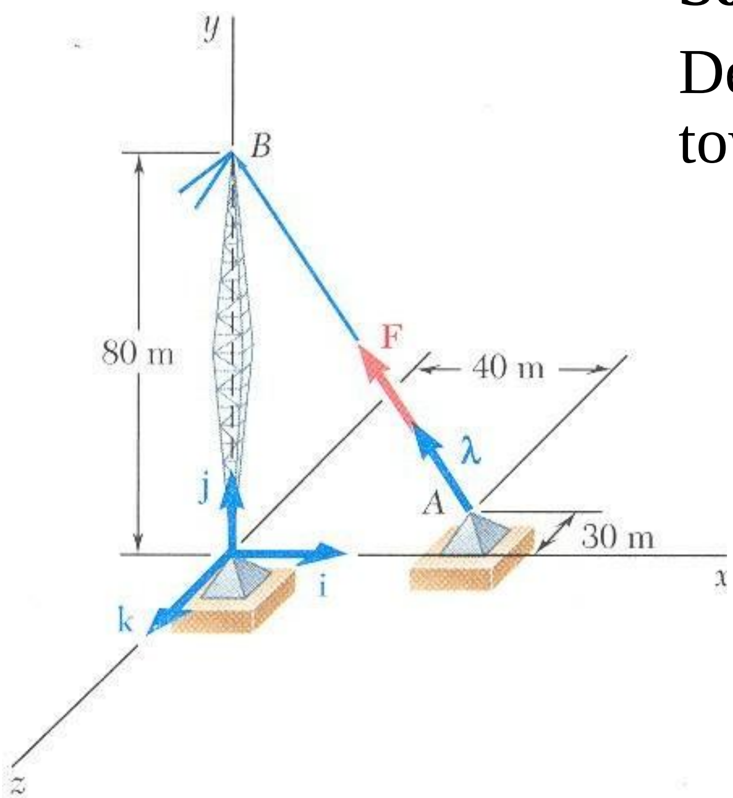
b) the angles θ_x, θ_y, θ_z defining the direction of the force



SOLUTION:

- Based on the relative locations of the points *A* and *B*, determine the unit vector pointing from *A* towards *B*.
- Apply the unit vector to determine the components of the force acting on *A*.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Rectangular Components in Space



Solution

Determine the unit vector pointing from *A* towards *B.*

$$\boldsymbol{AB} = -40\boldsymbol{i} - 80\boldsymbol{j} - 30\boldsymbol{k}$$

$$AB = \sqrt{(-40)^2 - (80)^2 - (30)^2} = 94.3$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{AB}}{AB} = \frac{-40\boldsymbol{i} - 80\boldsymbol{j} - 30\boldsymbol{k}}{94.3}$$

$$= -0.424\boldsymbol{i} - 0.848\boldsymbol{j} - 0.318\boldsymbol{k}$$

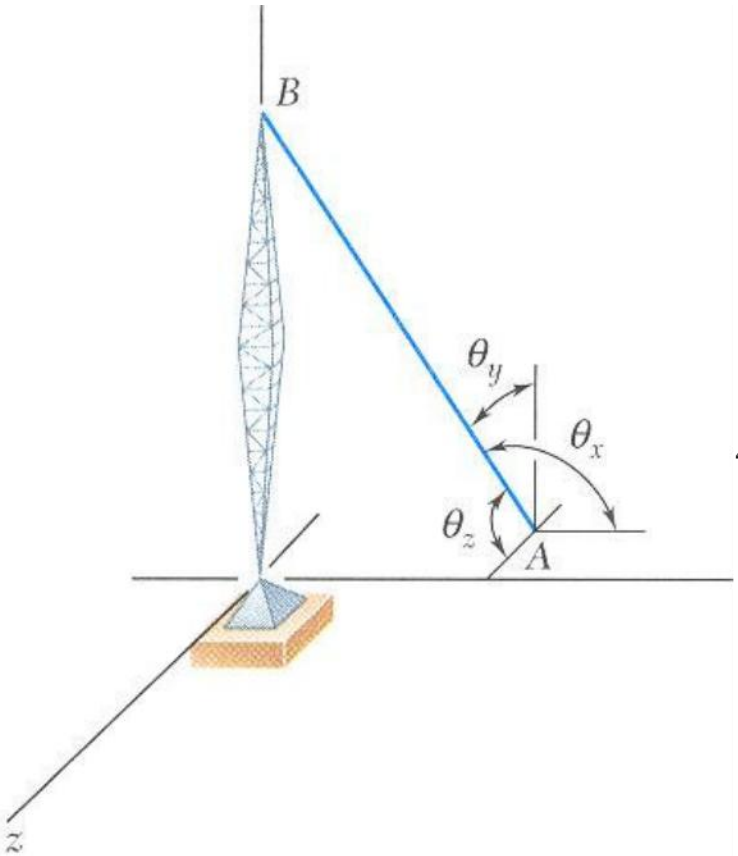
Determine the components of the force.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \qquad = 2500(-0.424\boldsymbol{i} - 0.848\boldsymbol{j} - 0.318\boldsymbol{k})$$

$$= -1060\boldsymbol{i} + 2120\boldsymbol{j} + 795\boldsymbol{k}$$

$$\boxed{\begin{aligned} F_x &= -1060\ N \\ F_y &= 2120\ N \\ F_z &= 795\ N \end{aligned}}$$

Rectangular Components in Space



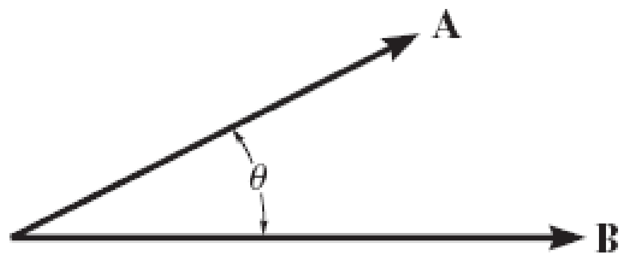
Solution

Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\boldsymbol{\lambda} = \cos\theta_x \boldsymbol{i} + \cos\theta_y \boldsymbol{j} + \cos\theta_z \boldsymbol{k}$$
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^0 \\ \theta_y &= 32.0^0 \\ \theta_z &= 71.5^0 \end{aligned}}$$

Vector Products



Dot Product $\boldsymbol{A}.\boldsymbol{B} = AB\cos\theta$

Applications:

to determine the angle between two vectors

to determine the projection of a vector in a specified direction

A.B = B.A (commutative)

A.(B+C) = A.B+A.C (distributive operation)

$$\boldsymbol{A}.\boldsymbol{B} = (A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}).(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k})$$

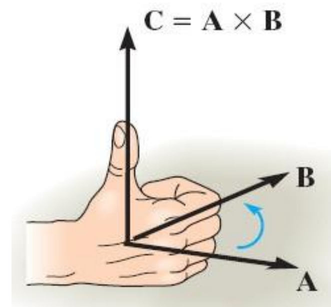
$$= A_xB_x + A_yB_y + A_zB_z$$

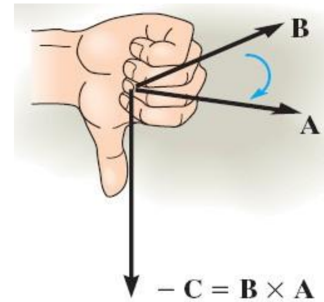
$$\boldsymbol{i}\ \boldsymbol{i} = 1$$

$$\boldsymbol{i}.\boldsymbol{j} = 0$$

Vector Products

Cross Product: $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{C} = ABsin\theta$



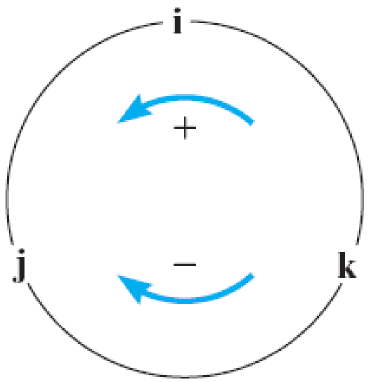


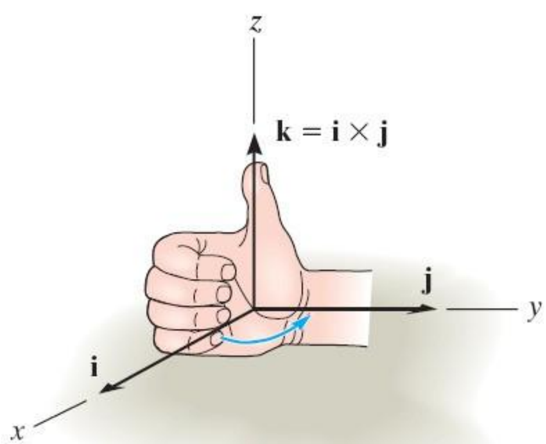
$$\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A})$$

$$\boldsymbol{A} \times \boldsymbol{B} = \left(A_x\boldsymbol{i} - A_y\boldsymbol{j} - A_z\boldsymbol{k}\right) \times \left(B_x\boldsymbol{i} - B_y\boldsymbol{j} - B_z\boldsymbol{k}\right)$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \left(A_yB_z - A_zB_y\right)\boldsymbol{i} + \left(A_zB_x - A_xB_z\right)\boldsymbol{j} + \left(A_xB_y - A_yB_x\right)\boldsymbol{k}$$

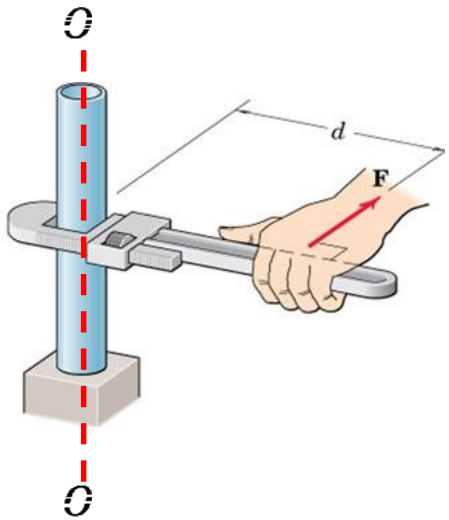
Cartesian Vector





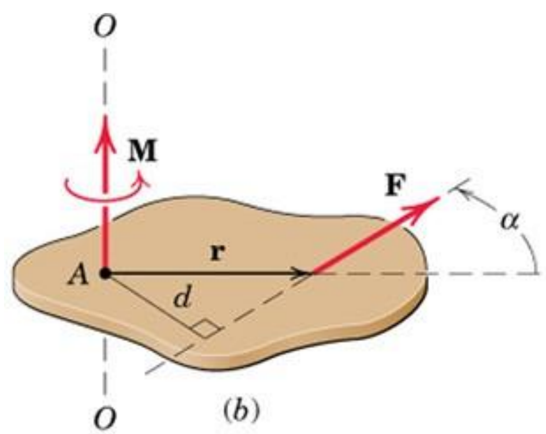
$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = \mathbf{0}$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = \mathbf{0}$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

Moment of a Force (Torque)



Moment about axis O-O is $M_o = Fd$

Magnitude of M_o measures tendency of **F** to cause rotation of the body about an axis along M_o.

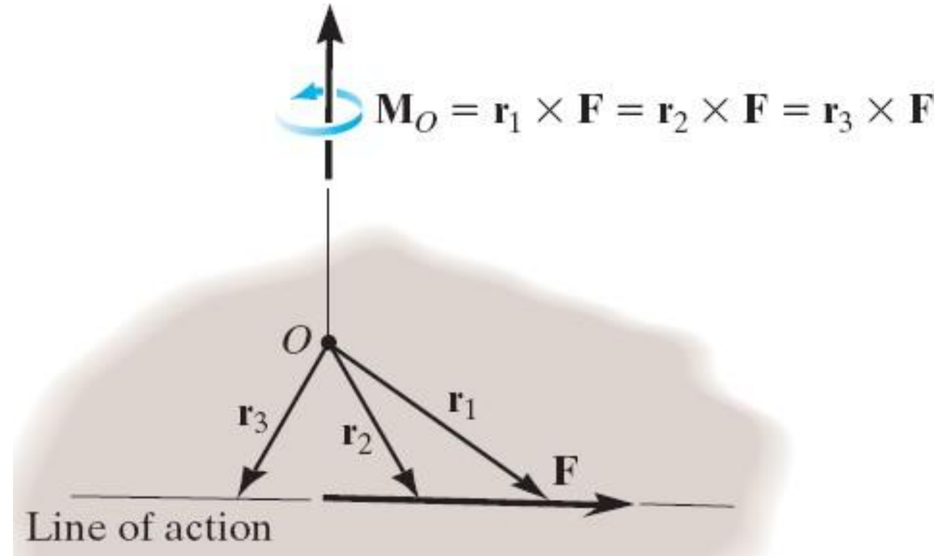


Moment about axis O-O is $M_o = Frsin\alpha$

$$M_o = r \times F$$

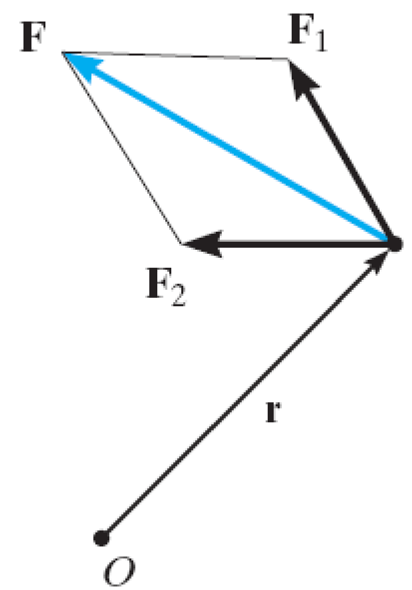
Sense of the moment may be determined by the right-hand rule

Moment of a Force



Principle of Transmissibility

Any force that has the same magnitude and direction as **F**, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



Varignon's Theorem (Principle of Moments)

Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

Rectangular Components of a Moment

The moment of $\boldsymbol{F}$ about O,

$$\boldsymbol{M}_o = \boldsymbol{r} \times \boldsymbol{F}$$

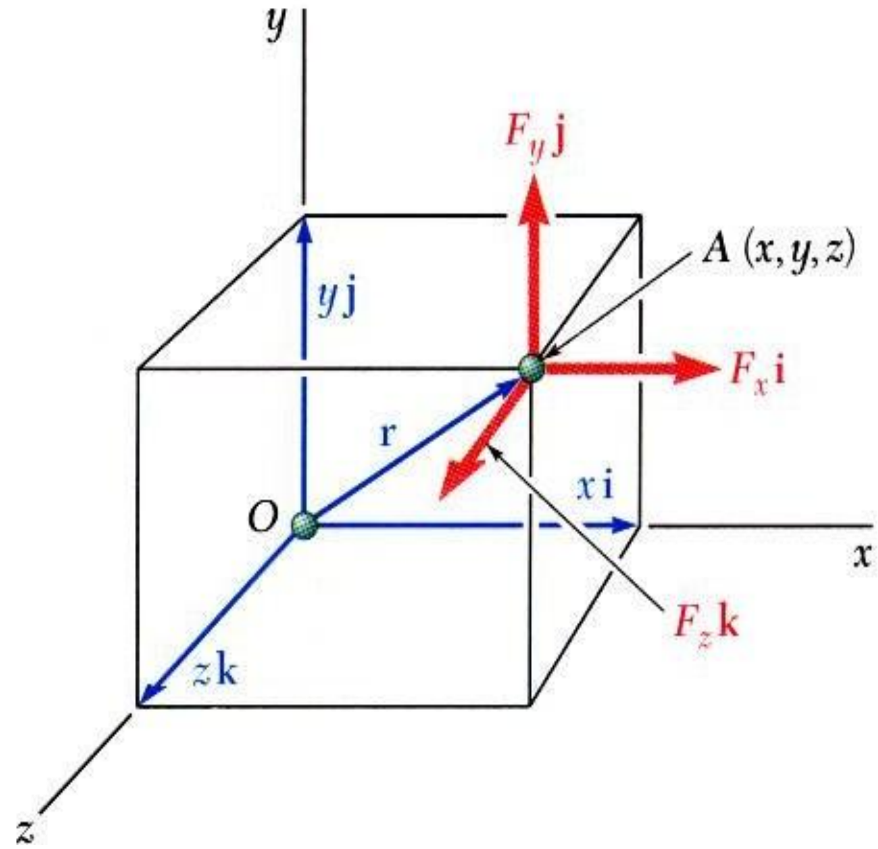
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

$$\boldsymbol{M}_o = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k}$$



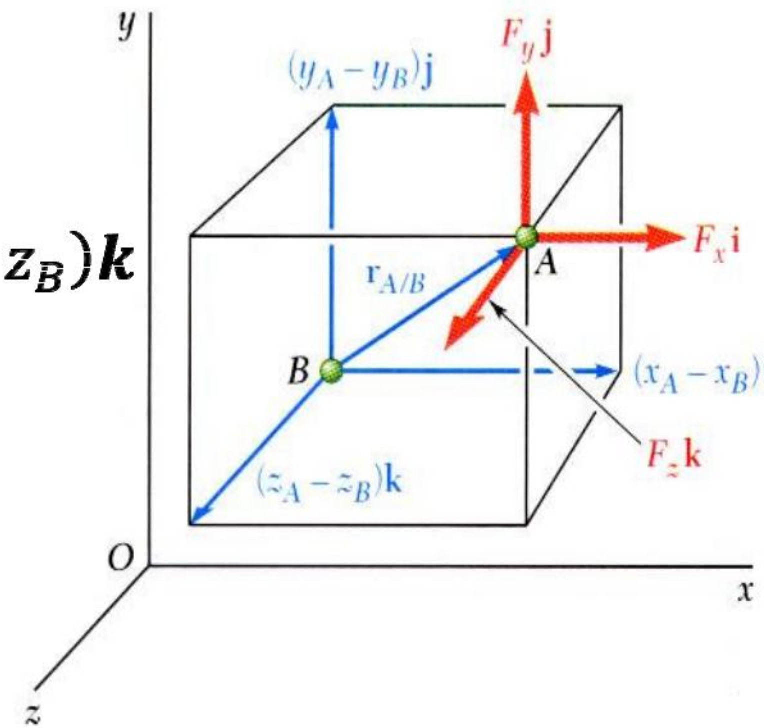
Rectangular Components of the Moment

The moment of $\boldsymbol{F}$ about B,

$$\boldsymbol{M}_{\boldsymbol{B}} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$\boldsymbol{r}_{AB} = (x_A - x_B)\boldsymbol{i} + (y_A - y_B)\boldsymbol{j} + (z_A - \boldsymbol{z}_{\boldsymbol{B}})\boldsymbol{k}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

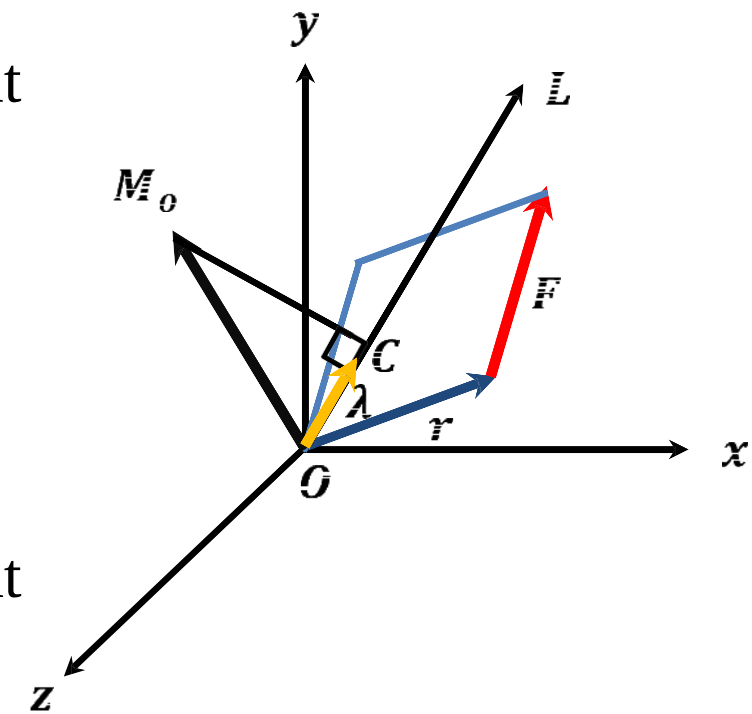


$$\boldsymbol{M}_{\boldsymbol{B}} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force About a Given Axis

Moment $\mathbf{M_O}$ of a force $\mathbf{F}$ applied at the point $\mathbf{A}$ about a point $\mathbf{O}$



$$\boldsymbol{M}_o = \boldsymbol{r} \times \boldsymbol{F}$$

Scalar moment M_{OL} about an axis **OL** is the projection of the moment vector $\mathbf{M_O}$ onto the axis,

$$M_{OL} = \boldsymbol{\lambda}.\boldsymbol{M}_o = \boldsymbol{\lambda}.(\boldsymbol{r} \times \boldsymbol{F})$$

Moments of **F** about the coordinate axes (using previous slide)

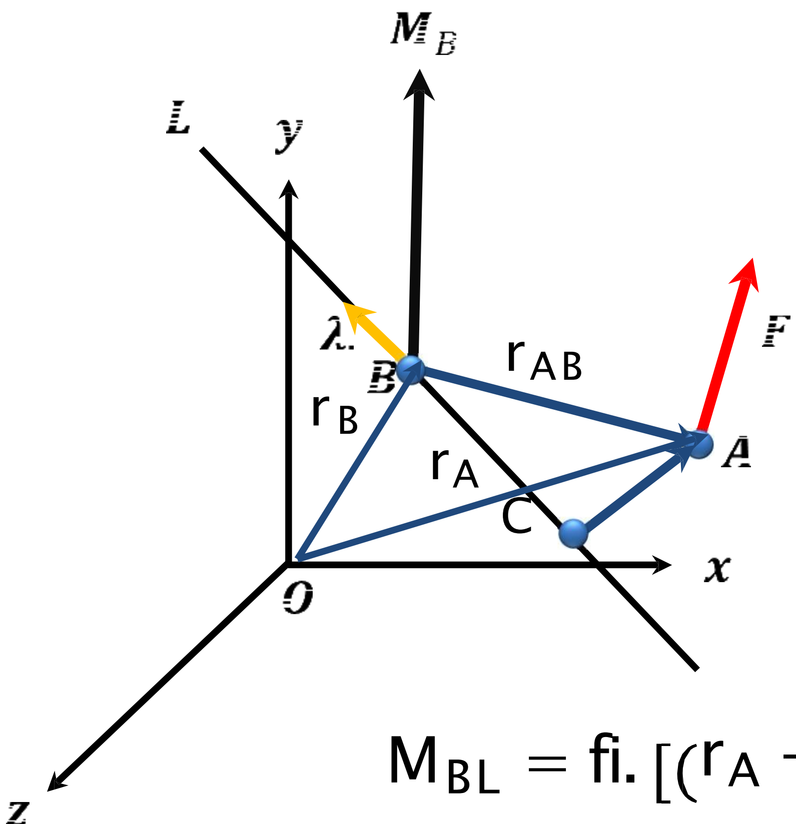
$$M_x = (yF_z - zF_y)$$

$$M_y = (zF_x - xF_z)$$

$$M_z = (xF_y - yF_x)$$

Moment of a Force About a Given Axis

Moment of a force about an arbitrary axis



$$\boldsymbol{M}_B = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$\boldsymbol{M}_{BL} = \lambda . \boldsymbol{M}_B = \lambda . (\boldsymbol{r}_{AB} \times \boldsymbol{F})$$

$$r_{AB} = r_A - r_B$$

If we take point C in place of point B

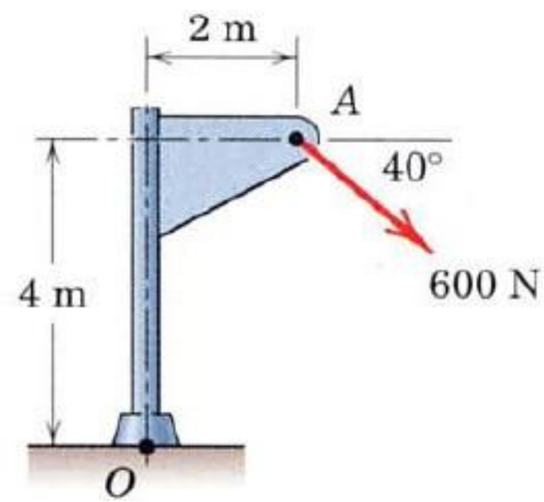
$$M_{BL} = \mathrm{fi}. [(r_A - r_C) \times F]$$

$$= \mathrm{fi}. [(r_A - r_B) \times F] + \boxed{\cancelto{0}{\mathrm{fi}. [(r_B - r_C) \times F]}}$$

$(r_B - r_C)$ and fi are in the same line

Moment: Example

Calculate the magnitude of the moment about the base point O of the 600 N force in different ways

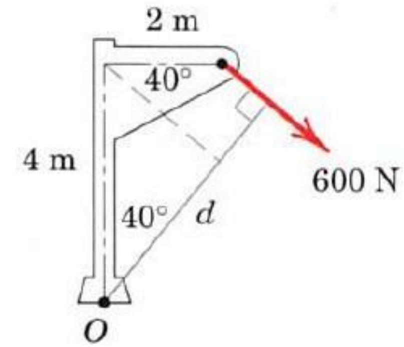


Solution 1.

Moment about O is

$$M_o = dF \qquad d = 4cos40^o + 2sin40^o = 4.35m$$

$$M_o = 600(4.35) = 2610\ N.m$$ ***Ans***

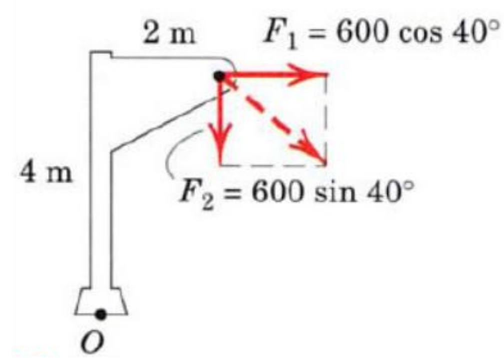


Solution 2.

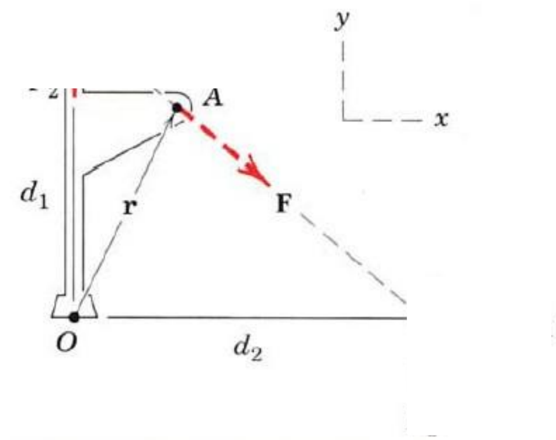
$$F_x = 600cos40^o = 460\ N$$

$$F_x = 600sin40^o = 386\ N$$

$$M_o = 460(4.00) + 386(2.00) = 2610\ N.m$$ ***Ans***



Moment: Example



Solution 3.

$$d_1 = 4 + 2tan40^o = 5.68m$$

$$M_o = 460(5.68) = 2610\ N.m \quad \textbf{Ans}$$

Solution 4.

$$d_2 = 2 + 4cot40^o = 6.77m$$

$$M_o = 386(6.77) = 2610\ N.m \quad \textbf{Ans}$$

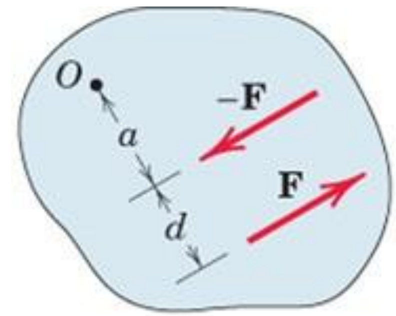
Solution 5.

$$\boldsymbol{M_0} = \boldsymbol{r} \times \boldsymbol{F} = (2\boldsymbol{i} + 4\boldsymbol{j}) \times 600(cos40^o\boldsymbol{i} - sin40^o\boldsymbol{j})$$

$$\boldsymbol{M_0} = -2610\ N.m \quad \textbf{Ans}$$

The minus sign indicates that the vector is in the negative z-direction

Moment of a Couple



Moment produced by two equal, opposite and non-collinear forces is called a *couple*.

Magnitude of the combined moment of the two forces about O:

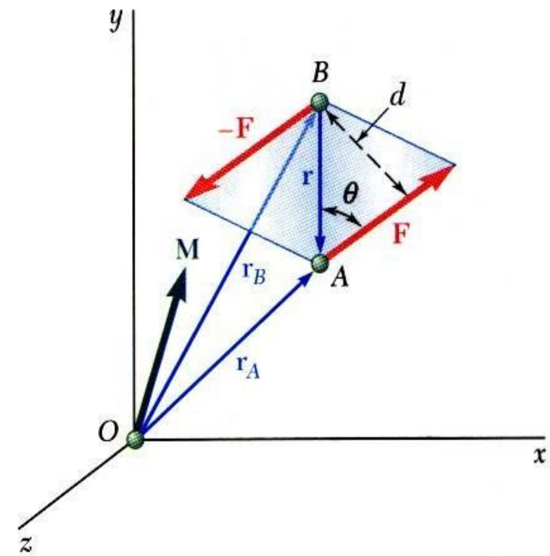
$$M = F(a + d) - Fa = Fd$$

$$M = r_A \times F + r_B \times (-F)$$

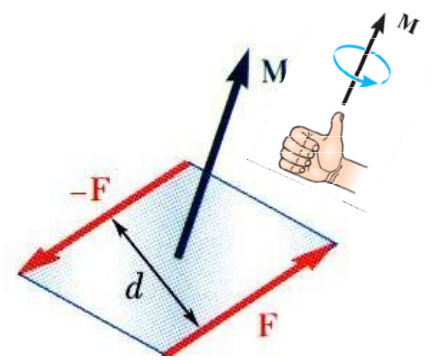
$$= (r_A - r_B) \times F$$

$$= r \times F$$

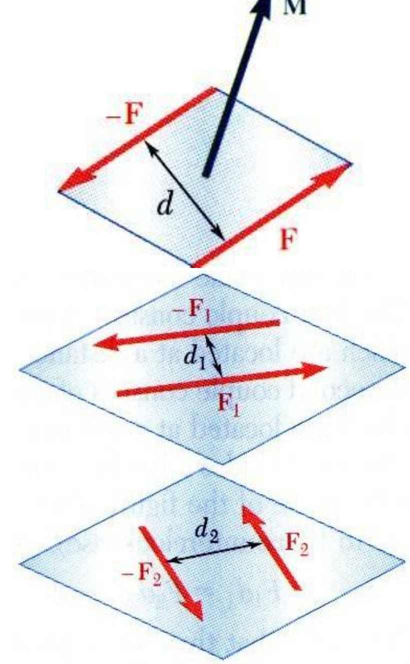
$$M = rF\sin 8 = Fd$$



The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.



Moment of a Couple

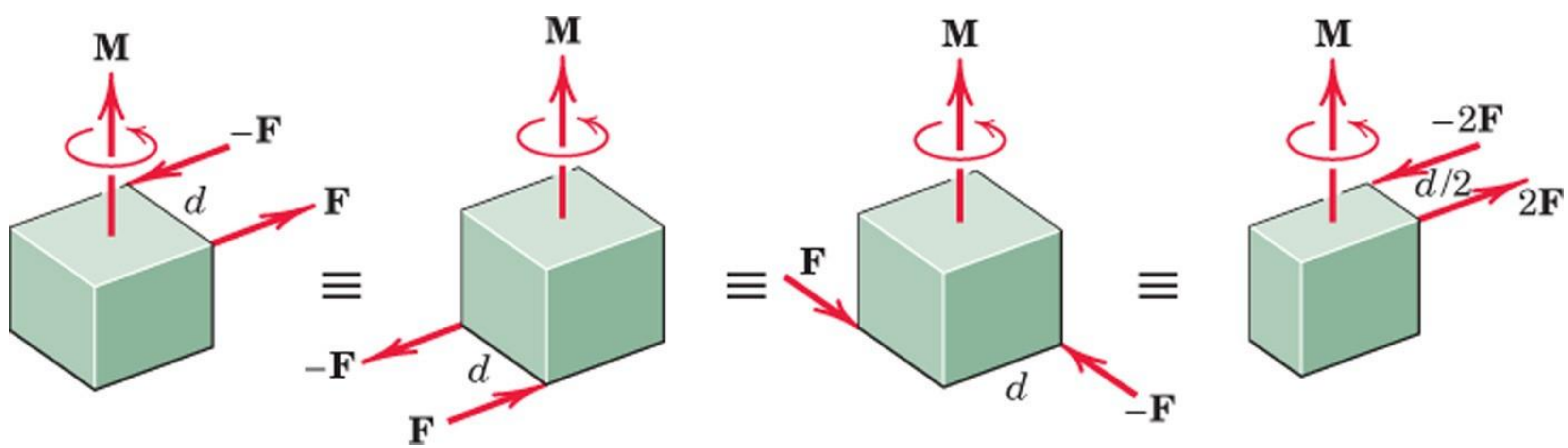


Two couples will have equal moments if $F_1 d_1 = F_2 d_2$

The two couples lie in parallel planes

The two couples have the same sense or the tendency to cause rotation in the same direction.

Examples:

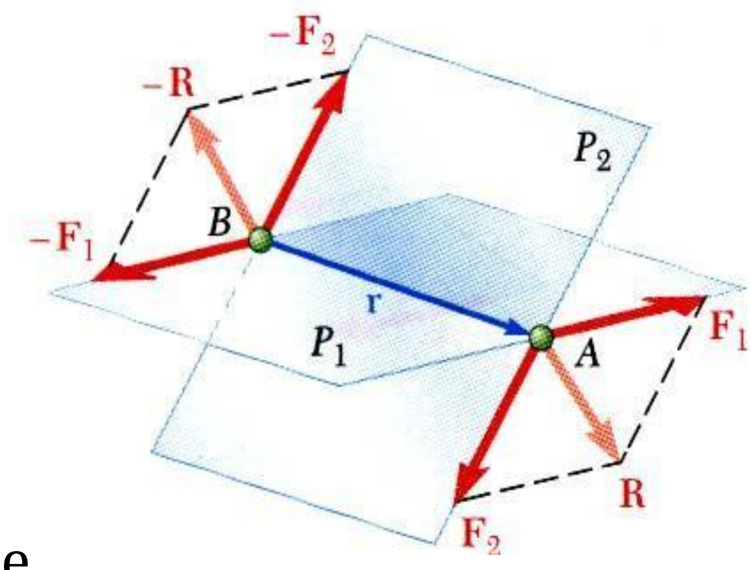


Addition of Couples

Consider two intersecting planes P_1 and P_2 with each containing a couple

$$\mathbf{M}_1 = \mathbf{r} \times \mathbf{F}_1 \quad \text{in plane } P_1$$

$$\mathbf{M}_2 = \mathbf{r} \times \mathbf{F}_2 \quad \text{in plane } P_2$$



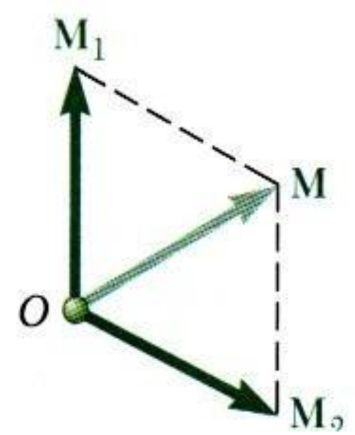
Resultants of the vectors also form a couple

$$\mathbf{M} = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2)$$

By Varigon's theorem

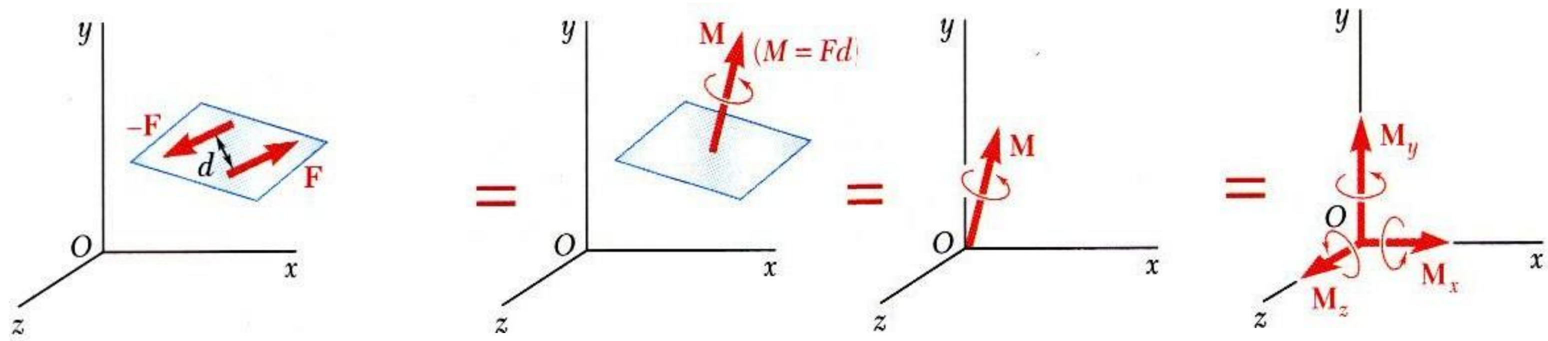
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

$$= \mathbf{M}_1 + \mathbf{M}_2$$



Sum of two couples is also a couple that is equal to the vector sum of the two couples

Couples Vectors



A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.

Couple vectors obey the law of addition of vectors.

Couple vectors are free vectors, i.e., the point of application is not significant.

Couple vectors may be resolved into component vectors.

Couple: Example

Moment required to turn the shaft connected at center of the wheel = 12 Nm

Case I: Couple Moment produced by 40 N forces = 12 Nm

Case II: Couple Moment produced by 30 N forces = 12 Nm

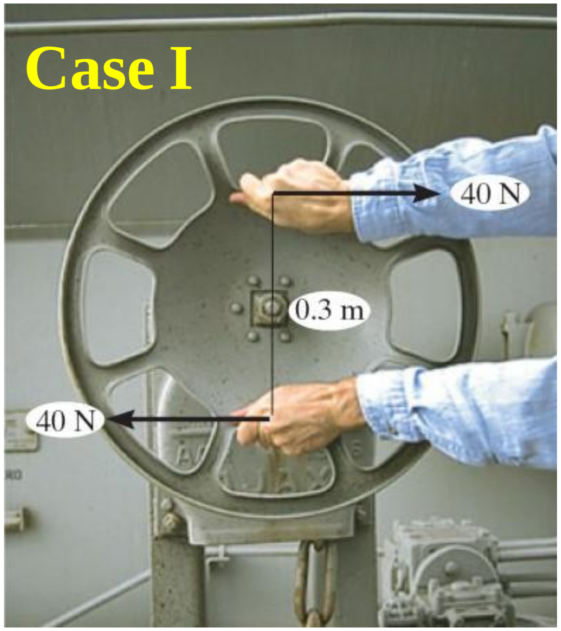
If only one hand is used?

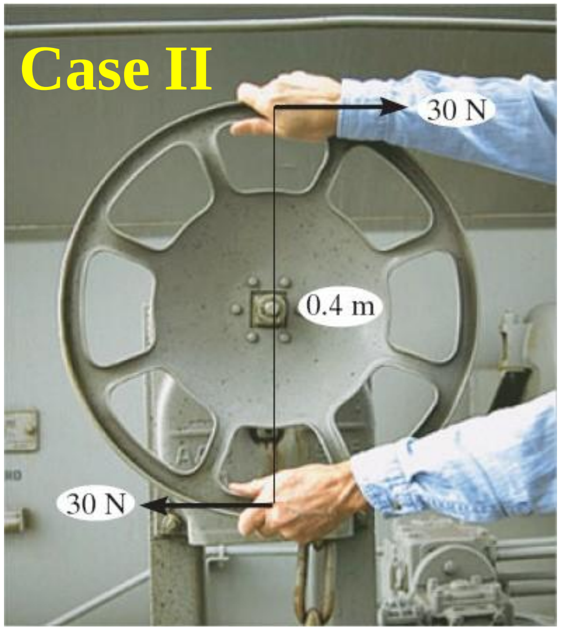
Force required for case I is 80N

Force required for case II is 60N

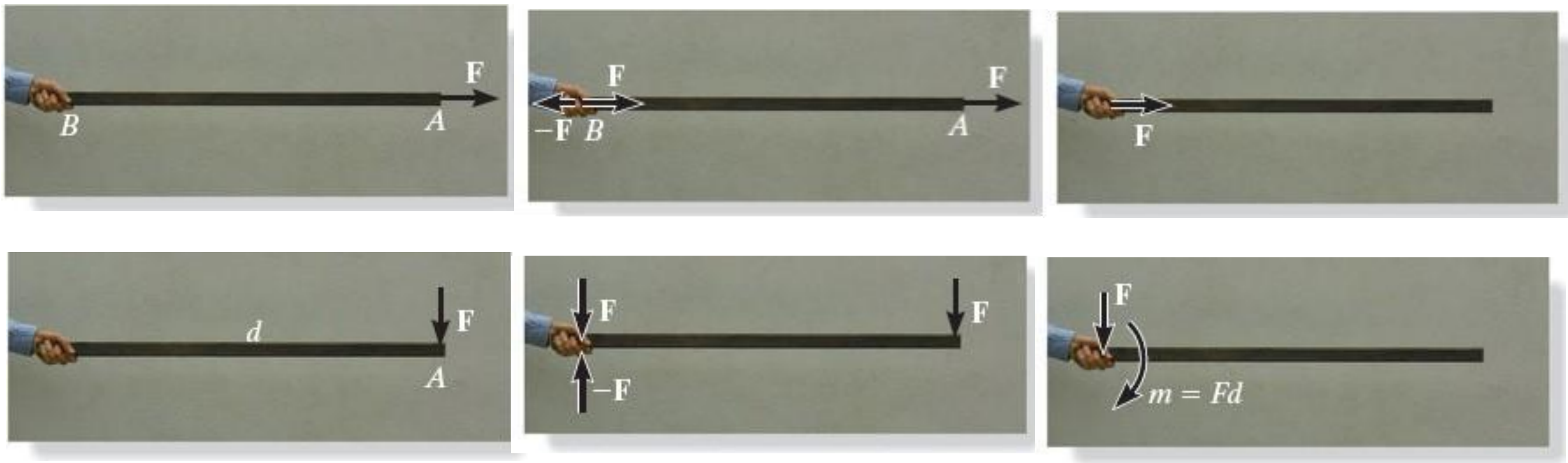
What if the shaft is not connected at the center of the wheel?

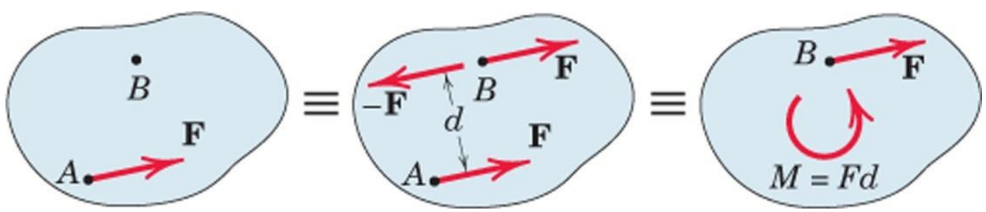
Is it a Free Vector?





Equivalent Systems



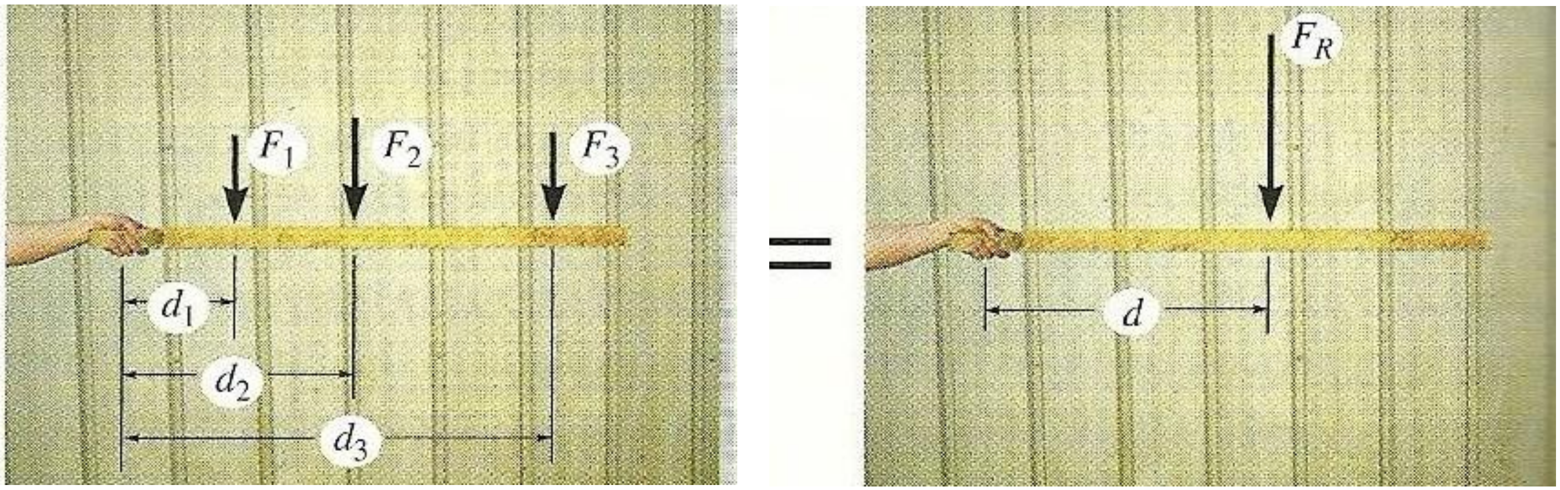


At support O

$$W_r = W_1 + W_2$$

$$M_o = W_1 d_1 + W_2 d_2$$

Equivalent Systems: Resultants



$F_R = F_1 + F_2 + F_3$

What is the value of d?

Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

$$\boxed{F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3}$$

Equilibrium Conditions

Equivalent Systems: Resultants

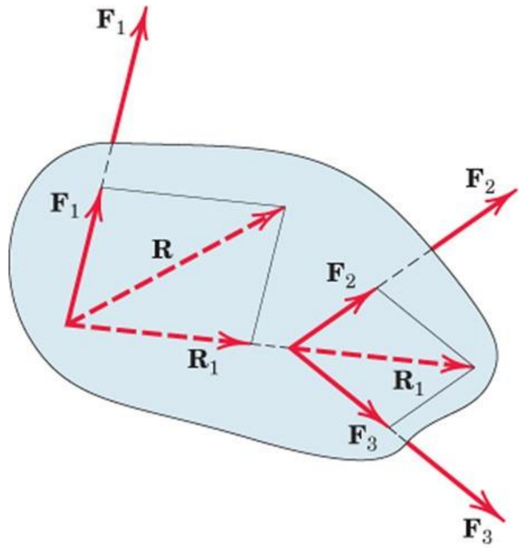
Equilibrium

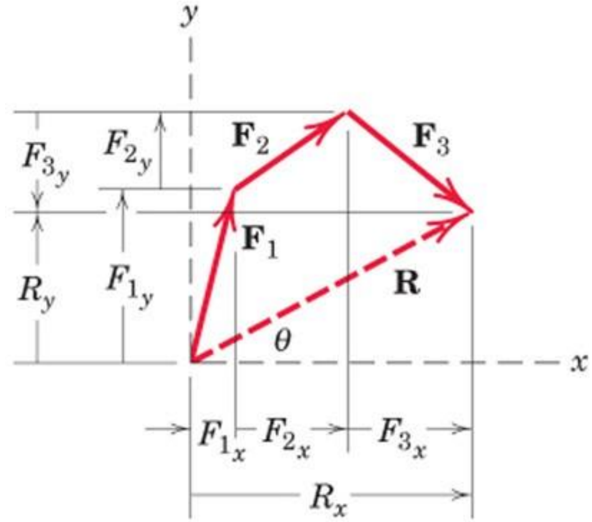
Equilibrium of a body is a condition in which the resultants of all forces acting on the body is zero.

Condition studied in Statics

Equivalent Systems: Resultants

Vector Approach: Principle of Transmissibility can be used





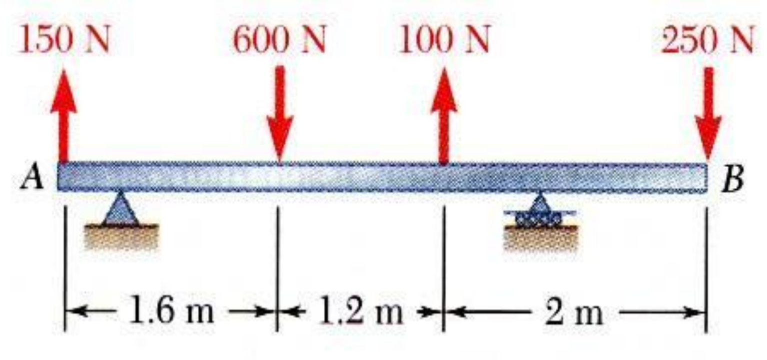
Magnitude and direction of the resultant force R is obtained by forming the force polygon where the forces are added head to tail in any sequence

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Equivalent Systems: Example



For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at *A*, (b) an equivalent force couple system at *B*, and (c) a single force or resultant.

Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

<u>Solution:</u>

a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

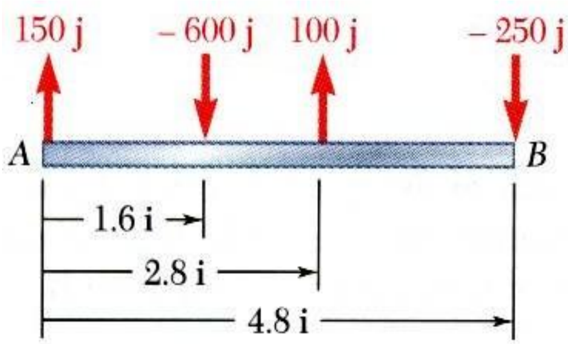
b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

Equivalent Systems: Example

SOLUTION

(a) Compute the resultant force and the resultant couple at *A*.



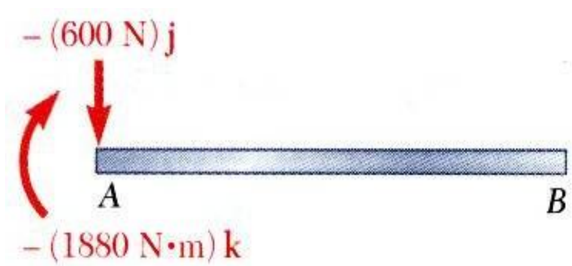
$$\vec{R} = \sum \vec{F} = 150\vec{j} - 600\vec{j} + 100\vec{j} - 250\vec{j}$$

$$\vec{R} = -(600\,\mathrm{N})\vec{j}$$

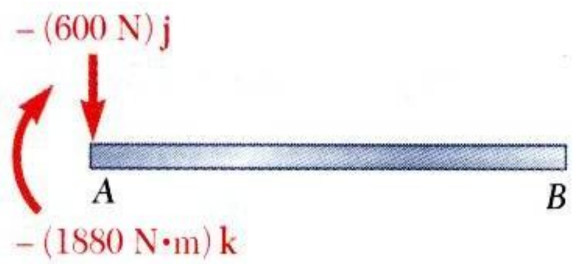
$$\vec{M}_A^R = \sum \left(\vec{r} \times \vec{F}\right)$$

$$= 1.6\vec{i} \times \left(-600\vec{j}\right) + 2.8\vec{i} \times \left(100\vec{j}\right) + 4.8\vec{i} \times \left(-250\vec{j}\right)$$

$$\vec{M}_A^R = -(1880\,\mathrm{N.n})\vec{k}$$



Equivalent Systems: Example

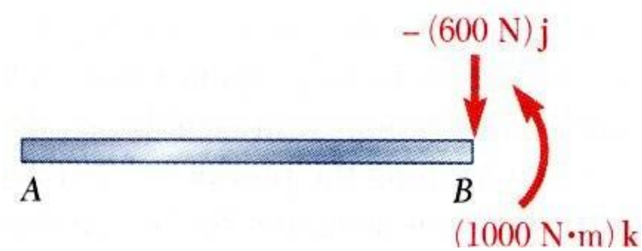


b) Find an equivalent force-couple system at B based on the force-couple system at A.

The force is unchanged by the movement of the force-couple system from A to B.

$$R = -(600N)j$$

The couple at B is equal to the moment about B of the force-couple system found at A.



$$M_B^R = M_A^R + r_{BA} \times R$$

$$= -1800k + (-4.8i) \times (-600j)$$

$$= (1000N.n\ )k$$

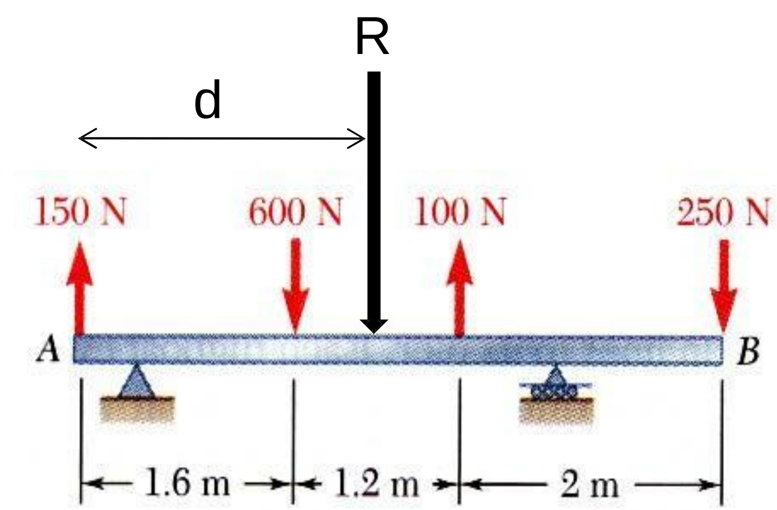
Equivalent Systems: Example

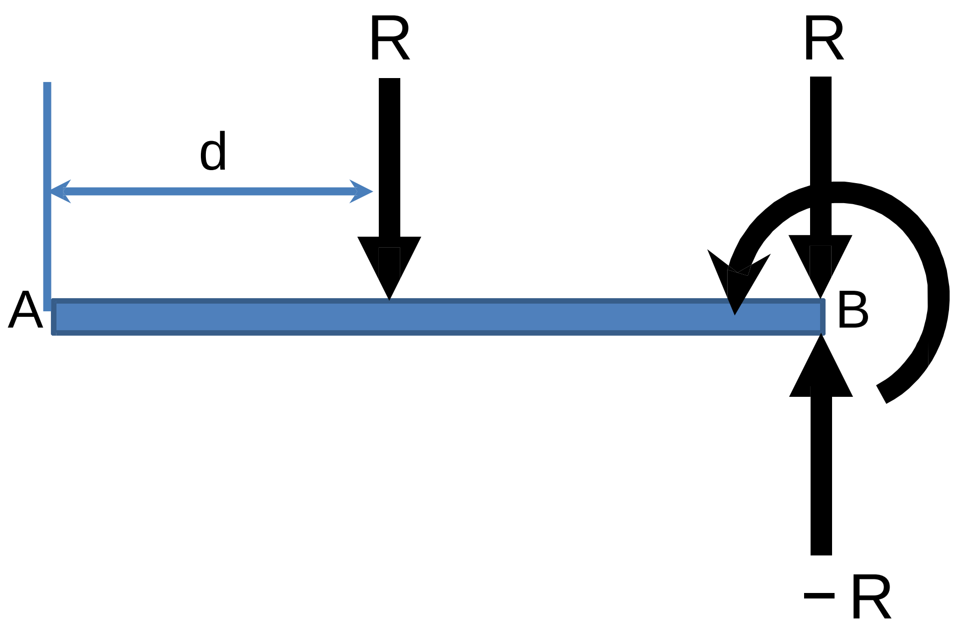
$R = F_1 + F_2 + F_3 + F_4$

$R = 150 - 600 + 100 - 250 = -600\text{ N}$

$Rd = F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4$

$d = 3.13\text{ n}$

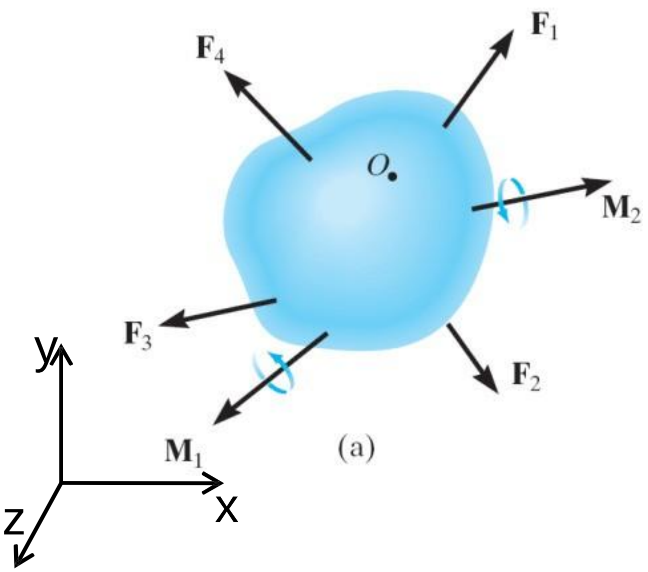




Rigid Body Equilibrium

A rigid body will remain in equilibrium provided

- sum of all the external forces acting on the body is equal to zero, and
- Sum of the moments of the external forces about a point is equal to zero

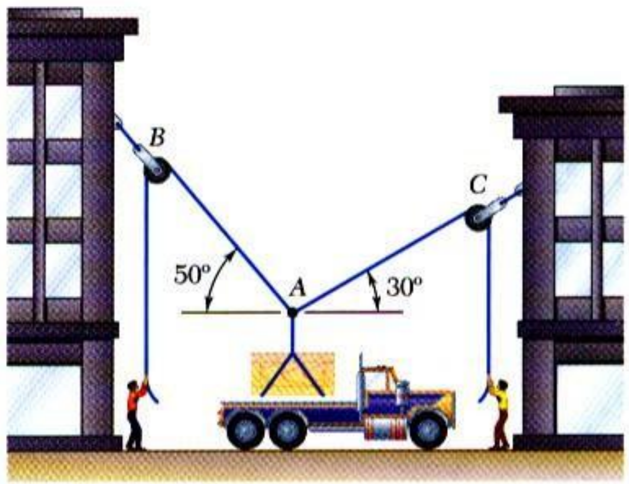


$$\boxed{\begin{aligned}\Sigma F_x &= 0\\ \Sigma F_y &= 0\\ \Sigma F_z &= 0\end{aligned}}$$

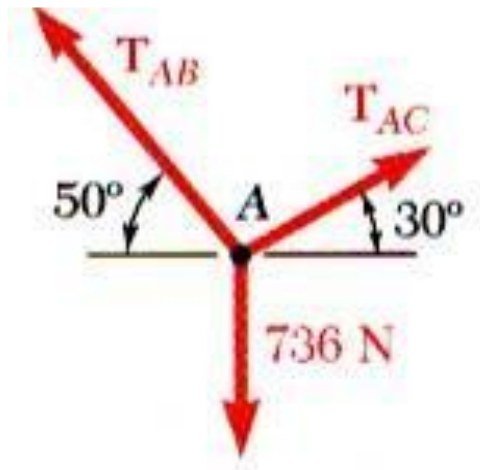
$$\boxed{\begin{aligned}\Sigma M_x &= 0\\ \Sigma M_y &= 0\\ \Sigma M_z &= 0\end{aligned}}$$

Rigid Body Equilibrium

Free-Body Diagrams



Space Diagram: A sketch showing the physical conditions of the problem.



Free-Body Diagram: A sketch showing only the forces on the selected particle.

Rigid Body Equilibrium

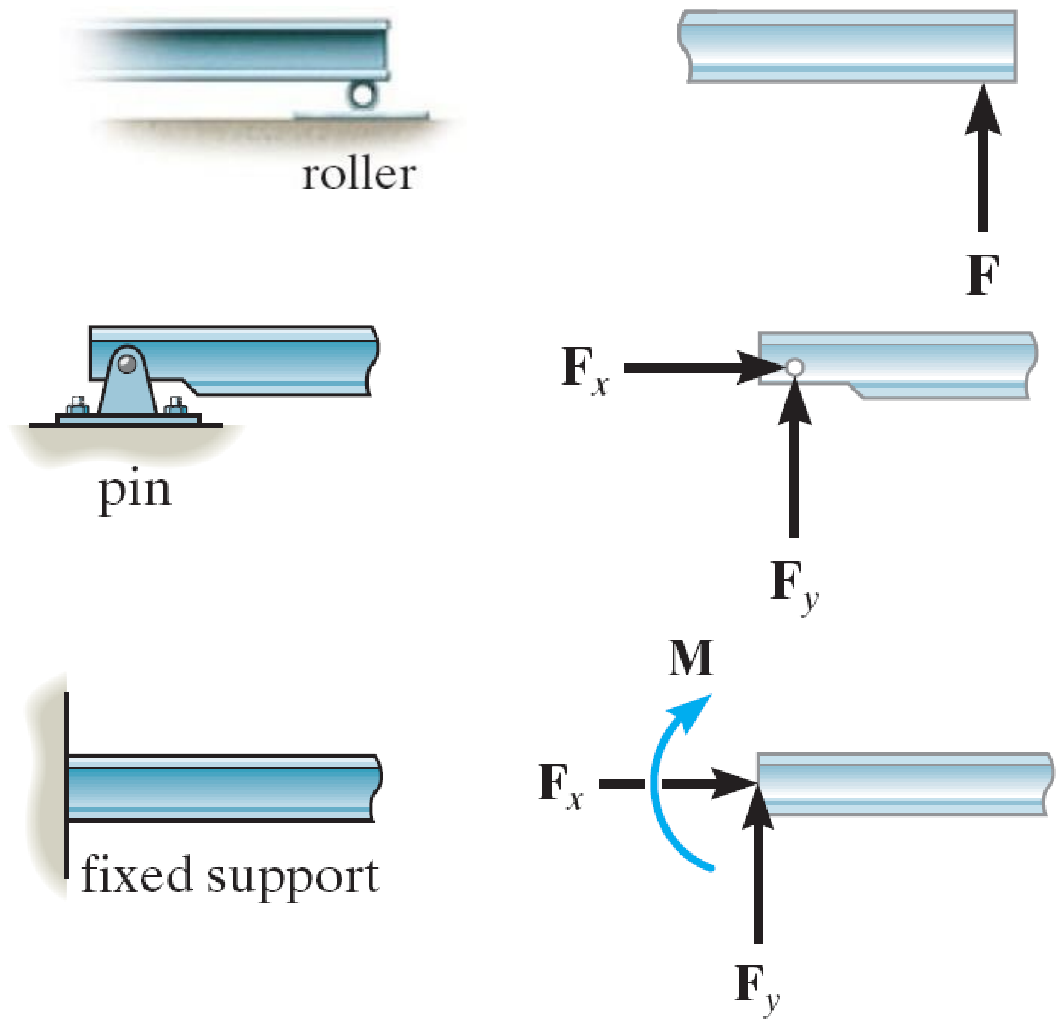
Support Reactions

Prevention of

Translation or

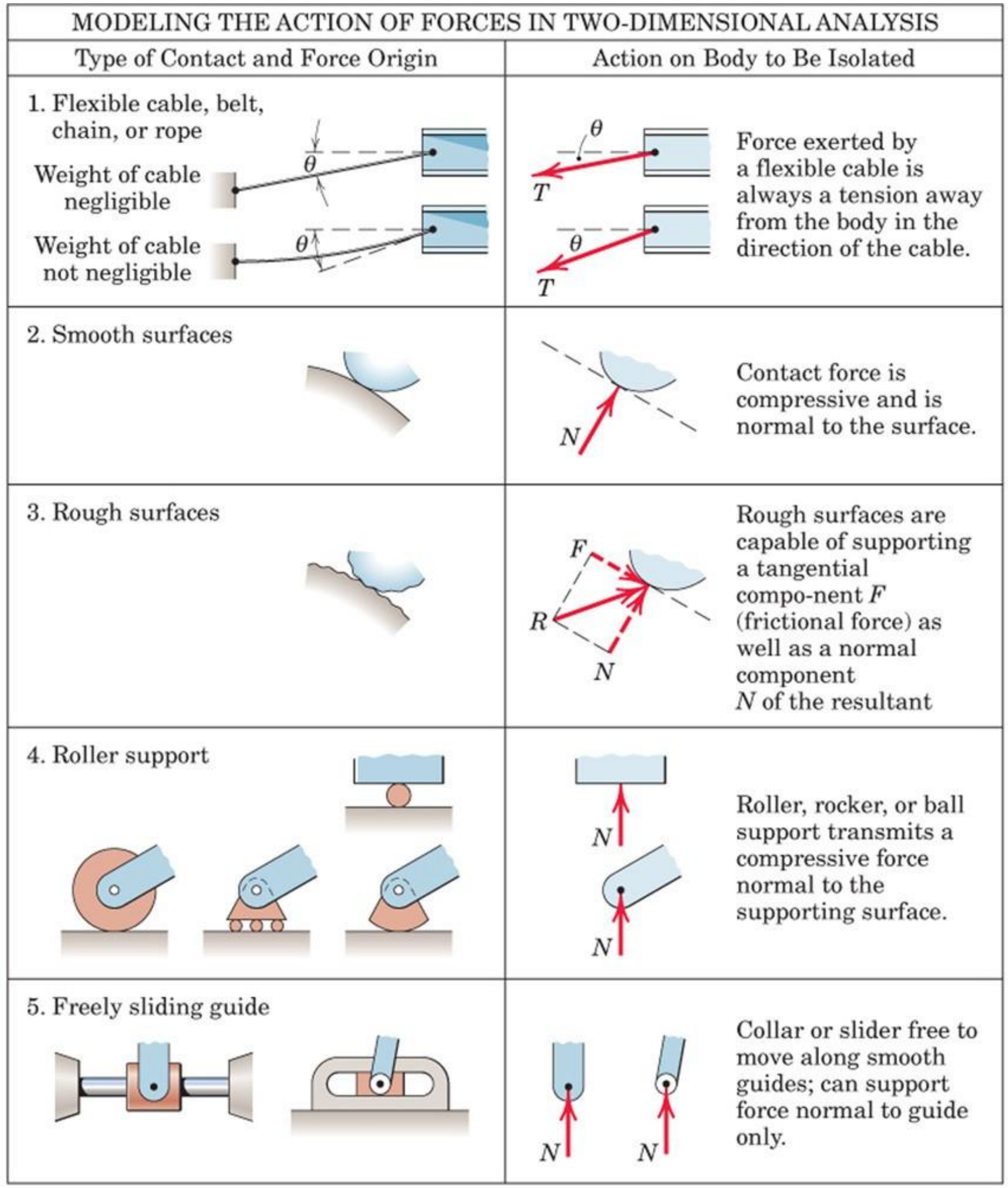
Rotation of a body

Restraints



Rigid Body Equilibrium

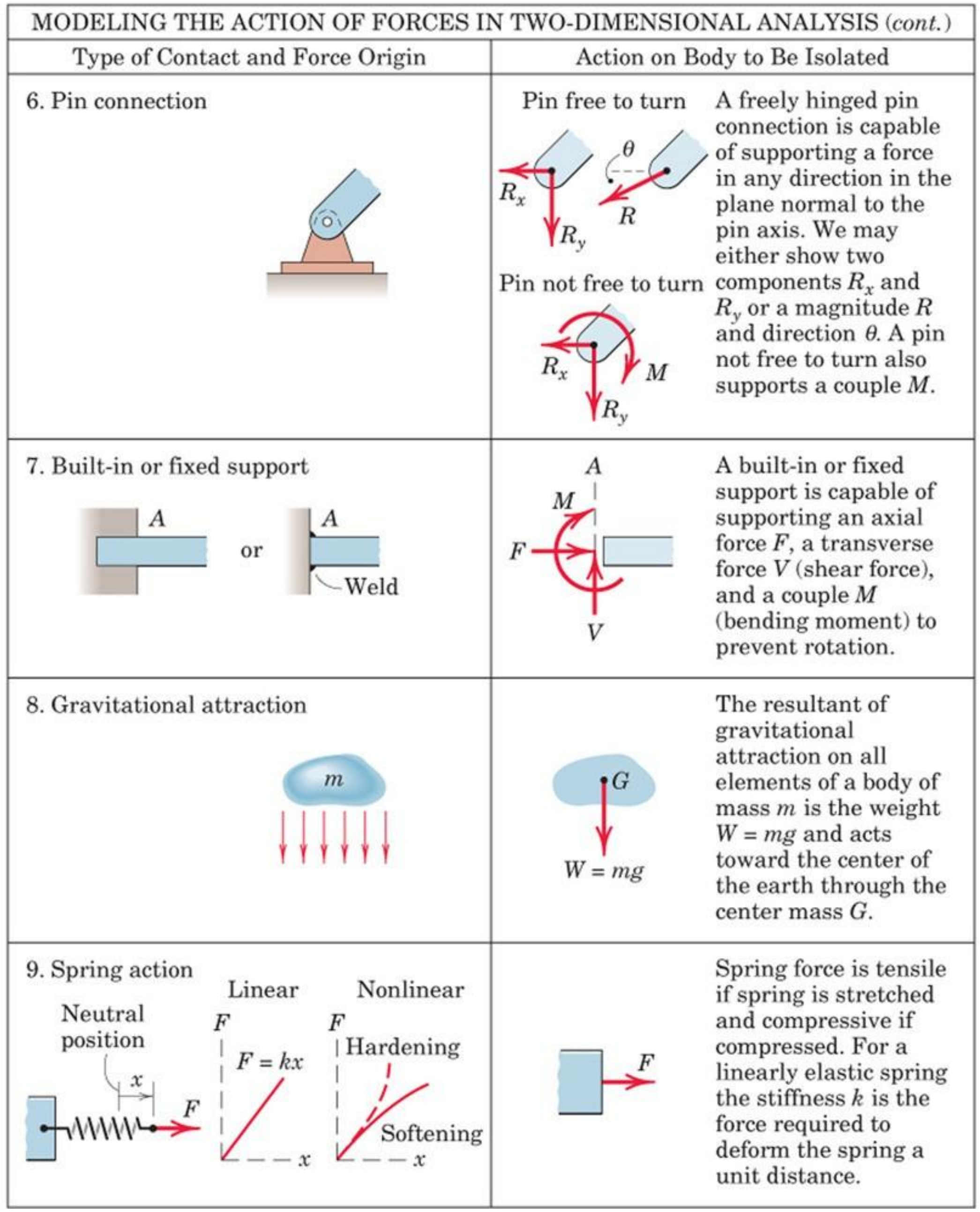
Various Supports
2-D Force Systems



MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible	Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces	Contact force is compressive and is normal to the surface.
3. Rough surfaces	Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant
4. Roller support	Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide	Collar or slider free to move along smooth guides; can support force normal to guide only.

Rigid Body Equilibrium

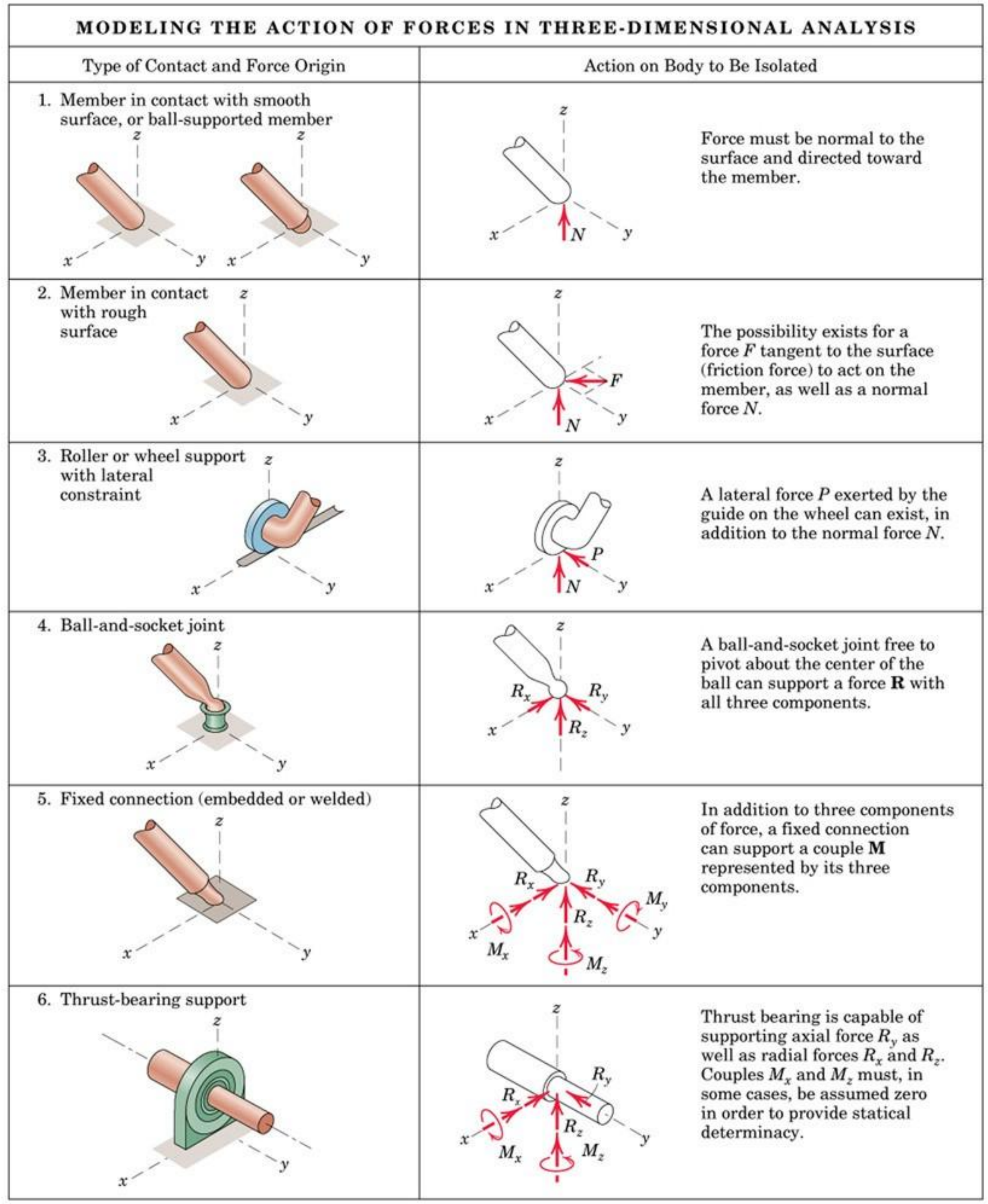
Various Supports
2-D Force Systems



MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)		
Type of Contact and Force Origin	**Action on Body to Be Isolated**	
6. Pin connection	Pin free to turn; Pin not free to turn	A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction	$W = mg$	The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action		Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

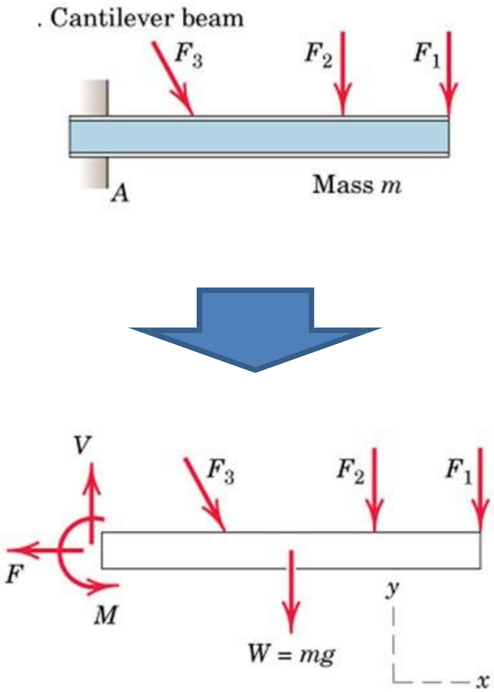
Rigid Body Equilibrium

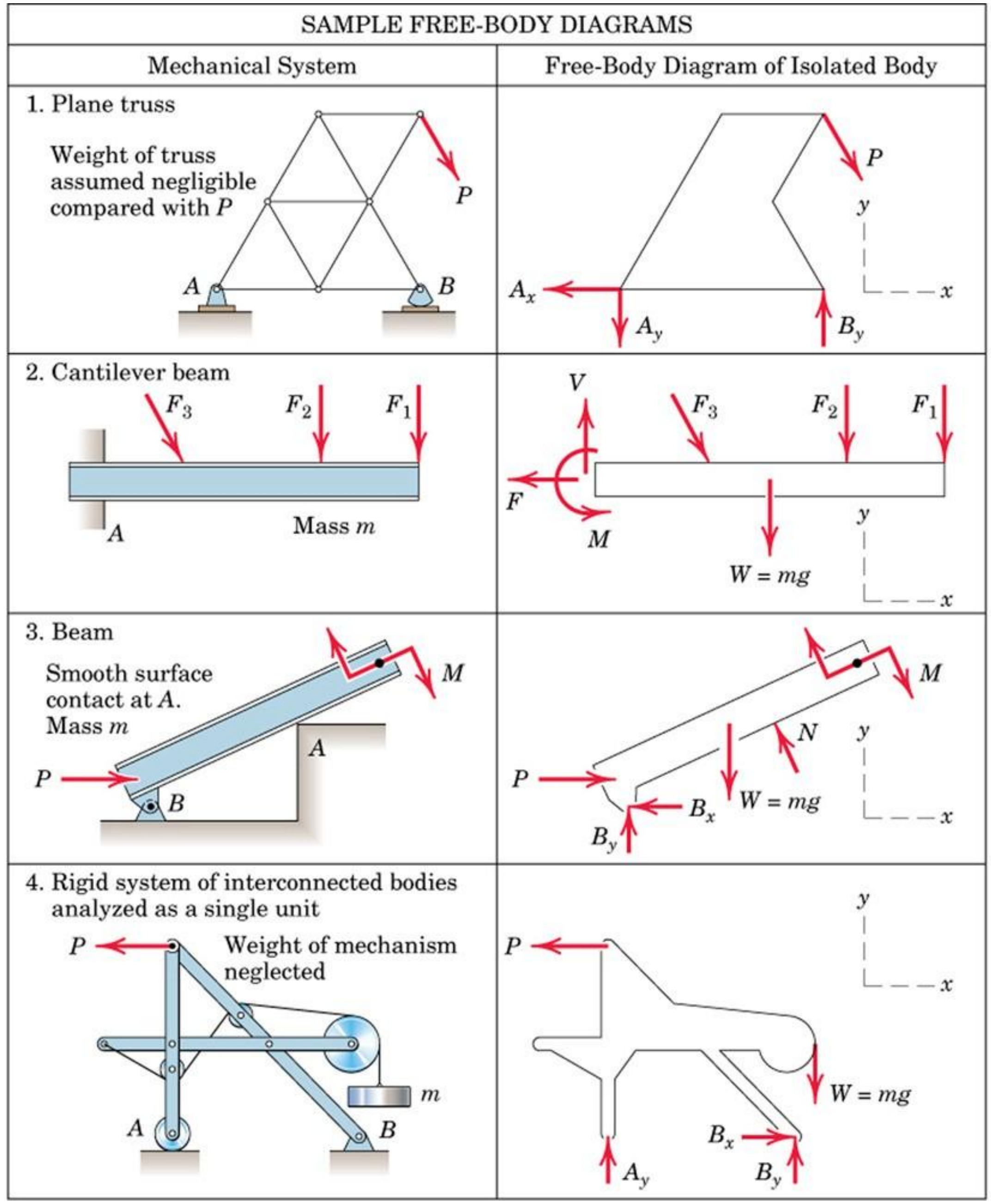
Various Supports
3-D Force
Systems



MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member	Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface	The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint	A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
4. Ball-and-socket joint	A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded)	In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support	Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

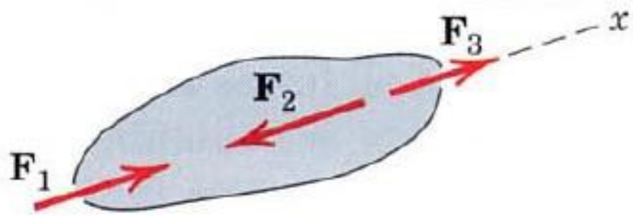
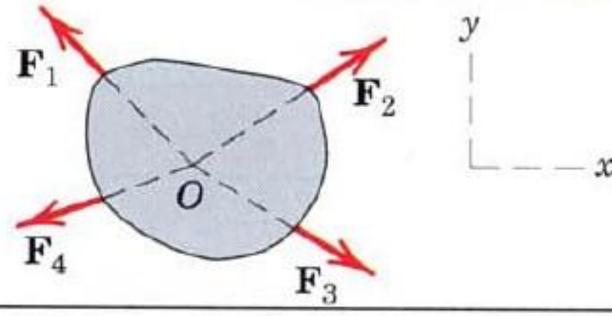
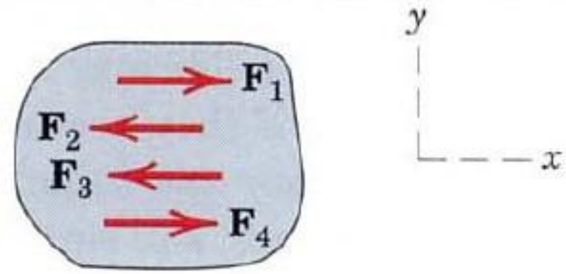
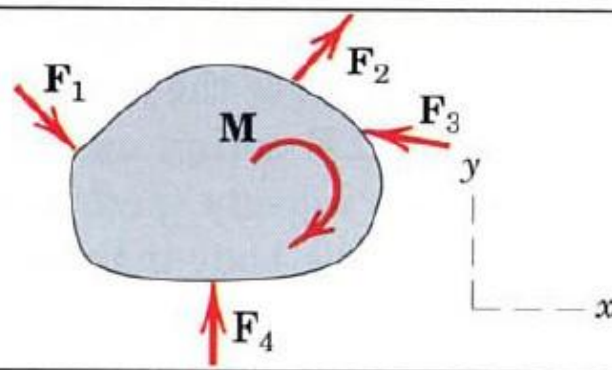
Free body diagram





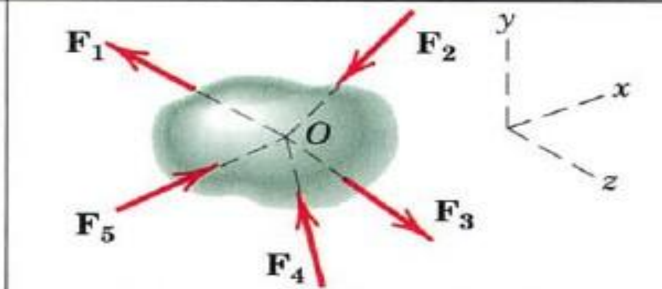
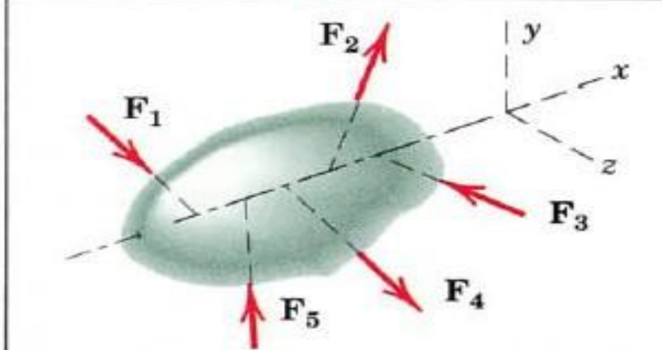
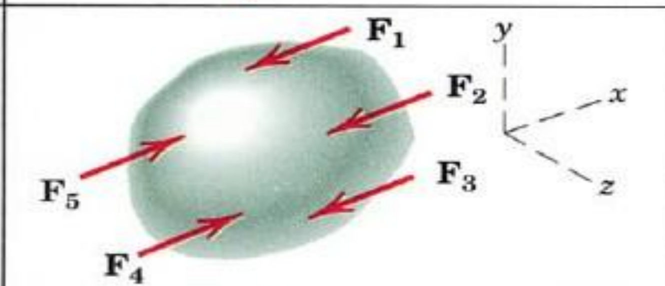
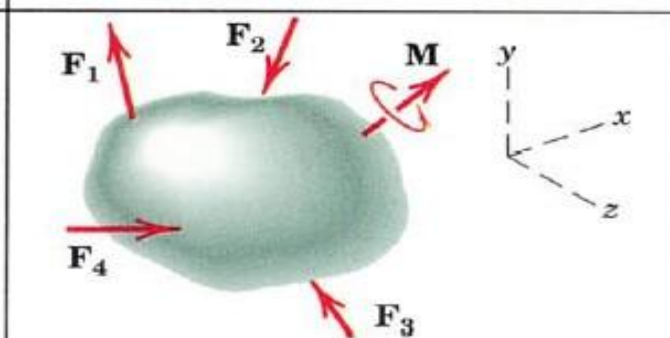
Rigid Body Equilibrium

Categories in 2-D

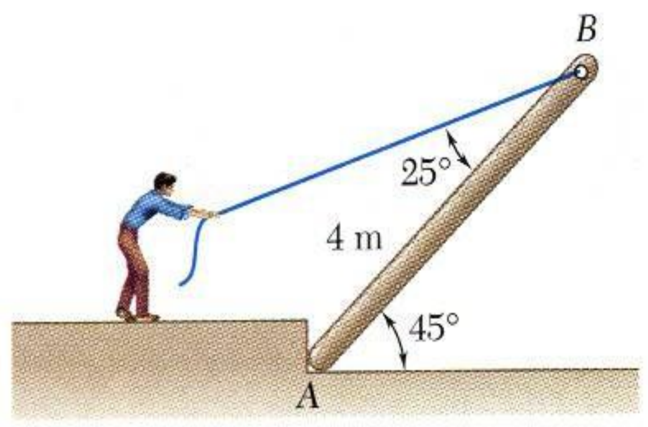
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Rigid Body Equilibrium

Categories in 3-D

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Rigid Body Equilibrium: Example



A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at A.

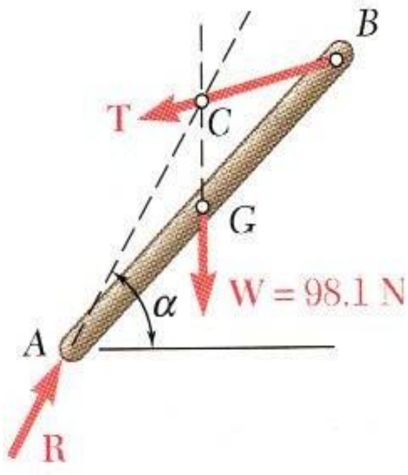
Solution:

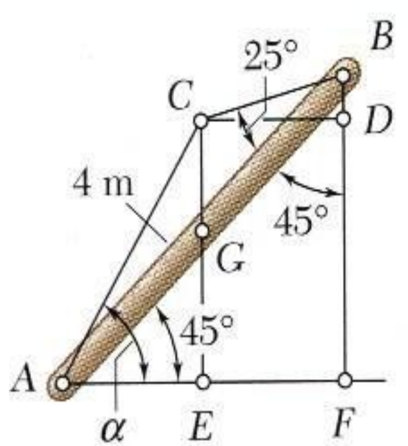
- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at A.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.

- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Rigid Body Equilibrium: Example





- Create a free-body diagram of the joist.
- Determine the direction of the reaction force R.

$$AF = AB\cos 45 = (4\text{m})\cos 45 = 2.828\text{m}$$

$$CD = AE = \tfrac{1}{2}AF = 1.414\text{m}$$

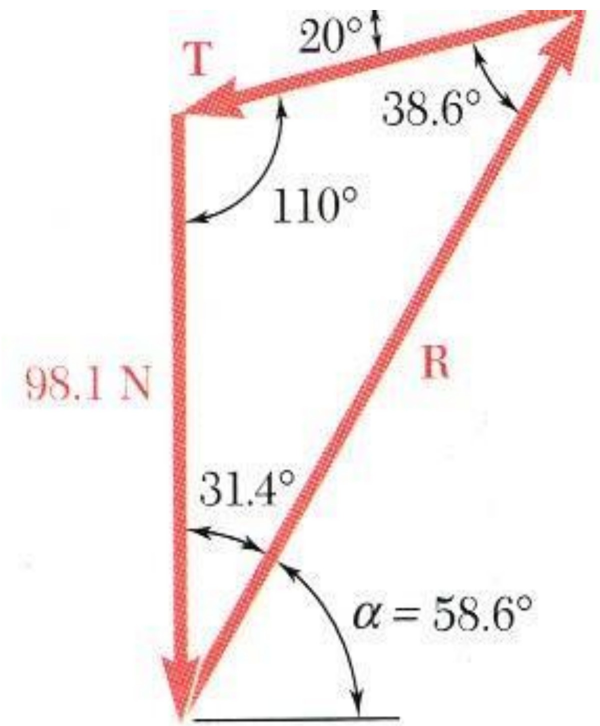
$$BD = CD\cot(45+20) = (1.414\text{m})\tan 20 = 0.515\text{m}$$

$$CE = BF - BD = (2.828 - 0.515)\text{m} = 2.313\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^0}$$

Rigid Body Equilibrium: Example



- Determine the magnitude of the reaction force R.

$$\frac{T}{\sin 31.4^{\circ}} = \frac{R}{\sin 110^{\circ}} = \frac{98.1\text{N}}{\sin 38.6^{\circ}}$$

$$\boxed{\begin{aligned} T &= 81.9\text{N} \\ R &= 147.8\text{N} \end{aligned}}$$