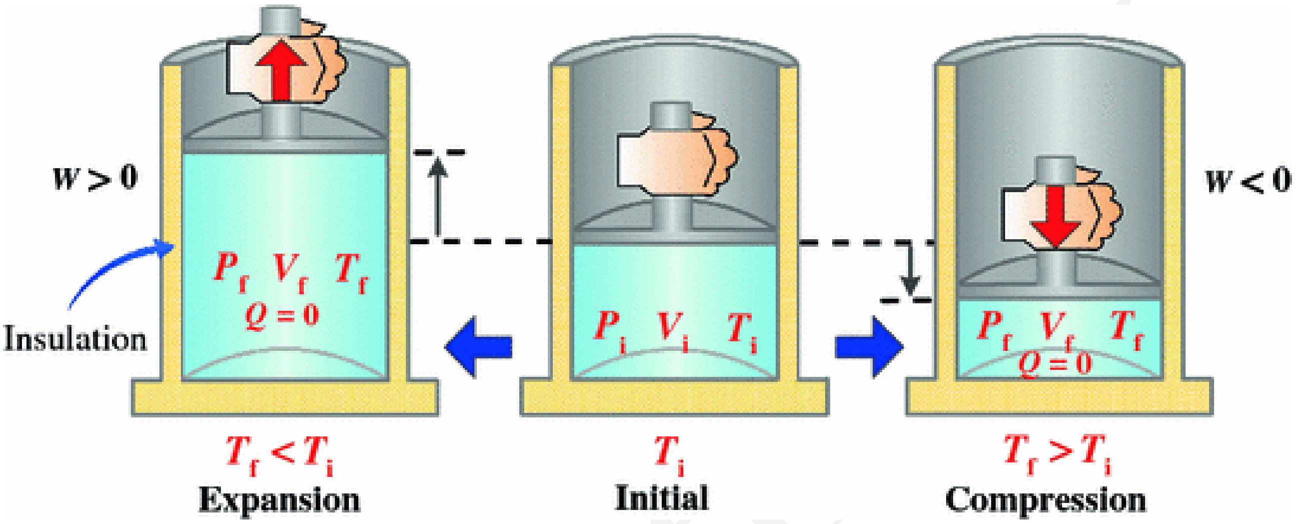
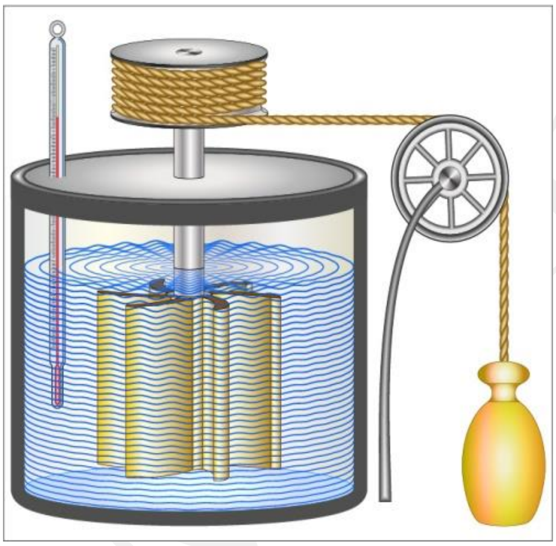
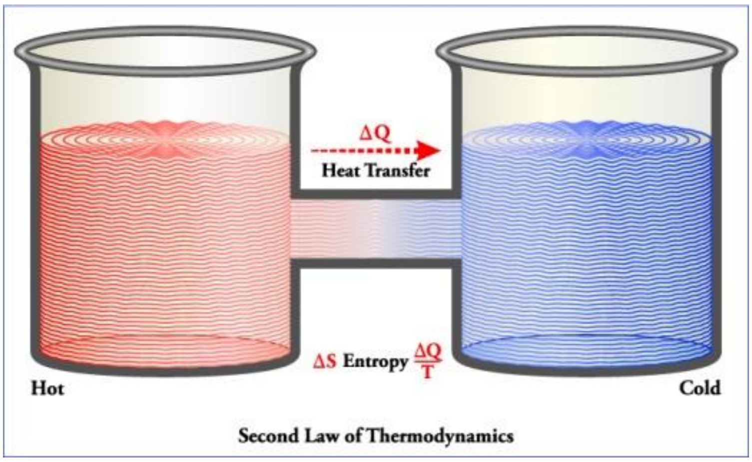
ME8391- ENGINEERING THERMODYNAMICS

UNIT I -BASIC CONCEPTS AND FIRST LAW







Thermodynamics

It is the science of the relations between heat, work and the properties of the systems.

Approaches in Thermodynamics:

In macroscopic approach, certain quantity of matter is considered, without considering the events occurring at the molecular level. These effects can be perceived by human senses or measured by instruments.

eg: pressure, temperature

Thermodynamics based on macroscopic approach is called Classical thermodynamics.

In microscopic approach, the effect of molecular motion is considered.

eg: At microscopic level the pressure of a gas is not constant, the temperature of a gas is a function of the velocity of molecules. Most microscopic properties cannot be measured with common instruments nor can be perceived by human senses

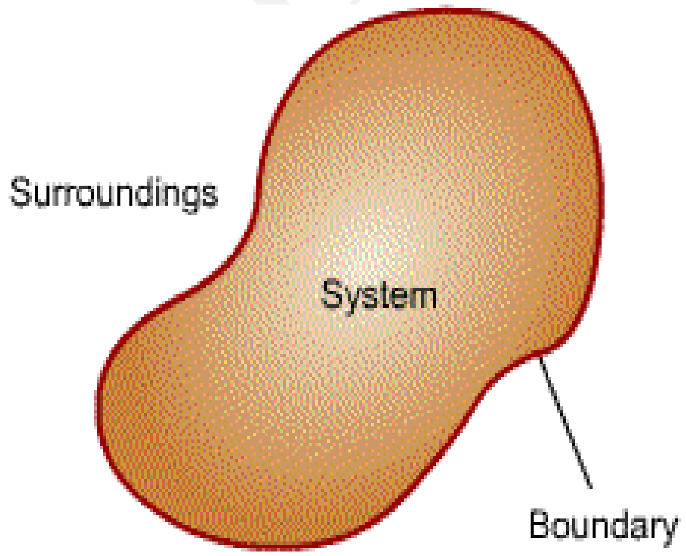
Thermodynamics based on microscopic approach is called Statistical Thermodynamics

System

The system is a quantity of matter or a region in space on which we focus our attention (eg: the water kettle or the aircraft engine).

Surroundings

The rest of the universe outside the system close enough to the system to have some perceptible effect on the system is called the surroundings.

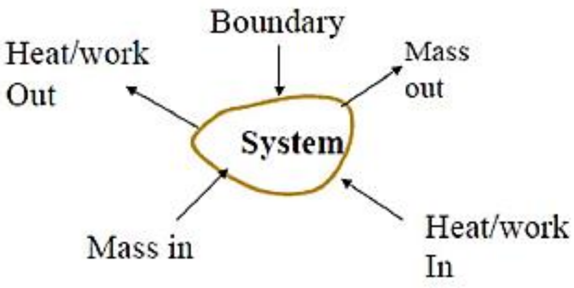


Boundaries

The surface which separates the system from the surroundings are called the boundaries as shown in fig below (eg: walls of the kettle, the housing of the engine).

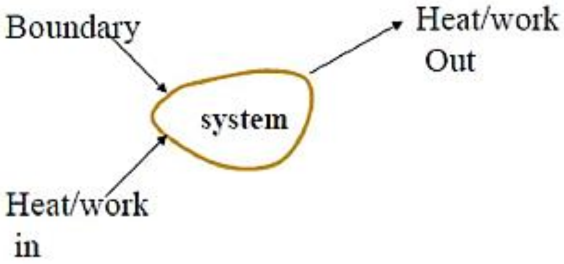
Open System

A system in which, mass and energy (work or heat) can be transferred across the boundary.



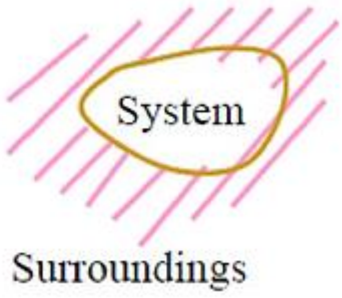
Closed System

A system in which there is no mass transfer but only energy transfer across the boundary.



Isolated System

A system in which there is no mass transfer and energy transfer across the boundary.



Properties

It is some characteristic of the system by which the condition of system is described.

Extensive property:

Properties whose value depends on the size or extent of the system

If mass is increased, the value of extensive property also increases. eg: volume, mass

Intensive property:

Properties whose value is independent of the size or extent of the system. eg: pressure, temperature (p, T).

State

A system is said to be in a state when it has definite values for properties. Any operation in which one or more properties change is called a 'change of state'.

Equilibrium State

It is a state of balance. A system is said to be in equilibrium state if it is under mechanical, chemical and thermal equilibrium.

Mechanical equilibrium: No unbalanced forces, ie no difference in pressure within the system

Chemical equilibrium: No chemical reaction within the system

Thermal equilibrium: No difference in temperature within the system

Process

The succession of states passed through during a change of state is called the *path of the system*.

A system is said to go through a process if it goes through a series of changes in state.

Consequently:

A system may undergo changes in some or all of its properties in a process.

Quasi-static Process

A process in which the intermediate states are equilibrium states is called quasistatic or quasi-equilibrium process. Generally quasi-static processes are slow processes.

Cycle

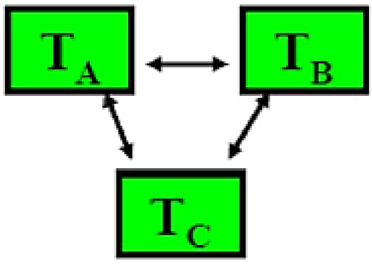
A series of processes at the end of which the system comes back to initial state.

Open and closed cycle

In a closed cycle, the same working substance will be undergoing the cycle again and again. In an open cycle, the working substance will be exhausted at the end of a series of processes which will be repeated again and again.

Zeroth Law of Thermodynamics

If two systems (say A and B) are in thermal equilibrium with a third system (say C) separately (that is A and C are in thermal equilibrium; B and C are in thermal equilibrium) then they are in thermal equilibrium themselves (that is A and B will be in thermal equilibrium.



Two systems are said to be equal in temperature, when there is no change in their respective observable properties when they are brought together. In other words, "when two systems are at the same temperature they are in thermal equilibrium"(They will not exchange heat).

Note: They need not be in thermodynamic equilibrium.

Point and path function

Path function: The function whose value is dependent on the path of the process. e.g. work transfer and heat transfer

Point function: The function the change in whose value is independent on the path of the process. e.g. pressure, temperature, etc.

Reversible and irreversible process

Reversible process: A reversible process is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states.

Irreversible process: An irreversible process is one which can be stopped at any stage and reversed but the system and surroundings are not restored to their initial states.

Work and its Sign convention

Work is transient quantity which appears at the boundary when a system changes its state due to the movement of a part of the boundary under the action of a force.

Sign convention:

If the work is done by the system, Work output of the system = + W

If the work is done on the system, Work input to system = – W

Heat and its Sign convention

Heat is transient quantity which appears at the boundary when a system changes its state due to a difference in temperature between the system and its surroundings.

Heat received by the system = + Q, Heat rejected or given up by the system = – Q.

Flow energy

Energy required to introduce a quantity of fluid in a pipe section is flow energy. It is equal to pV.

Internal Energy

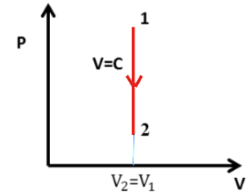
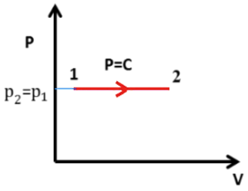
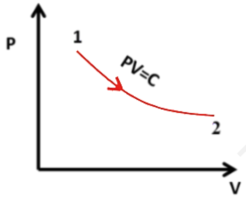
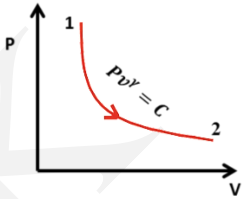
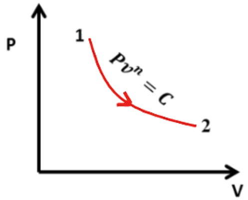
Internal energy of a gas is the energy stored in a gas due to its molecular interactions. It is denoted as U

Enthalpy of a system

It is the sum of internal energy and flow energy. i.e. H = U+ pV

Latent heat

Amount of heat required to cause a phase change in unit mass of a substance at constant pressure and temperature.

Process	**Units**	**Constant Volume (V=C)**	**Constant Pressure (p=C)**	**Isothermal (pV=C)**	**Adiabatic Process ($pV^{\gamma} = C$)**	**Polytropic Process ($pV^{n} = C$)**
p-V Diagram						
Work Done	**kJ**	W=0	$W = p_1(V_2 - V_1)$	$W = p_1V_1 \ln\left(\frac{V_2}{V_1}\right)$	$W = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$	$W = \frac{p_1V_1 - p_2V_2}{n - 1}$
Change In Internal Energy	**kJ**	$\Delta U = mC_v(T_2 - T_1)$	$\Delta U = mC_v(T_2 - T_1)$	$\Delta U = 0$	$\Delta U = \ -W$	$\Delta U = Q \ - W$
Heat Transfer	**kJ**	$Q = mC_v(T_2 - T_1)$	$Q = mC_P(T_2 - T_1)$	$Q = W = p_1V_1 \ln\left(\frac{V_2}{V_1}\right)$	$Q = 0$	$Q = \left[\frac{\gamma - n}{(\gamma - 1)}\right] \times W$
Change In Entropy	**kJ/K**	$\Delta S = mC_v \ln\left(\frac{T_2}{T_1}\right)$	$\Delta S = mC_P \ln\left(\frac{T_2}{T_1}\right)$	$\Delta S = mR \ln\left(\frac{V_2}{V_1}\right)$	$\Delta S = 0$	$\Delta S = mC_n \ln\left(\frac{T_2}{T_1}\right)$
p , V , T Relations		$\frac{T_2}{T_1} = \frac{p_2}{p_1}$	$\frac{T_2}{T_1} = \frac{V_2}{V_1}$	$\frac{V_2}{V_1} = \frac{p_1}{p_2}$	$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$	$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{n}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

- Where **C=Constant**, **1** and **2** indicates as **initial** and **final state**,
- Equation for Ideal Gas : **pV = mRT**
- Pressure ‘p’ – kN/m^2 (1 bar =1 X 10^2 kN/ m^2)
- Volume ‘ V ‘ - m^3
- Mass ‘m’ - kg
- Gas Constant ‘R’ - 0.287 kJ/kg.K
- Temperature ‘T’ - K (0°C = 273K)
- Enthalpy ‘h’ = u + Pv or h = C$_p$T

- **pv = RT**

Specific Volume $'v' = \frac{V}{m}$ **m^3/kg**

- **$p\dot{V} = \dot{m}RT$**

Volume flow rate $'\dot{V}' - m^3/s$
Mass flow rate ‘$\dot{m}$’ – kg/s

- **$p\bar{V} = \bar{n}RT$**

Molar volume V- $m^3/kgmol$
Number of moles ‘n’

Entropy for all process can also be calculated by:

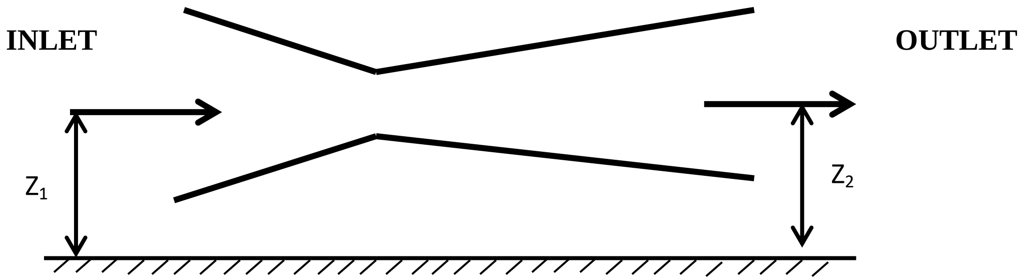
$\Delta S = mC_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right)$, $\Delta S = mC_P \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right)$

$\Delta S = mC_P \ln\left(\frac{V_2}{V_1}\right) + mC_v \ln\left(\frac{P_2}{P_1}\right)$

For Ideal Gas (Air):
Specific Heat ‘C$_p$’ = 1.005 kJ/kg.K, ‘C$_v$’ = 0.718 kJ/kg.K
For Water :
Specific Heat ‘C$_p$’ = 4.186 kJ/kg.K,

Index of Expansion $'\mathbf{n}' = \frac{\ln\left(\frac{p_2}{p_1}\right)}{\left(\ln\frac{V_1}{V_2}\right)}$

STEADY FLOW ENERGY EQUATION:



Inlet:

Kinetic energy + Potential energy+ Internal energy+ Flow energy + Heat

$$\frac{C_1^2}{2} + \mathbf{gz_1 + u_1 + p_1v_1 + Q} \quad \mathbf{(J/kg)}$$

Exit

Kinetic energy + Potential energy+ Internal energy+ Flow energy + Heat

$$\frac{C_2^2}{2} + \mathbf{gz_2 + u_2 + p_2v_2 + W \ (J/kg)}$$

According to the Energy Conservation Priciple

Inlet Energy = Exit Energy

$$\frac{\mathbf{C_1^2}}{\mathbf{2}} + \mathbf{gz_1 + u_1 + p_1v_1 + Q} = \frac{\mathbf{C_2^2}}{\mathbf{2}} + \mathbf{gz_2 + u_2 + p_2v_2 + W}$$

$$\mathbf{Q - W = \frac{C_2^2 - C_1^2}{2} + g(z_2 - z_1) + (u_2 - u_1) + (p_2v_2 - p_1v_1\ )} \qquad \text{(J/kg)}$$

$$\mathbf{Q - W = \frac{C_2^2 - C_1^2}{2000} + \frac{g(z_2 - z_1)}{1000} + (u_2 - u_1) + (p_2v_2 - p_1v_1)} \qquad \text{(kJ/kg)}$$

$$\mathbf{\dot{Q} - \dot{W} = \dot{m}\left[\frac{C_2^2 - C_1^2}{2000} + \frac{g(z_2 - z_1)}{1000} + (u_2 - u_1) + (p_2v_2 - p_1v_1)\right]} \ \text{(kJ/s)}$$

$$\mathbf{\dot{Q} - \dot{W} = \dot{m}\left[\frac{C_2^2 - C_1^2}{2000} + \frac{g(z_2 - z_1)}{1000} + (h_2 - h_1)\right]} \qquad \text{(kJ/s)}$$

<u>FOR OPEN SYSTEM:</u>

$$\mathbf{Q - W = \Delta E}$$

<u>FOR CLOSED SYSTEM:</u>

$$\mathbf{Q - W = \Delta U}$$

(i) Rate of heat transfer in the heat exchanger

From steady flow energy Equation:

$$\dot{\mathbf{m}}\left(\frac{C_1^2}{2000} + \frac{gz_1}{1000} + u_1 + p_1v_1\right) + \dot{Q} = \dot{\mathbf{m}}\left(\frac{C_2^2}{2000} + \frac{gz_2}{1000} + u_2 + p_2v_2\right) + \dot{W}$$

$z_1 = z_2$, $C_1 = 0$, $C_2 = 0$, W=0, $h_1 = u_1 + p_1v_1$ and $h_2 = u_2 + p_2v_2$

$$\mathbf{Q_{12} = \dot{m}(h_2 - h_1)} \qquad \text{(kJ/s)}$$

(ii) The power output from the turbine assuming no heat loss

From steady flow energy Equation:

$$\dot{\mathbf{m}}\left(\frac{C_1^2}{2000} + \frac{gz_1}{1000} + u_1 + p_1v_1\right) + \dot{Q} = \dot{\mathbf{m}}\left(\frac{C_2^2}{2000} + \frac{gz_2}{1000} + u_2 + p_2v_2\right) + \dot{W}$$

Where, $z_1 = z_2$, $h_1 = u_1 + p_1v_1$ and $h_2 = u_2 + p_2v_2$, $Q = 0$

$$\mathbf{W_{12} = \dot{m}\left[\frac{C_1^2 - C_2^2}{2000} + (h_1 - h_2)\right]} \qquad \text{(kJ/s)}$$

(iii) The power required to run the compressor assuming no heat loss

From steady flow energy Equation:

$$\dot{\mathbf{m}}\left(\frac{C_1^2}{2000} + \frac{gz_1}{1000} + u_1 + p_1v_1\right) + \dot{Q} = \dot{\mathbf{m}}\left(\frac{C_2^2}{2000} + \frac{gz_2}{1000} + u_2 + p_2v_2\right) + \dot{W}$$

Where, $z_1 = z_2$, $h_1 = u_1 + p_1v_1$ and $h_2 = u_2 + p_2v_2$, $Q = 0$

$$\mathbf{W_{12} = \dot{m}\left[\frac{C_1^2 - C_2^2}{2000} + (h_1 - h_2)\right]} \qquad \text{(kJ/s)} \qquad \textbf{(Negative work will attain)}$$

(iv) The velocity at exit of the nozzle

From steady flow energy Equation:

$$\dot{m}\left(\frac{C_1^2}{2000} + \frac{gz_1}{1000} + u_1 + p_1v_1\right) + \dot{Q} = m\left(\frac{C_2^2}{2000} + \frac{gz_2}{1000} + u_2 + p_2v_2\right) + \dot{W}$$

Where, $z_3 = z_4$, Q=0, W=0, $h_3 = u_3 + p_3v_3$ and $h_4 = u_4 + p_4v_4$

$$\frac{C_2^2}{2} = \frac{C_1^2}{2000} + (h_1 - h_2) \Rightarrow \qquad C_2 = \sqrt{C_1^2 + 2000\ (h_1 - h_2)}$$

$\Rightarrow$ $\mathbf{C_2 = 44.7\sqrt{(h_1 - h_2))}}$ (m/s) If C_1 is negligible

PROBLEMS ON FIRST LAW OF THERMODYNAMICS CLOSED SYSTEM:

1. **A mass of 1.5 kg of air is compressed in a quasi-static process from 0.1 MPa to 0.7 MPa for which pV = C. The initial density of air is 1.16 kg/m^3. Find the work done by the piston to compress the air.**

GIVEN: m=1.5 kg, P_1=0.1MPa, P_2=0.7MPa, ρ=1.16kg/m^3

FIND: The work done by the piston (W)

SOLUTION:

$$\rho = \frac{m}{V} \Longrightarrow V_1 = \frac{m}{\rho} = \frac{1.5}{1.16} \Longrightarrow \mathbf{V_1 = 1.293\ m^3}$$

For quasi-static process

$$W = \int pdV \Longrightarrow W = p_1V_1 \int_{V_1}^{V_2} \frac{dV}{V} \Longrightarrow W = p_1V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$\text{At } pV = C \longrightarrow p_1V_1 = p_2V_2 = C \longrightarrow \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

$$W = p_1V_1 \ln\left(\frac{V_2}{V_1}\right) \Rightarrow W = p_1V_1 \ln\left(\frac{p_1}{p_2}\right) \Rightarrow W = 0.1 \times 1.293 \ln\left(\frac{0.1}{0.7}\right) \Rightarrow \mathbf{W = -\,251.63\ kJ}$$

2. **A mass of gas is compressed in a quasi-static process from 80 kPa, 0.1 m^3 to 0.4 MPa, 0.03 m^3. Assuming that the pressure and volume are related by pV^n = constant, find the work done by the gas system.**

GIVEN: P_1=80 kPa, P_2=0.4MPa, V_1=0.1 m^3, V_2=0.03 m^3

FIND: (i) n, (ii) The work done (W)

SOLUTION:

$$\text{At } pV^n = C \Longrightarrow p_1V_1^n = p_2V_2^n$$

Taking Log on

$$\ln p_1 + n\ln V_1 = \ln p_2 + n \Longrightarrow n(\ln V_1 - \ln V_2) = \ln p_2 - \ln p_1 \Longrightarrow n\left(\ln\frac{V_1}{V_2}\right) = \ln\left(\frac{p_2}{p_1}\right)$$

$$n = \frac{\ln\left(\frac{p_2}{p_1}\right)}{\left(\ln\frac{V_1}{V_2}\right)} \Longrightarrow n = \frac{\ln\left(\frac{400}{80}\right)}{\left(\ln\frac{0.1}{0.03}\right)} \Longrightarrow \mathbf{n = 1.3367 \approx 1.34}$$

$$W = \int pdV \Longrightarrow W = \frac{p_1V_1 - p_2V_2}{n-1} \Longrightarrow W = \frac{80\times0.1-400\times0.03}{1.34-1} \Longrightarrow \mathbf{W = -11.764kJ}$$

3. **At the beginning of the compression stroke of a two-cylinder internal combustion engine the air is at a pressure of 101.325 kPa. Compression reduces the volume to 1/5 of its original volume, and the law of compression is given by pV$^{1.2}$ = constant. If the bore and stroke of each cylinder is 0.15 m and 0.25 m, respectively, determine the power absorbed in kW by compression strokes when the engine speed is such that each cylinder undergoes 500 compression strokes per minute.**

GIVEN: P_1=101.325 kPa, V_2=1/5 V_1, D=0.15 m, L=0.25m, N=500rpm and $pV^{1.2} = C$

FIND: The power absorbed in kW (P)

SOLUTION:

$$p_1V_1^{1.2} = p_2V_2^{1.2} \Rightarrow p_2 = \left(\frac{V_1}{V_2}\right)^{1.2} \times p_1 \Rightarrow (5)^{1.2} \times 101.325 \Rightarrow \mathbf{p_2 = 699.41kPa}$$

$$\textbf{Initial Volume } (\mathbf{V_1}) = \frac{\pi d^2}{4} \times L \Rightarrow V_1 = \frac{\pi \times 0.15^2}{4} \times 0.25 \Rightarrow \mathbf{V_1 = 0.00442\ m^3}$$

$$\textbf{Final Volume } (\mathbf{V_2}) = \frac{V_1}{5} \Rightarrow V_2 = \frac{0.00442}{5} \Rightarrow \mathbf{V_2 = 0.000884\ m^3}$$

$$\textbf{Work done W} = \frac{n}{n-1}[p_2V_2 - p_1V_1] \Rightarrow W = \frac{1.2}{1.2-1}[(699.41 \times 0.000884) - (101.325 \times 0.00442)]$$

$$\Rightarrow \mathbf{W = 3.59kJ}$$

4. **Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m^3, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate :(i) The final temperature ;(ii) The final volume ;(iii) The work done.**

GIVEN: P_1=1.02 bar, T_1=22°C+273= 295K, V_1= 0.015 m^3, P_2= 6.8 bar, $pV^n = C$

FIND : (i) T_2 , (ii) V_2, (iii) W

SOLUTION:

(i) Final temperature

From the relation

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 \Rightarrow T_2 = \left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}} \times 295 \Rightarrow \mathbf{T_2 = 507.24K}$$

(ii) Final Volume

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \Rightarrow \frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} \Rightarrow V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \times V_1 = \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}} \times 0.015 \Rightarrow \mathbf{V_2 = 0.00387\ m^3}$$

(iii) Work

$$W = \int pdV \Rightarrow W = \frac{p_1V_1 - p_2V_2}{\gamma-1} \Rightarrow W = \frac{102 \times 0.015 - 680 \times 0.00387}{1.4-1} \Rightarrow \mathbf{W = -2.754kJ}$$

5. **0.15 m^3 of an ideal gas at a pressure of 15 bar and 550 K is expanded isothermally to 4 times the initial volume. It is then cooled to 290 K at constant volume and then compressed back polytropically to its initial state. Calculate the net work done and heat transferred during the cycle.**

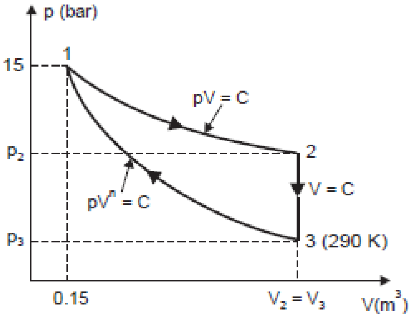
GIVEN: V_1= 0.15 m^3 **,** P_1=15 bar, T_1= T_2 =550K, $\frac{V_2}{V_1} = 4$

T_3=290K, $V_2 = 4 \times V_1 = 0.6\ m^3$

FIND : (i) W , (ii) Q

SOLUTION:

Process 1-2: Isothermal Process



$$\text{At } pV = C \Rightarrow p_1V_1 = p_2V_2 \Rightarrow p_2 = \frac{V_1}{V_2} \times p_1 = \frac{0.15}{4 \times 0.15} \times 15 \Rightarrow \mathbf{p_2 = 3.75bar}$$

$$\textbf{Work done } \mathbf{W_{12}} = p_1V_1 \ln\frac{V_2}{V_1} = (15 \times 10^5) \times 0.15 \ln(4) \Rightarrow \mathbf{W_{12} = 311.9kJ}$$

Process 2-3: Constant Volume Process, Work done $W_{23} = 0$

$$\text{At } \frac{p}{T} = C \Rightarrow p_2T_3 = p_3T_2 \Rightarrow p_3 = \frac{T_3}{T_2} \times p_2 = \frac{290}{550} \times 3.75 \Rightarrow \mathbf{p_3 = 1.98\ bar}$$

Process 3-1: Polytropic Process

$$\text{At } pV^n = C \Rightarrow p_3V_3^n = p_1V_1^n$$

Taking Log on

$$\ln p_3 + n \ln V_3 = \ln p_1 + n \ln V_1 \Rightarrow n(\ln V_3 - \ln V_1) = \ln p_3 - \ln p_1 \Rightarrow n\left(\ln \frac{V_3}{1}\right) = \ln\left(\frac{p_1}{p_3}\right)$$

$$n = \frac{\ln\left(\frac{p_1}{p_3}\right)}{\left(\ln\frac{V_3}{V_1}\right)} \Rightarrow n = \frac{\ln\left(\frac{15}{1.98}\right)}{(\ln(4))} \Rightarrow \mathbf{n = 1.46}$$

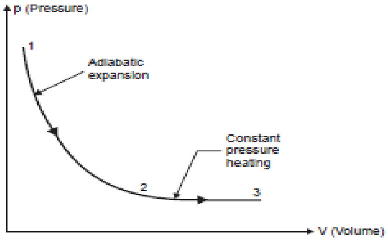
Work done:

$$W_{31} = \frac{p_3V_3 - p_1V_1}{n-1} \Rightarrow W_{31} = \frac{1.98\times10^5\times0.6 - 680\times10^5\times0.15}{1.46-1} \Rightarrow \mathbf{W_{31} = -230.87kJ}$$

Total Work done

$$\mathbf{W = W_{12} + W_{23} + W_{31} \Rightarrow W = 311.9 + 0 + (-230.87) \Rightarrow W = 81.03kJ}$$

6. **0.2 m^3 of air at 4 bar and 130°C is contained in a system. A reversible adiabatic expansion takes place till the pressure falls to 1.02 bar. The gas is then heated at constant pressure till enthalpy increases by 72.5 kJ. Calculate (i) The work done (ii) The index of expansion, if the above processes are replaced by a single reversible polytropic. Process giving the same work between the same initial and final states. Take C_p = 1 kJ/kg K, C_v = 0.714 kJ/kg K.**



GIVEN: V_1= 0.2 m^3 , P_1=4 bar, T_1= 130°C, P_2= 1.02 bar, $\Delta h = 72.5$kJ, C_p = 1 kJ/kgK, C_v = 0.714 kJ/kgK.

FIND : (i) W , (ii) n, (iii)

SOLUTION:

From the characteristic gas equation:

$$R = C_P - C_V \Rightarrow R = 1 - 0.714 \Rightarrow \mathbf{R = 0.286\ \frac{kJ}{kg}.K}$$

$$\gamma = \frac{C_P}{C_V} \Rightarrow \gamma = \frac{1}{0.714} \Rightarrow \boldsymbol{\gamma = 1.4}$$

Process 1-2: Reversible adiabatic process:

$$p_1V_1^\gamma = p_2V_2^\gamma \Rightarrow V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{I}{1.4}} \times V_1 = \left(\frac{4}{1.02}\right)^{\frac{I}{1.402}} \times 0.2 \Rightarrow \mathbf{V_2 = 0.53\ m^3}$$

$$\text{From the relation, } \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 \Rightarrow T_2 = \left(\frac{1.02}{4}\right)^{\frac{1.4-1}{1.4}} \times 403 \Rightarrow \mathbf{T_2 = 272.7K}$$

Work

$$W_{12} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} \Rightarrow W_{12} = \frac{(4\times10^5\times0.2) - (1.02\times\times10^5\times0.53)}{1.4-1} \Rightarrow \mathbf{W_{12} = 64.850kJ}$$

Process 2-3: Constant pressure process

$$Q_{23} = mC_p(T_3 - T_2) \Rightarrow 72.5 = 0.694 \times 1 \times (T_3 - 272.7)$$

$$\Rightarrow T_3 = \mathbf{377K}$$

$$m = \frac{P_1V_1}{RT_1} = \frac{(4 \times 10^5) \times 0.2}{286 \times 403}$$

$$\mathbf{m = 0.694kg}$$

$$\text{At } \frac{V}{T} = C \Rightarrow V_2T_3 = V_3T_2 \Rightarrow V_3 = \frac{T_3}{T_2} \times V_2 = \frac{377}{272.7} \times 0.53 \Rightarrow \mathbf{V_3 = 0.732m^2}$$

$$W_{23} = p_2(V_3 - V_2) \Rightarrow W_{23} = 1.02 \times 10^5(0.732 - 0.53) \Rightarrow \mathbf{W_{23} = 20.604kJ}$$

Total Work done:

$$\mathbf{W = W_{12} + W_{23}} \Rightarrow \mathbf{W = 64.85 + 20.604} \Rightarrow \mathbf{W = 85.454kJ}$$

Index of expansion, n: (If the work by the polytropic process is the same)

$$W_{13} = \frac{p_1V_1 - p_3V_3}{n-1} \Rightarrow 85.454 = \frac{(4\times10^2\times0.2)-(1.02\times\times10^2\times0.53)}{n-1} \Rightarrow \mathbf{n = 1.062}$$

7. **A cylinder contains 0.45 m^3 of a gas at 1 × 10^5 N/m^2 and 80°C. The gas is compressed to a volume of 0.13 m^3, the final pressure being 5 × 10^5 N/m^2. Determine :(i) The mass of gas (ii) The value of index 'n' for compression (iii) The increase in internal energy of the gas (iv) The heat received or rejected by the gas during compression.. Take γ = 1.4, R = 294.2 J/kg°C.**

GIVEN: V_1= 0.45 m^3 **,** P_1=1x 10^5 N/m^2 , T_1= T_2 =80°C, V_2= 0.13 m^3 , P_2=5x 10^5 N/m^2

FIND : (i) m , (ii) n , (iii) ΔU , (iv) Q

SOLUTION:

(i) The mass of gas :

$$m = \frac{P_1V_1}{RT_1} \Rightarrow m = \frac{(1\times10^5)\times0.45}{294.2\times353} \Rightarrow \mathbf{m = 0.433kg}$$

(ii) The value of index 'n' for compression

$$\text{At } pV^n = C \Rightarrow p_1V_1^n = p_2V_2^n$$

Taking Log on

$$\ln p_1 + n\ln V_1 = \ln p_2 + n\ln V_2 \Rightarrow n(\ln V_1 - \ln V_2) = \ln p_2 - \ln p_1 \Rightarrow n\left(\ln\frac{V_1}{V_2}\right) = \ln\left(\frac{p_2}{p_1}\right)$$

$$n = \frac{\ln\left(\frac{p_2}{p_1}\right)}{\left(\ln\frac{V_1}{V_2}\right)} \Rightarrow n = \frac{\ln\left(\frac{5}{1}\right)}{\left(\ln\frac{0.45}{0.13}\right)} \Rightarrow \mathbf{n = 1.296 \approx 1.3}$$

(iii) The increase in internal energy of the gas (polytropic process)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} \Rightarrow T_2 = \left(\frac{V_1}{V_2}\right)^{n-1} \times T_1 \Rightarrow T_2 = \left(\frac{0.45}{0.13}\right)^{1.3-1} \times 353 \Rightarrow \mathbf{T_2 = 509.7K}$$

$$\Delta U = mC_v(T_2 - T_1) \Rightarrow \Delta U = 0.433 \times \frac{0.2942}{1.4-1}(509.7 - 353) \Rightarrow \mathbf{\Delta U = 49.9kJ}$$

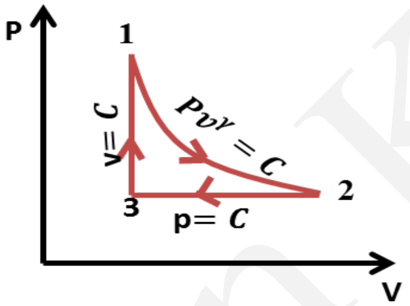
(iv) The heat received or rejected by the gas during compression

$$W = \int pdV \Rightarrow W = \frac{p_1V_1 - p_2V_2}{\gamma - 1} \Rightarrow W = \frac{1\times10^5\times0.45 - 5\times10^5\times0.13}{1.3-1} \Rightarrow \mathbf{W = -67.44kJ}$$

$$Q = \Delta U + W \Rightarrow Q = 49.9 - 67.44 \Rightarrow \mathbf{Q = -17.44kJ}$$

8. **Three grams of nitrogen gas at 6 atm and 160°C in a frictionless piston cylinder device is expanded adiabatically to double its initial volume, then compressed at constant pressure to its initial volume and then compressed again at constant volume to its initial state. Calculate the network done on the gas. Draw the P-V diagram for the processes.**

GIVEN:



$m = 3g = 3\times10^{-3}kg$

$P_1 = 6atm = 6.08\times10^2 kN/m^2$

$T_1 = 160°C + 273 = 433K$

$V_1 = V_3$

$V_2 = 2V_1$

Find:Network done?

Assume $\gamma = 1.4$

$$m = \frac{P_1V_1}{RT_1} \Rightarrow V_1 = \frac{mRT_1}{P_1} \Rightarrow V_1 = \frac{3\times10^{-3}\times0.287\times433}{6.08\times10^2} \Rightarrow \mathbf{V_1 = 0.00061m^3}$$

$$V_2 = 2\times0.00061 \Rightarrow \mathbf{V_2 = 1.22\times10^{-3}\ m^3}$$

Process 1-2: Revesible adiabatic expansion:

$$p_1V_1^\gamma = p_2V_2^\gamma \Rightarrow \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma \Rightarrow p_2 = 6.08\times10^2\times\left(\frac{0.00061}{1.22\times10^{-3}}\right)^{1.4} \Rightarrow \mathbf{p_2 = 2.5\times10^2 kN/m^2}$$

$$W = \frac{p_1V_1 - p_2V_2}{\gamma-1} \Rightarrow W = \frac{6.08\times10^2\times0.00061 - 2.5\times10^2\times1.22\times10^{-3}}{1.4-1} \Rightarrow \mathbf{W = 225.63kJ}$$

Process 2-3: Constant pressure cooling

$$W_{23} = P(V_3 - V_2) \Rightarrow W_{23} = 2.5\times10^2(0.00061 - 1.22\times10^{-3}) \Rightarrow \mathbf{W_{23} = -0.1525\ kJ}$$

Process 2-3: Constant volume compression

$\mathbf{W_{31} = 0}$

Network done:

$$W_{net} = 225.63 - 0.1525 + 0 \Rightarrow \mathbf{W_{net} = 225.48kJ}$$

9. **A gas undergoes a thermodynamic cycle consisting of three processes beginning at an initial state where $p_1 = 1$ bar, $V_1 = 1.5\ m^3$ and $U_1 = 512$ kJ. The processes are as follows:**
    - **(i) Process 1–2: Compression with pV = constant to $p_2 = 2$bar, $U_2 = 690$ kJ,**
    - **(ii) Process 2–3: $W_{23} = 0$, $Q_{23} = –150$ kJ, and**
    - **(iii) Process 3–1: $W_{31} = +50$ kJ. Neglecting KE and PE changes,**

Determine the heat interactions Q_{12} and Q_{31}.

GIVEN: P_1=1 bar , V_1= 1.5 m^3 , U_1= 512kJ, P_2=2 bar , U_2= 690kJ , W_{23} = 0,

Q_{23} = –150 kJ, W_{31} = +50 kJ

FIND : (i) Q_{12} , (ii) Q_{31}

SOLUTION:

Process 1–2: Isothermal Process

$$Q_{12} = \Delta U + \int \mathrm{pdV} \Rightarrow Q_{12} = U_2 - U_1 + \mathrm{p_1V_1} \ln\left(\frac{\mathrm{V_2}}{\mathrm{V_1}}\right) \Rightarrow Q_{12} = (690 - 512) + 100 \times 1.5 \ln\left(\frac{1}{2}\right)$$

$$Q_{12} = 178 - 103.972 \Rightarrow \mathbf{Q_{12} = 74.03kJ}$$

Process 2–3: Constant Volume Process

$\mathbf{W_{23} = 0}$, $\mathbf{Q_{23} = –150\ kJ}$

Process 3–1: Adiabatic Expansion Process

$$\mathrm{Q_{23}} = (\mathrm{U_2} - \mathrm{U_3}) + \mathrm{W} \Rightarrow -150 = (690 - \mathrm{U_3}) + 0 \Rightarrow \mathbf{U_3 = 540kJ}$$

$$Q_{31} = (\mathrm{U_3} - \mathrm{U_1}) + \mathrm{W} \Rightarrow Q_{31} = (540 - 512) + 50 \Rightarrow \mathbf{Q_{31} = 22kJ}$$

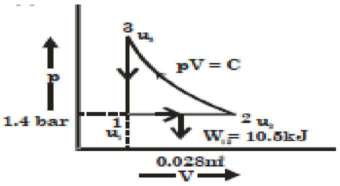
10. **A gas undergoes a thermodynamic cycle consisting of the following processes:**
    - i. **Process 1–2: Constant pressure p = 1.4 bar, V_1 = 0.028 m3, W_{12} = 10.5 kJ**
    - ii. **Process 2–3: Compression with pV = constant, U_3 = U_2**
    - iii. **Process 3–1: Constant volume, $U_1 – U_3$ = – 26.4 kJ.**

There are no significant changes in KE and PE. (a) Sketch the cycle on a p–V diagram, (b) Calculate the network for the cycle in kJ (c) Calculate the heat transfer for process 1–2, (d) Show that cycle ΣQ = ΣW.

GIVEN: P_1=1.4 bar , V_1= 0.028 m^3 , W_{12}= 10.5kJ,

U_3= U_2 $U_1 – U_3$ = – 26.4 kJ,

FIND: (i) Sketch the cycle, (ii) The net work for the cycle (iii) Q_{12} (iv) Show that cycle ΣQ = ΣW



SOLUTION:

Process 1–2: Constant Pressure Process

$$W_{12} = P(\mathrm{V_2} - \mathrm{V_1}) \Rightarrow 10.5 = 1.4 \times 100(\mathrm{V_2} - 0.028) \Rightarrow \mathbf{V_2 = 0.103m^3}$$

$$\mathrm{U_1} - \mathrm{U_3} = -26.4 \Rightarrow \mathrm{U_1} - \mathrm{U_2} = -26.4 \Rightarrow \mathbf{U_2 = U_1 + 26.4}$$

$$Q_{12} = \Delta E + \int \mathrm{pdV} \Rightarrow Q_{12} = U_2 - U_1 + W_{12} \Rightarrow Q_{12} = (26.4) + 10.5 \Rightarrow \mathbf{Q_{12} = 36.9kJ}$$

$$\Rightarrow \mathbf{W_{12} = 10.5kJ}$$

Process 2–3: Isothermal compression Process

$$\mathrm{W_{23}} = \mathrm{p_2V_2} \ln\left(\frac{\mathrm{V_3}}{\mathrm{V_2}}\right) \Rightarrow \mathrm{W_{23}} = 1.4 \times 100 \times 0.103 \ln\left(\frac{0.028}{0.103}\right) \Rightarrow \mathbf{W_{23} = -18.783\ kJ}$$

$$\mathrm{Q_{23}} = (\mathrm{U_2} - \mathrm{U_3}) + \mathrm{W_{23}} \Rightarrow \mathrm{Q_{23}} = (0) + (-18.783) \Rightarrow \mathbf{Q_{23} = -18.783kJ}$$

Process 3–1: Constant Volume Process $\Rightarrow$ $\mathbf{W_{31} = 0}$

$$\mathrm{Q_{31}} = (\mathrm{U_3} - \mathrm{U_1}) + \mathrm{W} \Rightarrow \mathrm{Q_{31}} = (-26.4) + 50 \Rightarrow \mathbf{Q_{31} = 26.4kJ}$$

(b) The net work for the cycle

$$W = W_{12} + W_{23} + W_{31} \Rightarrow W = 10.5 - 18.783 + 0 \Rightarrow \mathbf{W = -8.283kJ}$$

(d) Show that cycle ΣQ = ΣW

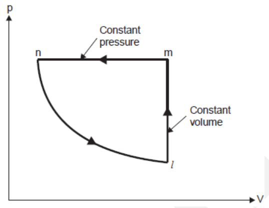
$$W_{12} + W_{23} + W_{31} = Q_{12} + Q_{23} + Q_{31} \Rightarrow -8.283 = 36.9 - 18.783 - 26.4$$

ΣQ = ΣW is proved

11. 90 kJ of heat are supplied to a system at a constant volume. The system rejects 95 kJ of heat at constant pressure and 18 kJ of work is done on it. The system is brought to original state by adiabatic process. Determine :

(i) The adiabatic work ;

(ii) The values of internal energy at all end states if initial value is 105 kJ.



GIVEN:

Heat supplied at constant volume =90kJ

Heat rejected at constant pressure=-95kJ

Work done on the system = -18kJ

Initial value of internal energy = 105kJ

SOLUTION:

Process *l*-m: Constant volume heat supplied

$$W_{l-m} = 0$$

$$Q_{l-m} = W_{l-m} + (U_m - U_l) \Rightarrow Q_{l-m} = (U_m - U_l) \Rightarrow 90 = (U_m - 105) \Rightarrow \mathbf{U_m = 195kJ}$$

Process m-n: Constant pressure heat rejected

$$Q_{m-n} = W_{m-n} + (U_n - U_m) \Rightarrow -95 = -18 + (U_n - U_m) \Rightarrow U_n - U_m = -77kJ \Rightarrow \mathbf{U_n = 118kJ}$$

Process n-*l*: Adiabatic Expansion $Q_{n-l} = 0$, for Adiabatic process

$$\int \delta Q = 90 - 95 = -5kJ$$

$$\oint \delta Q = \int \delta w \Rightarrow \int \delta W = -18 + W_{n-l} \Rightarrow -5 = -18 + W_{n-l} \Rightarrow \mathbf{W_{n-l} = 13kJ}$$

12. A fluid system, contained in a piston and cylinder machine, passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is -340 kJ. The system completes 200 cycles per min.

Process	Q (kJ/min)	W (kJ/min)	E (kJ/min)
1-2	0	4340	-
2-3	42,000	0	-
3-4	- 4,200	0	-73,200
4-1	-	-	-

Complete the above table showing the method for each item, and compute the net rate of work output in kW.

SOLUTION:

Sum of all heat transferred during the cycle = -340 kJ

Number of cycles completed by the system = 200 cycle/min

Process 1-2:

$Q_{12} = W_{12} + \Delta E_{12} \Rightarrow 0 = 4340 + \Delta E_{12} \Rightarrow \mathbf{\Delta E_{12} = -4340\ kJ/min}$

Process 2-3:

$Q_{23} = W_{23} + \Delta E_{23} \Rightarrow 42000 = 0 + \Delta E_{23} \Rightarrow \mathbf{\Delta E_{23} = 42000\ kJ/min}$

Process 3-4:

$Q_{34} = W_{34} + \Delta E_{34} \Rightarrow -4200 = W_{34} - 73200 \Rightarrow \mathbf{W_{34} = 69000\ kJ/min}$

Process 4-1:

$$\textstyle\sum_{\text{Cycle}} Q = -340\ \text{kJ/min}$$

The system completes 200 cycles / min

$Q_{12} + Q_{23} + Q_{34} + Q_{41} = 200 \times (-340) \Rightarrow 0 + 42000 - 4200 + Q_{41} = 200 \times (-340)$

$\Rightarrow \mathbf{Q_{41} = -105800\ kJ/min}$

Now, $\int dE = 0$, since cyclic integral of any property is zero

$\Delta E_{12} + \Delta E_{23} + \Delta E_{34} + \Delta E_{41} = 0 \Rightarrow -4340 + 42000 + (-73200) + \Delta E_{41} = 0$

$\Rightarrow \mathbf{\Delta E_{41} = 35540\ kJ/min}$

$Q_{41} = W_{41} + \Delta E_{41} \Rightarrow -105800 = W_{41} - 35540 \Rightarrow \mathbf{W_{41} = -141340\ kJ/min}$

Rate od work output $= -68000 \frac{\text{kJ}}{\text{min}} = -\frac{68000}{60} = \mathbf{1133.33\ kW}$

13. **680kg of fish at 5°C are to be frozen and stored at – 12°C. The specific heat of fish above freezing point is 3.182, and below freezing point is 1.717 kJ/kg K. The freezing point is – 2°C, and the latent heat of fusion is 234.5 kJ/kg. How much heat must be removed to cool the fish, and what per cent of this is latent heat?**

GIVEN: m=680 kg, T_1=5°C, T_2= – 2°C, T_3= – 12°C, $C_{afp}=3.182\frac{\text{kJ}}{\text{kg K}}$,

$C_{bfp}=1.717\frac{\text{kJ}}{\text{kg K}}$, h_{fg}=234.5 kJ/kg

FIND : (i) Heat removed to cool the fish, (ii) percentage of this is latent heat.

SOLUTION:

Heat to be removed above freezing point:

$Q_{af} = mC_{afp}(T_1 - T_2) \Rightarrow Q_{af} = 680 \times 3.182 \times (5 - (-2)) \Rightarrow \mathbf{Q_{af} = 15.146MJ}$

Latent heat to be removed:

$Q_{LH} = m \times h_{fg} \Rightarrow Q_{LH} = 680 \times 234.5 \Rightarrow \mathbf{Q_{LH} = 159.46MJ}$

Heat to be removed below freezing point:

$Q_{af} = mC_{bfp}(T_2 - T_3) \Rightarrow Q_{af} = 680 \times 1.717 \times (-1 - (-12)) \Rightarrow \mathbf{Q_{af} = 11.676MJ}$

Total Heat:

$Q = Q_{af} + Q_{LH} + Q_{af} \Rightarrow Q = 15.146 + 159.46 + 11.676 \Rightarrow \mathbf{Q = 186.2816MJ}$

14. **A gas of mass 1.5 kg undergoes a quasi static expansion, which follows a relationship P=a+bV, where 'a' and 'b' are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and the corresponding volumes are 0.2 m^3 and 1.2 m^3. The specific internal energy of the gas is given by the relation u=(1.5 pv-85) kJ/kg, where p is in kPa and V is in m^3/kg. Calculate the net heat transfer and the maximum internal energy of the gas attained during expansion.**

GIVEN: m=1.5 kg, P_1=1000kPa , P_2=200kPa , V_1=0.2 m^3, V_2=1.2 m^3,

FIND : (i) The net heat transfer, (ii) The maximum internal energy

SOLUTION:

p = a + bV, Where a and b are constants

Substitute P_1 V_1 and P_2, V_2, in the above quation to find the a and b are constants

$$1000 = a + 0.2b \longrightarrow 1$$

$$\underline{200 = a + 1.2b} \longrightarrow 2$$

$$b = -800$$

Substitute value b in the first equation

$1000 = a + 0.2b \Rightarrow 1000 = a + (0.2x(-800)) \Rightarrow a=1160$

The equation becomes

$$p = 1160 - 800V$$

Work done:

$$W = \int pdV \Rightarrow W = \int_{V_1}^{V_2}[1160 - 800V]\,dV \Rightarrow W = [1160V - 400V^2]_{V_1}^{V_2}$$

$$W = \left[1160(V_2 - V_1) - 400[V_2^2 - V_1^2]\right]$$

$$W = \left[1160(1.2 - 0.2) - 400[1.2^2 - 0.2^2]\right]$$

$$\mathbf{W = 600\ kJ}$$

Change in internal energy:

$$u_1 = 1.5p_1v_1 - 85 \Rightarrow u_1 = 1.5 \times 1000 \times \frac{0.2}{1.5} - 85 \Rightarrow \mathbf{u_1 = 215\ kJ/kg}$$

$$u_2 = 1.5p_2v_2 - 85 \Rightarrow u_2 = 1.5 \times 200 \times \frac{1.2}{1.5} - 85 \Rightarrow \mathbf{u_2 = 155\ kJ/kg}$$

$$\Delta U = m(u_1 - u_2) \Rightarrow \Delta U = 1.5(215 - 155) \Rightarrow \mathbf{\Delta U = 60kJ}$$

(i) The net heat transfer

$$Q = \Delta U + W \Rightarrow Q = 60 + 600 \Rightarrow \mathbf{Q = 660kJ}$$

(ii) The maximum internal energy

$$u = 1.5pv - 85 \Rightarrow u = 1.5(1160 - 800v)v - 85 \Rightarrow u = 1.5(1160v - 800v^2) - 85$$

$$\frac{\partial u}{\partial v} = 1.5(1160v - 1600v) - 85, \quad \text{For maximum u}, \frac{\partial u}{\partial v} = 0, \ v = \frac{1160}{1600} = 0.725$$

$$u_{max} = 1.5(1160 \times 0.725 - 800 \times 0.725^2) - 85 \Rightarrow \mathbf{u_{max} = 335.5\ kJ/kg}$$

$$U_{max} = m \times u_{max} \Rightarrow U_{max} = 1.5 \times 335.5 \Rightarrow \mathbf{U_{max} = 503.25kJ}$$

15. A system of volume V contains a mass m of gas at pressure p and temperature T. The macroscopic properties of the system obey the following relationship:

$$\left[P+\frac{a}{V^2}\right](V-b)=mRT$$

Where a, b, and R are constants. Obtain an expression for the displacement work done by the system during a constant-temperature expansion from volume V_1 to volume V_2.Calculate the work done by a system which contains 10 kg of this gas expanding from 1 m^3 to 10 m^3 at a temperature of 293 K. Use the values a = 15.7 ×10 Nm^4 , b = 1.07 $\times 10^{-2} m^3$, and R = 0.278 kJ/kg-K.

GIVEN: m=10kg, V_1=1 m^3,V_2=10 m^3, a = 15.7 ×10 Nm^4, b = 1.07 $\times 10^{-2} m^3$, R = 0.278 kJ/kg-K

FIND : The work done (W)

SOLUTION:

Displacement work done by the system during a constant-temperature expansion.

$$\left[P_1+\frac{a}{V_1^2}\right](V_1-b)=\left[P_2+\frac{a}{V_2^2}\right](V_2-b)=mRT=K$$

From the equation

$$\left[P+\frac{a}{V^2}\right](V-b)=K \Rightarrow P=\frac{K}{(V-b)}-\frac{a}{V^2}$$

Work done

$$W=\int pdV \Rightarrow W=\int_1^2\left[\frac{K}{(V-b)}-\frac{a}{V^2}\right]dV$$

$$\Rightarrow W=\int_1^2\left[K\left(\frac{1}{(V-b)}\right)-a\left(\frac{1}{V^2}\right)\right]dV$$

$$\Rightarrow W=\left[K\ln(V-b)+a\left(\frac{1}{V}\right)\right]_1^2$$

$$\Rightarrow W=K\ln\frac{(V_2-b)}{(V_1-b)}+a\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$$

$$\Rightarrow W=10\times 0.278\times 293\ln\frac{(10-1.07\times 10-2)}{(1-1.07\times 10-2)}+15.7\times 10\left(\frac{1}{10}-\frac{1}{1}\right)$$

$$\Rightarrow \mathbf{W=1742.14kJ}$$

16. The properties of a certain fluid are related as follows:

u = 196 + 0.718t,

pv = 0.287 (t + 273)

Where u is the specific internal energy (kJ/kg), t is in °C, p is pressure (kN/m^2), and v is specific volume (m^3/kg). For this fluid, find C_v and C_P.

Specific Heat at Constant Pressure:

$$C_P=\left(\frac{\partial h}{\partial T}\right)_P \Rightarrow C_P=\left(\frac{\partial(u+pv)}{\partial T}\right)_P \Rightarrow C_P=\left(\frac{\partial(196+0.718t+0.287(t+273))}{\partial T}\right)_P \Rightarrow \mathbf{T=t+273,\ \partial T=\partial t}$$

$$C_P=\left(\frac{0.718\,\partial t+0.287\,\partial t}{\partial t}\right)_P \Rightarrow \mathbf{C_P=1.005\ kJ/kg.K}$$

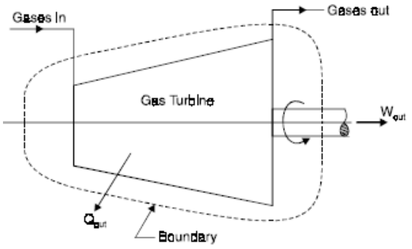
Specific Heat at Constant Volume:

$$C_v=\left(\frac{\partial u}{\partial T}\right)_P \Rightarrow C_v=\left(\frac{\partial(196+0.718t}{\partial T}\right)_P \Rightarrow \mathbf{C_v=0.718\ kJ/kg.K}$$

PROBLEMS ON FIRST LAW OF THERMODYNAMICS OPEN SYSTEM:

1. **In a gas turbine unit, the gases flow through the turbine is 15 kg/s and the power developed by the turbine is 12000 kW. The enthalpies of gases at the inlet and outlet are 1260 kJ/kg and 400 kJ/kg respectively, and the velocity of gases at the inlet and outlet are50 m/s and 110 m/s respectively. Calculate :**

(i) The rate at which heat is rejected to the turbine, and (ii) The area of the inlet pipe given that the specific volume of the gases at the inlet is 0.45 m^3/kg.



GIVEN: m=15kg/s, W=12000kW,

h_1=1260kJ/kg, h_2=400 kJ/kg,

C_1=50 m/s, C_2= 110 m/s

v=0.45 m^3/kg

FIND : (i) Q (ii) AREA

SOLUTION:

From steady flow energy Equation: (per unit mass)

$$\frac{c_1^2}{2000} + gz_1 + u_1 + p_1v_1 + q_{1-2} = \frac{c_2^2}{2000} + gz_2 + u_2 + p_2v_2 + w_{1-2}$$

Where,

$h_1 = u_1 + p_1v_1$ and $h_2 = u_2 + p_2v_2$, $z_1 = z_2$

The equation becomes

$$\frac{C_1^2}{2} + h_1 + q_{1-2} = \frac{C_2^2}{2} + h_2 + W \Rightarrow \frac{50^2}{2000} + 1260 + q_{1-2} = \frac{110^2}{2000} + 400 + \frac{12000}{15} \Rightarrow \mathbf{q_{1-2} = -55\ kJ/kg}$$

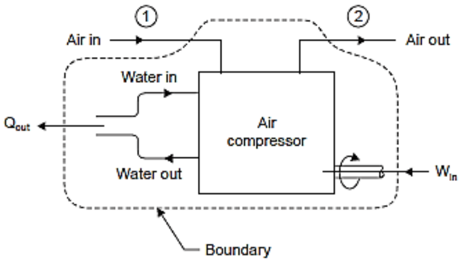
$$Q = \dot{m} \times -55\ \text{kJ/kg} \Rightarrow Q = 15\ \text{kg/s} \times 55\ \text{kJ/kg} \Rightarrow \mathbf{Q = 828\ kW}$$

Inlet Area

From continuity equation

$$\dot{m} = \frac{A_1C_1}{v_1} \Rightarrow 15 = \frac{A\times50}{0.45} \Rightarrow \mathbf{A = 0.135m^2}$$

2. **In an air compressor air flows steadily at the rate of 0.5 kg/s through an air compressor. It enters the compressor at 6 m/s with a pressure of 1 bar and a specific volume of 0.85 m^3/kg and leaves at 5 m/s with a pressure of 7 bar and a specific volume of 0.16 m^3/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in a jacket surrounding the cylinder absorbs heat from the air at the rate of 60 kJ/s. Calculate : (i) The power required to drive the compressor (ii) The inlet and output pipe cross-sectional areas.**



GIVEN: $m=0.5$ kg/s, $W=12000$kW,

$C_1=6$ m/s, $C_2= 5$ m/s

$P_1=1$ bar, $v_1=0.85$ m^3/kg

$P_2=7$ bar, $v_2=0.16$ m^3/kg

$\Delta u= - 90$ kJ/kg, $\dot{Q}= - 60$ kJ/s

FIND : (i) W (ii) A_1 and A_2

SOLUTION:

From steady flow energy Equation: (per unit mass)

$$\frac{c_1^2}{2000}+gz_1+u_1+p_1v_1+q_{1-2}=\frac{c_2^2}{2000}+gz_2+u_2+p_2v_2+w_{1-2}$$

Where, $z_1=z_2$

$$w=\frac{C_1^2-C_2^2}{2000}+(u_1-u_2)+(p_1v_1-p_2v_2)+\frac{\dot{Q}}{m}$$

$$w=\frac{6^2-5^2}{2000}+(-90)+\frac{(1\times10^5\times0.85)-(7\times10^5\times0.16)}{1000}+\left(\frac{-60}{0.5}\right)$$

$$\mathbf{w=-237\ kJ/kg}$$

Therefore,

$\dot{W}=\dot{m}\times237$ kJ/kg ⇨ $W=0.5$ kg/s × 237 kJ/kg ⇨ $\mathbf{W=118.5\ kW}$

Inlet Area and Exit Area

From continuity equation

$\dot{m}=\frac{A_1C_1}{v_1}$ ⇨ $0.5=\frac{A_1\times6}{0.85}$ ⇨ $\mathbf{A_1=0.0708m^2}$

$\dot{m}=\frac{A_2C_2}{v_2}$ ⇨ $0.5=\frac{A_1\times5}{0.16}$ ⇨ $\mathbf{A_2=0.016\ m^2}$

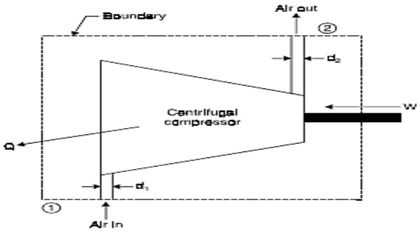
3. **12 kg of air per minute is delivered by a centrifugal air compressor. The inlet and outlet conditions of air are C_1 = 12 m/s, p_1 = 1 bar, v_1 = 0.5 m^3/kg and C_2 = 90 m/s,p_2 = 8 bar, v_2 = 0.14 m^3/kg. The increase in enthalpy of air passing through the compressor is150 kJ/kg and heat loss to the surroundings is 700 kJ/min. Find : (i) Motor power required to drive the compressor ; (ii) Ratio of inlet to outlet pipe diameter. Assume that inlet and discharge lines are at the same level.**

GIVEN: $m=0.2$ kg/s, $C_1= 12$ m/s, $C_2= 90$ m/s

$P_1=1$ bar, $v_1=0.5$ m^3/kg

$P_2= 8$ bar, $v_2=0.14$ m^3/kg

$\Delta h= 150$ kJ/kg, $Q= - 11.67$ kJ/s



FIND : (i) W, (ii) $\frac{d_1}{d_2}$

SOLUTION:

(i) Motor power required to drive the compressor

From steady flow energy Equation: (per unit time)

$$\dot{\mathbf{m}}\left(\frac{\mathbf{c}_1^2}{\mathbf{2000}}+\mathbf{gz}_1+\mathbf{u}_1+\mathbf{p}_1\mathbf{v}_1\right)+\dot{\mathbf{Q}}=\dot{\mathbf{m}}\left(\frac{\mathbf{c}_2^2}{\mathbf{2000}}+\mathbf{gz}_2+\mathbf{u}_2+\mathbf{p}_2\mathbf{v}_2\right)+\dot{\mathbf{W}}$$

Where, $z_1 = z_2$, $h_1 = u_1 + p_1v_1$ and $h_2 = u_2 + p_2v_2$

$$\dot{W} = \dot{m}\left[\frac{C_1^2-C_2^2}{2000} + (h_1 - h_2)\right] + \dot{Q} \Rightarrow \dot{W} = 0.2\left[\frac{12^2-90^2}{2000} + (-150)\right] + (-11.67)$$

$$\dot{\mathbf{W}} = -\mathbf{42.46\ kJ/s} \Rightarrow \mathbf{W} = -\mathbf{42.46\ kW}$$

(ii) Ratio of Inlet to outlet pipe diameter

From continuity equation

$$\dot{m} = \frac{A_1C_1}{v_1} = \frac{A_2C_2}{v_2} \Rightarrow \frac{A_1}{A_2} = \frac{C_2}{v_2} \times \frac{v_1}{C_1} \Rightarrow \frac{A_1}{A_2} = \frac{90}{0.14} \times \frac{0.5}{12} \Rightarrow \frac{\mathbf{A}_1}{\mathbf{A}_2} = \mathbf{26.78} \Rightarrow \frac{\mathbf{d}_1}{\mathbf{d}_2} = \mathbf{5.175}$$

4. At the inlet to a certain nozzle the enthalpy of fluid passing is 2800 kJ/kg, and the velocity is 50 m/s. At the discharge end the enthalpy is 2600 kJ/kg. The nozzle is horizontal and there is negligible heat loss from it.

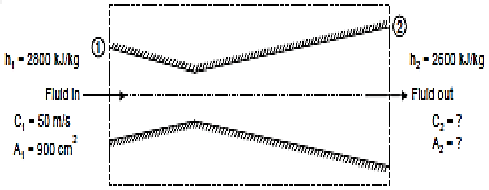
- **(i) Find the velocity at exit of the nozzle.**
- **(ii) If the inlet area is 900 cm^2 and the specific volume at inlet is 0.187 m^3/kg, find the mass flow rate.**
- **(iii) If the specific volume at the nozzle exit is 0.498 m^3/kg, find the exit area of nozzle**

GIVEN: h_1= 2800 kJ/kg, C_1= 50 m/s,

h_2= 2600 kJ/kg

A_1=900 cm^2, v_1=0.187 m^3/kg

v_2=0.498 m^3/kg



FIND : (i) C_2, (ii) $\dot{m}$, (iii) A_2

SOLUTION:

(i) The velocity at exit of the nozzle

From steady flow energy Equation: (per unit mass)

$$\frac{\mathbf{c}_1^2}{\mathbf{2000}}+\mathbf{gz}_1+\mathbf{u}_1+\mathbf{p}_1\mathbf{v}_1+\mathbf{q}_{1-2}=\frac{\mathbf{c}_2^2}{\mathbf{2000}}+\mathbf{gz}_2+\mathbf{u}_2+\mathbf{p}_2\mathbf{v}_2+\mathbf{w}_{1-2}$$

Where, $z_1 = z_2$, Q=0, W=0, $h_1 = u_1 + p_1v_1$ and $h_2 = u_2 + p_2v_2$

$$\frac{C_2^2}{2000} = \frac{C_1^2}{2000} + (h_1 - h_2) \Rightarrow C_2^2 = 50^2 + 2000(2800 - 2600) \Rightarrow \mathbf{C_2 = 634.4\ m/s}$$

(ii) Mass flow rate

From continuity equation

$$\dot{m} = \frac{A_1C_1}{v_1} \Rightarrow \dot{m} = \frac{900\times10^{-4}\times50}{0.187} \Rightarrow \boldsymbol{\dot{m} = 24.06\ m/s}$$

(iii) Area at the exit

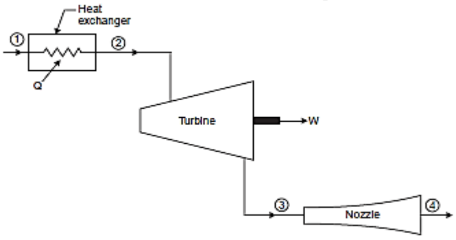
$$\dot{m} = \frac{A_2C_2}{v_2} \Rightarrow 24.06 = \frac{A_2\times634.4}{0.498} \Rightarrow \boldsymbol{A_2 = 0.018887\ cm^2}$$

5. Air at a temperature of 20°C passes through a heat exchanger at a velocity of 40 m/s where its temperature is raised to 820°C. It then enters a turbine with same velocity of 40 m/s and expands till the temperature falls to 620°C. On leaving the turbine, the air is taken at a velocity of 55 m/s to a nozzle where it expands until the temperature has fallen to510°C. If the air flow rate is 2.5 kg/s, calculate :

(i) Rate of heat transfer to the air in the heat exchanger

(ii) The power output from the turbine assuming no heat loss

(iii) The velocity at exit from the nozzle, assuming no heat loss



GIVEN: t_1= 20°C, C_1= 40 m/s,

t_2= 820°C, C_2= 40 m/s

t_3=620°C, C_3=55 m/s

t_4=510°C, $\dot{m} = 2.5\ kg/s$

FIND : (i) Q (ii) W , (iii) C_4

SOLUTION:

(i) Rate of heat transfer to the air in the heat exchanger

From steady flow energy Equation:

$$\dot{m}\left(\frac{C_1^2}{2000}+\frac{gz_1}{1000}+u_1+p_1v_1\right)+\dot{Q}_{12}=\dot{m}\left(\frac{C_2^2}{2000}+\frac{gz_2}{1000}+u_2+p_2v_2\right)+\dot{W}_{1-2}$$

Where, $z_1 = z_2$, $C_1 = 0$, $C_2 = 0$, W=0, $h_1 = u_1 + p_1v_1$ and $h_2 = u_2 + p_2v_2$

$$\dot{Q}_{12} = \dot{m}(h_2 - h_1) \Rightarrow \dot{Q}_{12} = \dot{m}C_P(t_2 - t_1) = 2.5 \times 1.005(820 - 20) \Rightarrow \dot{Q}_{12} = \mathbf{2010}kJ/s$$

(ii) The power output from the turbine assuming no heat loss

$$\dot{m}\left(\frac{C_2^2}{2000}+\frac{gz_2}{1000}+u_2+p_2v_2\right)+\dot{Q}_{23}=\dot{m}\left(\frac{C_3^2}{2000}+\frac{gz_3}{1000}+u_3+p_3v_3\right)+\dot{W}_{23}$$

Where, $z_2 = z_3$, $h_2 = u_2 + p_2v_2$ and $h_3 = u_3 + p_3v_3$, $Q = 0$

$$\dot{W}_{23} = \dot{m}\left[\frac{C_2^2-C_3^2}{2000} + (h_2 - h_3)\right] \Rightarrow \dot{W}_{23} = \dot{m}\left[\frac{C_2^2-C_3^2}{2000} + C_P(t_2 - t_3)\right]$$

$$\dot{W}_{23} = 2.5\left[\frac{40^2-55^2}{2000} + 1.005(820 - 620)\right] \Rightarrow \dot{W} = \mathbf{504.3\ kJ/s} \Rightarrow \dot{W} = \mathbf{504.3\ kW}$$

(iii) The velocity at exit of the nozzle

$$\dot{m}\left(\frac{c_3^2}{2000}+\frac{gz_3}{1000}+u_3+p_3v_3\right)+\dot{Q}_{34}=\dot{m}\left(\frac{c_4^2}{2000}+\frac{gz_4}{1000}+u_4+p_4v_4\right)+\dot{W}_{34}$$

Where, $z_3 = z_4$, Q=0, W=0, $h_3 = u_3 + p_3v_3$ and $h_4 = u_4 + p_4v_4$

$$\frac{c_4^2}{2} = \frac{c_3^2}{2} + C_P(t_2 - t_3) \Rightarrow \frac{c_4^2}{2} = \frac{55^2}{2000} + 1.005(620 - 510) \Rightarrow \mathbf{C_4 = 473.4\ m/s}$$

ADDITIONAL SOLVED PROBLEMS

1. **A piston-cylinder assembly contains air (ideal gas with γ=1.4) at 200 kPa and occupies a volume of 0.01 m^3. The piston is attached to one end of a spring and the other end of the spring is fixed to a wall. The force exerted by the spring on the piston is proportional to the decrease in the length of the spring from its natural length. The ambient atmospheric pressure is 100 kPa. Now, the air in the cylinder is heated till the volume is doubled and at this instant it is found that the pressure of the air in the cylinder is 500 kPa. Calculate the work done by the gas. [NOV/DEC 2015]**

SOLUTION:

We know that the force exerted by the spring on the piston is given by

$$F_S = KX = K\frac{(V-V_0)}{A}$$

Where K = spring constant

A = cross-sectional area of piston

V_0 = Volume of air if the spring is at its natural length, that is the spring is neither compressed nor elongated.

V = Volume of air if the spring is compressed by a distance X,

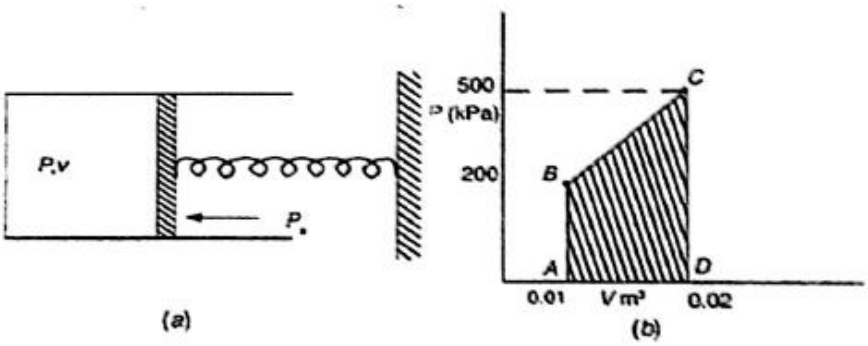


Fig.E 1.44 (*a*) Sketch for Example 1.44.
(*b*) P-v diagram in which shaded area represents the work done.

Force acting on the piston due to ambient atmosphere is given by

$$F_a = P_a A$$

Force balance on the piston when the air volume is V is given by

$$PA = P_a A + \frac{K(V-V_0)}{A}$$

or

$$P = P_a + \frac{K}{A^2}(V-V_0) \tag{A}$$

Force balance on the piston in the initial state gives

$$P_1 A = P_a A + K\left(\frac{V_1 - V_0}{A}\right) \tag{B}$$

Force balance on the piston in the final state gives

$$P_2 A = P_a A + K\left(\frac{V_2 - V_0}{A}\right) \qquad (C)$$

The work done by the air is given by

$$W = \int F.dx = \int PA.dX = \int PdV$$

$$= \int_1^2 \left\{P_a + \frac{K}{A^2}(V - V_0\right\}dV = P_a(V_2 - V_1) + \frac{K}{2A^2}\left\{(V_2 - V_0)^2 - (V_1 - V_0)^2\right\}$$

$$= P_a(V_2 - V_1) + \frac{K}{2A^2}\{(V_2 - V_0) + (V_1 - V_0)\}\{(V_2 - V_0) - (V_1 - V_0)\}$$

$$= P_a(V_2 - V_1) + \frac{K}{2A^2}(V_2 + V_1 - 2V_0)(V_2 - V_1)$$

$$= \left\{P_a + \frac{K}{2A^2}(V_2 + V_1 - 2V_0)\right\}(V_2 - V_1) \qquad (D)$$

From Eqns. (*B*) and (*C*), we get

$$\frac{P_1 + P_2}{2} = P_a + \frac{K}{2A^2}(V_2 + V_1 - 2V_0) \qquad (E)$$

Substituting Eqn.(*E*) in Eqn. (*D*), we obtain

$$W = \frac{(P_1 + P_2)}{2}(V_2 - V_1)$$

$$= \frac{(200 \times 10^3 + 500 \times 10^3)}{2} \times (0.02 - 0.01) = 3.5 \text{ kJ}$$

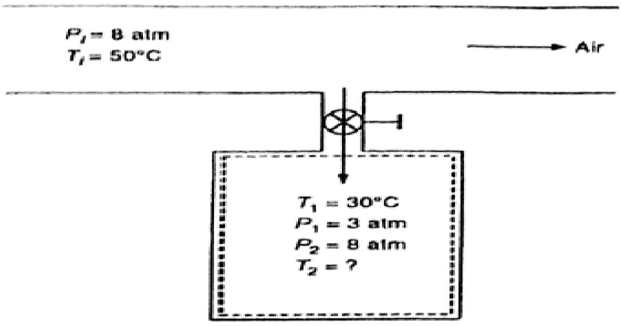
The shaded area of the trapezium ABCD in Fig.E 1.44 (*b*) represents the work done by air. It can be observed that the area of the trapezium ABCD is given by

$$\frac{(P_1 + P_2)}{2}(V_2 - V_1)$$

2. **An insulated rigid tank having 5 kg of air at 3 atm and 30°C is connected to an air supply line at 8 atm and 50°C through a valve. The valve is now slowly opened to allow the air from the supply line to flow into the tank until the tank pressure reaches 8 atm, and then the valve is closed. Determine the final temperature of the air in the tank. Also, find the amount of air added to the tank.**

SOLUTION: **[NOV/DEC 2015]**

The boundary of the control volume is indicated by dashed lines in the figure below. This is not a steady-flow process since the state of the control volume changes during the process.



The inlet conditions of the air during the process remain constant, and the state within the control volume may be assumed to be changing uniformly since the flow into the tank is let in slowly. Therefore, the flow process can be treated as a uniform-flow process. Let the subscripts 1 and 2 refer to the initial and final states of the process in the tank and the subscript i to the state of air at the inlet of the tank. For the present control volume with a single inlet, the mass conservation equation gives

$$m_i = (m_2 - m_1)_{CV}$$

The tank is insulated and no work is involved. Therefore, $Q = 0$ and $W = 0$. Thus, the conservation of energy Eq. (3.27) for this case reduces to

$$m_i h_i = (m_2 u_2 - m_1 u_1)_{CV}$$

For air, being an ideal gas, $u_1 = C_v T_1$, $u_2 = C_v T_2$ and $h_i = C_P T_i$. Therefore,

$$m_i C_P T_i = C_v(m_2 T_2 - m_1 T_1)$$

But $m_i = m_2 - m_1$. Thus,

$$(m_2 - m_1)C_P T_i = C_v(m_2 T_2 - m_1 T_1)$$

By state equation we have $PV = mRT$. Therefore,

$$m_1 = \frac{P_1 V_1}{RT_1}, \quad m_2 = \frac{P_2 V_2}{RT_2}$$

or

$$\frac{m_2}{m_1} = \frac{P_2}{P_1}\frac{T_1}{T_2} \qquad (\because V_1 = V_2)$$

$$= \frac{8}{3}\frac{T_1}{T_2}$$

Substituting this in the energy equation, we get

$$\left(\frac{8}{3}\frac{T_1}{T_2} m_1 \quad m_1\right) C_P T_i = C_v\left(\frac{8}{3}\frac{T_1}{T_2} T_2 m_1 - m_1 T_1\right)$$

or

$$\left(\frac{8}{3}\frac{T_1}{T_2} - 1\right) C_P T_i = C_v\left(\frac{8}{3}T_1 - T_1\right)$$

or

$$\left(\frac{8}{3}\frac{T_1}{T_2} - 1\right)\gamma T_i = \left(\frac{8}{3} - 1\right)T_1 \qquad \left(\because \gamma = \frac{C_P}{C_v}\right)$$

or

$$\frac{8}{3}\frac{303.15}{T_2} - 1 \quad \left(\frac{8}{3} - 1\right)\frac{303.15}{323.15}\frac{1}{1.4}, \qquad (\because \gamma = 1.4$$

$$1.117$$

Therefore,

$$T_2 = \boxed{381.86\ \text{K}}$$

Now,

$$\frac{m_2}{m_1} = \frac{8}{3}\frac{303.15}{381.86} = 2.117$$

Therefore,

$$m_2 = 2.117 \times 5 = 10.585\ \text{kg}$$

The mass of the air added $\Delta m = m_2 - m_1 = 10.585 - 5 = \boxed{5.585\ \text{kg}}$

Note that the final temperature of the air in the tank is more than the supply line temperature. This is due to the fact that the enthalpy $h = u + Pv$ contains the flow energy Pv, and this flow energy is converted to sensible internal energy once the flow comes to rest in the control volume and this is reflected as increase in temperature.

3. **A vessel of constant volume 0.3 m^3 contains air at 1.5 bar and is connected via a valve, to a large main carrying air at a temperature of 38°C and high pressure. The valve is opened allowing air to enter the vessel and raising the pressure therein to 7.5 bar. Assuming the vessel and valve to be thermally insulated, find the mass of air entering the vessel. [APR/MAY 2015]**

$P_i = 1.5$ bar $= 150$ kPa

$P_f = 7.5$ bar $= 750$ kPa

$V_i = V_f = 0.3$ m^3

$T_f = 38 + 273 = 311$ K

Solution :

It is the case of Tank filling p.

$Q = 0$ for insulated tank.

$$m_f = \frac{P_f V_f}{R T_f}$$

$$= \frac{750 \times 0.3}{0.287 \times 311}$$

$= \_\_\_\_\_\_$ kg

$$\frac{m_f}{m_i} = \frac{P_f T_f}{P_i T_f}$$

$$Q = (m_f u_f - m_i u_i) ~~+~~ - (m_f - m_i)\left[h_f + \frac{C_i^2}{2}\right]$$

$Q = 0$

$C_1 = 0$

$$(m_f u_f - m_i u_i) - (m_f - m_i) h_f = 0$$

$$m_f u_f - m_i u_i = (m_f - m_i)(h_f)$$

$$m_i\left[\frac{m_f}{m_i} u_f - u_i\right] = m_i\left[\frac{m_f}{m_i} - 1\right] h_f$$

$$m_i\left[\frac{P_f/T_f}{P_i/T_i}c_v T_f - c_v T_i\right]$$

$$= \left[\frac{P_f/T_f}{P_i/T_i} - 1\right](C_p T_s)$$

⇒ dividing throughout by c_v

$$\left[\frac{P_f}{P_i/T_i} - T_i\right] = \left[\frac{P_f/T_f}{P_i/T_i} - 1\right](\gamma T_s)$$

sub. all the values, and find T_i.

$$m_i = \frac{P_i V_i}{R T_i}$$

$m = (m_f - m_i)$ Ans

4. Prove that energy is a property of a system.

Property is characteristic nature of substance (system) expressed in terms of some numerical values followed by some unit. During process, properties get changed.

Energy can be classified as stored energy and transit energy. Stored energy (internal energy) is a property. Heat and work are transit energies

We can prove that energy is a property - by first law of TD. As per law, on completion of a cycle the net change in property value should be zero. $(\sum Q)_{cycle} = (\sum W)_{cycle}$ or $(\sum \Delta U)_{Cycle} = 0$

by applying 1st law of TD to the processes A, B and C which are carried out between the states 1&2

$$(\sum W)_{cycle} = (\sum Q)_{cycle}$$

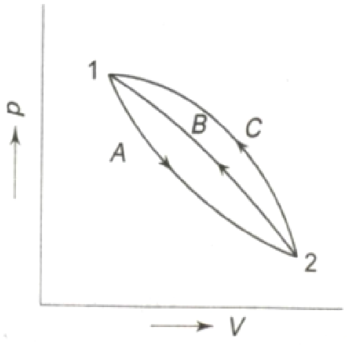
$$W_A + W_B = Q_A + Q_B$$

$$Q_A - W_A = W_B - Q_B$$

$$\Delta E_A = -\Delta E_B$$

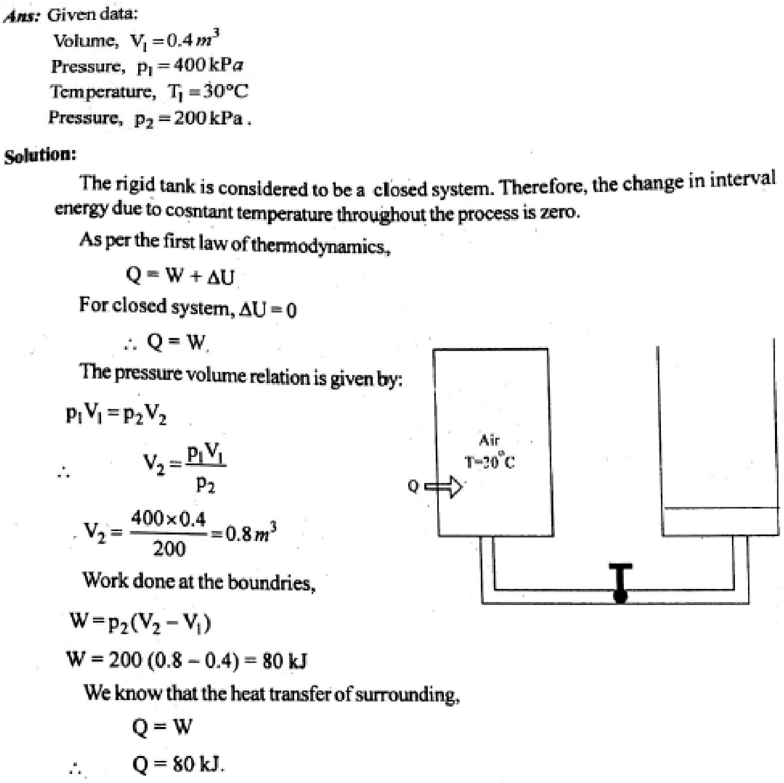
$$\therefore \Delta E_B = \Delta E_C$$

$$\Delta E_A = -\Delta E_C$$



Internal energy is not a path function. It is a point function. So, it is proved that energy is property

5. A rigid tank containing 0.4 m^3 of air at 400 kPa and 30°C is connected by a valve to a piston cylinder device with zero clearance. The mass of the piston is such that a pressure of 200kPa is required to raise the piston. The valve is opened slightly and air is allowed to flow into the cylinder until the pressure of the tank drops to 200 kPa. During this process, heat is exchanged with the surrounding such that the entire air remains at 30°C at all times. Determine the heat transfer for this process. [NOV/DEC 2010]



Ans: Given data:

Volume, $V_1 = 0.4\,m^3$
Pressure, $p_1 = 400\,kPa$
Temperature, $T_1 = 30°C$
Pressure, $p_2 = 200\,kPa$.

Solution:

The rigid tank is considered to be a closed system. Therefore, the change in interval energy due to cosntant temperature throughout the process is zero.

As per the first law of thermodynamics,

$$Q = W + \Delta U$$

For closed system, $\Delta U = 0$

$$\therefore\ Q = W$$

The pressure volume relation is given by:

$$p_1V_1 = p_2V_2$$

$$\therefore \quad V_2 = \frac{p_1V_1}{p_2}$$

$$V_2 = \frac{400 \times 0.4}{200} = 0.8\,m^3$$

Work done at the boundries,

$$W = p_2(V_2 - V_1)$$

$$W = 200\,(0.8 - 0.4) = 80\ kJ$$

We know that the heat transfer of surrounding,

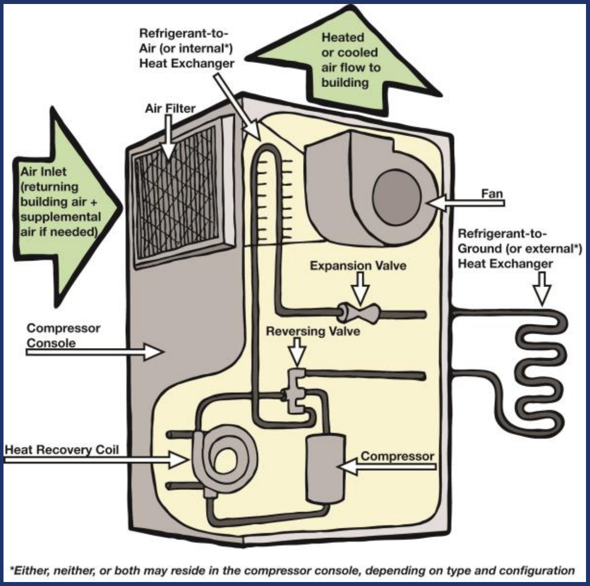
$$Q = W$$

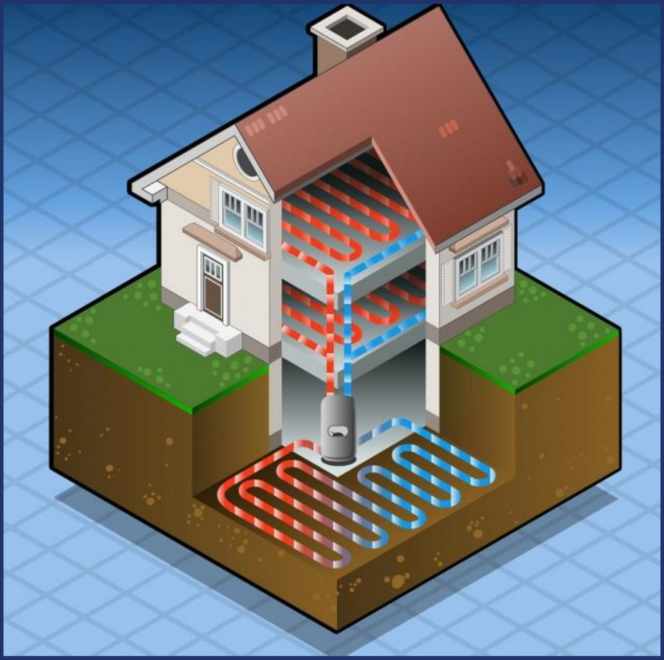
$$\therefore \quad Q = 80\ kJ.$$

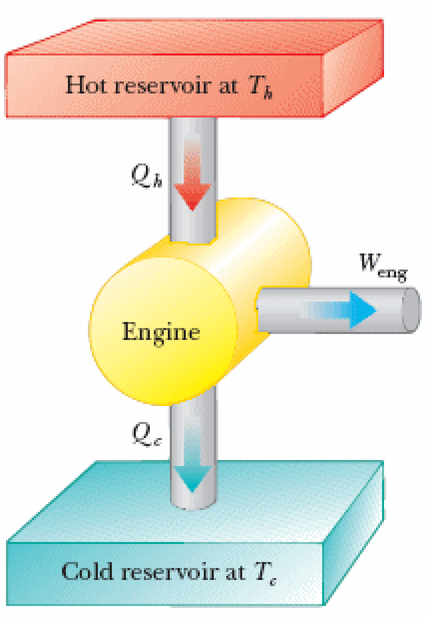
ME8391 – ENGINEERING THERMODYNAMICS

UNIT II

SECOND LAW AND AVAILABILITY ANALYSIS







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Limitations of First law of Thermodynamics.

First law does not indicate whether a process is possible or not. It does not give any information regarding the extent of conversion of heat into work.

PMM1

An imaginary machine which supplies work continuously without any other form of energy input is Perpetual Motion Machine of First kind (PMM1).

Perpetual motion machine of the second kind

Perpetual motion machine of second kind draws heat continuously from single reservoir and converts it into equivalent amount of work. Thus it gives 100% efficiency

Causes of Irreversibility

Lack of equilibrium during the process (ex: Heat transfer through a finite temperature difference)

Involvement of dissipative effects. (ex: Free Expansion)

Kelvin Planck's statement of Second Law:

- It is impossible to construct a device operating in cycle to produce net work while exchanging heat with single reservoir or reservoirs at single fixed temperature.
- It is implied from this statement that a heat engine cannot convert all the heat supplied to it into work. It has to necessarily reject heat.
- In other words, a heat engine cannot have zero heat rejection or 100% efficiency.

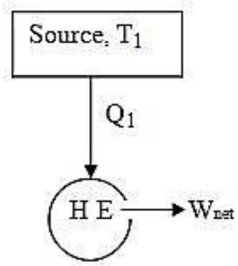


Figure 1 A heat engine which violates Kelvin-Planck's statement of II law (PMM2)*

Clausius' statement of Second Law:

It is impossible to construct a device operating in cycle to transfer heat from a lower temperature region to higher temperature without any external aid.

It is implied from this statement that a refrigerator or heat pump cannot function without work input.

Equivalence of Kelvin-Planck's and Clausius' Statements of II Law:

Both Kelvin-Planck's and Clausius' Statements of II Law are equivalent. This can not be directly proved. However, it can be shown that violation of either statement leads to the violation of other statement.

Violation of KP Statement leading to violation of Clausius' statement:

Consider a heat engine converting all the heat supplied to it into work as shown in Figure 2. This is in violation of KP statement.

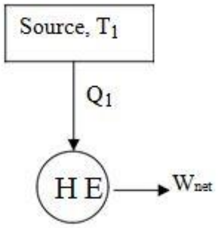


Figure 2

Now, it is possible to operate a heat pump with the work output W_{net} , from the heat engine without violating any statement of II law, as shown in Figure 3.

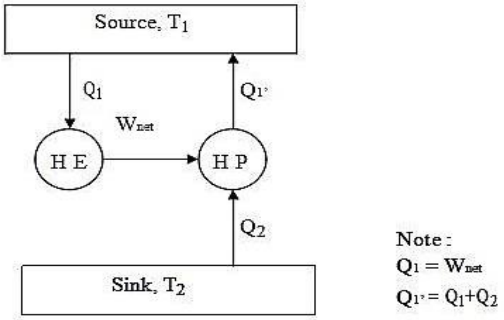


Figure 3

If the heat engine and heat pump are combined as a single system, the combined system transfers heat Q_2 from sink to source without external work input as shown in Figure 4.

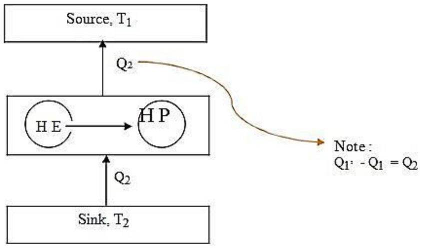


Figure 4

This is in violation of Clausius' statement of second law of thermodynamics.

Violation of Clausius' statement leading to violation of KP Statement:

Consider a heat pump transferring heat Q_1 from a sink at T_2 to a source at T_1 without any external work input as shown in Figure 5. This is in violation of Clausius' statement of second law of thermodynamics.

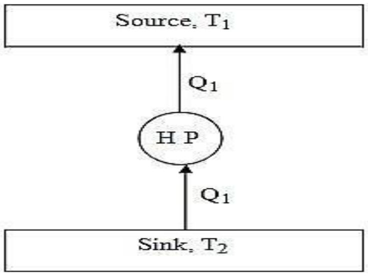


Figure 5

Now, it is possible to operate a heat engine between the source at T_1 and the sink at T_2 without violating any statement of II law, as shown in Figure 6. Let the heat engine take heat Q_1 from the source.

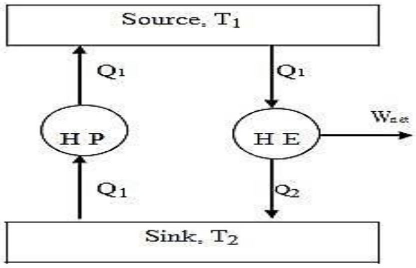


Figure 6

The heat engine and heat pump together constitute a single system, which acts as a heat engine as shown in Figure 7. As the heat transfer from the heat pump and heat transfer to the engine are of same the hot reservoir can be eliminated.

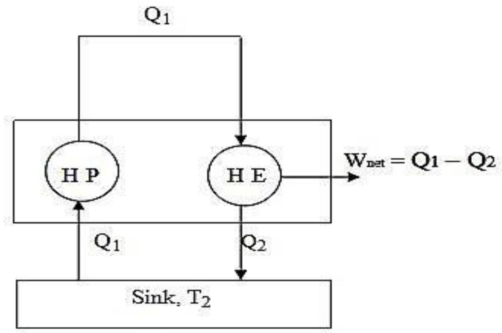


Figure 7

Now the combined system takes heat (Q_1 – Q_2) from the reservoir at T_2 (Now source) and converts it into work while exchanging with a single reservoir. This is in violation of Kelvin-Planck's statement of II law.

Carnot Theorem:

"Of all heat engines working between the same temperature levels, none has greater efficiency than a reversible engine."

Proof:

This cannot be directly proved. However, it can be shown that violation of Carnot theorem leads to the violation of second law of thermodynamics.

Consider two heat engines 'A' and 'B' operating between the same source and sink as shown in Figure 8. Let engine 'A' be reversible and 'B' irreversible. Let the heat supplied to both the engines be equal. i.e. $Q_{1A} = Q_{1B}$

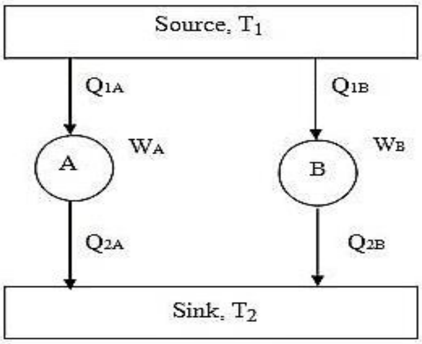


Figure 8

According to Carnot theorem, engine B, which is irreversible cannot have greater efficiency than engine A, which is reversible.

Let us assume that engine B is more efficient*.

i.e. $_B > _A$

Therefore $W_B > W_A$

Let engine 'A' be reversed as shown in Figure 9. Now the directions of Q_{1A} , Q_{2A} and W_A are reversed. As $W_B > W_A$, Engine 'B' can supply work W_A to the reversed engine 'A' (which now acts as a refrigerator/heat pump) and there is a net work out put of $(W_B - W_A)$. As $Q_{1A} = Q_{1B}$, the source can be eliminated.

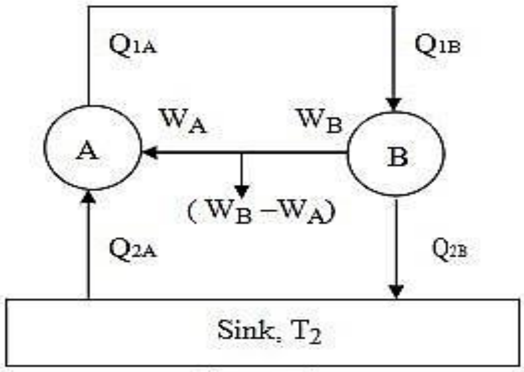


Figure 9

The heat engine 'B' and reversed heat engine 'A' together constitute a system which produces net work out put exchanging heat with a single reservoir at T_2. This is in violation of Kelvin-Planck's statement of second law of thermodynamics.

Hence the assumption that $_B >$ $_A$ is wrong. Therefore, $_A >$ $_B$.

Corollary of Carnot Theorem:

The efficiency of all reversible engines working between the same temperature levels is the same.

Proof:

Consider two reversible heat engines 'A' and 'B' operating between the same source and sink as shown in Figure 10. Let the heat supplied to both the engines be equal. i.e. $Q_{1A} = Q_{1B}$

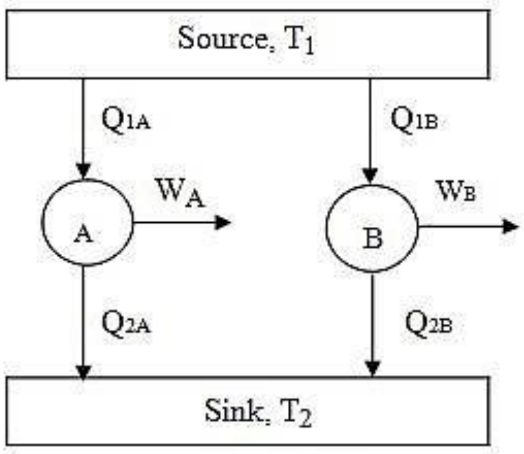


Figure 10

According to Carnot theorem, both the engines have same efficiency.

Let us assume that engine B is more efficient*.

i.e. $_B >$ $_{A,}$ Therefore $W_B > W_A$

Let engine 'A' be reversed as shown in Figure 11. Now the directions of Q_{1A} , Q_{2A} and W_A are reversed.

As $W_B > W_A$, Engine 'B' can supply work W_A to the reversed engine 'A' (which now acts as a refrigerator/heat pump) and there is a net work output of (W_B - W_A).

As $Q_{1A} = Q_{1B}$, the source can be eliminated.

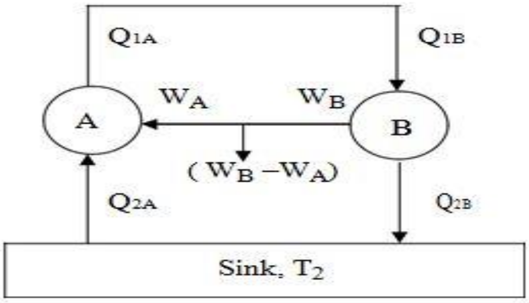


Figure 11

The reversible heat engine 'B' and reversed heat engine 'A' together constitute a system which produces net work out put exchanging heat with a single reservoir at T_2. This is in violation of Kelvin-Planck's statement of second law of thermodynamics.

Hence the assumption that $_B >\ _A$ is wrong.

Now, let us assume that engine A is more efficient*.

i.e. $_A >\ _B$

Therefore $W_A > W_B$

Let engine 'B' be reversed as shown in Figure 12. Now the directions of Q_{1B} , Q_{2B} and W_B are reversed. As $W_A > W_B$, Engine 'A' can supply work W_B to the reversed engine 'B' (which now acts as a refrigerator/heat pump) and there is a net work out put of $(W_A - W_B)$. As $Q_{1A} = Q_{1B}$, the source can be eliminated.

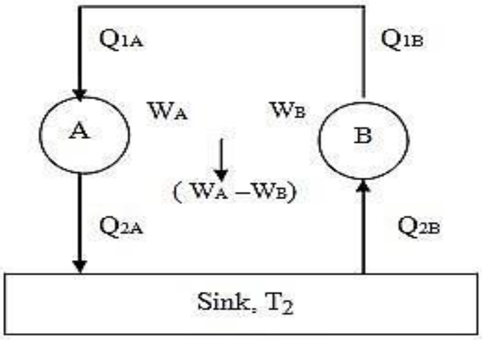
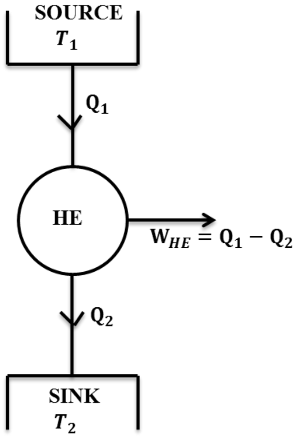
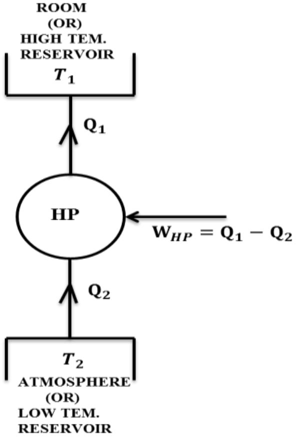
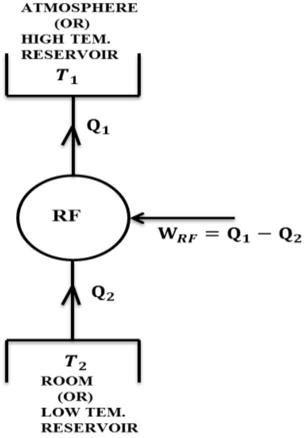
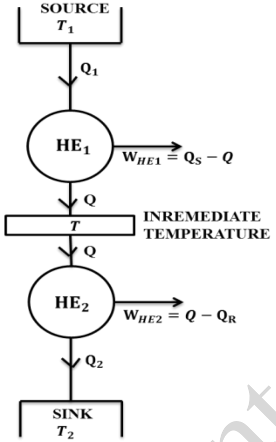
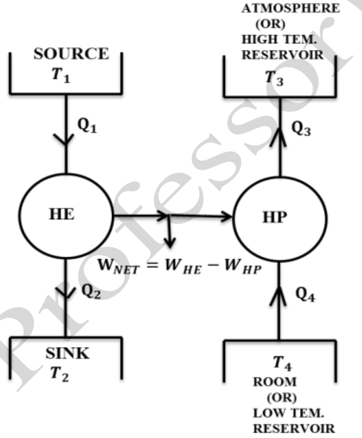
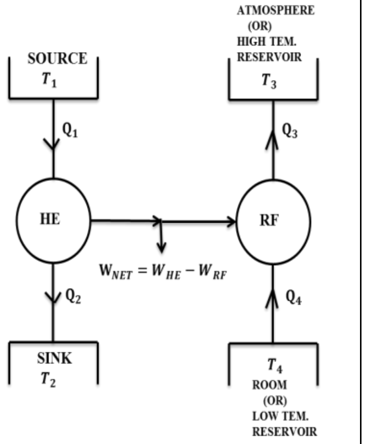


Figure 12

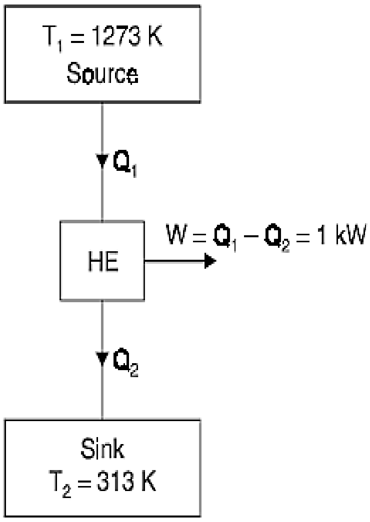
The reversible heat engine 'A' and reversed heat engine 'B' together constitute a system which produces net work out put exchanging heat with a single reservoir at T_2. This is in violation of Kelvin-Planck's statement of second law of thermodynamics.

Hence the assumption that $_A >\ _B$ is wrong. Therefore, $_A =\ _B$.

HEAT ENGINE	**HEAT PUMP**	**REFRIGERATOR**	**H.E CONNECTED IN SERIES**	**H.E OPERATES H.P**	**H.E OPERATES REFRIG.**
					
Work output $W_{HE} = Q_1 - Q_2$ **(Or)** $\eta \times Q_1$ **Heat Supplied to engine** $Q_1 = W_{HE} + Q_2$ **Heat Rejected from engine** $Q_2 = Q_1 - W_{HE}$ **Efficiency** $\boldsymbol{\eta} = \frac{W_{HE}}{Q_1} = 1 - \frac{Q_2}{Q_1}$ **For Reversible Heat Engine (Or) Carnot Engine :** **Efficiency** $\boldsymbol{\eta} = 1 - \frac{T_2}{T_1}$ **Heat Rejected from engine** $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$	**Work input** $W_{HP} = Q_1 - Q_2$ **(Or)** $\frac{Q_1}{COP}$ **Heat Supplied to room** $Q_1 = W_{HP} + Q_2$ **Heat obserbed from atm** $Q_2 = Q_1 - W_{HP}$ **Coefficient of perfom.** $COP_{HP} = \frac{Q_1}{W_{HP}} = \frac{Q_1}{Q_1 - Q_2}$ **Reversible H.P (Or) Carnot H.P :** **Efficiency** $\boldsymbol{\eta} = \frac{T_1}{T_1 - T_2}$ **Heat observed from atm.** $Q_2 = Q_1 \times \frac{T_2}{T_1}$	**Work input** $W_{REF} = Q_1 - Q_2$ **(Or)** $\frac{Q_2}{COP}$ **Heat delivered to atm** $Q_1 = W_{REF} + Q_2$ **Heat Rejected from room** $Q_2 = Q_1 - W_{REF}$ **For Actual Refrigerator:** **Coefficient of perfom.** $COP_{REF} = \frac{Q_2}{W_{REF}} = \frac{Q_2}{Q_1 - Q_2}$ **Reversible (Or) Carnot refrigerator :** **Efficiency** $COP_{REF} = \frac{T_2}{T_1 - T_2}$ **Heat Rejected from room** $Q_2 = Q_1 \times \frac{T_2}{T_1}$	**To find Intermediate Temperature** **Equal power output:** $\mathbf{W_{HE1} = W_{HE2}}$ $\eta_{HE1} \times Q_1 = \eta_{HE2} \times Q$ $\mathbf{T = \frac{T_1 + T_2}{2}}$ **Same Efficiency:** $\boldsymbol{\eta_{HE1} = \eta_{HE2}}$ $1 - \frac{T}{T_1} = 1 - \frac{T_2}{T}$ $\mathbf{T = \sqrt{T_1 \times T_2}}$	**Reversible Heat Engine:** **Efficiency** $'\boldsymbol{\eta}' = 1 - \frac{T_2}{T_1}$ **Work output** $W_{HE} = Q_1 - Q_2$ (Or) $\eta \times Q_1$ **Heat Supplied to engine** $Q_1 = W_{HE} + Q_2$ **Heat Rejected from engine** $Q_2 = Q_1 - W_{HE}$ $(OR) Q_1 \times \frac{T_2}{T_1}$ **Heat Pump:** **Coefficient of perfom.** $COP_{HP} = \frac{Q_3}{W_{HP}} = \frac{T_3}{T_3 - T_4}$ **Work input** $W_{HP} = Q_3 - Q_4$ (Or) $\frac{Q_3}{COP}$ **Heat Supplied to room** $Q_3 = W_{HP} + Q_4$ **Heat observed from atm** $Q_4 = Q_3 - W_{HP}$ $Q_4 = Q_3 \times \frac{T_4}{T_3}$	**Reversible Heat Engine:** **Efficiency** $'\boldsymbol{\eta}' = 1 - \frac{T_2}{T_1}$ **Work output** $W_{HE} = Q_1 - Q_2$ (Or) $\eta \times Q_1$ **Heat Supplied to engine** $Q_1 = W_{HE} + Q_2$ **Heat Rejected from engine** $Q_2 = Q_1 - W_{HE}$ $(OR) Q_1 \times \frac{T_2}{T_1}$ **Refrigeration:** **Coefficient of perfom.** $COP_{REF} = \frac{Q_4}{W_{REF}} = \frac{T_4}{T_3 - T_4}$ **Work input** $W_{REF} = Q_3 - Q_4$ (Or) $\frac{Q_4}{COP}$ **Heat Delivered to atm** $Q_3 = W_{REF} + Q_4$ **Heat Rejected from room** $Q_4 = Q_3 - W_{REF}$ $Q_4 = Q_3 \times \frac{T_4}{T_3}$

Problems on Heat Engine, Heat Pump and Refrigerator:

1. **A cyclic heat engine operates between a source temperature of 1000°C and a sink temperature of 40°C. what is the least rate of heat rejection per kW net output of the engine?**



Given:

T_1=1000°C+273=1273K, T_2=40°C+273=313K,

Find: (i) Rate of heat rejection per kW

Solution:

For a reversible heat engine, the rate of heat rejection will be minimum

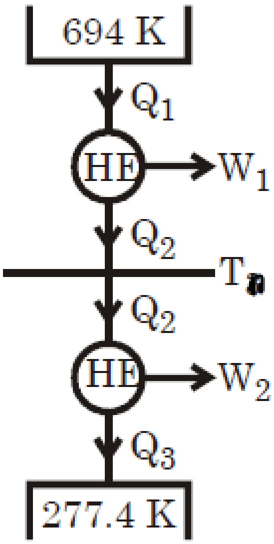
$$\eta_{max} = 1 - \frac{T_2}{T_1} \Rightarrow \eta_{max} = 1 - \frac{313}{1273} \Rightarrow \eta_{max} = 75.4\%$$

$$\eta_{max} = \frac{W_{HE}}{Q_1} \Rightarrow Q_1 = \frac{W_{HE}}{\eta_{max}} = \frac{1}{0.754} \Rightarrow \mathbf{Q_1 = 1.33\ kW}$$

The Rate of heat rejection:

$$Q_2 = Q_1 - W_{HE} \Rightarrow Q_2 = 1.33 - 1 \Rightarrow \mathbf{Q_2 = 0.33kW}$$

2. **Two reversible heat engines A and B are arranged in series, A rejecting heat directly to B. Engine A receives 200 kJ at a temperature of 421°C from a hot source, while engine B is in communication with a cold sink at a temperature of 4.4°C. If the work output of A is twice that of B, find (a) The intermediate temperature between A and B, (b) The efficiency of each engine (c) The heat rejected to the cold sink**



Given:

T_1=421°C+273=694K, T_3=4.4°C+273=277.4K, $Q_1 = 200$kJ

Find: (i) Rate of heat rejection per kW

Solution:

(a) The intermediate temperature between A and B

As the work output of A is twice that of B

$$W_1 = 2W_2 \Rightarrow Q_1 - Q_2 = 2\,(Q_2 - Q_3) \Rightarrow T_1 - T = 2\,(T - T_3)$$

$$694 - T = 2\,(T - 277.4) \Rightarrow 694 - T = 2\,T - 554.8) \Rightarrow \mathbf{T_2 = 416.27K}$$

(b) The efficiency of each engine

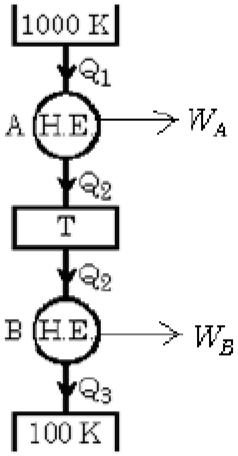
$$\eta_1 = 1 - \frac{T}{T_1} \Rightarrow \eta_1 = 1 - \frac{416.27}{694} \Rightarrow \mathbf{\eta_1 = 40.02\%}$$

$$\eta_2 = 1 - \frac{T_3}{T} \Rightarrow \eta_2 = 1 - \frac{277.4}{416.27} \Rightarrow \mathbf{\eta_2 = 33.36\%}$$

(c) The heat rejected to the cold sink

$$\frac{Q_3}{Q_1} = \frac{T_3}{T_1} \Rightarrow Q_3 = \frac{T_3}{T_1} \times Q_1 \Rightarrow Q_3 = \frac{277.4}{694} \times 200 \Rightarrow \mathbf{Q_3 = 79.94kJ}$$

4. **Two Carnot engines A and B are connected in series between two thermal reservoirs maintained at 1000 K and 100 K respectively. Engine A receives 1680 kJ of heat from the high-temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low-temperature reservoir. If engines A and B have equal thermal efficiencies, determine (a) The heat rejected by engine B , (b) The temperature at which heat is rejected by engine, A (c) The work done during the process by engines, A and B respectively. If engines A and B deliver equal work, determine, (d) The amount of heat taken in by engine B, (e) The efficiencies of engines A and B**



Given:

T_1=1000K, T_3=100K, $Q_1 = 1680\text{kJ}$

Find: (a) Q_3 (b) T (c) W_A , W_B (d) T if $W_A = W_B$ (e) η_1, η_2

Solution:

Engines A and B have equal thermal efficiencies:

(a) The heat rejected by engine B

$$\frac{Q_3}{Q_1} = \frac{T_3}{T_1} \Rightarrow Q_3 = \frac{T_3}{T_1} \times Q_1 \Rightarrow Q_3 = \frac{100}{1000} \times 1680 \Rightarrow \mathbf{Q_3 = 168kJ}$$

(b) The temperature at which heat is rejected by engine, A

$$\eta_1 = \eta_2 \Rightarrow 1 - \frac{T}{T_1} = 1 - \frac{T_3}{T} \Rightarrow \frac{T}{1000} = \frac{100}{T} \Rightarrow \mathbf{T = 316.3K}$$

As the work output of A is twice that of B

(c) The work done during the process by engines, A and B

$$\frac{Q_2}{Q_1} = \frac{T}{T_1} \Rightarrow Q_2 = \frac{T}{T_1} \times Q_1 \Rightarrow Q_2 = \frac{316.3}{1000} \times 1680 \Rightarrow \mathbf{Q_2 = 531.26kJ}$$

$$W_A = Q_1 - Q_2 \Rightarrow W_A = 1680 - 531.26 \Rightarrow \mathbf{W_A = 1148.74kJ}$$

$$W_B = Q_2 - Q_3 \Rightarrow W_B = 531.26 - 168 \Rightarrow \mathbf{W_B = 363.26kJ}$$

Engines A and B deliver equal work:

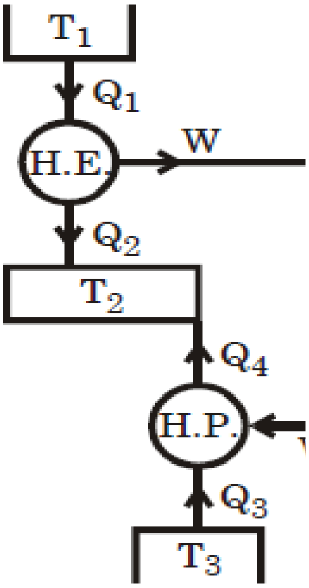
(d) The amount of heat taken in by engine B

$$T = \frac{T_1+T_2}{2} \Rightarrow T = \frac{1000+100}{2} \Rightarrow \mathbf{T = 550K}$$

$$\eta_1 = 1 - \frac{T}{T_1} \Rightarrow \eta_1 = 1 - \frac{550}{1000} \Rightarrow \mathbf{\eta_1 = 45\%}$$

$$\eta_2 = 1 - \frac{T_3}{T} \Rightarrow \eta_2 = 1 - \frac{100}{550} \Rightarrow \mathbf{\eta_2 = 81.82\%}$$

5. **A heat engine is used to drive a heat pump. The heat transfers from the heat engine and from the heat pump are used to heat the water circulating through the radiators of a building. The efficiency of the heat engine is 27% and the COP of the heat pump is 4. Evaluate the ratio of the heat transfer to the circulating water to the heat transfer to the heat engine.**



Given: $\eta_1 = 27\%$, COP=4

Find: (i) The ratio of the heat transfer

Solution:

The Heat Engine:

$$\eta_1 = 1 - \frac{Q_2}{Q_1} \Rightarrow 0.27 = 1 - \frac{Q_2}{Q_1} \Rightarrow \frac{Q_2}{Q_1} = 0.73 \Rightarrow \mathbf{Q_2 = 0.73Q_1}$$

$$\eta_1 = \frac{W_{HE}}{Q_1} \Rightarrow 0.27 = \frac{W_{HE}}{Q_1} \Rightarrow \mathbf{W_{HE} = 0.27Q_1}$$

The Heat Pump:

$$COP_{HP} = \frac{Q_4}{W_{HE}} \Rightarrow 4 = \frac{Q_4}{W_{HE}} \Rightarrow Q_4 = 4W_{HE} \Rightarrow \mathbf{Q_4 = 1.08Q_1}$$

Heat transfer to the circulating water:

$$Q_2 + Q_4 \Rightarrow 0.73Q_1 = 1.08Q_1 \Rightarrow \mathbf{Q_2 + Q_4 = 1.81Q_1}$$

$$= \frac{\text{The heat transfer to the circulating water}}{\text{The heat transfer to the heat engine}} \Rightarrow = \mathbf{1.81}$$

6. **A heat pump is run by a reversible heat engine operating between reservoirs at 800°C and 50°C. The heat pump working on Carnot cycle picks up 15 kW heat from reservoir at 10°C and delivers it to a reservoir at 50°C. The reversible engine also runs a machine that needs 25 kW. Determine the heat received from highest temperature reservoir and heat rejected to reservoir at 50°C.**

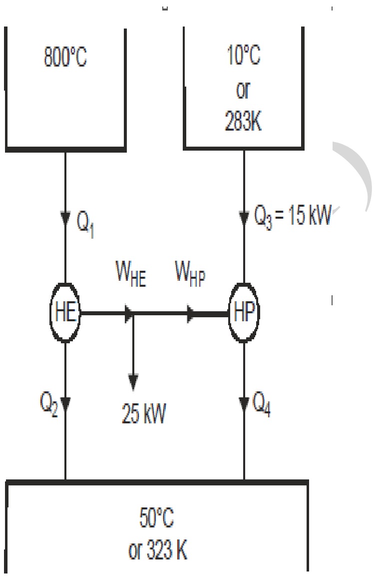
Given:

T_1=1073K, T_2= T_4=323K, T_3=283K, $Q_3 = 15\text{kW}$

Find: (a) Q_1 (b) $Q_2 + Q_4$

Solution:

Heat Pump:



$$\text{COP}_{HP} = \frac{T_4}{T_4 - T_3} \Rightarrow \text{COP}_{HP} = \frac{323}{323-283} \Rightarrow \mathbf{COP_{HP} = 8.075}$$

$$\text{COP}_{HP} = \frac{Q_4}{Q_4 - Q_3} \Rightarrow 8.075 = \frac{Q_4}{Q_4 - 15} \Rightarrow 8.075Q_4 - 121.13 = Q_4$$

$$\Rightarrow \mathbf{Q_4 = 17.12kW}$$

$$W_{HP} = Q_4 - Q_3 \Rightarrow W_{HP} = 17.12 - 15 \Rightarrow \mathbf{W_{HP} = 2.12kW}$$

Heat Engine:

$$W_{HE} = W_{HP} + 25 \Rightarrow W_{HE} = 2.12 + 25 \Rightarrow \mathbf{W_{HE} = 27.12kW}$$

$$\eta_{HE} = 1 - \frac{T_2}{T_1} \Rightarrow \eta_{HE} = 1 - \frac{323}{1073} \Rightarrow \mathbf{\eta_{HE} = 69.9\%}$$

$$\eta_1 = \frac{W_{HE}}{Q_1} \Rightarrow 0.699 = \frac{27.12}{Q_1} \Rightarrow \mathbf{Q_1 = 38.8kW}$$

$$W_{HE} = Q_1 - Q_2 \Rightarrow 27.12 = 38.8 - Q_2 \Rightarrow \mathbf{Q_2 = 11.68kW}$$

Heat rejected to reservoir at 50 °C

$$= Q_2 + Q_4 \Rightarrow = 11.68 + 17.12 \Rightarrow \mathbf{Q_2 + Q_4 = 28.8kW}$$

7. **A heat pump working on the Carnot cycle takes in heat from a reservoir at 5°C and delivers heat to a reservoir at 60°C. The heat pump is driven by a reversible heat engine which takes in heat from a reservoir at 840°C and rejects heat to a reservoir at 60°C. The reversible heat engine also drives a machine that absorbs 30 kW. If the heat pump extracts 17 kJ/s from the 5°C reservoir, determine (a) The rate of heat supply from the 840°C source (b) The rate of heat rejection to the 60°C sink.**

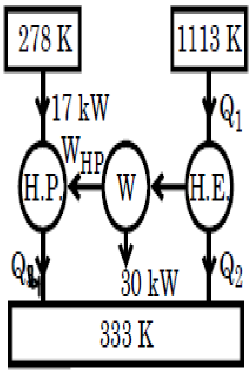
Given:

T_1=1113K, T_2= T_4=333K, T_3=278K, $Q_3 = 17\text{kW}$

Find: (a) Q_1 (b) $Q_2 + Q_4$

Solution:

Heat Pump:



$$\text{COP}_{HP} = \frac{T_4}{T_4 - T_3} \Rightarrow \text{COP}_{HP} = \frac{333}{333-278} \Rightarrow \mathbf{COP_{HP} = 6.055}$$

$$\text{COP}_{HP} = \frac{Q_4}{Q_4 - Q_3} \Rightarrow 6.055 = \frac{Q_4}{Q_4 - 17} \Rightarrow 6.055Q_4 - 102.94 = Q_4 \Rightarrow \mathbf{Q_4 = 20.36kW}$$

$$W_{HP} = Q_4 - Q_3 \Rightarrow W_{HP} = 20.36 - 17 \Rightarrow \mathbf{W_{HP} = 3.36kW}$$

Heat Engine:

$W_{HE} = W_{HP} + 30$ ⇨ $W_{HE} = 3.36 + 30$ ⇨ $\mathbf{W_{HE} = 33.36kW}$

$\eta_{HE} = 1 - \frac{T_2}{T_1}$ ⇨ $\eta_{HE} = 1 - \frac{333}{1113}$ ⇨ $\boldsymbol{\eta_{HE} = 70.08\%}$

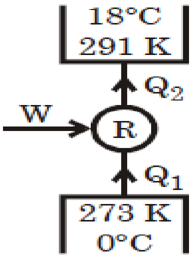
$\eta_1 = \frac{W_{HE}}{Q_1}$ ⇨ $0.708 = \frac{33.36}{Q_1}$ ⇨ $\mathbf{Q_1 = 47.12kW}$

$W_{HE} = Q_1 - Q_2$ ⇨ $33.36 = 47.12 - Q_2$ ⇨ $\mathbf{Q_2 = 13.76kW}$

Heat rejected to reservoir at 60 °C

$= Q_2 + Q_4$ ⇨ $= 13.76 + 20.36$ ⇨ $\mathbf{Q_2 + Q_4 = 34.12kW}$

8. **An ice-making plant produces ice at atmospheric pressure and at 0°C from water. The mean temperature of the cooling water circulating through the condenser of the refrigerating machine is 18°C. Evaluate the minimum electrical work in kWh required to produce 1 tonne of ice (The enthalpy of fusion of ice at atmospheric pressure is 333.5 kJ/kg).**



Given:

T_1=273K, T_2= 291K, $Q_1 = 333.5 \times 1000$

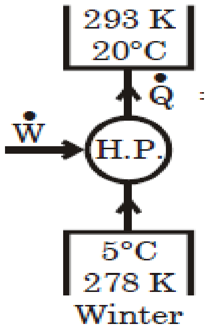
Find: (a) W_{min}

Solution:

$COP_{max} = \frac{T_1}{T_2 - T_1}$ ⇨ $COP_{max} = \frac{273}{291-273}$ ⇨ $\mathbf{COP_{max} = 15.2}$

$COP_{min} = \frac{Q}{W_{min}}$ ⇨ $15.2 = \frac{1000 \times 333.5}{W_{min}}$ ⇨ $\mathbf{W_{min} = 21.989MJ}$

9. **A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The interior temperature is to be maintained at 20°C. Heat transfer through the walls and roof is estimated to be 0.525 kJ/s per degree temperature difference between the inside and outside. (a) If the outside temperature in winter is 5°C, what is the minimum power required to drive the heat pump? (b) If the power output is the same as in part (a), what is the maximum outer temperature for which the inside can be maintained at 20°C?**



Given: T_1=293K, q=0.525kJ/s°C

Find: (a) W_{min} for heat pump if outside temperature in winter is 5°C

(b) The maximum outer temperature for which the inside can be maintained at 20°C (if power output is same for part a &b)

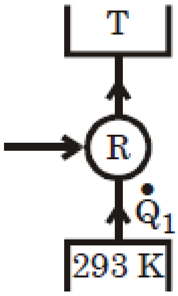
Solution:

Heat pump:

Estimated Heat Rate: $\dot{Q} = q \times (T_2 - T_1)$ ⇨ $\dot{Q} = 0.525 \times (20 - 5)$ ⇨ $\mathbf{\dot{Q} = 7.875kJ/s}$

$COP_{HP} = \frac{T_1}{T_2 - T_1}$ ⇨ $COP_{Hp} = \frac{293}{293-278}$ ⇨ $\mathbf{COP_{HP} = 19.53}$

$COP_{HP} = \frac{Q}{\dot{W}_{min}}$ ⇨ $19.53 = \frac{7.875}{W_{min}}$ ⇨ $\mathbf{\dot{W}_{min} = 403W}$



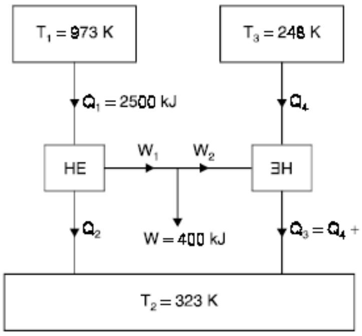
Refrigerator:

Heat Rate: $\dot{Q}_1 = q \times (T - T_1)$ ⇨ $\dot{Q}_1 = 0.525 \times (T - 293)$

$COP_{HP} = \frac{T_1}{T - T_1}$ ⇨ $COP_{Hp} = \frac{293}{T-293}$

$COP_{HP} = \frac{Q}{W_{min}}$ ⇨ $\frac{293}{T-293} = \frac{0.525 \times (T-293)}{0.403}$ ⇨ $\mathbf{T = 308K}$

10.A reversible heat engine operates between two reservoirs at temperatures 700°C and 50°C. The engine drives a reversible refrigerator which operates between reservoirs at temperatures of 50°C and – 25°C. The heat transfer to the engine is 2500 kJ and the net work output of the combined engine refrigerator plant is 400 kJ.(i) Determine the heat transfer to the refrigerant and the net heat transfer to the reservoir at 50°C (ii) Reconsider (i) given that the efficiency of the heat engine and the C.O.P. of the refrigerator are each 45 per cent of their maximum possible values.



Given:

T_1=973K, T_2= T_4=323K, T_3=248K, $Q_1 = 2500$kJ

Find: (a) Q_1 (b) $Q_2 + Q_4$

Solution:

Heat Engine:

$\eta_{HE} = 1 - \frac{T_2}{T_1} \Rightarrow \eta_{HE} = 1 - \frac{323}{973} \Rightarrow \boldsymbol{\eta_{HE} = 66.8\%}$

$\eta_1 = \frac{W_{HE}}{Q_1} \Rightarrow 0.668 = \frac{W_{HE}}{2500} \Rightarrow \mathbf{W_{HE} = 1670kJ}$

$W_{HE} = Q_1 - Q_2 \Rightarrow 1670 = 2500 - Q_2 \Rightarrow \mathbf{Q_2 = 830kJ}$

Refrigerator:

$W_{HE} = W + W_{ref} \Rightarrow 1670 = 400 + W_{ref} \Rightarrow \mathbf{W_{ref} = 1270kJ}$

$COP_{ref} = \frac{T_3}{T_4 - T_3} \Rightarrow COP_{ref} = \frac{248}{323-248} \Rightarrow \mathbf{COP_{ref} = 3.306}$

$COP_{ref} = \frac{Q_4}{W_{ref}} \Rightarrow 3.306 = \frac{Q_4}{1270} \Rightarrow \mathbf{Q_4 = 4198.6kJ}$

$W_{ref} = Q_3 - Q_4 \Rightarrow 1270 = Q_3 - 4198.6 \Rightarrow \mathbf{Q_3 = 5468.6kJ}$

Heat rejected to reservoir at 50 °C

$= Q_2 + Q_3 \Rightarrow = 830 + 5468.6 \Rightarrow \mathbf{Q_2 + Q_3 = 62986kJ}$

If the efficiency of the actual heat engine and COP is 45%

$\eta_{HE} = 45\% \times \eta_{max} \Rightarrow \eta_{HE} = 0.45 \times 0.668 \Rightarrow \boldsymbol{\eta_{HE} = 30\%}$

$\eta_1 = \frac{W_{HE}}{Q_1} \Rightarrow 0.3 = \frac{W_{HE}}{2500} \Rightarrow \mathbf{W_{HE} = 750kJ}$

$W_{HE} = Q_1 - Q_2 \Rightarrow 750 = 2500 - Q_2 \Rightarrow \mathbf{Q_2 = 1750kJ}$

Refrigerator:

$W_{HE} = W + W_{ref} \Rightarrow 750 = 400 + W_{ref} \Rightarrow \mathbf{W_{ref} = 350kJ}$

$COP_{ref} = 3.306 \times 0.45 \Rightarrow \mathbf{COP_{ref} = 1.48}$

$COP_{ref} = \frac{Q_4}{W_{ref}} \Rightarrow 1.48 = \frac{Q_4}{350} \Rightarrow \mathbf{Q_4 = 518kJ}$

$W_{ref} = Q_3 - Q_4 \Rightarrow 350 = Q_3 - 518 \Rightarrow \mathbf{Q_3 = 868kJ}$

Heat rejected to reservoir at 50 °C

$= Q_2 + Q_3 \Rightarrow = 1750 + 868 \Rightarrow \mathbf{Q_2 + Q_3 = 2618kJ}$

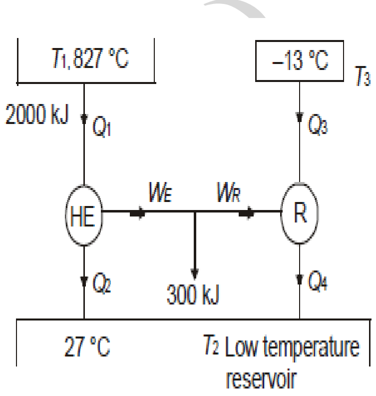
11.A reversible heat engine operates between two reservoirs at 827°C and 27°C. Engine drives a Carnot refrigerator maintaining –13°C and rejecting heat to reservoir at 27°C. Heat input to the engine is 2000 kJ and the net work available is 300 kJ. How much heat is transferred to refrigerant and total heat rejected to reservoir at 27°C.

Given: T_1=827°C, T_2= T_4=27°C, T_3= –13°C, $Q_1 = 2000\text{kJ}$

Find: (a) Q_4 (b) $Q_2 + Q_4$

Solution:

Heat Engine:



$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow Q_2 = \frac{T_2}{T_1} \times Q_1 = \frac{300}{1100} \times 2000 \Rightarrow \mathbf{Q_2 = 545.45kJ}$

$W_{HE} = Q_1 - Q_2 \Rightarrow W_{HE} = 2000 - 545.45 \Rightarrow W_{HE} = \mathbf{1454.55kJ}$

Refrigerator:

$W_{HE} = W + W_{ref} \Rightarrow 1454.55 = 300 + W_{ref} \Rightarrow \mathbf{W_{ref} = 1154.55kJ}$

$COP_{ref} = \frac{T_3}{T_4 - T_3} \Rightarrow COP_{ref} = \frac{260}{300-260} \Rightarrow \mathbf{COP_{ref} = 6.5}$

$COP_{ref} = \frac{Q_3}{W_{ref}} \Rightarrow 6.5 = \frac{Q_3}{1154.55} \Rightarrow \mathbf{Q_3 = 7504.58kJ}$

$W_{ref} = Q_4 - Q_3 \Rightarrow 1154.55 = Q_4 - 7504.58 \Rightarrow \mathbf{Q_4 = 8659.13kJ}$

Heat rejected to reservoir

$= Q_2 + Q_4 \Rightarrow = 545.45 + 8659.13 \Rightarrow \mathbf{Q_2 + Q_4 = 9204.58kJ}$

12.(a) A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 25°C. If the heat removal rate from the refrigerator is 1440 kJ/min, determine the C.O.P. of the machine and work input required.

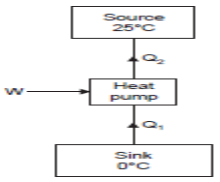
Given:

T_1=25°C+273=298K,T_2=0°C+273=273K,

$Q_1 = 1440\text{kJ/min}$= 24kJ/s

Find: (a) COP & W_{HP}**Find:** (a) Q_4 (b) $Q_2 + Q_4$

Solution:

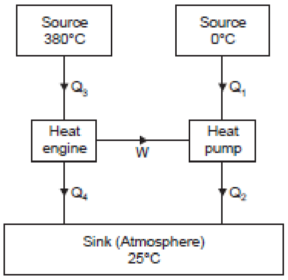


$COP_{HP} = \frac{T_1}{T_2 - T_1} \Rightarrow COP_{Hp} = \frac{298}{298-273} \Rightarrow \mathbf{COP_{HP} = 11.92}$

$COP_{HP} = \frac{Q}{\dot{W}_{HP}} \Rightarrow 11.92 = \frac{24}{W_{HP}} \Rightarrow \mathbf{\dot{W}_{HP} = 2.2kW}$

$W_{HP} = Q_2 - Q_1 \Rightarrow 2.2 = Q_2 - 24 \Rightarrow \mathbf{Q_2 = 26.2kJ/s}$

(b) If the required input to run the pump is developed by a reversible engine which receives heat at 380°C and rejects heat to atmosphere, then determine the overall C.O.P. of the system.



Heat Engine: $W_{HE} = W_{HP}$

$\frac{Q_3}{Q_4} = \frac{T_3}{T_4} \Rightarrow \frac{Q_4 + W_{HE}}{Q_4} = \frac{T_3}{T_4} \Rightarrow \frac{Q_4 + 2.2}{Q_4} = \frac{653}{298} \Rightarrow \mathbf{Q_4 = 1.847kJ/s}$

$W_{HE} = Q_3 - Q_4 \Rightarrow 2.2 = Q_3 - 1.847 \Rightarrow \mathbf{Q_3 = 4.047kJ/s}$

Heat Pump

$COP_{HP} = \frac{Q_1}{Q_3} \Rightarrow COP_{HP} = \frac{24}{1154.55} \Rightarrow COP_{HP} = \mathbf{5.98}$

The overall COP

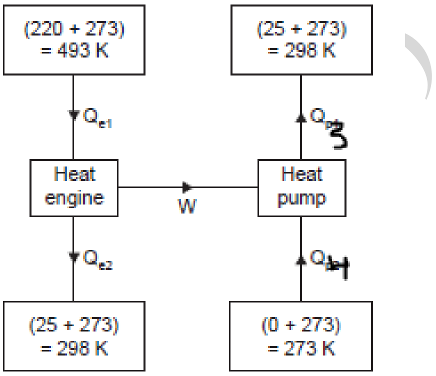
$= \frac{Q_2 + Q_4}{Q_3} \Rightarrow = \frac{26.2 + 1.847}{4.047} \Rightarrow = 6.98$

13. An ice plant working on a reversed Carnot cycle heat pump produces 15 tonnes of ice per day. The ice is formed from water at 0°C and the formed ice is maintained at 0°C. The heat is rejected to the atmosphere at 25°C. The heat pump used to run the ice plant is coupled to a Carnot engine which absorbs heat from a source which is maintained at 220°C by burning liquid fuel of 44500 kJ/kg calorific value and rejects the heat to the atmosphere. Determine :(i) Power developed by the engine (ii) Fuel consumed per hour. Take enthalpy of fusion of ice = 334.5 kJ/kg.

Given: T_1=493K, T_2= T_3=298K, T_4=273K,

Find: (a) W (b) Fuel consumed per hour

Solution:



Heat Pump:

$$COP_{HP} = \frac{T_3}{T_3 - T_4} \Rightarrow COP_{HP} = \frac{298}{298-273} \Rightarrow \mathbf{COP_{HP} = 11.92}$$

$$Q_4 = \frac{15\times1000\times334.5}{24\times60} \Rightarrow \mathbf{Q_4 = 58.07\ kJ/s}$$

$$COP_{HP} = \frac{Q_3}{Q_3 - Q_4} \Rightarrow 11.92 = \frac{Q_3}{Q_3 - 58.07} \Rightarrow 11.92Q_3 - 692.23 = Q_3$$

$$\Rightarrow \mathbf{Q_3 = 63.39\ kJ/s}$$

$$W_{HP} = Q_3 - Q_4 \Rightarrow W_{HP} = 63.39 - 58.07 \Rightarrow \mathbf{W_{HP} = 5.32kW}$$

Heat Engine:

$$\eta_{HE} = 1 - \frac{T_2}{T_1} \Rightarrow \eta_{HE} = 1 - \frac{298}{493} \Rightarrow \mathbf{\eta_{HE} = 39.55\%}$$

$$\eta_1 = \frac{W_{HE}}{Q_1} \Rightarrow 0.3955 = \frac{5.32}{Q_1} \Rightarrow \mathbf{Q_1 = 13.45kW}$$

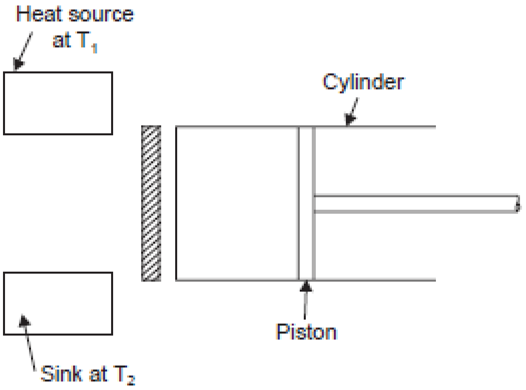
$$Q_1 = 13.45 \times 60 \times 60 \Rightarrow \mathbf{Q_1 = 48168\ kJ/hr}$$

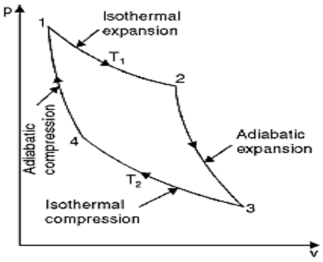
Quantity of fuel consumed/hour

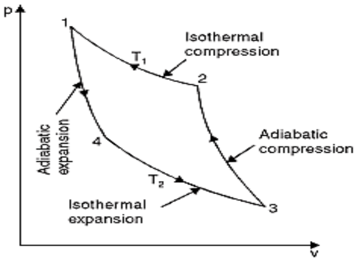
$$= \frac{\text{Heat Supplied to engine}}{\text{CV}} \Rightarrow = \frac{48168}{44500} \Rightarrow \mathbf{= 1.082kg/h}$$

14. Explain Carnot cycle with neat sketches.

- Any fluid may be used to operate the Carnot cycle which is performed in an engine cylinder the head of which is supposed alternatively to be perfect conductor or a perfect insulator of a heat.
- Heat is caused to flow into the cylinder by the application of high temperature energy source to the cylinder head during expansion, and to flow from the cylinder by the application of a lower temperature energy source to the head during compression.







The assumptions made for describing the working of the Carnot engine are as follows:

i. The piston moving in a cylinder does not develop any friction during motion.
ii. The walls of piston and cylinder are considered as perfect insulators of heat.
iii. The cylinder head is so arranged that it can be a perfect heat conductor or perfect heat insulator.
iv. The transfer of heat does not affect the temperature of source or sink.
v. Working medium is a perfect gas and has constant specific heat.
vi. Compression and expansion are reversible.

Following are the four stages of Carnot cycle :

Process 1-2: Hot energy source is applied. Heat Q_1 is taken in whilst the fluid expands isothermally and reversibly at constant high temperature T_1.

Process 2-3: The cylinder becomes a perfect insulator so that no heat flow takes place. The fluid expands adiabatically and reversibly whilst temperature falls from T_1 to T_2.

Process 3-4: Cold energy source is applied. Heat Q_2 flows from the fluid whilst it is compressed isothermally and reversibly at constant lower temperature T_2.

Process 4-1: Cylinder head becomes a perfect insulator so that no heat flow occurs. The compression is continued adiabatically and reversibly during which temperature is raised from T_2 to T_1.

The work delivered from the system during the cycle is represented by the enclosed area of the cycle. Again for a closed cycle, according to first law of the thermodynamics the work obtained is equal to the difference between the heat supplied by the source (Q_1) and the heat rejected to the sink (Q_2).

$$\therefore W = Q_1 - Q_2$$

Also, thermal efficiency, $\eta_{th} = \frac{\text{Work done}}{\text{Heat supplied by the source}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

Such an engine since it consists entirely of reversible processes, can operate in the reverse direction so that it follows the cycle shown in Fig and operates as a heat pump. Q_2 is being taken in at the lower temperature T_2 during the isothermal expansion (process 4-3) and heat Q_1 is being rejected at the upper temperature T_1 (process 2-1). Work W will be needed to drive the pump. Again, the enclosed area represents this work which is exactly equal to that flowing from it when used as engine.

The Carnot cycle cannot be performed in practice because of the following reasons :

1. It is impossible to perform a frictionless process.
2. It is impossible to transfer the heat without temperature potential.
3. Isothermal process can be achieved only if the piston moves very slowly to allow heat transfer so that the temperature remains constant. Adiabatic process can be achieved only if the piston moves as fast as possible so that the heat transfer is negligible due to very short time available. The isothermal and adiabatic processes take place during the same stroke therefore the piston has to move very slowly for part of the stroke and it has to move very fast during remaining stroke. This variation of motion of the piston during the same stroke is not possible.

ENTROPY:

Change in entropy (S_2-S_1) is:

$$\int_1^2 dS \geq \int_1^2 \frac{dQ}{T}$$

> sign holds good for irreversible process and = sign for reversible process.

Principle of increase of entropy

$$\Delta S_{universe} > 0$$

The change in entropy of a thermodynamic universe (system and surroundings) is always positive.

Change in Entropy :

Process	**Equation follows**	**Change In Entropy**
Constant Volume	(V=C)	$\Delta S = mC_v \ln\left(\frac{T_2}{T_1}\right)$
Constant Pressure	(p=C)	$\Delta S = mC_P \ln\left(\frac{T_2}{T_1}\right)$
Isothermal	(pV=C)	$\Delta S = mR \ln\left(\frac{V_2}{V_1}\right)$
Adiabatic Process	$(pV^\gamma = C)$	$\Delta S = 0$
Polytropic Process	$(pV^n = C)$	$\Delta S = mC_n \ln\left(\frac{T_2}{T_1}\right)$

Entropy for all process can also be calculated by:

$$\Delta S = mC_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right),$$

$$\Delta S = mC_P \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right) ,$$

$$\Delta S = mC_P \ln\left(\frac{V_2}{V_1}\right) + mC_v \ln\left(\frac{P_2}{P_1}\right)$$

For Ideal Gas (Air):

Specific Heat ‘C_p’ = 1.005 kJ/kg.K, ‘C_v’ = 0.718 kJ/kg.K

For Water :

Specific Heat ‘C_p’ = 4.186 kJ/kg.K,

1. **Air at 20°C and 1.05 bar occupies 0.025 m^3. The air is heated at constant volume until the pressure is 4.5 bar, and then cooled at constant pressure back to original temperature.**
Calculate :(i) The net heat flow from the air.(ii) The net entropy change.

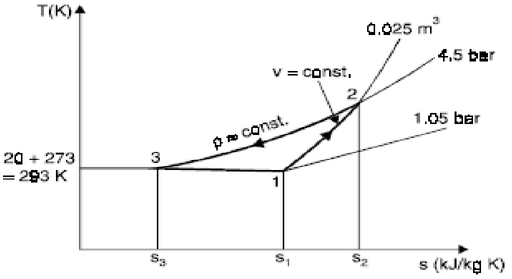
Given:

Temperature, $T_1 = 20 + 273 = 293$ K

Volume, $V_1 = V_3 = 0.025$ m^3

Pressure, $p_1 = 1.05$ bar $= 1.05 \times 10^5$ N/m^2

Pressure, $p_2 = 4.5$ bar $= 4.5 \times 10^5$ N/m^2.



Find: (i) Net heat flow (ii) Net entropy change

Solution:`

The mass of gas :

$$m = \frac{P_1V_1}{RT_1} \Rightarrow m = \frac{(1.05 \times 10^5) \times 0.025}{287 \times 293} \Rightarrow \mathbf{m = 0.0312kg}$$

Process1-2:Constant Volume Process

$$\text{At } \frac{p}{T} = C \Rightarrow p_1T_2 = p_2T_1 \Rightarrow T_2 = \frac{P_2}{P_1} \times T_1 = \frac{4.5}{1.05} \times 293 \Rightarrow \mathbf{T_2 = 1255.7K}$$

$$\mathbf{Q_{12}} = mC_v(T_2 - T_1) \Rightarrow \mathbf{Q_{12}} = 0.0312 \times 0.718(1255.7 - 293) \Rightarrow \mathbf{Q_{12} = 21.56kJ}$$

$$\Delta\mathbf{S_{12}} = mC_v \ln\left(\frac{T_2}{T_1}\right) \Rightarrow \Delta\mathbf{S_{12}} = 0.0312 \times 0.718 \ln\left(\frac{1255.7}{293}\right) \Rightarrow \Delta\mathbf{S_{12} = 0.0326kJ/K}$$

Process 2-3: Constant pressure process

$$Q_{23} = mC_p(T_3 - T_2) \Rightarrow Q_{23} = 0.0312 \times 1.005 \times (293 - 1255.7) \Rightarrow \mathbf{Q_{23} = -30.18kJ}$$

$$\Delta\mathbf{S_{23}} = mC_v \ln\left(\frac{T_2}{T_3}\right) \Rightarrow \Delta\mathbf{S_{23}} = 0.0312 \times 1.005 \ln\left(\frac{1255.7}{293}\right) \Rightarrow \Delta\mathbf{S_{23} = 0.0456kJ/K}$$

(i) The net heat flow

$$Q = Q_{12} + Q_{23} \Rightarrow Q = 21.56 - 30.18 \Rightarrow \mathbf{Q = -8.62kJ}$$

(ii) The Net decrease in entropy

$$\Delta S = S_{12} + S_{23} \Rightarrow \Delta S = 0.0326 - 0.0456 \Rightarrow \mathbf{\Delta S = 0.013kJ/K}$$

2. **An insulated cylinder of volume capacity 4 m^3 contains 20 kg of nitrogen. Paddle work is done on the gas by stirring it till the pressure in the vessel gets increased from 4 bar to 8 bar. Determine :(i) Change in internal energy,(ii) Work done,(iii) Heat transferred, and (iv) Change in entropy**

Given: $P_1 = 4$ bar $= 4 \times 10^5$ N/m^2 , $P_2 = 8$ bar $= 8 \times 10^5$ N/m^2, $V_1 = V_2 = 4$ m^3

C_p=1.04 kJ/kgK , C_V=0.7432 kJ/kgK

Find: (i) ΔU (ii) W (iii) Q (iv) ΔS

Solution:

$$R = C_P - C_V \Rightarrow R = 1.04 - 0.7432 \Rightarrow \mathbf{R = 0.2968\frac{kJ}{kg}.K}$$

(i) Change in internal energy

$$\Delta U = U_2 - U_1 \Rightarrow \Delta U = mC_v(T_2 - T_1) = mC_v(mT_2 - mT_1) \Rightarrow \Delta U = C_v\left(\frac{p_2V_2}{R} - \frac{p_1V_1}{R}\right)$$

$$\Delta U = 0.7432\left(\frac{8\times10^5\times4}{296.8} - \frac{4\times10^5\times4}{296.8}\right) \Rightarrow \mathbf{\Delta U = 4006.4kJ}$$

(ii) Work done, W

There is no change in system boundary or pdv work is absent. No heat is transferred to the system

$$Q_{12} = (U_2 - U_1) + W_{12} \Rightarrow 0 = (U_2 - U_1) + W_{12} \Rightarrow W_{12} = -(U_2 - U_1) \Rightarrow \mathbf{W_{12} = -4006.4kJ}$$

(iii) Heat transferred, $Q_{12} = 0$

(i) Change in entropy

$$\Delta S_{12} = mC_v \ln\left(\frac{T_2}{T_1}\right) \Rightarrow \Delta S_{12} = 20 \times 0.7432 \ln(2) \Rightarrow \mathbf{\Delta S_{12} = 10.3kJ/K}$$

3. **Find the change in entropy of steam generated at 400ºC from 5 kg of water at 27ºC and atmospheric pressure. Take specific heat of water to be 4.2 kJ/kg.K, heat of vaporization at 100ºC as 2260 kJ/kg and specific heat for steam given by; $c_p = R\,(3.5 + 1.2T + 0.14T^2)$ J/kgK.**

Given: T_1=400ºC, m=5 kg, T_2=27ºC, C_P=4.2kJ/kgK, h_{fg}=2260kJ/kg

Solution:

Total entropy change = Entropy change during water temperature rise (ΔS_1) + Entropy change during water to steam change (ΔS_2) + Entropy change during steam temperature rise (ΔS_3)

Entropy change during water temperature rise (ΔS_1):

$$\Delta S_1 = \frac{Q_1}{T_1} \Rightarrow \Delta S_1 = \frac{mC_p(T_2 - T_1)}{T_1} \Rightarrow \Delta S_1 = \frac{5 \times 4.2 \times (373 - 300)}{300} \Rightarrow \mathbf{\Delta S_1 = 5.11kJ/K}$$

Entropy change during water to steam change (ΔS_2):

$$\Delta S_2 = \frac{Q_2}{T_2} \Rightarrow \Delta S_2 = \frac{m \times h_{fg}}{T_2} \Rightarrow \Delta S_2 = \frac{5 \times 2260}{373.13} \Rightarrow \mathbf{\Delta S_2 = 30.28\ kJ/K}$$

Entropy change during steam temperature rise (ΔS_3):

For steam $R = \frac{8.314}{18}$ = 0.462 kJ/kg.K

Therefore, C_p for steam = $0.462\,(3.5 + 1.2 \cdot T + 0.14T^2) \times 10^{-3}$

$= (1.617 + 0.5544\,T + 0.065\,T^2) \times 10^{-3}$

$$\Delta S_3 = \int_{373.15}^{673.15} \frac{dQ}{T} \Rightarrow \Delta S_3 = \int_{373.15}^{673.15} 5 \text{ X } 10^{-3} \text{ X } \left(\frac{1.617}{T} + 0.5544 + 0.065T\right) dT$$

$$\Rightarrow \mathbf{\Delta S_3 = 51.84kJ/K}$$

Total entropy change = 5.11 + 30.28 + 51.84= **87.23 kJ/K**

4. **Determine the change in entropy of universe if a copper block of 1 kg at 150ºC is placed in a sea water at 25ºC. Take heat capacity of copper as 0.393 kJ/kg K.**

Given: m=1 kg, T_1=150 ºC+273=423 K, T_2=25ºC+273=298 K, C_P=0.393 kJ/kgK

Find: Change in entropy of universe

Solution:

$$\Delta S_{universe} = \Delta S_{block} + \Delta S_{water}$$

Here hot block is put into sea water, so block shall cool down upto sea water at 25ºC as sea may be treated as sink.

$$\Delta S_{block} = mC_v \ln\left(\frac{T_2}{T_1}\right) \Rightarrow \Delta S_{block} = 1 \times 0.393 \ln\left(\frac{298}{423}\right) \Rightarrow \mathbf{\Delta S_{block} = -0.1376\ kJ/K}$$

$$\Delta S_{water} = \frac{Q_1}{T_1} \Rightarrow \Delta S_{Water} = \frac{mC_p(T_2-T_1)}{T_1} = \frac{1\times0.393\times(423-298)}{298} \Rightarrow \boldsymbol{\Delta S_{water} = 0.165kJ/K}$$

Thus, $\Delta S_{universe} = -0.1376 + 0.165 \Rightarrow \boldsymbol{\Delta S_{universe} = 0.0274\ kJ/k}$ **or 27.4 J/K**

5. 1 kg of ice at – 5°C is exposed to the atmosphere which is at 25°C. The ice melts and comes into thermal equilibrium. (i) Determine the entropy increase of the universe (ii) What is the minimum amount of work necessary to convert the water back into ice at – 5°C ? Take : cp of ice = 2.093 kJ/kg°C, Latent heat of fusion of ice = 333.33 kJ/kg

Take : c_p *of ice = 2.093 kJ/kg°C*

Latent heat of fusion of ice = 333.33 kJ/kg.

Solution. Mass of ice, $m = 1$ kg

Temperature of ice = – 5°C (= – 5 + 273 = 268 K)

Temperature of atmosphere = 25°C (= 25 + 273 = 298 K)

Heat absorbed by ice from the atmosphere (Fig. 5.51)

= Heat absorbed in solid phase + latent heat + heat absorbed in liquid phase

$= 1 \times 2.093 \times [0 - (-5)] + 1 \times 333.33 + 1 \times 4.187 \times (25 - 0)$

$= 10.46 + 333.33 + 104.67 = 448.46$ kJ.

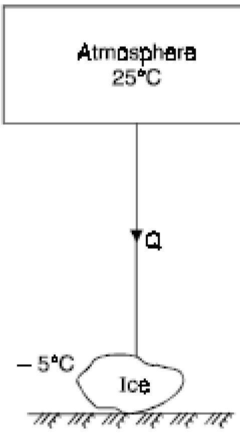


Fig. 5.51

(*i*) **Entropy increase of the universe,** $(\Delta s)_{universe}$:

Entropy change of the atmosphere,

$$(\Delta s)_{atm.} = -\frac{Q}{T} = -\frac{448.46}{298} = -1.5049 \text{ kJ/K}$$

Entropy change of system (ice) as it gets heated from – 5°C to 0°C,

$$(\Delta s_I)_{system} = \int_{268}^{273} mc_p \frac{dT}{T} = 1 \times 2.093 \log_e \frac{273}{268} = 0.0386 \text{ kJ/K}$$

Entropy change of the system as ice melts at 0°C to become water at 0°C.

$$(\Delta s_{II})_{system} = \frac{333.33}{273} = 1.2209 \text{ kJ/K}$$

Entropy change of water as it gets heated from 0°C to 25°C

$$(\Delta s_{III})_{system} = \int_{273}^{298} mc_p \frac{dT}{T} = 1 \times 4.187 \log_e \left(\frac{298}{273}\right) = 0.3668 \text{ kJ/K}$$

Total entropy change of ice as it melts into water

$$(\Delta s)_{total} = \Delta s_I + \Delta s_{II} + \Delta s_{III}$$
$$= 0.0386 + 1.2209 + 0.3668 = 1.6263 \text{ kJ/K}$$

Then temperature-entropy diagram for the system as ice at – 5°C converts to water at 25°C is shown in Fig. 5.52.

∴ **Entropy increase of the universe,**

$$(\Delta s)_{univ.} = (\Delta s)_{system} + (\Delta s)_{atm}.$$
$$= 1.6263 + (-1.5049) = \mathbf{0.1214\ kJ/K.} \quad \textbf{(Ans.)}$$

(*ii*) **Minimum amount of work necessary to convert the water back into ice at – 5°C, W$_{min.}$:**

To convert 1 kg of water at 25°C to ice at – 5°C, 448.46 kJ of heat have to be removed from it, and the system has to be brought from state 4 to state 1 (Fig. 5.52). A refrigerator cycle, as shown in Fig. 5.53, is assumed to accomplish this. The entropy change of the system would be the same, *i.e.*, $s_4 - s_1$, with the only difference that its sign will be negative, because heat is removed from the system (Fig. 5.52).

$$(\Delta s)_{system} = s_1 - s_4 \ (negative)$$

The entropy change of the working fluid in the refrigerator would be zero, since it is operating in a cycle, *i.e.*,

$$(\Delta s)_{refrigerator} = 0.$$

The entropy change of the atmosphere (*positive*),

$$(\Delta s)_{atm.} = \frac{Q+W}{T}$$

∴ Entropy change of the universe

$$(\Delta s)_{univ.} = (\Delta s)_{system} + (\Delta s)_{refrigerator} + (\Delta s)_{atm.}$$

$$= (s_1 - s_4) + 0 + \frac{Q+W}{T} = (s_1 - s_4) + \frac{Q+W}{T}$$

By the principle of increase of entropy

$$(\Delta s)_{univ.\ or\ isolated\ system} \geq 0$$

$$\therefore \quad \left[(s_1 - s_4) + \frac{Q+W}{T}\right] \geq 0$$

$$\therefore \quad \frac{Q+W}{T} \geq (s_4 - s_1)$$

$$W \geq T(s_4 - s_1) - Q$$

$$W_{(min)} = T(s_4 - s_1) - Q$$

Here $Q = 448.46$ kJ

$T = 298$ K

$$s_4 - s_1 = 1.6263 \text{ kJ/K}$$

$$\therefore \quad W_{(min)} = 298 \times 1.6263 - 448.46 = 36.17 \text{ kJ.} \quad \text{(Ans.)}$$

6. **Two tanks A and B are connected through a pipe with valve in between. Initially valve is closed and tanks A and B contain 0.6 kg of air at 90°C, 1 bar and 1 kg of air at 45°C, 2 bar respectively. Subsequently valve is opened and air is allowed to mix until equilibrium. Considering the complete system to be insulated determine the final temperature, final pressure and entropy change.**

Given: $P_A = 1$ bar, $T_A = 363$ K, $m_A = 0.6$ kg; $T_B = 318$K, $m_B = 1$kg, $p_B = 2$ bar

Find: Final temperature, Final pressure and Entropy change

Solution:

In this case due to perfectly insulated system, Q = 0, Also W = 0

$$\Delta Q = \Delta W + \Delta U$$

$$0 = 0 + \{(m_A + m_B)\, C_v.T_f - (m_A.C_vT_A) - (m_B.C_v.T_B)\}$$

$$T_f = \frac{(m_AC_vT_A + m_BC_vT_B)}{(m_A + m_B)\,C_v} \Rightarrow T_f = \frac{(0.6 \text{ X } 363 + 1\text{X}318)}{(0.6+1)} \Rightarrow \mathbf{T_f = 334.88\ K}$$

Using gas law for combined system after attainment of equilibrium,

$$p_f = \frac{(m_A + m_B)\,RT_f}{(V_A + V_B)} \Rightarrow p_f = \frac{(1+0.6) \text{ X } 0.287 \text{ X } 334.88}{(0.625+0.456)} \Rightarrow \mathbf{p_f = 142.25\ kPa}$$

$$V_A = \frac{m_A\,R\,T_A}{p_A}$$

$$\mathbf{V_A = 0.625\ m^3}$$

$$V_B = \frac{m_B\,R\,T_B}{p_B}$$

$$\mathbf{V_B = 0.456\ m^3}$$

Entropy change

$$\Delta S = \{((m_A + m_B).s_f) - (m_A.s_A + m_Bs_B)\}$$

$$= \{m_A(s_f - s_A) + m_B\,(s_f - s_B)\}$$

$$= \left\{m_A\left(c_p \ln\frac{T_f}{T_A} - R\ln\frac{p_f}{p_A}\right) + m_B\left(c_p \ln\frac{T_f}{T_B} - R\ln\frac{p_f}{p_B}\right)\right\}$$

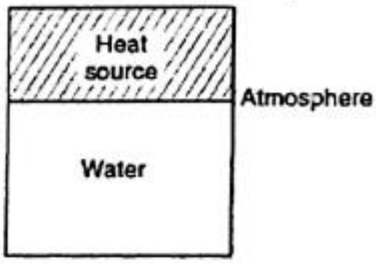
$$\Delta S = \left\{0.6\left(1.005\ln\frac{334.88}{363} - 0.287\ln\frac{142.25}{100}\right) + 1\left(1.005\ln\frac{334.88}{318} - 0.287\ln\frac{142.25}{200}\right)\right\}$$

$$\Delta S = \{-0.1093 + 014977\} \Rightarrow \mathbf{\Delta S = 0.04047\ kJ/K}$$

7. One kilogram of water at 273 K is brought into contact with a heat reservoir at 373 K. (i) When the water has reached 373 K, find the change in entropy of the water, of the heat reservoir, and of the universe. (ii) If the water had been heated from 273 K to 373K by first bringing it in contact with a reservoir at 323K and then with a reservoir at 373K, what would have been the change in entropy of the universe?

Solution

(a) Let us take the water and the heat source as our system and the surrounding atmosphere as the surroundings, as shown in the adjacent figure.



The heating of water can be treated as a constant volume process since water is incompressible. Thus, for water, the change in entropy is given by

$$\Delta S = \frac{\delta Q}{T} = mC_v \ln \frac{T_2}{T_1}$$

For water at 273 K, C_v = 4.217 kJ/(kg K) from standard tables. Thus,

$$\Delta S_{\text{water}} = 4217\left(\ln \frac{373}{273}\right) = \boxed{1316 \text{ J/K}}$$

The heat loss by the source δQ = heat gained by the water = $mC_v\Delta T$ = 4217 × 100
Therefore, the change in entropy of the heat source is

$$\Delta S_{\text{heat source}} = \frac{-\delta Q}{T}, \text{ since } \delta Q \text{ is negative.}$$

$$= \frac{-421,700}{373} = \boxed{-1130 \text{ J/K}}$$

The total change in entropy of the system = 1316 − 1130 = 186 J/K. This much entropy is added to the entropy of the universe, since the system considered is part of the universe. Thus, the entropy change of the universe is $\boxed{186 \text{ J/K}}$.

(b) Here the heat addition to water is done in two stages. In the first stage, the water temperature increases from 273 K to 323 K and in the second stage the temperature goes up from 323 K to 373 K. For stage one at 273 K, C_v = 4.217 kJ/(kg K) for water and for stage two at 323 K, C_v = 4.180 kJ/(kg K).

Therefore, for water

$$\Delta S_1 = mC_v \ln \frac{T_2}{T_1} = 4217 \ln \frac{323}{273} = 709.2 \text{ J/K}$$

and

$$\Delta S_2 = 4180 \ln \frac{373}{323} = 601.6 \text{ J/K}$$

Therefore, the net change in entropy of water is

$$\Delta S_{\text{water}} = \Delta S_1 + \Delta S_2 = 709.2 + 601.6 = \boxed{1310.8 \text{ J/K}}$$

For the heat source, the change in entropy for stage one is

$$\Delta S_1 = \frac{-\delta Q}{T} = \frac{-mC_v(\Delta T)}{T} = \frac{-4217 \times 50}{323} = -652.8 \text{ J/K}$$

And for stage two, the change in entropy is

$$\Delta S_2 = \frac{-4180 \times 50}{373} = -560.3 \text{ J/K}$$

Thus, the net change in entropy of the heat source is

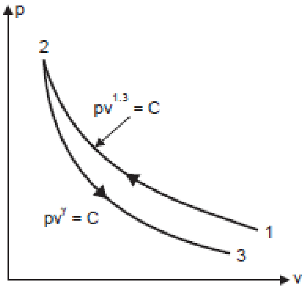
$$\Delta S_{\text{heat source}} = -652.8 - 560.3 = \boxed{-1213.1\ \text{J/K}}$$

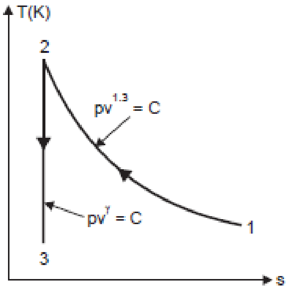
The change in entropy of the universe is, therefore = 1310.8 − 1213.1 = $\boxed{97.7\ \text{J/K}}$.

8. **5 m^3 of air at 2 bar, 27 °C is compressed up to 6 bar pressure following pV$^{1.3}$= C. It is subsequently expanded adiabatically to 2 bar. considering the two process to be reversible, determine the network, net heat transfer, change in entropy. Also plot the processes on T-S and P-V diagrams**

Given : $V_1 = 5\ m^3$; $p_1 = 2$ bar ; $T_1 = 27 + 273 = 300$ K ; $p_2 = 6$ bar ; $p_3 = 2$ bar

Net work :





Mass of air, $m = \frac{p_1V_1}{RT_1} = \frac{2 \times 10^5 \times 5}{287 \times 300} = 11.61$ kg.

Considering polytropic compression process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad \frac{T_2}{300} = \left(\frac{6}{2}\right)^{\frac{13-1}{13}} \quad \text{or} \quad T_2 = 386.5\ \text{K}$$

Considering isentropic process 2-3, we get

$$\frac{T_2}{T_3} = \left(\frac{p_2}{p_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{2}\right)^{\frac{14-1}{14}} = 1.369 \qquad (\because\ p_3 = p_1)$$

$$\therefore \qquad T_3 = \frac{T_2}{1369} = \frac{386.5}{1369} = 282.3\ \text{K}$$

Now, work done during polytropic *compression* 1-2,

$$W_{1-2} = \frac{mR(T_1 - T_2)}{n-1} = \frac{1161 \times 0.287(300 - 386.5)}{13 - 1} = -960.7\ \text{kJ}$$

and, work done during *adiabatic expansion* 2-3,

$$W_{2-3} = \frac{mR(T_2 - T_3)}{\gamma - 1} = \frac{1161 \times 0.287(386.5 - 282.3)}{14 - 1} = 868\ \text{kJ}$$

∴ Net work done = $W_{1-2} + W_{2-3} = -960.7 + 868 = -92.7$ kJ

Hence net work done on the air = 92.7 kJ. (Ans.)

9. **A metal block with m=5 kg, c=0.4 kJ/kgK at 40°C is kept in a room at 20°C. It is cooled in the following two ways: (i) Using a Carnot engine (executing internal number of cycles) with the room itself as the cold reservoir; (ii) Naturally. In each case, calculate the change in entropy of the block, of the air of the room and of the universe. Assume that the metal block has constant specific heat**

Given: m=5 kg, c_p=0.4 kJ/kgK, T_1=40°C, T_2=20°C

Solution:

(i) Cooling naturally

Heat absorbed by air, δQ=Heat released by the metal block

$$\delta Q = mc(T_1 - T_2) = 5 \times 0.4 \times (40 - 20)$$

$$\boldsymbol{\delta Q = 40kJ}$$

$$(\delta Q)_{BLOCK} = \int_{313}^{293} mc\frac{dT}{T} = 5 \times 0.4 \ln\frac{293}{313} \quad \Rightarrow \quad \boldsymbol{(\delta Q)_{BLOCK} = -0.132 kJ/K}$$

Entropy change of atm, $\delta Q_{air} = \frac{\delta Q}{T} = \frac{40}{293} \Rightarrow \boldsymbol{\delta Q_{air} = 0.1365\ kJ/K}$

Entropy of universe:

$$\delta Q_{universe} = \delta Q_{block} + \delta Q_{air}$$

$$\delta Q_{universe} = -0.132 + 0.1365$$

$$\boldsymbol{\delta Q_{universe} = 0.0045\ kJ/K}$$

(ii) Cooling using a Carnot Engine:

Entropy of Carnot engine, $\Delta S_{Carnot} = 0$

For Carnot engine, efficiency

$$\eta_{Carnot} = \frac{T_1 - T_2}{T_1} \Rightarrow \eta_{Carnot} = \frac{313-293}{313} \Rightarrow \boldsymbol{\eta_{Carnot} = 6.39\%}$$

$$\eta_{Carnot} = \frac{W}{\delta Q} \Rightarrow 0.0639 = \frac{W}{40} \Rightarrow \boldsymbol{W = 2.556\ kJ}$$

Entropy of air, $\Delta S_{air} = \frac{\delta Q + W}{T} = \frac{40+2.556}{293} \Rightarrow \boldsymbol{\Delta S_{air} = 0.1452\ kJ/K}$

$\Delta S_{Univ} = \Delta S_{block} + \Delta S_{carnot\ engine} + \Delta S_{air} \Rightarrow \boldsymbol{\Delta S_{Univ} = 0.2772\ kJ/K}$

10. 2 kg of water at 90°C is mixed with 3 kg of water at 10°C in an isolated system. Calculated the change of entropy due to the mixing process.

$$2 \times 4.18(90 - T_f) = 3 \times 4.18(T_f - 10)$$

$T_f = 42$ °C

$$\Delta S_1 = m_1 C_p \ln\left(\frac{T_f}{T_1}\right) = -1.1857 kJ/K$$

$$\Delta S_2 = m_1 C_p \ln\left(\frac{T_f}{T_2}\right) = 1.3433 kJ/K$$

$$\Delta S_{mixing} = \Delta S_1 + \Delta S_2 = 157 J/K$$

Availability and Irreversibiity:

	Between state 1-2	Dead State
Available Energy:	Initial State: $\Psi_1 = q_1 - T_o s_1 + P_o v_1$ Final State: $\Psi_2 = q_2 - T_o s_2 + P_o v_2$	Initial State: $\varphi_1 = u_1 - T_o s_1$ Final State: $\varphi_2 = u_2 - T_o s_2$
Change in Available Energy (W_{max})	$\Psi_1 - \Psi_2 = m[(q_1 - q_2) - T_o \Delta s] + P_o(v_1 - v_2)$	$\varphi_1 - \varphi_2 = m[(u_1 - u_2) - T_o \Delta s]$
Irreversibility	$I = W_{max} - W_{Actual}$	
Effectiveness	$I = \frac{W_{Actual}}{W_{max}}$	

1. **5 kg of air at 550K and 4 bar is enclosed in a closed vessel**

   (a) **Determine the availability of the system if the surrounding pressure and temperature are 1 bar and 290K.**

   (b) If the air is cooled at constant pressure to the atmospheric temperature, determine the availability and effectiveness.

Solution. Mass of air, $m = 5$ kg

Temperature, $T_1 = 550$ K

Pressure, $p_1 = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2$

Temperature, $T_2 = T_0 = 290$ K

Pressure, $p_2 = p_0 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$.

(*i*) Availability of the system :

Availability of the system is

$$= m[(u_1 - u_0) - T_0(s_1 - s_0)] = m[c_v(T_1 - T_0) - T_0\,\Delta s]$$

$$\Delta s = c_p \log_e \frac{T_1}{T_0} - R \log_e \left(\frac{p_1}{p_0}\right)$$

$$= 1.005 \log_e \left(\frac{550}{290}\right) - 0.287 \log_e \left(\frac{4}{1}\right)$$

$$= 0.643 - 0.397 = 0.246 \text{ kJ/kg K}$$

∴ **Availability of the system**

$$= m\,[c_v\,(T_1 - T_0) - T_0 \Delta s]$$

$$= 5[0.718\,(550 - 290) - 290 \times 0.246] = \mathbf{576.7\ kJ.}$$

(*ii*) Heat transferred during cooling

$$Q = m \times c_p \times (T_1 - T_0)$$

$$= 5 \times 1.005 \times (550 - 290)$$

$$= 1306.5 \text{ kJ}$$ ...... heat lost by the system

Change of entropy during cooling

$$\Delta S = m \times c_p \times \log_e \left(\frac{T_1}{T_0}\right)$$

$$= 5 \times 1.005 \times \log_e \left(\frac{550}{290}\right) = 3.216 \text{ kJ/K}$$

Unavailable portion of this energy

$$= T_0 \,.\, (\Delta S) = 290 \times 3.216 = 932.64 \text{ kJ}$$

∴ **Available energy** $= 1306.5 - 932.64 = \mathbf{373.86\ kJ.}$ (**Ans.**)

Effectiveness, $\epsilon = \dfrac{\text{Available energy}}{\text{Availability of the system}} = \dfrac{373.86}{576.7}$

$= \mathbf{0.648}$ **or 64.8%.** (**Ans.**)

2. **3 kg of air at 500 kPa, 90°C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of surroundings at 100 kPa and 10°C. Find maximum work, change in availability and irreversibility.**

Solution :

$P_1 = 500$ kPa

$T_1 = 90 + 273 = 363$ K

$T_2 = 10 + 273 = 283$ K

$T_0 = 283$ K

To find (ΔS) system

$$dS = \frac{dU}{T} + p\,\frac{dV}{T}$$

Intergrating on both sides.

$$\int^{2} dS = \int_{T_1}^{T_2} \frac{m C_v \, dT}{T} + \int_{V_1}^{V_2} \frac{mRT \, dV}{V \times T}$$

$$\left[\therefore pV = mRT \Rightarrow P = \frac{mRT}{V}\right]$$

$$(\Delta S)\text{ system} = S_2 - S_1 = mC_v \ln\left[\frac{T_2}{T_1}\right] + mR \ln\left[\frac{V_2}{V_1}\right]$$

$$= 3 \times 0.718 \ln\left[\frac{283}{363}\right] + 3 \times 0.287 \times \ln\left[\frac{2}{1}\right]$$

$$= -0.5363 + 0.5968 = 0.06055 \text{ KJ/K}$$

To find maximum work (W_{max})

$$W_{max} = (U_1 - U_2) - T_0 (S_1 - S_2)$$

$$= m\left[C_v (T_1 - T_2) + T_0 (S_2 - S_1)\right]$$

$$= 3\left[0.718 (363 - 283) + 283 (0.06055)\right]$$

$$= 3\left[57.44 + 17.14\right]$$

$$= 223.73 \text{ kJ}$$

To find change in availability:

$$= \phi_1 - \phi_2$$

$$= (U_1 - U_2) - T_0 (S_1 - S_2) + P_0 (V_1 - V_2)$$

$$= W_{max} + P_0 (V_1 - V_2)$$

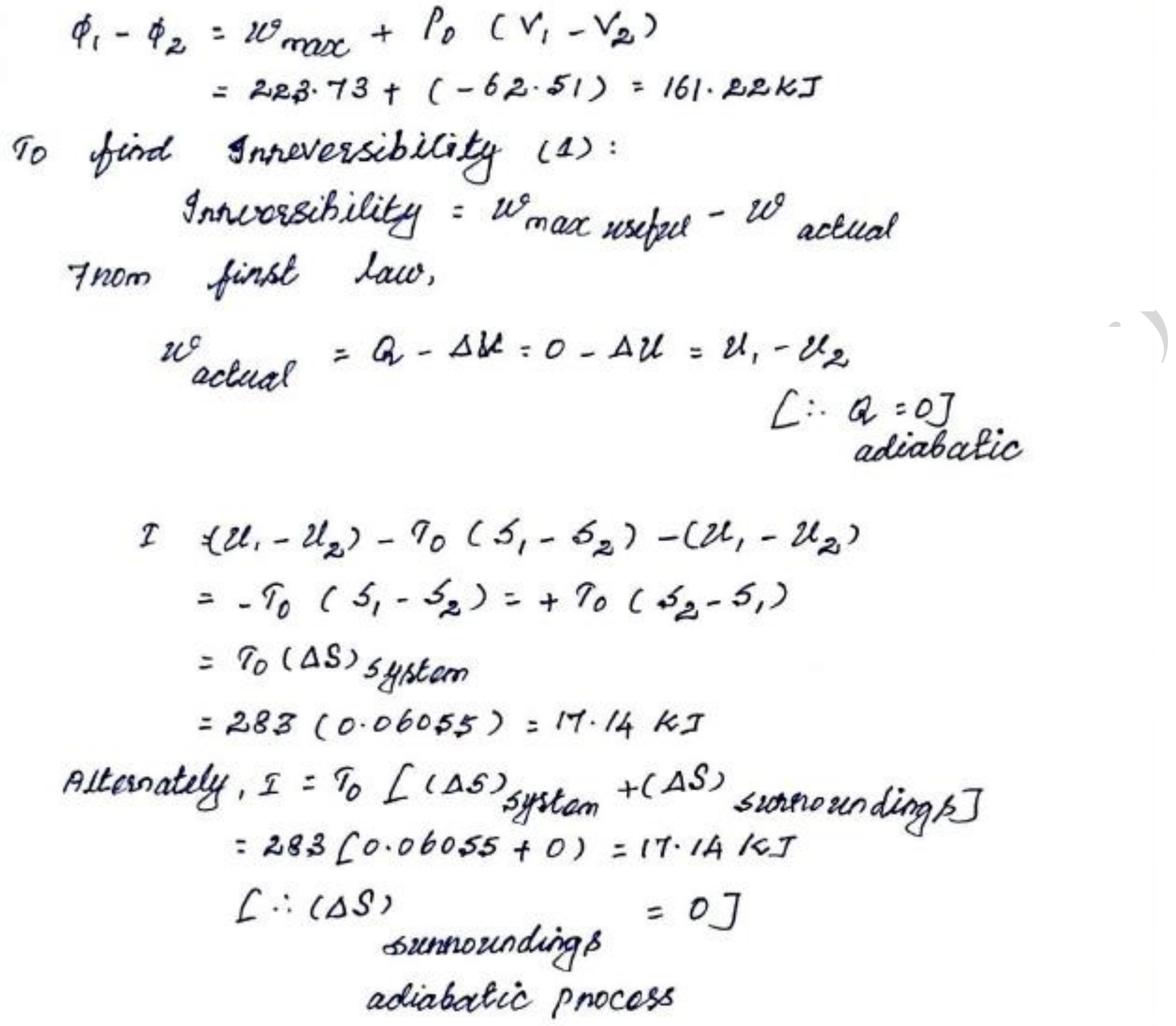
$$\left[\therefore W_{max} = U_1 - U_2) - T_0 (S_1 - S_2)\right]$$

$$P_0 (V_1 - V_2) = P_0 (V_1 - 2V_1)$$

$$= -P_0 V_1$$

$$= -100 \times \left[\frac{3 \times 0.287 \times 363}{500}\right] = -62.51 \text{ KJ}$$

$$\left[\therefore P_1 V_1 = mRT_1 \qquad V_1 = \frac{mRT_1}{P_1} = \frac{3 \times 0.287 \times 363}{500}\right]$$



$\phi_1 - \phi_2 = W_{max} + P_0 (V_1 - V_2)$

$= 223.73 + (-62.51) = 161.22\,kJ$

To find Irreversibility (I):

Irreversibility = $W_{max\ useful} - W_{actual}$

From first law,

$W_{actual} = Q - \Delta U = 0 - \Delta U = U_1 - U_2$

[∵ Q = 0]
adiabatic

$I = (U_1 - U_2) - T_0 (S_1 - S_2) - (U_1 - U_2)$

$= -T_0 (S_1 - S_2) = +T_0 (S_2 - S_1)$

$= T_0 (\Delta S)_{system}$

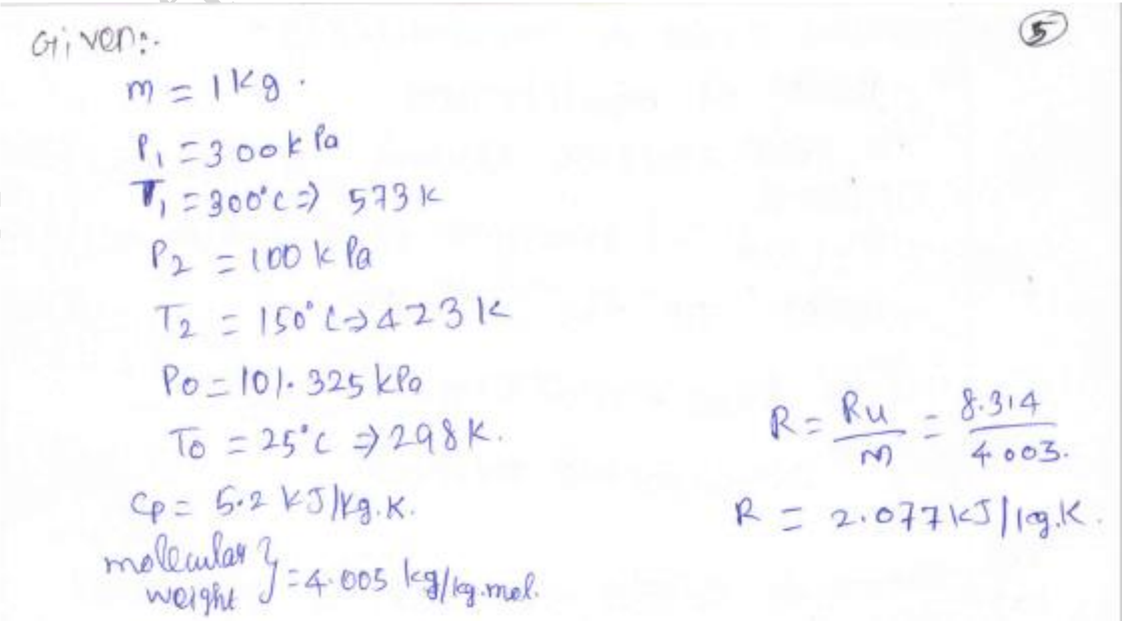
$= 283 (0.06055) = 17.14\,kJ$

Alternately, $I = T_0 [(\Delta S)_{system} + (\Delta S)_{surroundings}]$

$= 283 [0.06055 + 0) = 17.14\,kJ$

[∵ $(\Delta S)_{surroundings} = 0$]
adiabatic process

3. Helium enters an actual turbine at 300 kPa, 300°C and expands to 100 kPa, 150°C. Heat transfer to atmosphere at 101.325 kPa, 25°C amounts to 7 kJ/kg. Calculate the entering stream availability, leaving stream availability and the maximum work. For helium, Cp=5.2kJ/kg and molecular wt=4.003kg/kg-mol



⑤

Given:-

$m = 1\,kg$.

$P_1 = 300\,kPa$

$T_1 = 300°C \Rightarrow 573\,K$

$P_2 = 100\,kPa$

$T_2 = 150°C \rightarrow 423\,K$

$P_0 = 101.325\,kPa$

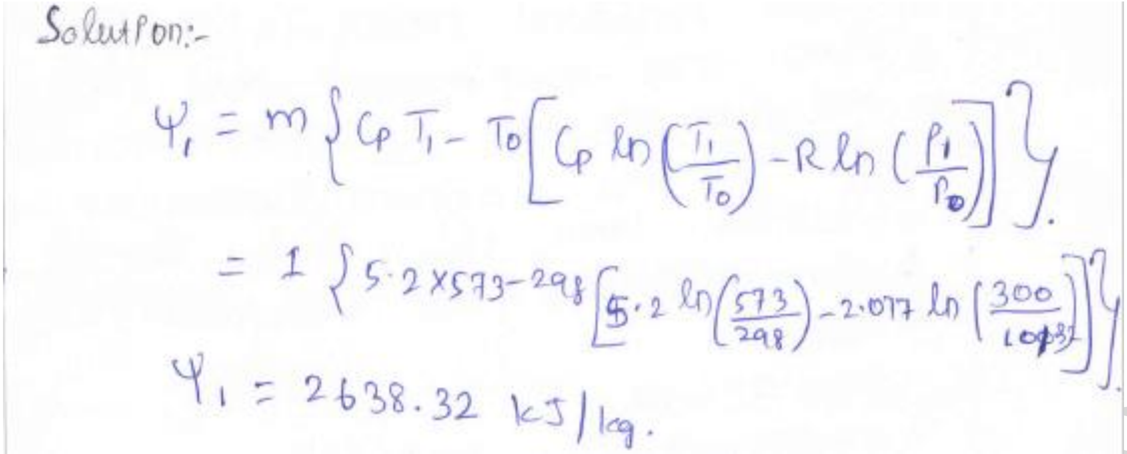
$T_0 = 25°C \Rightarrow 298\,K$.

$C_p = 5.2\,kJ/kg.K$.

molecular weight } = 4.005 kg/kg.mol.

$R = \frac{R_u}{M} = \frac{8.314}{4.003}$.

$R = 2.077\,kJ/kg.K$.

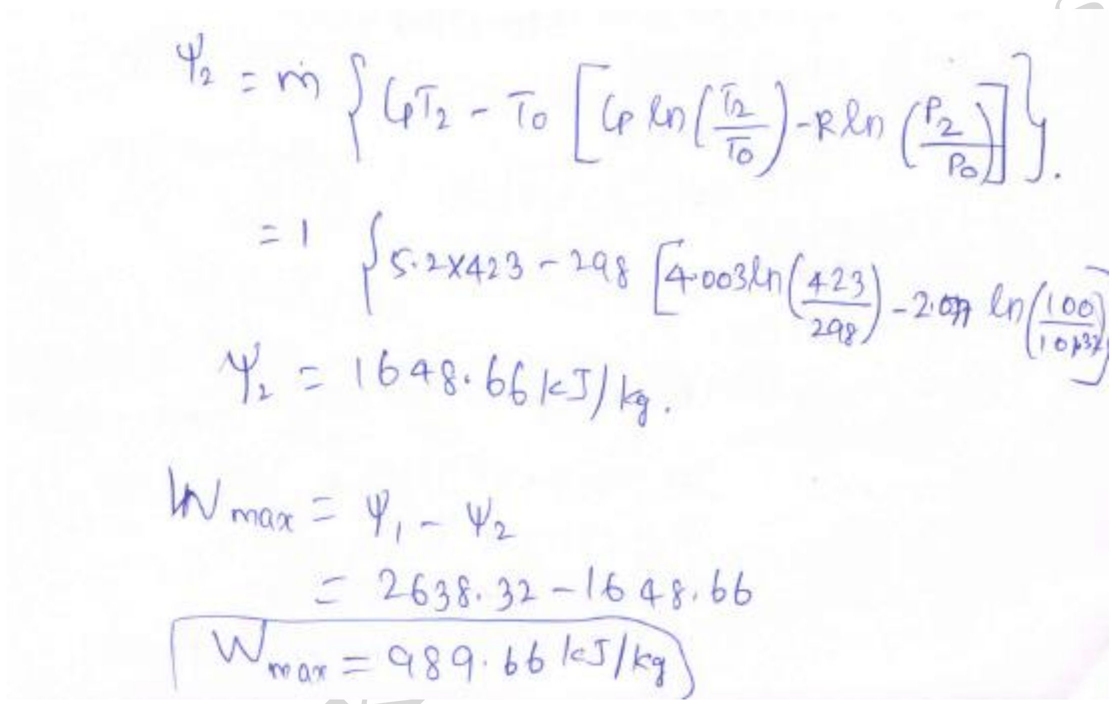


Solution:-

$$\psi_1 = m\left\{C_p T_1 - T_0\left[C_p \ln\left(\frac{T_1}{T_0}\right) - R\ln\left(\frac{P_1}{P_0}\right)\right]\right\}.$$

$$= 1\left\{5.2\times573 - 298\left[5.2\ln\left(\frac{573}{298}\right) - 2.077\ln\left(\frac{300}{101.32}\right)\right]\right\}.$$

$$\psi_1 = 2638.32 \text{ kJ/kg}.$$



$$\psi_2 = m\left\{C_p T_2 - T_0\left[C_p \ln\left(\frac{T_2}{T_0}\right) - R\ln\left(\frac{P_2}{P_0}\right)\right]\right\}.$$

$$= 1\left\{5.2\times423 - 298\left[4.003\ln\left(\frac{423}{298}\right) - 2.077\ln\left(\frac{100}{101.32}\right)\right]\right\}$$

$$\psi_2 = 1648.66 \text{ kJ/kg}.$$

$$W_{max} = \psi_1 - \psi_2$$

$$= 2638.32 - 1648.66$$

$$\boxed{W_{max} = 989.66 \text{ kJ/kg}}$$

4. (a) Define the terms 'Irreversible process' and 'Reversible process'. Give an e.g. of each.

Irreversible process

Processes that are not reversible are called irreversible processes. Once having taken place, these processes cannot reverse themselves spontaneously and restore the system to its initial state. For this reason, they are classified as irreversible processes.

Example for irreversible process: Once a cup of hot coffee cools, it will not heat up by retrieving the heat it lost from the surroundings.

Reversible process

A reversible process is defined as a process that can be reversed without leaving any trace on the surroundings. That is, both the system and the surroundings are returned to their initial states at the end of the reverse process. This is possible only if the net heat and net work exchange between the system and the surroundings is zero for the combined (original and reverse) process.

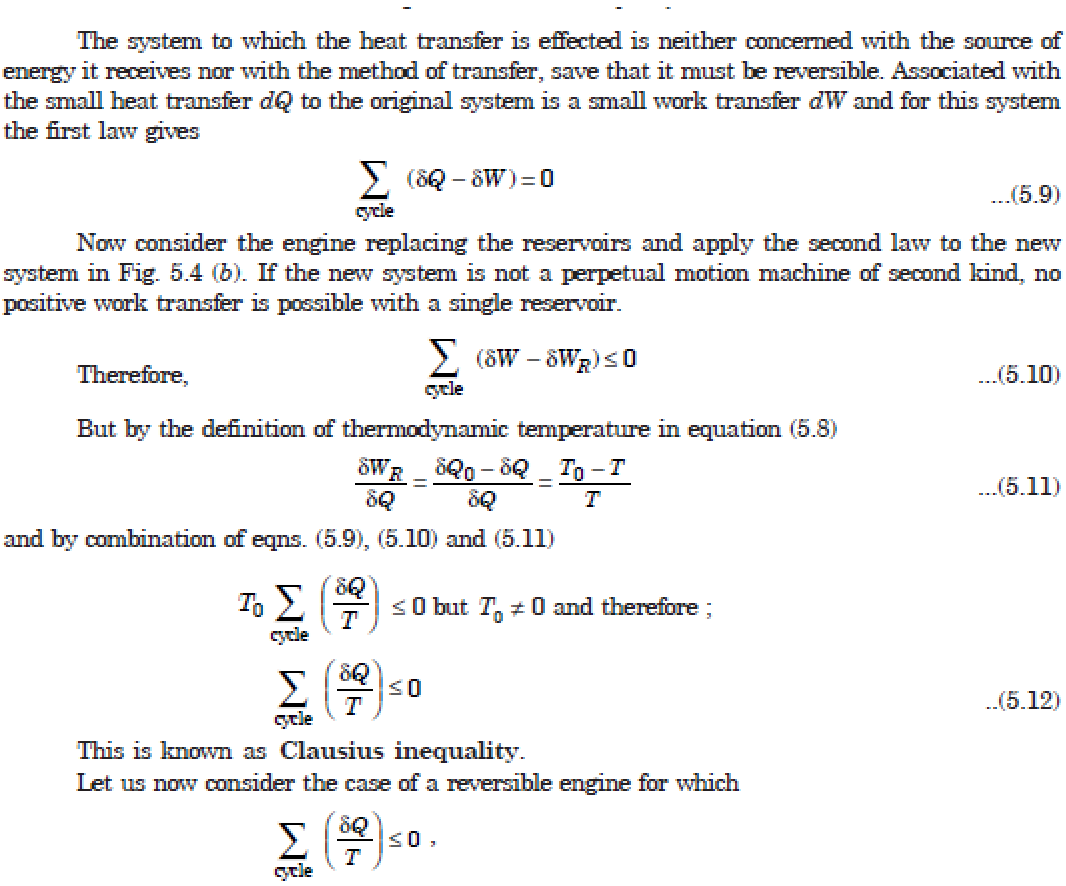
Examples for reversible process are

i) Quasi equilibrium expansion and compression of gas

ii) Frictionless pendulum

5. State and prove Clausius inequality.

The Clausius Inequality applies to any real engine cycle and implies a negative change in entropy on the cycle. That is, the entropy given to the environment during the cycle is larger than the entropy transferred to the engine by heat from the hot reservoir



The system to which the heat transfer is effected is neither concerned with the source of energy it receives nor with the method of transfer, save that it must be reversible. Associated with the small heat transfer dQ to the original system is a small work transfer dW and for this system the first law gives

$$\sum_{\text{cycle}} (\delta Q - \delta W) = 0 \qquad \text{...(5.9)}$$

Now consider the engine replacing the reservoirs and apply the second law to the new system in Fig. 5.4 (*b*). If the new system is not a perpetual motion machine of second kind, no positive work transfer is possible with a single reservoir.

Therefore, $$\sum_{\text{cycle}} (\delta W - \delta W_R) \le 0 \qquad \text{...(5.10)}$$

But by the definition of thermodynamic temperature in equation (5.8)

$$\frac{\delta W_R}{\delta Q} = \frac{\delta Q_0 - \delta Q}{\delta Q} = \frac{T_0 - T}{T} \qquad \text{...(5.11)}$$

and by combination of eqns. (5.9), (5.10) and (5.11)

$$T_0 \sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \text{ but } T_0 \ne 0 \text{ and therefore ;}$$

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \qquad \text{..(5.12)}$$

This is known as **Clausius inequality**.

Let us now consider the case of a reversible engine for which

$$\sum_{\text{cycle}} \left(\frac{\delta Q}{T}\right) \le 0 \text{ ,}$$

6. Mention the Clasius inequality for open, closed and isolated systems.

i. $\sum_{\text{Cycle}} \frac{\delta Q}{T} < 0$, Cycle is irreversible

ii. $\sum_{\text{Cycle}} \frac{\delta Q}{T} = 0$, Cycle is reversible

iii. $\sum_{\text{Cycle}} \frac{\delta Q}{T} < 0$, Cycle is impossible

7. 300 kJ/s of heat is supplied at a constant fixed temperature of 290°C to a heat engine. The heat rejection takes place at 8.5°C. The following results were obtained : (i) 215 kJ/s are rejected. (ii) 150 kJ/s are rejected. (iii) 75 kJ/s are rejected. Classify which of the result report a reversible cycle or irreversible cycle or impossible results.

Given: Heat supplied at 290°C= 300 kJ/s , Heat rejected at 8.5°C

Find: (i) 215kJ/s, (ii) 150 kJ/s, (iii) 75 kJ/s

Solution:

Applying clausius inequality to the cycle

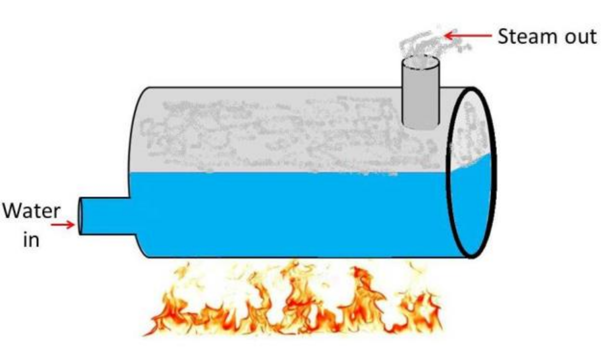
(ii) $\sum_{\text{Cycle}} \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} \Rightarrow \sum_{\text{Cycle}} \frac{\delta Q}{T} = \frac{300}{563} - \frac{215}{281.5} \Rightarrow \sum_{\text{Cycle}} \frac{\delta Q}{T} = -2309 < 0$, **Cycle is irreversible**

(iii) $\sum_{\text{Cycle}} \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} \Rightarrow \sum_{\text{Cycle}} \frac{\delta Q}{T} = \frac{300}{563} - \frac{150}{281.5} \Rightarrow \sum_{\text{Cycle}} \frac{\delta Q}{T} = 0$, **Cycle is reversible**

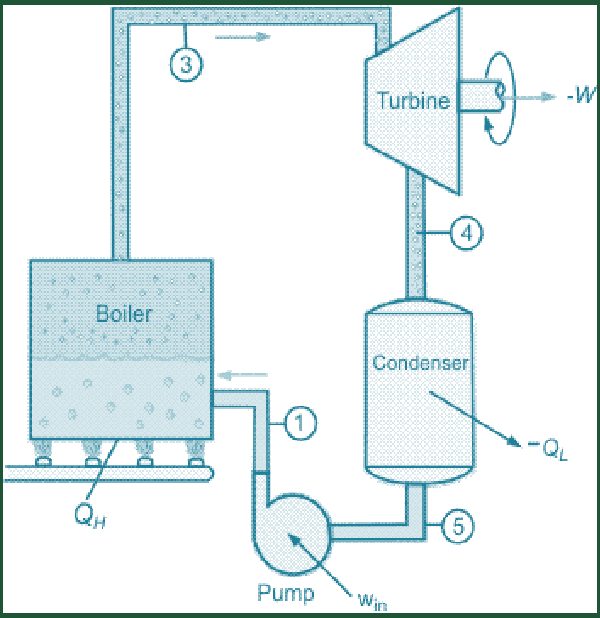
(iv) $\sum_{\text{Cycle}} \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2} \Rightarrow \sum_{\text{Cycle}} \frac{\delta Q}{T} = \frac{300}{563} - \frac{75}{281.5} \Rightarrow \sum_{\text{Cycle}} \frac{\delta Q}{T} = 0.2664 < 0$, **Cycle is impossible**

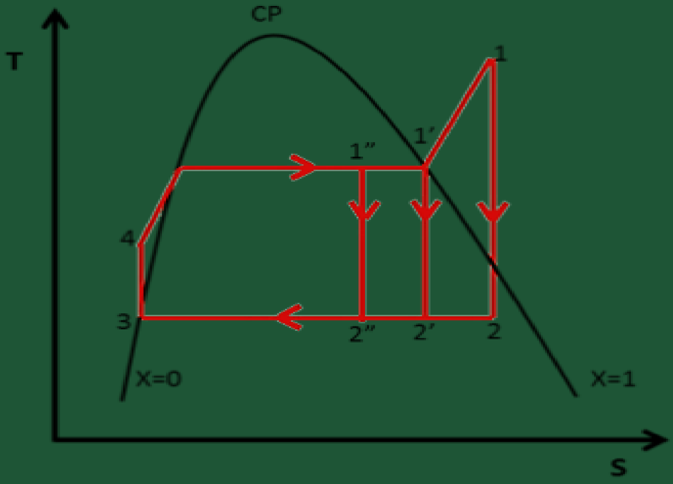
ENGINEERING THERMODYNAMICS

PROPERTIES OF PURE SUBSTANCE
AND
STEAM POWER CYCLE



Principle of Boiler





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PURE SUBSTANCE

A pure substance is a substance of constant chemical composition throughout its mass. A pure substance does not have to be of a single chemical element. A mixture of various chemical elements is also called as pure substance as long as the mixture is homogeneous in composition, homogeneous in chemical aggregation and invariable in chemical aggregation. e.g. H_2O.

SATURATION STATES

A saturation state is a state from which a change of phase may occurs without a change of pressure or temperature. The state of a substance at which a phase transformation begins or ends.

SUB COOLED LIQUID

The state of a pure substance at which the temperature is less than the saturation temperature corresponding to the pressure is known as sub-cooled liquid state

COMPRESSED LIQUID

The pressure on the liquid water is greater than the saturated pressure at a given temperature. In this condition, the liquid water is known as the compressed liquid

SUPER HEATING' AND 'SUBCOOLING'

Heating steam beyond its saturation temperature and cooling of liquid below saturation temperatures are known as superheating and subcooling.

DEGREE OF SUPERHEAT

The difference between the superheated temperature and the saturated temperature at the given pressure is called the degree of superheat.

CRITICAL STATE. THE TERM CRITICAL PRESSURE , CRITICAL TEMPERATURE, AND CRITICAL VOLUME OF WATER.

The state at which the transition from liquid to vapour phase suddenly takes place. The specific volume of the saturated liquid and of the saturated vapour is the same. Such a state of the substance is called the critical state. The properties like pressure, temperature and volume at critical state are known as critical pressure, critical temperature, and critical volume. The corresponding values for water are: 221.2 bar, 374.15°C and 0.00317 m3/kg

TRIPLE POINT. FOR A PURE SUBSTANCE, HOW MANY DEGREES OF FREEDOM ARE THERE AT TRIPLE POINT?

The state at which all the three phases-solid ,liquid and vapour coexist in equilibrium is called the triple point. At the triple point, C=1, P=3 and application of the phase rule gives F=C+2-P=1+2-3+0. Hence, the number of degrees of freedom at the triple point is equal to zero. In other words the triple point is invariant (F=0). One cannot arbitrarily assign either temperature or pressure for the triple point. The triple point exists at u definite pressure and temperature. For example, the triple point of water is P=0.611kPa and t=0.01°C.

PHASE RULE FOR PURE SUBSTANCE

Phase rule gives F=C+2-P, Hence, the number of degrees of freedom at the triple point is equal to zero. In other words the triple point is invariant (F=0). One cannot arbitrarily assign either temperature or pressure for the triple point. The triple point exists at u definite pressure and temperature. For example, the triple point of water is P=0.611kPa and t=0.01°C.

LIST THE ADVANTAGES OF SUPERHEATED STEAM

- ✓ Its heat content and hence its capacity to do work is increased without having to increase its pressure
- ✓ High temperature of superheated steam results in an increase in thermal efficiency
- ✓ Superheating is done in a superheated which obtains its heat from waste furnace gases which would have otherwise uselessly up the chimney

DRYNESS FRACTION

The term dryness fraction is related with wet steam. It is defined as the ratio of the mass of actual dry steam to the mass of steam containing it. It is expressed by the symbol 'x'.

HEAT OF SUPERHEAT

The additional amount of heat supplied to the steam during superheating is called as heat of superheat. It can be calculated by using the specific heat of superheated steam at constant pressure.

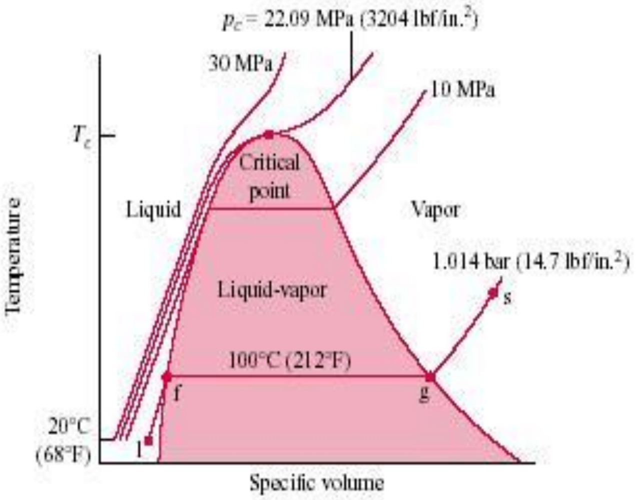
LATENT HEAT

The heat being supplied does not show any rise of temperature but results in change of state is known as latent heat.

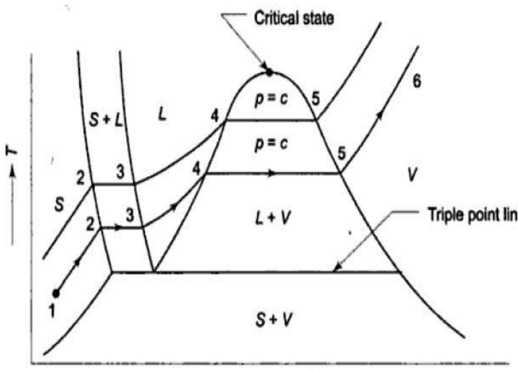
SENSIBLE HEAT OF WATER.

It is defined as the quantity of heat absorbed by 1 kg of water when it is heated from 0°C to boiling point. It is also called total heat of water.

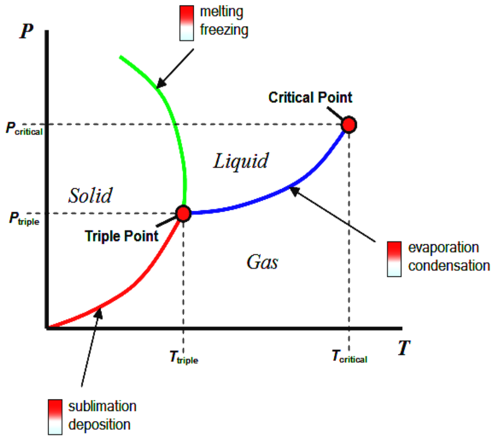
PHASE EQUILIBRIUM DIAGRAM ON T-V COORDINATES



PHASE EQUILIBRIUM DIAGRAM FOR A PURE SUBSTANCES ON T-S PLOT



P-T DIAGRAM FOR WATER



RANKINE CYCLE:

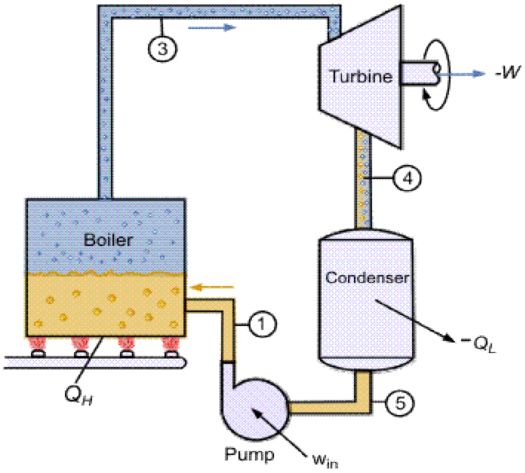
A simple Rankine cycle has the following processes for the boiler, turbine, condenser and pump of a steam power cycle:

Process 1–2: Isentropic expansion of the working fluid through the turbine from saturated vapour at state 1 to the condenser pressure.

Process 2–3: Heat transfer from the working fluid as it flows at constant pressure through the condenser with saturated liquid at state 3.

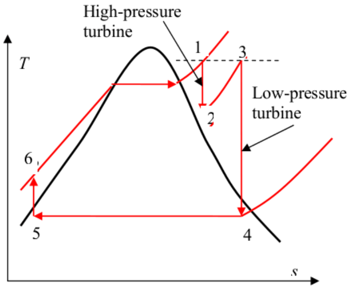
Process 3–4: Isentropic compression in the pump to state 4 in the compressed liquid region.

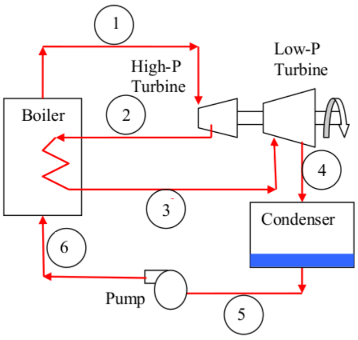
Process 4–1: Heat transfer to the working fluid as it flows at constant pressure through the boiler to complete the cycle.



REHEAT RANKINE CYCLE:

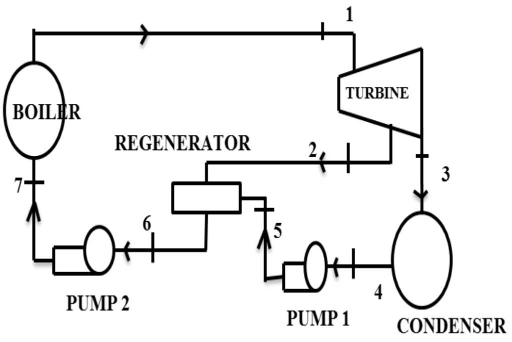
Increasing the boiler pressure can increase the thermal efficiency of the Rankine cycle, but it also increases the moisture content at the exit of the turbine to an unacceptable level. To correct this side effect, the simple Rankine cycle is modified with a reheat process. The schematic of an ideal reheat Rankine cycle with its T-s diagram. In this reheat cycle, steam is expanded isentropically to an intermediate pressure in a high-pressure turbine (stage I) and sent back to the boiler, where it is reheated at constant pressure to the inlet temperature of the high-pressure turbine. Then the steam is sent to a low-pressure turbine and expands to the condenser pressure (stage II) . The total heat input and total work output is

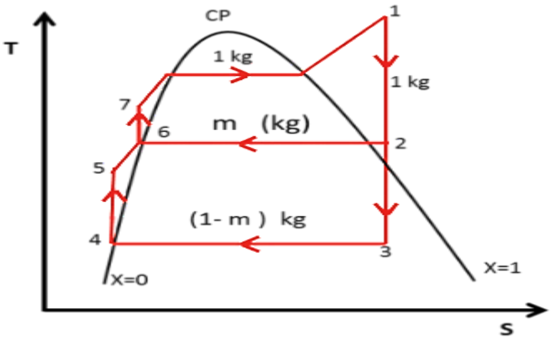




REGENERATIVE RANKINE CYCLE:

In regenerative Rankine cycle, some amount of steam is bled off during expansion in the turbine and mixed with feed water before it enters the boiler to reduce the heat input. In the below both reheat and regeneration (Two feed water heaters) are incorporated.





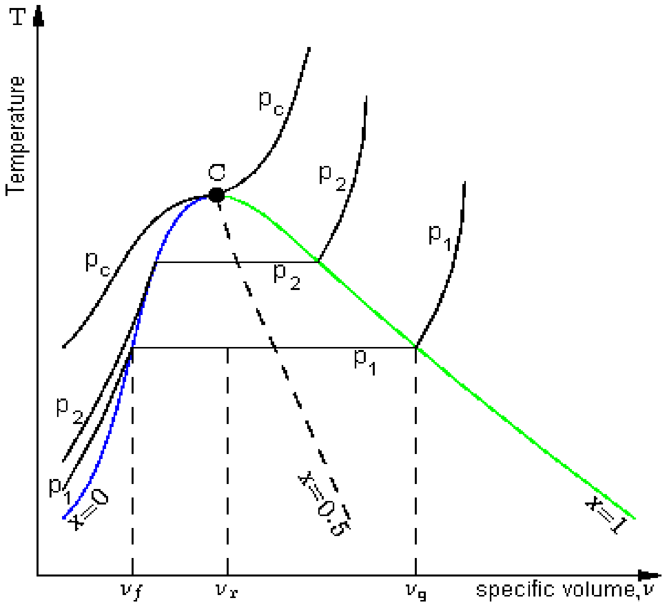
RANKINE CYCLE IS MODIFIED?

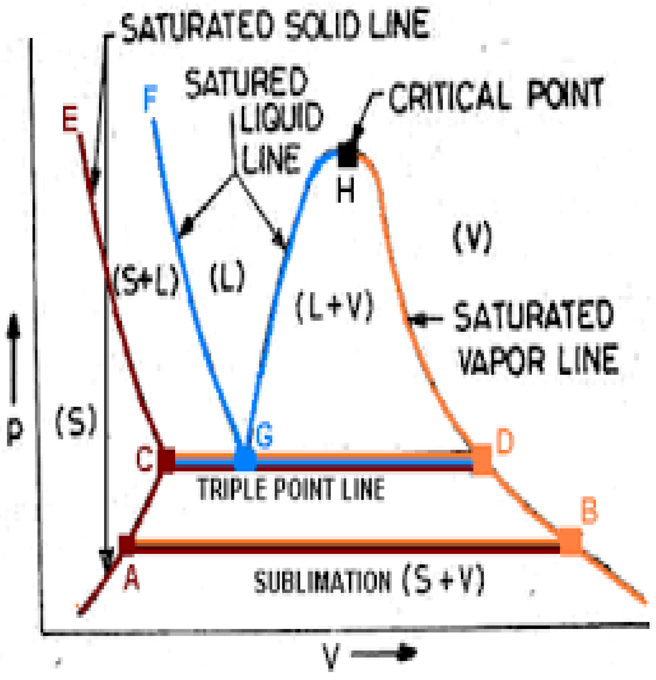
The work obtained at the end of the expansion is very less. The work is too inadequate to overcome the friction. Therefore the adiabatic expansion is terminated at the point before the end of the expansion in the turbine and pressure decreases suddenly, while the volume remains constant.

IMPROVE THERMAL EFFICIENCY OF THE RANKINE CYCLE.

- ✓ Reheating of steam
- ✓ Regenerative feed water heating
- ✓ By water extraction
- ✓ Using binary vapour

PROPERTIES OF PURE SUBSTANCE:





TYPES OF STEAM:

WET STEAM: **From Steam Table at given dryness fraction and Pressure (or Temperature)**

$v_f = ?$, $v_g = ?$, $h_f = ?$, $h_{fg} = ?$, $s_f = ?$, $s_{fg} = ?$

$$v = v_f + (xv_{fg})$$

$$h = h_f + (xh_{fg})$$

$$s = s + (xs_{fg})$$

Table 1 for given temperature
or
Table 2 for given pressure

if Dryness fraction is not given

QUALITY OF THE MIXTURE,

$$x = \frac{m_g}{m_g + m_f}$$ or find out from above formula

DRY OR SATURATED STEAM: **From Steam Table at given Pressure or Temperature**

$$v = v_f + v_{fg} \Rightarrow v_g$$

$$h = h_f + h_{fg} \Rightarrow h_g$$

$$s = s + s_{fg} \Rightarrow s_g$$

Table 1 for given temperature
or
Table 2 for given pressure

SUPERHEATED STEAM: **From Steam Table at given Pressure and Temperature**

$$v_{sup} = \frac{v_g}{T_s} \text{ X } T_{sup} \Rightarrow$$ Table 3

$$h = h_g + C_P(T_{Sup} - T_{Sat}) \Rightarrow$$ Table 4

$$s = s_g + C_P \ln\left(\frac{T_{Sup}}{T_{Sat}}\right) \Rightarrow$$ Table 5

For superheated steam, **use formula or Table**
(To find out T_{sup} , use formula which is easier)

Interpolate the valve and find exact one if not available in steam table
Ex: Temperature vs Entropy (Find T)

$$\frac{T_r - T_a}{T_b - T_a} = \frac{s_r - s_a}{s_b - s_a}$$

IF A VESSEL CONTAINS WATER AND VAPOR

The total mass of mixture, $m = m_f + m_g$

Total Volume $V = V_f + V_g \Rightarrow V = (m_f \times v_f) + (m_g \times v_g)$

Volume of liquid, $V_f = m_f \times v_f$

Volume of vapour $V_g = m_g \times v_g$

Mass of vapour, $m_f = \frac{V_f}{v_f}$

Mass of vapour, $m_g = \frac{V_g}{v_g}$

QUALITY OF THE MIXTURE,

$$x = \frac{m_g}{m_g + m_f}$$

THE SPECIFIC INTERNAL ENERGY, U :

$$u = h - pv$$

FROM SUPERHEATED STEAM TO WET STEAM EXPANSION:

From Superheated steam tables, At 10 bar & 300°C.

$h_{sup} =$ … … … …

$v_{sup} =$ … … … … …

INTERNAL ENERGY OF SUPERHEATED STEAM

$$u_1 = h_{sup} - pv_{sup}$$

From steam tables, at exit pressure (wet steam)

$v_g = ?$, $h_f = ?$, $h_{fg} = ?$,

ENTHALPY OF WET STEAM (AFTER EXPANSION)

$$h_2 = h_{f2} + x_2 h_{fg2}$$

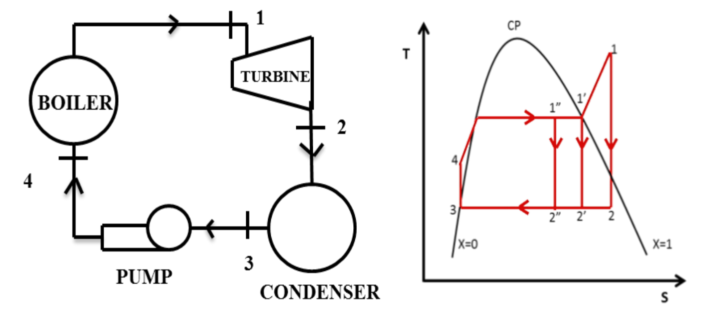
INTERNAL ENERGY OF THIS STEAM,

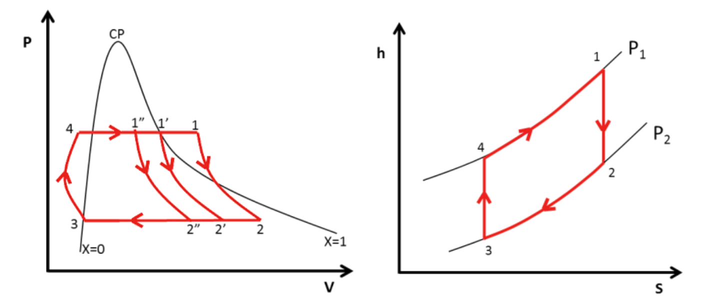
$$u_2 = h_2 - p_2 x_2 v_{g2}$$

HENCE CHANGE OF INTERNAL ENERGY PER KG

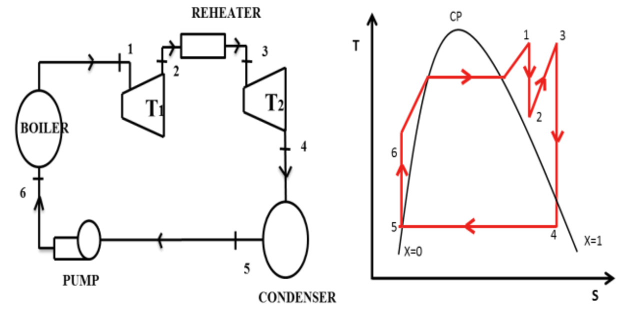
$$\Delta u = (u_2 - u_1)$$

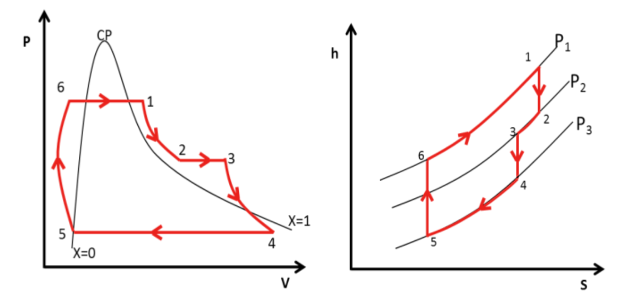
SIMPLE RANKINE CYCLE



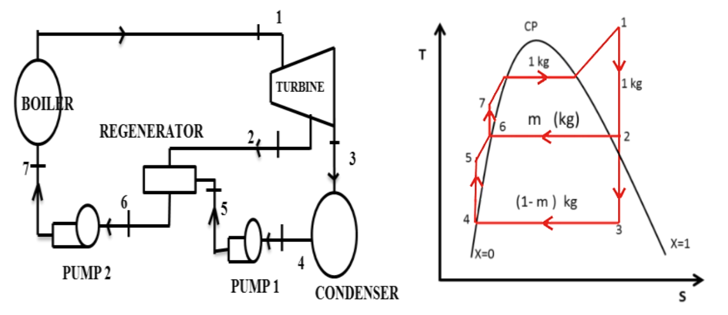


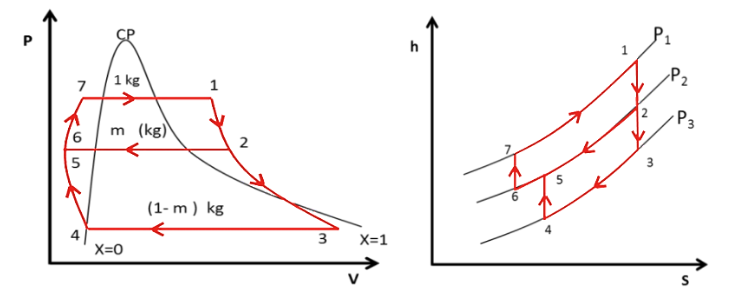
REHEAT RANKINE CYCLE





REGENERATION RANKINE CYCLE





COMMON CONDITIONS TO THREE CYCLES

NOTE:

1. Boiler oulet steam condition may be Dry saturated or Super heated steam which enters to the Turbine.(Use saturated (h_g) or superheated steam table (Table 3,4,5) to find out properties.)
2. Turbine exit steam condition is wet steam which enters to the Condenser. (Use formulas $\left[s = s + \left(x s_{fg}\right)\right]$ to find out Dryness Fraction).
3. **For Reheater and Regenerative cycle**, Turbine exit at state 2, steam condition may be Dry Saturated Or Superheated Condition.
4. Condenser outlet condition is Saturated liquid always which enters to the Pump.(at a given Pressure, the enthalpy h_f)
5. Pump outlet condition is subcooled liquid which enters to the boiler. (Ex: Simple Rankine Cycle, Use formulas $\left[W_p = V_{f3}(P_4 - P_3)\right]$ to find out pump work and for enthalpy $h_4 = W_p + h_3 \left(\frac{kJ}{kg}\right)$. (not possible to use steam table at this condition.)

NOTATIONS IN THE STEAM TABLE:

f – Liquid water
g – Dry or sautated vapour
fg – During the phase change from liquid to vapor

NOTE: (FOR REHEAT AND REGENERATION AT STATE 2)

Turbine inlet entropy = Turbine exit entropy

$$(s_1 = s_2)$$

at a given turbine exit pressure find entropy in the saturated pressure table s_{g2}

if

$s_{g2} = s_2$ then the steam is dry saturated steam

$s_{g2} < s_2$ then the steam is superheated steam

$s_{g2} > s_2$ then the steam is wet steam

SIMPLE RANKINE CYCLE

Process 1-2: Reversible adiabatic expansion $(s_1 = s_2)$

$$W_T = h_1 - h_2 \left(\frac{kJ}{kg}\right)$$

h_1 – Turbine inlet enthalpy

h_2 – Turbine exit and condenser intet enthalpy

Process 2-3:Constant pressure heat rejection $(P_2 = P_3)$

$$Q_R = h_2 - h_3 \left(\frac{kJ}{kg}\right)$$

$h_3 = h_{f2} = h_{f3}$ Condenser exit and pump inlet enthalpy

Process 3-4: Reversible adiabatic pumping $(s_3 = s_4)$

$$W_p = h_4 - h_3 \left(\frac{kJ}{kg}\right)$$

h_4 – Pump exit & boiler inlet enthalpy

$$W_p = V_{f3}(P_4 - P_3) \left(\frac{kJ}{kg}\right)$$

$$W_p = V_{f3}(P_1 - P_2) \left(\frac{kJ}{kg}\right)$$

$$h_4 = W_p + h_3 \left(\frac{kJ}{kg}\right)$$

Process 4-1: Constant pressure heat supplied $(P_4 = P_1)$

$$Q_s = h_1 - h_4 \left(\frac{kJ}{kg}\right)$$

Efficiency :

$$\eta = \frac{W_T - W_p}{Q_s} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4}$$

REHEAT RANKINE CYCLE

Process 1-2: Reversible adiabatic expansion $(s_1 = s_2)$

$$W_{T1} = h_1 - h_2 \left(\frac{kJ}{kg}\right)$$

h_1 – Turbine1 inlet enthalpy

h_2 – Turbine exit and reheater intet enthalpy

Process 2-3: Constant pressure reheating $(P_2 = P_3)$

$$Q_{s1} = h_3 - h_2 \left(\frac{kJ}{kg}\right)$$

h_3 – Turbine1 inlet enthalpy

Process 3-4: Reversible adiabatic expansion $(s_3 = s_4)$

$$W_{T2} = h_3 - h_4 \left(\frac{kJ}{kg}\right)$$

h_4 – Turbine exit and condenser intet enthalpy

Process 4-5:Constant pressure heat rejection $(P_4 = P_5)$

$$Q_R = h_4 - h_5 \left(\frac{kJ}{kg}\right)$$

$h_5 = h_{f4} = h_{f5}$ Condenser exit *enthalpy*

Process 5-6:Reversible adiabatic pumping $(s_5 = s_6)$

$$W_p = h_6 - h_5 \left(\frac{kJ}{kg}\right)$$

h_6 – Pump exit & boiler inlet *enthalpy*

$$W_p = V_{f5}(P_6 - P_5) \left(\frac{kJ}{kg}\right)$$

$$h_6 = W_p + h_5 \left(\frac{kJ}{kg}\right)$$

Process 6-1:Constant pressure heat supplied $(P_6 = P_1)$

$$Q_{s2} = h_1 - h_6 \left(\frac{kJ}{kg}\right)$$

Efficiency :

$$\eta = \frac{W_{T1} + W_{T2} - W_p}{Q_{s1} + Q_{s2}}$$

$$\eta = \frac{(h_1 - h_2) + (h_4 - h_3) - (h_6 - h_5)}{(h_3 - h_2) + (h_1 - h_6)}$$

REGENERATION RANKINE CYCLE

Turbine Work: Reversible adiabatic expansion $(s_1 = s_2 = s_3)$

$$W_T = 1kg(h_1 - h_2) + (1 - m)(h_2 - h_3)$$

h_1 – Turbine1 inlet enthalpy

h_2 – bypass regeneration enthalpy

Process 3-4: Constant pressure heat rejection $(P_3 = P_4)$

$$Q_R = h_3 - h_4 \left(\frac{kJ}{kg}\right)$$

h_3 – Turbine exit and condenser intet enthalpy

$h_4 = h_{f3} = h_{f4}$ Condenser exit *enthalpy*

To find bypass steam mass: energy balance

$$m((h_2 - h_6) = (1 - m)(h_6 - h_5)$$

Process 4-5: Reversible adiabatic pumping $(s_4 = s_5)$

$$W_{p1} = (1 - m)h_4 - h_5 \left(\frac{kJ}{kg}\right)$$

Process 6-7: Reversible adiabatic pumping $(s_6 = s_7)$

$$W_{p2} = 1kg\,(h_7 - h_6) \left(\frac{kJ}{kg}\right)$$

$$W_p = V_{f6}(P_7 - P_6) \left(\frac{kJ}{kg}\right)$$

$$h_7 = W_p + h_6 \left(\frac{kJ}{kg}\right)$$

Process 7-1: Constant Pressure Heat Supplied $(P_7 = P_1)$

$$Q_{s1} = 1kg\,(h_1 - h_7) \left(\frac{kJ}{kg}\right)$$

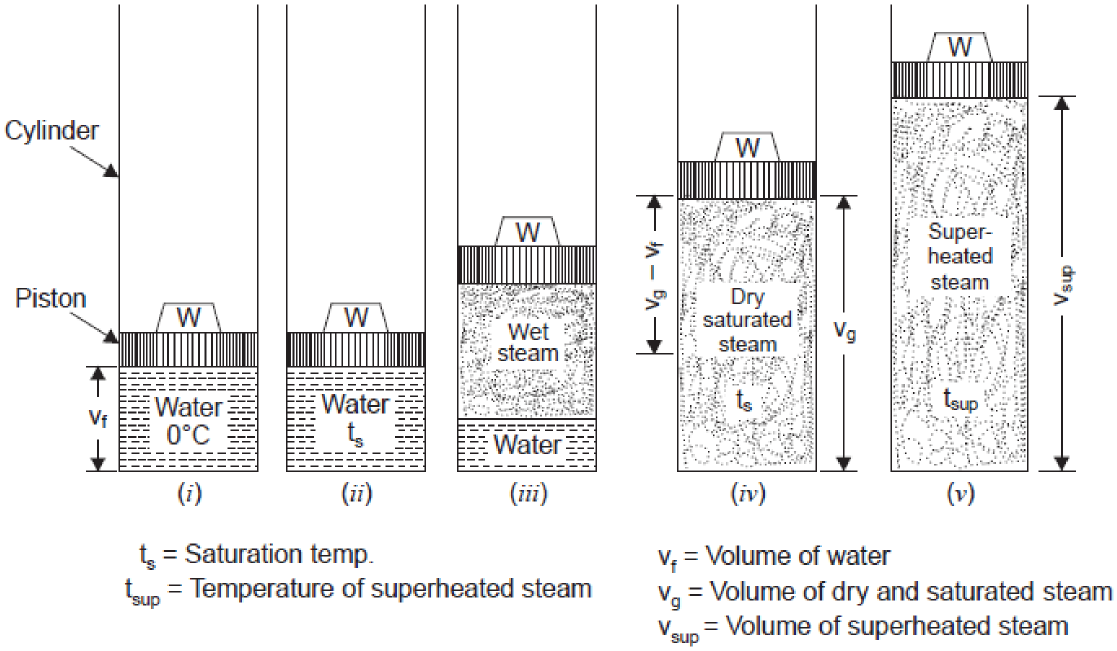
Efficiency :

$$\eta = \frac{W_T}{Q_s} = \frac{1kg(h_1 - h_2) + (1 - m)(h_2 - h_3)}{1kg(h_1 - h_7)}$$

Note: Here Pump work is negligible

PART - B

1. Explain steam formation with relevant sketch and label all salient points and explain every point in detail.



- ➢ Consider a cylinder fitted with a piston which can move freely upwards and downwards in it.
- ➢ Let, for the sake of simplicity, there be 1 kg of water at 0°C with volume v_f (m^3) under the piston. Further let the piston is loaded with load W to ensure heating at constant pressure.
- ➢ Now if the heat is imparted to water, a rise in temperature will be noticed and this rise will continue till boiling point is reached.
- ➢ The temperature at which water starts boiling depends upon the pressure and as such for each pressure (under which water is heated) there is a different boiling point.
- ➢ This boiling temperature is known as the temperature of formation of steam or saturation temperature.
- ➢ It may be noted during heating up to boiling point that there will be slight increase in volume of water due to which piston moves up and hence work is obtained as shown in Fig. (ii).
- ➢ This work, however, is so small that is can be neglected. Now, if supply of heat to water is continued it will be noticed that rise of temperature after the boiling point is reached nil but piston starts moving upwards which indicates that there is increase is volume which is only possible if steam formation occurs.
- ➢ The heat being supplied does not show any rise of temperature but changes water into vapour state (steam) and is known as latent heat or hidden heat.
- ➢ So long as the steam is in contact with water, it is called wet steam [Fig. (iii)] and if heating of steam is further progressed [as shown in Fig. (iv)] such that all the water particles associated with steam are evaporated, the steam so obtained is called dry and saturated steam.

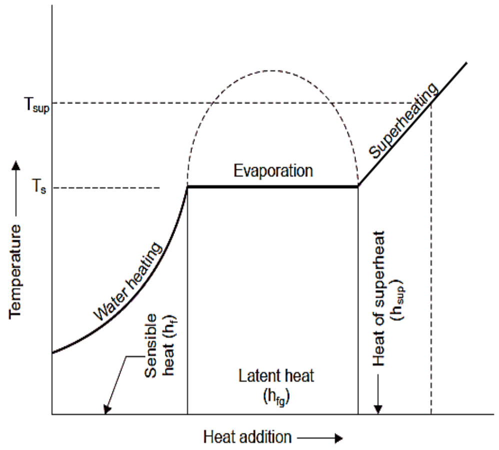
- If v_g (m^3) is the volume of 1 kg of dry and saturated steam then work done on the piston will be

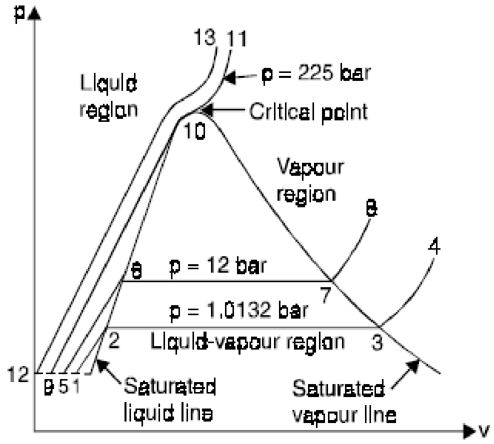
$$\mathbf{p(v_g - v_f)}\ .$$

where,

p is the constant pressure (due to weight ‘W’ on the piston).

- Again, if supply of heat to the dry and saturated steam is continued at constant pressure there will be increase in temperature and volume of steam. The steam so obtained is called superheated steam and it behaves like a perfect gas. This phase of steam formation is illustrated in Fig. (v).





- The amount of heat required to convert the liquid water completely into vapour under this condition is called the **heat of vapourisation**. The temperature at which vapourisation takes place at a given pressure is called the **saturation temperature** and the given pressure is called the **saturation pressure.**
- The pressure on the liquid water is greater than the saturation pressure at a given temperature. In this condition, the liquid water is known as the **compressed liquid.**
- When the temperature increases above the saturation temperature (in this case 100°C), the vapour is known as the **superheated vapour** and the temperature at this state is called the **superheated temperature.**
- The difference between the superheated temperature and the saturation temperature at the given pressure is called the **degree of superheat.**
- The specific volume of the saturated liquid and of the saturated vapour is the same, i.e., vf = vg. Such a state of the substance is called the **critical state.** The parameters like temperature, pressure, volume, etc. At such a state are called **critical parameters.**
- When the pressure is greater than the critical pressure. At this state, the liquid water is directly converted into superheated steam. As there is no definite point at which the liquid water changes into superheated steam, it is generally called liquid water when the temperature is less than the critical temperature and superheated steam when the temperature is above the critical temperature.

2. **A vessel having a capacity of 0.05 m^3 contains a mixture of saturated water and saturated steam at a temperature of 245°C. The mass of the liquid present is 10 kg. Find the following, (i) The pressure, (ii) The mass, (iii) The specific volume, (iv) The specific enthalpy, (v) The specific entropy, and (vi) The specific internal energy.**

GIVEN: $V=0.05\ m^3$, $T_1=245°C$, $m_f=10$ kg

FIND : (i) p (ii) m (iii) v (iv) h (v) s (vi) u

SOLUTION:

From steam tables, corresponding to 245°C :

$p_{sat} = 36.5$ bar, $v_f = 0.001239\ m^3/kg$, $v_g = 0.0546\ m^3/kg$

$h_f = 1061.4$ kJ/kg, $h_{fg} = 1740.2$ kJ/kg,

$s_f = 2.7474$ kJ/kg K $s_{fg} = 3.3585$ kJ/kg K.

(I) THE PRESSURE **'P' = 36.5 bar**

(II) THE MASS

Volume of liquid, $V_f = m_f \times v_f \Rightarrow V_f = 10 \times 0.001239 \Rightarrow \mathbf{V_f = 0.01239\ m^3}$

Volume of vapour $V_g = V - V_f \Rightarrow V_g = 0.05 - 0.01239 \Rightarrow \mathbf{V_g = 0.03761\ m^3}$

$\therefore$ Mass of vapour, $m_g = \frac{V_g}{v_g} \Rightarrow m_g = \frac{0.03761}{0.0546} \Rightarrow \mathbf{m_g = 0.688\ kg}$

$\therefore$ The total mass of mixture, $m = m_f + m_g \Rightarrow m = 10 + 0.688 \Rightarrow \mathbf{m = 10.688\ kg}$

(III) THE SPECIFIC VOLUME

Quality of the mixture,

$$x = \frac{m_g}{m_g + m_f} \Rightarrow x = \frac{0.688}{0.688+10} \Rightarrow \mathbf{x = 0.064}$$

$$v = v_f + xv_{fg} \Rightarrow v = 0.001239 + 0.064 \times (0.0546 - 0.001239) \Rightarrow \mathbf{v = 0.004654 m^3/kg}$$

(IV) THE SPECIFIC ENTHALPY

$$h = h_f + xh_{fg} \Rightarrow h = 1061.4 + (0.064 \times 1740.2\,) \Rightarrow \mathbf{h = 1172.77\ kJ/kg}$$

(V) THE SPECIFIC ENTROPY

$$s = s + xs_{fg} \Rightarrow s = 2.7474 + (0.064 \times 3.3585\,) \Rightarrow \mathbf{s = 2.9623\ kJ/kg\ K}$$

(VI) THE SPECIFIC INTERNAL ENERGY

$$u = h - pv \Rightarrow u = 1172.77 - 36.5 \times 10^2 \times 0.004654 \Rightarrow \mathbf{u = 1155.78 kJ/kg}$$

3. **A pressure cooker contains 1.5 kg of saturated steam at 5 bar. Find the quantity of heat which must be rejected so as to reduce the quality to 60% dry. Determine the pressure and temperature of the steam at the new state.**

GIVEN: **m**= 1.5 kg , p = 5 bar, $x_1 = 1$, $x_2 = 0.6$

FIND: Pressure and temperature of the steam at the new state :

SOLUTION:

From steam tables, At 5 bar.

$t_s = 151.8°C$, $h_f = 640.1$ kJ/kg , $h_{fg} = 2107.4$ kJ/kg, $v_g = 0.375\ m^3/kg$

INTERNAL ENERGY OF STEAM per kg at initial point 1,

$$u_1 = h_1 - p_1 v_1 \Rightarrow u_1 = \left(h_f + h_{fg}\right) - p_1 v_{g1}$$

$$u_1 = (640.1 + 2107.4) - 5 \times 10^5 \times 0.375 \times 10^{-3}) \Rightarrow u_1 = 2747.5 - 187.5$$

$$\mathbf{u_1 = 2560\ kJ/kg}$$

THE VOLUME OF PRESSURE COOKER, $\mathbf{V_1} = m \times v_g \Rightarrow \mathbf{V_1} = 1.5 \times 0.375 \Rightarrow \mathbf{V_1 = 0.5625m^2}$

We know that $\mathbf{V_1 = V_2,\ v_{f2}}$ is negligible

$$V_2 = m\left[(1 - x_2)v_{f2} + x_2 v_{g2}\right] \Rightarrow V_2 = 1.5x_2 v_{g2} \Rightarrow 0.5625 = 1.5 \times 0.6 \times v_{g2} \Rightarrow \mathbf{v_{g2} = 0.625\ \frac{m^3}{kg}}$$

From steam tables at $\mathbf{v_{g2} = 0.625}$

$\mathbf{p_2 = 2.9\ bar}$, $\mathbf{t_2 = 132.4}$ °C, $h_{f2} = 556.5$ kJ/kg , $h_{fg2} = 2166.6$ kJ/kg

INTERNAL ENERGY OF STEAM per kg, at final point 2,

$$u_2 = h_2 - p_2 v_2 \Rightarrow u_2 = \left(h_{f2} + h_{fg2}\right) - p_2 v_{g2}$$

$$u_2 = (556.5 + 2166.6) - 2.9 \times 10^5 \times 0.625 \times 10^{-3}) \Rightarrow u_2 = 1856.46 - 108.75$$

$$\mathbf{u_2 = 1747.71\ kJ/kg}$$

HEAT TRANSFERRED AT CONSTANT VOLUME PER KG

$$Q = \Delta u + W \Rightarrow Q = \Delta u \Rightarrow Q = u_2 - u_1 \Rightarrow Q = 1747.71 - 2560 \Rightarrow \mathbf{Q = -\ 812.29\ kJ/kg}$$

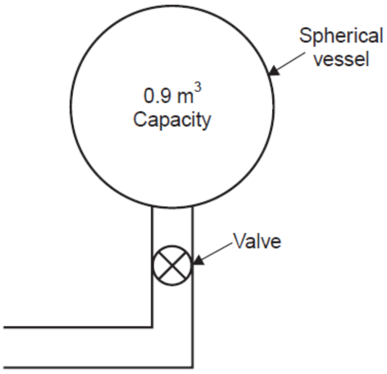
TOTAL HEAT TRANSFERRED

$$Q = m \times Q \Rightarrow Q = -\ 812.29 \times 1.5 \Rightarrow \mathbf{Q = -\ 1218.43\ kJ}$$

4. **A spherical vessel of 0.9 m^3 capacity contains steam at 8 bar and 0.9 dryness fraction. Steam is blown off until the pressure drops to 4 bar. The valve is then closed and the steam is allowed to cool until the pressure falls to 3 bar. Assuming that the enthalpy of steam in the vessel remains constant during blowing off periods, determine :(i) The mass of steam blown off (ii) The dryness fraction of steam in the vessel after cooling ;(iii) The heat lost by steam per kg during cooling.**

GIVEN: V = 0.9 m^3, p_1 = 8 bar, x_1 = 0.9, p_2 = 4 bar, p_3 = 3 bar

FIND : (i) m (ii) x_2, (iii) Q



SOLUTION:

From steam tables, At 8 bar.

$h_{f1} = 720.9$ kJ/kg , $h_{fg1} = 2046.5$ kJ/kg, $v_{g1} = 0.24\ m^3/kg$

(I) THE MASS OF STEAM BLOWN OFF :

$$V_1 = m_1 x_1 v_{g1} \Rightarrow m_1 = \frac{V_1}{x_1 v_{g1}} = \frac{0.9}{0.9 \times 0.24} \Rightarrow \mathbf{m_1 = 4.167\ kg}$$

THE ENTHALPY OF STEAM BEFORE BLOWING OFF (per kg)

$$h_1 = h_{f1} + x h_{fg1} \Rightarrow h_1 = 720.9 + (0.9 \times 2046.5\ ) \Rightarrow \mathbf{h_1 = 2562.75\frac{kJ}{kg} = h_2}$$

From steam tables, At 4 bar.

$h_{f1} = 604.7$ kJ/kg , $h_{fg1} = 2133$ kJ/kg, $v_{g1} = 0.462\ m^3/kg$

ENTHALPY BEFORE BLOWING OFF = ENTHALPY AFTER BLOWING OFF

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow 2562.75 = 604.7 + (x_2 \times 2133) \Rightarrow \mathbf{x_2 = 0.918}$$

NOW THE MASS OF STEAM IN THE VESSEL AFTER BLOWING OFF,

$$V_2 = m_2 x_2 v_{g2} \Rightarrow m_2 = \frac{V_2}{x_2 v_{g2}} = \frac{0.9}{0.918 \times 0.462} \Rightarrow \mathbf{m_2 = 2.122\ kg}$$

Mass of steam blown off, $m = m_1 + m_2 \Rightarrow m = 4.167 - 2.122 \Rightarrow \mathbf{m = 2.045\ kg}$

From steam tables, At 4 bar.

$h_{f3} = 604.7$ kJ/kg , $h_{fg3} = 2133$ kJ/kg, $v_{g3} = 0.606\ m^3/kg$

(II) DRYNESS FRACTION OF STEAM IN THE VESSEL AFTER COOLING

As it is constant volume cooling

$$x_2 v_{g2} = x_3 v_{g3} \Rightarrow 0.918 \times 0.462 = x_3 \times 0.606 \Rightarrow \mathbf{x_3 = 0.699}$$

(III) HEAT LOST DURING COOLING

$$u_2 = h_2 - p_2 x_2 v_{g2} \Rightarrow u_2 = (h_{f2} + x_2 h_{fg2}) - p_2 x_2 v_{g2}$$

$$u_2 = (604.7 + 0.918 \times 2133) - 4 \times 10^5 \times 0.918 \times 0.462 \times 10^{-3})$$

$$u_2 = 2562.79 - 169.65 \Rightarrow \mathbf{u_2 = 2393.14\ kJ/kg}$$

$$u_3 = h_3 - p_3 x_3 v_{g3} \Rightarrow u_3 = (h_{f3} + x_3 h_{fg3}) - p_3 x_3 v_{g3}$$

$$u_3 = (561.4 + 0.669 \times 2163.2) - 3 \times 10^5 \times 0.699 \times 0.606 \times 10^{-3})$$

$$u_3 = 2073.47 - 127.07 \Rightarrow \mathbf{u_3 = 1946.4\ kJ/kg}$$

HEAT TRANSFERRED DURING COOLING

$$Q = m(u_2 - u_1) \Rightarrow Q = 2.045(1946.4 - 2393.14) \Rightarrow \mathbf{Q = -\ 913.6\ kJ/kg}$$

5. **Calculate the internal energy per kg of superheated steam at a pressure of 10 bar and a temperature of 300°C. Also find the change of internal energy if this steam is expanded to 1.4 bar and dryness fraction 0.8.**

GIVEN: $p_1 = 10$ bar, $T_1 = 300°C$, $x_1 = 0.8$, $p_2 = 1.4$ bar

FIND : (i) Δu

SOLUTION:

From steam tables, At 10 bar

$v_g = 0.194\ m^3/kg$, $T_s = 179.9 + 273 = 452.9K$

From Superheated steam tables, At 10 bar & 300°C.

$h_{sup} = 3051.2$ kJ/kg ,

$$v_{sup} = \frac{v_g}{T_s} X\, T_{sup} \Rightarrow v_{sup} = \frac{0.194}{452.9} X\, 573 \Rightarrow v_{sup} = 0.245\ m^3/kg$$

INTERNAL ENERGY OF SUPERHEATED STEAM AT 10 BAR,

$$u_1 = h_{sup} - p v_{sup} \Rightarrow u_1 = 3051.2 - 10 \times 10^5 \times 0.245 \times 10^{-3} \Rightarrow \mathbf{u_1 = 2806.2\ kJ/kg}$$

From steam tables, at 1.4 bar :

$v_g = 1.236\ m^3/kg$, $h_f = 458.4$ kJ/kg, $h_{fg} = 2231.9$ kJ/kg,

$s_f = 2.7474$ kJ/kg K, $s_{fg} = 3.3585$ kJ/kg K.

ENTHALPY OF WET STEAM (AFTER EXPANSION)

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h_2 = 458.4 + (0.8 \times 2231.9) \Rightarrow \mathbf{h_2 = 2243.92\,\frac{kJ}{kg}}$$

INTERNAL ENERGY OF THIS STEAM,

$$u_2 = h_2 - p_2 x_2 v_{g2} \Rightarrow u_2 = 2243.92 - (1.4 \times 10^{5} \times 0.8 \times 1.236 \times 10^{-3})$$

$$\mathbf{u_2 = 2105.49\ kJ/kg}$$

HENCE CHANGE OF INTERNAL ENERGY per kg

$$\Delta u = (u_2 - u_1) \Rightarrow \Delta u = (2105.49 - 2806.2) \Rightarrow \mathbf{\Delta u = -\ 700.7\ kJ/kg}$$

Negative sign indicates decrease in internal energy.

6. Steam initially at 0.3 Mpa, 250°C is cooled at constant volume. At what temperature will the steam become saturated vapour? What is the steam quality at 80°C? Also find what is the heat transferred per kg of steam in cooling from 25°C to 80°C.

From Steam Table

At P_1=30 bar and T_1=250 °C

v_1=0.7964 m^3/kg, h_1=2967.6 kJ/kg

we know that , $v_1 = v_2 = v_3$=0.7964 m^3/kg

From steam Table

At $v_2 = v_g$ = 0.7964 m^3/kg

v_g = 0.8919 m^3/kg, T_{sat}=120°C

v_g = 0.7706 m^3/kg, T_{sat}=125°C

$$\frac{T_r - T_b}{T_a - T_b} = \frac{v_r - v_b}{v_a - v_a} \Rightarrow \frac{T_r - 120}{125 - 120} = \frac{0.7964 - 0.8919}{0.7706 - 0.8919} \Rightarrow \mathbf{T_{sat} = 123.9}\text{℃}$$

State 2

From Steam Table

At $T_2 = 80$℃

v_f = 0.001029 m^3/kg , v_g = 3.0407 m^3/kg, h_f = 334.91 kJ/kg, h_{fg} = 2308.8 kJ/kg, p_{sat} = 47.039bar

$$v_2 = v_{f2} + x_2 v_{fg2} \Rightarrow 0.7964 = 0.001029 + x\ (3.0407 - 0.001029) \Rightarrow \mathbf{x_2 = 0.234}$$

THE SPECIFIC ENTHALPY

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h = 334.91 + (0.234 \times 2308.8) \Rightarrow \mathbf{h_2 = 875.9\ kJ/kg}$$

THE SPECIFIC INTERNAL ENERGY

$$u_1 = h_1 - p_1 v_1 \Rightarrow u_1 = 2967.6 - 30 \times 10^2 \times 0.7964 \Rightarrow \mathbf{u_1 = 578.4\ kJ/kg}$$

$$u_2 = h_2 - p_2 v_2 \Rightarrow u_2 = 875.9 - 4.71 \times 10^2 \times 0.7964 \Rightarrow \mathbf{u_2 = -2875.1 kJ/kg}$$

HEAT TRANSFER

From first law of thermodynamics

$$Q_{1-2} = \Delta U + W_{1-2} \quad , \quad W_{1-2} = 0$$

Then, $Q_{1-2} = \Delta U$

$$Q_{1-2} = \Delta U \Rightarrow Q_{1-2} = U_2 - U_1 \Rightarrow Q_{1-2} = -2875.1 - 578.4 \Rightarrow \mathbf{Q_{1-2} = -3453.5\ kJ/kg}$$

7. **Steam at 30 bar and 350 °C is expanded in a non-flow isothermal process to a pressure of 1 bar. The temperature and pressure of the surroundings are 25 °C and 100 kPa respectively. Determine the maximum work that can be obtained from this process per kg of steam. Also find the maximum useful work.**

GIVEN : Non-flow system (Closed system)

SYSTEM AT STATE 1:

From steam table

p_1= 30 bar and T_1=350 °C (super-heated steam since sat T= 233.8°C for 30 bar sat pr.)

$v_1 = 0.09053\ m^3/kg$

SYSTEM AT STATE 2:

From steam table

p_2= 1 bar and $T_2 = T_1 = 350$ °C (super-heated steam since sat T= 99.63°C for 1 bar sat pr.)

$v_1 = 2.871\ m^3/kg$

Surrounding is at are 25 °C and 100 kPa

Consider mass of steam as 1 kg. Find maximum work that can be obtained from this process per kg of steam.

$$\mathrm{W_{max}} = \mathrm{Pv}\ln\left(\frac{v_2}{v_1}\right) \Rightarrow \mathrm{W_{max}} = 30 \times 10^2 \times 0.09053 \ln\left(\frac{2.871}{0.09053}\right) \Rightarrow \mathbf{W_{max}} = \mathbf{938.8\,\frac{kJ}{kg}}$$

Maximum useful work. [reversible adiabatic (isentropic) work]

From steam table

p_1= 30 bar and T_1=350 °C (super-heated steam since sat T= 233.8°C for 30 bar sat pr.)

$h_1 = 3117.5$ kJ/kg

$s_1 = 6.747$ kJ/kg.k

From steam tables at 1 bar

$s_{f2} = 1.303\ \mathrm{kJ/kgK}$, $s_{fg2} = 6.057\ \mathrm{kJ/kgK}$,

$h_{f2} = 417.5\ \mathrm{kJ/kg}$ $h_{fg2} = 2257.9\ \mathrm{kJ/kg}$,

ENTHALPY OF WET STEAM (AFTER EXPANSION) $(s_1 = s_2)$

$$s_1 = s_2 = s_{f2} + x_2 s_{fg2} \Rightarrow 6.747 = 1.303 + (\mathbf{x_2} \times 6.057) \Rightarrow \mathbf{x_2 = 0.8987}$$

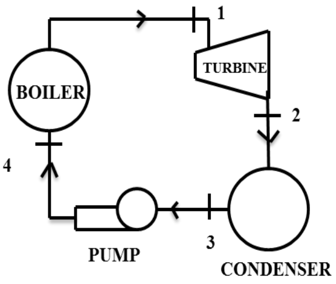
$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h_2 = 417.5 + (0.8987 \times 2257.9) \Rightarrow \mathbf{h_2 = 2446.67\,\frac{kJ}{kg}}$$

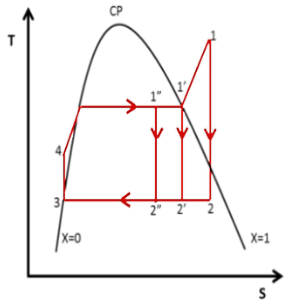
MAXIMUM USEFUL WORK:

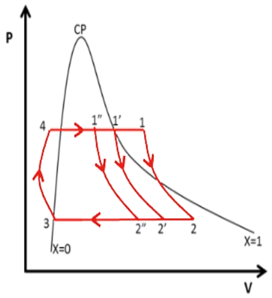
$$w_{1-2} = h_1 - h_2 \Rightarrow w_{1-2} = 3117.5 - 2446.67 \Rightarrow \mathbf{w_{1-2} = 670.83\,\frac{kJ}{kg}}$$

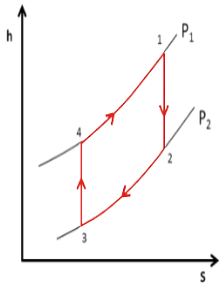
RANKINE CYCLE

1. **Draw the P-v, T-s, h -s, diagrams and theoretical lay out for Rankine cycle and hence deduce the expression for its efficiency.**









Process 1-2: Turbine Work : Reversible adiabatic expansion ($s_1 = s_2$)

$$W_T = h_1 - h_2 \ (\frac{kJ}{kg})$$

h_1 – Turbine inlet (**wet or dry or superheated steam**) enthalpy

h_2 – Turbine exit and condenser intet (**wet or dry steam**) enthalpy

Process 2-3: Condenser heat rejection : Constant pressure heat rejection (wet or dry steam convert into liquid) ($P_2 = P_3$)

$$Q_R = h_2 - h_3 \ (\frac{kJ}{kg})$$

$h_3 = h_{f2} = h_{f3}$ Condenser exit and pump inlet (saturated liquid) *enthalpy*

Process 3-4: Pump Work: Reversible adiabatic pumping ($s_3 = s_4$)

$$W_p = h_3 - h_4 \ (\frac{kJ}{kg})$$

h_4 – Pump exit & boiler inlet (subcooled liquid) *enthalpy*

$$W_p = V_{f3}(P_4 - P_3) \ (\frac{kJ}{kg})$$

$$h_4 = W_p + h_3 \ (\frac{kJ}{kg})$$

Process 4-1: Boiler heat supplied (Constant pressure heat supplied) : Subcooled liquid to dry or superheated steam ($P_4 = P_1$)

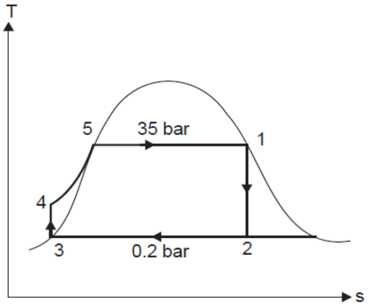
$$Q_s = h_1 - h_4 \ (\frac{kJ}{kg})$$

Efficiency :

$$\eta = \frac{W_T - W_p}{Q_s}$$

$$= \frac{(h_1 - h_2) - (h_3 - h_4)}{h_1 - h_4}$$

2. **In a Rankine cycle, the steam at inlet to turbine is saturated at a pressure of 35 bar and the exhaust pressure is 0.2 bar. Determine : (i) The turbine work (ii) The condenser heat flow, (iii) The pump work, (iv) Heat Supplied to the boiler, (v) Network done, (vi)The Rankine efficiency, (vii) The dryness at the end of expansion, (viii) Carnot efficiency (ix) SSC, (x)Work ratio, (xi) Turbine power output if flow rate of 9.5 kg/s.**



GIVEN: $p_1 = 35$ bar, $p_2 = 0.2$ bar

FIND : (i) W, (ii) η

SOLUTION:

State 1

From steam tables at 35 bar

$s_1 = s_g = 6.1228$ kJ/kgK, $h_1 = h_g = 2802$ kJ/kg

State 2

From steam tables at 0.2 bar

$s_{f2} = 0.8321$ kJ/kgK , $s_{fg2} = 7.0773$kJ/kgK ,

$h_{f2} = 251.5$ kJ/kg $h_{fg2} = 2358.4$ kJ/kg ,

$v_{f2} = 0.001017$ m^3/kg

Enthalpy of wet steam (after expansion) $(\mathbf{s_1 = s_2})$

$$s_1 = s_2 = s_{f2} + x_2 s_{fg2} \Rightarrow 6.1228 = 0.8321 + (\mathbf{x_2} \times 7.0773) \Rightarrow \mathbf{x_2 = 0.747}$$

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h_2 = 251.5 + (0.747 \times 2358.4) \Rightarrow \mathbf{h_2 = 2013 \frac{kJ}{kg}}$$

State 3

$$h_3 = h_{f2} = h_{f3} = \mathbf{251.5 \frac{kJ}{kg}}$$

State 4

$$W_p = V_{f3}(P_4 - P_3) \Rightarrow W_p = 0.001017\ (35 - 0.2) \times 10^2 \Rightarrow \mathbf{W_p = 3.54\ kJ/kg}$$

$$h_4 = W_p + h_3 \Rightarrow h_4 = 3.54 + 251.5 \Rightarrow \mathbf{h_4 = 255.04\ kJ/kg}$$

PROCESS 1-2: TURBINE WORK : REVERSIBLE ADIABATIC EXPANSION

$$W_T = (h_1 - h_2) \Rightarrow W_T = (2802 - 2013) \Rightarrow \mathbf{W_T = 789 \frac{kJ}{kg}}$$

PROCESS 2-3: CONDENSER HEAT REJECTION: CONSTANT PRESSURE HEAT REJECTION $(P_2 = P_3)$

$$Q_R = h_2 - h_3 \Rightarrow Q_R = 2013 - 251.5 \Rightarrow \mathbf{Q_R = 1761.5 \frac{kJ}{kg}}$$

PROCESS 3-4: PUMP WORK: REVERSIBLE ADIABATIC PUMPING $(s_3 = s_4)$

$$W_p = V_{f3}(P_4 - P_3) \Rightarrow W_p = 0.001017\ (35 - 0.2) \times 10^2 \Rightarrow \mathbf{W_p = 3.54\ kJ/kg}$$

PROCESS 4-1: BOILER HEAT SUPPLIED: CONSTANT PRESSURE HEAT SUPPLIED $(P_4 = P_1)$

$$Q_s = h_1 - h_4 \Rightarrow Q_s = 2802 - 255.04 \Rightarrow \mathbf{Q_s = 2546.96\ kJ/kg}$$

NET WORK DONE:

$$W_{Net} = W_T - W_p \Rightarrow W_{Net} = 789 - 3.54 \Rightarrow \mathbf{W_{Net} = 785.46\ kJ/kg}$$

EFFICIENCY :

$$\eta = \frac{W_{Net}}{Q_s} \Rightarrow \eta = \frac{785.46}{2546.96} \Rightarrow \boldsymbol{\eta = 30.84\%}$$

CARNOT EFFICIENCY:

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \eta = 1 - \frac{333.9}{515.5} \Rightarrow \boldsymbol{\eta = 35\%}$$

SPECIFIC STEAM CONSUMPTION:

$$SSC = \frac{3600}{W_{net}} \Rightarrow SSC = \frac{3600}{785.46} \Rightarrow \mathbf{SSC = 4.58\ kg/kWh}$$

WORK RATIO:

$$W_r = \frac{W_{net}}{W_T} \Rightarrow W_r = \frac{785.46}{789} \Rightarrow \boldsymbol{W_r = 0.995}$$

TURNINE POWER OUTPUT:

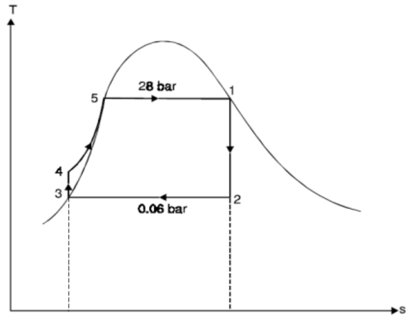
$$P = \dot{m}(h_1 - h_2) \Rightarrow P = 9.5(2802 - 2013) \Rightarrow \mathbf{P = 7495.5\ kW}$$

3. **A simple Rankine cycle works between pressures 28 bar and 0.06 bar, the initial condition of steam being dry saturated. Calculate the cycle efficiency, work ratio and specific steam consumption.**

GIVEN: $p_1 = 28$ bar, $p_2 = 0.06$ bar

FIND : (i) W_r , (ii) η, (iii) SSC

SOLUTION:



State 1

From Superheated steam tables at 28 bar

$s_1 = s_g = 6.2104$ kJ/kgK, $h_1 = h_g = 2802$ kJ/kg

State 2

From steam tables at 0.06 bar

$s_{f2} = 0.521$ kJ/kgK , $s_{fg2} = 7.809$ kJ/kgK ,

$h_{f2} = 151.5$ kJ/kg $h_{fg2} = 2415.9$ kJ/kg ,

$v_{f2} = 0.001 m^3/kg$

Enthalpy of wet steam (after expansion) $(\mathbf{s_1 = s_2})$

$$s_1 = s_2 = s_{f2} + x_2 s_{fg2} \Rightarrow 6.2104 = 0.521 + (\mathbf{x_2} \times 7.809) \Rightarrow \mathbf{x_2 = 0.728}$$

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h_2 = 151.5 + (0.728 \times 2415.9) \Rightarrow \mathbf{h_2 = 1910.27\,\frac{kJ}{kg}}$$

State 3

$$h_3 = h_{f2} = h_{f3} = \mathbf{151.5\,\frac{kJ}{kg}}$$

State 4

$$W_p = V_{f3}(P_4 - P_3) \Rightarrow W_p = 0.001\ (28 - 0.06) \times 10^2 \Rightarrow \mathbf{W_p = 2.79\ kJ/kg}$$

$$h_4 = W_p + h_3 \Rightarrow h_4 = 2.79 + 151.5 \Rightarrow \mathbf{h_4 = 154.29\ kJ/kg}$$

PROCESS 1-2: TURBINE WORK : REVERSIBLE ADIABATIC EXPANSION

$$W_T = (h_1 - h_2) \Rightarrow W_T = (2802 - 1910.27) \Rightarrow \mathbf{W_T = 891.73\,\frac{kJ}{kg}}$$

PROCESS 2-3: CONDENSER HEAT REJECTION: CONSTANT PRESSURE HEAT REJECTION $(P_2 = P_3)$

$$Q_R = h_2 - h_3 \Rightarrow Q_R = 1910.27 - 151.5 \Rightarrow \mathbf{Q_R = 1758.77\ \frac{kJ}{kg}}$$

PROCESS 3-4: PUMP WORK: REVERSIBLE ADIABATIC PUMPING $(s_3 = s_4)$

$$W_p = V_{f3}(P_4 - P_3) \Rightarrow W_p = 0.001\ (28 - 0.06) \times 10^2 \Rightarrow \mathbf{W_p = 2.79\ kJ/kg}$$

PROCESS 4-1: BOILER HEAT SUPPLIED: CONSTANT PRESSURE HEAT SUPPLIED $(P_4 = P_1)$

$$Q_s = h_1 - h_4 \Rightarrow Q_s = 2802 - 154.29 \Rightarrow \mathbf{Q_s = 2647.71\ kJ/kg}$$

NET WORK DONE:

$$W_{Net} = W_T - W_p \Rightarrow W_{Net} = 891.73 - 2.79 \Rightarrow \mathbf{W_{Net} = 888.94\ kJ/kg}$$

EFFICIENCY :

$$\eta = \frac{W_{Net}}{Q_s} \Rightarrow \eta = \frac{888.94}{2647.71} \Rightarrow \boldsymbol{\eta = 33.57\%}$$

SPECIFIC STEAM CONSUMPTION:

$$SSC = \frac{3600}{W_{net}} \Rightarrow SSC = \frac{3600}{888.94} \Rightarrow \mathbf{SSC = 4.049\ kg/kWh}$$

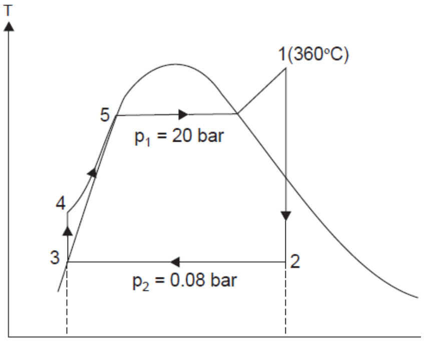
WORK RATIO:

$$W_r = \frac{W_{net}}{W_T} \Rightarrow W_r = \frac{888.94}{891.73} \Rightarrow \mathbf{W_r = 0.997}$$

4. **In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam the net work and the cycle efficiency.**

GIVEN: $p_1 = 20$ bar, $T_1 = 360°C$, $p_2 = 0.08$ bar

FIND : (i) W, (ii) η



SOLUTION:

From Superheated steam tables at 20 bar, 360°C

By interpolation

$$\frac{h_R - h_b}{h_a - h_b} = \frac{T_R - T_b}{T_a - T_b} \Rightarrow \frac{h_R - 3138.6}{3248.7 - 3138.6} = \frac{360 - 350}{400 - 350} \Rightarrow \mathbf{h_r = 3159.3\ \frac{kJ}{kg} = h_1}$$

$$\frac{s_R - s_b}{s_a - s_b} = \frac{T_R - T_b}{T_a - T_b} \Rightarrow \frac{s_R - 6.960}{7.130 - 6.960} = \frac{360 - 350}{400 - 350} \Rightarrow \mathbf{s_r = 6.991\ \frac{kJ}{kg.k} = s_1}$$

From steam tables at 0.08 bar

$s_{f2} = 0.5926$ kJ/kgK , $s_{fg2} = 7.6361$ kJ/kgK ,

$h_{f2} = 173.88$ kJ/kg $h_{fg2} = 2403.1$ kJ/kg ,

$v_{f2} = 0.001008\ m^3/kg$

Enthalpy of wet steam (after expansion) $(s_1 = s_2)$

$$s_2 = s_{f2} + x_2 s_{fg2} \Rightarrow 6.991 = 0.5926 + (\mathbf{x_2} \times 7.6361) \Rightarrow \mathbf{x_2 = 0.838}$$

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h_2 = 173.88 + (0.838 \times 2403.1) \Rightarrow \mathbf{h_2 = 2187.68\ \frac{kJ}{kg}}$$

PROCESS 1-2: TURBINE WORK : REVERSIBLE ADIABATIC EXPANSION

$$W_T = (h_1 - h_2) \Rightarrow W_T = (3159.3 - 2187.68) \Rightarrow \mathbf{W_T} = 971.62\ \frac{\mathbf{kJ}}{\mathbf{kg}}$$

PROCESS 2-3: CONDENSER HEAT REJECTION: CONSTANT PRESSURE HEAT REJECTION $(P_2 = P_3)$

$$Q_R = h_2 - h_3 \Rightarrow Q_R = 2187.62 - 173.88 \Rightarrow \mathbf{Q_R = 2013.74\ \frac{kJ}{kg}}$$

$h_3 = h_{f2} = h_{f3}$ Condenser exit and pump inlet (saturated liquid) enthalpy

PROCESS 3-4: PUMP WORK: REVERSIBLE ADIABATIC PUMPING $(s_3 = s_4)$

$$W_p = V_{f3}(P_4 - P_3) \Rightarrow W_p = 0.00108\ (20 - 0.08) \times 10^2 \Rightarrow \mathbf{W_p = 2.008\ kJ/kg}$$

$$h_4 = W_p + h_3 \Rightarrow h_4 = 2.008 + 173.88 \Rightarrow \mathbf{h_4 = 175.89\ kJ/kg}$$

h_4 – Pump exit & boiler inlet (subcooled liquid) enthalpy

PROCESS 4-1: BOILER HEAT SUPPLIED: CONSTANT PRESSURE HEAT SUPPLIED $(P_4 = P_1)$

$$Q_s = h_1 - h_4 \Rightarrow Q_s = 3159.3 - 175.89 \Rightarrow \mathbf{Q_s = 2983.41\ kJ/kg}$$

NET WORK DONE:

$$W_{Net} = W_T - W_p \Rightarrow W_{Net} = 971.62 - 2.008 \Rightarrow \mathbf{W_{Net} = 969.61\ kJ/kg}$$

EFFICIENCY :

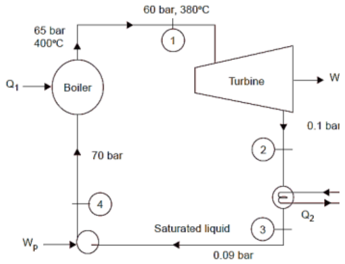
$$\eta = \frac{W_{Net}}{Q_s} \Rightarrow \eta = \frac{969.61}{2983.41} \Rightarrow \mathbf{\eta = 32.5\%}$$

5. **The following data refer to a simple steam power plant :**

   Calculate :(i) Power output of the turbine, (ii) Heat transfer per hour in the boiler and condenser separately, (iii) Mass of cooling water circulated per hour in the condenser. Choose the inlet temperature of cooling water 20°C and 30°C at exit from the condenser, (iv) Diameter of the pipe connecting turbine with condenser.

S. No.	*Location*	*Pressure*	*Quality/temp.*	*Velocity*
1.	*Inlet to turbine*	*6 MPa (= 60 bar)*	*380°C*	—
2.	*Exit from turbine inlet to condenser*	*10 kPa (= 0.1 bar)*	*0.9*	*200 m/s*
3.	*Exit from condenser and inlet to pump*	*9 kPa (= 0.09 bar)*	*Saturated liquid*	—
4.	*Exit from pump and inlet to boiler*	*7 MPa (= 70 bar)*	—	—
5.	*Exit from boiler*	*6.5 MPa (= 65 bar)*	*400°C*	—
	Rate of steam flow = 10000 kg/h.			

SOLUTION:



(I) POWER OUTPUT OF THE TURBINE, P :

From Superheated steam tables

At 60 bar, 380°C (380°C is not available in steam table)

By interpolation

$$\frac{h_R - h_b}{h_a - h_b} = \frac{T_R - T_b}{T_a - T_b} \Rightarrow \frac{h_R - 3045.8}{3180.1 - 3045.8} = \frac{380 - 350}{400 - 350} \Rightarrow \mathbf{h_r = 3123.5\ \frac{kJ}{kg} = h_1}$$

From steam tables at 0.1 bar

$h_{f2} = 191.8\ kJ/kg$

$h_{fg2} = 2392.8\ kJ/kg$, $x_2 = 0.9$

$v_{g2} = 14.67\ m^3/kg$

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h_2 = 191.8 + (0.9 \times 2392.8) \Rightarrow \mathbf{h_2 = 2345.3\ kJ/kg}$$

$$W_T = m(h_1 - h_2) \Rightarrow W_T = \frac{10000}{3600}(3123.5 - 2392.8) \Rightarrow \mathbf{W_T = 2162kW}$$

HEAT TRANSFER PER HOUR IN THE BOILER AND CONDENSER :

From steam tables at 70 bar

$h_{f4} = 1267.4$ kJ/kg

From Superheated steam tables at 65 bar, 400°C

$h_{a1} = 3167.6$ kJ/kg (by interpolation)

HEAT TRANSFER PER HOUR IN THE BOILER,

$$Q_1 = m(h_{a1} - h_{f4}) \Rightarrow Q_1 = \frac{10000}{3600}(3167.6 - 1267.4) \Rightarrow \mathbf{Q_1 = 5277.78kW}$$

From steam tables at 70 bar

$h_{f3} = 183.3$ kJ/kg

HEAT TRANSFER PER HOUR IN THE CONDENSER,

$$Q_2 = m(h_2 - h_{f3}) \Rightarrow Q_2 = \frac{10000}{3600}(2345.3 - 183.3) \Rightarrow \mathbf{Q_2 = 6000\ kW}$$

MASS OF COOLING WATER CIRCULATED PER HOUR IN THE CONDENSER, m_w :

Heat lost by steam = Heat gained by the cooling water

$$Q_2 = m_w C_P (t_2 - t_1) \Rightarrow Q_2 = m_w \times 4.18(30 - 20) \Rightarrow \mathbf{m_w = 3100\ kg/s}$$

DIAMETER OF THE PIPE CONNECTING TURBINE WITH CONDENSER, d :

$$m_s = \rho AC \Rightarrow m_s = \frac{AC}{x_2 v_{g2}} \Rightarrow \frac{m_s x_2 v_{g2}}{C} = \frac{\pi}{4} d^2 \Rightarrow \frac{2.78 \times 0.9 \times 14.7}{200} = \frac{\pi}{4} d^2 \Rightarrow \mathbf{d = 483mm}$$

6. A Rankine cycle operates between pressures of 80 bar and 0.1 bar. The maximum cycle temperature is 600°C. If the steam turbine and condensate pump efficiencies are 0.9 and 0.8 respectively, calculate the specific work and thermal efficiency. Relevant steam table extract is given below.

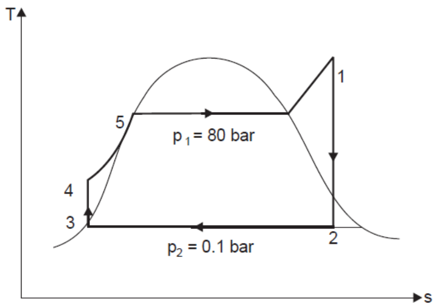
p(bar)	t(°C)	Specific volume (m³/kg)		Specific enthalpy (kJ/kg)			Specific entropy (kJ/kg K)		
		v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
0.1	45.84	0.0010103	14.68	191.9	2392.3	2584.2	0.6488	7.5006	8.1494
80	295.1	0.001385	0.0235	1317	1440.5	2757.5	3.2073	2.5351	5.7424

80 bar, 600°C	v	$0.486\ m^3/kg$
Superheat	h	3642 kJ/kg
table	s	7.0206 kJ/kgK

GIVEN: $p_1 = 80$ bar, $T_1 = 600°C$, $p_2 = 0.1$ bar

FIND : (i) W, (ii) η

SOLUTION:



State 1

From Superheated steam tables at 80 bar, 600°C

$s_1 = 7.0206$ kJ/kgK, $h_1 = 3642$ kJ/kg , $v_{g2} = 0.486\ m^3/kg$

State 2

From steam tables at 0.1 bar

$s_{f2} = 0.6488 \text{ kJ/kgK}, \quad s_{fg2} = 7.5006 \text{ kJ/kgK},$

$h_{f2} = 191.9 \text{ kJ/kg} \quad h_{fg2} = 2392.3 \text{ kJ/kg},$

$v_{f2} = 0.0010103 \text{ m}^3\text{/kg}$

Enthalpy of wet steam (after expansion)

$$s_2 = s_{f2} + x_2 s_{fg2} \Rightarrow 7.0206 = 0.6488 + (\mathbf{x_2} \times 7.006) \Rightarrow \mathbf{x_2 = 0.85}$$

$$h_2 = h_{f2} + x_2 h_{fg2} \Rightarrow h_2 = 191.9 + (0.85 \times 2392.3) \Rightarrow \mathbf{h_2 = 2225.36\ \frac{kJ}{kg}}$$

State 3

$$h_3 = h_{f2} = h_{f3} = \mathbf{191.9\frac{kJ}{kg}}$$

State 4

$$W_p = V_{f3}(P_4 - P_3) \Rightarrow W_p = 0.486\,(60 - 0.1) \times 10^2 \Rightarrow \mathbf{W_p = 8.072\ kJ/kg}$$

$$\text{Actual pump work} = \frac{8.072}{\eta_{pump}} = \frac{8.072}{0.8} \Rightarrow \mathbf{W_p = 10.09\ kJ/kg}$$

$$h_4 = W_p + h_3 \Rightarrow h_4 = 10.09 + 191.9 \Rightarrow \mathbf{h_4 = 201.99\ kJ/kg}$$

PROCESS 1-2: TURBINE WORK : REVERSIBLE ADIABATIC EXPANSION

$$W_T = (h_1 - h_2) \Rightarrow W_T = (3642 - 2225.36) \Rightarrow \mathbf{W_T = 1275\ \frac{kJ}{kg}}$$

PROCESS 2-3: CONDENSER HEAT REJECTION: CONSTANT PRESSURE HEAT REJECTION $(P_2 = P_3)$

$$Q_R = h_2 - h_3 \Rightarrow Q_R = 2225.36 - 191.9 \Rightarrow \mathbf{Q_R = 2033.76\ \frac{kJ}{kg}}$$

PROCESS 3-4: PUMP WORK: REVERSIBLE ADIABATIC PUMPING $(s_3 = s_4)$

$$\mathbf{W_p = 10.09\ kJ/kg}$$

PROCESS 4-1: BOILER HEAT SUPPLIED: CONSTANT PRESSURE HEAT SUPPLIED $(P_4 = P_1)$

$$Q_s = h_1 - h_4 \Rightarrow Q_s = 3642 - 201.99 \Rightarrow \mathbf{Q_s = 3440.01\ kJ/kg}$$

NET WORK DONE:

$$W_{Net} = W_T - W_p \Rightarrow W_{Net} = 1275 - 10.09 \Rightarrow \mathbf{W_{Net} = 1264.91\ kJ/kg}$$

EFFICIENCY :

$$\eta = \frac{W_{Net}}{Q_s} \Rightarrow \eta = \frac{1264.91}{3440.01} \Rightarrow \mathbf{\eta = 36.8\%}$$

SPECIFIC STEAM CONSUMPTION:

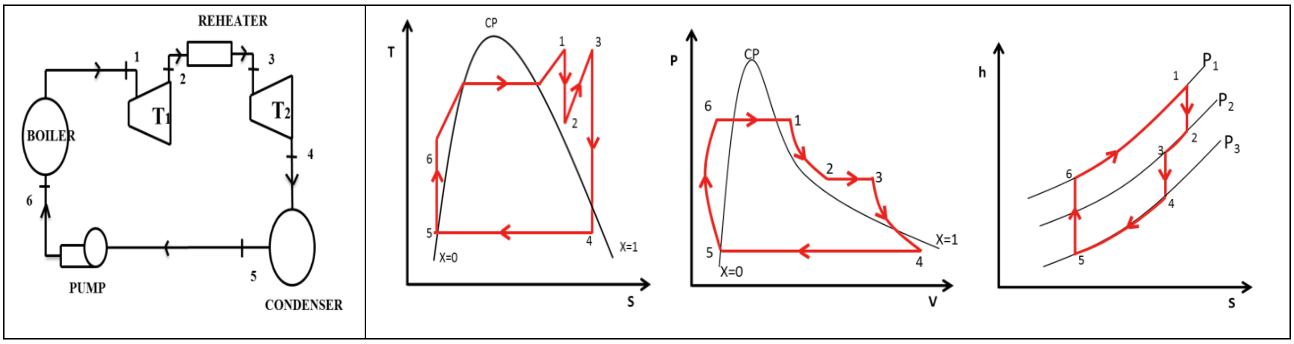
$$SSC = \frac{3600}{W_{net}} \Rightarrow SSC = \frac{3600}{1264.91} \Rightarrow \mathbf{SSC = 2.85\ kg/kWh}$$

WORK RATIO:

$$W_r = \frac{W_{net}}{W_T} \Rightarrow W_r = \frac{1264.91}{1275} \Rightarrow \boldsymbol{W_r = 0.992}$$

7. Draw the P-V, T-S, h -s, diagrams and theoretical lay out for Reheat Rankine cycle and hence deduce the expression for its efficiency.

SOLUTION:



PROCESS 1-2: TURBINE WORK : REVERSIBLE ADIABATIC EXPANSION $(s_1 = s_2)$

$$\mathbf{W_{T1} = h_1 - h_2} \left(\frac{kJ}{kg}\right)$$

h_1 − Turbine1 inlet (**dry or superheated steam**) enthalpy

h_2 − Turbine exit and reheater intet (**wet or dry steam**) enthalpy

PROCESS 2-3: CONSTANT PRESSURE REHEATING$(P_2 = P_3)$

$$\mathbf{Q_{s1} = h_3 - h_2} \left(\frac{kJ}{kg}\right)$$

h_3 − Turbine1 inlet (**dry or superheated steam**) enthalpy

PROCESS 3-4: TURBINE WORK2 : REVERSIBLE ADIABATIC EXPANSION $(s_3 = s_4)$

$$\mathbf{W_{T2} = h_3 - h_4} \left(\frac{kJ}{kg}\right)$$

h_4 − Turbine exit and condenser intet (**wet or dry steam**) enthalpy

PROCESS 4-5: CONDENSER HEAT REJECTION: CONSTANT PRESSURE HEAT REJECTION $(P_4 = P_5)$

$$\mathbf{Q_R = h_4 - h_5} \left(\frac{kJ}{kg}\right)$$

$h_5 = h_{f4} = h_{f5}$ Condenser exit (saturated liquid) enthalpy

PROCESS 5-6: PUMP WORK: REVERSIBLE ADIABATIC PUMPING $(s_5 = s_6)$

$$\mathbf{W_p = h_6 - h_5} \left(\frac{kJ}{kg}\right)$$

h_6 − Pump exit & boiler inlet (subcooled liquid) enthalpy

$$\mathbf{W_p = V_{f5}(P_6 - P_5)} \left(\frac{kJ}{kg}\right)$$

$$\mathbf{h_6 = W_p + h_5} \left(\frac{kJ}{kg}\right)$$

PROCESS 6-1: BOILER HEAT SUPPLIED: CONSTANT PRESSURE HEAT SUPPLIED $(P_6 = P_1)$

$$\mathbf{Q_{s2} = h_1 - h_6} \left(\frac{kJ}{kg}\right)$$

EFFICIENCY :

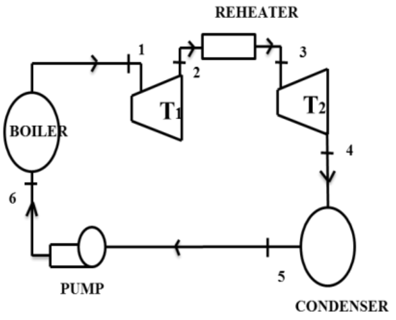
$$\eta = \frac{W_{T1} + W_{T2} - W_p}{Q_{s1} + Q_{s2}} = \frac{(h_1 - h_2) + (h_4 - h_3) - (h_6 - h_5)}{(h_3 - h_2) + (h_1 - h_6)}$$

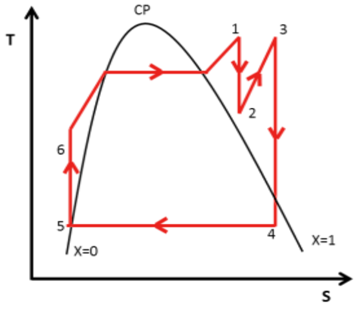
8. A reheat Rankine cycle receives steam at 35 bar and 0.1 bar. Steam enters the first stage steam turbine 350 °C. If reheating is done at 8 bar and 350 °C, calculate the specific steam consumption and reheat Rankine cycle efficiency.

GIVEN:

P_1=35 bar, T_1=350 °C, P_3=8 bar,

T_3=350 °C, P_4=0.1 bar





SOLUTION:

PROCESS 1-2: ADIABATIC EXPANSION PROCESS

From the superheated steam table at 35 bar and 350 °C

By interpolation

$$\frac{h_r-h_b}{h_a-h_b}=\frac{p_r-p_b}{p_a-p_b} \Rightarrow \frac{h_r-3104.2}{3108.7-3104.2}=\frac{35-34}{36-34} \Rightarrow \mathbf{h_r = 3106.45\frac{kJ}{kg} = h_1}$$

$$\frac{s_r-s_b}{s_a-s_b}=\frac{p_r-p_b}{p_a-p_b} \Rightarrow \frac{s_r-6.647}{6.679-6.647}=\frac{35-34}{36-34} \Rightarrow \mathbf{s_r = 6.663\frac{kJ}{kg.k} = s_1}$$

From saturated steam table at 8 bar

$$s_g = 6.660\frac{kJ}{kg.K}$$

Where, $\mathbf{s_1} = \mathbf{s_2} = s_g$ **,** so the exit of turbine is saturated steam

From saturated steam table at 8 bar

$$\mathbf{h_2 = 2767.4\frac{kJ}{kg} = h_1}$$

TURBINE WORK

$$w_{T1} = h_1 - h_2 \Rightarrow w_{T1} = 3106.45 - 2767.4 \Rightarrow \mathbf{w_{T1} = 339.05 kJ/kg}$$

PROCESS 2-3: CONSTANT PRESSURE HEAT ADDITION

From superheated steam table at 8 bar and 350 °C

$$\mathbf{h_3 = 3162.4\frac{kJ}{kg},\quad s_3 = 7.411\frac{kJ}{kg.K}}$$

HEAT SUPPLIED TO REHEATER

$$q_{s2} = h_3 - h_2 \Rightarrow q_{s2} = 3162.4 - 2767.4 \Rightarrow \mathbf{q_{s2} = 395\frac{kJ}{kg.K}}$$

PROCESS 3-4: ADIABATIC EXPANSION PROCESS IN TURBINE 2: $s_3 = s_4$

From saturated steam table at 0.1 bar

$$s_{g4} = 8.151\frac{kJ}{kg.K}$$

$\mathbf{s_3} = \mathbf{s_4} < s_{g4}$, so the exit of turbine is wet steam

From saturated steam table at 0.1 bar

$$s_{f4} = 0.649\frac{kJ}{kg.K},\quad s_{fg4} = 7.502\frac{kJ}{kg.K},\quad h_{f4} = 191.8\frac{kJ}{kg},\quad h_{fg4} = 2392.8\frac{kJ}{kg}$$

$$s_4 = s_{f4} + x_4 s_{fg4} \Rightarrow 7.411 = 0.649 + (\mathbf{x_4} \times 7.502) \Rightarrow \mathbf{x_4 = 0.9}$$

$\mathrm{h_4 = h_{f4} + x_4 h_{fg4}} \Rightarrow \quad \mathrm{h_4 = 191.8 + (0.9 \times 2392.8)} \Rightarrow \quad \mathbf{h_4 = 2345.32\frac{kJ}{kg}}$

$\mathrm{w_{T2} = h_3 - h_4} \Rightarrow \quad \mathrm{w_{T2} = 3162.4 - 2345.32} \Rightarrow \quad \mathbf{w_{T2} = 817.08 kJ/kg}$

PROCESS 4-5: CONSTANT PRESSURE HEAT REJECTION

$\mathbf{h_{f4} = h_{f5} = h_5 = 191.8\ kJ/kg}$

PROCESS 5-6: ADIABATIC PUMPING

$\mathrm{W_p = v_{f4}}(P_6 - P_5) \Rightarrow \quad \mathrm{W_p = 0.001010 \times (35 - 0.1) \times 100} \Rightarrow \mathbf{W_p = 3.53\ kJ/kg}$

$\mathrm{W_p = h_6 - h_5} \Rightarrow \mathrm{3.53 = h_6 - 191.8} \Rightarrow \quad \mathbf{h_6 = 195.33\ kJ/kg}$

PROCESS 5-6: CONSTANT PRESSURE HEAT ADDITION

$\mathrm{q_{s1} = h_1 - h_6} \Rightarrow \quad \mathrm{q_{s1} = 3106.45 - 195.33} \Rightarrow \quad \mathbf{q_{s1} = 2911.12\frac{kJ}{kg.K}}$

RANKINE CYCLE EFFICIENCY

$\eta = \frac{\mathrm{W_{T1}+W_{T2}-W_p}}{\mathrm{Q_{s1}+Q_{s2}}} = \frac{339.05+817.08-3.53}{395+2911.12} \Rightarrow \quad \boldsymbol{\eta = 34.86\%}$

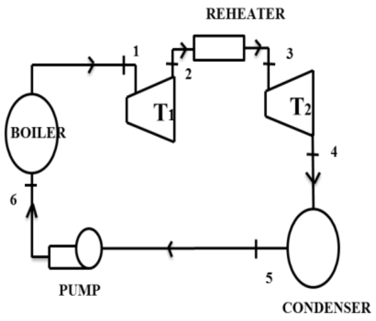
SPECIFIC STEAM CONSUMPTION

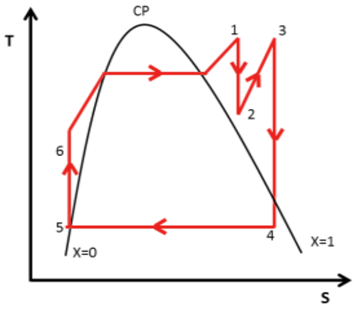
$SSC = \frac{3600}{W_{net}} \Rightarrow \quad SSC = \frac{3600}{339.05+817.08} \Rightarrow \quad \mathbf{SSC = 3.12\ kg/kWh}$

9. **A steam power plant operates on a theoretical reheat cycle. Steam at boiler at 150bar, 550 °C expands through the high pressure turbine. It is reheated at a constant pressure of 40 bar to 550 °C and expands through the low pressure turbine to a condenser at 0.1 bar. Draw T-s and h-s diagram. Find (i) Quality of steam at turbine exhaust (ii) Cycle efficiency (iii) Steam Rate in kg/kWh.**

GIVEN:

P_1=150 bar, T_1=550 ºC, P_3=40 bar, T_3=550 ºC, P_4=0.1 bar





SOLUTION:

PROCESS 1-2: ADIABATIC EXPANSION PROCESS

State 1

From the superheated steam table at 150 bar and 550 ºC

$\mathbf{h_1 = 3445.2\frac{kJ}{kg}} \quad , \quad \mathbf{s_1 = 6.5125\frac{kJ}{kg.k}}$

State 2

From saturated steam table at 40 bar

$\mathrm{s_g} = 6.069\frac{kJ}{kg.K}$

Where, $\mathbf{s_1 = s_2} > \mathrm{s_g}$ **,** so the exit of turbine is Superheated steam

From saturated steam table at $\mathbf{s_2 = 6.5125\frac{kJ}{kg.k}}$ **and 40 bar**

By interpolation

$\frac{T_r - T_b}{T_a - T_b} = \frac{s_r - s_b}{s_a - s_b}$ ⇨ $\frac{T_r - 300}{350 - 300} = \frac{6.5125 - 6.364}{6.587 - 6.364}$ ⇨ $\mathbf{T_r = 332℃ = T_2}$

$\frac{h_r - h_b}{h_a - h_b} = \frac{T_r - T_b}{T_a - T_b}$ ⇨ $\frac{h_r - 2962}{3095.1 - 2962} = \frac{332 - 300}{350 - 300}$ ⇨ $\mathbf{h_r = 3047.18 \frac{kJ}{kg} = h_2}$

TURBINE WORK

$w_{T1} = h_1 - h_2$ ⇨ $w_{T1} = 3445.2 - 3047.18$ ⇨ $\mathbf{w_{T1} = 398.02\ kJ/kg}$

PROCESS 2-3: CONSTANT PRESSURE HEAT ADDITION

From superheated steam table at 8 bar and 350 °C

$\mathbf{h_3 = 3558.9 \frac{kJ}{kg}}$, $\mathbf{s_3 = 7.229 \frac{kJ}{kg.K}}$

HEAT SUPPLIED TO REHEATER

$q_{s2} = h_3 - h_2$ ⇨ $q_{s2} = 3558.9 - 3047.18$ ⇨ $\mathbf{q_{s2} = 511.72 \frac{kJ}{kg.K}}$

PROCESS 3-4: ADIABATIC EXPANSION PROCESS IN TURBINE 2: $s_3 = s_4$

From saturated steam table at 0.1 bar

$$s_{g4} = 8.151 \frac{kJ}{kg.K}$$

$\mathbf{s_3 = s_4} < s_{g4}$, so the exit of turbine is wet steam

From saturated steam table at 0.1 bar

$s_{f4} = 0.649 \frac{kJ}{kg.K}$, $s_{fg4} = 7.502 \frac{kJ}{kg.K}$

$h_{f4} = 191.8 \frac{kJ}{kg}$, $h_{fg4} = 2392.8 \frac{kJ}{kg}$

$s_4 = s_{f4} + x_4 s_{fg4}$ ⇨ $7.411 = 0.649 + (\mathbf{x_4} \times 7.502)$ ⇨ $\mathbf{x_4 = 0.9}$

$h_4 = h_{f4} + x_4 h_{fg4}$ ⇨ $h_4 = 191.8 + (0.9 \times 2392.8)$ ⇨ $\mathbf{h_4 = 2345.32 \frac{kJ}{kg}}$

$w_{T2} = h_3 - h_4$ ⇨ $w_{T2} = 3558.9 - 2345.32$ ⇨ $\mathbf{w_{T2} = 1213.58\ kJ/kg}$

PROCESS 4-5: CONSTANT PRESSURE HEAT REJECTION

$$\mathbf{h_{f4} = h_{f5} = h_5 = 191.8\ kJ/kg}$$

PROCESS 5-6: ADIABATIC PUMPING

$W_p = v_{f4}(P_6 - P_5)$ ⇨ $W_p = 0.001010 \times (150 - 0.1) \times 100$ ⇨ $\mathbf{W_p = 15.14\ kJ/kg}$

$W_p = h_6 - h_5$ ⇨ $15.14 = h_6 - 191.8$ ⇨ $\mathbf{h_6 = 206.94\ kJ/kg}$

PROCESS 5-6: CONSTANT PRESSURE HEAT ADDITION

$q_{s1} = h_1 - h_6$ ⇨ $q_{s1} = 3445.2 - 206.94$ ⇨ $\mathbf{q_{s1} = 3238.26 \frac{kJ}{kg.K}}$

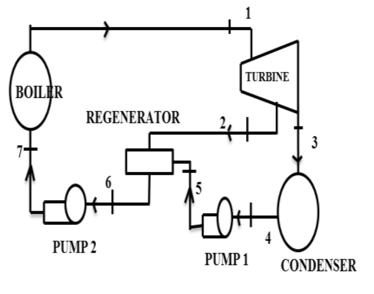
RANKINE CYCLE EFFICIENCY

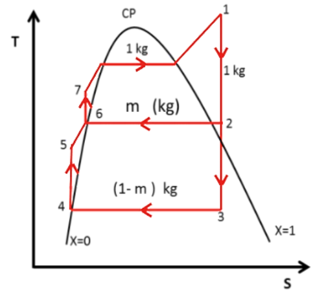
$\eta = \frac{W_{T1} + W_{T2} - W_p}{Q_{s1} + Q_{s2}} = \frac{398.02 + 1213.58 - 15.14}{511.72 + 3238.26}$ ⇨ $\mathbf{\eta = 42.57\%}$

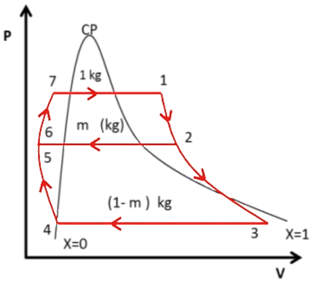
SPECIFIC STEAM CONSUMPTION

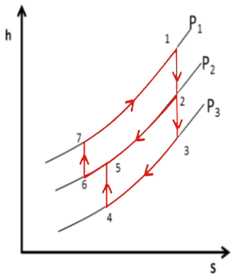
$SSC = \frac{3600}{W_{net}}$ ⇨ $SSC = \frac{3600}{398.02 + 1213.58 - 15.14}$ ⇨ $\mathbf{SSC = 2.254\ kg/kWh}$

10.Draw the P-V, T-S, h -s, diagrams and theoretical lay out for Regeneration Rankine cycle and hence deduce the expression for its efficiency.









PROCESS 1-2 & 1-3 TURBINE WORK

$$\mathbf{W_T = 1kg(h_1 - h_2) + (1 - m)(h_2 - h_3)}$$

h_1 – Turbine1 inlet enthalpy

h_2 – bypass regeneration enthalpy

PROCESS 3-4: CONDENSER HEAT REJECTION: CONSTANT PRESSURE HEAT REJECTION ($P_3 = P_4$)

$$\mathbf{Q_R = h_3 - h_4} \left(\frac{kJ}{kg}\right)$$

h_3 – Turbine exit and condenser intet enthalpy

$h_4 = h_{f3} = h_{f4}$ Condenser exit enthalpy

TO FIND BYPASS STEAM MASS: ENERGY BALANCE

$$\mathbf{m((h_2 - h_6) = (1 - m)(h_6 - h_5)}$$

PROCESS 4-5: PUMP WORK1: REVERSIBLE ADIABATIC PUMPING ($s_4 = s_5$)

$$\mathbf{W_{p1} = (1 - m)h_4 - h_5} \left(\frac{kJ}{kg}\right)$$

PROCESS 6-7: PUMP WORK2: REVERSIBLE ADIABATIC PUMPING ($s_6 = s_7$)

$$\mathbf{W_{p2} = 1kg\ (h_7 - h_6\ )} \left(\frac{kJ}{kg}\right)$$

$$\mathbf{W_p = V_{f6}(P_7 - P_6\ )} \left(\frac{kJ}{kg}\right)$$

$$\mathbf{h_7 = W_p + h_6} \left(\frac{kJ}{kg}\right)$$

PROCESS 7-1: BOILER HEAT SUPPLIED: CONSTANT PRESSURE HEAT SUPPLIED ($P_7 = P_1$)

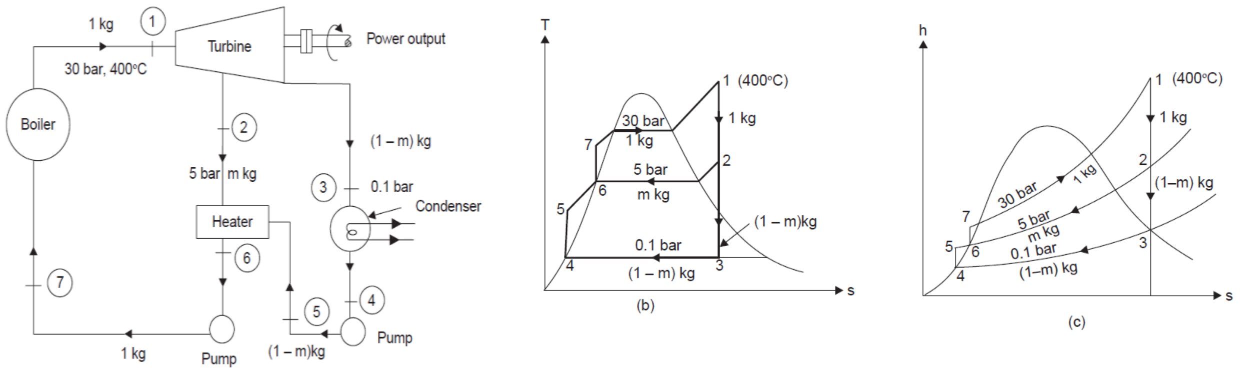
$$\mathbf{Q_{s1} = 1kg\ (h_1 - h_7\ )} \left(\frac{kJ}{kg}\right)$$

EFFICIENCY

$$\boldsymbol{\eta} = \frac{\mathbf{W_T}}{\mathbf{Q_s}} = \frac{\mathbf{1kg(h_1 - h_2) + (1 - m)(h_2 - h_3)}}{\mathbf{1kg(h_1 - h_7)}}$$

Note: Here Pump work is negligible

11.In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Find : (i) The efficiency and the steam rate of the cycle. (ii) The increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration). Pump work may be neglected.



(b)

(c)

PROCESS 1-2: ADIABATIC EXPANSION PROCESS

State 1

From the superheated steam table at 30 bar and 400 °C

$\mathbf{h_1 = 3232.5\frac{kJ}{kg}}$, $\mathbf{s_1 = 6.925\frac{kJ}{kg.k}}$

State 2

From saturated steam table at 5 bar

$$s_g = 6.819\frac{kJ}{kg.K}$$

Where, $\mathbf{s_1} = \mathbf{s_2} > s_g$ **,** so the exit of turbine is Superheated steam

From saturated steam table at $\mathbf{s_2 = 6.925\frac{kJ}{kg.k}}$ **and 5 bar**

By interpolation

$\frac{T_r - T_b}{T_a - T_b} = \frac{s_r - s_b}{s_a - s_b}$ ⇨ $\frac{T_r - 151.8}{200 - 151.8} = \frac{6.925 - 6.819}{7.059 - 6.819}$ ⇨ $\mathbf{T_r = 173.08°C = T_2}$

$\frac{h_r - h_b}{h_a - h_b} = \frac{T_r - T_b}{T_a - T_b}$ ⇨ $\frac{h_r - 2747.5}{2855.1 - 2747.5} = \frac{173.08 - 151.8}{200 - 151.8}$ ⇨ $\mathbf{h_r = 2795.02\frac{kJ}{kg} = h_2}$

State 3

From saturated steam table at 0.1 bar

$$s_{g3} = 8.151\frac{kJ}{kg.K}$$

$\mathbf{s_1} = \mathbf{s_3} < s_{g3}$, so the exit of turbine is wet steam

From saturated steam table at 0.1 bar

$s_{f3} = 0.649\ \frac{kJ}{kg.K}$, $s_{fg3} = 7.502\ \frac{kJ}{kg.K}$

$h_{f3} = 191.8\ \mathbf{\frac{kJ}{kg}}$, $h_{fg3} = 2392.8\ \mathbf{\frac{kJ}{kg}}$

$s_3 = s_{f3} + x_3 s_{fg3}$ ⇨ $6.925 = 0.649 + (\mathbf{x_3} \times 7.502)$ ⇨ $\mathbf{x_3}$ **=0.837**

$h_3 = h_{f3} + x_3 h_{fg3}$ ⇨ $h_3 = 191.8 + (0.837 \times 2392.8)$ ⇨ $\mathbf{h_3 = 2194.57\frac{kJ}{kg}}$

State 4 and 6

Since pump work is neglected

$\mathbf{h_4 = h_{f4} = 191.8\ kJ/kg = h_5}$, $\mathbf{h_6 = h_{f6} = 640.1\ kJ/kg\ (at\ 5\ bar) = h_7}$

ENERGY BALANCE FOR HEATER GIVES

$$m\,(h_2 - h_6) = (1 - m)\,(h_6 - h_5)$$

$$m\,(2796 - 640.1) = (1 - m)\,(640.1 - 191.8) = 448.3\,(1 - m)$$

$$2155.9\ m = 448.3 - 448.3\ m$$

$$\therefore \mathbf{m = 0.172\ kg}$$

TURBINE WORK,

$W_T = (h_1 - h_2) + (1 - m)\,(h_2 - h_3)$ ⇨ $W_T = (3230.9 - 2796) + (1 - 0.172)\,(2796 - 2192.2)$

$W_T = 434.9 + 499.9$ $\mathbf{W_T = 934.8\ kJ/kg}$

HEAT SUPPLIED,

$Q_1 = h_1 - h_{f6}$ ⇨ $Q_1 = 3230.9 - 640.1$ $\mathbf{Q_1 = 2590.8\ kJ/kg.}$

EFFICIENCY OF CYCLE, η_{cycle} :

$\eta_{cycle} = \frac{W_T}{Q_1}$ ⇨ $\eta_{cycle} = \frac{934.8}{2590.8}$ $\mathbf{\eta_{cycle} = 0.3608\ or\ 36.08\%}$

STEAM RATE

$SSC = \frac{3600}{W_T}$ ⇨ $SSC = \frac{3600}{934.8}$ $\mathbf{SSC = 3.85\ kg/kWh}$

THE INCREASE IN MEAN TEMPERATURE OF HEAT ADDITION

$T_{m1} = \frac{h_1 - h_{f7}}{s_1 - s_7}$ ⇨ $T_{m1} = \frac{3230.9 - 191.8}{6.921 - 0.649}$ $\mathbf{T_{m1} = 484.5\ K}$ or 211.5°C.

INCREASE IN T_{m1} DUE TO REGENERATION

Increase in $T_{m1} = 238.9 - 211.5$ **Increase in $T_{m1} = 27.4°C$**

WITHOUT REGENERATION:

TURBINE WORK

$W_T = h_1 - h_3$ ⇨ $W_T = 3230.9 - 2192.2$ $\mathbf{W_T = 1038.7\ kJ/kg}$

STEAM RATE WITHOUT REGENERATION

$SSC = \frac{3600}{W_T}$ ⇨ $SSC = \frac{3600}{1038.7}$ $\mathbf{SSC = 3.46\ kg/kWh}$

INCREASE IN STEAM RATE DUE TO REGENERATION

Increase in steam rate = 3.85 – 3.46 **Increase in steam rate = 0.39 kg/kWh**

EFFICIENCY OF CYCLE, η_{cycle} :

$\eta_{cycle} = \frac{h_1 - h_3}{h_1 - h_{f4}}$ $\mathbf{\eta_{cycle} = 0.3418\ or\ 34.18\%}$

INCREASE IN CYCLE EFFICIENCY DUE TO REGENERATION

$\boldsymbol{\eta}_{\mathbf{cycle}} = 36.08 - 34.18$ $\mathbf{\eta_{cycle} = 1.9\%}$

12. A steam turbine is fed with steam having an enthalpy of 3100 kJ/kg. It moves out of the turbine with an enthalpy of 2100 kJ/kg. Feed heating is done at a pressure of 3.2 bar with steam enthalpy of 2500 kJ/kg. The condensate from a condenser with an enthalpy of 125 kJ/kg enters into the feed heater. The quantity of bled steam is 11200 kg/h. Find the power developed by the turbine. Assume that the water leaving the feed heater is saturated liquid at 3.2 bar and the heater is direct mixing type. Neglect pump work.

SOLUTION:

From the steam table at 3.2 bar

h_{f6} = 570.9 kJ/kg.

Consider **m** kg out of 1 kg is taken to the feed heater

Energy balance for the feed heater is written as :

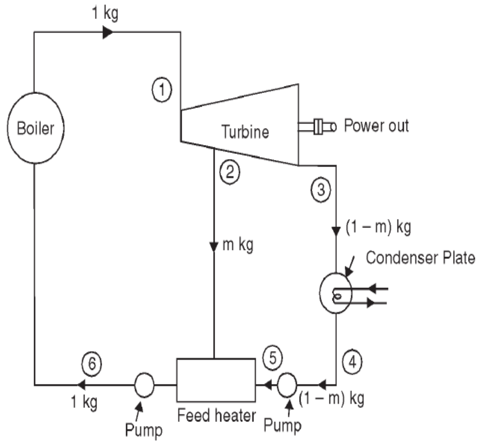
$$(m \times h_2) + (1 - m)\, h_{f5} = 1 \times h_{f6}$$

$$m \times 2100 + (1 - m) \times 125 = 1 \times 570.9$$

$$2100\, m + 125 - 125\, m = 570.9$$

$$1975\, m = 570.9 - 125$$

m = 0.226 kg per kg of steam supplied to the turbine



STEAM SUPPLIED TO THE TURBINE PER HOUR

$$= \frac{11200}{0.226}$$

= 49557.5 kg/h

NET WORK DEVELOPED PER KG OF STEAM

$$= 1\text{ kg}\,(h_1 - h_2) + (1 - m)\,(h_2 - h_3)$$

$$= (3100 - 2500) + (1 - 0.226)\,(2500 - 2100)$$

$$= 600 + 309.6$$

= 909.6 kJ/kg

POWER DEVELOPED BY THE TURBINE

= 12521.5 kW

13. Steam enters the turbine at 3 Mpa and 400°C and is condensed at 10kPa. Some quantity of steam leaves the turbine at 0.5 Mpa and enters feed water heater. Compute the fraction of the steam extracted per kg of steam and cycle thermal efficiency.

From steam tables :

At 30 bar, 400°C : h_1 = 3230.9 kJ/kg,
s_1 = 6.921 kJ/kg K = s_2 = s_3,
At 5 bar : s_f = 1.8604,
s_g = 6.8192 kJ/kg K,
h_f = 640.1 kJ/kg

Since $s_2 > s_g$, the state 2 must lie in the superheated region. From the table for superheated steam

$t_2 = 172°C$,

$h_2 = 2796$ kJ/kg.

At 0.1 bar : $s_f = 0.649$,

$s_{fg} = 7.501$,

$h_f = 191.8$,

$h_{fg} = 2392.8$

Now, $s_2 = s_3$

i.e., $6.921 = s_{f3} + x_3 \, s_{fg3}$

$x_3 = 0.836$

$h_3 = h_{f3} + x_3 \, h_{fg3}$

$= 2192.2$ kJ/kg

Since pump work is neglected

$h_{f4} = 191.8$ kJ/kg $= h_{f5}$

$h_{f6} = 640.1$ kJ/kg (at 5 bar) $= h_{f7}$

Energy balance for heater gives

$$m\,(h_2 - h_{f6}) = (1 - m)\,(h_{f6} - h_{f5})$$

$\therefore m = 0.172$ kg

$\therefore$ Turbine work, $W_T = (h_1 - h_2) + (1 - m)\,(h_2 - h_3)$

$= 934.8$ kJ/kg

Heat supplied, $Q_1 = h_1 - h_{f6}$

$= 2590.8$ kJ/kg.

Efficiency of cycle, ηcycle :

$$\eta_{cycle} = \frac{W_T}{Q_1}$$

$= 0.3608$ or 36.08%. (Ans.)

Steam rate = 3.85 kg/kWh. (Ans.)

14. In a steam generator compressed liquid water at 10 MPa, 30°C enters a 30 m diameter tube at the Rate of 3 litres/sec. Steam at 9MPa, 400°C exits the tube. Find the rate of heat transfer to the water.

GIVEN:

In boiler Pressurized water enters at section 1 (p_1=100 bar, 30°C)

At section 2 it exits the tube as Steam at 90 bar, 400°C.

Check the condition of water at 1. (find $v1$)

From volume flow rate find mass flow rate of water

and find h_1

Check the condition of water at 2. (find h_2)

Find H.T rate $Q = m(h_2 - h_1)$ in kJ/min

Given:

In boiler Pressurized water enters at section 1 (p_1=100 bar, 30°C)

At section 2 it exits the tube as Steam at 90 bar, 400°C.

Check the condition of water at 1. (find $v1$)

From volume flow rate find mass flow rate of water

and find h_1

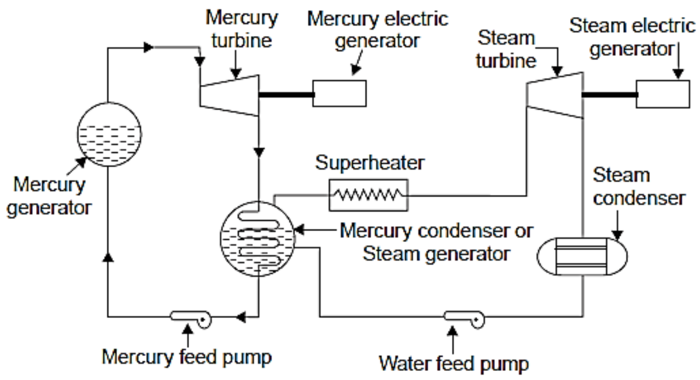
Check the condition of water at 2. (find h_2)

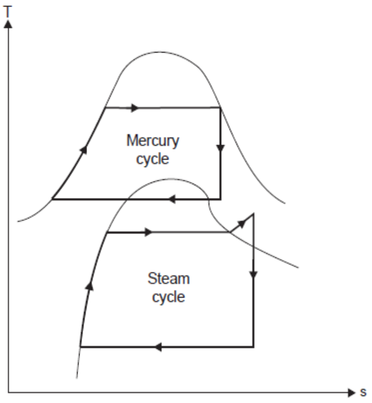
Find H.T rate $Q = m(h_2 - h_1)$ in kJ/min

15. BINARY VAPOUR CYCLE

A Binary Vapor Rankine Power Cycle consists of two separate Rankine Vapor Power Cycles that use different working fluids. The two cycles only interact through a shared heat exchanger. In this HEX, the sub-cooled liquid leaving the pump in the low-temperature cycle absorbs heat from the turbine effluent of the high-temperature cycle. If necessary, the working fluid in the low-temperature cycle can receive additional heat from the boiler before entering the low-temperature turbine.

- If we use steam as the working medium the temperature rise is accompanied by rise in pressure and at critical temperature of 374.15°C the pressure is as high as 225 bar which will create many difficulties in design, operation and control.
- An ideal fluid for this purpose should have a very high critical temperature combined with low pressure. Mercury, diphenyl oxide and similar compounds, aluminium bromide and zinc ammonium chloride are fluids which possess the required properties in varying degrees.
- Mercury is the only working fluid which has been successfully used in practice. It has high critical temperature (588.4°C) and correspondingly low critical pressure (21 bar abs.).
- The mercury alone cannot be used as its saturation temperature at atmospheric pressure is high (357°C). Hence binary vapour cycle is generally used to increase the overall efficiency of the plant. Two fluids (mercury and water) are used in cascade in the binary cycle for production of power.





The few more properties required for an ideal binary fluid used in high temperature

- It should have high critical temperature at reasonably low pressure.
- It should have high heat of vaporisation to keep the weight of fluid in the cycle to minimum.
- Freezing temperature should be below room temperature.
- It should have chemical stability through the working cycle.
- It must be non-corrosive to the metals normally used in power plants.
- It must have an ability to wet the metal surfaces to promote the heat transfer.
- The vapour pressure at a desirable condensation temperature should be nearly atmospheric which will eliminate requirement of power for maintenance of vacuum in the condenser.
- After expansion through the prime mover the vapour should be nearly saturated so that a desirable heat transfers co-efficient can be obtained which will reduce the size of the condenser required.

- It must be available in large quantities at reasonable cost.
- It should not be toxic and, therefore, dangerous to human life.

Although mercury does not have all the required properties, it is more favourable than any other fluid investigated. It is most stable under all operating conditions.

Although, mercury does not cause any corrosion to metals, but it is extremely dangerous to human life, therefore, elaborate precautions must be taken to prevent the escape of vapour. The major disadvantage associated with mercury is that it does not wet surface of the metal and forms a serious resistance to heat flow.

Working:

- ✓ A binary vapour power cycle is two power cycles working together.
- ✓ The high-temperature cycle absorbs heat from the hot reservoir and the low-temperature cycle rejects heat to the cold reservoir.
- ✓ The interesting part is that the heat rejected from the high-temperature cycle, Q_{C1} is used as the heat INPUT to the low-temperature cycle, Q_{H2}.
- ✓ The transfer of heat out of the high-temperature working fluid and into the low temperature working fluid takes place in a heat exchanger, labeled “HEX” in my diagram.
- ✓ If the flow rates of the two working fluids are controlled properly, the low-temperature cycle does not need to receive heat from the boiler.
- ✓ Otherwise, the low temperature working fluid can be heated further in the boiler as I have shown here.
- ✓ The key is that the working fluids in the two cycles CANNOT be the same.
- ✓ If the two working fluids were the same, there would be no advantage to using a binary vapour cycle.
- ✓ So, now, let’s try to determine what advantage there might be in using a binary cycle.

Thermal properties of mercury :

- ✓ Its freezing point is – 3.3°C and boiling point is – 354.4°C at atmospheric pressure.
- ✓ The pressure required when the temperature of vapour is 540°C is only 12.5 bar (app.) and, therefore, heavy construction is not required to get high initial temperature.
- ✓ Its critical temperature is so far removed from any possible upper temperature limit with existing metals as to cause no trouble.

Some undesirable properties of mercury are listed below :

- ✓ Since the latent heat of mercury is quite low over a wide range of desirable condensation temperatures, therefore, several kg of mercury must be circulated per kg of water evaporated in binary cycle.
- ✓ The cost is a considerable item as the quantity required is 8 to 10 times the quantity of water circulated in binary system.
- ✓ Mercury vapour in larger quantities is poisonous, therefore, the system must be perfect and tight.

UNIT-V

PSYCHROMETRY

3.1 INTRODUCTION

The psychrometric is that branch of engineering science which deals with the study of moist air i.e., dry air mixed with water vapour or humidity. It also includes the study of behavior of dry air and water vapour mixture under various sets of conditions. Though the earth's atmosphere is a mixture of gases including nitrogen (N_2), oxygen (O_2), argon (Ar) and carbon dioxide (CO_2), yet for the purpose of psychrometric, it is considered to be a mixture of dry air and water vapour only.

3.2 PSYCHOMETRIC TERMS

Though there are many psychometric terms, yet the following are important from the subject point of view :

1. ***Dry air***. The pure dry air is a mixture of a number of gases such as nitrogen, oxygen, carbon dioxide, hydrogen, argon, neon, helium etc. But the nitrogen and oxygen have the major portion of the combination. The dry air is considered to have the composition as given in the following table:

Table .1 Composition of dry air

S.No.	***Constituent***	***By volume***	***By mass***	***Molecular Mass***
1	Nitrogen (N_2)	78.03%	75.47%	28
2	Oxygen (O_2)	20.99%	23.19%	32
3	Argon (Ar)	0.94%	1.29%	40
4	Carbon dioxide (CO_2)	0.03%	0.05%	44
5	Hydrogen (H_2)	0.01%	-	2

The molecular mass of dry air is taken as 28.966 and the gas constant of air (R_a) is equal 0.287 kJ / kg K or 287 J/kg K.

The molecular mass of water vapour is taken as 18.016 and the gas constant for water vapour (k) is equal to 0.461-kJ/kg K or 461 J/kg K.

Notes: (a) The pure dry air does not ordinarily exist in nature because it always contains some water vapout

(b) The term air, wherever used in this text, means dry air containing moisture in the vapour form.

(c) Both dry air and water vapour can be considered as perfect gases because both exist in the atmosphere at low pressure. Thus all the perfect gas terms can be applied to them individually.

(d) The density of dry air is taken as 1.293 kg/m3 at pressure 1.0135 bar or 101.35 1(11/m2 and at temperature 0°C (273 K).

2. ***Moist air***. It is a mixture of dry air and water vapour. The amount of water vapour present. in the air depends upon the absolute pressure and temperature of the mixture.

3. ***Saturated air***. It is mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it. The water vapours, usually, occur in the form of superheated steam as an invisible gas. However, when the saturated air is cooled, the water vapour in the air starts condensing, and the same may be visible in the form of moist, fog or condensation on cold surfaces.

4. ***Degree of saturation***. It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature.

5. ***Humidity***. It is the mass of water vapour present in 1 kg of dry air, and is generally expressed in terms of gram per kg of dry air (g / kg of dry air). It is also called specific humidity or humidity ratio.

6. ***Absolute humidity***. It is the mass of water vapour present in 1 m3 of dry air, and is generally expressed in terms of gram per cubic metre of dry air (g /m3 of dry air). It is also expressed in terms of grains per cubic metre of dry air. Mathematically, one kg of water vapour is equal to 15 430 grains.

7. ***Relative humidity***. It is the ratio of actual mass of water vapour in a given_volume of moist air to the mass of water vapour in the same volume of saturated air at the same temperature and pressure. It is briefly written as RH.

8. ***Dry bulb temperature***. It is the temperature of air recorded by a thermometer, when it is not affected by the moisture present in the air. The dry bulb temperature (briefly written as DBT) is generally denoted by t_d or t_{db}.

9. ***Wet bulb temperature***. It is the temperature of air recorded by a thermometer, when its bulb is surrounded by a wet cloth exposed to the air. Such a thermometer is called *wet bulb thermometer. The wet bulb temperature (briefly written as WBT) is generally denoted by t_w or t_{wb}.

10. ***Wet bulb depression***. It is the difference between dry bulb temperature and wet bulb temperature at any point. The wet bulb depression indicates relative humidity of the air.

11. ***Dew point temperature***. It is the temperature of air recorded by a thermometer, when the moisture (water vapour) present in it begins to condense. In other words, the dew point temperature is the saturation temperature (t_{sat}). corresponding to the partial pressure of water vapour (P_v) It is, usually, denoted by t_{dp}. Since p_v. is very smaIl, therefore the saturation temperature by water vapour at pv is also low (less than the atmospheric or dry bulb temperature). Thus the water vapour in air exists in the superheated state and the moist air containing moisture in such a form (i.e., superheated state) is said to be unsaturated air. This condition is shown by point A on temperature-entropy (T-s) diagram as shown in Fig.1. When the partial pressure of water vapour (Pv) is equal to the saturation pressure (Ps) the water vapour is in dry condition and the air will be saturated air

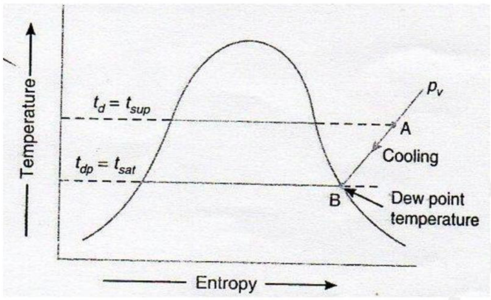


Fig.1. T-s diagram

If a sample of unsaturated air, containing superheated water vapour, is cooled at constant pressure, the partial pressure (pr) of each constituent remains constant until the water vapour reaches the saturated state as shown by point B in Fig.1. At this point 8, the first drop of dew will' be formed and hence the temperature at point B is called dew paint temperature. Further cooling will cause condensation of water vapour.

From the above we see that the dew point temperature is the temperature at which the water vapour begins to condense.

Note: For saturated air, the dry bulb temperature, wet bulb temperature and dew point temperature is same.

12. ***Dew point depression***. It is the difference between the dry bulb temperature and dew point temperature of air.

13. ***Psychrometer***. There are many types of psychrometers, but the sling psychrometer, as shown in Fig..2, is widely used. It consists of a dry bulb thermometer and a wet bulb thermometer mounted side by side in a protective case that is attached to a handle by a swivel connection so that the case can be easily rotated. The dry bulb thermometer is directly exposed to air and measures the actual temperature of the air. The bulb of the wet bulb thermometer is covered; by a wick thoroughly wetted by distilled water. The temperature measured by this wick covered bulb of a thermometer is the temperature of liquid water in the wick and is called wet Nib j temperature.

The sling psychrometer is rotated in the air for approximately one minute after which HO readings from both the thermometers are taken. This process is repeated several times to assure': that the lowest possible wet bulb temperature is recorded.

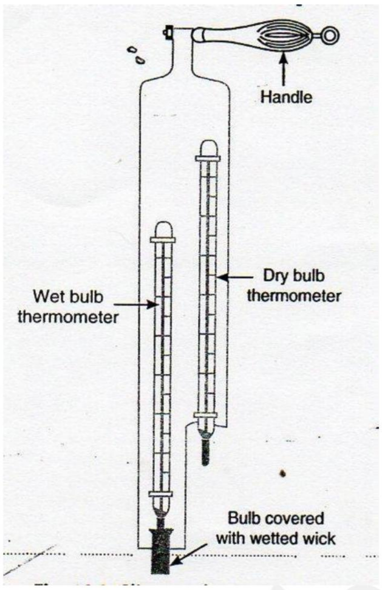


Fig.2, Sling psychrometer

3.3 DALTON'S LAW OF PARTIAL PRESSURES

It states, The total pressure exerted by the mixture of air and water vapour is equal to the sum of the pressures, which each constituent Fould exert, if it occupied the same space by itself. In other words, the total pressure exerted by air and water vapour mixture is equal to the barometric pressure. Mathematically, barometric pressure of the mixture,

$$P_b = P_a + P_v,$$

where P_a = Partial pressure of dry air, and

P_v= Partial pressure of water vapour.

3.4 PSYCHROMETRIC RELATIONS

We have already discussed some psychrometric terms in Art. These terms have some relations between one another. The following psychrometric relations are important from the subject point of view:

1. ***Specific humidity***, humidity ratio or moisture content. It is the mass of water vapour present in 1 kg of dry air (in the air-vapour mixture) and is generally expressed in g /kg of dry air. It may also be defined as the ratio of mass of water vapour to the mass of dry air in a given volume of the air-vapour mixture.

Let P_a, V_a, Ta, m_a and R_a = Pressure, volume, absolute temperature, mass and gas constant respectively for dry air, and

P_v, V_v, m_v and R_v = Corresponding values for the water vapour.

Assuming that the dry air and water vapour behave as perfect gases, we have for dry air,

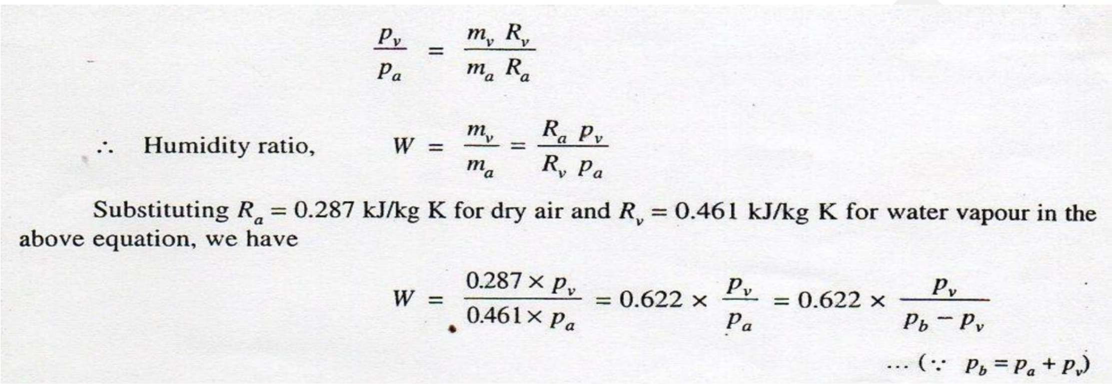
$$P_a v_a = m_a R_a T_a$$

and for water vapour, $P_v v_v = m_v R_v T_v$,

Also $v_a = v_v$

and $T_a = T_v = T_d$ ... (where Td is dry bulb temperature)

From equations (i) and (ii), we have



$$\frac{p_v}{p_a} = \frac{m_v R_v}{m_a R_a}$$

$\therefore$ Humidity ratio, $W = \frac{m_v}{m_a} = \frac{R_a\, p_v}{R_v\, p_a}$

Substituting $R_a = 0.287$ kJ/kg K for dry air and $R_v = 0.461$ kJ/kg K for water vapour in the above equation, we have

$$W = \frac{0.287 \times p_v}{0.461 \times p_a} = 0.622 \times \frac{p_v}{p_a} = 0.622 \times \frac{p_v}{p_b - p_v}$$

$$\ldots (\because\ p_b = p_a + p_v)$$

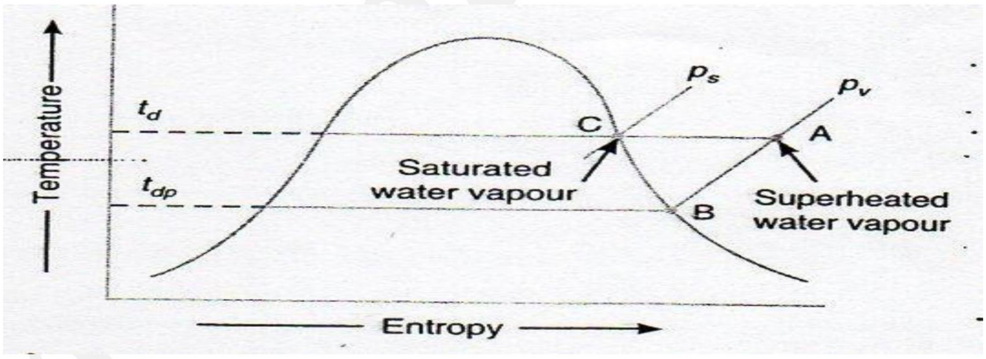


Fig.3 T-s diagram

Consider unsaturated air containing superheated vapour at dry bulb temperature t_d and partial pressure p_v as shown by point A on the *T-s* diagram in Fig. 3. If water is added into this unsaturated air, the water will evaporate which will increase the moisture content (specific humidity) of the air and the partial pressure p_v increases. This will continue until the water vapour becomes saturated at that temperature, as shown by point C in Fig.3, and there will be more evaporation of water. The partial pressure p_v, increases to the saturation pressure p_s and it is maximum partial pressure of water vapour at temperature t_d. The air containing moisture in such a state (point *C*) is called saturated air.

For saturated air (i.e. when the air is holding maximum amount of water vapour), the humidity ratio or maximum specific humidity,

$$W_s = W_{max} = 0.622 \times \frac{p_s}{p_b - p_s}$$

where P_s = Partial pressure of air corresponding to saturation temperature (i.e. dry bulb temperature td).

2. ***Degree of saturation or percentage humidity***. We have already discussed that the degree of saturation is the ratio of vapour in a unit mass of water air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature (dry bulb temperature), it may be defined as the ratio of actual specific humidity to the specific humidity of saturated air at the same dry bulb temperature. It is, usually, denoted by μ. Mathematically, degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{\dfrac{0.622\, p_v}{p_b - p_v}}{\dfrac{0.622\, p_s}{p_b - p_s}} = \frac{p_v}{p_s}\left(\frac{p_b - p_s}{p_b - p_v}\right) = \frac{p_v}{p_s}\left[\frac{1 - \dfrac{p_s}{p_b}}{1 - \dfrac{p_v}{p_b}}\right]$$

Notes: (a) The partial pressure of saturated air (P_s) is obtained from the steam tables corresponding to dry bulb temperature t_d.

(b) If the relative humidity,Ø) = *Pv* / *Ps* is equal to zero, then the humidity ratio, W = 0, i.e. for dry air, $\mu = 0$.

(c) If the relative humidity, Ø) *Pv* / *Ps* is equal to 1, then W = Ws and $\mu = 1$. Thus p. varies between 0 and 1.

3. ***Relative humidity***. We have already discussed that the relative humidity is the ratio of actual mass of water vapour (m_v) in a given volume of moist air to the mass of water vapour (m_s) in the same volume of saturated air at the same temperature and pressure. It is usually denoted by Ø. Mathematically, relative humidity,

$$\phi = \frac{m_v}{m_s}$$

Let p_v, v_v, T_v, m_v and R_v = Pressure, volume, temperature, mass and gas constant respectively for

water vapour in actual conditions, and

p_s, v_s, T_s, m_s and *Rs* = Corresponding values for water vapour in saturated air.

We know that for water vapour in actual conditions,

$$P_v v_v = m_v\ R_v\ T_v \qquad \text{.....(i)}$$

Similarly, for water vapour in saturated air,

$$Ps\ vs = ms\ Rs\ Ts \qquad \text{...(ii)}$$

According to the definitions,

$$v_v = v_s$$

$$Tv = Ts$$

Also $Rv = Rs = 0.461$ kJ/kg K

∴ From equations (i) and (ii), relative humidity,

$$\phi = \frac{m_v}{m_s} = \frac{p_v}{p_s}$$

Thus, the relative humidity may also be defined as the ratio of actual partial pressure of water vapour in moist air at a given temperature (dry bulb temperature) to the saturation pressure of water vapour (or partial pressure of water vapour in saturated air) at the same temperature.

The relative humidity may also be obtained as discussed below:

We know that degree of saturation,

$$\mu = \frac{p_v}{p_s}\left[\frac{1-\dfrac{p_s}{p_b}}{1-\dfrac{p_v}{p_b}}\right] = \phi\left[\frac{1-\dfrac{p_s}{p_b}}{1-\phi\times\dfrac{p_s}{p_b}}\right] \qquad \cdots \left(\because \phi = \frac{p_v}{p_s}\right)$$

$$\phi = \frac{\mu}{1-(1-\mu)\dfrac{p_s}{p_b}}$$

4. ***Pressure of water vapour***. According to Carrier's equation, the partial pressure of water vapours,

$$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44\,t_w}$$

Where p_w, = Saturation pressure corresponding to wet bulb temperature (from steam tables),

P_b = Barometric pressure,

t_d = Dry bulb temperature, and

t_w = Wet bulb temperature.

5. ***Vapour density or absolute humidity***. We have already discussed that the vapour density or absolute humidity is the mass of water vapour present in 1 m^3 of dry air.

Let v_v = Volume of water vapour in m^3/kg of dry air at its partial pressure,

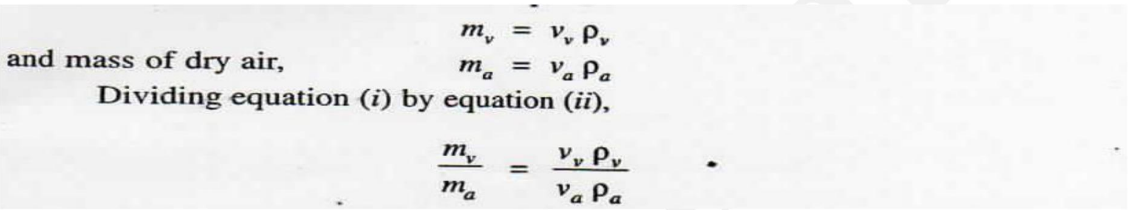
v_a = Volume of dry air in m^3/kg of dry air at its partial pressure,

ρ_v, = Density of water vapour in kg/m^3 corresponding to its partial pressure and dry bulb

temperature t_d, and

ρ_a = Density of dry air in kg/m^3 of dry air.

We know that mass of water vapour,



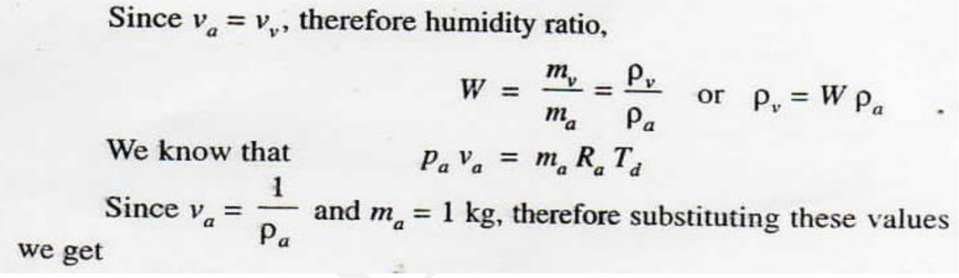
$$m_v = v_v \rho_v$$

and mass of dry air,

$$m_a = v_a \rho_a$$

Dividing equation (*i*) by equation (*ii*),

$$\frac{m_v}{m_a} = \frac{v_v \rho_v}{v_a \rho_a}$$



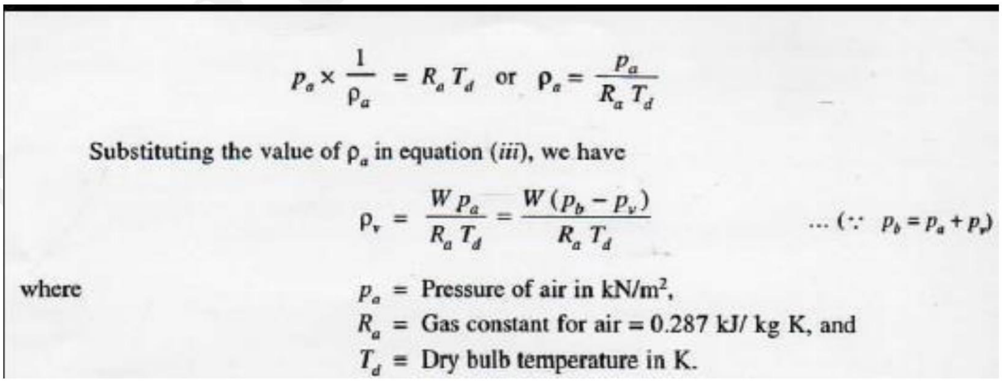
Since $v_a = v_v$, therefore humidity ratio,

$$W = \frac{m_v}{m_a} = \frac{\rho_v}{\rho_a} \quad \text{or} \quad \rho_v = W \rho_a$$

We know that

$$p_a v_a = m_a R_a T_d$$

Since $v_a = \frac{1}{\rho_a}$ and $m_a = 1$ kg, therefore substituting these values we get



$$p_a \times \frac{1}{\rho_a} = R_a T_d \quad \text{or} \quad \rho_a = \frac{p_a}{R_a T_d}$$

Substituting the value of ρ_a in equation (*iii*), we have

$$\rho_v = \frac{W p_a}{R_a T_d} = \frac{W(p_b - p_v)}{R_a T_d} \qquad \ldots (\because \; p_b = p_a + p_v)$$

where

p_a = Pressure of air in kN/m^2,

R_a = Gas constant for air = 0.287 kJ/ kg K, and

T_d = Dry bulb temperature in K.

Example.1. The readings from a sling psychrometer are as follows ry bulb temperature = 30° C ; Barometer reading 740mm of Hg Using steam tables, determine : I. Dew point temperature ; 2. Relative humidity ; 3. Specific humidity ; 4. Degree of-saturation ; 5. Vapour density ; and 6. Enthalpy of mixture per kg of dry air.

Solution given: $t_d = 30°C$; $t_{w.} = 20°C$; $P_4 = 740$ mm of Hg

1.Dew point temperature

First of all, let us find the partial pressure of water vapour (P_v).

From steam tables, we find that the saturation pressure corresponding to wet bulb temperature of 20° C is

$$P_w = 0.023\ 37 \text{ bar}$$

We know that barometric pressure,

ph = 740 mm of Hg ... (Given)

$= 740 \times 133.3 = 98\ 642 \text{ N/m}^2$ … ($\because$ mm of Hg = 133.3 N/m^2)

$= 0.986\ 42$ bar …. $\because 1 \text{ bar} = 10^5 \text{ N/m}^2$)

∴ Partial pressure of water vapour,

$$\begin{aligned} p_v &= p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44\, t_w} \\ &= 0.023\ 37 - \frac{(0.986\ 42 - 0.02337)(30 - 20)}{1544 - 1.44 \times 20} \\ &= 0.023\ 37 - 0.006\ 36 = 0.017\ 01 \text{ bar} \end{aligned}$$

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (P_v), therefore from steam tables, we find that corresponding to pressure 0.017 01 bar, the dew point temperature is

$$t_{dp} = 15°C \text{ Ans}$$

2. Relative humidity

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 30°C is

$$Ps = 0.042\ 42 \text{ bar}$$

We know the relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{0.01701}{0.04242} = 0.40 \text{ or } 40\% \text{ Ans.}$$

3. ***Specific humidity***

We know that specific humidity,

$$W = \frac{0.622\,p_v}{p_b - p_v} = \frac{0.622 \times 0.01701}{0.986\,42 - 0.01701}$$

$$= \frac{0.01058}{0.96941} = 0.010\,914 \text{ kg/kg of dry air}$$

$$= 10.914 \text{ g/kg of dry air} \text{ Ans.}$$

4. ***Degree of saturation***

We know that specific humidity of saturated air,

$$W_s = \frac{0.622\,p_s}{p_b - p_s} = \frac{0.622 \times 0.04242}{0.98642 - 0.04242}$$

$$= \frac{0.02638}{0.944} = 0.027\,945 \text{ kg/kg of dry air}$$

We know that degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{0.010\,914}{0.027\,945} = 0.391 \text{ or } 39.1\% \text{ Ans.}$$

Note : The degree of saturation (μ) may also be calculated from the following relation :

$$\mu = \frac{p_v}{p_s}\left(\frac{p_b - p_s}{p_b - p_v}\right)$$

$$= \frac{0.01701}{0.04242}\left[\frac{0.98642 - 0.04242}{0.98642 - 0.01701}\right]$$

$$= 0.391 \text{ or } 39.1\% \text{ Ans.}$$

5. ***Vapour density***

We know that vapour density,

$$\rho_v = \frac{W\,(p_b - p_v)}{R_a\,T_d} = \frac{0.010\,914\,(0.98642 - 0.01701)\,10^5}{287\,(273 + 30)}$$

$$= 0.012\,16 \text{ kg/m}^3 \text{ of dry air} \text{ Ans.}$$

6. ***Enthalpy of mixture per kg of dry air***

From steam tables, we find that the latent heat of vaporisation of water at dew point temperature of 15°C is

$$h_{fgdp} = 2466.1 \text{ kJ/kg}$$

$\therefore$ Enthalpy of mixture per kg of dry air,

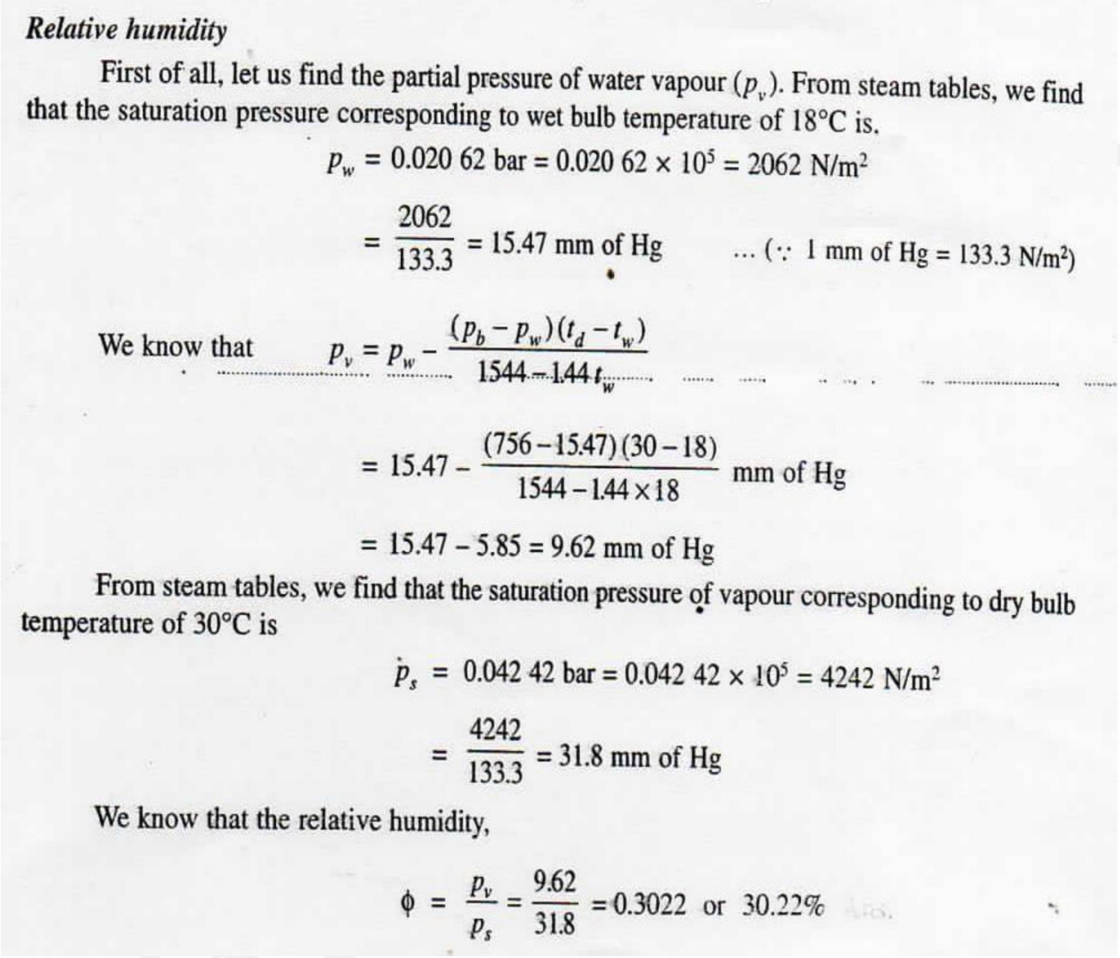
$$h = 1.022\,t_d + W\,[h_{fgdp} + 2.3\,t_{dp}]$$

$$= 1.022 \times 30 + 0.010\,914\,[2466.1 + 2.3 \times 15]$$

$$= 30.66 + 27.29 = 57.95 \text{ kJ/kg of dry air} \text{ Ans.}$$

Example.2: On a particular day, the atmospheric air was found to have a dry bulb temperature of 30°C and a wet bulb temperature of 18°C. The barometric pressure was observed to b 756mm of Hg. Using the tables of psychrometric properties of air, determine the relative humidity, the specific humidity, the dew point temperature, the enthalpy of air per kg of dry air and the volume of mixture per kg of dry air.

Solution: Given: $t_d = 30°C$; t_w - 18°C; $P_b = 756$ *mm* of *Hg*



Relative humidity

First of all, let us find the partial pressure of water vapour (p_v). From steam tables, we find that the saturation pressure corresponding to wet bulb temperature of 18°C is.

$$p_w = 0.020\,62 \text{ bar} = 0.020\,62 \times 10^5 = 2062 \text{ N/m}^2$$

$$= \frac{2062}{133.3} = 15.47 \text{ mm of Hg} \qquad \ldots (\because \ 1 \text{ mm of Hg} = 133.3 \text{ N/m}^2)$$

We know that $$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44\,t_w}$$

$$= 15.47 - \frac{(756 - 15.47)(30 - 18)}{1544 - 1.44 \times 18} \text{ mm of Hg}$$

$$= 15.47 - 5.85 = 9.62 \text{ mm of Hg}$$

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 30°C is

$$p_s = 0.042\,42 \text{ bar} = 0.042\,42 \times 10^5 = 4242 \text{ N/m}^2$$

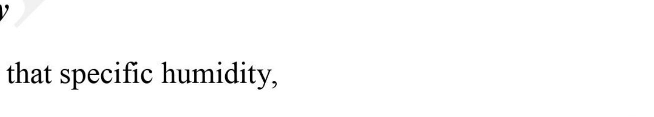
$$= \frac{4242}{133.3} = 31.8 \text{ mm of Hg}$$

We know that the relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{9.62}{31.8} = 0.3022 \text{ or } 30.22\% \quad \text{Ans.}$$

Specific humidity

We know that specific humidity,



$$W = \frac{0.622\,p_v}{p_b - p_v} = \frac{0.622 \times 9.62}{756 - 9.62} = 0.008 \text{ kg/kg of dry air} \quad \text{Ans.}$$

Dew point temperature

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (P_v), therefore from steam tables, we find that corresponding to 9.62 mm of Hg or 9.62 x 133.3 = 1282.3 N/m^2 = 0.012 823 bar, the dew point temperature is,

$$t_{dp} = 10.6° \text{ C Ans.}$$

Enthalpy of air per kg of dry air

From steam tables, we also find that latent heat of vaporization of water at dew point temperature of 10.6°C,

$$h_{fgdp} = 2476.5 \text{ kJ/kg}$$

We know that enthalpy of air per kg of dry air,

$h = 1.022\, t_d + W\,(h_{fgdp} + 2.3\, t_{dp})$

$= 1.022 \times 30 + 0.008\,(2476.5 + 2.3 \times 10.6)$

$= 30.66 + 20 = 50.66$ kJ/kg of dry air Ans.

Volume of the mixture per kg of dry air

From psychrometric tables, we find that specific volume of the dry air at 760 mm of Hg and 30°C dry bulb temperature is 0.8585 m^3/kg of dry air. We know that one kg of dry air at a partial pressure of (756 — 9.62) mm of Hg occupies the same volume as W = 0.008 kg of vapour at its partial pressure of 9.62 mm of Hg. Moreover, the mixture occupies the same volume but at a total pressure of 756 mm of Hg.

∴ Volume of the mixture (v) at a dry bulb temperature of 30°C and a pressure of 9.62 mm of Hg

= Volume of 1 kg of dry air (v_a) at a pressure of (756 — 9.62) or 746.38 mm of Hg

$$= 0.8585 \times \frac{760}{746.38} = 0.8741 \text{ kg/kg of dry air Ans.}$$

Note : The volume of mixture per kg of dry air may be calculated as discussed below :

We know that $$v = v_a = \frac{R_a T_d}{p_a}$$

where R_a = Gas constant for air = 287 J/kg K

T_d = Dry bulb temperature in K

= 30 + 273 = 303 K, and

p_a = Pressure of air in N/m^2

$= P_b - P_v = 756 - 9.62 = 746.38$ mm of Hg

$= 746.38 \times 133.3 = 994\ 92$ N/m^2

Substituting the values in the above equation,

$$v = \frac{287 \times 303}{994\ 92} = 0.8741 \text{ m}^3\text{/kg of dry air} \quad \text{Ans.}$$

Example.3. The humidity ratio of atmospheric air at 28°C dry bulb temperature and 760 mm of mercury is 0.016 kg / kg of dry air. Determine: 1. partial pressure of Water vapour; 2.relative humidity; 3. dew point temperature; 4. specific enthalpy; and 5. vapour density.

Solution: Given: $t_d = 28$°C ; $P_b = 760$ mm of Hg ; W = 0.016 kg/ kg of dry air

1. Partial pressure of water vapour

Let P_v = Partial pressure of water vapour.
We know that humidity ratio (W),

$$0.016 = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622\, p_v}{760 - p_v}$$

$12.16 - 0.016\, P_v = 0.622\, P_v$ or $0.638\, P_v = 12.16$

$P_v = 12.16/0.638 = 19.06$ mm of Hg

$= 19.06 \times 133.3 = 2540.6$ N/m^2 Ans.

2. Relative humidity

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 28`C is

$P_s = 0.03778$ bar $= 3778$ N/m^2

∴ Relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{2540.6}{3778} = 0.672 \text{ or } 67.2\% \quad \text{Ans.}$$

3. Dew point temperature

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (P_v), therefore from steam tables, we find that corresponding to a pressure of 2540.6 N/m^2 (0.025406 bar), the dew point temperature is,

$t_{dp} = 21.1$° C Ans.

4. Specific enthalpy

From steam tables, latent heat of vaporization of water corresponding to a dew point temperature of 21.1° C,

$$h_{fgdp} = 2451.76 \text{ kJ/kg}$$

We know that specific enthalpy.

$$h = 1.022\, t_d + W\, (h_{\text{fgdp}} + 2.3\, t_{\text{dp}})$$

$$= 1.022 \times 28 + 0.016\, (2451.76 + 2.3 \times 21\_1)$$

$$= 28.62 + 40 - 68.62 \text{ kJ/kg of dry air Ans.}$$

5. Vapour density

We know that vapour density,

$$\rho_v = \frac{W\,(p_b - p_v)'}{R_a\, T_d} = \frac{0.016\,(760 - 19.06)\,133.3}{287\,(273 + 28)}$$

$$= 0.0183 \text{ kg/m}^3 \text{ of dry air.}$$

3.5 THERMODYNAMIC WET BULB TEMPERATURE OR ADIABATIC SATURATION TEMPERATURE

The thermodynamic wet bulb temperature or adiabatic saturation temperature is the temperature at which the air can be brought to saturation state, adiabatically, by the evaporation of water into the flowing air.

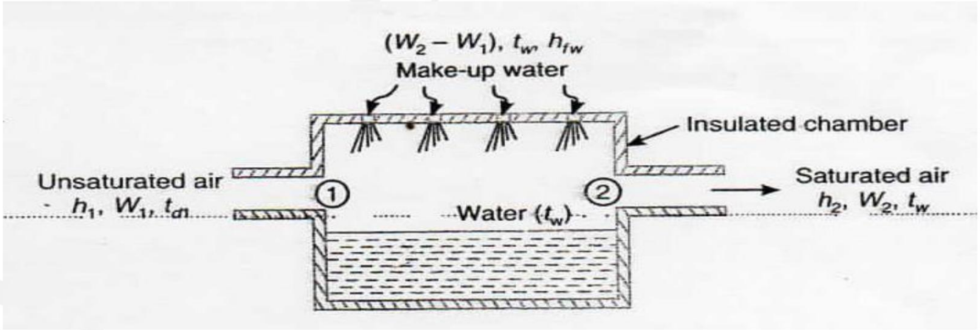


Fig.4 Adiabatic saturation of air.

The equipment used for the adiabatic saturation of air, in its simplest form, consists of an insulated chamber containing adequate quantity of water. There is also an arrangement for extra water (known as make-up water) to flow into the chamber from its top, as shown in Fig.4.

Let the unsaturated air enters the chamber at section 1. As the air passes through the chamber over a long sheet of water, the water evaporates which is carried with the flowing stream of air, and the specific humidity of the air increases. The make-up water is added to the chamber at this temperature to make the water level constant. Both the air and water are cooled as the evaporation takes place. This process continues until the energy transferred from the air to the water is equal to the energy required to vaporize the water. When steady conditions are reached, the air flowing at section 2 is saturated with water vapour. The temperature of the saturated air at section 2 is known as *thermodynamic wet bulb temperature or adiabatic saturation temperature.*

The adiabatic saturation process can be represented on *T-s* diagram as shown by the curve 1-2 in Fig.5.

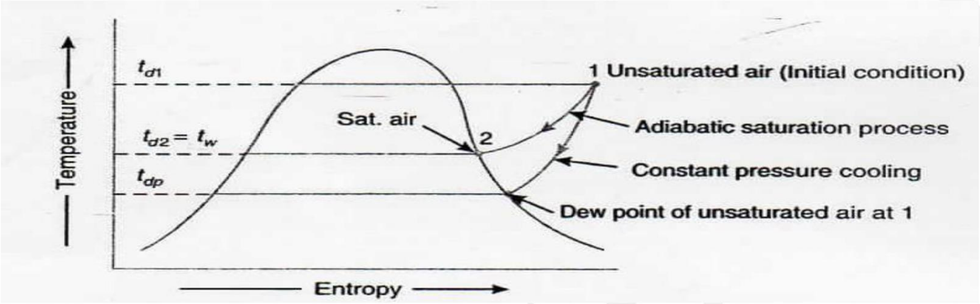


Fig.5. T-s diagram for adiabatic saturation process

During the adiabatic saturation process, the partial pressure of vapour increases, although the total pressure of the air-vapour mixture. The unsaturated air initially at dry bulb temperature t_{d2}, is cooled adiabatically to dry bulb temperature td, which is equal to the adiabatic saturation temperature t_w. It may be noted that the adiabatic saturation temperature is taken equal to the wet bulb temperature for all practical purposes.

Let h_1 = Enthalpy of unsaturated air at section 1,

W_1 = Specific humidity of air at section 1,

h_2, W_2 = Corresponding values of saturated air at section 2, and

h_{fw}= Sensible heat of water at adiabatic saturation temperature.

Balancing the enthalpies of air at inlet and outlet (i.e. at sections 1 and 2),

$$h_1 + (W_2 - W_1)\, h_{fw} = h_2 \qquad \dots (i)$$

or

$$h_1 - W_1 h_{fw} = h_2 - W_2 h_{fw} \qquad \dots (ii)$$

The term $(h_2 - W_2 h_{fw})$ is known as *sigma heat* and remains constant during the adiabatic process.

We know that $h_1 = h_{a1} + W_1 h_{s1}$

and $h_2 = h_{a2} + W_2 h_{s2}$

where h_{a1} = Enthalpy of 1 kg of dry air at dry bulb temperature t_{d1},

$*h_{s1}$ = Enthalpy of superheated vapour at t_{d1} per kg of vapour,

h_{a2} = Enthalpy of 1 kg of air at wet bulb temperature t_w, and

h_{s2} = Enthalpy of saturated vapour at wet bulb temperature t_w per kg of vapour.

Now the equation (ii) may be written as :

$$(h_{a1} + W_1 h_{s1}) - W_1 h_{fw} = (h_{a2} + W_2 h_{s2}) - W_2 h_{fw}$$

$$W_1 (h_{s1} - h_{fw}) = W_2 (h_{s2} - h_{fw}) + h_{a2} - h_{a1}$$

$$\therefore \qquad W_1 = \frac{W_2 (h_{s2} - h_{fw}) + h_{a2} - h_{a1}}{h_{s1} - h_{fw}}$$

3.6 PSYCHROMETRIC CHART

It is a graphical representation of the various thermodynamic properties of moist air. The psychrometric chart is very useful for finding out the properties of air (which are required in the field of air conditioning) and eliminate lot of calculations. There is a slight variation in the charts prepared by different air-conditioning manufactures but basically they are all alike. The psychrometric chart is normally drawn for standard atmospheric pressure of 760 mm of Hg (or 1.01325 bar).

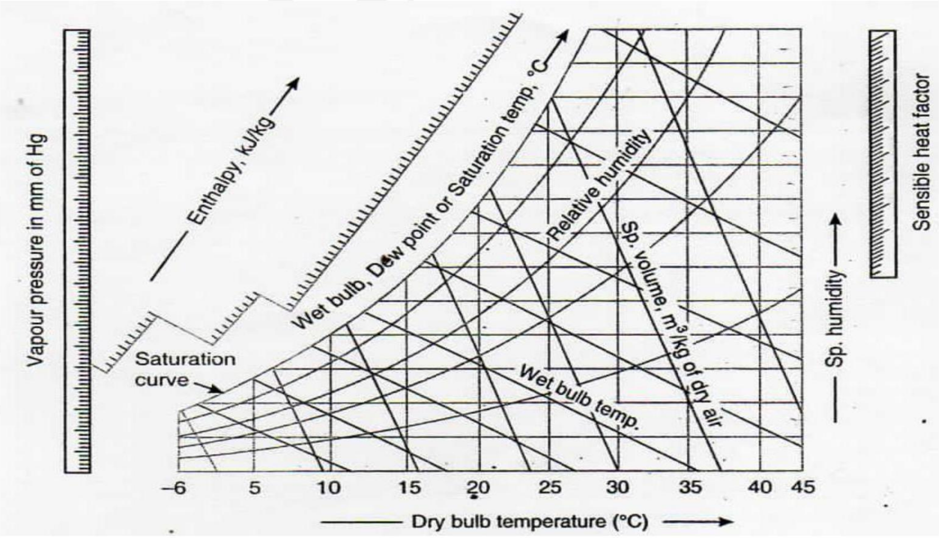


Fig. 6 Psychrometric chart.

In a psychrometric chart, dry bulb temperature is taken as abscissa and specific humidity i.e. moisture contents as ordinate, as shown in Fig. 6. Now the saturation curve is drawn by plotting the various saturation points at corresponding dry bulb temperatures. The saturation curve represents 100% relative humidity at various dry bulb temperatures. It also represents the wet bulb and dew point temperatures.

Though the psychrometric chart has a number of details, yet the following lines are important frpm the subject point of view :

1. ***Dry bulb temperature lines***. The dry bulb temperature lines are vertical i.e. parallel to the ordinate and uniformly spaced as shown in Fig. 7. Generally the temperature range of these lines on psychrometric chart is from - 6° C to 45° C. The dry bulb temperature lines are drawn with difference of every 5°C and up to the saturation curve as shown in the figure. The values of dry bulb temperatures are also shown on the saturation curve.

2. ***Specific humidity or moisture content lines***. The specific humidity (moisture content) lines are horizontal i.e. parallel to the abscissa and are also uniformly spaced as shown in Fig. 16.8. Generally, moisture content range of these lines on psychrometric chart is from 0 to 30 g / kg of dry air (or from 0 to 0.030 kg / kg of dry air). The moisture content lines are drawn with a difference of every 1 g (or 0.001 kg) and up to the saturation curve as shown in the figure.

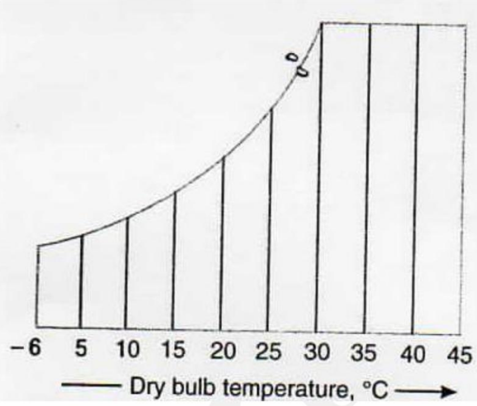


Fig.7. Dry bulb temperature lines.

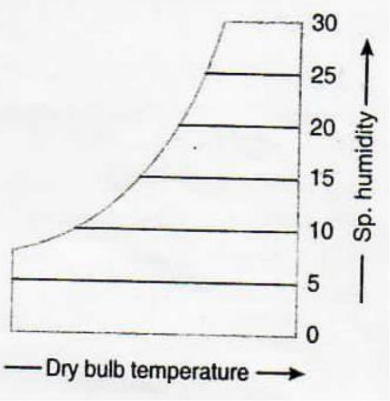
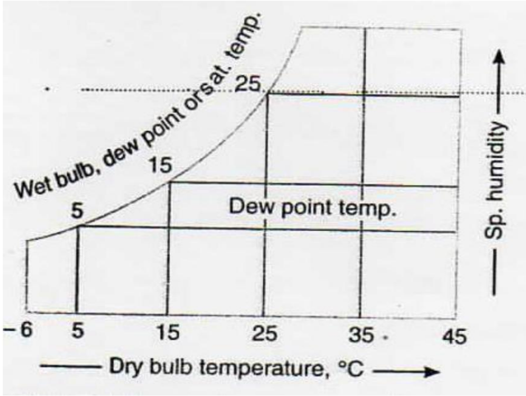


Fig. 8. Specific humidity lines.

3. ***Dew point temperature lines***. The dew point temperature lines are horizontal i.e. parallel to the abscissa and non-uniformly spaced as shown in Fig. 16.9. At any point on the saturation curve, the dry bulb and dew point temperatures are equal.

The values of dew point temperatures are generally given along the saturation curve of the chart as shown in the figure.



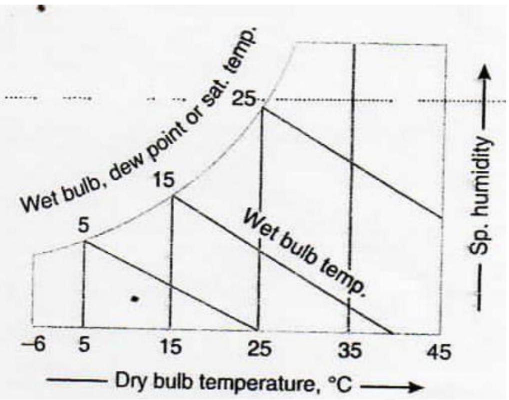


Fig. 9 Dew point temperature lines.

Fig.10 Wet bulb temperature lines.

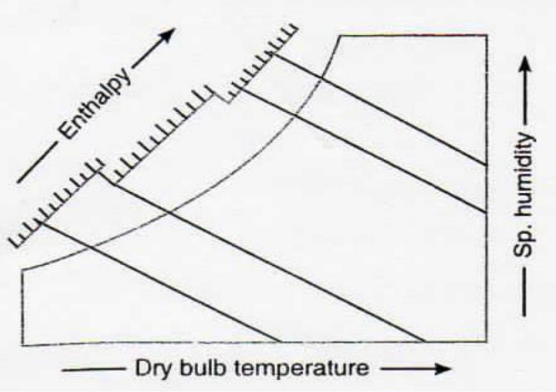
4. ***Wet bulb temperature lines***. The wet bulb temperature lines are inclined straight lines and non-uniformly spaced as shown in Fig.10. At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.

The values of wet bulb temperatures are generally given along the saturation curve of the chart as shown in the figure.

5. ***Enthalpy (total heat) lines***. The enthalpy (or total heat) lines are inclined straight lines and uniformly spaced as shown in Fig.11. These lines are parallel to the wet bulb temperature lines, and are drawn up to the saturation curve. Some of these lines coincide with the wet bulb temperature lines also.

The values of total enthalpy are given on a scale above the saturation curve as shown in the figure.

6. ***Specific volume lines.*** The specific volume lines are obliquely inclined straight lines and uniformly spaced as shown in Fig.12. These lines are drawn up to the saturation curve. The values of volume lines are generally given at the base of the chart.



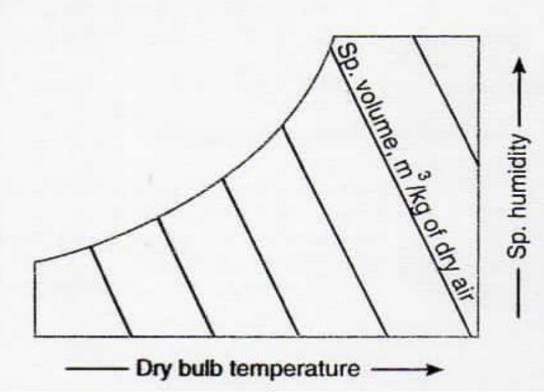


Fig. 11. Enthalpy lines.

Fig. 12. Specific volume lines.

7. ***Vapour pressure lines***. The vapour pressure lines are horizontal and uniformly spaced. Generally, the vapour pressure lines are not drawn in the main chart. But a scale showing vapour pressure in mm of Hg is given on the extreme left side of the chart as shown in Fig.13.

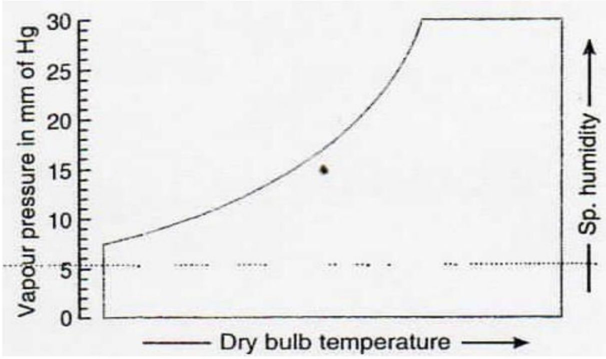


Fig. 13. Vapour pressure lines.

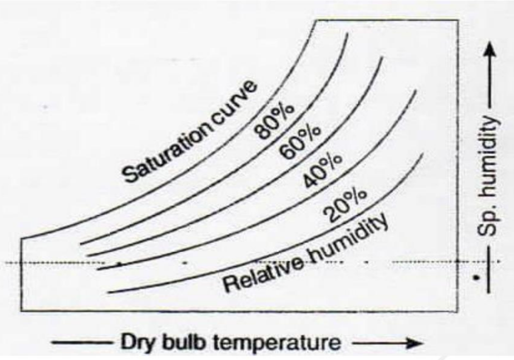


Fig. 14. Relative humidity lines.

8. ***Relative humidity lines***. The relative humidity lines are curved lines and follow the saturation curve. Generally, these lines are drawn with values 10%, 20%, 30% etc. and up to 100%. The saturation curve represents 100% relative humidity. The values of relative humidity lines are generally given along the lines themselves as shown in Fig. 14.

3.7 PSYCHROMETRIC PROCESSES

The various psychrometric processes involved in air conditioning to vary the psychrometric properties of air according to the requirement are as follows:

1. Sensible heating, 2. Sensible cooling, 3. Humidification and dehumidification, 4. Cooling and adiabatic humidification, 5. Cooling and humidification by water injection, 6. Heating and humidification, 7. Humidification by steam injection, 8. Adiabatic chemical dehumidification, 9. Adiabatic mixing of air streams.

We shall now discuss these psychrometric processes, in detail, in the following pages.

3.71 Sensible Heating

The heating of air, without any-change in its specific humidity, is known as sensible heating. Let air at temperature td, passes over a heating coil of temperature t_{d3}, as shown in Fig. 15 (a). It may be noted that the temperature of air leaving the heating coil (t_{d2}) will be less than t_{d3}. The process of sensible heating, on the psychrometric chart, is shown by a horizontal line 1-2 extending from left to right as shown in Fig.15 (b). The point 3 represents the surface temperature of the heating coil.

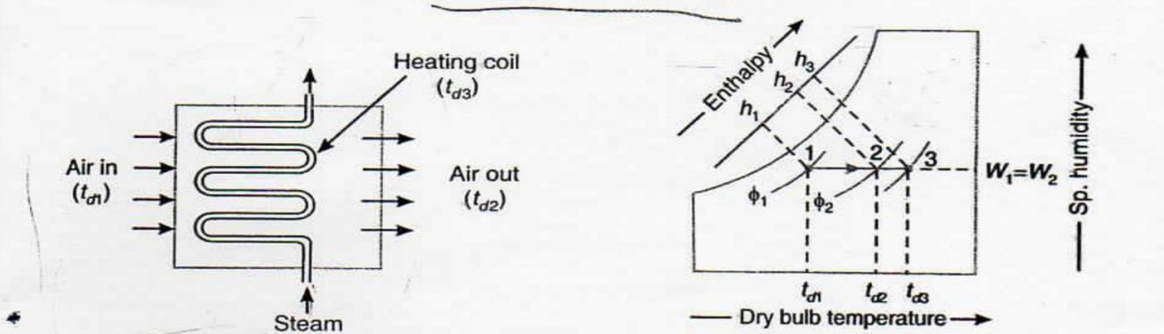
The heat absorbed by the air during sensible heating may be obtained from the psychrometric chart by the enthalpy difference (h_2 - h_1) as shown in Fig. 15 (b). It may be noted that the specific humidity during the sensible heating remains constant (i.e. $W_1 = W_2$).

The dry bulb temperature increases from t_{d1}, to t_{d2} and relative humidity reduces from $\emptyset_1$, to $\emptyset_2$ as shown in Fig. 15 (b). The amount of heat added during sensible heating may also be obtained from the relation:

$$\begin{aligned}\text{Heat added,} \quad q &= h_2 - h_1\\ &= c_{pa}(t_{d2} - t_{d1}) + W c_{ps}(t_{d2} - t_{d1})\\ &= (c_{pa} + W c_{ps})(t_{d2} - t_{d1}) = c_{pm}(t_{d2} - t_{d1})\end{aligned}$$

The term ($c_{pa} + W c_{ps}$) is called *humid specific heat* (c_{pm}) and its value is taken as 1.022 kJ /kg K.

$$\therefore \text{ Heat added,} \quad \underline{q = 1.022\,(t_{d2} - t_{d1}) \text{ kJ/kg}}$$



(*a*) Psychrometric process. (*b*) Psychrometric chart.

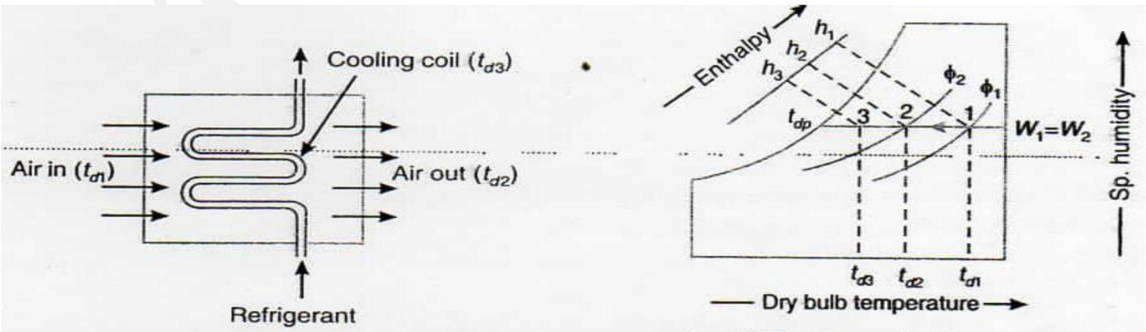
Fig.15 Sensible heating

Notes: 1. For sensible heating, steam or hot water is passed through the heating coil. The heating coil may be electric resistance coil.

2. The sensible heating of moist air can be done to any desired temperature.

3.72 Sensible Cooling

The cooling of air without any change in its specific humidity, is known as sensible cooling. Let air at temperature rd, passes over a cooling coil of temperature t_{d3} as shown in Fig. 16 (a). It may be noted that the temperature of air leaving the cooling coil (t_{d2}) will be more than t_{d3}. The process of sensible cooling, on the psychrometric chart, is shown by a horizontal line 1-2 extending from right to left as shown in Fig. 16



(*a*) Psychrometric process. (*b*) Psychrometric chart.

Fig. 16 Sensible cooling.

The heat rejected by air during sensible cooling may be obtained from the psychrometric chart by the enthalpy difference (h1 — h,) as shown in Fig. 16(b).

It may be noted that the specific humidity during the sensible cooling remains constant (i.e.$W_1 = W_2$). The dry bulb temperature reduces from t_{d1} to t_{d2} and relative humidity increases from $\emptyset_1$ to $\emptyset_2$ as shown in Fig. 16(b). The amount of heat rejected during sensible cooling may also be obtained from the relation:

Heat rejected, $q = h_1 - h_2$

$$= C_{pa}(t_{d1} - t_{d2}) + W\,C_{ps}(t_{d1} - t_{d2})$$

$$= (C_{pa} + W\,C_{ps})(t_{d1} - t_{d2}) = C_{pm}(t_{d1} - t_{d2})$$

The term ($C_{pa} + W\,C_{ps}$) is called humid specific heat (C_{pm}) and its value is taken as 1.022 kJ /kg K.

∴ Heat rejected, $q = 1.022\,(t_{d1} - t_{d2})$ kJ/kg

For air conditioning purposes, the sensible heat per minute is given as

$$SH = m_a\,C_{pm}\,\Delta t = v\rho\,C_{pm}\,\Delta t \text{ kJ/min} \qquad \ldots (\because m = v\rho)$$

where v = Rate of dry air flowing in m^3/min,

ρ = Density of moist air at 20° C and 50% relative humidity

= 1.2 kg / m3 of dry air,

C_{pm} = Humid specific heat = 1.022 kJ /kg K, and

$\Delta t = t_{d1} - t_{d2}$ = Difference of dry bulb temperatures between the entering and leaving conditions of air in ° C.

Substituting the values of p and cp,,, in the above expression, we get

$$SH = v \times 1.2 \times 1.022 \times \Delta t = 1.2264\, v \times \Delta t \text{ kJ/min}$$

$$= \frac{1.2264\, v \times \Delta t}{60} = 0.020\,44\, v \times \Delta t \text{ kJ/s or kW} \qquad \ldots (\because 1 \text{ kJ/s} = 1 \text{ kW})$$

3.73 By-pass Factor of Heating and Cooling Coil

The temperature of the air corning out of the apparatus (t_{d2}) will be less than *t_{d3} in case the coil is a heating coil and more than t_{d3} in case the coil is a cooling coil.

Let 1 kg of air at temperature t_{d1} is passed over the coil having its temperature (i.e. coil surface temperature) t_{d3} as shown in Fig. 17.

A little consideration will show that when air passes over a coil, some of it (say x kg) just by-passes unaffected while the remaining (1 - x) kg comes in direct contact with the coil. This by-pass process of air is measured in terms of a by-pass factor. The amount of air that by-passes or the by-pass factor depends upon the following factors :

1. The number of fins provided in a unit length i.e. the pitch of the cooling coil fins ;
2. The number of rows in a coil in the direction of flow; and
3. The velocity of flow of air.

It may be noted that the by-pass factor of a cooling coil decreases with decrease in fin spacing and increase in number of rows.

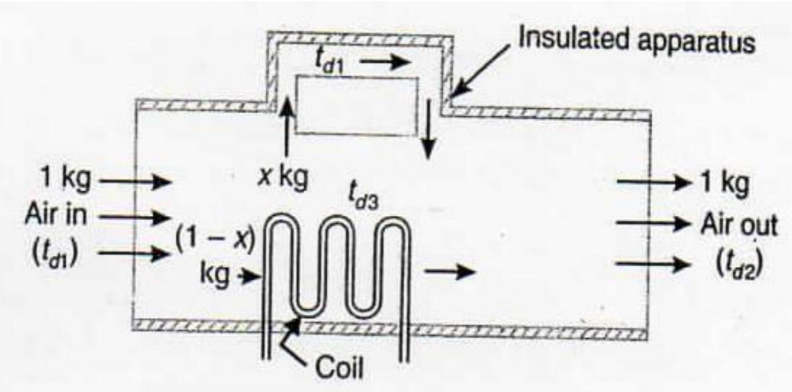
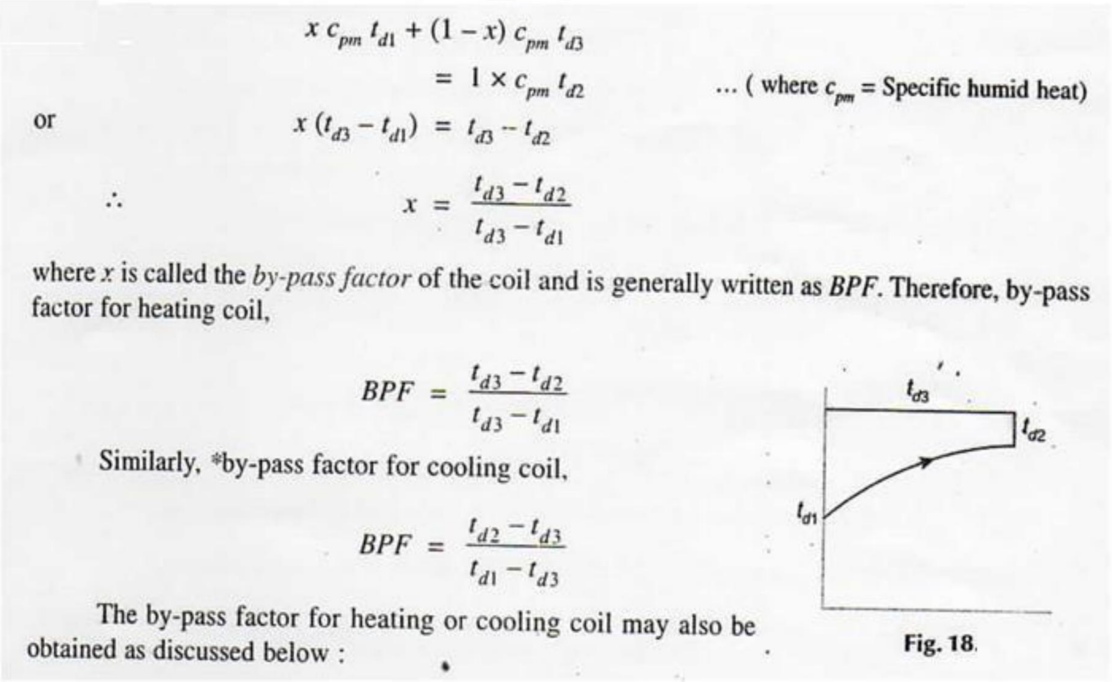


Fig.17. By-pass factor

Balancing the enthalpies, we get



$$x\, c_{pm}\, t_{d1} + (1-x)\, c_{pm}\, t_{d3} = 1 \times c_{pm}\, t_{d2} \qquad \text{... (where } c_{pm} = \text{Specific humid heat)}$$

or $$x\,(t_{d3} - t_{d1}) = t_{d3} - t_{d2}$$

$$\therefore \qquad x = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

where x is called the *by-pass factor* of the coil and is generally written as *BPF*. Therefore, by-pass factor for heating coil,

$$BPF = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

Similarly, *by-pass factor for cooling coil,

$$BPF = \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}}$$

The by-pass factor for heating or cooling coil may also be obtained as discussed below :

Fig. 18.

Let the air passes over a heating coil. Since the temperature distribution of air passing through the heating coil is as shown in Fig.18. therefore sensible heat given out by the coil.

$$Q_s = U A_c t_m \qquad \ldots (i)$$

Air hanling system

where

U = Overall heat transfer coefficient,

A_c = Surface area of the coil, and

t_m = Logarithmic mean temperature difference.

We know that logarithmic mean temperature difference,

$$t_m = \frac{t_{d2} - t_{d1}}{\log_e \left[\dfrac{t_{d3} - t_{d1}}{t_{d3} - t_{d2}} \right]}, \text{ and } BPF = \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}}$$

$$\therefore \qquad t_m = \frac{t_{d2} - t_{d1}}{\log_e (1/BPF)}$$

Now the equation (i) may be written as

$$Q_s = U \times A_c \times \frac{t_{d2} - t_{d1}}{\log_e (1/BPF)} \qquad \ldots (ii)$$

We have already discussed that the heat added during sensible heating,

$$Q_s = m_a \, c_{pm} (t_{d2} - t_{d1}) \qquad \ldots (iii)$$

where c_{pm} = Humid specific heat = 1.022 kJ/kg K, and

m_a = Mass of air passing over the coil.

Equating equations (ii) and (iii), we have

$$UA_c = m_a \, c_{pm} \log_e (1/BPF)$$

$$\log_e \left(\frac{1}{BPF} \right) = \frac{UA_c}{m_a \, c_{pm}}$$

or

$$\log_e (BPF) = -\frac{UA_c}{m_a \, c_{pm}}$$

$$\therefore \qquad BPF = e^{-\left(\frac{UA_c}{m_a \, c_{pm}} \right)} = e^{-\left(\frac{UA_c}{1.022 \, m_a} \right)} \qquad \ldots (iv)$$

Proceeding in the same• way as discussed above, we can derive the equation (iv) for a cooling coil.

Note: The performance of a heating or cooling coil is measured in terms of a by-pass factor. A coil with low by-pass factor has better performance.

3.74 Efficiency of Heating and Cooling Coils

The term (1 - BPF) is known as efficiency of coil or contact factor.

∴ Efficiency of the heating coil,

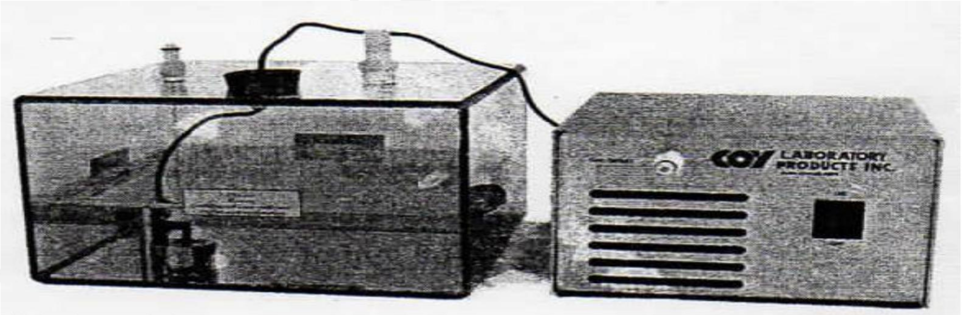
$$\eta_H = 1 - BPF = 1 - \frac{t_{d3} - t_{d2}}{t_{d3} - t_{d1}} = \frac{t_{d2} - t_{d1}}{t_{d3} - t_{d1}}$$

Similarly, efficiency of the cooling coil,

$$\eta_C = 1 - \frac{t_{d2} - t_{d3}}{t_{d1} - t_{d3}} = \frac{t_{d1} - t_{d2}}{t_{d1} - t_{d3}}$$

3.75 Humidification and Dehumidification

The addition of moisture to the air, without change in its dry bulb temperature, is known as *humidification*. Similarly, removal of moisture from the air, without change in its dry bulb temperature, is known as *dehumidification*. The heat added during humidification process and heat removed during dehumidification process is shown on the psychrometric chart in Fig. 19 (a) and (b) respectively.



Ultrasonic humidification system

It may be noted that in humidification, the relative humidity increases from $\varnothing_1$ to $\varnothing_2$ and specific humidity also increases from W_1 to W_2 as shown in Fig. 19 (a). Similarly, in dehumidification, the relative humidity decreases from $\varnothing_1$ to $\varnothing_2$ and specific humidity also decreases from W_1 to W_2 as shown in Fig. 19 (b).

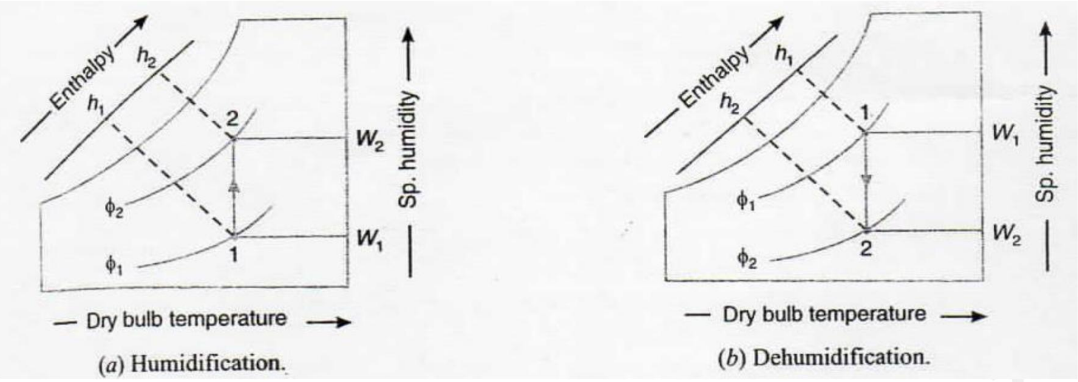
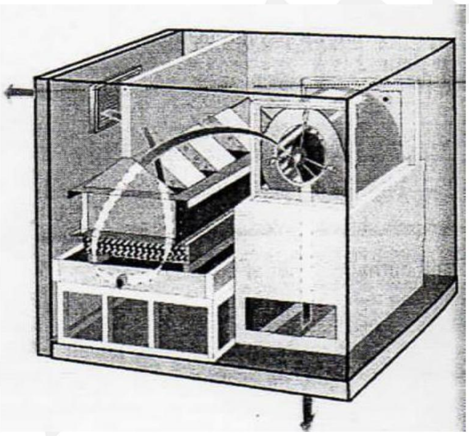


Fig. 19 Humidification and dehumidification

It may be noted that in humidification, change in enthalpy is shown by the intercept (h_2 - h_1) on the psychrometric chart. Since the dry bulb temperature of air during the humidification remains constant, therefore its sensible heat also remains constant. It is thus obvious that the change in enthalpy per kg of dry air due to the increased moisture content equal to (W_2 - W_1) kg per kg of dry air is considered to cause a latent heat transfer (LH). Mathematically,



Multiple small plate dehumidification system

LH = (h_2 - h_1) = h_{fg} (W_2 - W_1) where h_{fg} is the latent heat of vaporization at dry bulb temperature (t_{dt}).

Notes: 1. For dehumidification, the above equation may be written as:

$$LH = (h_1 - h_2) = h_{fg} (W_1 - W_2)$$

2. Absolute humidification and dehumidification processes are rarely found in practice. These are always accompanied by heating or cooling processes.

3. In air conditioning, the latent heat load per minute is given as

$$LH = m_a \Delta h = m_a h_{fg} \Delta W = v \rho h_{fg} \Delta W \qquad \ldots (\because m_a = v \rho)$$

where v = Rate of dry air flowing in m^3/min,

ρ = Density of moist air = 1.2 kg/m^3 of dry air,

h_{fg} = Latent heat of vaporization = 2500 kJ /kg, and

AW = Difference of specific humidity between the entering and leaving conditions of

air = $(W_2 - W_1)$ for humidification and $(W_1 – W_2)$ for dehumidification.

Substituting these values in the above expression, we get

$$LH = v \text{ x } 1.2 \text{ x } 2500 \text{ x } \Delta W = 3000 \text{ v x } \Delta W \text{ kJ/min}$$

$$= \frac{3000\, v \times \Delta W}{60} = 50\ v \times \Delta W \text{ kJ/s or kW}$$

3.8 Methods of Obtaining Humidification and Dehumidification

The humidification is achieved either by supplying or spraying steam or hot water or cold water into the air. The humidification may be obtained by the following two methods:

1. **Direct method**. In this method, the water is sprayed in a highly atomized state into the room to be air-conditioned. This method of obtaining humidification is not very effective.

2. **Indirect method**. In this method, the water is introduced into the air in the air-conditioning plant, with the help of an air-washer, as shown in Fig. 20. This -conditioned air is then supplied to the room to be air-conditioned. The air-washer humidification may be accomplished in the following three ways:

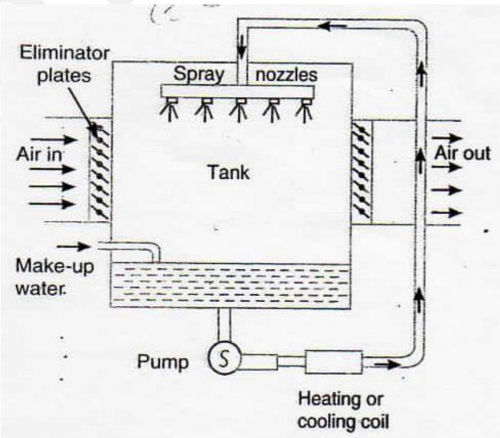


Fig. 20. Air-washer.

(a) by using re-circulated spray water without prior heating of air,

(b) by pre-heating the air and then washing if with re-circulated water, and

(c) by using heated spray water.

The dehumidification may be accomplished with the help of an air-washer or by using chemicals. In the air-washer system the outside or entering air is cooled below its dew point temperature so that it loses moisture by condensation. The moisture removal is also accomplished when the spray water is chilled water and its temperature is lower than the dew point temperature of the entering air. Since the air leaving the air-washer has its dry bulb temperature much below the desired temperature in the room, therefore a heating coil is placed after the air-washer. The dehumidification may also be achieved by using chemicals which have the capacity to absorb moisture in them. Two types of chemicals known as absorbents (such as calcium chloride) and adsorbents (such as silica gel and activated alumina) are commonly used for this purpose.

Sensible Heat Factor

As a matter of fact, the heat added during a psychrometric process may be split up into sensible heat and latent heat. The ratio of the *sensible heat to the total heat is known us *sensible heat factor* (briefly written as SHF) or *sensible heat ratio* (briefly written as SHR). Mathematically,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{SH + LH}$$

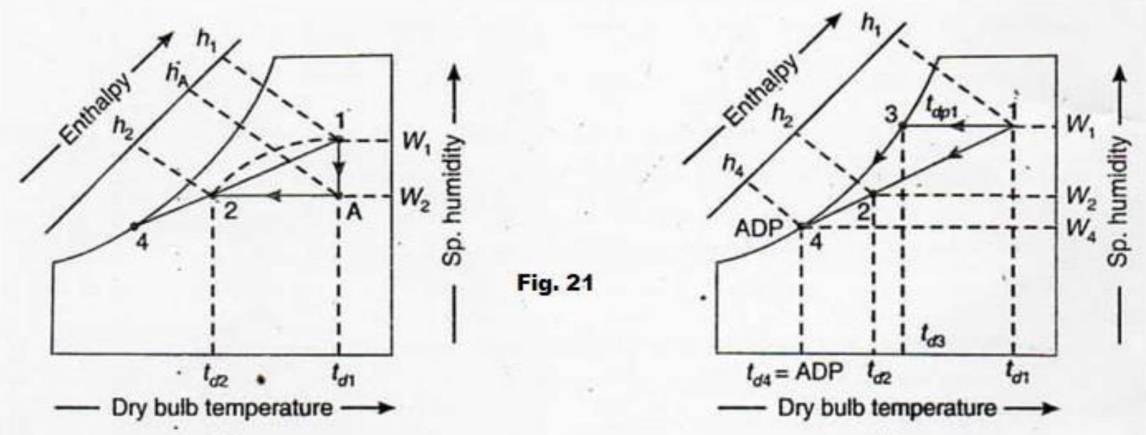
where SH = Sensible heat, and

LH = Latent heat.

The sensible heat factor scale is shown on the right hand side of the psychrometric chart.

3.9 Cooling and Dehumidification

This process is generally used in summer air conditioning to cool and dehumidify the air. The air is passed over a cooling coil or through a cold water spray. In this process, the dry bulb temperature as well as the specific humidity of air decreases. The final relative humidity of the air is generally higher than that of the entering air. The dehumidification of air is only possible wh6n the effective surface temperature of the cooling coil (i.e.t_{d4}) is less than the dew point temperature of the air entering the coil (i.e., t_{dpt}.). The effective surface temperature of the coil is known as *apparatus dew point* (briefly written as ADP). The cooling and dehumidification process is shown in Fig. 21.



(a) **Fig. 21** Cooling and dehumidification. (b)

t_{d1} = Dry bulb temperature of air entering the coil,

t_{dp1} = Dew point temperature of the entering air = t_{d3} and

t_{d4} = Effective surface temperature or ADP of the coil.

Under ideal conditions, the dry bulb temperature of the air leaving the cooling coil (i.e. t_{d4}) should be equal to the surface temperature of the cooling coil (i.e. ADP), but it is never possible due to inefficiency of the cooling coil. Therefore, the resulting condition of air coming out of the coil is shown by a point 2 on the straight line joining the points 1 and 4. The by-pass factor in this case is given by

$$BPF = \frac{t_{d2} - t_{d4}}{t_{d1} - t_{d4}} = \frac{t_{d2} - ADP}{t_{d1} - ADP}$$

Also

$$BPF = \frac{W_2 - W_4}{W_1 - W_4} = \frac{h_2 - h_4}{h_1 - h_4}$$

Actually, the cooling and dehumidification process follows the path as shown by a dotted curve in Fig. 21(a), but for the calculation of psychrometric properties, only end points are important. Thus the cooling and dehumidification process shown by a line 1-2 may be assumed to have followed a path 1-A (i.e. dehumidification) and A-2 (i.e. cooling) as shown in Fig. 21 (a). We see that the total heat removed from the air during the cooling and dehumidification process is

$$q = h_1 - h_2 = (h_1 - h_A) + (h_A - h_2) = LH + SH$$

where LH = $h_1 - h_A$ = Latent heat removed due to condensation of vapour of the reduced moisture content ($W_1 - W_2$), and

SH = $h_A - h_2$ = Sensible heat removed.

We know that sensible heat factor,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{SH}{LH + SH} = \frac{h_A - h_2}{h_1 - h_2}$$

Note: The line 1-4 (i.e. the line joining the point of entering air and the apparatus dew point) in Fig. 21 (b) is known as sensible heat factor line.

Example 1: In a cooling application, moist air enters a refrigeration coil at the rate of 100 kg of dry air per minute at 35° C and 50% RH. The apparatus dew point of coil is 5° C and by-pass factor is 0.15. Determine the outlet state of moist air and cooling capacity of coil in TR.

Solution Given: $m_a = 100$ kg/min; $t_{dt} = 35°C$; $\emptyset = 50\%$; ADP = 5°C; BPF = 0.15

Outlet state of moist air

Let t_{d2}, and $\emptyset_2$ = Temperature and relative humidity of air leaving the cooling coil.

First of all, mark the initial condition of air, i.e. 35° C dry bulb temperature and 50% relative humidity on the psychrometric chart at point 1, as shown in Fig. 22. From the psychrometric chart, we find that the dew point temperature of the entering air at point 1,

$$t_{dpt} = 23°C$$

Since the coil or apparatus dew point (ADP) is less than the dew point temperature of entering air, therefore it is a process of cooling and dehumidification.

We know that by-pass factor,

$$BPF = \frac{t_{d2} - t_{d4}}{t_{d1} - t_{d4}} = \frac{t_{d2} - ADP}{t_{d1} - ADP}$$

$$0.15 = \frac{t_{d2} - 5}{35 - 5} = \frac{t_{d2} - 5}{30}$$

$$t_{d2} = 0.15 \times 30 + 5 = 9.5°C \text{ Ans.}$$

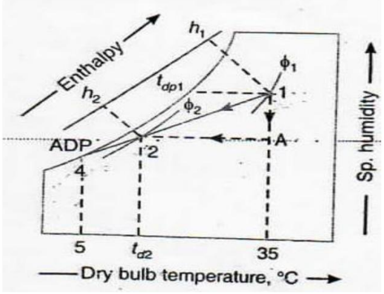


Fig.22

From the psychrometric chart, we find that the relative humidity corresponding to a dry bulb temperature (t_{d2},) of 9.5°Con the line 1-4 is $\emptyset_2 = 99\%$. Ans.

Cooling capacity of the coil

The resulting condition of the air coming out of the coil is shown by point 2, on the line joining the points 1 and 4, as shown in Fig. 22. The line 1-2 represents the cooling and dehumidification process which may be assumed to have followed the path 1-A (i.e. dehumidification) and A-2 (i.e. cooling). Now from the psychrometric chart, we find that enthalpy of entering air at point 1,

h_1= 81 kJ/kg of dry air

and enthalpy of air at point 2,

h_2= 28 kJ/kg of dry air

We know that cooling capacity of the coil

$= m_a(h_1 - h_2) = 100\ (81 - 28) = 5300$ kJ/min

$= 5300/210 = 25.24$ TR Ans. ….($\because$ 1 TR = 210 kJ/min)

Example 2. 39.6 m^3/min of a mixture of re-circulated room air and outdoor air enters cooling coil at 31°C dry bulb temperature and 18.5°C wet bulb temperature. The effective surface temperature of the coil is 4.4°C. The surface area of the coil is such as would give 12.5 kW of refrigeration with the given entering air state. Determine the dry and wet bulb temperatures of the air leaving the coil and the by-pass factor.

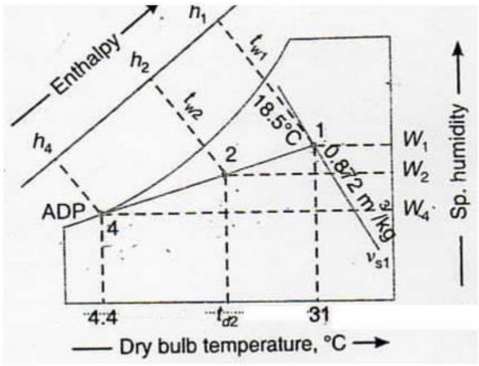
Solution: Given: v_1= 39.6 m^3/min; t_{dt} = 31°C; t_{wt} = I8.5°C; ADP= t_{d4}= 4.4°C; Q= 12.5 kW = 12.5 kJ/s = 12.5 x 60 kJ/min

Dry and wet bulb temperature of the air leaving the coil

Let t_{d2} and t_{w2} = Dry and wet bulb temperature of the air leaving the coil.

First of all, mark the initial condition of air, i.e. 31°C dry bulb temperature and 18.5°C wet bulb temperature on the psychrometric chart at point 1, as shown in Fig. 23. Now mark the effective surface temperature (ADP) of the coil at 4.4°C at point 4.

From the psychrometric chart, we find that enthalpy at point 1



h_1 = 52.5 kJ / kg of dry air

Enthalpy at point 4,

h_4 = 17.7kJ/kg of dry air

Specific humidity at point 1

W_1 = 0.0082 kg / kg of dry air

Specific humidity at point 4,

W_4= 0.00525 kg / kg of dry air

Specific volume at point

v_{s1}, = 0.872m^3/ kg

We know that mass flow rate of dry air at point 1,

$$m_a = \frac{v_1}{v_{s1}} = \frac{39.6}{0.872} = 44.41 \text{ kg/min}$$

and cooling capacity of the coil,

$$Q = m_a(h_1 - h_2)$$

or

$$h_1 - h_2 = \frac{Q}{m_a} = \frac{12.5\times 60}{44.41} = 16.89 \text{ kJ / kg of dry air}$$

$\therefore$

$$h_2 = h_1 - 16.89 = 52.5 - 16.89 = 35.61 \text{ kJ / kg of dry air}$$

The equation for the condition line 1–2–4 is given as

$$\frac{W_2 - W_4}{W_1 - W_4} = \frac{h_2 - h_4}{h_1 - h_4}$$

$$\frac{W_2 - 0.005\,25}{0.0082 - 0.005\,25} = \frac{35.61 - 17.7}{52.5 - 17.7}$$

$\therefore$

$$W_2 = 0.006\,77 \text{ kg / kg of dry air}$$

Now plot point 2 on the psychrometric chart such as enthalpy, h_2 = 35.61 kJ/kg of dry air and specific humidity, W_2 = 0.00677 kg/kg of dry air. At point 2, we find that

t_{d2} = 18.5°C; and t_{w2} = 12.5°C Ans.

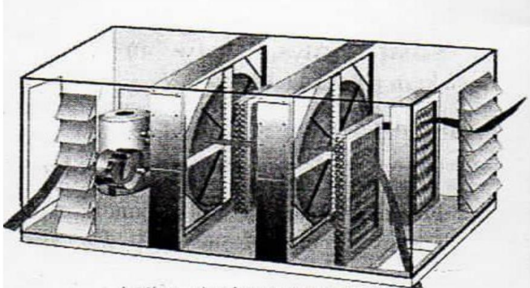
By-pass factor

We know that by-pass factor,

$$BPF = \frac{h_2 - h_4}{h_1 - h_4} = \frac{35.61 - 17.7}{52.5 - 17.7} = 0.5146 \text{ Ans.}$$

3.10 Heating and Humidification

This process is generally used in winter air conditioning to warm and humidify the air. It is the reverse process of cooling and -- dehumidification. When air is passed through a humidifier having spray water temperature higher than the dry bulb temperature of the entering air, the unsaturated air will reach the condition of saturation and thus the air becomes hot. The heat of vaporization of water is absorbed from the spray water itself and hence it gets cooled. In this way, the air becomes heated and humidified. The process of heating and humidification is shown by line 1-2 on the psychrometric chart as shown in Fig. 24.



Active desiccant system.

The air enters at condition 1 and leaves at condition 2. In this process, the dry bulb temperature as well as specific humidity of air increases. The final relative humidity of the air can be lower or higher than that of the entering air.

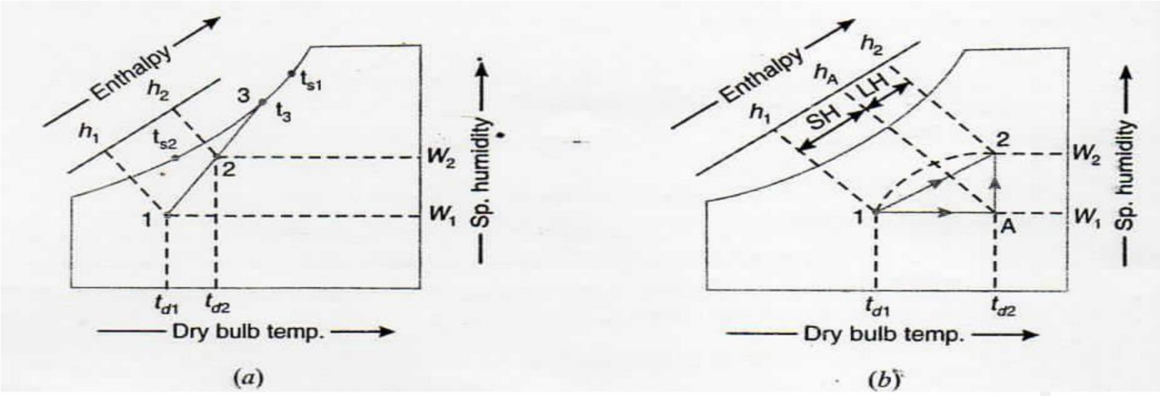


Fig.24 heating and humidification

Let m_{w1} and m_{w2} = Mass of spray water entering and leaving the humidifier in kg,

h_{fw1} and h_{fw2} = Enthalpy of spray water entering and leaving the humidifier in kJ/kg,

W_1 and W_2 = Specific humidity of the entering and leaving air in kg/kg of dry

h_1 and h_2 = Enthalpy of entering and leaving air in kJ/kg of dry air, and

m_a = Mass of dry air entering in kg.

For mass balance of spray water,

$$(m_{w1} - m_{w2}) = m_a (W_2 - W_1)$$

$$m_{w2} = m_{w1} - m_a (W_2 — W_1) \qquad \text{…. }(i)$$

or and for enthalpy balance,

$$m_{w1} h_{fw1} = m_{w2} h_{fw2} = m_a(h_2 - h_1) \qquad \text{….}(ii)$$

Substituting the value of mw2 from equation (i), we have

$$m_{w1} h_{fw1} - [\, m_{w1} - m_a (W_2 - W_1)]\, h_{fw2}$$

$$= m_a (h_2 - h_1)$$

$$\therefore \qquad h_2 - h_1 = \frac{m_{w1}}{m_a} (h_{fw1} - h_{fw2}) + (W_2 - W_1)\, h_{fw2}$$

The temperatures t_{s1} and t_{s2} shown in Fig. 24 (a) denote the temperatures of entering and leaving spray water respectively. The temperature 13 is the mean temperature of the spray water which the entering air may be assumed to approach.

Actually, the heating and humidification process follows the path as shown by dotted curve in Fig. 24(b), but for the calculation of psychrometric properties, only the end points are important. Thus, the heating and humidification process shown by a line 1-2 on the psychrometric chart may be assumed to have followed the path 1-A (i.e. heating) and A-2 (i.e. humidification), as shown in Fig. 24(b). We see that the total heat added to the air during heating and humidification is

$$q = h_2 - h_1 = (h_2 - h_1) + (h_A - h_i) = q_t + q_s$$

where $q_t = (h_2 - h_A)$ = Latent heat of vaporization of the increased moisture

content $(W_2 - W_1)$, and

$q_s = (h_A - h_i)$ = Sensible heat added

We know that sensible heat factor,

$$SHF = \frac{\text{Sensible heat}}{\text{Total heat}} = \frac{q_S}{q} = \frac{q_S}{q_S + q_L} = \frac{h_A - h_1}{h_2 - h_1}$$

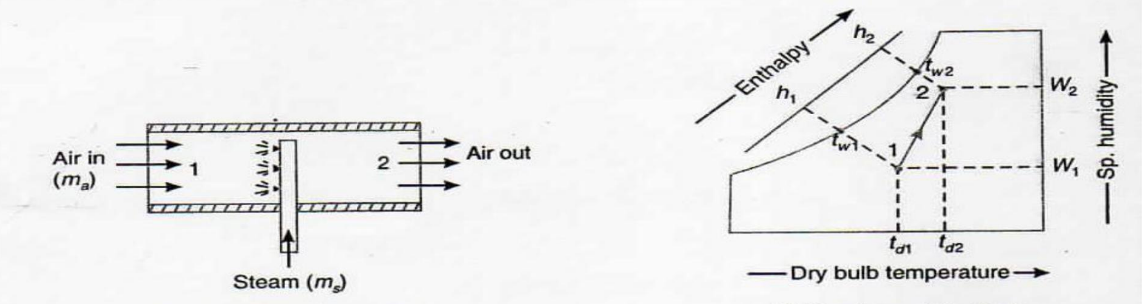
Note: The line 1-2 in Fig. 24 (b) is called sensible heat factor line.

3.11 Heating and Humidification by Steam Injection

The steam is normally injected into the air in order to increase its specific humidity as shown in Fig. 25 (a). This process is used for the air conditioning of textile mills where high humidity is to be maintained. The dry bulb temperature of air changes very little during this process, as shown on the psychrometric chart in Fig. 25 (b).

Let m_s = Mass of steam supplied,

m_a = Mass of dry air entering,



(a) Psychrometric process. (b) Psychrometric chart.

Fig.25 heating and humidification by steam injection

W_1= Specific humidity of air entering,

W_2 = Specific humidity of air leaving,

h_1 = Enthalpy of air entering,

h_2 = Enthalpy of air leaving, and

h_s = Enthalpy of steam injected into the air.

Now for the mass balance,

$$W_2 = W_1 + \frac{m_s}{m_a} \qquad \ldots.(i)$$

and for the heat balance,

$$h_2 = h_1 + \frac{m_s}{m_a} \times h_s = h_1 + (W_2 - W_1)\, h_s \qquad \text{... [From equation } (i)\text{]}$$

Example 3: Atmospheric air at a dry bulb temperature of 16° C and 25% relative humidity passes through a furnace and then through a humidifier, in such a way that the final dry bulb temperature is 30° C and 50% relative humidity. Find the heat and moisture added to the air. Also determine the sensible heat factor of the-process.

Solution: Given: t_{dt} = 16° C; Ø_1 = 25% ; t_{d2} = 30° C; 42 = 50%

Heat added to the air

First of all, mark the initial condition of air i.e. at 16°C dry bulb temperature and 25% relative humidity on the psychrometric chart at point 1, as shown in Fig. 16.47. Then mark the final condition of air at 30° C dry bulb temperature and 50% relative humidity on the psychrometric chart at point 2. Now locate the point A by drawing horizontal line through point 1 and vertical line through point 2. From the psychrometric chart, we find that enthalpy of air at point 1,

h_1 = 23 k1/kg of dry air

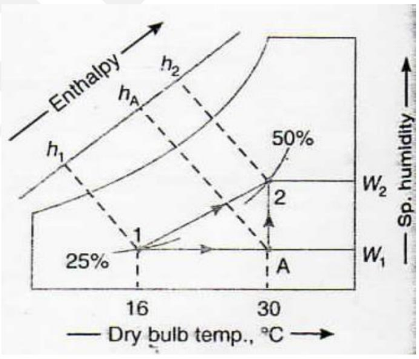


Fig.26

Enthalpy of air at point A,

h_A = 38 kJ/kg of dry air

and enthalpy of air at point 2,

h_2 = 64 la/kg of dry air

∴ Heat added to the air

= h_2- h_1= 64 - 23 = 41 kJ/kg of dry air Ans.

Moisture added to the air

From the psychrometric chart, we find that the specific humidity in the air at point 1,

W_1 = 0.0026 kg/kg of dry air

and specific humidity in the air at point 2,

W_2 = 0.0132 kg /kg of dry air

∴ Moisture added to the air

= $W_2 - W_1$ = 0.0132 - 0.0026 = 0.0106 kg/kg of dry air Ans.

Sensible heat factor of the process

We know that sensible heat factor of the process,

$$SHF = \frac{h_A - h_1}{h_2 - h_1} = \frac{38 - 23}{64 - 23} = 0.366 \text{ Ans.}$$

Example 4: Air at 10°C dry bulb temperature and 90% relative humidity is to be ated and humidified to 35°C dry bath temperature and 22.5°C wet bulb temperature. The air is pre-heated sensibly before passing to the air washer in which water is re-circulated. The relative humidity of the air coming out of the air washer is 90%. This air is again reheated sensibly to obtain the final desired condition. Find: 1. the temperature to which the air should be preheated. 2. the total heating required; 3. the makeup water required in the air washer ; and 4. the humidifying efficiency of the air washer.

Solution: Given : t_{dt} = 10°C; $\emptyset_1$ = 90%; $t_{d2,}$=35°C; t_{w2}=22.5°C

First of all, mark the initial condition of air i.e. at 10°C dry bulb temperature and 90% relative humidity, on the psychrometric chart at point 1, as shown in Fig. 16.48. Now mark the final condition of air i.e. at 35° C dry bulb temperature and 22.5° C wet bulb temperature at point 2.

From point I , draw a horizontal line to represent sensible heating and from point 2 draw horizontal line to intersect 90% relative humidity curve at point B. Now from point B, draw a constant wet bulb temperature line which intersects the horizontal line drawn through point 1 at point A. The line 1-A represents preheating of air, line AB represents humidification and line 8-2 represents reheating to final condition.

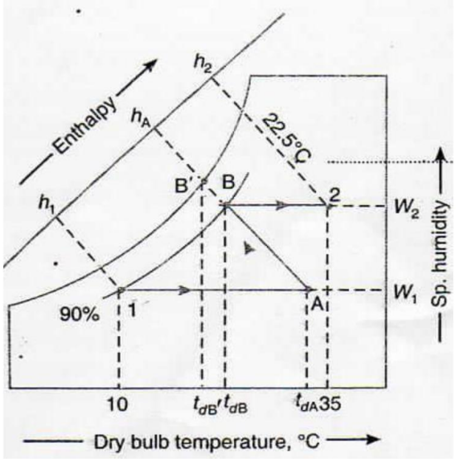


Fig.27

1. Temperature to which the air should be preheated

From the psychrometric chart, the temperature to which the air should be preheated (corresponding to point A) is 4 $t_{dA} = 32.6°C$ Ans.

2. Total heating required

From the psychrometric chart, we find that enthalpy of,air at point 1.

$$h_1 = 27.2 \text{ kJ /kg of dry air}$$

Enthalpy of air at point A,

$$h_A = 51 \text{ kJ/kg of dry air}$$

and enthalpy of air at point 2,

$$h_2 = 68 \text{ kJ/kg of dry air}$$

We know that heat required for preheating of air

$$= h_A - h_1 = 51 - 27.2 = 23.8 \text{ kJ/kg of dry air}$$

and heat required for reheating of air

$$= h_2 - h_B = 68 - 51 = 17 \text{ kJ/kg of dry air}$$

∴ Total heat required $= 23.8 + 17 = 40.8$ kJ/kg of dry air Ans.

3. Make up water required in the air washer

From the psychrometric chart, we find that specific humidity of entering air,

$$W_1 = 0.0068 \text{ kg /kg of dry air}$$

and specific humidity of leaving air,

$$W_2 = 0.0122 \text{ kg /kg of dry air}$$

∴ Make up water required in the air washer

$$= W_B - W_A = W_2 - W_1$$

$$= 0.0122 - 0.0068 = 0.0054 \text{ kg/kg of dry air} \quad \text{Ans.}$$

4. Humidifying efficiency of the air washer

From the psychrometric chart, we find that

$$t_{dB} = 19.1°C \text{ and } t_{dB} = 18°C$$

We know that humidifying efficiency of the air washer,

$$\eta_H = \frac{\text{Actual drop in DBT}}{\text{Ideal drop in DBT}} = \frac{t_{dA} - t_{dB}}{t_{dA} - t_{dB'}}$$

$$= \frac{32.6 - 19.1}{32.6 - 18} = \frac{13.5}{14.6} = 0.924 \text{ or } 92.4\% \quad \text{Ans.}$$

3.12 Heating and Dehumidification -Adiabatic Chemical Dehumidification

This process is mainly used in industrial air conditioning and can also be used for some comfort air conditioning installations requiring either a low relative humidity or low dew point temperature in the room.

In this process, the air is passed over chemicals which have an affinity for moisture. As the air comes in contact with these chemicals, the moisture gets condensed out of the air and gives up its latent heat. Due to the condensation, the specific humidity decreases and the heat of condensation supplies sensible heat for heating the air and thus increasing its dry bulb temperature.

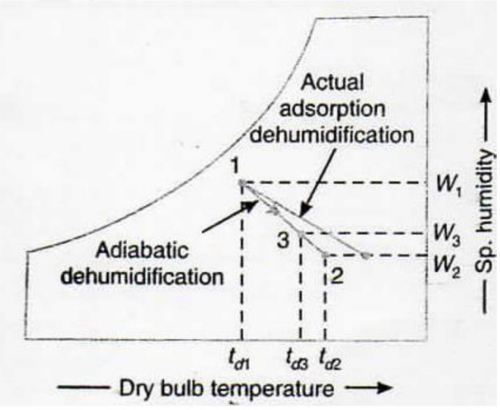


Fig.28

The process, which is the reverse of adiabatic saturation process, is shown by the line 1-2 on the psychrometric chart as shown in Fig. 28. The path followed during the process is along the constant wet bulb temperature line or-constant enthalpy line.

The effectiveness or efficiency of the dehumidifier is given as

$$\eta_H = \frac{\text{Actual increase in dry bulb temperature}}{\text{Ideal increase in dry bulb temperature}} = \frac{t_{d3} - t_{d1}}{t_{d2} - t_{d1}}$$

Notes: 1. In actual practice, the process is accompanied with a release of heat called heat of adsorption, which is very large. Thus the sensible heat gain of air exceeds the loss of latent heat and the process is shown above the constant wet bulb temperature line in Fig. 28.

2. Two types of chemicals used for dehumidification are absorbents and adsorbents. The absorbents are substances which can take up moisture from air and during this process change it chemically, physically or in both respects. These include water solutions or brines of calcium chloride, lithium chloride, lithium bromide and ethylene glycol. These are used as air dehydrators by spraying or otherwise exposing a large surface of the solution in the air stream.

The adsorbents are substances in the solid state which can take up moisture from the air and during this process do not change it chemically or physically. These include silca gel (which is a form of silicon dioxide prepared by mixing fused sodium silicate and sulphuric acid) and activated alumina (which is a porous amorphous form of aluminum oxide).

Example 5: Saturated air at 21° C is passed through a drier so that its final relative humidity is 20%. The drier uses silica gel adsorbent. The air is then passed through a cooler until its final temperature is 21° C without a change in specific humidity. Determine : 1. the temperature of air at the end of the drying process; 2. the heat rejected during the cooling process ; 3. the relative humidity at the end of cooling process; 4. the dew point temperature at the end of the drying process ; and 5. the moisture removed during the drying process.

Solution: Given: $t_{d1}, = t_{d3} = 21°C$; $\emptyset_2 = 20\%$

1. Temperature of air at the end of drying process

First of all, mark the initial condition of air i.e. at 21°C dry bulb temperature upto the saturation curve (because the air is saturated) on the psychrometric chart at point 1, as shown in Fig. 29. Since the drying process is a chemical dehumidification process, therefore-. it follows a path along-the-constant wet bulb temperature or the constant enthalpy line as shown by the line 1- 2 in Fig. 29. Now mark the point 2 at relative humidity of 20%. From the psychrometric chart, the temperature at the end of drying process at point 2, $t_{d2} = 38.5°C$ Ans.

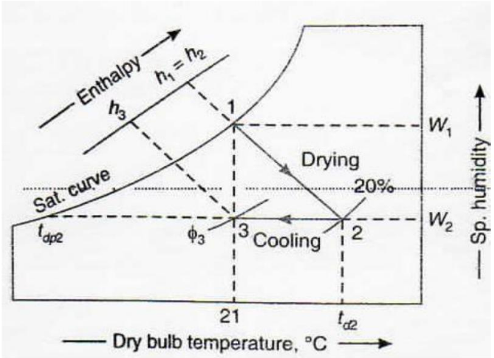


Fig.29

2. Heat rejected during the cooling process

The cooling process is shown by the line 2-3 on the psychrometric chart as shown in Fig.29. From the psychrometric chart. we find that enthalpy of air at point 2.

h_2 = 61 kJ /kg of dry air

and enthalpy of air at point 3,

h_3 = 43 kJ/kg of dry air

∴ Heat rejected during the cooling process

= $h_2 - h_3$ = 61— 43 = 18 kJ/kg of dry air Ans.

3. Relative humidity at the end of cooling process

From the psychrometric chart, we find that relative humidity at the end of cooling process (i.e. at point 3),

$\emptyset_3$ = 55% Ans.

4. Dew point temperature at the end of drying process

From the psychrometric chart, we find that the dew point temperature at the end of the drying process,

t_{dp2} = 11.6° C Ans.

5. Moisture removed during the drying process

From the psychrometric chart, we find that moisture in air before the drying process at point 1,

W_1 = 0.0157 kg/kg of dry air

and moisture in air after the drying process at point 2,

W_2 = 0.0084 kg/kg of dry air

∴ Moisture removed during the drying process

= W, — W, = 0.0157 0.0084 = 0.0073 kg/kg of dry air Ans.