ME3491
Theory of Machine

Unit – 2
GEARS and GEAR TRAINS

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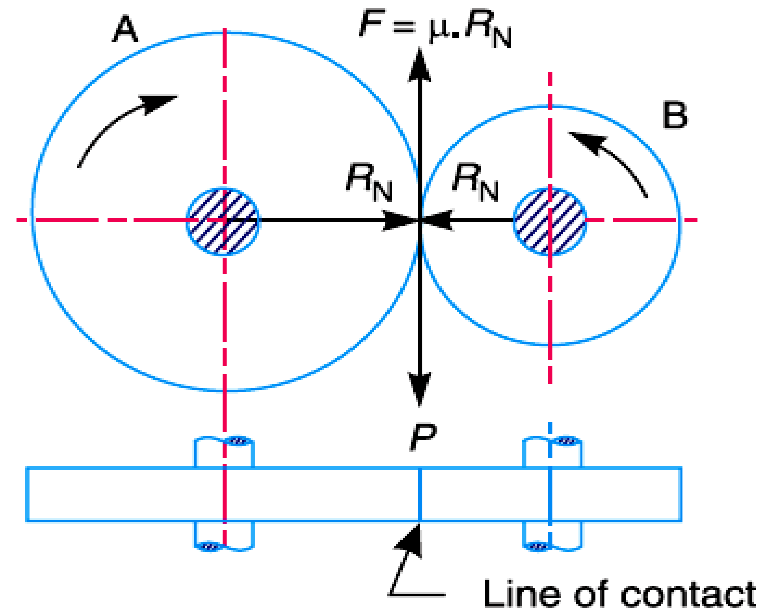
UNIT II GEARS AND GEAR TRAINS (9)

Spur gear – law of toothed gearing – involute gearing – Interchangeable gears – Gear tooth action interference and undercutting – nonstandard teeth – gear trains – parallel axis gears trains – epicyclic gear trains – automotive transmission gear trains.

Introduction

- **Friction Wheel**

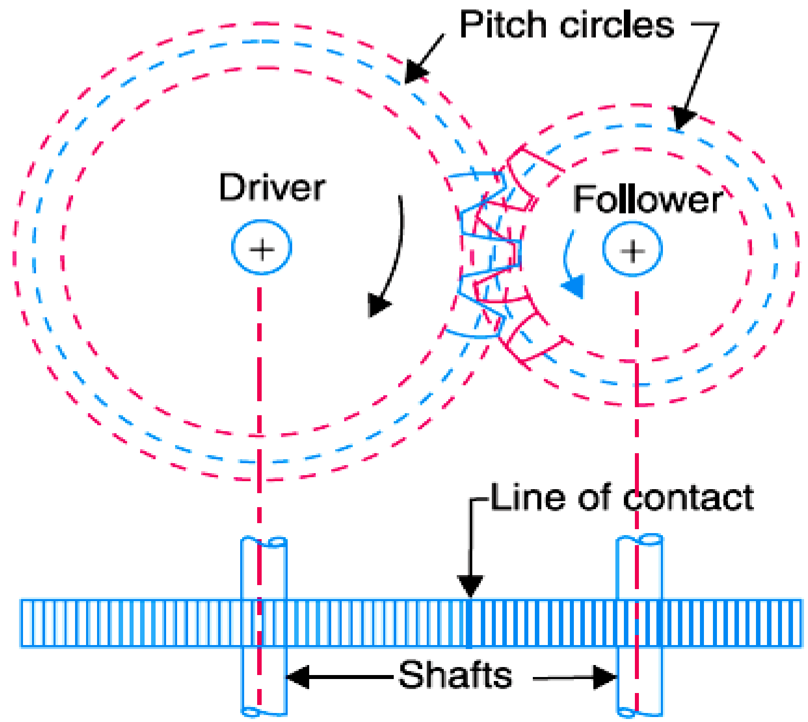
The wheel is rotated so long as the tangential force exerted by the other wheel. But when the tangential force (P) exceeds the **frictional resistance (F), slipping will take place between the two** wheels. Thus the friction drive is not a positive drive



(*a*) Friction wheels.

Toothed Wheel

To avoid the slipping, a number of projections (called teeth) are provided on the periphery of the wheel which will fit into the corresponding recesses on the periphery of the other wheel friction wheel with the teeth cut on it is known as toothed wheel or gear.



(*b*) Toothed wheels.

Advantages and Disadvantages of Gear Drive

- *Advantages*

  It transmits exact velocity ratio.

  It may be used to transmit large power.

  It has high efficiency.

  It has reliable service.

  It has compact layout.

- *Disadvantages*

  The manufacture of gears require special tools and equipment.

  The error in cutting teeth may cause vibrations and noise during operation.

Application of Gear Drive

- **Small** – Wrist watch, Toys, Home appliences, etc.,
- **Medium** – Lathe, Milling machine, Shaping machine, Automotive gear box, Simple power transmission system, etc.,
- **Large** – Turbines, Marine, Aero space, Cranes, etc.,

Classification of Toothed Gears

1. *According to the position of axes of the shafts.*

   (a) Parallel, (b) Intersecting, and (c) Non-intersecting and non-parallel.

2. *According to the peripheral velocity of the gears.*

   (a) Low velocity (< 3m/s), (b) Medium velocity (3-15 m/s), and (c) High velocity (>15m/s).

3. *According to the type of gearing*

(a) External gearing, (b) Internal gearing, and (c) Rack and pinion

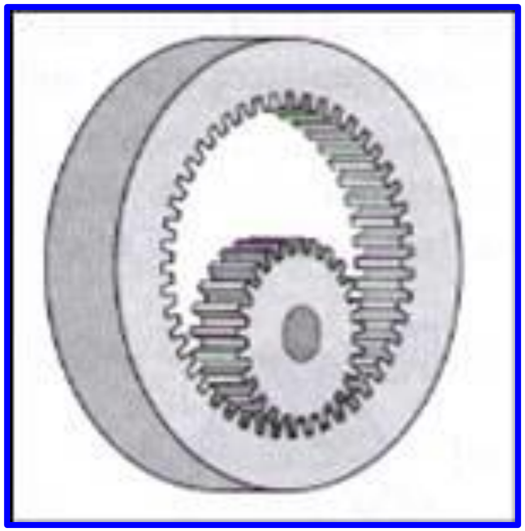
4. *According to position of teeth on the gear surface*

(a) Straight, (b) Inclined, and (c) Curved.

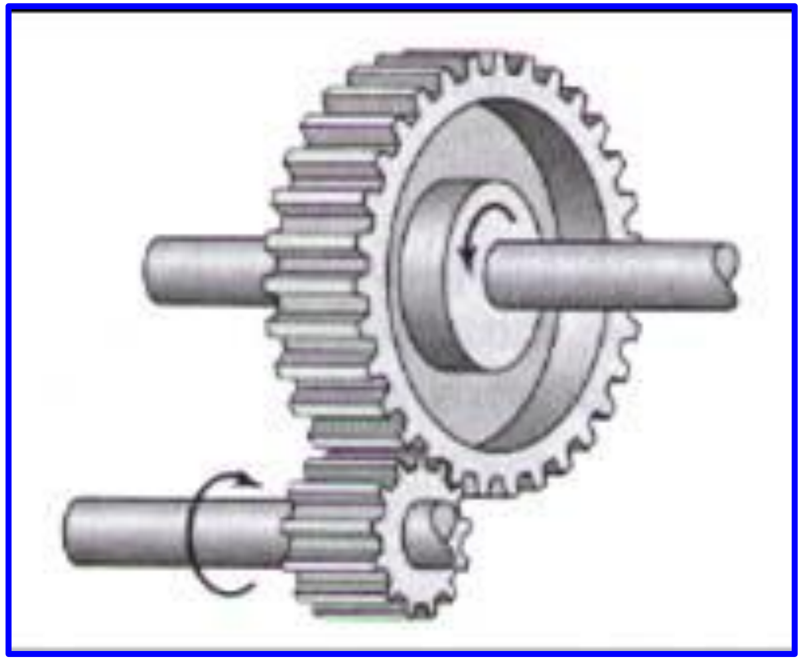
Types of Gears

- **Spur Gear** - Tooth profile is parallel to the axis of rotation, transmits motion between parallel shafts.
  - External spur gear
  - Internal Spur gear

Internal gears

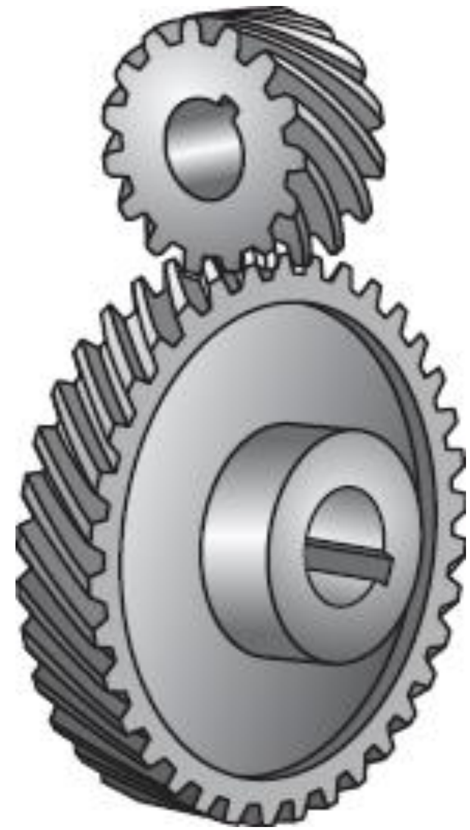


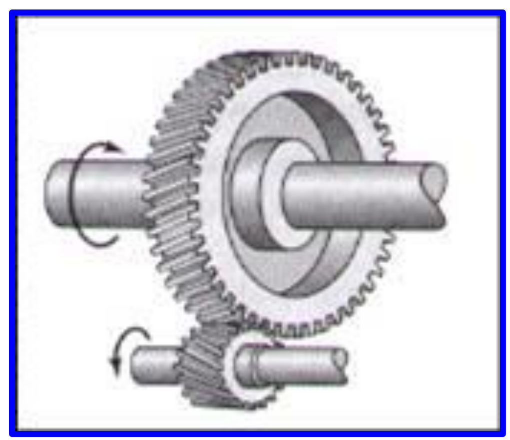
Gear (large gear)



Types of Gears

- **Helical Gear -** – teeth are inclined to the axis of rotation, the angle provides more gradual engagement of the teeth during meshing, transmits motion between parallel shafts.
  - Single Helical gear
  - Double Helical gear (Herringbone gear)



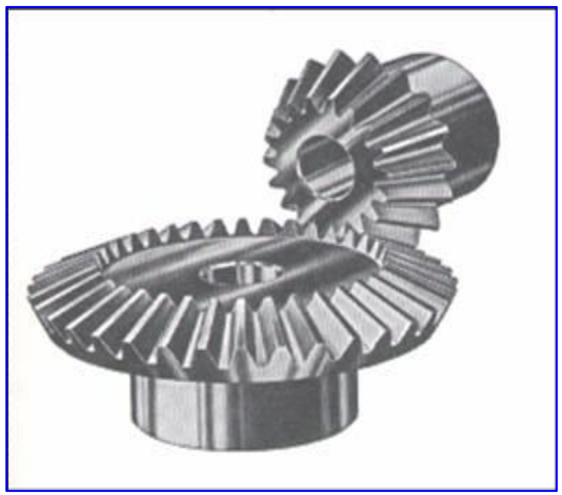


Types of Gears

- **Bevel Gear -** Teeth are formed on a conical surface, used to transfer motion between non-parallel and intersecting shafts.
  – Straight bevel gear
  – Spiral bevel gear

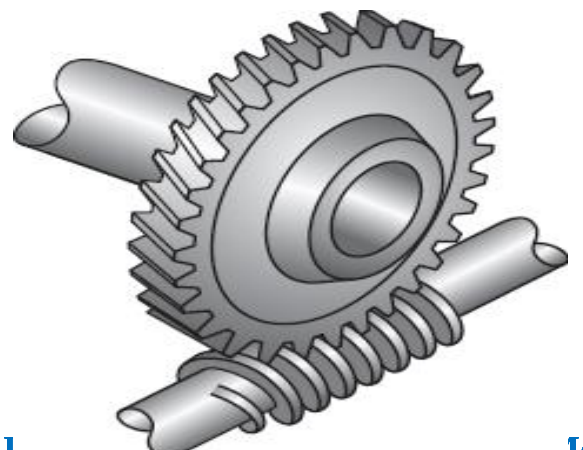
Spiral bevel gear

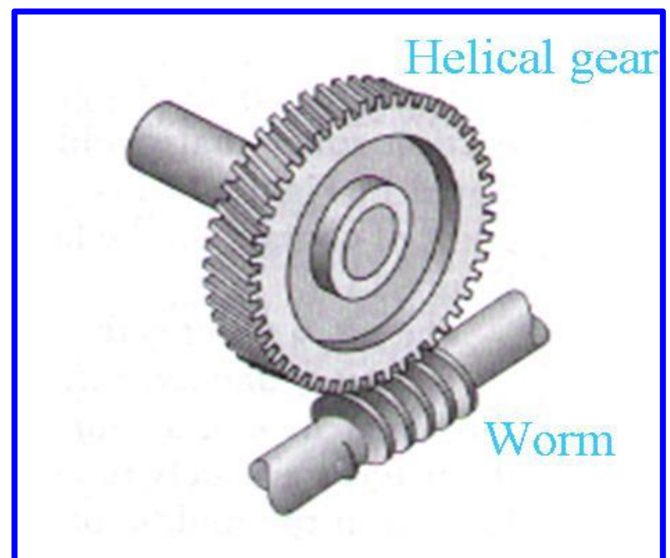
Straight bevel gear



Types of Gears

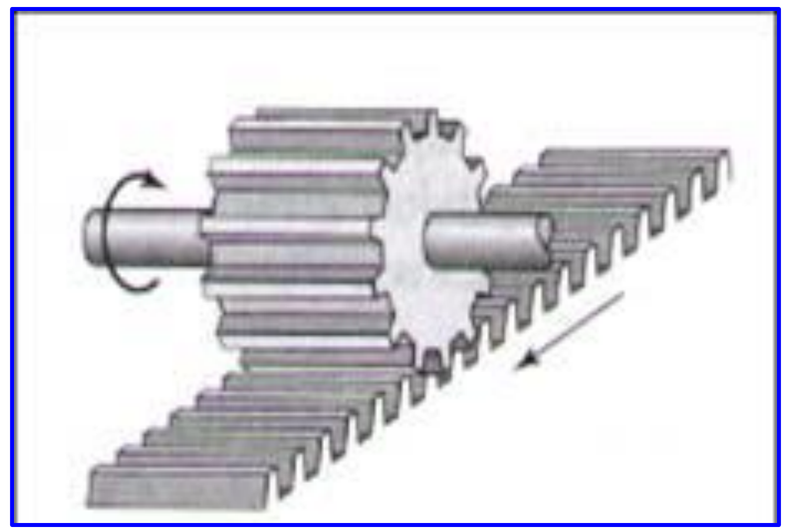
- **Worm and Worm wheel** - Consists of a helical gear and a power screw (worm), used to transfer motion between non-parallel and non-intersecting shafts.





Types of Gears

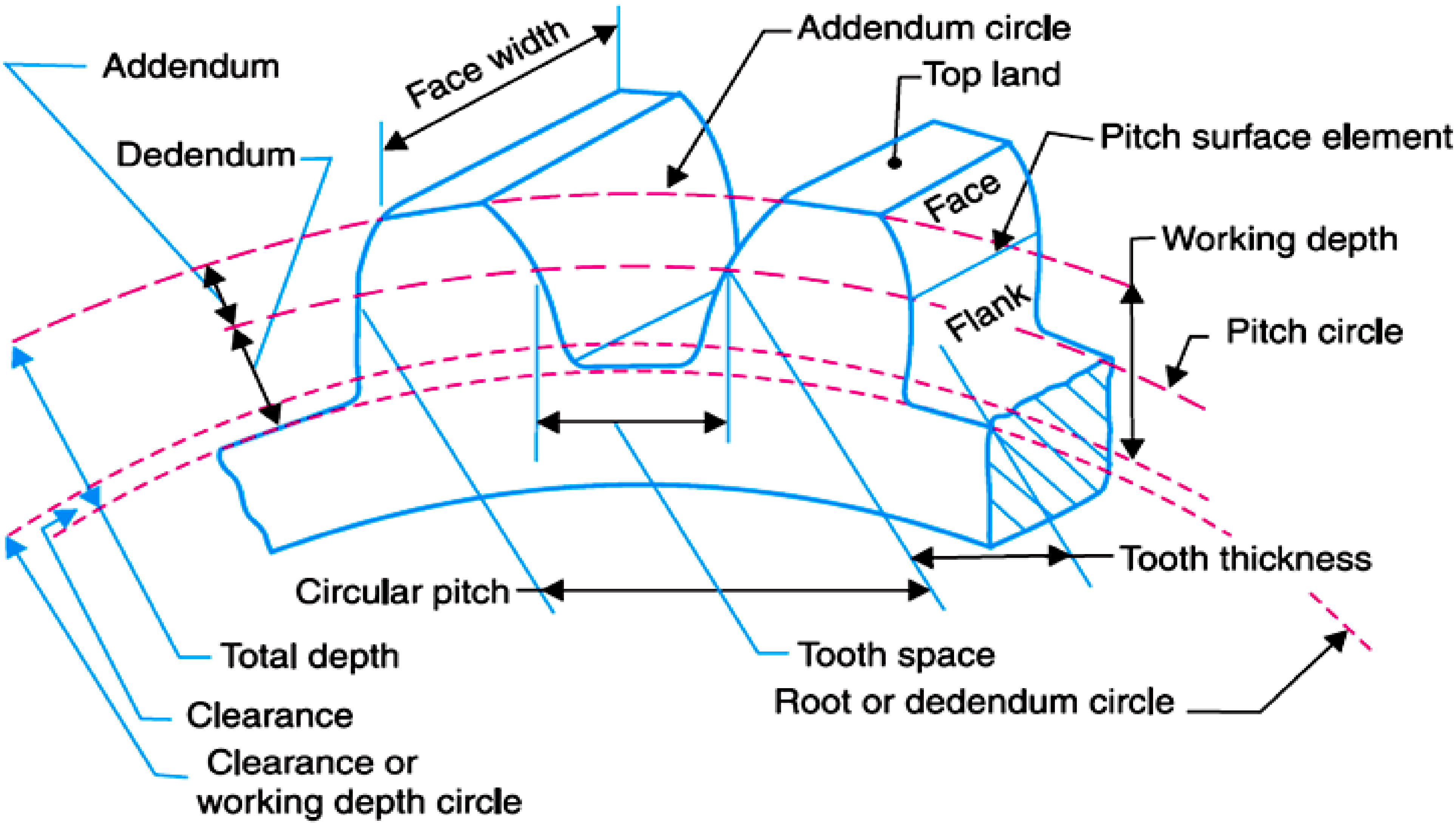
- **Rack and Pinion** - A special case of spur gears with the gear having an infinitely large diameter, the teeth are laid flat.



Gear Materials

- The material used for the manufacture of gears depends upon the strength and service conditions like wear, noise etc
- The gears may be manufactured from metallic or non-metallic materials
- Metallic gears - cast iron, steel and bronze.
- Nonmetallic – wood, compressed paper, Composite materials, PVC, nylon

Nomenclature of Gear



1. **Pitch circle**. It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.
2. **Pitch circle diameter**. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.
3. **Pitch point**. It is a common point of contact between two pitch circles.
4. **Pressure angle or angle of obliquity**. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by The standard pressure angles are 14 ° and 20°

5. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth..

6. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

7. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

8. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

9. Circular pitch. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by pc.

- Mathematically,

Circular pitch, pc = $\pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel..

10. Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by pd

Mathematically,

Diametral pitch, $$p_d = \frac{T}{D} = \frac{\pi}{p_c} \quad \left(\because\ p_c = \frac{\pi D}{T} \right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

11. Module. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m.

Mathematically, Module, $m = D / T$

12. Clearance. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as clearance circle.

13. Total depth. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

14. **Working depth.** It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

15. **Tooth thickness.** It is the width of the tooth measured along the pitch circle.

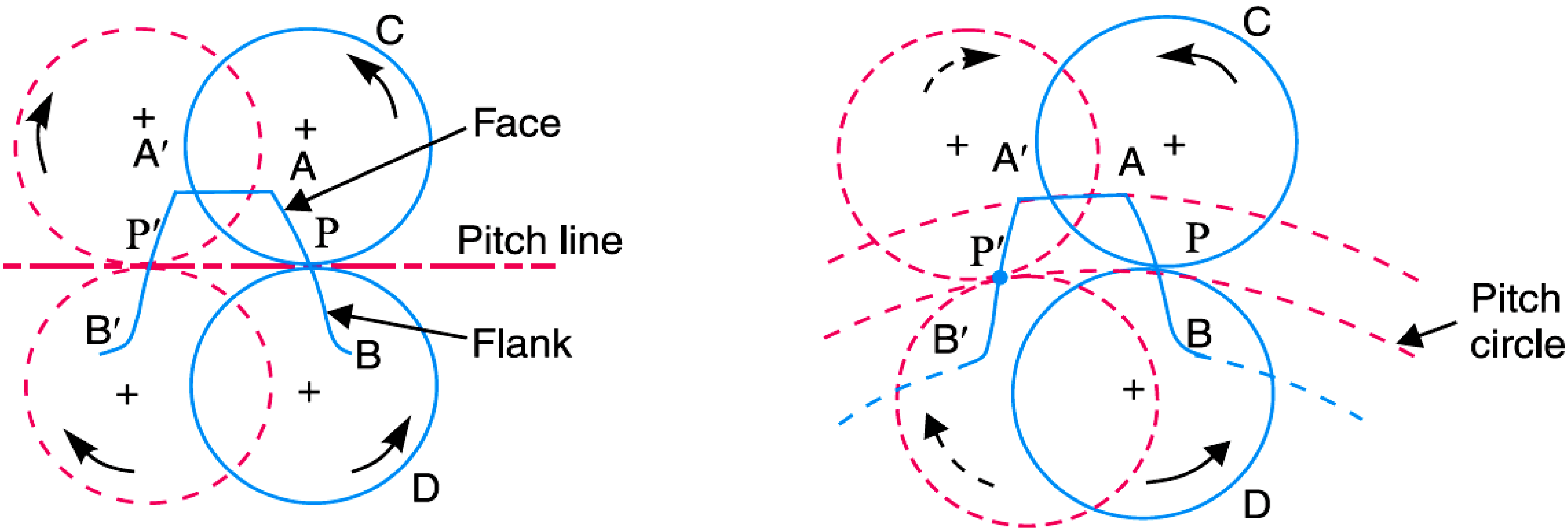
16. **Tooth space** . It is the width of space between the two adjacent teeth measured along the pitch circle..

17. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

18. Interference & Undercutting

Gear tooth forms

1. Cycloidal teeth



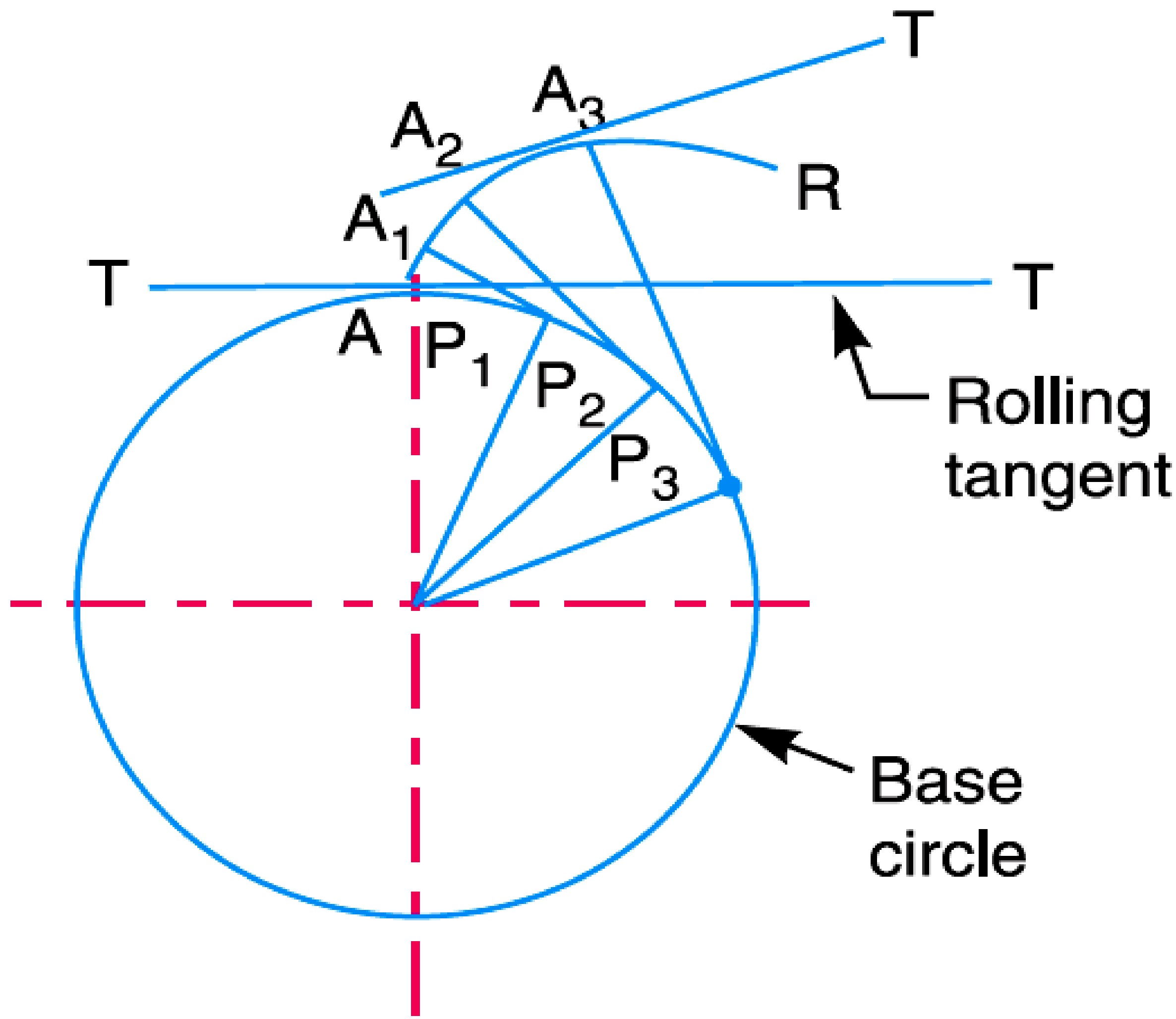
(*a*) (*b*)

Advantages of Cycloidal teeth

- Cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears
- Less wear in cycloidal gears as compared to involute gears
- Interference does not occur at all
- Exact centre distance to be maintained
- Pressure angle varies during the engagement
- Manufacturing cost is more

Gear tooth forms

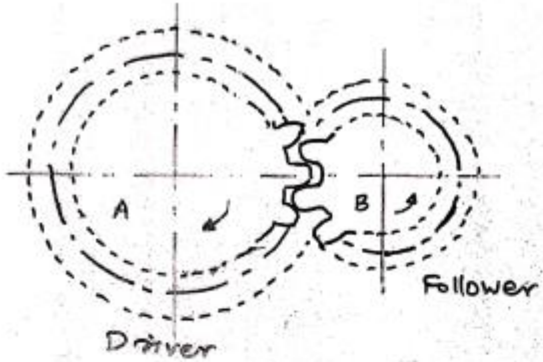
2. Involute teeth



Advantages of Involute teeth

- Centre distance for a pair may be varied with in the limits
- Pressure angle is constant through out the engagement
- Easy to manufacture
- Interference occurs

* VELOCITY RATIO OF A SIMPLE GEAR DRIVE



Let N_1 = Speed of the driver

T_1 = No. of teeth on the driver

D_1 = Pitch circle diameter of the driver

11^{ly} N_2, T_2, D_2 = Corresponding values of the follower

If the Surface Velocity of Driver and follower are equal then

$$\pi D_1 N_1 = \pi D_2 N_2$$

$$\therefore \frac{D_1}{D_2} = \frac{N_2}{N_1}$$

Since Pitch of Both wheels are same

$$\text{Then } \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2}$$

$$\therefore \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

$$\therefore \text{Velocity ratio} = \frac{N_2}{N_1} = \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

* Power Transmitted by Simple Gear

Power transmitted (or) Workdone = Force · x distance

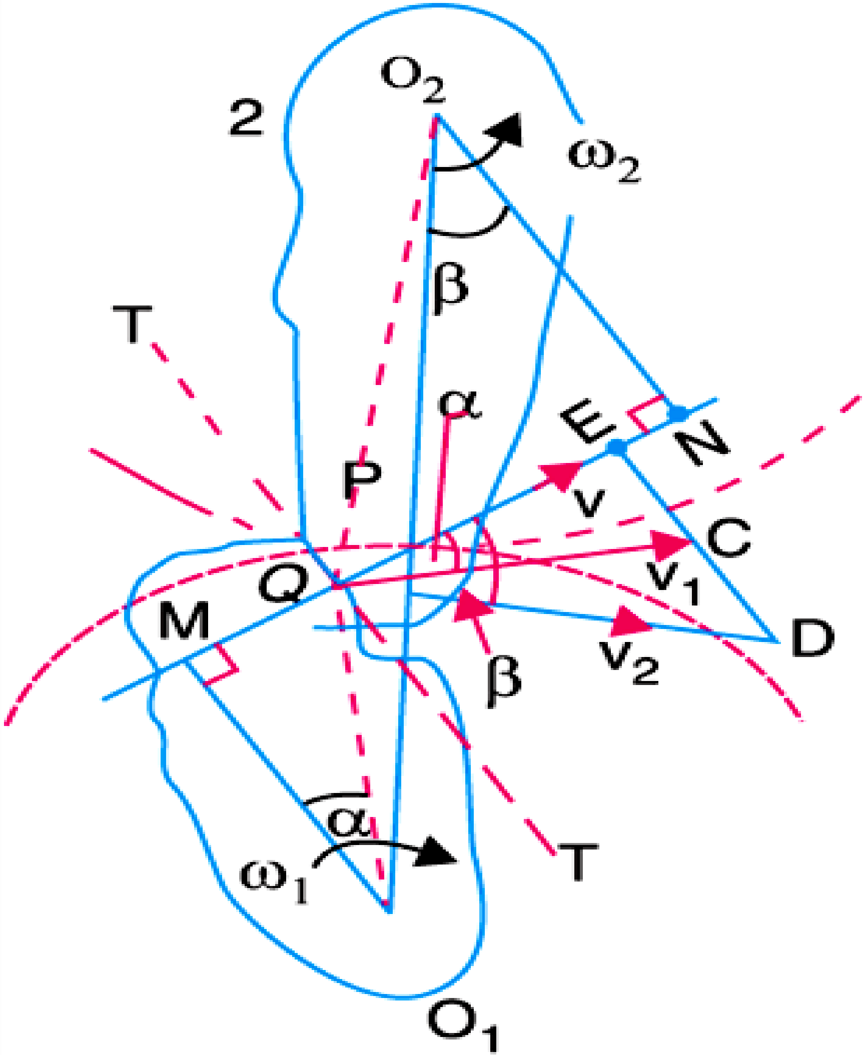
$$P = F \times v$$

F = Tangential Force excerted by the driver

v = Peripheral velocity of the driver

$$(w) \quad v = \frac{\pi D N}{60}$$

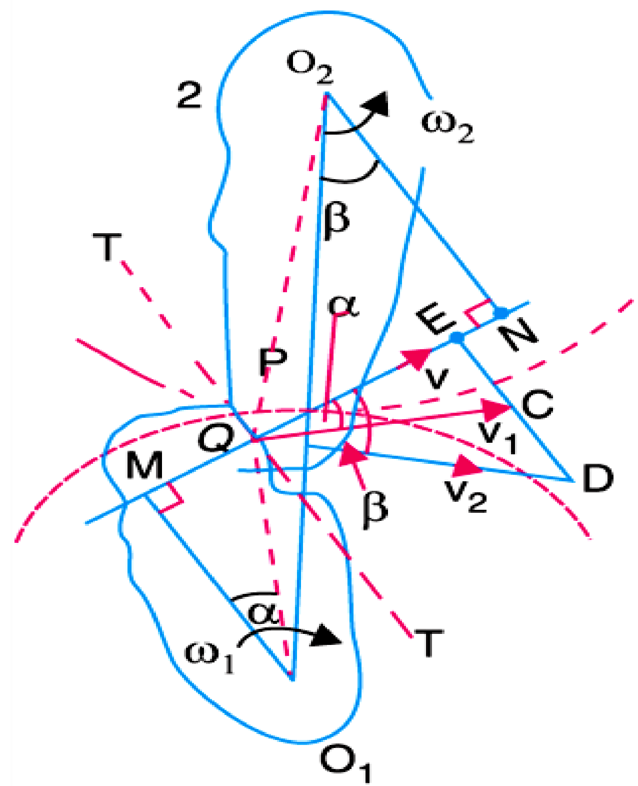
Law of Gearing



- The common normals of both contact points still pass through the same pitch point
- The common normal of the tooth profiles, at all contact points within the mesh, must always pass through a fixed point on the line of centers, called the pitch point..
- The fundamental law of gearing states that **the angular velocity ratio of all gears must remain constant throughout the gear mesh**.

Let $T\ T$ be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2, draw O_1M and O_2N perpendicular to MN. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let V_1 and V_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal

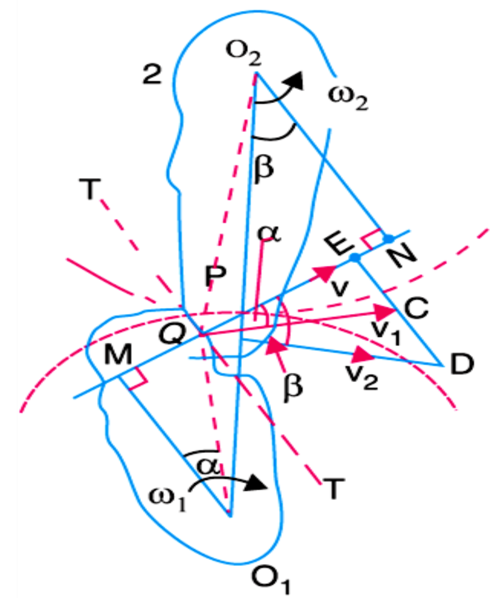


$$\therefore \quad v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1 Q)\cos\alpha = (\omega_2 \times O_2 Q)\cos\beta$$

$$(\omega_1 \times O_1 Q)\frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q)\frac{O_2 N}{O_2 Q} \quad \text{or} \quad \omega_1 \times O_1 M = \omega_2 \times O_2 N$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$$



Also from similar triangles O_1MP and O_2NP,

$$\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

Combining equations (*i*) and (*ii*), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P} \qquad = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

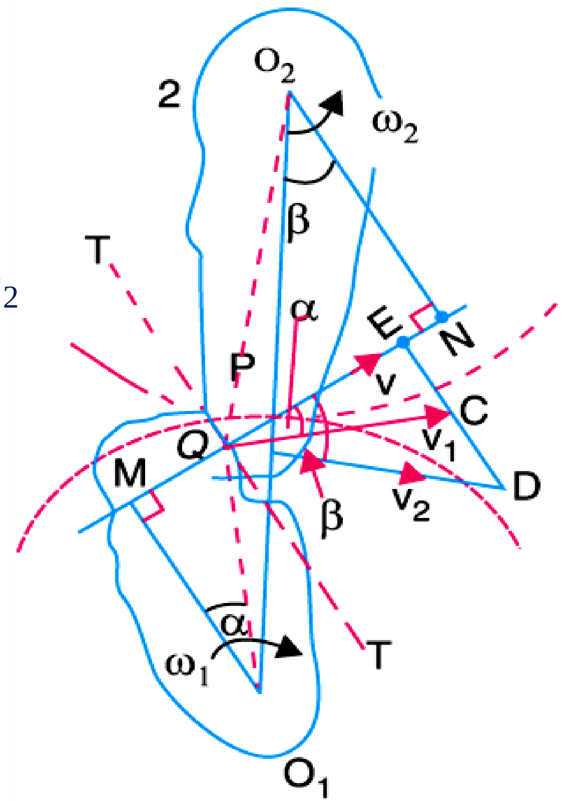
The angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres O1 and O2

Velocity of Sliding teeth

The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact

The velocity of point Q, considered as a point on wheel 1, along the common tangent TT is represented by EC. From similar triangles QEC and O_1MQ,

$$\frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_1 \qquad \text{or} \qquad EC = \omega_1 . MQ$$



Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tangent TT is represented by ED. From similar triangles QCD and O_2 NQ,

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega_2 \qquad \text{or} \qquad ED = \omega_2 . QN$$

v_S = Velocity of sliding at Q.

$$v_S = ED - EC = \omega_2 . QN - \omega_1 . MQ$$
$$= \omega_2 (QP + PN) - \omega_1 (MP - QP)$$
$$= (\omega_1 + \omega_2) QP + \omega_2 . PN - \omega_1 . MP$$

Since $\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$ or $\omega_1 . MP = \omega_2 . PN$,

$$v_S = (\omega_1 + \omega_2) QP$$

D

Length of path of contact

The length of path of contact is the length of common normal cut off by the addendum circles of the wheel and pinion

From the fig.

KL = Length of path of contact

KL = KP+PL

KP = Length of path of approach

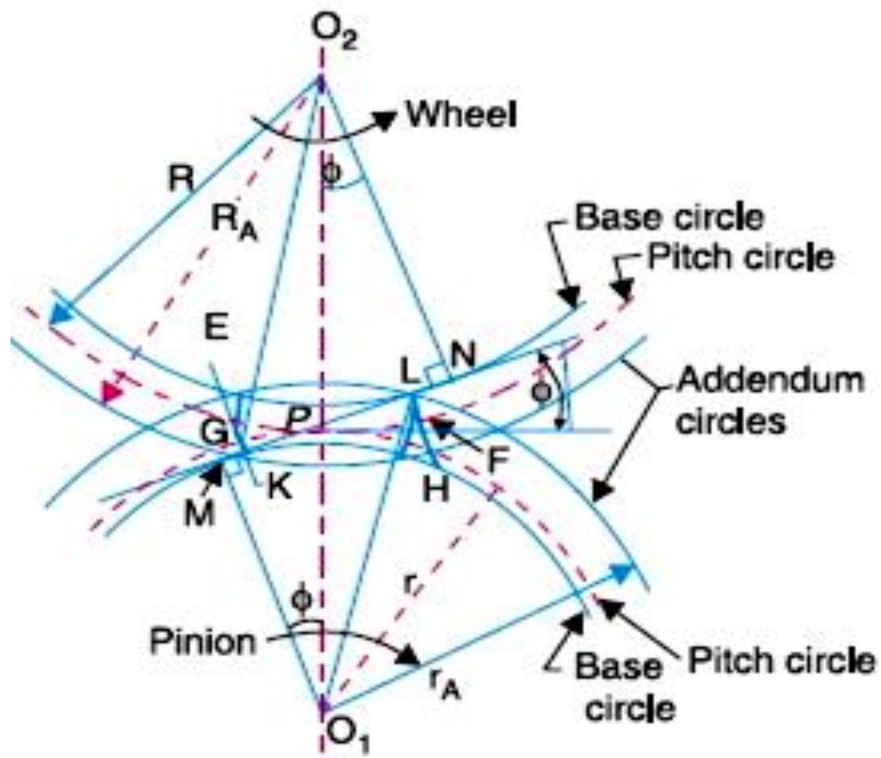
PL = Length of path of recess

r_A = O_1L Radius of addendum circle of the pinion

R_A = O_1K Radius of addendum circle of the gear

r = O_1P Radius of PCD of the pinion

R = O_2P Radius of PCD circle of the gear



From the fig

Radius of the base circle of the pinion = $O_1M = O_1P\cos\phi = r\cos\phi$

Radius of the base circle of the gear = $O_2N = O_2P\cos\phi = R\cos\phi$

Now from right angled triangle O_2KN,

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

$\therefore$ Length of the part of the path of contact, or the path of approach,

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

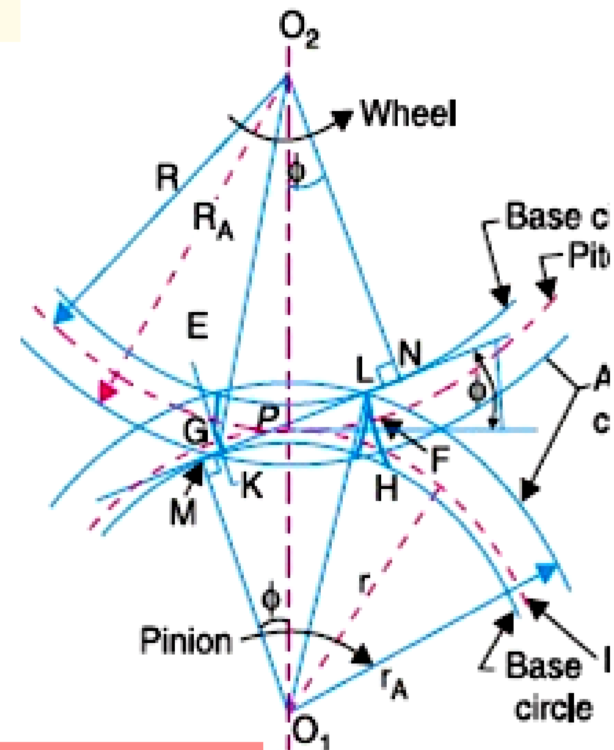
Similarly from right angled triangle O_1ML,

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

$\therefore$ Length of the part of the path of contact, or path of recess,

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$



$\therefore$ Length of the path of contact,

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

Length of Arc of Contact

The arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth

- The length of path of contact is the sum of arc of approach and arc of recess
- The length of the arc of approach $= \dfrac{\text{Length of path of approach}}{\cos\phi} = \dfrac{KP}{\cos\phi}$
- The length of the arc of recess $= \dfrac{\text{Length of path of recess}}{\cos\phi} = \dfrac{PL}{\cos\phi}$
- Length of the arc of contact $= \dfrac{\text{Length of path of contact}}{\cos\phi} = \dfrac{KL}{\cos\phi}$

Contact Ratio

The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact to the circular pitch**

Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of the arc of contact}}{p_c}$$

$$p_c = \text{Circular pitch} = \pi m, \text{ and}$$

$$m = \text{Module.}$$

Interference in Involute Gears

- The tip of tooth on the pinion undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as ***interference.*** In brief, ***The phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference***
- *Interference may be prevented, if the* **addendum circles of the two mating gears** *cut the common tangent to the base circles between the points of tangency*.

- When interference is just avoided, the maximum length of path of contact is *MN*
- Maximum length of path of approach $MP = r \sin \phi$
- Maximum length of path of recess $PN = R \sin \phi$
- Maximum length of path of contact

$$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$$

- maximum length of arc of contact

$$\frac{(r + R) \sin \phi}{\cos \phi} = (r + R) \tan \phi$$

Minimum Number of Teeth on the Pinion to Avoid Interference

Let

t = Number of teeth on the pinion,,

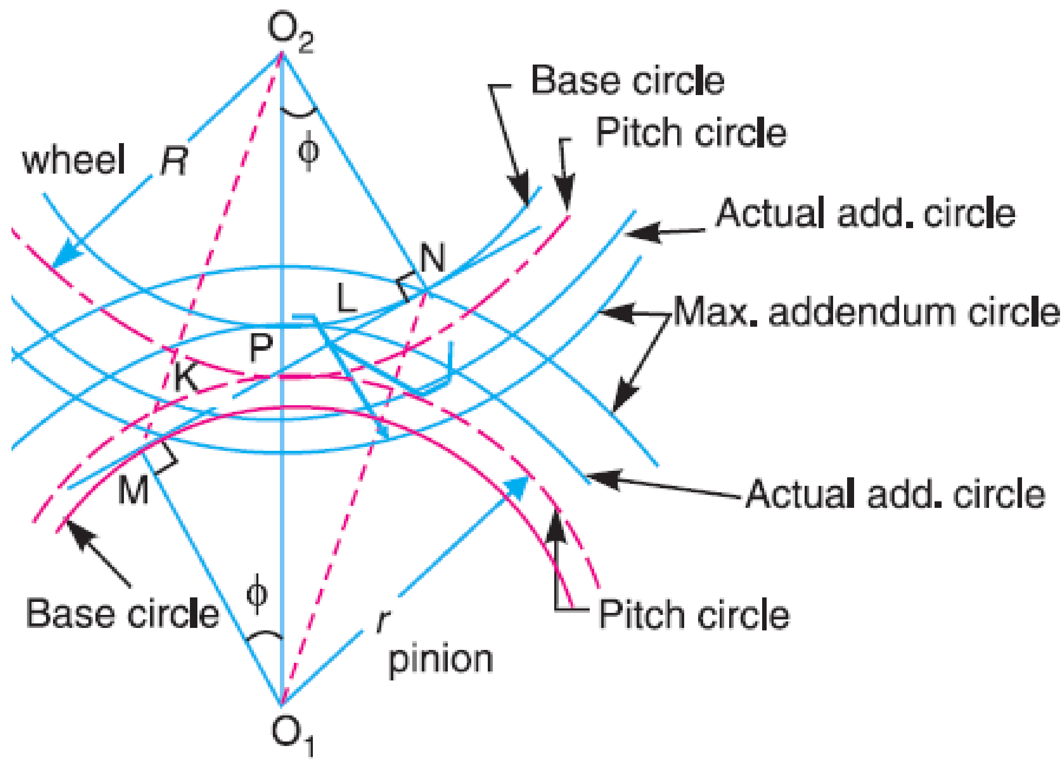
T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $m.t / 2$

G = Gear ratio = $T / t = R / r$

ϕ = Pressure angle or angle of obliquity.



From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

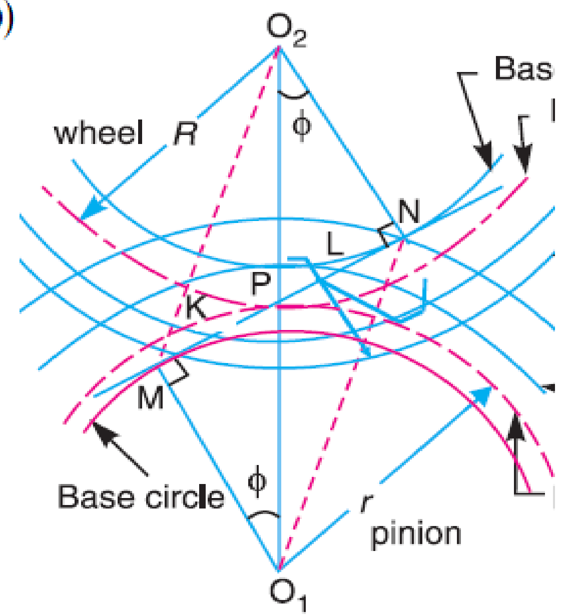
$$...(\because\ PN = O_2P \sin\phi = R\sin\phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$\therefore$ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$



Let $A_p m$ = Addendum of the pinion, where A_p is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$\therefore \qquad A_P.m = \frac{m.t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{m.t}{2} \qquad ...(\because O_1P = r = m.t/2)$$

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$A_P = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore \qquad t = \frac{2A_P}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

Minimum Number of Teeth on the Wheel to Avoid Interference

$$T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

Problem 1 *A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.*

Solution. Given : t = 30 ; T = 80 ; ϕ = 20° ; m = 12 mm ; Addendum = 10 mm

Length of path of contact We know that pitch circle radius of pinion, r = m.t / 2 = 12 × 30 / 2 = 180 mm

and pitch circle radius of gear,

$$R = m.T / 2 = 12 \times 80 / 2 = 480 \text{ mm}$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum} = 180 + 10 = 190 \text{ mm}$$

and radius of addendum circle of gear,

$$R_A = R + \text{Addendum} = 480 + 10 = 490 \text{ mm}$$

We know that length of the path of approach,

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$= \sqrt{(490)^2 - (480)^2 \cos^2 20^\circ} - 480 \sin 20^\circ = 191.5 - 164.2 = 27.3 \text{ mm}$$

and length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(190)^2 - (180)^2 \cos^2 20^\circ} - 180 \sin 20^\circ = 86.6 - 61.6 = 25 \text{ mm}$$

We know that length of path of contact,

$$KL = KP + PL = 27.3 + 25 = 52.3 \text{ mm} \quad \textbf{Ans.}$$

Length of arc of contact

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos \phi} = \frac{52.3}{\cos 20^\circ} = 55.66 \text{ mm} \quad \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$p_c = \pi . m = \pi \times 12 = 37.7 \text{ mm}$$

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of arc of contact}}{p_c} = \frac{55.66}{37.7} = 1.5 \text{ say } 2 \quad \textbf{Ans.}$$

Problem 2 *Two involute gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5 mm and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find :*

1. *The angle turned through by pinion when one pair of teeth is in mesh ; and*

2. *The maximum velocity of sliding.*

Solution. Given : $\phi = 20°$; $t = 20$; $G = T/t = 2$; $m = 5$ mm ; $v = 1.2$ m/s ; addendum = 1 module = 5 mm

1. ***Angle turned through by pinion when one pair of teeth is in mesh***

We know that pitch circle radius of pinion,

$$r = m.t / 2 = 5 \times 20 / 2 = 50 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T / 2 = m.G.t / 2 = 2 \times 20 \times 5 / 2 = 100 \text{ mm} \qquad \ldots (\because T = G.t)$$

∴ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum} = 50 + 5 = 55 \text{ mm}$$

and radius of addendum circle of wheel,

$$R_A = R + \text{Addendum} = 100 + 5 = 105 \text{ mm}$$

We know that length of the path of approach (*i.e.* the path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$= \sqrt{(105)^2 - (100)^2 \cos^2 20°} - 100 \sin 20°$$

$$= 46.85 - 34.2 = 12.65 \text{ mm}$$

and the length of path of recess (*i.e.* the path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(55)^2 - (50)^2 \cos^2 20°} - 50 \sin 20° = 28.6 - 17.1 = 11.5 \text{ mm}$$

∴ Length of the path of contact,

$$KL = KP + PL = 12.65 + 11.5 = 24.15 \text{ mm}$$

and length of the arc of contact

$$= \frac{\text{Length of path of contact}}{\cos \phi} = \frac{24.15}{\cos 20°} = 25.7 \text{ mm}$$

We know that angle turned through by pinion

$$= \frac{\text{Length of arc of contact} \times 360°}{\text{Circumference of pinion}} = \frac{25.7 \times 360°}{2\pi \times 50} = 29.45° \ \textbf{Ans.}$$

2. *Maximum velocity of sliding*

Let ω_1 = Angular speed of pinion, and

ω_2 = Angular speed of wheel.

We know that pitch line speed,

$$v = \omega_1 . r = \omega_2 . R$$

$\therefore$ $\omega_1 = v/r = 120/5 = 24$ rad/s

and $\omega_2 = v/R = 120/10 = 12$ rad/s

$\therefore$ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\, KP \qquad ...(\because KP > PL)$$

$$= (24 + 12)\, 12.65 = 455.4 \text{ mm/s}$$ **Ans.**

Problem 3: The following data relate to a pair of 20° involute gears in mesh :
Module = 6 mm, Number of teeth on pinion = 17, Number of teeth on gear = 49 ;
Addendum on pinion and gear wheel = 1 module.
Find : 1. The number of pairs of teeth in contact ; 2. The angle turned through by the pinion and the gear wheel when one pair of teeth is in contact, and 3. The ratio of sliding to rolling motion when the tip of a tooth on the larger wheel (i) is just making contact, (ii) is just leaving contact with its mating tooth, and (iii) is at the pitch point.

Solution. Given : $\phi = 20°$; $m = 6$ mm ; $t = 17$; $T = 49$; Addenda on pinion and gear wheel = 1 module = 6 mm

Pitch circle radius of the pinion $r = m.t / 2 = 6 \times 17 / 2 = 51$ mm

Pitch circle radius of the pinion $R = m.T / 2 = 6 \times 49 / 2 = 147$ mm

Radius of addendum circle of pinion, $r_A = r$ + Addendum = 51 + 6 = 57 mm
Radius of addendum circle of gear, $R_A = R$ + Addendum = 147 + 6 = 153 mm

We know that the length of path of approach (*i.e.* the path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$= \sqrt{(153)^2 - (147)^2 \cos^2 20°} - 147 \sin 20°$$

$$= 65.8 - 50.3 = 15.5 \text{ mm}$$

length of path of recess (*i.e.* the path of contact when disengagement occurs)

$$PL = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

$$= \sqrt{(57)^2 - (51)^2 \cos^2 20°} - 51 \sin 20°$$

$$= 30.85 - 17.44 = 13.41 \text{ mm}$$

∴ Length of path of contact,

$$KL = KP + PL = 15.5 + 13.41 = 28.91 \text{ mm}$$

and length of arc of contact $= \dfrac{\text{Length of path of contact}}{\cos\phi} = \dfrac{28.91}{\cos 20°} = 30.8 \text{ mm}$

We know that circular pitch,

$$p_c = \pi.m = \pi \times 6 = 18.852 \text{ mm}$$

∴ Number of pairs of teeth in contact (or contact ratio)

$$= \frac{\text{Length of arc of contact}}{\text{Circular pitch}} = \frac{30.8}{18.852} = 1.6 \text{ say } 2 \quad \textbf{Ans.}$$

2. ***Angle turned through by the pinion and gear wheel when one pair of teeth is in contact***

We know that angle turned through by the pinion

$$= \frac{\text{Length of arc of contact} \times 360°}{\text{Circumference of pinion}} = \frac{30.8 \times 360}{2\pi \times 51} = 34.6° \quad \textbf{Ans.}$$

and angle turned through by the gear wheel

$$= \frac{\text{Length of arc of contact} \times 360°}{\text{Circumference of gear}} = \frac{30.8 \times 360}{2\pi \times 147} = 12° \quad \textbf{Ans.}$$

3. *Ratio of sliding to rolling motion*

Let ω_1 = Angular velocity of pinion, and

ω_2 = Angular velocity of gear wheel.

We know that $\omega_1 / \omega_2 = T / t$ or $\omega_2 = \omega_1 \times t / T = \omega_1 \times 17 / 49 = 0.347\ \omega_1$

and rolling velocity, $v_R = \omega_1 . r = \omega_2 . R = \omega_1 \times 51 = 51\ \omega_1$ mm/s

(*i*) At the instant when the tip of a tooth on the larger wheel is just making contact with its mating teeth (*i.e.* when the engagement commences), the sliding velocity

$$v_S = (\omega_1 + \omega_2)\ KP = (\omega_1 + 0.347\ \omega_1)\ 15.5 = 20.88\ \omega_1 \text{ mm/s}$$

∴ Ratio of sliding velocity to rolling velocity,

$$\frac{v_S}{v_R} = \frac{20.88\ \omega_1}{51 \omega_1} = 0.41$$ **Ans.**

(*ii*) At the instant when the tip of a tooth on the larger wheel is just leaving contact with its mating teeth (*i.e.* when engagement terminates), the sliding velocity,

$$v_S = (\omega_1 + \omega_2)\ PL = (\omega_1 + 0.347\ \omega_1)\ 13.41 = 18.1\ \omega_1 \text{ mm/s}$$

∴ Ratio of sliding velocity to rolling velocity

$$\frac{v_S}{v_R} = \frac{18.1\ \omega_1}{51\ \omega_1} = 0.355$$ **Ans.**

(*iii*) Since at the pitch point, the sliding velocity is zero, therefore the ratio of sliding velocity to rolling velocity is zero. **Ans.**

Problem 4. *Two mating gears have 20 and 40 involute teeth of module 10 mm and 20° pressure angle. The addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point has half the maximum possible length. Determine the addendum height for each gear wheel, length of the path of contact, arc of contact and contact ratio.*

Solution. Given : t = 20 ; T = 40 ; m = 10 mm ; φ = 20°

Addendum height for each gear wheel

pitch circle radius of the smaller gear wheel, r = m.t / 2 = 10 × 20 / 2 = 100 mm

pitch circle radius of the larger gear wheel, R = m.T / 2 = 10 × 40 / 2 = 200 mm

RA = Radius of addendum circle for the larger gear wheel, and

rA = Radius of addendum circle for the smaller gear wheel.

Since the addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point (i.e. the path of approach and the path of recess) has half the maximum possible length, therefore

Path of approach, $KP = \frac{1}{2} MP$

$$\sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin\phi = \frac{r.\sin\phi}{2}$$

$$\sqrt{(R_A)^2 - (200)^2 \cos^2 20°} - 200 \sin 20° = \frac{100 \times \sin 20°}{2} = 50 \sin 20°$$

$$\sqrt{(R_A)^2 - 35\ 320} = 50 \sin 20° + 200 \sin 20° = 250 \times 0.342 = 85.5$$

$$(R_A)^2 - 35\ 320 = (85.5)^2 = 7310 \quad \text{...(Squaring both sides)}$$

$$(R_A)^2 = 7310 + 35\ 320 = 42\ 630 \quad \text{or} \quad R_A = 206.5 \text{ mm}$$

∴ Addendum height for larger gear wheel

$$= R_A - R = 206.5 - 200 = 6.5 \text{ mm}$$ **Ans.**

Now path of recess, $PL = \frac{1}{2} PN$

$$\sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin\phi = \frac{R.\sin\phi}{2}$$

$$\sqrt{(r_A)^2 - (100)^2 \cos^2 20°} - 100 \sin 20° = \frac{200 \sin 20°}{2} = 100 \sin 20°$$

$$\sqrt{(r_A)^2 - (100)^2 \cos^2 20°} = 100 \sin 20° + 100 \sin 20° = 200 \times 0.342 = 68.4$$

$$(r_A)^2 - 8830 = (68.4)^2 = 4680 \quad \text{...(Squaring both sides)}$$

$$(r_A)^2 = 4680 + 8830 = 13\ 510 \quad \text{or} \quad r_A = 116.2 \text{ mm}$$

∴ Addendum height for smaller gear wheel

$$= r_A - r = 116.2 - 100 = 6.2 \text{ mm}$$ **Ans.**

Length of the path of contact

We know that length of the path of contact

$$= KP + PL = \frac{1}{2}MP + \frac{1}{2}PN = \frac{(r+R)\sin\phi}{2}$$

$$= \frac{(100+200)\sin 20^\circ}{2} = 51.3 \text{ mm} \quad \textbf{Ans.}$$

Length of the arc of contact

We know that length of the arc of contact

$$= \frac{\text{Length of the path of contact}}{\cos\phi} = \frac{51.3}{\cos 20^\circ} = 54.6 \text{ mm} \quad \textbf{Ans.}$$

Contact ratio

We know that circular pitch,

$$P_c = \pi m = \pi \times 10 = 31.42 \text{ mm}$$

$$\therefore \quad \text{Contact ratio} = \frac{\text{Length of the path of contact}}{p_c} = \frac{54.6}{31.42} = 1.74 \text{ say } 2 \quad \textbf{Ans.}$$

Problem 5. *Determine the minimum number of teeth required on a pinion, in order to avoid interference which is to gear with,*
1. a wheel to give a gear ratio of 3 to 1 ; and 2. an equal wheel.
The pressure angle is 20° and a standard addendum of 1 module for the wheel may be assumed.

Solution. Given : $G = T / t = 3$; $\phi = 20°$; $A_W = 1$ module

1. *Minimum number of teeth for a gear ratio of 3 : 1*

We know that minimum number of teeth required on a pinion,

$$t = \frac{2 \times A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

$$= \frac{2 \times 1}{3\left[\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20°} - 1\right]} = \frac{2}{0.133} = 15.04 \text{ or } 16$$ **Ans.**

2. Minimum number of teeth for equal wheel

We know that minimum number of teeth for equal wheel,

$$t = \frac{2 \times A_W}{\sqrt{1 + 3 \sin^2 \phi} - 1} = \frac{2 \times 1}{\sqrt{1 + 3 \sin^2 20°} - 1} = \frac{2}{0.162}$$

$$= 12.34 \text{ or } 13$$ **Ans.**

Problem 6. *A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is 1.75, find in order that the interference is just avoided ; **1.** the addenda on pinion and gear wheel ; **2.** the length of path of contact ; and **3.** the maximum velocity of sliding of teeth on either side of the pitch point*

Solution. Given : $\phi = 16°$; $m = 6$ mm ; $t = 16$; $N_1 = 240$ r.p.m. or
$\omega_1 = 2\pi \times 240/60 = 25.136$ rad/s ; $G = T/t = 1.75$ or $T = G.t = 1.75 \times 16 = 28$

1. *Addenda on pinion and gear wheel*

We know that addendum on pinion

$$= \frac{m.t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1 + \frac{28}{16}\left(\frac{28}{16} + 2\right)\sin^2 16°} - 1\right]$$

$= 48\ (1.224 - 1) = 10.76$ mm **Ans.**

addendum on wheel $= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$

$$= \frac{6 \times 28}{2}\left[\sqrt{1 + \frac{16}{28}\left(\frac{16}{28} + 2\right)\sin^2 16^\circ} - 1\right]$$

$$= 84\ (1.054 - 1) = 4.56 \text{ mm} \quad \textbf{Ans.}$$

2. *Length of path of contact*

We know that the pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 28/2 = 84 \text{ mm}$$

and pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 16/2 = 48 \text{ mm}$$

∴ Addendum circle radius of wheel,

$$R_A = R + \text{Addendum of wheel} = 84 + 10.76 = 94.76 \text{ mm}$$

and addendum circle radius of pinion,

$$r_A = r + \text{Addendum of pinion} = 48 + 4.56 = 52.56 \text{ mm}$$

We know that the length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(94.76)^2 - (84)^2\cos^2 16^\circ} - 84\sin 16^\circ$$

$$= 49.6 - 23.15 = 26.45 \text{ mm}$$

length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(52.56)^2 - (48)^2\cos^2 16°} - 48\sin 16°$$

$$= 25.17 - 13.23 = 11.94 \text{ mm}$$

∴ Length of the path of contact,

$$KL = KP + PL = 26.45 + 11.94 = 38.39 \text{ mm}$$ **Ans.**

3. *Maximum velocity of sliding of teeth on either side of pitch point*

Let ω_2 = Angular speed of gear wheel.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t} = 1.75$ or $\omega_2 = \frac{\omega_1}{1.75} = \frac{25.136}{1.75} = 14.28$ rad/s

∴ Maximum velocity of sliding of teeth on the left side of pitch point *i.e.* at point K

$$= (\omega_1 + \omega_2)\,KP = (25.136 + 14.28)\,26.45 = 1043 \text{ mm/s}$$ **Ans.**

and maximum velocity of sliding of teeth on the right side of pitch point *i.e.* at point L

$$= (\omega_1 + \omega_2)\,PL = (25.136 + 14.28)\,11.94 = 471 \text{ mm/s}$$ **Ans.**

GEAR TRAIN

- Two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ***gear train*** or ***train of toothed wheels*.**

- The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears

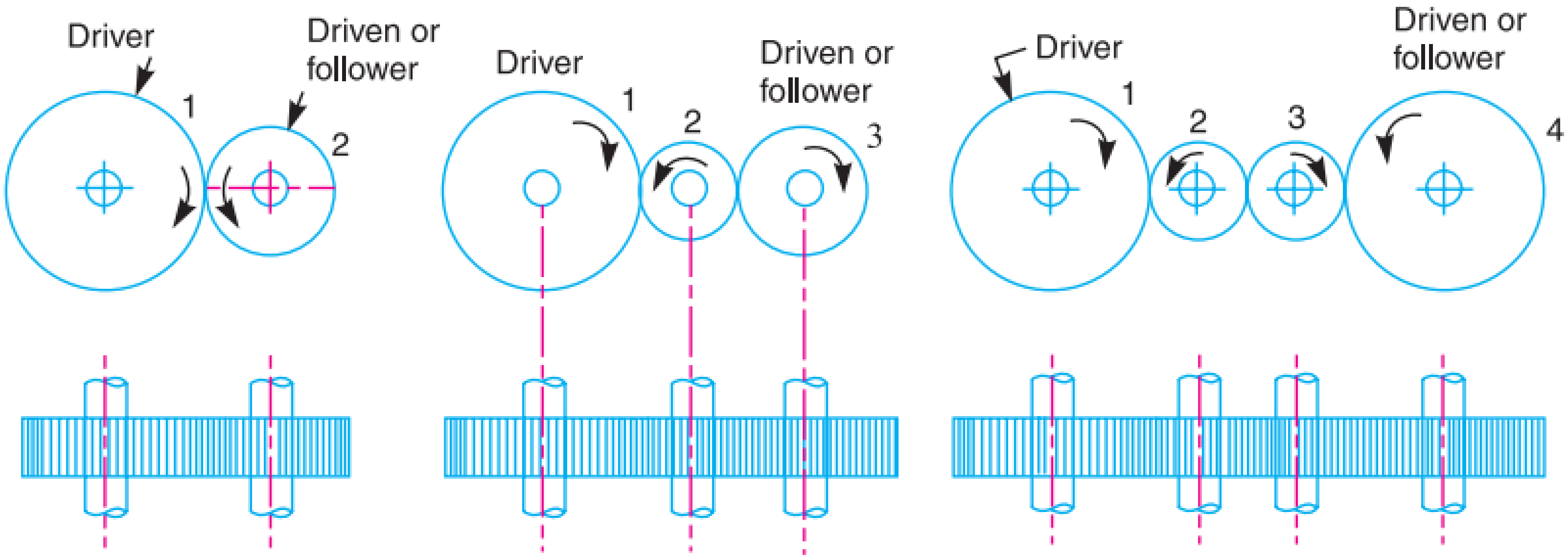
Types of Gear Trains

1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other.

But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

Simple gear train



(a) (b) (c)

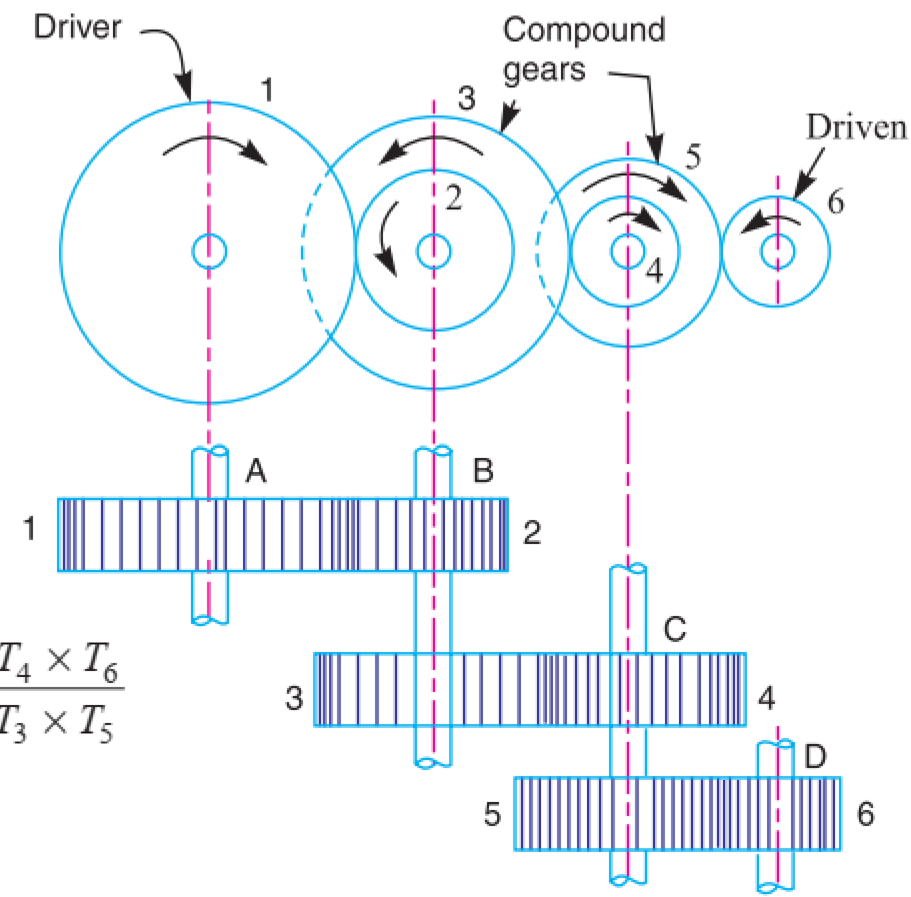
Let N_1 = Speed of gear 1(or driver) in r.p.m.,
N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,
T_1 = Number of teeth on gear 1, and
T_2 = Number of teeth on gear 2.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Ratio of the speed of the driven or follower to the speed of the driver is known as ***train value*** of the gear train.

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

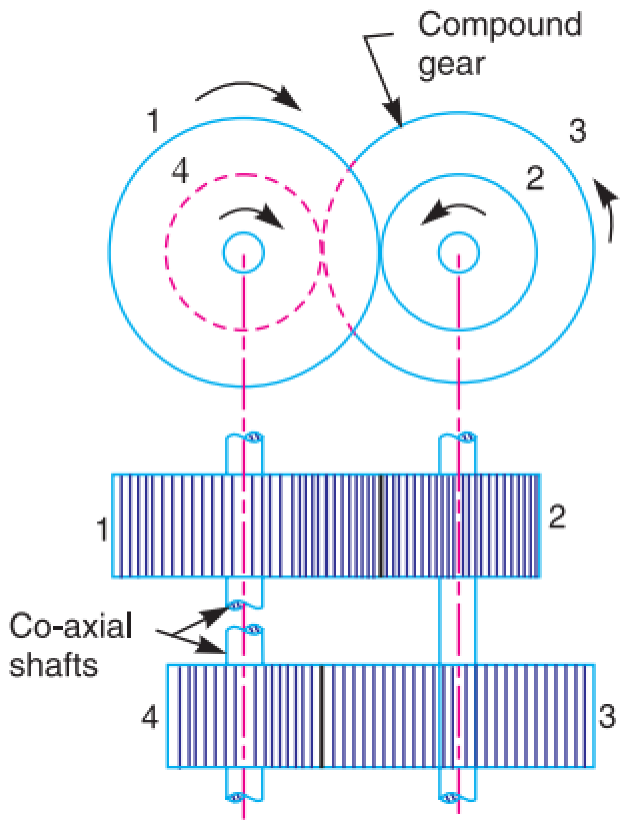
Compound gear train



The speed ratio of compound gear train is obtained by

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

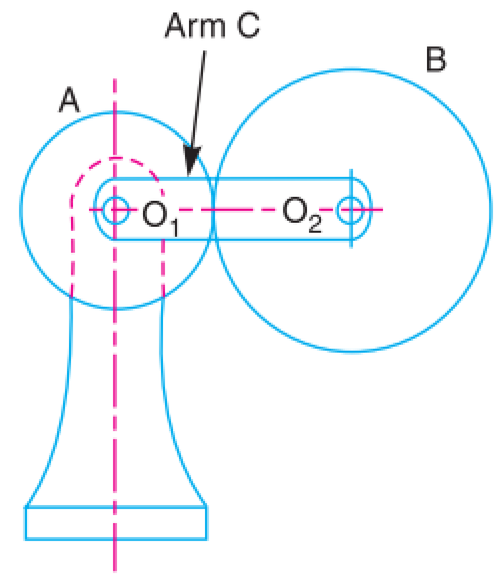
Reverted gear train



Epicyclic gear train

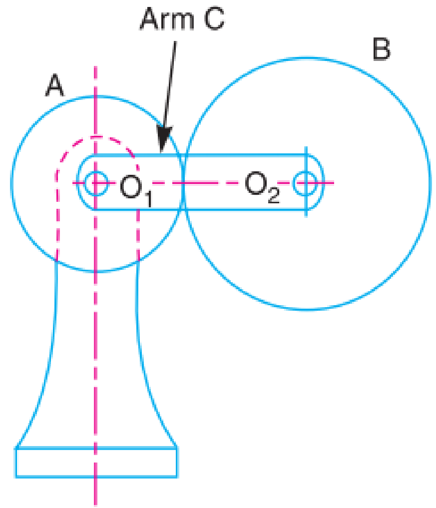
Where a gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or **vice- versa**, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O_1), then the gear B is forced to rotate **upon** and **around** gear A.

Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as **epicyclic gear trains**



Velocity Ratios of Epicyclic Gear Train

1.Tabular method, and **2.** Algebraic method.



Tabular method.

- Consider an epicyclic gear train as shown in Fig.

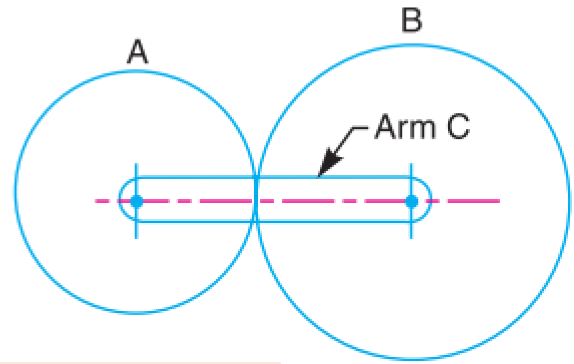
Let T_A = Number of teeth on gear A, and

T_B = Number of teeth on gear B.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Problem 1. In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?

Solution. Given : TA = 36 ; TB = 45 ; NC = 150 r.p.m. (anticlockwise)



Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +\,150 \text{ r.p.m.}$$

Also the gear A is fixed, therefore

$$x + y = 0 \qquad \text{or} \qquad x = -y = -\,150 \text{ r.p.m.}$$

$\therefore$ Speed of gear B, $N_B = y - x \times \dfrac{T_A}{T_B} = 150 + 150 \times \dfrac{36}{45} = +\,270$ r.p.m.

$= 270$ r.p.m. (anticlockwise) **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

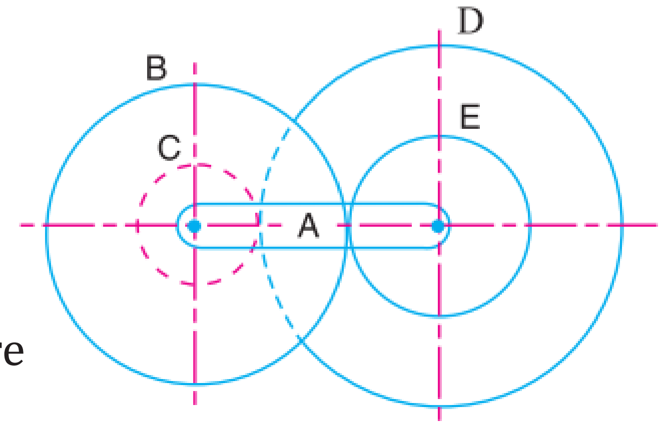
$$x + y = -\,300 \qquad \text{or} \qquad x = -\,300 - y = -\,300 - 150 = -\,450 \text{ r.p.m.}$$

$\therefore$ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +\,510 \text{ r.p.m.}$$

$= 510$ r.p.m. (anticlockwise) **Ans.**

Example 2. *In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.*



Solution. Given : $T_B = 75$; $T_C = 30$; $T_D = 90$; $N_A = 100$ r.p.m. (clockwise)

From the geometry of the figure,

$d_B + d_E = d_C + d_D$

Since the number of teeth on each gear, for the same module, are proportional to their pitch circle diameters, therefore

$T_B + T_E = T_C + T_D$

$T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D-E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D-E* rotated through + *x* revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Since the gear *B* is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_E}{T_B} = 0 \quad \text{or} \quad y - x \times \frac{45}{75} = 0$$

$$\therefore \quad y - 0.6 = 0$$

Also the arm *A* makes 100 r.p.m. clockwise, therefore

$$y = -100$$

Substituting $y = -100$ in equation **(*i*)**, we get

$$-100 - 0.6\,x = 0 \quad \text{or} \quad x = -100 / 0.6 = -166.67$$

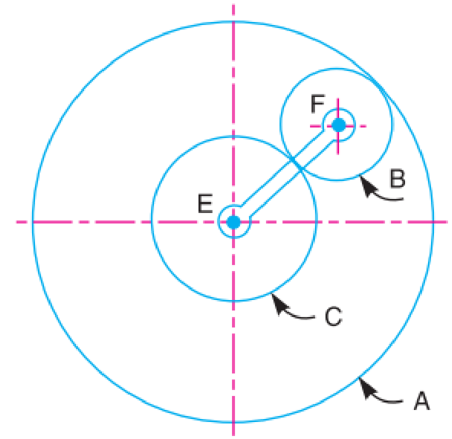
From the fourth row of the table, speed of gear *C*,

$$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = +400 \text{ r.p.m.}$$

$$= 400 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

Problem 3. *An epicyclic gear consists of three gears A, B and C as shown in Fig. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.*

Solution. Given : $T_A = 72$; $T_C = 32$; Speed of arm EF = 18 r.p.m.



Speed of gear C

We know that the speed of the arm is 18 r.p.m. therefore, $y = 18$ r.p.m. and the gear A is fixed,

Step No.	Conditions of motion	*Revolutions of elements*			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+ y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$

$$y - x \times \frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x \times \frac{32}{72} = 0$$

$$x = 18 \times 72 / 32 = 40.5$$

Speed of gear C $= x + y = 40.5 + 18$

$= + 58.5$ r.p.m.

$= 58.5$ r.p.m. in the direction of arm. **Ans.**

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. Therefore, from the geometry of Fig. 13.10,

$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \quad \text{or} \quad 2\,d_B + d_C = d_A$$

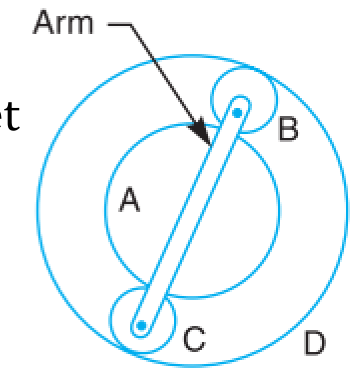
Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2\,T_B + T_C = T_A \quad \text{or} \quad 2\,T_B + 32 = 72 \quad \text{or} \quad T_B = 20$$

$\therefore$ Speed of gear B $= y - x \times \frac{T_C}{T_B} = 18 - 40.5 \times \frac{32}{20} = -46.8$ r.p.m.

= 46.8 r.p.m. in the opposite direction of arm. **Ans.**

Problem 4. *An epicyclic train of gears is arranged as shown in Fig. How many revolutions does the arm, to which the pinions B and C are attached, make :*
1. *when A makes one revolution clockwise and D makes half a revolution anticlockwise, and*
2. *when A makes one revolution clockwise and D is stationary ? The number of teeth on the gears A and D are 40 and 90 respectively*.



Solution. Given : $T_A = 40$; $T_D = 90$

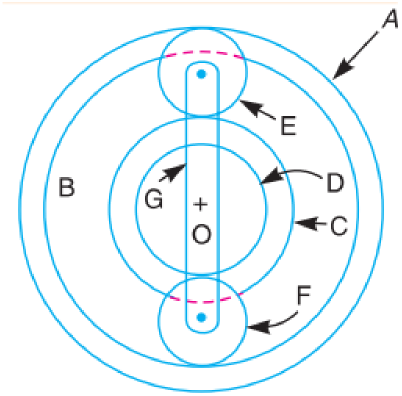
First of all, let us find the number of teeth on gears B and C (i.e. T_B and T_C). Let d_A, d_B, d_C and d_D be the pitch circle diameters of gears A, B, C and D respectively. Therefore from the geometry of the figure,

$d_A + d_B + d_C = d_D$ or $d_A + 2\, d_B = d_D$ ...($\because d_B = d_C$)

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$T_A + 2\, T_B = T_D$ or $40 + 2\, T_B = 90$

$\therefore T_B = 25$, and $T_C = 25$ ...($\because T_B = T_C$)



Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $=-\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $=+\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2.	Arm fixed-wheel *A* rotates through + *x* revolutions	0	$+x$	$+x\times\frac{T_A}{T_E}$	$-x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_A}{T_E}$	$y-x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

3. *Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed*

Since the arm G makes 100 r.p.m. clockwise, therefore from the fourth row of the table,

$$y = -100 \qquad \ldots(i)$$

Also, the wheel A is fixed, therefore from the fourth row of the table,

$$x + y = 0 \quad \text{or} \quad x = -y = 100 \qquad \ldots(ii)$$

$$\therefore \quad \text{Speed of wheel } B = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} = -100 + 100 \times \frac{64}{28} \times \frac{26}{62} = -100 + 95.8 \text{ r.p.m.}$$

$$= -4.2 \text{ r.p.m.} = 4.2 \text{ r.p.m. clockwise}$$ **Ans.**

4. *Speed of wheel B when arm G makes* 100 *r.p.m. clockwise and wheel A makes* 10 *r.p.m. counter clockwise*

Since the arm G makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100 \qquad \ldots(iii)$$

Also the wheel A makes 10 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$x + y = 10 \quad \text{or} \quad x = 10 - y = 10 + 100 = 110 \qquad \ldots(iv)$$

$$\therefore \quad \text{Speed of wheel } B = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} = -100 + 110 \times \frac{64}{28} \times \frac{26}{62} = -100 + 105.4 \text{ r.p.m.}$$

$$= +5.4 \text{ r.p.m.} = 5.4 \text{ r.p.m. counter clockwise}$$ **Ans.**

GEAR TRAINS


Thank You