ME 8693 HEAT AND MASS TRANSFER (L T P C – 3 2 0 4)

OBJECTIVES:

- To understand the mechanisms of heat transfer under steady and transient conditions.
- To understand the concepts of heat transfer through extended surfaces.
- To learn the thermal analysis and sizing of heat exchangers and to understand the basic concepts of mass transfer.

(Use of standard HMT data book permitted)

UNIT I CONDUCTION **9+6**

General Differential equation of Heat Conduction– Cartesian and Polar Coordinates – One Dimensional Steady State Heat Conduction — plane and Composite Systems – Conduction with Internal Heat Generation – Extended Surfaces – Unsteady Heat Conduction – Lumped Analysis – Semi Infinite and Infinite Solids –Use of Heisler's charts.

UNIT II CONVECTION **9+6**

Free and Forced Convection - Hydrodynamic and Thermal Boundary Layer. Free and Forced Convection during external flow over Plates and Cylinders and Internal flow through tubes .

UNIT III PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS **9+6**

Nusselt's theory of condensation - Regimes of Pool boiling and Flow boiling. Correlations in boiling and condensation. Heat Exchanger Types - Overall Heat Transfer Coefficient – Fouling Factors - Analysis – LMTD method - NTU method.

UNIT IV RADIATION **9+6**

Black Body Radiation – Grey body radiation - Shape Factor – Electrical Analogy – Radiation Shields. Radiation through gases.

UNIT V MASS TRANSFER **9+6**

Basic Concepts – Diffusion Mass Transfer – Fick's Law of Diffusion – Steady state Molecular Diffusion – Convective Mass Transfer – Momentum, Heat and Mass Transfer Analogy –Convective Mass Transfer Correlations.

TOTAL : 75 PERIODS

OUTCOMES: Upon the completion of this course the students will be able to

CO1 Apply heat conduction equations to different surface configurations under steady state and transient conditions and solve problems

CO2 Apply free and forced convective heat transfer correlations to internal and external flows through/over various surface configurations and solve problems

CO3 Explain the phenomena of boiling and condensation, apply LMTD and NTU methods of thermal analysis to different types of heat exchanger configurations and solve problems

CO4 Explain basic laws for Radiation and apply these principles to radiative heat transfer between different types of surfaces to solve problems

CO5 Apply diffusive and convective mass transfer equations and correlations to solve problems for different applications

TEXT BOOKS:

1. Holman, J.P., "Heat and Mass Transfer", Tata McGraw Hill, 2000
2. Yunus A. Cengel, "Heat Transfer A Practical Approach", Tata McGraw Hill, 5th Edition 2015

REFERENCES:

1. Frank P. Incropera and David P. Dewitt, "Fundamentals of Heat and Mass Transfer", John Wiley & Sons, 1998.
2. Kothandaraman, C.P., "Fundamentals of Heat and Mass Transfer", New Age International, New Delhi, 1998.
3. Nag, P.K., "Heat Transfer", Tata McGraw Hill, New Delhi, 2002
4. Ozisik, M.N., "Heat Transfer", McGraw Hill Book Co., 1994.
5. R.C. Sachdeva, "Fundamentals of Engineering Heat & Mass transfer", New Age International Publishers, 2009

7. Kothandaraman, C.P., "Fundamentals of Heat and Mass Transfer", New Age International, New Delhi, 1998.
8. Yadav, R., "Heat and Mass Transfer", Central Publishing House, 1995.
9. M.Thirumaleshwar : Fundamentals of Heat and Mass Transfer, "Heat and Mass Transfer", First Edition, Dorling Kindersley, 2009

UNIT I CONDUCTION

1.1 Heat

"Heat is defined as the transmission of energy from one region to another as a result of temperature gradient."

It is a vector quantity, flowing in the direction of decreasing temperature, with a negative temperature gradient. In the science of thermodynamics, the important parameter is the quantity of heat transferred during a process. Thermodynamics is concerned with the transition of a system from one equilibrium state to another, and is based principally on the two laws of nature, the first law of thermodynamics and second law of thermodynamics.

The application of Heat transfer:

i) Design of thermal and nuclear power plants including heat engines, steam generators , condensers and other heat exchange equipments , catalytic converters, heat shields for space vehicles ,furnaces, electronic equipments etc.
ii) Internal combustion engines
iii) Refrigeration and air conditioning units
iv) Design of cooling systems for electric motors, generators and transformers.
v) Heating and cooling of fluids.
vi) Construction of dams and structures.
vii) Heat treatment of metals.

Modes of heat transfer:

Heat transfer takes places by the following three modes

i) Conduction
ii) Convection
iii) Radiation

Conduction:-

Conduction is the transfer of heat from one part of a substance to another part of the same substance or from one substance to another in physical contact with it

Heat is conducted by

1. Atomic vibration

2. By transport of free electrons

Fourier's law of heat conduction:-

The conduction heat transfer through a simple homogeneous solid is directly proportional to

1. The area of section at right angle to the direction of heat flow.

2. The change in temperature in between the two faces of the slab

3. Inversely proportional to the thickness of the slab.

Mathematically,

$$Q \alpha A \frac{dT}{dx}$$

Q = Heat flow through a body per unit time(W)

A = surface area of heat flow (Perpendicular to the direction of flow)m^2

dT= Temperature difference of the faces of block (°C or K)

dx = Thickness of body in the direction of flow (m)

$$Q = - KA \frac{dT}{dx}$$

K = thermal conductivity

-ve sign Kis to take care of the decreasing temperature along with the direction of increasing thickness or the direction of heat flow dT/dx always –ve so Qis positive

.

Thermal conductivity

The amount of energy conducted through a body of unit area , and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference.

Unit of thermal conductivity

K= $\frac{Q}{A}\ \frac{dx}{dT}$

= $\frac{W}{m^2}\ \frac{m}{K}$

= $\frac{W}{mK}$

Thermal resistance(R_{th})

Q = $\frac{dT}{\left(\frac{dx}{KA}\right)}$

Q = $\frac{dT}{R_{th}}$

The quantity $\left(\frac{dx}{KA}\right)$ is called thermal conduction resistance

The reciprocal of the thermal resistance is called thermal conductance

Heat flux $\left(\frac{Q}{A}\right)$

It is defined as heat transfer per unit area is directly proportional to the change in temperature and inversely proportional to the thickness of the slab.

Unit is $\frac{W}{m^2}$

. .

Heat transfer by Convection

When fluid flows over a solid surface or inside a channel while temperature of the fluid and the solid surface are different. Heat transfer between the fluid and the solid surface takes place as a consequence of the motion of fluid relative to the surface. This mechanism of heat transfer is called convection

This convection is classified in to two types. They are,

i) Free convection

ii) Forced convection

Newton's law of cooling:

The rate equation for the convective heat transfer between a surface and an adjacent fluid is prescribed by Newton's law of cooling.

$Q = hA(T_s - T_f)$

A = Area of exposed to heat transfer

Q =Rate of convective heat transfer

T_s =Surface Temperature.

T_f = Fluid Temperature

h = Convective heat transfer co-efficient

Convective heat transfer coefficient

The amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time

Unit of heat transfer coefficient

$h = Q/A(T_s - T_f) = \frac{W}{mK}$

Heat transfer coefficient depends on the following factors:

i) Nature of fluid flow

ii) Geometry of the surface

iii) Viscosity of fluid

iv) Density of fluid

Thermal resistance

$$Q = \frac{(Ts - Tf)}{1/hA}$$

$$Q = \frac{(Ts - Tf)}{R_{th}}$$

R_{th} = Convective thermal resistance

Heat transfer by Radiation:

The mode of heat transfer which continuously takes place without the necessity of intervening medium is called radiation. The most important example of thermal radiation is the transport of heat from the sun to the earth.

Stefan-Boltzmann law :

The states that the emissive power of a black body is directly proportional to fourth power of its absolute temperature.

$$Q \propto T^4$$

$$Q = F\sigma A(T_1^4 - T_2^4)$$

F = A factor depending on geometry and surface properties

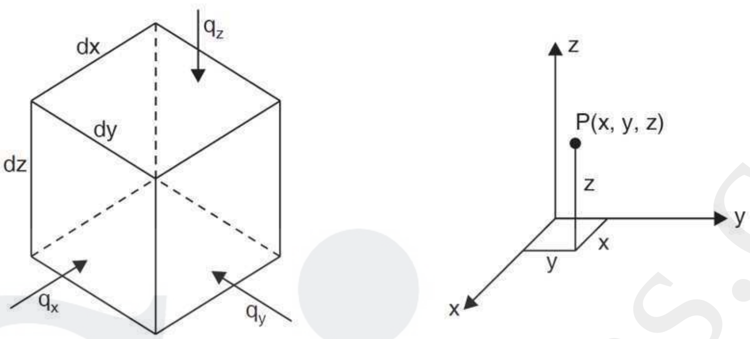
σ =Stefan-Boltzmann constant = 5.67×10^{-8} W/m^{2}K^4

A = Area m^2

T_1 ,T_2 = higher temperature and lower temperature in K(or) C

General heat conduction equation:-

Let us consider a small volume element of sides dx,dy,and dz respectively three axes x,y,z.



T = Temperature at the left face ABCD this temperature may be assumed uniform over the entire surface

$$\frac{\partial T}{\partial x} \text{ =Temperature changes along x- direction}$$

$$\left(\frac{\partial T}{\partial x}\right)dx \text{ =Change of temperature through distance dx}$$

$$T+\left(\frac{\partial T}{\partial x}\right)dx \text{ =temperature on the right EFGH}$$

K_x,K_y,K_z = Thermal conductivities along x,y,z axis

q_g = heat generated per unit volume per unit time.

Energy balance for volume element

Net heat accumulated in the element due to conduction of heat from all the co-ordinate direction (A) + heat generated with in the element (B) = Energy stored in the element (C)

Heat flow Along x- direction:-

dt = time interval

According to Fourier's Law

At left face

$$Q_x = - K_x(dy\ dx) \frac{\partial T}{\partial x} dt$$

At right face

$$Q_{(x+dx)} = Q_x + \frac{\partial}{\partial x}(Qx)\ dx$$

Heat accumulated in x- direction

$$dQ_x = Q_x - Q_{(x+dx)}$$

$$= Q_x - \left(Q_x + \frac{\partial}{\partial x}(Qx)\ dx\right)$$

$$= - \frac{\partial}{\partial x}(Qx)\ dx$$

$$= - \frac{\partial}{\partial x}\left(- K_x(dy\ dx) \frac{\partial T}{\partial x} dt\right) dx$$

$$dQ_x = K_x\ dxdydx \frac{\partial^2 T}{\partial x^2} dt \quad \text{-------------------} \quad (1)$$

Similarly along y- direction

$$dQ_y = K_y\ dxdydx \frac{\partial^2 T}{\partial y^2} dt \quad \text{-------------------} \quad (2)$$

Similarly along z- direction

$$dQ_z = K_z\ dxdydx \frac{\partial^2 T}{\partial z^2} dt \quad \text{-------------------} \quad (3)$$

.

Net heat accumulated in the element due to conduction of heat from all the co-ordinates

A) $\Rightarrow$ (1)+(2)+(3)

K_x dxdydx $\frac{\partial^2 T}{\partial x^2}$ dt + K_y dxdydx $\frac{\partial^2 T}{\partial y^2}$ dt + K_z dxdydx $\frac{\partial^2 T}{\partial z^2}$ dt ---------(A)

Heat generated with in the element (B)

Qg = q_g dxdydz .dt ----------(B)

Energy stored in the element (C)

Q_E = m $\times$ Cp $\times$ (temperature difference)

Q_E =ρV $\times$ Cp $\times$ $\frac{\partial T}{\partial t}$ dt

Q_E =ρ (dxdydx)Cp $\frac{\partial T}{\partial t}$ dt ----------(C)

Energy balance equation

(A) + (B) = (C)

$K_x$dxdydx $\frac{\partial^2 T}{\partial x^2}$ dt + $K_y$dxdydx $\frac{\partial^2 T}{\partial y^2}$ dt +$K_z$dxdydx $\frac{\partial^2 T}{\partial z^2}$ dt +q_g dxdydz .dt = ρ(dxdydx)Cp $\frac{\partial T}{\partial t}$ dt

$K_x = K_y = K_z$ (For isotropic material and homogeneous material)

$$K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + q_g \right) = \rho Cp \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{\rho Cp}{K} \frac{\partial T}{\partial t} \quad \text{--------------------(4)}$$

$\alpha = \frac{K}{\rho Cp}$ [α = Thermal Diffusivity = $\frac{\text{Thermal conductivi ty}}{\text{Thermal Capacity}}$]

. .

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The above equation is called "general heat conduction equation for unsteady state three dimensional with internal heat generation"

Steady state, $\frac{\partial T}{\partial t} = 0$

1) Three dimensional steady state heat conduction equation with out heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

This equation is called Laplace equation

2) Three dimensional steady state heat conduction equation with heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{K} = 0$$

3) Three dimensional Unsteady state heat conduction equation with out heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

This equation is called Fourier's equation

4) Two dimensional steady state heat conduction equation with out heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

5) Two dimensional steady state heat conduction equation with heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

6) Two dimensional steady state heat conduction equation with heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{q_g}{K} = 0$$

7) One dimensional Unsteady state heat conduction equation with heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

8) One dimensional steady state heat conduction equation with heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{K} = 0$$

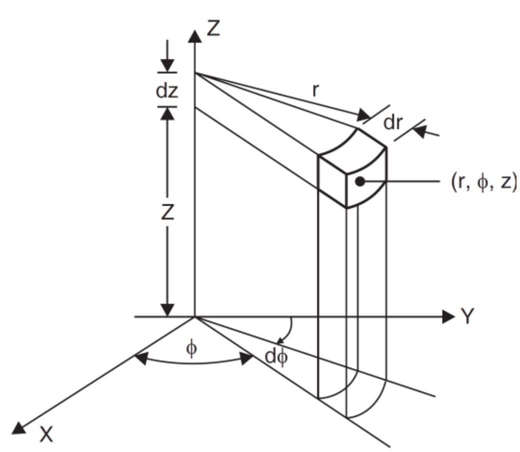
9) One dimensional steady state heat conduction equation with out heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0$$

10) One dimensional Unsteady state heat conduction equation with out heat generation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

General heat conduction in cylindrical co-ordinates:-



The volume of element = rdϕ .dr.dz

Energy balance equation

Net heat accumulated in the element due to conduction of heat from all the co-ordinate direction (A) + heat generated with in the element (B) = Energy stored in the element (C)

A)

Heat flow in radial direction (x-ϕ plane)

According to Fourier's Law

At left face

$$Q_r = -K(rd\phi\,dz)\frac{\partial T}{\partial r}dt$$

At right face

$$Q_{(r+dr)} = Q_r + \frac{\partial}{\partial r}(Qr)\,dr$$

Heat accumulated in r- direction

$$dQ_r = Q_r - Q_{(r+dr)}$$

$$= Q_r - \left(Q_r + \frac{\partial}{\partial r}(Qr)\,dr\right)$$

$$= -\frac{\partial}{\partial r}(Qr)\,dr$$

$$= -\frac{\partial}{\partial r}\left(-K(rd\phi\,dz)\frac{\partial T}{\partial r}dt\right)dr$$

$$= K(dr\,d\phi\,dz)\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)dt$$

.

$$= K(dr\, d\phi\, dz)\ (r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r})dt$$

$$dQ_r = K(dr\, rd\phi\, dz)\ (\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r})dt \qquad \text{--------------------(1)}$$

Heat flow in angular direction

$$Q\phi = -K(dr.dz)\ \frac{\partial T}{r\partial \phi}\ dt$$

$$Q_{(\phi + d\phi)} = Q\phi + \frac{\partial}{r\partial\phi}\ (Q\phi)\ r.d\phi$$

Heat accumulated

$$dQ\phi = Q\phi - Q_{(\phi + d\phi)}$$

$$= -\frac{\partial}{r\partial\phi}\ (Q\phi)\ r.d\phi$$

$$= -\frac{\partial}{r\partial\phi}\ (-K(dr.dz)\ \frac{\partial T}{r\partial\phi}\ dt).\ r.d\phi$$

$$dQ\phi = K\ (dr.\ rd\phi.dz)\ \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}\ dt \qquad \text{--------------------(2)}$$

Heat flow in Z direction (r-ϕ) plane

$$Q_z = -K(dr.rd\phi)\ \frac{\partial T}{\partial z}\ dt$$

$$Q_{(z+dz)} = Q_z + \frac{\partial}{\partial z}(Qz)\ dz$$

Heat accumulated

. .

$dQ_z = Q_z - Q_{(z+dz)}$

$$= -\frac{\partial}{\partial z}(Q_z)\,dz$$

$$= -\frac{\partial}{\partial z}\left(-K(dr.rd\phi)\frac{\partial T}{\partial z}dt\right)dz$$

$$dQ_z = K(dr.\,rd\phi.dz)\frac{\partial^2 T}{\partial z^2}dt \qquad \text{---------------------(3)}$$

Net heat accumulated in the element due to conduction of heat from all the co-ordinate direction

A) $\Rightarrow$ (1)+(2)+(3)

$$K(dr\,rd\phi\,dz)\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)dt + K(dr.\,rd\phi.dz)\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}dt + K(dr.\,rd\phi.dz)\frac{\partial^2 T}{\partial z^2}dt \qquad \text{---------(A)}$$

Heat generated with in the element (B)

$$Q_g = q_g(dr.rd\phi.dz).dt \qquad \text{----------(B)}$$

Energy stored in the element (C)

$$Q_E = m \times Cp \times (\text{temperature difference})$$

$$Q_E = \rho V \times Cp \times \frac{\partial T}{\partial t}dt$$

$$Q_E = \rho(dr.rd\phi.dz)\,Cp\frac{\partial T}{\partial t}dt \qquad \text{----------(C)}$$

Energy balance equation

(A) + (B) = (C)

$$K(dr\, rd\phi\, dz)\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)dt + K(dr.\, rd\phi .dz)\ \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}\ dt + K(dr.\, rd\phi .dz)\ \frac{\partial^2 T}{\partial z^2}\ dt\ + q_g\ (dr.rd\phi .dz)\ .dt$$

$$= \rho\ (dr.rd\phi .dz)\ Cp \frac{\partial T}{\partial t}\ dt$$

$$K\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}\ )+ q_g = \rho\ Cp \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{K} = \frac{\rho Cp}{K}\frac{\partial T}{\partial t}$$

$$\boxed{\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

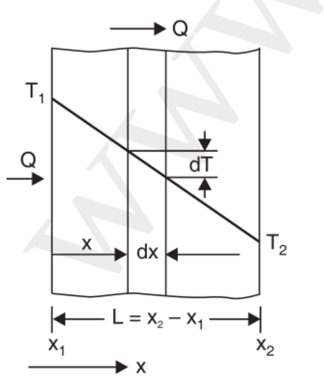
The above equation is called "general heat conduction equation for unsteady state three dimensional with internal heat generation" in cylindrical coordinate system

General heat conduction in spherical coordinate system:-

$$\frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \Phi^2}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{q_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

One dimensional heat flow:-

Heat conduction through a plane wall:



Consider a plane wall of homogeneous material which heat is flowing only in x-direction

L = thickness of the plane wall

A = Cross sectional area of the wall

K = thermal conductivity of the wall material

T_1, T2 = Temperature maintained at two faces.

At x = 0 $T=T_1$ (Initial condition)

At x = L $T=T_2$ (Boundary Condition)

One dimensional steady state without heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0$$

(Or)

$$\frac{d^2 T}{dx^2} = 0$$

By integrating the above equation twice

$$\frac{dT}{dx} = C1$$

$$T = C_1 x + C_2 \qquad \text{--------------(a)}$$

Appling initial condition

$$C_2 = T_1$$

Appling boundary condition

$$T_2 = C1(L) + C_2$$

.

$$C_1 = \frac{T_2 - T_1}{L}$$

C_1&C_2 values substitute in equation (a)

$$T = \left(\frac{T_2 - T_1}{L}\right) x + T_1$$

According to Fourier's Law

$$Q = -KA \frac{\partial T}{\partial x}$$

$$= -KA \frac{\partial}{\partial x} \left[\left(\frac{T_2 - T_1}{L}\right) x + T_1\right]$$

$$= -KA \left(\frac{T_2 - T_1}{L}\right)$$

$$Q = KA \left(\frac{T_1 - T_2}{L}\right)$$

$$Q = \frac{T_1 - T_2}{\left(L/KA\right)}$$

$$Q = \frac{T_1 - T_2}{(R_{th})_{cond}}$$

Heat conduction through a plane wall:

Consider A, Band C composite wall

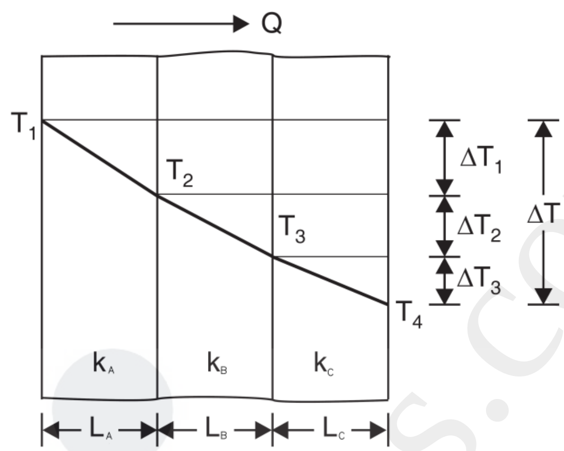
L_A, L_B, L_C = thickness of slabs A,Band C respectively,

K_A, K_B, K_C = Thermal conductivities of the slabs A,BandC respectively,

. .

T_1, T_4 = Temperature at the wall surface 1 and 4 respectively,

T_2, T_3 = Temperature at the interface 2 and 3 respectively



Perfect contact between layers so no temperature drop.

$$Q = K_A A\left(\frac{T_1 - T_2}{L_A}\right) = K_B A\left(\frac{T_2 - T_3}{L_B}\right) = K_C A\left(\frac{T_3 - T_4}{L_C}\right)$$

$$T_1 - T_2 = \frac{QL_A}{K_A A} \quad \text{---------(i)}$$

$$T_2 - T_3 = \frac{QL_B}{K_B A} \quad \text{---------(ii)}$$

$$T_3 - T_4 = \frac{QL_C}{K_C A} \quad \text{---------(ii)}$$

By adding equation (i),(ii),(iii)

$$T_1 - T_4 = \frac{QL_A}{K_A A} + \frac{QL_B}{K_B A} + \frac{QL_C}{K_C A}$$

$$T_1 - T_4 = Q\left[\frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A}\right]$$

$$Q = \frac{T_1 - T_4}{\frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A}}$$

$$Q = \frac{T_1 - T_4}{R_A + R_B + R_C}$$

Heat Flux(Q/A)

$$\frac{Q}{A} = \frac{T_1 - T_4}{\frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C}} \quad (\frac{W}{m^2})$$

Composite wall 'n' layers,

$$Q = \frac{T_1 - T_{n+1}}{\sum_{n=1}^{n} \frac{L}{KA}}$$

$$Q = \frac{(\Delta T)_{Overall}}{\sum R}$$

The overall heat transfer coefficient

While dealing with the problems of fluid to fluid heat transfer across a metal boundary, it is usual to adopt an overall heat transfer coefficient U which gives the heat transmitted per unit area per unit time per degree temperature different between the bulk fluids on each side of the metal.

$$Q = h_{hf} A(T_{hf} - T_1) = K A(\frac{T_1 - T_2}{L}) = h_{cf} A(T_2 - T_{cf})$$

$$T_{hf} - T_1 = \frac{Q}{h_{hf} A} \qquad \text{---------(i)}$$

$$T_1 - T_2 = \frac{QL}{K\ A} \qquad \text{---------(ii)}$$

$T_2-T_{cf} = \frac{Q}{h_{Cf} A}$ ---------(ii)

By adding equation (i),(ii),(iii)

$$T_{hf}-T_{cf} = \frac{Q}{h_{hf} A} + \frac{QL}{K\ A} + \frac{Q}{h_{Cf} A}$$

$$T_{hf}-T_{cf} = Q[\frac{1}{h_{hfA} A} + \frac{L}{K\ A} + \frac{1}{h_{Cf} A}]$$

$$Q = \frac{T_{hf} - T_{cf}}{\frac{1}{h_{hf} A} + \frac{L}{KA} + \frac{1}{h_{Cf} A}}$$

$$Q = \frac{T_{hf} - T_{cf}}{R_1 + R_2 + R_3}$$

$$Q = \frac{A(T_{hf} - T_{cf})}{\frac{1}{h_{hf}} + \frac{L}{K} + \frac{1}{h_{Cf}}}$$

$$Q = U_0 A (T_{hf}-T_{cf})$$

$$U_o = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{K} + \frac{1}{h_{Cf}}}$$

U_o = Overall heat transfer coefficient

Heat conduction through hollow cylinder:-

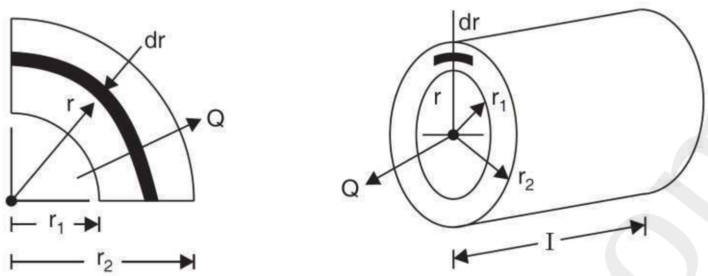
Consider a hollow cylinder of homogeneous material which heat is flowing only in radial direction

L = length of the cylinder wall

A = Cross sectional area of the cylinder

K = thermal conductivity of the wall material

T_1, T2 = Temperature maintained at two faces.



At $r = r_1$ $T=T_1$ (Initial condition)

At $r = r_2$ $T=T_2$ (Boundary Condition)

One dimensional steady state without heat generation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0$$

(Or)

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

By integrating the above equation twice

$$\frac{1}{r}\left[\frac{d}{dr}\left(r\frac{dT}{dr}\right)\right] = 0$$

$$\frac{1}{r} \neq 0 \;\therefore\; \frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$r\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r}$$

.

$$T = C_1 \ln(r) + C_2 \qquad \text{-----------------------(A)}$$

Appling initial condition

$$T_1 = C_1 \ln(r_1) + C_2 \qquad \text{----------------------- (1)}$$

Appling boundary condition

$$T_2 = C_1 \ln(r_2) + C_2 \qquad \text{----------------------- (2)}$$

By subtracting equation (1) and (2)

$$C_1 = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)}$$

Substitute C_1 value in equation (1)

$$T_1 = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} \ln(r_1) + C_2$$

$$C_2 = T_1 - \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} \ln(r_1)$$

Substitute C_1 & C_2 value in equation (A)

$$T = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} \ln(r) + T_1 - \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} \ln(r_1)$$

$$T = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} [\ln(r) - \ln(r_1)] + T_1$$

. .

$$T = \frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} [\ln(\frac{r}{r_1})] + T_1$$

According Fourier's Law

$$Q = -KA\frac{\partial T}{\partial r}$$

$$Q = -KA\frac{\partial}{\partial r}[\frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} [\ln(\frac{r}{r_1})] + T_1]$$

$$Q = -KA\frac{T_1 - T_2}{\ln\left(\frac{r_1}{r_2}\right)} (\frac{r_1}{r})(\frac{1}{r_1})$$

$$Q = KA\frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)} (\frac{1}{r})$$

A = 2π r L (Surface Area of the Cylinder)

$$Q = K(2\pi r L)\frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)} (\frac{1}{r})$$

$$Q = 2\pi K L\frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$Q = \frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right) / 2\pi KL}$$

$$Q = \frac{T_1 - T_2}{R}$$

Thermal resistance $R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi KL}$

Heat conduction thorough a composite cylinder:-

Consider Inside and out side convection.

$$Q = h_{hf} A_i(T_{hf} - T_1) = 2\pi K_A L \frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)} = 2\pi K_B L \frac{T_2 - T_3}{\ln\left(\frac{r_3}{r_2}\right)} = h_{cf} A_o(T_3 - T_{cf})$$

A_i and A_o are inside and outside surface area,

$$Q = h_{hf} (2\pi r_1 L)(T_{hf} - T_1) = 2\pi K_A L \frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)} = 2\pi K_B L \frac{T_2 - T_3}{\ln\left(\frac{r_3}{r_2}\right)} = h_{cf} (2\pi r_3 L)(T_3 - T_{cf})$$

$$T_{hf} - T_1 = \frac{Q}{h_{hf}\, 2\pi\, r_1 L} \quad \text{---------(i)}$$

$$T_1 - T_2 = \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{2\pi K_A L} \quad \text{---------(ii)}$$

$$T_2 - T_3 = \frac{Q \ln\left(\frac{r_3}{r_2}\right)}{2\pi K_B L} \quad \text{---------(iii)}$$

$$T_3 - T_{cf} = \frac{Q}{h_{Cf}\, 2\pi r_3 L} \quad \text{---------(iv)}$$

By adding equations

.

$$(T_{hf} - T_{cf}) = \frac{Q}{h_{hf}\, 2\pi\, r_1 L} + \frac{Q \ln\left(\frac{r_2}{r_1}\right)}{2\pi K_A L} + \frac{Q \ln\left(\frac{r_3}{r_2}\right)}{2\pi K_B L} + \frac{Q}{h_{Cf}\, 2\pi r_3 L}$$

$$(T_{hf} - T_{cf}) = Q[\frac{1}{h_{hf}\, 2\pi\, r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_A L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_B L} + \frac{1}{h_{Cf}\, 2\pi r_3 L}]$$

$$Q = (T_{hf} - T_{cf}) \,/\, [\frac{1}{h_{hf}\, 2\pi\, r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_A L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_B L} + \frac{1}{h_{Cf}\, 2\pi r_3 L}]$$

$$Q = \frac{T_{hf} - T_{cf}}{\frac{1}{h_{hf}\, 2\pi r_1 L} + \frac{\ln \frac{r_2}{r_1}}{2\pi K_A L} + \frac{\ln \frac{r_3}{r_2}}{2\pi K_B L} + \frac{1}{h_{Cf}\, 2\pi r_3 L}}$$

$$Q = \frac{2\pi L (T_{hf} - T_{cf})}{\frac{1}{h_{hf} r_1} + \frac{\ln \frac{r_2}{r_1}}{K_A} + \frac{\ln \frac{r_3}{r_2}}{K_B} + \frac{1}{h_{Cf} r_3}}$$

Add r_1 in denominator and numerator

$$Q = \frac{2\pi r_1 L (T_{hf} - T_{cf})}{\frac{1}{h_{hf}} + \frac{r_1 \ln \frac{r_2}{r_1}}{K_A} + \frac{r_1 \ln \frac{r_3}{r_2}}{K_B} + \frac{r_1}{h_{Cf} r_3}}$$

$$Q = \frac{A_i (T_{hf} - T_{cf})}{\frac{1}{h_{hf}} + \frac{r_1 \ln \frac{r_2}{r_1}}{K_A} + \frac{r_1 \ln \frac{r_3}{r_2}}{K_B} + \frac{r_1}{h_{Cf} r_3}}$$

We know ,

. .

.

Q = $U_i \; A_i(T_{hf} - T_{cf})$

U_i = Over all heat transfer coefficient based on inner side

$$U_i = \frac{1}{h_{hf}} + \frac{r_1}{K_A}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{K_B}\ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{h_{Cf} r_3}$$

Add r_3 in denominator and numerator

$$Q = \frac{2\pi r_3 L(T_{hf} - T_{cf})}{\frac{r_3}{h_{hf}} + \frac{r_3 \ln \frac{r_2}{r_1}}{K_A} + \frac{r_3 \ln \frac{r_3}{r_2}}{K_B} + \frac{1}{h_{Cf}}}$$

$$Q = \frac{A_o(T_{hf} - T_{cf})}{\frac{r_3}{h_{hf}} + \frac{r_3 \ln \frac{r_2}{r_1}}{K_A} + \frac{r_3 \ln \frac{r_3}{r_2}}{K_B} + \frac{1}{h_{Cf}}}$$

We know ,

Q = $U_o \; A_o(T_{hf} - T_{cf})$

U_o= Over all heat transfer coefficient based on outer side

$$U_o = \frac{r_3}{h_{hf} r_1} + \frac{r_3}{K_A}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_3}{K_B}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_{Cf}}$$

Heat conduction through hollow sphere

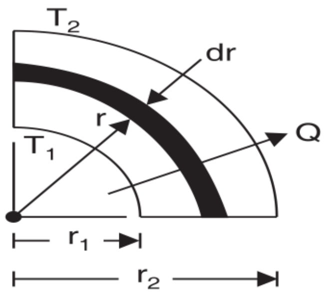
Consider a hollow sphere of homogeneous material which heat is flowing only in radial direction

At $r = r_1$ $T = T_1$ (Initial condition)

At $r = r_2$ $T = T_2$ (Boundary condition)

. .

General heat conduction equation for One dimensional steady state heat conduction with out heat generation



$$\frac{1}{r2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$r^2\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T = -\frac{C_1}{r^2} + C_2 \qquad \text{------------------------(A)}$$

Appling initial condition

$$T_1 = -\frac{C_1}{r^2} + C_2 \qquad \text{------------------------(i)}$$

Appling Boundary condition

$$T_2 = -\frac{C_1}{r_2} + C_2 \qquad \text{-----------------------(ii)}$$

By subtracting equations

$$T_1 - T_2 = -\frac{C_1}{r^2} + \frac{C_1}{r_2}$$

$$C_1 = \frac{T_1 - T_2}{\left(\frac{r_1 - r_2}{r_1 . r_2}\right)}$$

$$C_2 = T_1 + \frac{r_2 (T_1 - T_2)}{r_1 - r_2}$$

C_1, C2Substitute in equation (A)

$$T = -\frac{T_1 - T_2}{r\left(\frac{r_1 - r_2}{r_1 . r_2}\right)} + T_1 + \frac{r_2 (T_1 - T_2)}{r_1 - r_2}$$

According Fourier's Law

$$Q = -KA \frac{\partial T}{\partial r}$$

$$Q = -KA \frac{\partial}{\partial r} \left[-\frac{T_1 - T_2}{r\left(\frac{r_1 - r_2}{r_1 . r_2}\right)} + T_1 + \frac{r_2 (T_1 - T_2)}{r_1 - r_2} \right]$$

$$Q = -KA \left[\frac{T_1 - T_2}{r^2\left(\frac{r_1 - r_2}{r_1 . r_2}\right)} \right]$$

A = surface Area of the sphere $= 4\pi r^2$

$$Q = -K 4\pi r^2 \left[\frac{T_1 - T_2}{r^2\left(\frac{r_1 - r_2}{r_1 . r_2}\right)} \right]$$

.

$$Q = 4\pi K \frac{(T_1 - T_2)}{\left(\frac{r_2 - r_1}{r_1 . r_2}\right)}$$

$$Q = \frac{(T_1 - T_2)}{\left(\frac{r_2 - r_1}{4\pi K r_1 . r_2}\right)}$$

$$Q = \frac{(\Delta T)_{overall}}{R}$$

R= Thermal resistance = $\frac{r_2 - r_1}{4\pi K r_1 . r_2}$

Heat conduction through Composite sphere:-

$$Q = \frac{(T_{hf} - T_{cf})}{\frac{1}{h_{hf} 4\pi r_1^2} + \left(\frac{r_2 - r_1}{4\pi K_A r_1 . r_2}\right) + \frac{r_3 - r_2}{4\pi K_B r_2 . r_3} + \frac{1}{h_{cf} 4\pi r_3^2}}$$

$$Q = \frac{4\pi (T_{hf} - T_{cf})}{\frac{1}{h_{hf} r_1^2} + \frac{r_2 - r_1}{K_A r_1 . r_2} + \frac{r_3 - r_2}{K_B r_2 . r_3} + \frac{1}{h_{cf} r_3^2}} \quad \text{---------------------(B)}$$

Multiply r1 in denominator and numerator

$$Q = \frac{4\pi r_1^2 (T_{hf} - T_{cf})}{\frac{1}{h_{hf}} + \frac{r_2 - r_1}{K_A}\left(\frac{r_1}{r_2}\right) + \frac{(r_3 - r_2) r_1}{K_B r_2 . r_3} + \frac{r_1^2}{h_{cf} r_3^2}}$$

$$Q = \frac{A_i (T_{hf} - T_{cf})}{\frac{1}{h_{hf}} + \frac{r_2 - r_1}{K_A}\left(\frac{r_1}{r_2}\right) + \frac{(r_3 - r_2) r_1}{K_B r_2 . r_3} + \frac{r_1^2}{h_{cf} r_3^2}}$$

We know

. .

$Q = U_i A_i (T_{hf} - T_{cf})$

U_i = Overall heat transfer coefficient based on inner side

$$U_i = \frac{1}{\frac{1}{h_{hf}} + \frac{r_2 - r_1}{K_A}\left(\frac{r_1}{r_2}\right) + \frac{(r_3 - r_2) r_1}{K_B r_2 . r_3} + \frac{r_1^2}{h_{cf} r_3^2}}$$

Multiply r_3 in denominator and numerator in eqn.(B)

$$Q = \frac{4\pi r_3^2 (T_{hf} - T_{cf})}{\frac{r_3^2}{h_{hf} r_1^2} + \frac{(r_2 - r_1) r_3^2}{K_A r_1 r_2} + \frac{(r_3 - r_2) r_3}{K_B r_2} + \frac{1}{h_{cf}}}$$

$$Q = \frac{A_o (T_{hf} - T_{cf})}{\frac{r_3^2}{h_{hf} r_1^2} + \frac{(r_2 - r_1) r_3^2}{K_A r_1 r_2} + \frac{(r_3 - r_2) r_3}{K_B r_2} + \frac{1}{h_{cf}}}$$

We know

$Q = U_o A_o (T_{hf} - T_{cf})$

U_o= Overall heat transfer coefficient based on outer side

$$U_o = \frac{1}{\frac{r_3^2}{h_{hf} r_1^2} + \frac{(r_2 - r_1) r_3^2}{K_A r_1 r_2} + \frac{(r_3 - r_2) r_3}{K_B r_2} + \frac{1}{h_{cf}}}$$

Critical thickness of insulation

Insulation

Purpose of insulation is,

1. it prevents the heat flow from the system to the surroundings
2. it prevents the heat flow from the surroundings to the system

Critical thickness of insulation

The thickness up to which heat flow increase and after which heat flow decrease is termed as critical thickness. Incase of cylinders and spheres is called "critical radius"

Application :

1. Boilers and steam pipes
2. Air-conditioning system
3. Food preserving stores and refrigerators
4. Insulating bricks
5. Preservation of liquid gases etc,

Critical thickness of insulation for cylinder:-

Consider a solid cylinder of radius r_1 insulated with an insulation of thickness (r_2-r_1) as shown in fig.

L = length of the cylinder

T_1 = Surface temperature of the cylinder

T_a =Temperature of air

K = Thermal conductivity of insulating material

$$Q = \frac{2\pi L(T_1 - T_a)}{\frac{\ln \frac{r_2}{r_1}}{K} + \frac{1}{h_0 r_2}}$$

Q becomes maximum when denominator becomes minimum

$$\frac{\partial}{\partial r_2}\left(\frac{\ln \frac{r_2}{r_1}}{K} + \frac{1}{h_0 r_2}\right) = 0$$

$$\frac{1}{K} \times \frac{r_1}{r_2} \times \frac{1}{r_1} - \frac{1}{h_0 r_2^{\,2}} = 0$$

$$\frac{1}{K} \times \frac{1}{r_2} - \frac{1}{h_0 r_2^{\,2}} = 0$$

$$h_0 r_2 = K$$

$$r_2 = \frac{K}{h_0}$$

$$r_2 = r_c = \frac{K}{h}$$

In the physical sense we may arrived at the following conclusions:-

1. For cylindrical bodies with $r_1<r_c$ the heat transfer by adding insulation till $r_2 = r_c$. When r_1 is small and r_c is large, the thermal conductivity of insulation K is high (poor insulating material) and h_0 is low

    Application:

    Electric cables – Good insulating for current ,Poor insulating for heat
2. For cylindrical bodies with $r_1>r_c$ the heat transfer by adding insulation till $r_2 = r_c$. When r_1 is large and r_c is small, the thermal conductivity of insulation K is low and h_0 is high

    Application:

    Steam pipes – Good insulating for heat

Critical thickness of insulation for sphere:

$$Q = \frac{(T_1 - T_a)}{\frac{(r_2 - r_1)}{4\pi K r_1 r_2} + \frac{1}{4\pi h_0 r_2^2}}$$

Q becomes maximum when denominator becomes minimum

$$\frac{\partial}{\partial r_2}\left(\frac{(r_2 - r_1)}{4\pi K r_1 r_2} + \frac{1}{4\pi h_0 r_2^2}\right) = 0$$

$$\frac{\partial}{\partial r_2}\left(\frac{1}{4\pi K r_1} - \frac{1}{4\pi K r_2} + \frac{1}{4\pi h_0 r_2^2}\right) = 0$$

$$\frac{1}{4\pi K r_2^2} - \frac{2}{4\pi h_0 r_2^3} = 0$$

$$\frac{1}{4\pi K r_2^2} = \frac{2}{4\pi h_0 r_2^3}$$

.

$$r_2 = \frac{2K}{h_0}$$

$$r_2 = r_c = \frac{2K}{h}$$

Problems:-

1. Calculate the critical radius of insulation for asbestos (K=0.172 W/mK) surrounding a pipe and exposed to room air at 300K with h = 2.8 W/m^2K. Calculate the heat loss from a 475K, 60 mm diameter pipe when covered with the critical radius of insulation and without insulation.

Given:

K =0.172 W/mK

h = 2.8 W/m^2K

T_1 = 475 K

T_a = 300 K

Solution:-

$$r_c = \frac{K}{h} = \frac{0.172}{2.8} = 61.42\text{mm}$$

$$Q \text{ (with insulation)} = \frac{2\pi L(T_1 - T_a)}{\frac{\ln \frac{r_c}{r_1}}{K} + \frac{1}{h_0 r_c}}$$

$$= 110.16\ W/m$$

Q (with out insulation) = ho $2\pi r_1 L(T_1 - T_a)$

. .

$= 92.36 \; W/m$

2. A small electric heating application uses wire of 2mm diameter with 0.8 mm thick insulation (K=0.12 W/mK).the heat transfer coefficient of the insulated surface is 35 W/m^2K . Determine the critical thickness of insulation in this case and the percentage change in the heat transfer rate if the critical thickness is used. Assuming the temperature difference between the surface of the wire and surrounding air remains unchanged.

Given:

$K = 0.12 \; W/mK$

$h = 35 \; W/m^2K$

$r_1 = 1mm = 1X10^{-3}$ m

$r_2 = 1+0.8mm = 1.8X10^{-3}$ m

Solution:-

$$r_c = \frac{K}{h} = \frac{0.172}{2.8} = 61.42mm$$

Case - I

$$Q_1 \text{ (with critical insulation)} = \frac{2\pi L(T_1 - T_a)}{\frac{\ln \frac{r_c}{r_1}}{K} + \frac{1}{h_0 r_c}}$$

.

$$= \quad (T_1 - T_a)$$

Case - II

$$Q_2 \text{ (with insulation)} = \frac{2\pi L(T_1 - T_a)}{\frac{\ln\frac{r_2}{r_1}}{K} + \frac{1}{h_0 r_2}}$$

$$= \quad (T_1 - T_a)$$

$$\% \text{ increase} = \frac{Q_1 - Q_2}{Q_1} \times 100$$

= 11.6 %

3. A wire of 6.5mm diameter at a temperature of 60ºC is to be insulated by a material having K=0.174 W/mK .convection heat transfer coefficient h= 8.722 W/m^2K . The ambient temperature is 20 ºC. for maximum heat loss, what is the minimum thickness of insulation and heat loss per meter length? Also find percentage increase in the heat dissipation too.

Given:

K =0.174 W/mK

h = 8.722 W/m^2K

T_1 = 60 ºC

T_a = 20 ºC

r_1 = 3.25 mm

. .

.

Solution:-

$$r_c = \frac{K}{h} = \frac{0.174}{8.722} = 19.95 \text{ mm}$$

$$Q \text{ (with insulation)} = \frac{2\pi L(T_1 - T_a)}{\frac{\ln \frac{r_c}{r_1}}{K} + \frac{1}{h_0 r_c}}$$

$$= 15.537 \, W/m$$

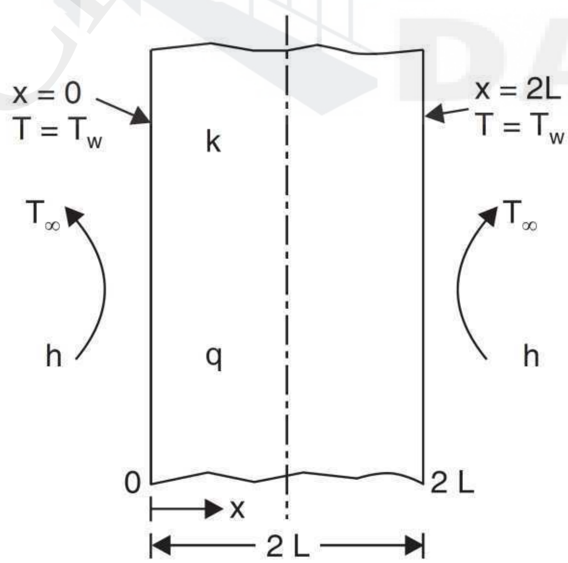
$$Q \text{ (with out insulation)} = h_o \, 2\pi r_1 L(T_1 - T_a)$$

$$= 7.124 \, W/m$$

$$\% \text{ increase} = \frac{Q_1 - Q_2}{Q_1} \times 100$$

$$= 118.09 \, \%$$

<u>Heat conduction with internal heat generation</u>:



.   .

Following are some of the cases where heat generation and heat conduction are encountered

1. Fuel rod – Nuclear reactor
2. Electrical conductors
3. Chemical and combustion process
4. Drying and settling of concrete

Plane wall with uniform heat generation :-

Heat conducted

$$Q_x = -KA\frac{dT}{dx}$$

Heat generated in the element:

$$Q_g = A.dx.q_g$$

q_g = Heat generated per unit volume per unit time in the element

Heat conducted x+dx distance

$$Q_{(x+dx)} = Q_x + \frac{\partial}{\partial x}(Qx)\,dx$$

$$Q_{(x+dx)} = Q_x + Q_g$$

$$Q_x + \frac{\partial}{\partial x}(Qx)\,dx = Q_x + Q_g$$

$$Q_g = \frac{\partial}{\partial x}(Qx)\,dx$$

$$A.dx.q_g = \frac{\partial}{\partial x}(-KA\frac{\partial T}{\partial x})\,dx$$

$$= -KA\frac{\partial^2 T}{\partial x^2}\,dx$$

$$q_g = -K\frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial^2 T}{\partial x^2} = -\frac{q_g}{K}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{K} = 0$$

Integrating twice

$$\frac{\partial T}{\partial x} = -\frac{q_g}{K}x + C_1$$

$$T = -\frac{q_g}{2K}x^2 + C_1x + C_2$$

Case -1

Both the surface having the same temperature:

At x = 0 $\quad$ $T=T_w$ (initial condition)

At x =L $\quad$ $T=T_w$ (Boundary condition)

Appling initial condition

$$C_2 = T_w$$

Appling Boundary condition :

$$T_w = -\frac{q_g}{2K}L^2 + C_1L + T_w$$

$$C_1 = \frac{q_g}{2K}L$$

$$T = -\frac{q_g}{2K}x^2 + \frac{q_g}{2K}L\,x + T_w$$

$$T = \frac{q_g}{2K}(L-x)x + T_w$$

The location of the maximum temperature

$$\frac{\partial T}{\partial x} = \frac{q_g}{2K}L - 2x\frac{q_g}{2K}$$

$$\frac{\partial T}{\partial x} = \frac{q_g}{2K}(L-2x)$$

$$\frac{\partial T}{\partial x} = 0$$

$$\frac{q_g}{2K}(L-2x) = 0$$

$$x = \frac{L}{2}$$

$$T_{max} = \frac{q_g}{2K}(L-\frac{L}{2})\frac{L}{2} + T_w$$

$$T_{max} = \frac{q_g}{2K}(\frac{L^2}{4}) + T_w$$

$$T_{max} = \frac{q_g}{8K}L^2 + T_w$$

According to Fourier's Law

$$Q_x = -KA\frac{d}{dx}\left(\frac{q_g}{2K}(L-x)x + T_w\right)$$

.

$$= -KA\left(\frac{q_g}{2K}(L - 2x)\right)$$

At x=L

$$Q = \frac{AL}{2}q_g$$

When both surface are considered

$$Q = 2x\frac{AL}{2}q_g = ALq_g$$

Problems:-

1. the rate of heat generation in a slab of thickness 160mm (K= 180 W/mK) is 1.2x10^6 W/m .If the temperature of each of the surface of the solid is 120 ºC determine.

i) The temperature at the mid and quarter planes,

ii) the heat flow rate

Given data :

T_w = 120 ºC

q_g = 1.2x10^6 W/m

K = 180 W/mK

L = 160 mm = 0.16 m

To find :-

(i) $T_{(x=L/2)}$, & $T_{(x=L/4)}$

(ii) $Q_{(x=L/2)}$ & $Q_{(x=L/4)}$

Solution :-

. .

(i) $T = \frac{q_g}{2K}(L - x)x + T_w$

$T_{(x=L/2)} = \frac{q_g}{2K}(L^2/4) + T_w = 141.33\ ºC$

$T_{(x=L/4)} = \frac{q_g}{2K}(3L^2/16) + T_w = 136\ ºC$

(ii) $Q_{(x=L/2)} = Axq_g = 96000\ W/m^2$

$Q_{(x=L/4)} = Axq_g = 48000\ W/m^2$

Extended Surface :- (Fins)

The fins enhance the heat transfer rate from a surface by exposing large surface area to convection. The fins are normally thin strips of highly conducting metals such aluminium, copper, brass etc.

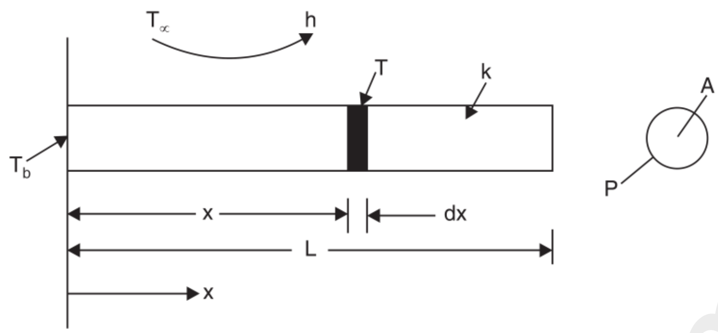
Types of fins:-

i) uniform straight fin
ii) Tapered straight fin
iii) Splines
iv) Annular fin
v) Pin fins(spines)

Heat flow through rectangular Fin:-

T_o = Temperature at the base of the fin

T_a = ambient temperature



Heat conducted into the element

$$Q_x = -K A_{cs} \frac{dT}{dx}$$

Heat conducted at distance (x+dx)

$$Q_{(x+dx)} = Q_x + \frac{d}{dx}(Qx).dx$$

Heat convected out of the element between the planes x and (x+dx)

$$Q_{conv} = hAdT$$

$$= hx(Pxdx).(T-T_a)$$

Appling an energy balance on the element

$$Q_x = Q_{(x+dx)} + Q_{conv}$$

$$Q_x = Q_x + \frac{d}{dx}(Qx).dx + h(Pdx).(T-T_a)$$

$$-\frac{d}{dx}(Qx).dx = h(Pdx).(T-T_a)$$

$$-\frac{d}{dx}(-KA\frac{dT}{dx}).dx = h(Pdx).(T-T_a)$$

$$K A \frac{d^2T}{dx^2} = hP.(T-T_a)$$

$$\frac{d^2T}{dx^2} = \frac{hp}{KA}.(T-T_a)$$

(Or)

$$\frac{d^2(T-T_a)}{dx^2} = \frac{hp}{KA}.(T-T_a)$$

$$\frac{d^2(T-T_a)}{dx^2} - \frac{hp}{KA}.(T-T_a) = 0$$

$T-T_a = \theta$

$$\frac{d^2\theta}{dx^2} - \frac{hp}{KA}.(\theta) = 0$$

$$\frac{hp}{KA} = m^2 \quad m = \sqrt{\frac{hp}{KA}}$$

$$\frac{d^2\theta}{dx^2} - m^2.(\theta) = 0$$

$$\theta = C_1e^{mx}+C_2e^{-mx}$$

$$T-T_a = C_1e^{mx}+C_2e^{-mx}$$

The following cases may be considered

Case – 1 (Long fin)

The fin is infinitely long and the temperature a tend of the fin ,is equal to the ambient.

Case – 2 (Short fin end is insulated)

The fin is short and the end of the fin is insulated.

Case – 3 (short fin end is not insulated)

The fin is short and the end of the fin is not insulated.(loses by convection)

Heat dissipation from an infinitely long fin

$L = \infty$

At $x = 0$, $T=T_o$ $\theta = \theta_0$, (Initial condition)

At $x = \infty$, $T=T_a$ $\theta = 0$ (boundary condition)

Applying initial condition

$$\theta = C_1e^{mx}+C_2e^{-mx}$$

$$\theta_0 = C_1+C_2$$

applying boundary condition

$$\theta = C_1e^{mx}+C_2e^{-mx}$$

$$0 = C_1e^{m\infty}+C_2e^{-m\infty}$$

$$C_1 = 0$$

$$C_2 = \theta_0$$

$$\theta = C_1e^{mx}+C_2e^{-mx}$$

$$\theta = \theta_0e^{-mx}$$

$$T-T_a = (T_0-T_a)\ e^{-mx}$$

$$T = (T_0-T_a)\ e^{-mx}+T_a$$

According to Fourier's law

$$Q_{fin} = -K\,A\frac{dT}{dx}$$

$$= - K A \frac{d}{dx}\{(T_0-T_a)\ e^{-mx}+T_a\ )\ \}$$

$$= - K A(-m)(T_0-T_a)\ e^{-mx}$$

$$= K A m\ (T_0-T_a)\ e^{-mx}$$

$$m = \sqrt{\frac{hp}{KA}}$$

At x = 0

$$Q_{fin} = K A \sqrt{\frac{hp}{KA}}\ (T_0-T_a)\ e^{-m(0)}$$

$$Q_{fin} = \sqrt{hpKA}\ (T_0-T_a)$$

<u>Heat dissipation from a fin insulated at the tip :- (Short fin end insulated)</u>

At $x = 0$,$T=T_o$ $\theta = \theta_0$, (Initial condition)

At $x = L$, $\frac{dT}{dx} = 0$ (boundary condition)

$$\theta = C_1e^{mx}+C_2e^{-mx}$$

Appling initial condition

$$\theta_0= C_1+C_2$$

Appling boundary condition

$$T-T_a = C_1e^{mx}+C_2e^{-mx}$$

Differentiating both side

$$\frac{dT}{dx} = mC_1e^{mx} - mC_2e^{-mx}$$

$$mC_1e^{mL} - mC_2e^{-mL} = 0$$

$$mC_1e^{mL} - m(\theta_0 - C_1)e^{-mL} = 0$$

$$C_1e^{mL} - (\theta_0 - C_1)e^{-mL} = 0$$

$$C_1(e^{mL} + e^{-mL}) = \theta_0\, e^{-mL}$$

$$C_1 = \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right)\theta_0$$

$$C_2 = \theta_0\left(1 - \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right)\right)$$

Substitute C_1, C2 Values

$$T - T_a = \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right)\theta_0\, e^{mx} + \left[1 - \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right)\right]\theta_0 e^{-mx}$$

$$T - T_a = \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right)(T_0 - T_a)\, e^{mx} + (T_0 - T_a)\left(1 - \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right)\right]\, e^{-mx}$$

$$\left(\frac{T - T_a}{T_0 - T_a}\right) = \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right) e^{mx} + 1 - \left(\frac{e^{-ml}}{e^{mL} + e^{-mL}}\right) e^{-mx}$$

$$\left(\frac{T - T_a}{T_0 - T_a}\right) = \left(\frac{e^{m(x-l)}}{e^{mL} + e^{-mL}}\right) + \left(\frac{e^{m(l-x)}}{e^{mL} + e^{-mL}}\right)$$

$$\left(\frac{T - T_a}{T_0 - T_a}\right) = \left(\frac{e^{m(x-l)} + e^{m(l-x)}}{e^{mL} + e^{-mL}}\right)$$

$$\left(\frac{T - T_a}{T_0 - T_a}\right) = \left(\frac{e^{m(l-x)} + e^{-m(l-x)}}{e^{mL} + e^{-mL}}\right)$$

$$\left(\frac{T - T_a}{T_0 - T_a}\right) = \left(\frac{\cosh m(l-x)}{\cosh ml}\right)$$

According to Fourier's law

$$Q_{fin} = -KA\frac{dT}{dx}$$

$$= -KA\frac{d}{dx}\left[\left(\frac{\cosh m(l-x)}{\cosh ml}\right)(T_o - T_a) + T_a\right]$$

$$= KAm\left(\frac{\sinh m(l-x)}{\cosh ml}\right)(T_o - T_a)$$

$$Q_{fin} = KA\sqrt{\frac{hp}{KA}}\left(\frac{\sinh m(l-x)}{\cosh ml}\right)(T_o - T_a)$$

$$\boxed{Q_{fin} = \sqrt{hpKA}\,(T_0 - T_a)\left(\frac{\sinh m(l-x)}{\cosh ml}\right)}$$

At x = 0

$$Q_{fin} = \sqrt{hpKA}\left(\frac{\sinh ml}{\cosh ml}\right)(T_o - T_a)$$

$$Q_{fin} = \sqrt{hpKA}\,(T_o - T_a)\tanh ml$$

Heat dissipation from a fin losing heat at the tip (short fin end is not insulated)

Temperature distribution

$$\left(\frac{T - T_a}{T_0 - T_a}\right) = \frac{\cosh m(l-x) + {h}/{km}\sinh\ m(l-x)}{\cosh ml + {h}/{km}\sinh\ ml}$$

$$Q_{fin} = \sqrt{hpKA}\ (T_o - T_a)\left(\frac{\tanh ml + {h}/{km}}{1 + {k}/{km}\tanh ml}\right)$$

4.4.1. Fin Effectiveness, ∑f: Fins are used to increase the heat transfer from a surface by increasing the effective surface area. When fins are not present, the heat convected by the base area is given by *Ah*(*To* – *T*∞), where *A* is the base area. When fins are used the heat transferred by the fins, *qf*, is calculated using equations. The ratio of these quantities is defined as fin effectiveness.

$$\varepsilon_f = \frac{q_f}{hA\,(T_b - T_\infty)}$$

Fin efficiency, ηf: This quantity is more often used to determine the heat flow when variable area fins are used. **Fin efficiency is defined as the ratio of heat transfer by the fin to the heat transfer that will take place if the whole surface area of the fin is at the base temperature.**

$$\eta_f = \frac{q_f}{hA_s(T_b - T_\infty)}$$

CIRCUMFERENTIAL FINS AND PLATE FINS OF VARYING SECTIONS

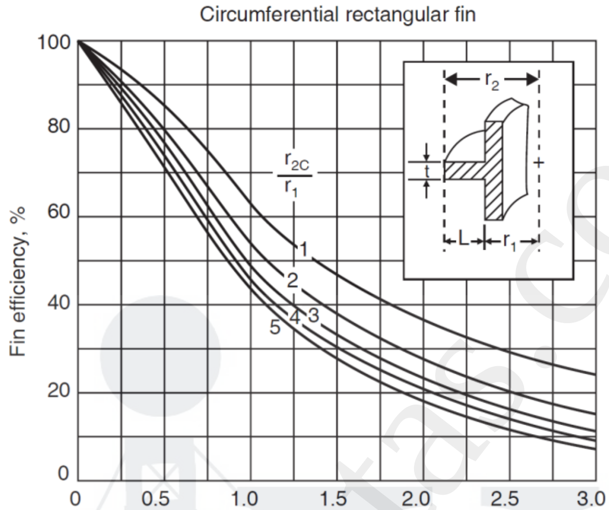
Circumferential fins and plate fins of varying sections are in common use. The preceding analysis has not taken this into account. As already mentioned the fin efficiency is correlated to the combination of parameters *L*, *t*, *h* and *k* (length, thickness, convection coefficient and thermal conductivity). Once these are specified, the chart can be entered by using the parameter to determine efficiency. The value of efficiency, the surface area, temperature and convection coefficient provide the means to calculate the heat dissipated.

Q = fin efficiency. $As\ h\ (Tb - T\infty)$

Charts are available for constant thickness circumferential fins, triangular section plate fins and pin fins of different types. The parameters used for these charts are given in the charts.

The fin efficiency chart for circumferential fins is given below:

$$Q = \eta A_s h (T_b - T_\infty)$$



$$L_c^{1.5}\left(\frac{h}{kA_p}\right)^{0.5}$$

$$L_c = L + \frac{t}{2} \qquad r_{2c} = r_1 + L_c$$

$$A_p = t(r_{2c} - r_1) \qquad A_s = 2\pi(r_{2c}^2 - r_1^2)$$

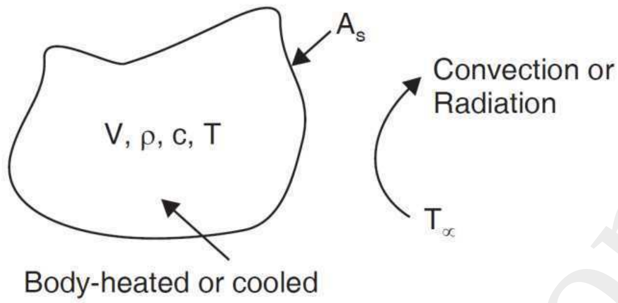
TRANSIENT HEAT CONDUCTION

Heat transfer equipments operating at steady state is only one phase of their functioning. These have to be started and shut down as well as their performance level may have to be altered as per external requirements. A heat exchanger will have to operate at different capacities. This changes the conditions at the boundary of heat transfer surfaces. Before a barrier begins to conduct heat at steady state the barrier has to be heated or cooled to the temperature levels that will exist at steady conditions. Thus the study of transient conduction situation is an important component of conduction studies. This study is a little more complicated due to the introduction of another variable namely time to the parameters affecting conduction. This means that temperature is not only a function of location but also a function of time, τ, i.e. $T = T(x, y, z, \tau)$. In addition heat capacity and heat storage (as internal energy) become important parameters of the problem. The rate of temperature change at a location and the spatial temperature distribution at any time are the important parameters to be determined in this study. This automatically provides information about the heat conduction rate at any time or position through the application of Fourier law.

LUMPED PARAMETER MODEL

It is also known as lumped heat capacity system. This model is applicable when a body with a known or specified temperature level is exposed suddenly to surroundings at a different temperature level and when the temperature level in the body as a whole increases or decreases without any difference of temperature

within the body. *i.e.*, $T = T(\tau)$ only. Heat is received from or given to the surroundings at the surface and this causes a temperature change
instantly all through the body. The model is shown in Fig.



The body with surface area *As*, volume *V*, density ρ, specific heat *c* and temperature *T* at the time instant zero is exposed suddenly to the surroundings at *T*∞ with a convection coefficient *h* (may be radiation coefficient *hr*). This causes the body temperature *T* to change to $T + dT$ in the time interval $d\tau$. The relationship between *dT* and $d\tau$ can be established by the application of the energy conservation principle.

Heat convected over the boundary = Change in internal energyover a time period $d\tau$ during this time

If *dT* is the temperature change during the time period $d\tau$ then the following relationship results:
(*As*-Surface area)

$$h\,As\,(T - T\infty)\,d\tau = \rho\,c\,VdT$$

This equation can be integrated to obtain the value of *T* at any time. The integration is possible after introducing a new variable.

$\theta = T - T\infty$

The equation now becomes

$hA_s\theta d\tau = \rho cVd\theta$

$$\frac{hA_s}{\rho cV} d\tau = d\theta$$

Separating the variables and integrating and using the initial conditions that at $\tau=0$, $\theta=\theta_o$ and denoting $V/A_s=L$, we get

$$ln\frac{\theta}{\theta_o} = \frac{hA_s}{\rho cV}\tau$$

Substituting for θ and θ_o and taking the antilog

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{hA_s}{\rho cV}\tau} = e^{-\frac{h}{\rho cL}\tau}$$

Heat flow up to time τ

$$\theta = \rho cV\,(T_\tau - T_i)$$

SEMI INFINITE SOLID

Theoretically a solid which extends in both the positive and negative *y* and *z* directions to infinity and in the positive *x* direction to infinity is defined as a semi infinite body. There can be no such body in reality. If one surface of a solid with a particular temperature distribution is suddenly exposed to convection conditions or has its surface temperature changed suddenly, conduction will produce a change in the temperature distribution along the thickness of the body. If this change does not reach the other side or surface of the solid under the time under consideration, then the solid may be modeled as semi infinite solid. A thick slab with a low value of thermal diffusivity exposed to a different environment on its surface can be treated as semi infinite body, provided heat does not penetrate to the full depth in the time under consideration. A road surface exposed to solar heat or chill winds can be cited as an example of a semi infinite body. There are a number of practical applications in engineering for the semi infinite medium conduction.

The differential equation applicable is the simplified general heat conduction equation: in rectangular coordinates, (excluding heat generation) eqn.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

There are three types of boundary conditions for which solutions are available in a simple form. These are (*i*) at time $\tau = 0$, the surface temperature is changed and maintained at a specified value, (*ii*) at time $\tau = 0$, the surface exposed to convection at $T\infty$ and (*iii*) at time $\tau = 0$, the surface is exposed to a constant heat flux q.

at $\tau = 0$, $T(x, \tau) = Ti$, or $T(x, 0) = \mathrm{T}i$

For $\tau > 0$, $T(0, \tau) = Ts$ *i.e.* at $x = 0$, $T = Ts$ at all times.

The analytical solution for this case is given by derivation available in specialized texts on conduction

$$\frac{T_{x,\tau} - T_s}{T_i - T_s} = \mathrm{erf}\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$$

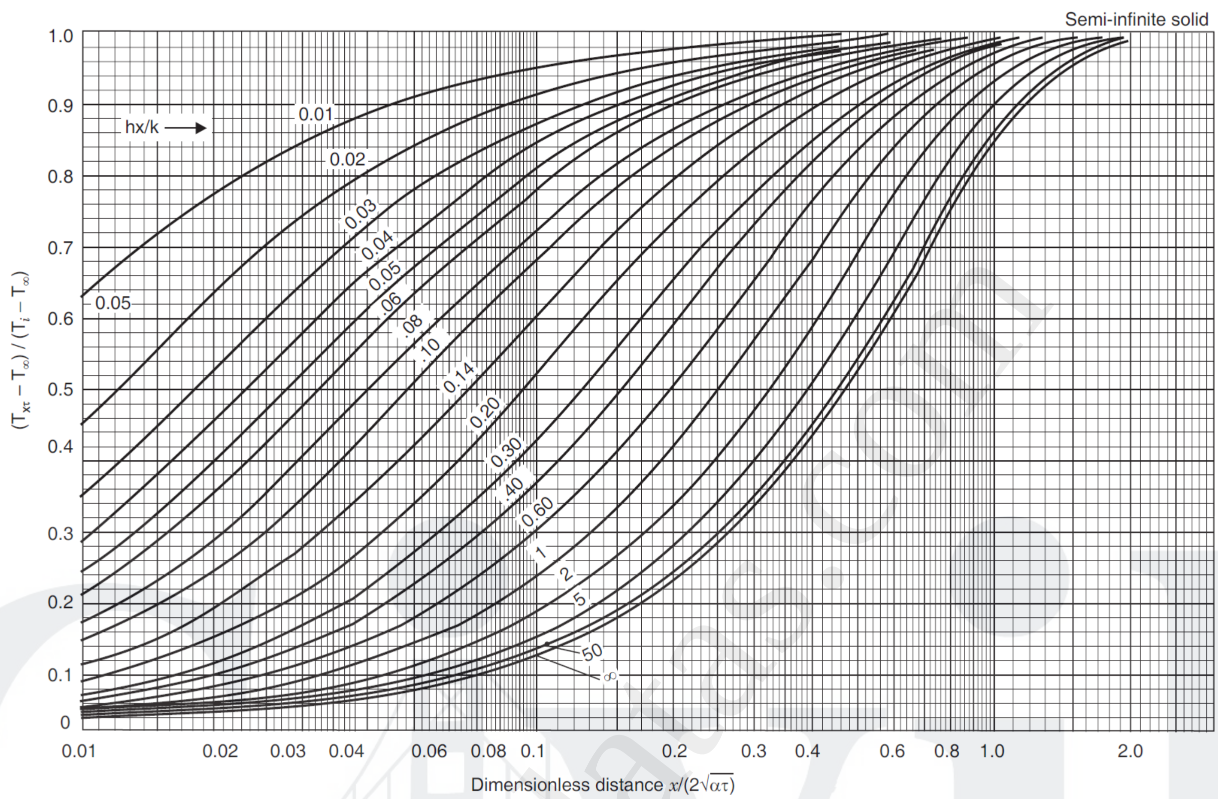
where, erf indicates "error function of" and the definition of error function is generally available in mathematical texts. Usually tabulations of error function values are available in handbooks. (Refer appendix).

The heat flow at the surface at any time is obtained using Fourier's equation $-k\mathrm{A}\,(d\mathrm{T}/dx)$. The surface heat flux at time τ is

$$q_s(\tau) = k(T_s - T_i)/\sqrt{\pi\alpha\tau}$$

The total heat flow during a given period can be obtained by integrating $qs(\tau)\,d\tau$ between the limits of 0 and τ

$$\mathrm{Q}_\tau = 2k \times A\,(T_s - T_i)\,\sqrt{\tau / \pi\alpha}$$



TRANSIENT HEAT CONDUCTION IN LARGE SLAB OF LIMITED THICKNESS, LONG CYLINDERS AND SPHERES

This model is the one which has a large number of applications in heating and cooling processes a special case being heat treatment. The general solution process attempts to estimate the temperature at a specified location in a body (which was at a specified initial temperature) after exposure to a different temperature surroundings for a specified time. The other quantity of interest is the change in the internal energy of the body after such exposure.

The differential equation applicable for a slab extending to ∞ in the *y* and *z* directions and thickness $2L$ in the *x* direction with both surfaces suddenly exposed to the surroundings is equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

The initial condition at time zero is
$T = Ti$ all through the solid. *i.e.* $x = -L$ to $x = L$.
The boundary condition is

$$h\,(T_\infty - T_L) = -k\frac{\partial T}{\partial x} \text{ at } x = L \text{ and } x = -L$$

at $x = L$ and $x = -L$

The equation is solved using a set of new variables X and θ defining $T = X.\theta$ (X is a function of x only and θ is a function of τ only). The algebra is long and tedious.

The solution obtained is given below :

$$\frac{T_{x,\tau} - T_\infty}{T_i - T_\infty} = \sum_{n=1}^{\infty} \frac{2\text{Bi}\sin(\delta_n)\cos(\delta_n x/L)}{\delta_n(\text{Bi} + \sin^2\delta_n)}.e^{-\delta_n{}^2.\text{Fo}}$$

The temperature essentially is a function of *Bi*, *Fo* and *x/L or T =f(Bi, Fo, x/L)*

Where $T_{x,\tau}$ *= the temperature at x and* τ

T_i *= initial temperature*

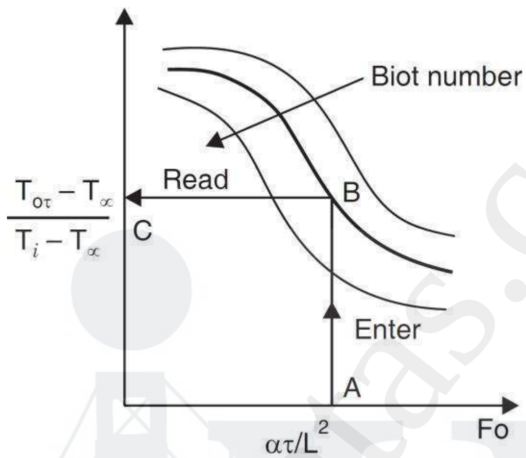
T_∞ *= surrounding temperature*

Bi=hL/k=Biot number

Fo= Fourier number = $\alpha\tau/L^2$

δ_n *= roots of the equation* $\delta_n \tan \delta_n = Bi$

The solution using calculating devices is rather tedius and the results in a graphical form, was first published by Heisler in 1947, using the parameters Biot number and Fourier number

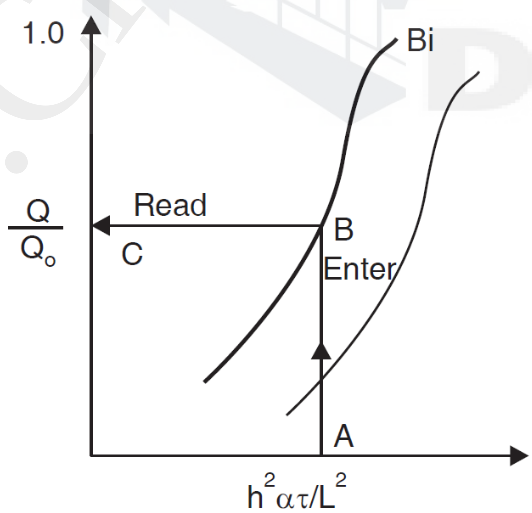


Heat Transfer during a given time period: The total heat transfer can be obtained by using

$$Q = \int_0^\tau h(T_{L,\tau} - T_\infty)\, d\tau$$

and substituting for $T_{L,\tau}$ from equation (6.21). As the resulting expression indicates that it is a function of $h^2\alpha\tau/k^2$ and hL/k these solutions have been presented by Heisler as shown in the skeleton form in Fig as Q/Q_o. where Q-heat transferred over the given period, and

$$Q_o = \rho\, cV\,(T_i - T_\infty)$$



Heat and mass Transfer

Unit I

November 2008

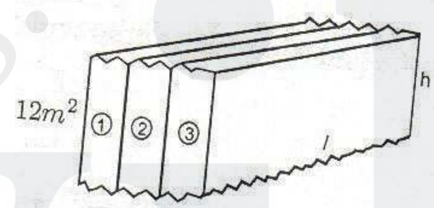
1. **Calculate the rate of heat loss through the vertical walls of a boiler furnace of size 4 m by 3 m by 3 m high. The walls are constructed from an inner fire brick wall 25 cm thick of thermal conductivity 0.4 W/mK, a layer of ceramic blanket insulation of thermal conductivity 0.2 W/mK and 8 cm thick, and a steel protective layer of thermal conductivity 55 W/mK and 2 mm thick. The inside temperature of the fire brick layer was measured at 600^o C and the temperature of the outside of the insulation 60^0 C. Also find the interface temperature of layers.**

Given:

Composite Wall

l= 4m b= 3m h= 3m

Area of rectangular wall lb = 4x3 = 12m^2



L_1 = 25 cm, k_1 = **0.4 W/mK** — Fire brick

L_2 =0.002m, k_2 = **54 W/mK** — Steel

L_3 = 0.08 m, k_1 = **0.2 W/mK** — insulation

T_1 = 600^0 C

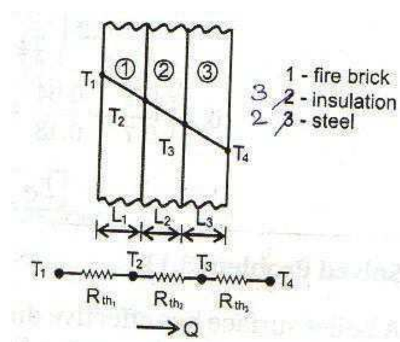
T_2 = 60^0 C

Find

(i) Q (ii) (T_3 – T_4)

Solution

We know that,



$$Q = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

Here

(ΔT) overall = $T_1 - T_4$

And $\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

$$R_{th1} = \frac{L_1}{k_1 A} = \frac{0.25}{0.4x12} = 0.0521 K/W$$

$$R_{th2} = \frac{L_2}{k_2 A} = \frac{0.08}{0.2x12} = 0.0333 K/W$$

$$R_{th3} = \frac{L_3}{k_3 A} = \frac{0.002}{54x12} = 0.0000031 K/W$$

$$Q = \frac{T_1 - T_4}{R_{th1} + R_{th2} + R_{th3}}$$

$$= \frac{600 - 60}{0.0521 + 0.0000031 + 0.0333}$$

Q = 6320.96 W

(i) To find temperature drop across the steel layer (T_2 - T_3)

$$Q = \frac{T_2 - T_3}{R_{th3}}$$

T_3- T_4 = Q× R_{th2}

= 6320.96× 0.0000031

T_3- T_4 = 0.0196 K .

2. A spherical container of negligible thickness holding a hot fluid at 140^0 and having an outer diameter of 0.4 m is insulated with three layers of each 50 mm thick insulation of k_1 = 0.02: k_2 = 0.06 and k_3 = 0.16 W/mK. (Starting from inside). The outside surface temperature is 30^0C. Determine (i) the heat loss, and (ii) Interface temperatures of insulating layers.

Given:

OD = 0.4 m

r_1 = 0.2 m

r_2 = r_1 + thickness of 1st insulation

= 0.2+0.05

r_2 = 0.25m

r_3 = r_2 + thickness of 2nd insulation

= 0.25+0.05

r_3 = 0.3m

r_4 = r_3 + thickness of 3rd insulation

= 0.3+0.05

r_4 = 0.35m

T_{hf} = 140^o C, T_{cf} = 30^o C,

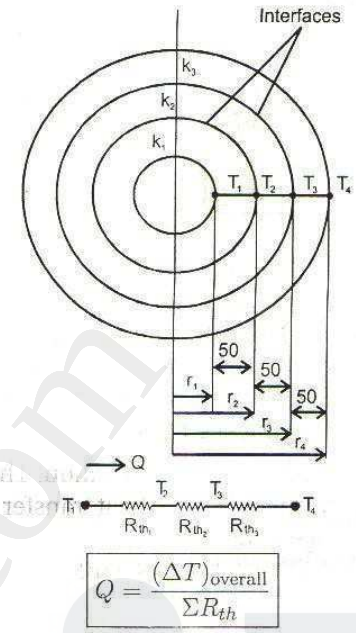
k_1 = 0.02 W/mK

k_2 = 0.06 W/mK

k_3 = 0.16 W/mK.

Find (i) Q (ii) T_2, T_3

Solution



$$Q = \frac{(\Delta T)_{overall}}{\Sigma R_{th}}$$

$\Delta T = T_{hf} - T_{cf}$

$\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

$R_{th1} = \frac{r_2 - r_1}{4\pi k_1 r_2 r_1} = \frac{(0.25-0.20)}{4\pi\, x0.02x0.25x0.2} = 3.978^{o}$ C/W

$R_{th2} = \frac{r_3 - r_2}{4\pi k_2 r_3 r_2} = \frac{(0.30-0.25)}{4\pi\, x0.06x0.3x0.25} = 0.8842^{o}$ C/W

$R_{th1} = \frac{r_4 - r_3}{4\pi k_3 r_4 r_3} = \frac{(0.35-0.30)}{4\pi\, x0.16x0.35x0.30} = 0.23684^{o}$ C/W

$$Q = \frac{140 - 30}{0.0796 + 0.8842 + 0.23684}$$

Q = 21.57 W

To find interface temperature (T_2 , T_3)

$Q = \frac{T_2 - T_3}{R_{th1}}$

$T_2 = T_1 - [Q \text{ x } R_{th1}]$

= 140 – [91.62×0.0796]

$T_2 = 54.17^{0}$C

$Q = \frac{T_2 - T_3}{R_{th1}}$

$T_3 = T_2 - [Q \times R_{th2}]$

= 132.71- [91.62×0.8842]

$T_3 = 35.09^{o}$ C

3. May 2008

A steel tube with 5 cm ID, 7.6 cm OD and k=15W/m ^{o} C is covered with an insulative covering of thickness 2 cm and k 0.2 W/m ^{o}C. A hot gas at 330^{o} C with h = 400 W/m^{2o}C flows inside the tube. The outer surface of the insulation is exposed to cooler air at 30^{o}C with h = 60 W/m^{2o}C. Calculate the heat loss from the tube to the air for 10 m of the tube and the temperature drops resulting from the thermal resistances of the hot gas flow, the steel tube, the insulation layer and the outside air.

Given:

Inner diameter of steel, d_1 = 5 cm =0.05 m

Inner radius,r_1 = 0.025m

Outer diameter of steel, d_2 = 7.6 cm = 0.076m

Outer radius,r_2 = 0.025m

Radius, $r_3 = r_2$ + thickness of insulation

= 0.038+0.02 m

$r_3 = 0.058$ m

Thermal conductivity of steel, $k_1 = 15 W/m^o C$

Thermal conductivity of insulation, $k_2 = 0.2 W/m^oC$

Hot gas temperature, $T_{hf} = 330^o C + 273 = 603 K$

Heat transfer co-efficient at innear side, $h_{hf} = 400 W/m^{2o}C$

Ambient air temperature, $T_{cf} = 30^oC + 273 = 303 K$

Heat transfer co-efficient at outer side $h_{cf} = 60 W/m^{2o}C$.

Length, L = 10 m

To find:

(i) Heat loss (Q)

(ii) Temperature drops $(T_{hf} - T_1)$, $(T_1 - T_2)$, $(T_2 - T_3)$, $(T_3 - T_{cf})$,

Solution:

Heat flow $Q = \frac{\Delta T_{overall}}{\Sigma R_{th}}$

Where

$\Delta T_{overall} = T_{hf} - T_{cf}$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_{hf}r_1} + \frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{k_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{k_3}\ln\left[\frac{r_4}{r_3}\right] + \frac{1}{h_{cf}r_4}\right]$$

$$Q = \frac{T_{hf} - T_{cf}}{\frac{1}{2\pi L}\left[\frac{1}{h_{hf}r_1} + \frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{k_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{h_{cf}r_3}\right]}$$

$$Q = \frac{603 - 303}{\frac{1}{2\pi \times 10}\left[\frac{1}{400 \times 0.025} + \frac{1}{15}\ln\left[\frac{0.038}{0.025}\right] + \frac{1}{0.2}\ln\left[\frac{0.058}{0.038}\right] + \frac{1}{60 \times 0.058}\right]}$$

Q = 7451.72 W

We know that,

$$Q = \frac{T_{hf} - T_1}{R_{th\ conv.}}$$

$$= \frac{T_{hf} - T_1}{\frac{1}{2\pi L} \times \frac{1}{h_{hf}r_1}}$$

$$7451.72 = \frac{T_{hf} - T_1}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{400 \times 0.025}}$$

$$T_{hf} - T_1 = 11.859K$$

$$Q = \frac{T_1 - T_2}{R_{th1}}$$

$$= \frac{T_1 - T_2}{\frac{1}{2\pi L} \times \left[\frac{1}{k_1}\ln\left[\frac{r_2}{r_1}\right]\right]}$$

$$7451.72 = \frac{T_1 - T_2}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{15} \ln \left[\frac{0.038}{0.025}\right]}$$

$$T_1 - T_2 = 3.310\ K$$

$$Q = \frac{T_2 - T_3}{R_{th2}}$$

$$= \frac{T_2 - T_3}{\frac{1}{2\pi L} \times \left[\frac{1}{k_2} \ln\left[\frac{r_3}{r_2}\right]\right]}$$

$$7451.72 = \frac{T_2 - T_3}{\frac{1}{2 \times \pi \times 10} \times \frac{1}{0.2} \ln \left[\frac{0.058}{0.038}\right]}$$

$$T_2 - T_3 = 250.75\ K$$

$$Q = \frac{T_3 - T_{cf}}{R_{th\ conv.}}$$

$$= \frac{T_3 - T_{cf}}{\frac{1}{2\pi L} \times \frac{1}{h_{cf} r_3}}$$

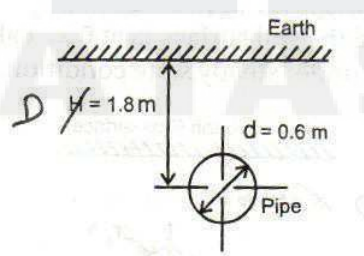
$$7451.72 = \frac{T_3 - T_{cf}}{\frac{1}{2 \times \pi \times 10} \times \left[\frac{1}{60 \times 0.058}\right]}$$

$$T_3 - T_{cf} = 34.07K$$

Nov 2009

4. A long pipe of 0.6 m outside diameter is buried in earth with axis at a depth of 1.8 m. the surface temperature of pipe and earth are 95^0 C and 25^0 C respectively. Calculate the heat loss from the pipe per unit length. The conductivity of earth is 0.51W/mK.

Given



r= $\frac{0.6}{2}$ = 0.3 m

L = 1 m

T_p = 95° C

T_e = 25° C

D = 1.8 m

k = 0.51W/mK

Find

Heat loss from the pipe (Q/L)

Solution

We know that

$$\frac{Q}{L} = k.S(T_p - T_e)$$

Where S = Conduction shape factor =

$$\frac{2\pi L}{\ln\left(\frac{2D}{r}\right)}$$

$$= \frac{2\pi x\ 1}{\ln\left(\frac{2x\ 1.8}{0.3}\right)}$$

S = 2.528m

$$\frac{Q}{L} = 0.51x2.528(95-25)$$

$$\frac{Q}{L} = 90.25W/m$$

Nov.2010

5. A steam pipe of 10 cm ID and 11 cm OD is covered with an insulating substance k = 1 W/mK. The steam temperature is 200^0 C and ambient temperature is 20^0 C. If the convective heat transfer coefficient between insulating surface and air is 8 W/m^2K, find the critical radius of insulation for this value of r_c. Calculate the heat loss per m of pipe and the outer surface temperature. Neglect the resistance of the pipe material.

Given:

$$r_{i=}\frac{ID}{2} = \frac{10}{2} = 5\ cm = 0.05m$$

$$r_{0=}\frac{OD}{2} = \frac{11}{2} = 5.5\ cm = 0.055m$$

k =1 W/mK

$T_i = 200^oC$ $T_\infty = 20^o$ C

h_0 =8 W/m^2K

Find

(i) r_c

(ii) If $r_c = r_o$ then Q/L

(iii) T_o

Solution

To find critical radius of insulation (r_c)

$$r_{0=}\frac{k}{h_0} = \frac{1}{8} = 0.125m$$

When $r_c = r_o$

Kpipe, h_{hf} not given

$$\frac{Q}{L} = \frac{2\pi(T_0 - T_\infty)}{\frac{\ln\left(\frac{r_c}{r_o}\right)}{k} + \frac{1}{h_o r_o}}$$

$$= \frac{2\pi(200-20)}{\frac{\ln\left(\frac{0.125}{0.050}\right)}{1} + \frac{1}{8\,x\,0.125}}$$

$$\frac{Q}{L} = 621\, W/m$$

To Find T_o

$$\frac{Q}{L} = \frac{T_0 - T_\infty}{R_{thconv}}$$

$$T_0 = T_\infty + \frac{Q}{L}\,(R_{thconv})$$

$$= 20 + 621 \times \left(\frac{1}{8 \times 2\pi \times 0.125}\right)$$

$$T_0 = 118.72^0 C$$

November 2011.

6. The temperature at the inner and outer surfaces of a boiler wall made of 20 mm thick steel and covered with an insulating material of 5 mm thickness are 300^0 C and 50^0 C respectively. If the thermal conductivities of steel and insulating material are 58W/m^{0}C and 0.116 W/m^{0}C respectively, determine the rate of flow through the boiler wall.

$L1 = 20 \times 10^{-3}$ m

$k_1 = 58$ W/m^{0}C

$L_2 = 5 \times 10^{-3}$ m

$k_2 = 0.116$ W/m^{0}C

$T_1 = 300^0$ C

$T_2 = 50^0$ C

Find

(i) Q

Solution

$$Q = \frac{(\Delta T)overall}{\Sigma Rth} = \frac{T_1 - T_3}{R_{th1} - R_{th2}}$$

$$R_{th1} = \frac{L1}{k1A} = \frac{0.20 \times 10^{-3}}{58 \times 1} = 3.45 \times 10^{-4}\ {}^0C/W$$

$$R_{th2} = \frac{L2}{k2A} = \frac{5 \times 10^{-3}}{0.116 \times 1} = 0.043\ {}^0C/W$$

$$Q = \frac{300 - 50}{3.45 \times 10{-}4 + 0.043} = 5767.8\ W$$

Q = 5767.8 W

7. A spherical shaped vessel of 1.2 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 200^o C. Thermal conductivity of material is 0.3 kJ /mh^{o}C.

Given

d_1 =1.2 m

r_1 = 0.6 m

r_2 = r_1 + thick

= 0.6 + 0.1

r_2 = 0.7 m

ΔT =200^{0}C

K = 0.3 kJ /mhr ^{o}C = 0.0833 W/m^o C

Find

Q

Solution:

$$Q = \frac{\Delta T}{R_{th}} = \frac{T_1 - T_2}{R_{th}}$$

$$R_{th} = \frac{r_2 - r_1}{4\pi r_2 r_1} = \frac{(0.7-0.6)}{4\pi\times0.0833\times0.6\times0.7} = 0.2275\ K/W$$

$$Q = \frac{\Delta T}{R_{th}} = \frac{200}{0.2275} = 879.132W$$

November 2011 (old regulation)

8. A steel pipe (K = 45.0 W/m.K) having a 0.05m O.D is covered with a 0.042 m thick layer of magnesia (K = 0.07W/m.K) which in turn covered with a 0.024 m layer of fiberglass insulation (K = 0.048 W/m.K). The pipe wall outside temperature is 370 K and the outer surface temperature of the fiberglass is 305K. What is the interfacial temperature between the magnesia and fiberglass? Also calculate the steady state heat transfer.

Given:

OD = 0.05 m

d_1= 0.05 m

r_1 = 0.025 m

k_1 = 45 W/mK

r_2 = r_1 + thick of insulation 1

r_2 = 0.025+0.042

r_2 = 0.067 m

k_2 = 0.07 W/mK

$k_3 = 0.048$ W/mK

$r_3 = r_2$ + thick of insulation 2

$= 0.067+0.024$

$r_3 = 0.091$ m

$T_1 = 370$ K

$T_3 = 305$ K

To find

(i) T_2

(ii) Q

Solution

Here thickness of pipe is not given; neglect the thermal resistance of pipe.

$$Q = \frac{(\Delta T)overall}{\Sigma Rth}$$

Here

$$(\Delta T)overall = T_1 - T_3 = 370 - 305 = 65\ K$$

$$\Sigma R_{th} = R_{th1} + R_{th2}$$

$$R_{th1} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{2L}} = \frac{\ln\left(\frac{0.067}{0.025}\right)}{2\pi \times 0.07 \times 1} = 2.2414 \text{ K/W}$$

$$R_{th2} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{3L}} = \frac{\ln\left(\frac{0.091}{0.067}\right)}{2\pi \times 0.48 \times 1} = 1.0152 \text{ K/W}$$

$$Q = \frac{65}{2.2414+1.0152} = 19.959 \text{ W/m}$$

To find T_2

$$Q = \frac{T_1 - T_2}{R_{th1}}$$

$T_2 = T_1 - [Q \times R_{th1}]$

$= 370- [19.959 \times 2.2414]$

$T_3 = 325.26$K

9. A motor body is 360 mm in diameter (outside) and 240 mm long. Its surface temperature should not exceed 55 °C when dissipating 340W. Longitudinal fins of 15 mm thickness and 40 mm height are proposed. The convection coefficient is 40W/m² °C. determine the number of fins required. Atmospheric temperature is 30°C. thermal conductivity = 40 W/m°C.

Given:

$D = 360 \times 10^{-3}$ m

$L = 240 \times 10^{-3}$ m

$T_b = 55°C$

$Q_{generating} = 340$W

Longitudinal fin

$t_{fin} = 15 \times 10^{-3}$ m

$h_{fin} = 40 \times 10^{-3}$ m

$h = 40 W/m^2 °C$

$k = 40 W/m °C$.

$T_\infty = 30 °C$

To find:

No of fins required (N)

Solution:

Here length (or) height of fin is given. It is short fin(assume end insulated)

$N = \frac{Q_{gen}}{Q_{per\ fin}}$

From HMT Data book,

$$Q = \sqrt{hPkA}\ (T_b - t_\infty).\tan h(mL)$$

$$m = \sqrt{\frac{hP}{kA}}\ m^{-1}$$

Perimeter (P) = 2L = 2 x 0.24 = 0.48 m

(for longitudinal fin fitted on the cylinder)

Area (A) = Lt = 0.24 x 0.015

A = 0.0036m²

$$m = \sqrt{\frac{40 \times 0.48}{40 \times 0.0036}} = 11.55\ m^{-1}$$

$$Q_{fin} = \sqrt{40 \times 0.48 \times 40 \times 0.0036}\ (55 - 30).\tan h(11.55 \times 0.04)$$

$Q_{fin} = 4.718$ W

$$N = \frac{340}{4.718} = 72.06 = 72\ fins.$$

May 2012

10. A mild steel tank of wall thickness 10 mm contains water at 90° C. The thermal conductivity of mild steel is 50 W/m°C , and the heat transfer coefficient for inside and outside of the tank area are 2800 and 11 W/m² °C, respectively. If the atmospheric temperature is 20°C , calculate

(i) The rate of heat loss per m² of the tank surface area.

(ii) The temperature of the outside surface tank.

Given

$L = 10 \times 10^{-3} m$

$T_{hf} = 90\ ^oC$

$k = 50\ W/m\ ^oC$

$h_{hf} = 2800\ W/m^2\ ^oC$

$h_{cf} = 11\ W/m^2\ ^oC$

$T_{cf} = 20^o\ C$

To find

(i) Q/m^2

(ii) T_2

Solution

$$Q = \frac{(\Delta T) overall}{\Sigma Rth}$$

Here $(\Delta T)_{overall} = T_{hf} - T_{cf} = 90 - 20 = 70^oC$

$$\sum R_{th} = R_{th_{conv_{hf}}} + R_{th1} + R_{th_{conv_{cf}}}$$

$$R_{th_{conv_{hf}}} = \frac{1}{h_{hf}.A} = \frac{1}{2800 \times 1} 0.00036\ K/W$$

$$R_{th} = \frac{L}{kA} = \frac{10 \times 10^{-3}}{50 \times 1} = 0.0002\ K/W$$

$$R_{th_{conv_{cf}}} = \frac{1}{h_{cf}.A} = \frac{1}{11 \times 1} 0.09091\ K/W$$

$$Q = \frac{70}{0.091469} = 765.29\ W/m^2$$

To find T_2

$$Q = \frac{T_{hf} - T_2}{R_{conv_{hf}+R_{th\,1}}}$$

$$T_{2=} T_{hf} - \left[Q \times R_{conv_{hf}+R_{th\,1}}\right]$$

$= 90 - [765x\ 0.00056]$

$T_2 = 89.57\ ^0C$

11. A 15 cm outer diameter steam pipe is covered with 5 cm high temperature insulation (k = 0.85 W/m °C) and 4 cm of low temperature (k = 0.72 W/m° C). The steam is at 500 °C and ambient air is at 40 °C. Neglecting thermal resistance of steam and air sides and metal wall calculate the heat loss from 100 m length of the pipe. Also find temperature drop across the insulation.

Given

d_1 = 15 cm

r_1 = 7.5×10^{-2} m

r_2 = r_1 + thick of high temperature insulation

r_2 = $7.5 + 5 = 12.5 \times 10^{-2}$ m

r_3 = r_2 + thick of low temperature insulation

r_3 = $12.5 + 4 = 16.5 \times 10^{-2}$ m

k_{ins1} = 0.85 w/m° C

k_{ins2} = 0.72 w/m° C

T_{hf} = 500 ° C

T_{cf} = 40 ° C

To find

(i) Q if L = 1000mm = 1 m

Solution:

$$Q = \frac{(\Delta T)overall}{\Sigma Rth}$$

Here

$$\Delta T = T_1 - T_3$$

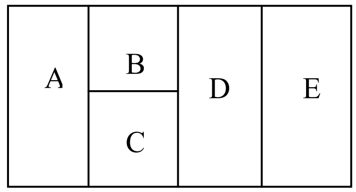
$$\Sigma R_{th} = R_{th1} + R_{th2}$$

$$R_{th1} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{1L}} = \frac{\ln\left(\frac{0.125}{0.075}\right)}{2\pi \times 0.85 \times 1} = 0.09564 \text{ K/W or } ^\circ\text{C/W}$$

$$R_{th2} = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{2L}} = \frac{\ln\left(\frac{0.165}{0.125}\right)}{2\pi \times 0.72 \times 1} = 0.06137 \text{ K/W or } ^\circ\text{C/W}$$

$$Q = \frac{500-40}{0.09564+0.06137} = 2929.75 \text{W/m}$$

12. Determine the heat transfer through the composite wall shown in the figure below. Take the conductives of A, B, C, D & E as 50, 10, 6.67, 20& 30 W/mK respectively and assume one dimensional heat transfer. Take of area of A =D= E = 1m^2 and B=C=0.5 m^2. Temperature entering at wall A is 800 ^{o} C and leaving at wall E is 100^o C.



Given:

$T_i = 800^o C$

$T_o = 100^o C$

k_A = 50 W/mK

k_B = 10 W/mK

k_c = 6.67 W/mK

k_D = 20 W/mK

k_E = 30 W/mK

$A_A = A_D = A_E = 1 m^2$

$A_B = A_C = 0.5\ m^2$

Find

(i) Q

Solution

$$Q = \frac{(\Delta T) overall}{\Sigma Rth}$$

$$R_{th1=}\ R_{thA=}\ \frac{L_A}{k_A\ A}$$

Parallel $\frac{1}{R_{th2}} = \frac{1}{R_{thB}} + \frac{1}{R_{thC}} = \frac{R_{thB}+R_{thC}}{R_{thB}R_{thC}}$

$$R_{th2} = \frac{R_{thB}R_{thC}}{R_{thB}+R_{thC}}$$

$$R_{thB} = \frac{L_B}{k_B A_B}$$

$$R_{thC} = \frac{L_C}{k_C A_C}$$

$$R_{th4} = R_{thE} = \frac{L_E}{k_E A_E}$$

$$R_{th3} = R_{thD} = \frac{L_D}{k_D A_D}$$

$$R_{th1=}\ R_{thA=}\ \frac{1}{50 \times 1} = 0.02\ K/W$$

$$R_{thB} = \frac{1}{10 \times 0.5} = 0.2\ K/W$$

$$R_{thC} = \frac{1}{6.67 \times 0.5} = 0.2969\ K/W$$

$$R_{th2} = \frac{R_{thB}R_{thC}}{R_{thB} + R_{thC}} = \frac{0.2 \times 0.299}{0.2 + 0.299} = \frac{0.0598}{0.499}$$

$$R_{th2} = 0.1198\ K/W$$

$$R_{th3} = R_{thD} = \frac{L_D}{K_D A_D} = \frac{1}{20 \times 1} = 0.05\ K/W$$

$$R_{th4} = R_{thE} = \frac{L_E}{K_E A_E} = \frac{1}{30 \times 1} = 0.0333\ K/W$$

$$Q = \frac{T_i - T_o}{\sum R_{th}} = \frac{800 - 100}{0.02 + 0.1198 + 0.05 + 0.0333} = 3137.61W$$

$$Q = 3137.61W$$

13. A long carbon steel rod of length 40 cm and diameter 10 mm (k = 40 w/mK) is placed in such that one of its end is 400^o C and the ambient temperature is 30^o C. the flim co-efficient is 10 w/m^{2}K. Determine

(i) Temperature at the mid length of the fin.

(ii) Fin efficiency

(iii) Heat transfer rate from the fin

(iv) Fin effectiveness

Given:

$l = 40 \times 10^{-2}$ m

d = 10×10^{-3} m

k = 40 W/mK

$T_b = 400^o$ C

$T_\infty = 30^o$ C

H = 10 w/m^{2}K

To find

(i) T , x = L/2

(ii) η_{fin}

(iii) Q_{fin}

Solution

It is a short fin end is insulated

From H.M.T Data book

$$Q = \sqrt{hPkA}\ (T_b - T_\infty).\tan h(mL)$$

$$m = \sqrt{\frac{hP}{kA}}\ m^{-1}$$

Perimeter = $\pi d = \pi \times 10 \times 10^{-3} = 0.0314$ m

$$Area = \frac{\pi}{4} d^2 = \frac{\pi}{4} (10 \times 10^{-3})^2 = 0.0000785\ m^2$$

$$m = \sqrt{\frac{10 \times 0.0314}{40 \times 0.0000785}} = 10\ m^{-1}$$

$$Q = \sqrt{10 \times 0.0314 \times 40 \times 0.0000785}\ (400 - 30).\tan h(10 \times 40 \times 10^{-2})$$

$$Q = 0.115 \text{ W}$$

From H.M.T Data book

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos hm\ (L - x)}{\cos h\ (mL)}$$

$$\frac{T - 30}{400 - 30} = \frac{\cos h\ 10\ (0.4 - 0.2)}{\cos h\ (10 \times 0.4)}$$

$$\frac{T - 30}{400 - 30} = \frac{3.762}{27.308}$$

$$\frac{T - 30}{370} = 0.13776$$

$$T = 50.97 + 30$$

$$T = 80.97\ ^oC$$

14. A wall furnace is made up of inside layer of silica brick 120 mm thick covered with a layer of magnesite brick 240 mm thick. The temperatures at the inside surface of silica brick wall and outside the surface of magnesite brick wall are 725°C and 110°C respectively. The contact thermal resistance between the two walls at the interface is 0.0035°C/w per unit wall area. If thermal conductivities of silica and magnesite bricks are 1.7 W/m°C and 5.8 W/m°C, calculate the rate of heat loss per unit area of walls.

Given:

$L_1 = 120 \times 10^{-3}$ m

$k_1 = 1.7$ W/m^{0}C

$L_2 = 240 \times 10^{-3}$ m

$k_2 = 5.8$ W/m^{0}C

$T_1 = 725\ ^0$C

$T_4 = 110^0$ C

$(R_{th})_{contact} = 0.0035\ ^oC/W$

Area = 1 m^2

Find

(i) Q

Solution

$$Q = \frac{(\Delta T)overall}{\Sigma Rth} = \frac{T_1 - T_4}{\text{Rth1} + (R_{th})_{cont} + \text{Rth2}}$$

Here $T_1 - T_4 = 725 - 110 = 615^o$ C

$$R_{th1} = \frac{L1}{k1A} = \frac{120 \text{ x } 10^{-3}}{1.7 \times 1} = 0.0706^0 \text{ C /W}$$

$$R_{th2} = \frac{L2}{k2A} = \frac{240 \text{x } 10^{-3}}{5.8 \times 1} = 0.0414\ ^0 \text{ C /W}$$

$$Q = \frac{615}{0.0706 + 0.0035 + 0.0414} = 5324.67 \text{ W/m}^2$$

Q = 5324.67 W/m

15. A furnace walls made up of three layers , one of fire brick, one of insulating brick and one of red brick. The inner and outer surfaces are at 870^o C and 40^o C respectively. The respective co- efficient of thermal conduciveness of the layer are 1.0, 0.12 and 0.75 W/mK and thicknesses are 22 cm, 7.5, and 11 cm. assuming close bonding of the layer at their interfaces, find the rate of heat loss per sq.meter per hour and the interface temperatures.

Given

Composite wall (without convection)

$L_1 = 22 \text{ x}10^{-2}$ m

$k_1 = 1$ W/mK

$L_2 = 7.5 \text{ x}10^{-2}$ m

$k_2 = 0.12$ W/mK

$L_3 = 11\text{x}10^{-2}$ m

$k_3 = 0.75$ W/mK

$T_1 = 870^o$ C

$T_4 = 40^o$ C

Find

(i) Q / hr (ii) T_2, T_3

Solution

We know that,

$$Q = \frac{(\Delta T)overall}{\Sigma Rth}$$

Here

(ΔT) overall = $T_1 - T_4$

= 870 – 40

= 830 ^{o}C

And $\Sigma R_{th} = R_{th1} + R_{th2} + R_{th3}$

(assume A = 1 m^2)

$R_{th1} = \frac{L1}{k1A} = \frac{22 \text{ x}10-2}{1 \times 1} = 22 \text{ x}10^{-2}$ K/W

$R_{th2} = \frac{L2}{k2A} = \frac{7.5 \text{ x}10-2}{0.12 \times 1} = 0.625$ K/W

$R_{th3} = \frac{L3}{k3A} = \frac{11\text{x}10-2}{0.75 \times 1} = 0.1467$ K/W

$$Q = \frac{T1 - T4}{Rth1 + Rth2 + Rth3}$$

$$= \frac{870 - 40}{0.9917}$$

Q = 836.95 W/m^2

Q = 3.01X 10^5 J/h

Nov 2010

16. A 12 cm diameter long bar initially at a uniform temperature of 40°C is placed in a medium at 650°C with a convective co efficient of 22 W/m²K calculate the time required for the bar to reach255°C. Take k = 20W/mK, ρ = 580 kg/m³ and c = 1050 J/kg K.

Given : Unsteady state

D = 12 cm = 0.12 m

R = 0.06 m

T_o = 40 + 273 = 313 K

T_∞ = 650 + 273 = 923 K

T = 255 + 273 =528 K

h = 22 W/m²K

k = 20 W/mK

ρ = 580 Kg/m³

c = 1050 J/kg K

Find:

Time required to reach 255°C (τ)

Solution

Characteristic length for cylinder = $L_c = \frac{R}{2}$

$$L_c = \frac{0.06}{2} = 0.03 \text{ m}$$

We know that

$$B_i = \frac{hL_c}{k} = \frac{22 \times 0.03}{20}$$

$$B_i = 0.033 < 0.1$$

Biot number is less than 0.1. Hence this is lumped heat analysis type problem.

For lumped heat parameter, from HMT data book.

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{\left[-\frac{h\,A}{c\,V\,\rho} \times \tau\right]}$$

We know that

$$L_c = \frac{V}{A}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{\left[\frac{-h}{c\,L_c\rho} \times \tau\right]}$$

$$\frac{528 - 923}{313 - 923} = e^{\left[\frac{-22}{1050 \times 0.03 \times 580} \times \tau\right]}$$

$$\ln\left[\frac{528 - 923}{313 - 923}\right] = \frac{22}{1050 \times 0.03 \times 580} \times \tau$$

$$\tau = 360.8 \text{ sec}$$

17. A aluminium sphere mass of 5.5 kg and initially at a temperature of 290°Cis suddenly immersed in a fluid at 15 °C with heat transfer co efficient 58 W/m² K. Estimate the time required to cool the aluminium to 95° C for aluminium take ρ = 2700 kg/m³ , c = 900 J /kg K, k = 205 W/mK.

Given:

M = 5.5 kg

$T_o = 290 + 273 = 563$ K

$T_\infty = 15 + 273 = 288$ K

T = 95 + 273 =368 K

h = 58 W/m²K

k = 205 W/mK

ρ = 2700 kg/m³

c = 900 j/kg K

To find:

Time required to cool at 95° C (τ)

Solution

$$\text{Density} = \rho = \frac{\text{mass}}{\text{volume}} = \frac{\text{m}}{\text{v}}$$

$$V = \frac{m}{p} = \frac{5.5}{2700}$$

$$V = 2.037 \text{ X } 10^{-33}$$

For sphere,

$$\text{Characteristic length} \quad L_c = \frac{R}{3}$$

$$\text{Volume of sphere} \quad V = \frac{4}{3}\pi R^3$$

$$R = \sqrt[3]{\frac{3V}{4\pi}}$$

$$= \sqrt[3]{\frac{3\times 2.03\times 10^{-3}}{4\pi}}$$

$$R = 0.0786 \text{ m}$$

$$L_c = \frac{0.0786}{3} = 0.0262\, m$$

$$\text{Biot number } B_i = \frac{hL_c}{k}$$

$$= \frac{58 \times 0.0262}{205}$$

$$B_i = 7.41 \text{ X } 10^{-3} < 0.1$$

B_i < 0.1 this is lumped heat analysis type problem.

UNIT II CONVECTION

The process of heat transfer between a surface and a fluid flowing in contact with it is called convection. If the flow is caused by an external device like a pump or blower, it is termed as forced convection. If the flow is caused by the buoyant forces generated by heating or cooling of the fluid the process is called as natural or free convection.

In the previous chapters the heat flux by convection was determined using equation

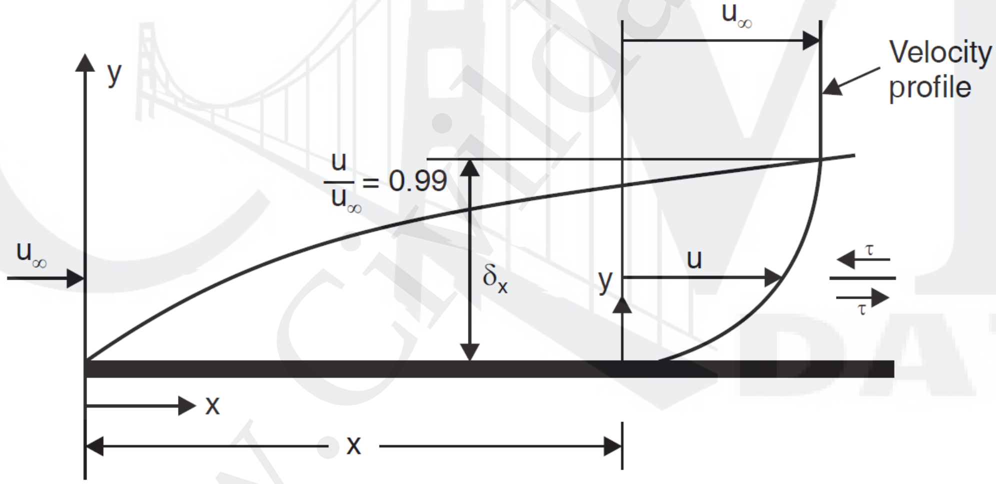
$$q = h\,(T_s - T_\infty)$$

q is the heat flux in W/m^2, T_s is the surface temperature and T_∞ is the fluid temperature of the free stream, the unit being °C or K. Hence the unit of convective heat transfer coefficient h is W/m^2 K or W/m^2 °C both being identically the same. In this chapter the basic mechanism of convection and the method of analysis that leads to the correlations for convection coefficient are discussed. In this process the law of conservation of mass, First law of themodynamics and Newtons laws of motion are applied to the system.

THE CONCEPT OF VELOCITY BOUNDARY LAYER

We have seen that in the determination of the convective heat transfer coefficient the key is the determination of the temperature gradient in the fluid at the solid-fluid interface. The velocity gradient at the surface is also involved in the determinations. This is done using the boundary layer concept to solve for $u = f(y)$, $T = f'(y)$. The simplest situation is the flow over a flat plate. The fluid enters with a uniform velocity of u_∞ as shown in Fig. When fluid particles touch the surface of the plate the velocity of these particles is reduced to zero due to viscous forces. These particles in turn retard the velocity in the next layer, but as these two are fluid layers, the velocity is not reduced to zero in the next layer. This retardation process continues along the layers until at some distance y the scale of retardation becomes negligible and the velocity of the fluid is very nearly the same as free stream velocity u at this level. The retardation is due to shear stresses along planes parallel to the flow.

The value of y where velocity $u = 0.99\, u_\infty$ is called hydrodynamic boundary layer thickness denoted by δ**.** The velocity profile in the boundary layer depicts the variation of u with y, through the boundary layer. This is shown in Fig.



The model characterizes the flow as consisting of two distinct regions (*i*) a thin boundary layer in which the velocity gradients and shear stresses are large and (*ii*) the remaining region outside of the boundary layer where the velocity gradients and shear stresses are negligibly small. This is also called potential flow. The boundary layer thickness increases along the direction of flow over a flat plate as effects of viscous drag is felt farther into the free stream.

This is called the velocity boundary layer model as this describes the variation of velocity in the boundary layer.

The direct application of velocity boundary layer is in fluid mechanics for the determination of the wall shear stress and then the dimensionless drag coefficient. The net shear over the plate in flow is the wall shear and shear stress beyond the boundary layer is zero.

The wall shear is given by the equation

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}$$

It may be seen that the velocity gradient can be determined if a functional relationship such as $u = f(y)$ is available. Such a relationship is obtained using the boundary layer model and applying the continuity and Newtons laws of motion to the flow. The friction coefficient C_f is defined as below.

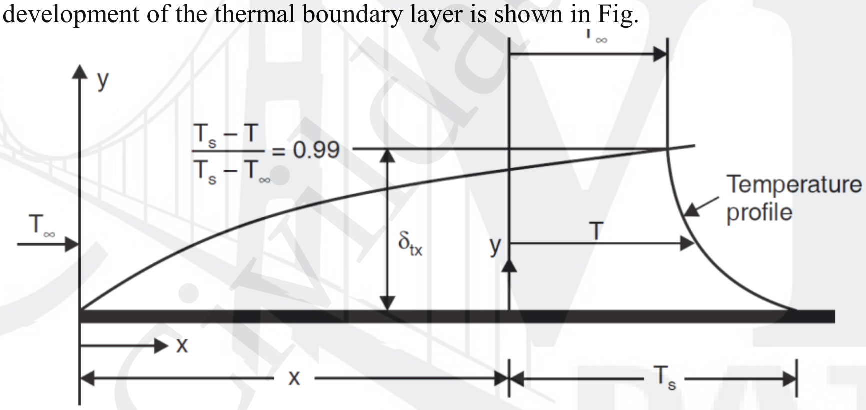
$$C_f = \tau_s/(\rho u_\infty^2/2)$$

There are local and average values for both τ_s and C_f denoted as $\tau_{s.x}$. $\overline{\tau_s}$ C_{fx} and $\overline{C_f}$. In heat transfer the friction coefficient by analogy is found to provide a value for Nusselt number and hence its importance. Measured values of C_f are also available for various values of an important parameter, namely Reynolds number. Curve fitted equations are also available for c_f.

THERMAL BOUNDARY LAYER

Velocity boundary layer automatically forms when a real fluid flows over a surface, but thermal boundary layer will develop only when the fluid temperature is different from the surface temperature. Considering the flow over a flat plate with fluid temperature of T_∞ and surface temperature T_s the temperature of the fluid is T_∞ all over the flow till the fluid reaches the leading edge of the surface. The fluid particles coming in contact with the surface is slowed down to zero velocity and the fluid layer reaches equilibrium with the surface and reaches temperature T_s. These particles in turn heat up the next layer and a tmperature gradient develops. At a distance y, the temperature gradient becomes negligibly small. **The distance y at which the ratio $[(T_s - T)/(T_s - T_\infty)] = 0.99$ is defined as thermal boundary layer thickness δ_t.** The flow can now be considered to consist of two regions.

A thin layer of thickness δ_t in which the temperature gradient is large and the remaining flow where the temperature gradient is negligible. As the distance from the leading edge increases the effect of heat penetration, increases and the thermal boundary layer thickness increases. The heat flow from the surface to the fluid can be calculated using the temperature gradient at the surface. The temperature gradient is influenced by the nature of free stream flow. The development of the thermal boundary layer is shown in Fig.



The thermal and velocity boundary layers will not be identical except in a case where $Pr = 1$. Additional influencing factors change the thickness of the thermal boundary layer as compared to the thickness of the velocity boundary layer at any location. Note that both boundary layers exist together. Similar development of boundary layer is encountered in convective mass transfer also.

LAMINAR AND TURBULENT FLOW

The formation of the boundary layer starts at the leading edge. In the starting region the flow is well ordered. The streamlines along which particles move is regular. The velocity at any point remains steady. This type of flow is defined as laminar flow. There is no macroscopic mixing between layers. The momentum or heat transfer is mainly at the molecular diffusion level. After some distance in the flow, macroscopic mixing is found to occur. Large particles of fluid is found to move from one layer to another. The motion of particles become irregular. The velocity at any location varies with respect to a mean value. The flow is said to be turbulent. Due to the mixing the boundary layer thickness is larger. The energy flow rate is also higher. The velocity and temperature profiles are flatter, but the gradient at the surface is steeper due to the same reason. This variation is shown in Fig.

$$Re_x = \rho\, u_\infty\, x/\mu \quad \text{or} \quad u_\infty\, x/\nu$$

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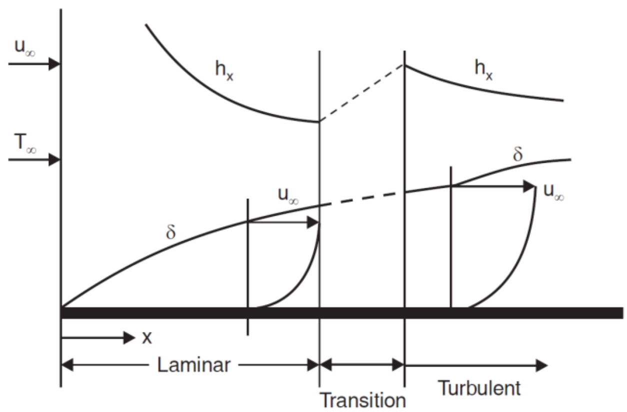


The changeover does not occur at a sharp location. However for calculations some location has to be taken as the change over point. In the velocity boundary layer, this transition is determined by a dimensionless group, Reynolds number-defined for flow over a plate by the equation
For flow in a tube or across a tube or sphere it is given by the equation.

$$Re = \rho\, u_\infty D/\mu \quad \text{or} \quad u_\infty D/\nu$$

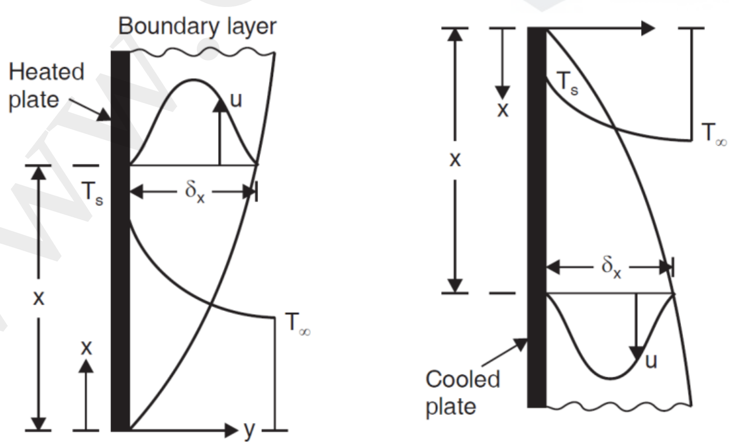
The grouping represents the ratio of inertia and viscous forces. Up to a point the inertia forces keep the flow in order and laminar flow exists. When the viscous forces begin to predominate, movement of particles begin to be more random and turbulence prevails. The **transition Reynolds number** for flow over a flat plate depends on many factors and may be anywhere from 10^5 to 3×10^6. Generally the value **is taken as** $\mathbf{5 \times 10^5}$ unless otherwise specified.

For **flow through tubes the transition value is 2300, unless otherwise specified.** In the quantitative estimation of heat flow, the correlation equations for the two regions are distinctly different and hence it becomes necessary first to establish whether the flow is laminar or turbulent. Turbulent flow is more complex and exact analytical solutions are difficult to obtain. Analogical model is used to obtain solutions.

FORCED AND FREE CONVECTION

When heat transfer occurs between a fluid and a surface, if the flow is caused by a fan, blower or pump or a forcing jet, the process is called **forced convection.** The boundary layer development is similar to the descriptions in the previous section. When the temperature of a surface immersed in a stagnant fluid is higher than that of the fluid, the layers near the surface get heated and the density decreases in these layers.

The surrounding denser fluid exerts buoyant forces causing fluid to flow upwards near the surface. This process is called **free convection flow** and heating is limited to a layer, as shown in Fig.The heat transfer rate will be lower as the velocities and temperature gradients are lower. If the surface temperature is lower, the flow will be in the downward direction.



FLOW OVER FLAT PLATES

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In this chapter additional practical correlations are introduced. Though several types of boundary conditions may exist, these can be approximated to three basic types. These are (i) constant wall temperature, (as may be obtained in evaporation, condensation etc., phase change at a specified pressure) (ii) constant heat flux, as may be obtained by electrical strip type of heating and (iii) flow with neither of these quantities remaining constant, as when two fluids may be flowing on either side of the plate.

Distinct correlations are available for constant wall temperature and constant heat flux. But for the third case it may be necessary to approximate to one of the above two cases

Laminar flow:

The condition is that the Reynolds number should be less than 5×10^5 or as may be stated otherwise. For the condition that the plate temperature is constant the following equations are valid with fluid property values taken at the film temperature.

Hydrodynamic boundary layer thickness

$$\delta_x = 5x/Re_x^{0.5}$$

Thermal boundary layer thickness

$$\delta_{tx} = \delta_x\, Pr^{-0.333}$$

Displacement thickness and Momentum thickness are not directly used in heat transfer calculations. However, it is desirable to be aware of these concepts.

Displacement thickness is the difference between the boundary layer thickness and the thickness with uniform velocity equal to free stream velocity in which the flow will be the same as in the boundary layer. For laminar flow displacement thickness is defined as

$$\int_0^{\delta}\left(1-\frac{u}{u_\infty}\right)dy$$

$$\delta_d = \delta_x/3$$

Momentum thickness is the difference between the boundary layer thickness and the layer thickness which at the free stream velocity will have the same momentum as in the boundary layer.

Momentum thickness δ_m in the laminar region is defined by

$$\int_0^{\delta}\left[\frac{u}{u_\infty}-\left(\frac{u}{u_\infty}\right)^2\right]dy$$

$$\delta_m = \delta_x/7$$

Friction coefficient defined as $\tau s/(\rho u_\infty^2/2)$ is given by

$$C_{fx} = 0.664\, Re_x^{-0.5}$$

The average value of C_f in the laminar region for a length L from leading edge is given by

$$C_{fL} = 1.328\, Re_L^{-0.5}$$

The value of **local Nusselt number** is given by

$$Nu_x = 0.332\, Re_x^{0.5}\, Pr^{0.33}$$

$$\overline{N}u_L = 2NuL = 0.664\, Re_L^{0.5}\, Pr^{1/3}$$

TURBULENT FLOW

$Re_x > 5 \times 10^5$ are as specified. In flow over flat plate, the flow is initially laminar and after some distance turns turbulent, the value of Reynolds number at this point being near 5×10^5. However, there are circumstances under which the flow turns turbulent at a very short distance, due to higher velocities or due to disturbances, roughness etc. The critical Reynolds number in these cases is low and has to be specified. In the turbulent region the velocity **boundary layer thickness** is given by

$$\delta_x = 0.381\, x \times Re_x^{-0.2}$$
$$\delta_t \approx \delta_x$$

The **displacement and momentum thickness** are much thinner. The **displacement thickness is**

$$\delta_d = \delta_x/8$$

Momentum thickness is

$$\delta_m = (7/72)\, \delta_x$$

The local friction coefficient defined as $\tau_w/(\rho u_\infty^2/2)$ is given for the range Re_x from 5×10^5 to 10^7 by

$$C_{fx} = 0.0592\, Re_x^{-0.2}$$

For higher values of Re in the range 10^7 to 10^9

$$C_{fx} = 0.37\, [\log_{10} Re_x]^{-2.584}$$

The **local Nusselt number** is given by

$$Nu_x = 0.0296\, Re_x^{0.8} Pr^{0.33}$$

The **average Nusselt number** is given by

$$\overline{Nu} = 0.037\, Re^{0.8}\, Pr^{0.33}$$

The assumption that the flow is turbulent althrough (from start) may not be acceptable in many situations. **The average values are now found by integrating the local values up to the location where Re = 5×10^5 using laminar flow relationship and then integrating the local value beyond this point using** the turbulent flow relationship and then taking the average. This leads to the following relationship for constant wall temperature

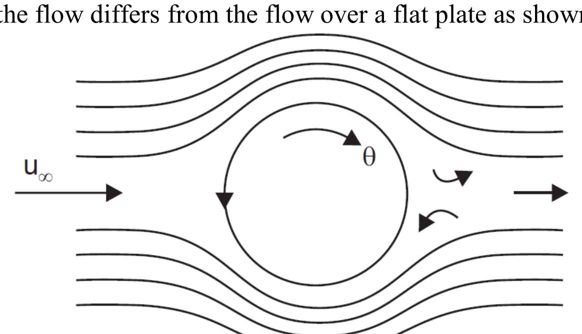
$$\delta_x = 0.381x \times Re_x^{-0.2} - 10256x \times Re_x^{-1.0}$$
$$C_{fL} = 0.074\, Re_L^{0.2} - 1742\, Re_L^{-1.0}$$

A more general relationship can be used for other values of critical Reynolds number.

$$Nu_x = Pr^{0.333}\, [0.037 Re_L^{0.8} - 871]$$

FLOW ACROSS CYLINDERS

The other type of flow over surfaces is **flow across cylinders** often met with in heat exchangers and hot or cold pipe lines in the open. An important difference is the velocity distribution along the flow. The obstruction by the cylinder causes a closing up of the streamlines and an increase in pressure at the stagnation point. The velocity distribution at various locations in the flow differs from the flow over a flat plate as shown in Fig.



That the averaging out the convection coefficient is difficult. The experimental values measured by various researchers plotted using common parameters Re_D and Nu_D (log log plot) is shown in Fig. 8.4. It can be seen that scatter is high at certain regions and several separate straight line correlations are possible for various ranges. Some researchers have limited their correlations for specific ranges and specific fluids. Thus a number of correlations are available and are listed below.

A very widely used correlation is of the form

$$Nu_D = CRc_D^m \, Pr^{0.333}$$

Where C and *m* are tabulated below. The applicability of this correlation for very low values of Prandtl number is doubtful. The length parameter in Nusselt number is diameter *D* and Nusselt number is referred as Nu_D.

The properties are to be evaluated at the film temperature

Re_D	C	m
0.4–4.0	0.989	0.330
4–40.0	0.91	0.385
40–4000	0.683	0.466
4000–40000	0.193	0.618
40000–400000	0.0266	0.805

FLOW ACROSS SPHERES

There are a number of applications for flow over spheres in industrial processes. As in the case of flow across cylinders, the flow development has a great influence on heat transfer. Various correlations have been obtained from experimental measurements and these are listed in the following paras.

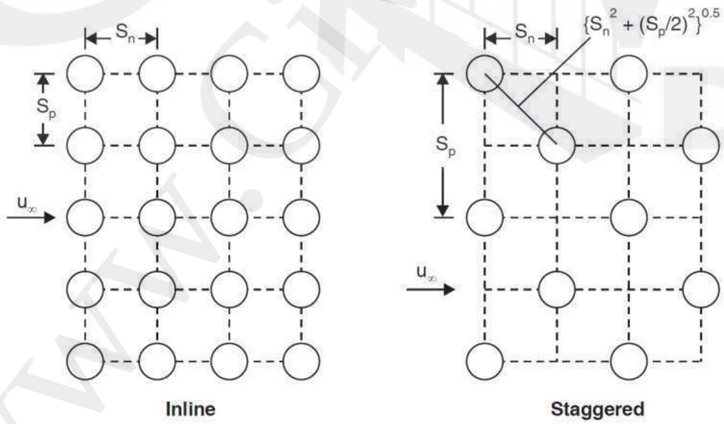
The following three relations are useful for air with $Pr = 0.71$ (1954)

$$Nu = 0.37\, Re^{0.6} \quad 17 < Re < 7000$$

With Properties evaluated at film temperature.

FLOW ACROSS BANK OF TUBES

In most heat exchangers in use, tube bundles are used with one fluid flowing across tube bundles. First it is necessary to define certain terms before discussing heat transfer calculations. Two types of tube arrangement are possible. (*i*) in line and (*ii*) staggered. The distance between tube centers is known as pitch. The pitch along the flow is known as (S_n) and the pitch in the perpendicular direction is called (S_p). These are shown in Fig.



Due to the obstruction caused by the tubes, the velocity near the tube increases and this increased value has to be used in the calculation of Reynolds number. In the case of in line the actual velocity near the tubes

$$V_{max} = [S_p/(S_p - D)]u_\infty$$

In the case of staggered arrangement the larger of the value given by 8.53 and 8.54 is to be used

$$V_{max} = [S_p/2\,(S_D - D)]u_\infty$$

where

$$S_D = \left[S_n^{\ 2} + \left(\frac{S_p}{2} \right)^2 \right]^{0.5}$$

This is because of the larger obstruction possible in the staggered arrangement.

For number of rows of tubes of 10 or more

$$Nu = 1.33\ C\ Re^n .\ Pr^{0.33}$$

$$N \geq 10,\ 2000 < Re < 40000$$

Reynolds number to be calculated based on Vmax. The property values should be at T_f. The value of C and n are tabulated below in Table 8.1. For larger values of S_p/D, tubes can be considered as individual tubes rather than tube bank

S_p/D	1.25 C	n	1.5 C	n	2.0 C	n	3.0 C	n
In line								
1.25	0.348	0.592	0.275	0.608	0.100	0.704	0.0633	0.752
1.5	0.367	0.586	0.250	0.620	0.101	0.702	0.0678	0.744
2.0	0.418	0.570	0.299	0.602	0.229	0.632	0.1980	0.648
3.0	0.290	0.601	0.357	0.584	0.374	0.581	0.2860	0.608
Staggered								
0.6	—	—	—	—	—	—	0.213	0.636
0.9	—	—	—	—	0.446	0.571	0.401	0.581
1.0	—	—	0.497	0.558	—	—	—	—
1.125	—	—	—	—	0.478	0.565	0.518	0.560
1.25	0.518	0.556	0.505	0.554	0.519	0.556	0.552	0.562
1.5	0.451	0.568	0.460	0.562	0.452	0.568	0.488	0.568
2.0	0.404	0.572	0.416	0.568	0.482	0.556	0.449	0.570
0.3	0.310	0.592	0.356	0.580	0.440	0.562	0.421	0.574

FORCED CONVECTION

The internal flow configuration is the most convenient and popularly used geometry for heating or cooling of fluids in various thermal and chemical processes. There are basic differences in the development of boundary layer between the external flow geometry and internal flow geometry. In the case of internal flow, the fluid is confined by a surface, and the boundary layer after some distance cannot develop further. **This region is called entrance region.** The region beyond this point is known as **fully developed region.** Another important difference is that the flow does not change over at a location from laminar to turbulent conditions, but is **laminar or turbulent from the start**, depending upon the value of Reynolds number (based on diameter) being greater or less than about 2300.

HYDRODYNAMIC BOUNDARY LAYER DEVELOPMENT

The development of hydrodynamic boundary layer in a pipe, together with velocity distributions at various sections for laminar and turbulent flows are shown in Fig. for the shape of the profile in laminar flow given by

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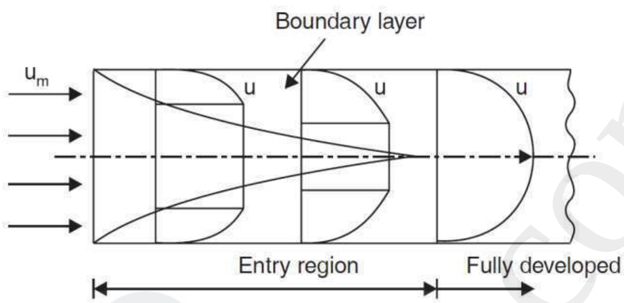
$$\frac{u_r}{u_{\max}} = 1 - \left(\frac{r}{R}\right)^2$$

where $u_{\max}$ is the velocity at the centreline.

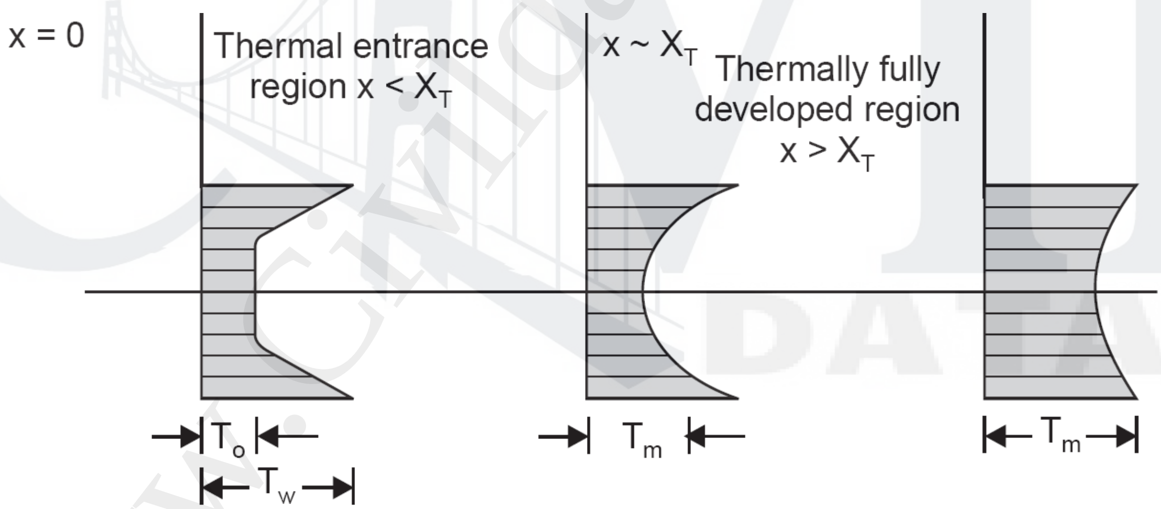
$$u_{\max} = 2u_m.$$



The velocity distribution beyond the entry region will remain invariant. But the actual distribution will be affected by the fluid property variation during heating or cooling. If heating or cooling causes reduction in the viscosity near the wall, the velocity profile flattens out as compared to isothermal flow. If viscosity increases, then the velocity near the wall will be reduced further and the velocity distribution will be more peaked. This is shown in Fig.Such distortion will affect the heat transfer correlations to some extent

THERMAL BOUNDARY LAYER

The development of thermal boundary layer is somewhat similar to the development of velocity profile. As shown in fig.



i) As the temperature increases continuously the direct plot of temperature will vary with x location. However the plot of dimensionless temperature ratio will provide a constant profile in the fully developed region. The bulk mean temperature T_m varies along the length as heat is added/removed along the length. The ratio $(T_w - T_r)/(T_w - T_m)$ remains constant along the x direction in the fully developed flow. T_r is the temperature at radius r and T_m is the bulk mean temperature.

ii) The length of entry region will be different as compared to the velocity boundary development.

iii) Boundary conditions are also different–constant wall temperature and constant heat flux.

iv) The development of both boundary layers may be from entry or heating may start after the hydrodynamic boundary layer is fully developed.

These are in addition to the laminar and turbulent flow conditions. Thus it is not possible to arrive at a limited number of correlations for convection coefficient.

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In the case of internal flow, there are four different regions of flow namely (*i*) Laminar entry region (*ii*) Laminar fully developed flow (*iii*) Turbulent entry region and (*iv*) Turbulent fully developed region.

LAMINAR FLOW

Constant Wall Temperature: ($Re_d < 2300$) Reynolds number is defined as below

$$Re = Du_m/\nu = 4G/\pi D\mu$$

It is to be noted that **for long tubes Nusselt number** does not vary with length and **is constant** as given by equation

$$\mathbf{Nu = 3.66}$$

TURBULENT FLOW

The development of boundary layer is similar except that the **entry region** length is between **10 to 60 times the diameter.** The convective heat transfer coefficient has a higher value as compared to laminar flow.
The friction factor for smooth pipes is given by eqn.

$$f = 0.184\ Re^{-0.2}$$

$$f = [0.7 \ln Re - 1.64]^{-2}$$

$$f = 4[1.58 \ln Re - 3.28]^{-2}$$

The more popular correlation for fully developed flow in smooth tubes is due to **Dittus and Boelter (1930) (modified Colburn)**

$$Nu = 0.023\ Re^{0.8}\ Pr^n$$

n = 0.3 for cooling and 0.4 for heating of fluids

NATURAL CONVECTION

When a surface is maintained in still fluid at a temperature higher or lower than that of the fluid, a layer of fluid adjacent to the surface gets heated up or cooled. A density difference is created between this layer and the still fluid surrounding it. The density difference introduces a buoyant force causing flow of the fluid near the surface. Heat transfer under such conditions is known as natural or free convection. Usually a thin layer of flowing fluid forms over the surface. The fluid beyond this layer is essentially still, and is at a constant temperature of T_∞.

The flow velocities encountered in free convection is lower compared to flow velocities in forced convection. Consequently the value of convection coefficient is lower, generally by one order of magnitude. Hence for a given rate of heat transfer larger area will be required. As there is no need for additional devices to force the fluid, this mode is used for heat transfer in simple devices as well as for devices which have to be left unattended for long periods.

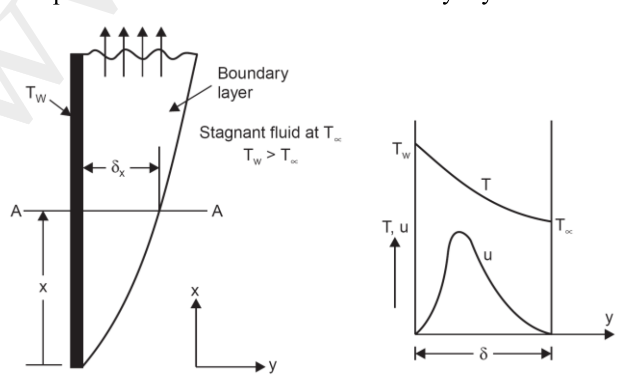
The heat transfer rate is calculated using the general convection equation given below

$$Q = h\ A(T_w - T_\infty)$$

Q—heat transfer in W, h—convection coefficient – W/m^{2}K.
A—area in m^2, T_w—surface temperature
T_∞—fluid temperature at distances well removed from the surface (here the stagnant fluid temperature)

BASIC NATURE OF FLOW UNDER NATURAL CONVECTION CONDITIONS

The layer of fluid near the surface gets heated or cooled depending on the temperature of the solid surface. A density difference is created between the fluid near the surface and the stagnant fluid. This causes as in a chimney a flow over the surface. Similar to forced convection a thin boundary layer is thus formed over the surface. Inertial, viscous and buoyant forces are predominant in this layer. Temperature and velocity gradients exist only in this layer. The velocity and temperature distributions in the boundary layer near a hot vertical surface are shown in Fig



The velocity is zero at the surface and also at the edge of the boundary layer. As in the case of forced convection the temperature gradient at the surface is used in the determination of heat flow (heat is transferred at the surface by conduction mode).

$$h = -k \left. \frac{\partial \left(\frac{T - T_\infty}{T_w - T_\infty} \right)}{\partial \left(\frac{y}{\delta} \right)} \right|_{y=0} .$$

The temperature gradient at the surface can be evaluated using either the solution of differential equations or by assumed velocity and temperature profiles in the case of integral method of analysis. This leads to the identification of Nusselt number and Prand the number as in the case of forced convection. These numbers have the same physical significance as in forced convection.

The buoyant forces play an important role in this case, in addition to the viscous and inertia forces encountered in forced convection. This leads to the identification of a new dimensionless group called Grashof number

$$Gr = \frac{g\beta\,(T_w - T_\infty)\,.\,L^3}{\nu^2}$$

where β is the coefficient of cubical expansion having a dimension of 1/Temperature. For gases $\beta = 1/T$ where T is in K. For liquids β can be calculated if variation of density with temperature at constant pressure is known. The other symbols carry the usual meaning.

The physical significance of this number is given by

$$\mathbf{Gr} = \frac{\textbf{Inertia force}}{\textbf{Viscous force}} \cdot \frac{\textbf{Buoyant force}}{\textbf{Viscous force}}$$

The flow turns turbulent for value of $Gr\,Pr > 10^9$. As in forced convection the microscopic nature of flow and convection correlations are distinctly different in the laminar and turbulent regions.

CONSTANT HEAT FLUX CONDITION—VERTICAL SURFACES

Here the value of wall temperature is not known. So ΔT is unspecified for the calculation of Grashof number. Though a trial solution can be attempted, it is found easier to eliminate ΔT by q which is known in most cases. This is done by multiplying Grashof number by Nusselt number and equating $q = h\Delta T$.

This product is known as **modified Grashof number, Gr***

$$Gr_x^* = Gr_x\,Nu_x = \frac{g\,\beta\,\Delta t\,x^3}{\nu^2} \cdot \frac{hx}{k} = \frac{g\,\beta\,q\,x^4}{k\,\nu^2}$$

The correlation for laminar range is given by

$$Nu_x = 0.60\,[Gr_x^*\,Pr]^{0.2}$$

$$10^5 < Gr^* < 10^{11}$$

Constant Heat Flux, Horizontal Surfaces

For horizontal surfaces, the correlations are given in table 10.1 for constant wall temperature conditions. For constant heat flux conditions the following correlations are available. The property values except β in these cases are to be evaluated at T_∞.

The characteristic length L = Area/ perimeter generally. For circle 0.9 D and for Rectangle $(L + W)/2$

For heated face facing upwards or cooled face facing downwards: laminar conditions

$$Nu = 0.54\,(Gr\,Pr)^{1/4},\ Gr\,Pr \to 10^5 \text{ to } 2 \times 10^7$$

$$\overline{Nu} = 0.14\,(Gr\,Pr)^{1/3}$$

$$Gr\,Pr \to 2 \times 10^7 \text{ to } 3 \times 10^{10}.$$

For heated surface facing downward

$$Nu = 0.27\,(Gr\,Pr)^{1/4}$$

$$Gr\,Pr \to 3 \times 10^5 \text{ to } 3 \times 10^{10}$$

$$Nu = 0.58\,(Gr\,Pr)^{0.2}$$

$$10^6 < Gr\,Pr < 10^{11}.$$

HORIZONTAL CYLINDERS

A more general correlation as compared to the ones given in table for the laminar range, $Gr\,Pr < 10^9$ the correlation is

$$Nu = 0.36 + \frac{0.518\,(Gr\;Pr)^{0.25}}{[1 + (0.559/Pr)^{9/16}]^{4/9}}$$

For spheres:

The general correlation is

$$Nu = 2 + 0.43\,(Gr\;Pr)^{0.25}$$

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$$\frac{T - T_\infty}{T_o - T_\infty} = e^{\left[\frac{-h}{c\,L_c\rho}\times\tau\right]}$$

$$\frac{368 - 288}{536 - 288} = e^{\left[\frac{58}{900\times0.0262\times2700}\times\tau\right]}$$

$$\tau = 1355.4 \text{ sec}$$

Unit II

May 2012

1. Air at 25 °C flows past a flat plate at 2.5 m/s. the plate measures 600 mm X 300 mm and is maintained at a uniform temperature at 95 °C. Calculate the heat loss from the plate, if the air flows parallel to the 600 mm side. How would this heat loss be affected if the flow of air is made parallel to the 300 mm side.

Given:

Forced convection (air)

Flat plate

T_∞ =25° C

U = 25 m/s

T_w = 95 °C

L = 600 mm = 600 X 10^{-3} m

W = 300 mm = 300 X 10^{-3} m

Find

(i) Q if air flows parallel to 600 mm side

(ii) Q if air flows parallel to 300 mm side and % of heat loss.

Solution:

$$T_f = \frac{T_W - T_\infty}{2} = \frac{95-25}{2} = \frac{120}{2} = 60^o\ C$$

Take properties of air at T_f = 60° C from H.M.T data book (page no 34)

Pr = 0.696

γ = 1897 x 10^{-6} m²/s

k = 0.02896

$$Re = \frac{UL}{\gamma} = \frac{2.5\times0.6}{18.97\times10^{-6}}$$

$$Re = 7.91\times10^4 < 5\times10^5$$

This flow is laminar.

From H.M.T data book

$$Nu_x = 0.332 {Re_x}^{0.5}\, pr^{0.333}$$

(or) $$Nu_L = 0.332 {Re_L}^{0.5}\, pr^{0.333}$$

.

$= 0.332 \times (7.91 \times 10^{4})^{0.5} (0.696)^{0.333}$

$Nu_L = 82.76$

$$\overline{N_u} = 2Nu_L = 2 \times 82.76$$

$$\overline{N_u} = 165.52$$

$$\overline{N_u} = \frac{\bar{h}L}{k}$$

$$h\ (or)\bar{h} = \frac{\overline{N_u}k}{L} = \frac{165.52\ \times 0.02896}{0.6}$$

$$h\ (or)\bar{h} = 7.989\ W/m^2K$$

$$Q = \ \bar{h}A(\Delta T)(or)h(w.L)(T_w - T_\infty)$$

$$Q_1 = 7.989\ (0.6 \times 0.3)(95 - 25)$$

$Q_1 = 100.66$ W

(iii) If L = 0.3 m and W = 0.6 m (parallel to 300 mm side)

$$R_e = \frac{UL}{\gamma} = \frac{2.5 \times 0.3}{18.97\ \times 10^{-6}} = 3.95 \times 10^4$$

$$R_e = 3.95 \times 10^4\ < 5 \times 10^5$$

$$the\ flow\ is\ laminar$$

From H.M.T Data book

$$Nu_x = 0.332x^{0.5}Pr^{0.333}$$

$$(or)Nu_L = 0.332{Re_L}^{0.5}Pr^{0.333}$$

$$Nu_L = 0.332(3.95 \times 10^4)^{0.5}(0.696)^{0.333}$$

$Nu_L = 58.48$

$$\overline{Nu} = 2Nu_L = 2 \times 58.48 = 116.96$$

$$\overline{N_u} = \frac{\bar{h}L}{k}$$

$$\bar{h} = \frac{\overline{N_u}k}{L} = \frac{116.96\ \times 0.02896}{0.3}$$

$$h\ (or)\bar{h} = 11.29\ W/m^2K$$

$$Q_2 = hA(\Delta T)(or)h(w.L)(T_w - T_\infty)$$

$$Q_2 = 11.29\ (0.6 \times 0.3)(95 - 25)$$

$Q_2 = 142.25$W

$\%\ \text{heat loss} = \frac{Q_2 - Q_1}{Q_1} \times 100$

$= \frac{142.25 - 100.66}{100.66} \times 100$

$$\%\ \text{heat loss} = \ 41.32\%$$

2. When 0.6 kg of water per minute is passed through a tube of 2 cm diameter, it is found to be heated from 20°C to 60°C. the heating is achieved by condensing steam on the surface of the tube and subsequently the surface temperature of the tube is maintained at 90° C. Determine the length of the tube required for fully developed flow.

Given:

Mass, m = 0.6kg/min = 0.6/60 kg/s

= 0.01 kg/s

Diameter, D = 2 cm = 0.02m

Inlet temperature, T_{mi} = 20° C

Outlet temperature, T_{mo} = 60°C

Tube surface temperature , T_w= 90°C

To find

length of the tube,(L).

Solution:

Bulk mean temperature = $T_m = \frac{T_{mi}+T_{mo}}{2} = \frac{20+60}{2} = 40^oC$

Properties of water at 40°C:

(From H.M.T Data book, page no 22, sixth edition)

P = 995 kg/m^3

V = 0.657x10^{-6} m^2/s

Pr = 4.340

K = 0.628W/mK

C_p = 4178J/kgK

Mass flow rate, $\dot{m} = \rho AU$

$$U = \frac{\dot{m}}{\rho A}$$

$$U = \frac{0.01}{995 \times \frac{\pi}{4}(0.02)^2}$$

$$\text{velocity}, U = 0.031 m/s$$

Let us first determine the type of flow

$$Re = \frac{UD}{v} = \frac{0.031 \times 0.02}{0.657x10^{-6}}$$

$$Re = 943.6$$

Since Re < 2300, the flow is laminar.

For laminar flow,

Nusselt Number, Nu = 3.66

We know that

$$Nu = \frac{hD}{k}$$

$$3.66 = \frac{h \times 0.02}{0.628}$$

$$h = 114.9\ W/m^2K$$

Heat transfer, $Q = mc_p \Delta T$

$$Q = mc_p(T_{mo} - T_{mi})$$

$$= 0.01 \times 4178 \times (60 - 20)$$

Q = 1671.2 W

We know that $Q = hA\Delta T$

$$Q = h \times \pi \times D \times L \times (T_w - T_m)$$

$$1671.2 = 114.9 \times \pi \times 0.02 \times L \times (90 - 40)$$

Length of tube , L = 4.62m

November 2012

3. Water is to be boiled at atmospheric pressure in a polished copper pan by means of an electric heater. The diameter of the pan is 0.38 m and is kept at 115° C. calculate the following

1. Surface heat flux
2. Power required to boil the water
3. Rate of evaporation
4. Critical heat flux

Given:

Diameter, d = 0.38 m

Surface temperature, T_w = 115°C

To find

1. Q/A
2. P
3. $\dot{m}$
4. $(Q/A)_{max}$

Solution:

We know that, Saturation temperature of water is 100° C

i.e. T_{sat} = 100°C

Properties of water at 100°C:

(From H.M.T Data book, page no 22, sixth edition)

Density, ρ_l = 961 kg/m^3

Kinematic viscosity, v = 0.293x10^{-6} m^2/s

Prandtl Number, Pr = 1.740

Specific heat, C_{pl} = 4216 J/kgK

Dynamic viscosity, $\mu_l = \rho_l \times v = 961 \times 0.293 \times 10^{-6}$

= 281.57 X 10^{-6} Ns/m^2

From Steam table [R.S khurmi steam table]

At 100° C

Enthalpy of evaporation, h_{fg} = 2256.9 kJ/kg.

h_{fg} = 2256.9 x 10^3 J/kg

Specific volume of vapour, v_g = 1.673 m^3/kg

Density of vapour, $\rho_v = \frac{1}{v_g}$

$$\rho_v = \frac{1}{1.673}$$

$$\rho_v = 0.597\ kg/m^3$$

$$\Delta T = excess\ temperature = T_w - T_{sat} = 115^o - 100^o = 15^oC$$

$$\Delta T = 15^oC < 50^oC. So\ this\ is\ Nucleate\ pool\ boiling\ process.$$

Power required to boil the water,

For *Nucleate pool boiling*

Heat flux, $\frac{Q}{A} = \mu_l \times h_{fg}\left[\frac{g\times(\rho_l-\rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{Cpl\times\Delta T}{C_{sf}\times h_{fg}{P_r}^n}\right]^3$ ….(1)

(From H.M.T Data book)

Where $\sigma = surface\ tension\ for\ liquid\ vapour\ interface$

At 100°C

$\sigma = 0.0588\ N/m$ (From H.M.T Data book)

For water – copper → C_{sf} = surface fluid constant = 0.013

N = 1 for water (From H.M.T Data book)

Substitute

$\mu_l, h_{fg}, \rho_l, \rho_v, \sigma, Cpl, \Delta T, C_{sf}, n, h_{fg}, p_r$ values in eqn (1)

$$\frac{Q}{A} = 281.57 \times 10^{-6} \times 2256.9 \times 10^{3\times}\left[\frac{9.81\times(961-0.597)}{0.0588}\right]^{0.5}$$

$$\times\left[\frac{4216\times 15}{0.013\times 2256.9\times 10^3\times(1.74)^1}\right]^3$$

$$Surface\ Heat\ flux\ , \frac{Q}{A} = 4.83 \times 10^5 W/m^2$$

$$Heat\ transfer, , Q = 4.83 \times 10^5 \times A$$

$$= 4.83 \times 10^5 \times \frac{\pi}{4} d^2$$

$$= 4.83 \times 10^5 \times \frac{\pi}{4} (0.38)^2$$

$$Q = 54.7 \text{ x}10^3 \text{ W}$$

$$Q = 54.7 \text{ x}10^3 = P$$

$$\text{Power} = 54.7 \text{ x}10^3 \text{ W}$$

2. Rate of evaporation, ($\dot{m}$)

We know that,

$$\text{Heat transferred, } Q = \dot{m} \times h_{fg}$$

$$\dot{m} = \frac{Q}{h_{fg}} = \frac{54.7 \times 10^3}{2256.9 \times 10^3}$$

$$\dot{m} = 0.024\ kg/s$$

3. Critical heat flux, (Q/A)

For Nucleate pool boiling, critical heat flux,

$$\frac{Q}{A} = 0.18 h_{fg} \times \rho_v \left[\frac{\sigma \times g \times (\rho_l - \rho_v)}{{\rho_v}^2}\right]^{0.25}$$

(From H.M.T Data book)

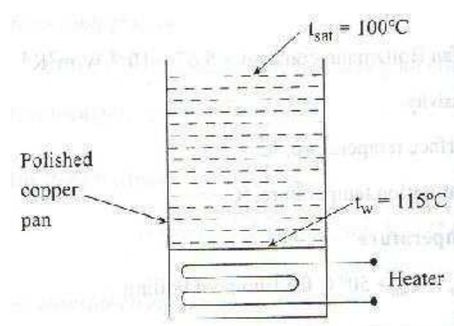
$$= 0.18 \times 2256.9 \times 10^3 \times 0.597 \times \left[\frac{0.0588 \times 9.81 \times (961 - 0.597)}{(0.597)^2}\right]^{0.25}$$

$$Critical\ heat\ flux\ , q = \frac{Q}{A} = 1.52 \times 10^6 W/m^2$$

May 2013

4. A thin 80 cm long and 8 cm wide horizontal plate is maintained at a temperature of 130°C in large tank full of water at 70°C. Estimate the rate of heat input into the plate necessary to maintain the temperature of 130°C.

Given:



Horizontal plate length, L = 80 cm = 0.08m

Wide, W = 8 cm = 0.08 m,

Plate temperature, T_w = 130°C

Fluid temperature, T_∞ = 70°C

To find:

Rate of heat input into the plate,Q.

Solution:

Flim temperature, $T_f = \frac{T_w - T_\infty}{2} = \frac{130+70}{2} = 100^o\ C$

Properties of water at 100°C:

(From H.M.T Data book, page no 22, sixth edition)

ρ= 961 kg/m^3

v = 0.293x10^{-6} m^2/s

Pr = 1.740

k = 0.6804W/mK

$\beta_{water} = 0.76 \times 10^{-3} K^{-1}$

(From H.M.T Data book, page no 30, sixth edition)

We know that,

$$Grashof\ number, Gr = \frac{g \times \beta \times {L_c}^3 \times \Delta T}{V^2}$$

For horizontal plate:

$$\mathrm{L_c} = \text{Characteristic length} = \frac{W}{2}$$

$$\mathrm{L_c} = \frac{0.08}{2}$$

$$\mathrm{L_c} = 0.04\ m$$

$$Grashof\ number, Gr = \frac{9.81 \times 0.76 \times 10^{-3} \times (0.04)^3 \times (130 - 70)}{(0.293 \times 10^{-6})^2}$$

$$Gr = 0.333 \times 10^9$$

$$GrPr = 0.333 \times 10^9 \times 1.740$$

$$GrPr = 0.580 \times 10^9$$

GrPr value is in between 8x10^6 and 10^{11}

i.e., 8x10^6 < GrPr<10^{11} So, for horizontal plate, upper surface heated,

Nusselt number, Nu = 0.15(GrPr)$^{0.333}$

(From H.M.T Data book, page no 136, sixth edition)

Nu = 0.15(0580 x 10^9)$^{0.333}$

Nu = 124.25

$$\text{Nusselt number, Nu} = \frac{\mathrm{h_u L_c}}{\mathrm{k}}$$

$$124.25 = \frac{\mathrm{h_u} \times 0.04}{0.6804}$$

$$\mathrm{h_u} = 2113.49\ \mathrm{W/m^2K}$$

Heat transfer coefficient for upper surface heated h_u = 2113.49 W/m^{2}K

For horizontal plate, Lower surface heated:

Nusselt number, Nu_l = 0.27(GrPr)$^{0.25}$

(From H.M.T Data book, page no 137, sixth edition)

$$= 0.27[0.580x10^9]^{0.25}$$

$$Nu_l = 42.06$$

We know that,

$$\text{Nusselt number}, Nu_l = \frac{h_l L_c}{k}$$

$$42.06 = \frac{h_l \times 0.04}{0.6804}$$

$$h_l = 715.44\ W/m^2K$$

Heat transfer coefficient for lower surface heated $h_l = 715.44\ W/m^2K$

$$\text{Total heat transfer, } Q = (h_u + h_l) A\,\Delta T$$

$$= (h_u + h_l) \times W \times L \times [T_w - T_\infty]$$

$$= (2113.49 + 715.44) \times (0.08 \times 0.8) \times [130 - 70]$$

$$Q = 10.86 \times 10^3 W$$

5. A vertical pipe 80 mm diameter and 2 m height is maintained at a constant temperature of 120 ° C. the pipe is surrounded by still atmospheric air at 30° . Find heat loss by natural convection.

Given:

Vertical pipe diameter D = 80 mm = 0.080m

Height (or) length L = 2 m

Surface temperature $T_S = 120\,^oC$

Air temperature $T_\infty = 30\,^oC$

To find

heat loss (Q)

Solution:

We know that

Flim temperature , $T_f = \frac{T_w + T_\infty}{2} = \frac{120+30}{2} = 75^o\ C$

Properties of water at 75 °C:

$\rho = 1.0145\ kg/m^3$

$\nu = 20.55\ x10^{-6}\ m^2/s$

Pr = 0.693

$k = 30.06 \times 10^{-3}\ W/mK$

We know

$$\beta = \frac{1}{T_f\, in\ K}$$

$$\beta = \frac{1}{75 + 273} = 2.87 \times 10^{-3} K^{-1}$$

We know

$$Grashof\ number, Gr = \frac{g \times \beta \times L^3 \times \Delta T}{V^2}$$

$$= \frac{9.81 \times 2.87 \times 10^{-3} \times (0.08)^3 \times (120 - 30)}{(20.55 \times 10^{-6})^2}$$

$$Gr = 4.80 \times 10^{10}$$

$$GrPr = 4.80 \times 10^{10} \times 0.693$$

$$GrPr = 3.32 \times 10^{10}$$

Since GrPr>10^9, flow is turbulent.

For turbulent flow, from HMT data book

$$Nu = 0.10(GrPr)^{0.333}$$

$$Nu = 0.10(3.32 \times 10^{10})^{0.333}$$

Nu = 318.8

We know that,

$$Nusselt\ number, Nu = \frac{hL}{k}$$

$$318.8 = \frac{h \times 2}{30.06 \times 10^{-3}}$$

$$Heat\ transfer\ cofficient, h = 4.79\ W/m^2K$$

Heat loss, $Q = h \times A \times \Delta T$

$$= h \times \pi \times D \times L \times (T_s - T_\infty)$$

$$= 4.79 \times \pi \times 0.080 \times 2 \times (120 - 30)$$

Q = 216.7 W

Heat loss Q = 216.7.

November 2012

6. Derive an equation for free convection by use of dimensional analysis.

$$Nu = C(Pr^n.Gr^m)$$

Assume, h = f {ρ, μ, Cp, k, Σ,(β, ΔT)}

The heat transfer co efficient in case of natural or free convection, depends upon the variables, V, ρ, k,μ, Cp and L, or D. Since the fluid circulation in free convection is owing to difference in density between the various fluids layers due to temperature gradient and not by external agency.

Thus heat transfer coefficient ‘h’ may be expressed as follows:

$h = f(\rho, L, \mu, c_p, k, \beta\, g\, \Delta T)$ ....(i)

$f_1\,(\rho, L, \mu, k, h, c_p, \beta\, g\, \Delta T)$ ....(ii)

[This parameter (β g ΔT) represents the buoyant force and has the dimensions of LT^{-2}.]

Total number of variables, n = 7

Fundamental dimensions in the problem are M,L,T, θ and hense m = 4

Number of dimensionless π- terms = (n –m) = 7-4= 3

The equation (ii) may be written as

$$f_1\,(\pi_1, \pi_2, \pi_3) = 3$$

We close ρ, L, μ and k as the core group (repeating variables) with unknown exponents. The groups to be formed are now represented as the following π groups.

$$\pi_1 = \rho^{a_1}.L^{b_1}.\mu^{c_1}.k^{d_1}.h$$

$$\pi_2 = \rho^{a_2}.L^{b_2}.\mu^{c_2}.k^{d_2}.c_p$$

$$\pi_3 = \rho^{a_3}.L^{b_3}.\mu^{c_3}.k^{d_3}.\beta\; g\; \Delta t$$

π_1- term:

$$M^O L^0 T^0 \theta^O = (ML^{-3})^{a_1}.(L)^{b_1}.(ML^{-1}T^{-1})^{c_1}.(MLT^{-3}\theta^{-1})^{d_1}.(ML^{-3}\theta^{-1})$$

Equating the exponents of M,L,T and θ respectively, we get

For M: $0 = a_1 + c_1 + d_1 + 1$

For L: $0 = -3a_1 + b_1 - c_1 + d_1$

For T: $0 = -c_1 + 3d_1 - 3$

For T: $\theta = -d_1 - 1$

Solving the above equations, we get

$a_1 = 0, b_1 = 1, c_1 = 0, d_1 = -1$

$$\pi_1 = Lk^{-1}h\;(or)\;\pi_{1=}\frac{hL}{k}$$

π_2- Term:

$$M^O L^0 T^0 \theta^O = (ML^{-3})^{a_2}.(L)^{b_2}.(ML^{-1}T^{-1})^{c_2}.(MLT^{-3}\theta^{-1})^{d_2}.(L^2T^{-2}\theta^{-1})$$

Equating the exponents of M, L, T and θ respectively, we get

For M: $0 = a_2 + c_2 + d_2$

For L: $0 = -3a_2 + b_2 - c_2 + d_2 + 2$

For T: $0 = -c_2 - 3d_2 - 2$

For T: $\theta = -d_2 - 1$

Solving the above equations, we get

$a_2 = 0, b_2 = 0, c_2 = 1, d_2 = -1$

$$\pi_2 = \mu.k^{-1}.c_p\;(or)\pi_2 = \frac{\mu c_p}{k}$$

π_3- Term:

$$M^O L^0 T^0 \theta^O = (ML^{-3})^{a_3}.(L)^{b_3}.(ML^{-1}T^{-1})^{c_3}.(MLT^{-3}\theta^{-1})^{d_3}.(LT^{-2})$$

Equating the exponents of M, L, T and θ respectively, we get

For M: $0 = a_3 + c_3 + d_{32}$

For L: $0 = -3a_3 + b_3 - c_3 + d_3 + 1$

For T: $0 = -c_3 - 3d_3 - 2$

For T: $\theta = -d_3$

Solving the above equations, we get

$a_3 = 2, b_3 = 3, c_3 = -2, d_3 = 0$

$$\pi_3 = \rho^2.L^3\mu^{-2}.(\beta g\Delta t)$$

or $\pi_3 = \dfrac{(\beta g\Delta t)\rho^2.L^3}{\mu^2} = \dfrac{(\beta g\Delta t)L^3}{v^2}$

or $Nu = \emptyset(Pr)(Gr)$

or $Nu = C(Pr)^n(Gr)^m(where\ Gr = Grashoff\ number)$

Here C, n and m are constants and may be evaluated experimentally.

Unit – 3 PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS

Unit – 3 Boiling and Condensation

Boiling

Boiling is a convection process involving a change in phase from liquid to vapour. Boiling may occur when a liquid is in contact with a surface maintained at a temperature higher than the saturation temperature of the liquid

Heat is transferred from the solid surface to the liquid according to the law

$$q = h\,(T_s - T_{sat}) \quad = h\,\Delta T_e$$

$\Delta T_e = (T_s - T_{sat})$ is known as the excess temperature

Application:-

Boiling process finds wide application as mentioned below

i) steam production (steam and nuclear power plant)
ii) Heat absorption in refrigeration an air conditioning system.
iii) Distillation, and refining of liquids
iv) Concentration, dehydration and drying of foods and materials.
v) Cooling of nuclear reactors and rocket motors.

The boiling heat transfer phenomenon may occur in the following forms:-

i) Pool boiling :-

In the case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

The pool boiling occurs in steam boilers involving natural convection.

(values are for water boiling at 100°C)

1. Purely convective region $\Delta T < 5°C$

2. Nucleate Boiling $5 < \Delta T < 50°C$

3. Unstable (nucleate ⇔ film) boiling $50°C < \Delta T < 200°C$

4. Stable film boiling $\Delta T > 200°C$.

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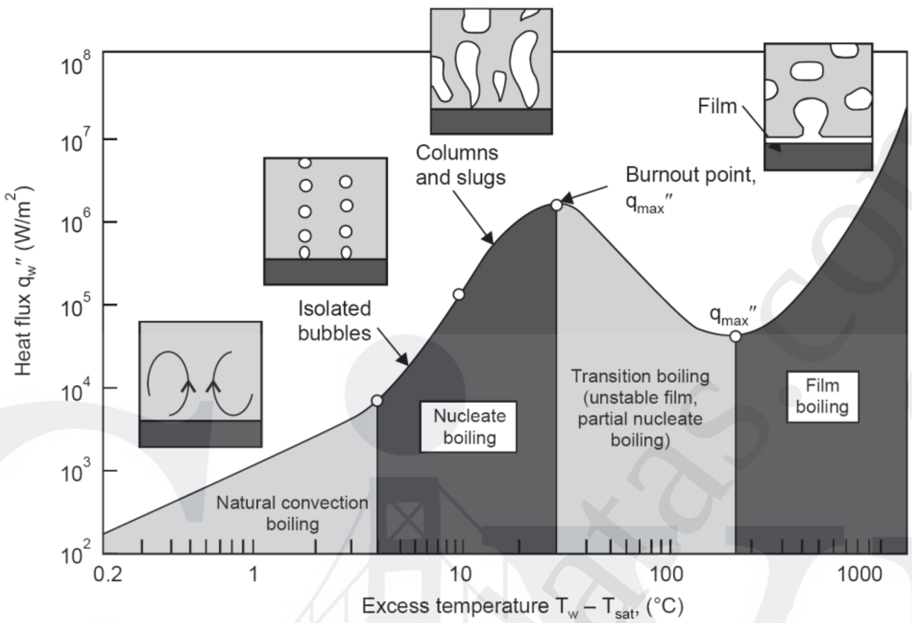
Note that the temperature values are indicative only.

Boiling regimes:-



Pool boiling process has following six regimes

i) interface evaporation (free convection) – Region I

ii) Nucleate boiling – Region II & III

iii) Film boiling – Region IV,V&VI

The different regimes of boiling are indicated in figure. This specific curve has been obtained from an electrically heated platinum wire submerged and measuring the surface heat flux (q_s).

(i) <u>Interface evaporation (Free convection) Region I</u>

Region I called the free convection zone, the excess temperature ΔT_e is very small ($\cong$ 5ºC). here the liquid near the surface is superheated and evaporation takes place at the liquid surface.

(ii) <u>Nucleate boiling – Region II & III</u>

As the excess temperature is further increased bubbles are formed more rapidly and rise to the surface of the liquid resulting in rapid evaporation. Nucleate boiling exists up to ΔT_e = 50 ºC. at the end

. .

of the nucleate boiling the heat flux is maximum. This heat flux, known as the critical heat flux (or) Burnout point.

(iii) Film boiling

Region –IV

Further increase of heat flux with increase in excess temperature up to region III. After region III heat flux decrease.This region the bubbles formation is very rapid , and the bubbles collapses and form a vapour film which covers the surface completely. With in the temperature range 50 ºC< ΔT_e < 150 ºC, condition oscillate between nucleate and film boiling this region is called unstable film boiling.

Region – V

With further increase in ΔT_e the vapour film is stabilized and the heating surface is completely covered by a vapour blanket and the heat flux is the lowest as shown in region V

Region – VI

This region heat flux slowly increases with the increase in excess temperature. The surface temperature required to maintained a stable film are high and under these conditions a sizeable amount of heat is lost by the surface due to radiation.

Flow boiling:-

Flow or forced convection boiling may occur when a liquid is forced through a pipe or over a surface which is maintained at a temperature higher than the saturation temperature of the liquid

Application :-

Design of steam generators for nuclear power plants and space power plants.

Boiling correlations:-

Nucleate pool boiling:-

Heat flux q_s = $\frac{Q}{A}$ = $\mu_l h_{fg}\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2}\left(\frac{C_{pl}\Delta T}{C_{sf}h_{fg}P_r^{1.7}}\right)^3$

μ_l = Liquid viscosity (Dynamic) Ns/m^2

h_{fg} = enthalpy of vaporization (J/kg)

ρ_l =Density of saturated liquid (kg/m^3)

ρ_v =Density of saturated vapour (kg/m^3)

σ = surface tension of the liquid vapour interface (N/m)

C_{pl} = Specific heat capacity at constant pressure

C_{sf} = Surface fluid constant

ΔT = (T_s - $T_{sat)}$

T_s = surface temperature ºC

T_{sat} = saturation temperature

g = acceleration due to gravity (9.81)

Critical heat flux :-

$$\frac{Q}{A} = 0.18h_{fg}\rho_v\left(\frac{\sigma g(\rho_l-\rho_v)}{\rho v^2}\right)^{1/4}$$

ΔT = (T_s - $T_{sat)}$) <50 ºC for nucleate pool boiling

Film pool boiling

ΔT = (T_s - $T_{sat)}$) > 50 ºC for film boiling

h = h_{conv}+0.75h_{rad}

$$h_{conv} = 0.62\left(\frac{K_v^{\,3}\rho_v(\rho_l-\rho_v)g(h_{fg}+0.4C_{p_v}\Delta T)}{\mu_v D\Delta T}\right)^{1/4}$$

where ,

K_V = Thermal conductivity of vapour W/mK

h_{fg} = Enthalpy of vaporization (J/kg)

ρ_l = Density of saturated liquid (kg/m^3)

ρ_v = Density of saturated vapour (kg/m^3)

μ_v = Dynamic viscosity of vapour Ns/m^2

C_{pv} = Specific heat of vapour at constant pressure (kJ/kgK)

D = Diameter ,m

ΔT = (T_s - T_{sat})

T_s = surface temperature ºC

T_{sat} = saturation temperature

g = acceleration due to gravity (9.81)

$$h_{rad} = \sigma\varepsilon\left(\frac{T_s^{\,4}-T_{sat}^{\,4}}{T_s-T_{sat}}\right)$$

Where,

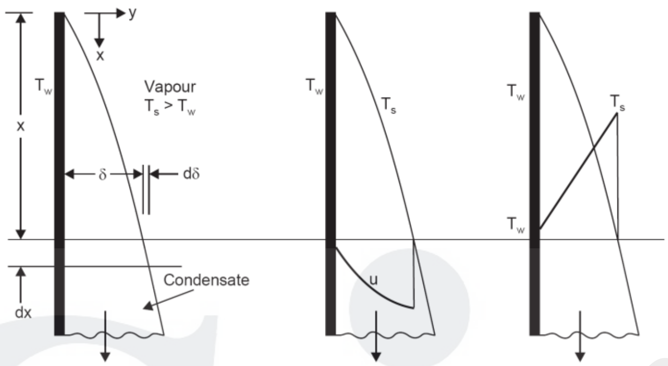
σ = Stefan boltzmann constant = 5.67x10^{-8} W/m^{2}K4

ε = Emissivity

T_s = surface temperature ºC

T_{sat} = saturation temperature

Condensation heat transfer:-



The condensation process is the reverse of boiling process. Whenever a saturation vapour comes in contact with a surface whose temperature is lower than the saturation temperature corresponding to the vapour pressure

The change of phase from vapour to liquid state is known as condensation.

Types of condensation

There are two types of condensation

i) Film wise condensation

ii) Drop wise condensation

Film wise condensation:-

In which the condensate wets the surface forming a continuous film which covers the entire surface.

Drop wise condensation:-

In which the vapour condenses into small liquid droplets of vapours sizes which fall down the surface in a random fashion.

Nusselt's analysis of film condensation :-

i) the plate is maintained at a uniform temperature (T_s), which is less than the saturation temperature (T_{sat}) of the vapour

ii) the condensate flow is laminar

iii) the fluid properties are constant

iv) the shear stress at the liqid vapour interface is negligible.

v) The acceleration of fluid with in the condensate layer is negligible

vi) The heat transfer across the condensate layer is by pure conduction and the temperature distribution is linear

Correlation for film wise condensing process:-

i) Film thickness for laminar flow vertical surface

$$\delta x = \left(\frac{4\mu_l K_l x (T_{sat} - T_s)}{g h_{fg} \rho_l^2} \right)^{1/4}$$

δx = Boundary layer thickness (m)

K_l = Thermal conductivity of fluid (W/mK)

h_{fg} = Enthalpy of vaporization (J/kg)

ρ_l = Density of fluid(kg/m^3)

μ_l = Dynamic viscosity of fluid (Ns/m^2)

C_{pv} = Specific heat of vapour at constant pressure (kJ/kgK)

X = Distance along the surface,(m)

T_s = surface temperature ºC

T_{sat} = saturation temperature ºC

g = acceleration due to gravity (9.81)

ii) Local heat transfer co-efficient (h_x) for vertical surface ,laminar flow

$$h_x = \frac{K}{\delta x}$$

iii) Average heat transfer coefficient (h) for vertical surface , laminar flow

$$h_L = 0.943\left(\frac{K^3 \rho_l(\rho_l - \rho_v) g h_{fg}}{\mu L (T_{sat} - T_s)}\right)^{1/4}$$

Since the experimental values of h_L are usually 20% (or) higher than those predicted by h_L, it has been suggested by Mc Adams that the constant 0.943 be replaced by 1.13 hence

$$h_L = 1.13\left(\frac{K_l^3 \rho_l(\rho_l - \rho_v) g h_{fg}}{\mu_l L (T_{sat} - T_s)}\right)^{1/4}$$

iv) Film temperature T_f $= \frac{(T_{sat} + T_s)}{2}$

h_{fg} should be taken at T_{sat}

v) Film wise condensation on horizontal tubes:-

$$h_D = 0.725\left(\frac{K_l^3 \rho_l(\rho_l - \rho_v) g h_{fg}}{\mu_l D (T_{sat} - T_s)}\right)^{1/4}$$

vi) Average heat transfer coefficient for the bank of tubes :-

$$h_D = 0.725\left(\frac{K_l^3 \rho_l(\rho_l - \rho_v) g h_{fg}}{N \mu_l D (T_{sat} - T_s)}\right)^{1/4}$$

vii) For laminar flow $R_e < 1800$

viii) For turbulent flow $R_e > 1800$

ix) $R_e = \frac{4\dot{m}}{\mu P}$

x) Average heat transfer coefficient for vertical surface , turbulent flow

$$h = 0.0077\,(R_e)^{0.4}\left(\frac{K_l^{\,3}\rho_l^{\,2}g}{\mu_l^{\,2}}\right)^{0.333}$$

Solved problems on Boiling:-

1. Water is boiled at the rate of 25kg/hr in a polished copper pan, 280mm in diameter at atmospheric pressure. Assuming nucleate boiling condition, calculate the temperature of the bottom surface of the pan

Given data:

$\dot{m}$ = 25 kg/hr = 25/3600

$\dot{m}$ = 6.6x10^{-3} kg/hr

d = 300mm = 0.3 m

To find :-

Surface temperature (T_s)

Solution:-

We know saturation temperature (T_s)

T_{sat} = 100 ºC

Saturated water Properties at 100 ºC

[From HMT data book Page No: 21 – 6th edition]

ν = 0.293x10^{-6} m^2/s

ρ_l = 961 kg/m^3

C_{pl} = 4216 J/kg

P_r = 1.74

$\mu_l = \rho_l \times \nu$

μ_l = 281.57x10^{-6} Ns/m^2

From steam table, Fro 100 ºC

h_{fg} = 2256.9 kJ/kg

v_g = 1.673 m^3/kg

$$\rho_v = \frac{1}{v_g}$$

ρ_v = 0.597 kg/m^3

At 100 ºC

σ = 0.0588 N/m (From HMT data book page no 144)

C_{sf} = 0.013 (From HMT data book page no 145)

Q = m x h_{fg}

$$\frac{Q}{A} = \frac{\text{m x h}_{fg}}{A}$$

$$\frac{Q}{A} = \frac{\text{m x h}_{fg}}{\frac{\pi}{4}d^2}$$

$$\frac{Q}{A} = \frac{6.6\text{x}10\text{ - }3\text{ x }2257\times10^3}{\frac{\pi}{4}(.28)^2}$$

$\frac{Q}{A}$ = 254.52x10^3

Heat flux q_s = $\frac{Q}{A}$ = $\mu_l h_{fg}\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2}\left(\frac{C_{pl}\Delta T}{C_{sf}h_{fg}P_r^{1.7}}\right)^3$

254.52x10^3 =

$$281.57\times10^{-6}\times2257\times10^3\left(\frac{9.81(961-0.597)}{0.0588}\right)^{1/2}\left(\frac{4216\Delta T}{0.013\times2257\times10^3\times1.74}\right)^3$$

254.52x10^3 = 44.7678 x ΔT^3

ΔT = 12.08 ºC

ΔT = (T_s - T_{sat}) =12.08

T_s = 12.08 + T_{sat}

T_s = 12.08 + 100

T_s = 112.08 ºC

Result :

Surface temperature (T_s)= 112.08 ºC

2. water at atmospheric pressure is to be boiled in polished copper pan. The diameter of the pan is 350mm.and is kept at 115 ºC calculate the following.
    i) power of the burner
    ii) Rate of evaporation in kg/hr
    iii) Critical heat flow for this condition.

Given data:

D = 350 mm = 0.35 m

T_s = 115 ºC

To find :-

i) power of the burner

ii) Rate of evaporation in kg/hr

iii) Critical heat flow for this condition.

Solution:-

We know saturation temperature (T_s)

T_{sat} = 100 ºC

Saturated water Properties at 100 ºC

[From HMT data book Page No: 21 – 6th edition]

ν = 0.293x10^{-6} m^2/s

ρ_l = 961 kg/m^3

C_{pl} = 4216 J/kg

P_r = 1.74

$\mu_l = \rho_l \times \nu$

μ_l = 281.57x10^{-6} Ns/m^2

From steam table, Fro 100 ºC

h_{fg} = 2256.9 kJ/kg

v_g = 1.673 m^3/kg

$$\rho_v = \frac{1}{v_g}$$

ρ_v = 0.597 kg/m^3

ΔT = T_s - T_{sat}

ΔT = 115 – 100 = 15

At 100 ºC

σ = 0.0588 N/m (From HMT data book page no 144)

C_{sf} = 0.013 (From HMT data book page no 145)

1) Power required :

Heat flux q_s = $\frac{Q}{A}$ = $\mu_l h_{fg}\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2}\left(\frac{C_{pl}\Delta T}{C_{sf}h_{fg}P_r^{1.7}}\right)^3$

$\frac{Q}{A}$ =

$$281.57\times10^{-6}\times2257\times10^3\left(\frac{9.81(961-0.597)}{0.0588}\right)^{1/2}\left(\frac{4216\times15}{0.013\times2257\times10^3\times1.74}\right)^3$$

$\frac{Q}{A}$ = 4.8x10^5

Q = 4.8x10^5 x $\frac{\pi}{4}(0.35)^2$

Q = 46.181 kW

ii) Rate of evaporation(m)

Q = m x h_{fg}

m = $\frac{Q}{hfg}$

$$m = \frac{46.181\times10^3}{2257\times10^3}$$

$m = 0.0204$ kg/s

$m = 73.66$ kg/hr

iii) Critical heat flux:

$$\frac{Q}{A} = 0.18 h_{fg}\rho_v\left(\frac{\sigma g(\rho_l-\rho_v)}{\rho v^2}\right)^{1/4}$$

$$\frac{Q}{A} = 0.18\times2257\times10^3\times0.597\left(\frac{0.0588\times9.81(961-0.597)}{0.597^2}\right)^{1/4}$$

$$\frac{Q}{A} = 1.522\times10^6 \text{ W/m}^2$$

Result :

i) Power (P)		= 46.181 kW
ii) Rate of evaporation (m)		=73.66 kg/hr
iii) Critical heat flux		= 1.522 x 10^6 W/m^2

3. It is desired to generate 100 kg/hr of saturated steam at 100 ºC using a heating element of copper of surface are 5m^2. Calculate the convective heat transfer coefficient and the temperature of the heating surface.

Given data:

$\dot{m}$ = 100 kg/hr = 100/3600

$\dot{m}$ = 0.0277 kg/s

A = 5m^2

Saturation temperature (T_s) = 100 ºC

To find:-

i) heat transfer coefficient

ii) surface temperature (T_s)

Solution:-

Saturated water Properties at 100 ºC

[From HMT data book Page No: 21 – 6th edition]

ν = 0.293x10^{-6} m^2/s

ρ_l = 961 kg/m^3

C_{pl} = 4216 J/kg

P_r = 1.74

μ_l = ρ_l x ν

μ_l = 281.57x10^{-6} Ns/m^2

From steam table, Fro 100 ºC

h_{fg} = 2256.9 kJ/kg

v_g = 1.673 m^3/kg

$$\rho_v = \frac{1}{v_g}$$

ρ_v = 0.597 kg/m^3

At 100 ºC

σ = 0.0588 N/m (From HMT data book page no 144)

C_{sf} = 0.013 (From HMT data book page no 145)

i) Surface temperature (T_s)

$Q = m \times h_{fg}$

$$\frac{Q}{A} = \frac{\text{m x h}_{fg}}{A}$$

$$\frac{Q}{A} = \frac{\text{m x h}_{fg}}{\frac{\pi}{4}d^2}$$

$$\frac{Q}{A} = \frac{0.0277 \text{ x } 2257\text{x}10^3}{5}$$

$$\frac{Q}{A} = 12.538\text{x}10^3 \text{ W/m}^2$$

$$\text{Heat flux } q_s = \frac{Q}{A} = \mu_l h_{fg}\left(\frac{g(\rho_l - \rho_v)}{\sigma}\right)^{1/2}\left(\frac{C_{pl}\Delta T}{C_{sf} h_{fg} P_r^{1.7}}\right)^3$$

$$12.538\text{x}10^3 = 281.57\times10^{-6}\times2257\times10^3\left(\frac{9.81(961-0.597)}{0.0588}\right)^{1/2}\left(\frac{4216\Delta T}{0.013\times2257\times10^3\times1.74}\right)^3$$

$$12.538\text{x}10^3 = 44.7678 \text{ x } \Delta T^3$$

$$\Delta T = 4.5 \text{ ºC}$$

$$\Delta T = (T_s - T_{sat}) = 4.5$$

$$T_s = 4.5 + T_{sat}$$

$$T_s = 4.5 + 100$$

$$T_s = 104.5 \text{ ºC}$$

ii) Heat transfer coefficient (h):

Q = h A dT

$$h = \frac{Q}{A \times \Delta T}$$

$$h = \frac{12.538 \times 10^3}{4.5}$$

h = 2786 W/m^{2}K

Result :

i) the surface Temperature (T_s) = 104.5 ºC

ii) Heat transfer coefficient (h) = 2786 W/m^{2}K

4. A metal –clad heating element of 10mm diameter and of emissivity 0.92 is submerged in a water bath horizontally. If the surface temperature of the metal is 260

Heat exchanger

Heat exchanger may be defined as an equipment which transfer the energy from a hot fluid to cold fluid.

Examples of heat exchanger:

i) refrigerating and air-conditioning systems
ii) power systems
iii) food processing systems
iv) chemical reactors
v) space or aeronautical application
vi) steam power plants
vii) radiators in cars

Types of heat exchangers:-

Heat exchangers are classified on the basis of

i) Nature of heat exchange process
ii) Relative direction of fluid motion
iii) Design and constructional features
iv) Physical state of fluids

i) Nature of heat exchange process
   a) Direct contact heat exchangers
   b) Indirect contact heat exchangers

a) Direct contact heat exchangers

In a direct contact (or) open heat exchanger the exchange of heat takes place by direct mixing of hot and mass take place simultaneously.

Examples:-

i) Cooling towers
ii) Jet condensers

b) Indirect contact heat exchanger:-

In this type of heat exchanger the heat transfer between two fluids could be carried out by transmission through wall which separates the two fluids.

Examples :

i) Automobile radiators
ii) Oil coolers, intercoolers, air preheater, economizers ,super heaters

ii) Relative direction of fluid motion:-

According to the relative directions of two fluid streams the heat exchange are classified in to the following three categories

a) Parallel flow (or) unidirectional flow
b) Counter flow
c) Cross flow

a) Parallel flow heat exchanger :

Two fluid streams (hot and cold) travel in same direction.

b) Counter flow heat exchanger :

The two fluids flow in opposite direction. The hot and cold fluids enter at the opposite ends.

c) Cross flow heat exchanger:

The two fluids (hot and cold) cross one another in space. usually at right angle.

i) One fluid mixed other un mixed
ii) Both fluid unmixed

1. Design and construction :-

i) concentric tubes :

in this type two concentric tubes are used each carrying one of the fluids the direction of flow may be parallel or counter.

ii) Shell and tube

One of the fluids flows through a bundle of tubes enclosed by a shell.

a)Two tube pass one shell pass type
b) Four tube pass , two shell pass

iii) Multiple shell and tube pass

iv) Compact heat exchanger

These are special purpose heat exchanger and have a very large transfer surface area per unit volume of the exchanger. They are generally employed when convective heat transfer coefficient associated with one of the fluids is much smaller than that associated with the other fluid.

Example: Plate fin, flattened fin tube exchanger etc..

2. Physical state of fluids:

Depending upon the physical state of fluids the heat exchangers are classified as follows

i) Condensers

ii) Evaporators

Condensers:-

In a condenser, the condensing fluid (hot fluid) remains at constant temperature throughout the exchanger while the temperature of the colder fluid gradually increases from inlet to outlet.

Heat exchanger Analysis:-

For designing (or) predicting the performance of a heat exchanger it is necessary that the total heat transfer may be related with its governing parameters.

i) U (overall heat transfer coefficient)

ii) A total surface area of the heat transfer

iii) Inlet and outlet fluid temperature.

m = mass flow rate , kg/s

Cp = Specific heat of fluid at constant pressure J/kgK.

T = Temperature of fluid ºC

ΔT = Temperature drop (or) rise of the fluid across the heat exchanger

Heat lost by the hot fluid

$$Q = m_h C_{ph}(T_{h1}-T_{h2})$$

Heat gain by the cold fluid

$$Q = m_c C_{pc}(T_{c2}-T_{c1})$$

Total heat transfer rate in the heat exchanger

$$Q = U\,A\,\Delta T_m$$

ΔT_m = Logarithmic Mean Temperature Difference (LMDT)

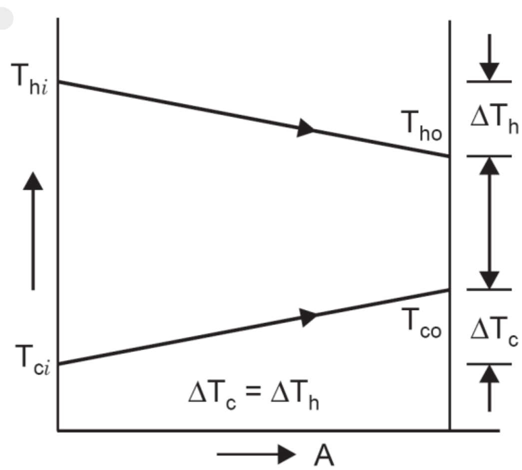
Logarithmic Mean Temperature Difference (LMDT)

The following assumptions are made :-

1. the overall heat transfer coefficient U is constant
2. the flow conditions are steady
3. the specific heats and mass flow rates of both fluids are constant
4. There is no change of phase either of the fluid during the heat transfer.
5. there is no loss of heat to the surroundings ,due to the heat exchanger being perfectly insulated.
6. The changes in potential and kinetic energies are negligible.
7. Axial condition along the tubes of the heat exchanger is negligible.

Logarithmic Mean Temperature Difference (LMDT) for "Parallel flow"

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary are,



.

dQ = U dA (T_h-T_c)

dQ = U dA ΔT_m

in a Parallel flow system , the temperature of hot fluid decrease in the direction of heat exchanger length, hence the – Ve sign

Heat lost by the hot fluid

dQ = $m_h C_{ph}(T_{h1}-T_{h2})$

dQ = - $m_h C_{ph}(T_{h2}-T_{h1})$

dQ = - $m_h C_{ph}$ dT_h

dQ = - $m_h C_{ph}$ dT_h

$$dT_h = -\frac{dQ}{m_h C_{ph}}$$

$$dT_h = -\frac{dQ}{C_h}$$

C_h = Heat capacity of hot fluid

Heat gain by the cold fluid

dQ = $m_c C_{pc}(T_{c2}-T_{c1})$

dQ = $m_c C_{pc}(T_{c2}-T_{c1})$

dQ = $m_c C_{pc}$ dT_c

dQ = $m_c C_{pc}$ dT_c

$$dT_c = \frac{dQ}{m_c C_{pc}}$$

. .

.

$$dT_c = \frac{dQ}{C_c}$$

C_c = Heat capacity of cold fluid

$$dT_h - dT_c = -\frac{dQ}{C_h} - \frac{dQ}{C_c}$$

$$= -\,dQ\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

$$= -\,UdA(T_h-T_c)\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

$$dQ = -\,U\,dA\,\theta\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

$$\frac{d\theta}{\theta} = -\,U\,dA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

Integrating between inlet and outlet conditions

Area A= 0 to A = A

$$\int_1^2 \frac{d\theta}{\theta} = \int_0^A -\,U\,dA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

$$\left[\ln(\theta)\right]_1^2 = -\,U\left(\frac{1}{C_h}+\frac{1}{C_c}\right)\left[A\right]_0^A$$

$$\ln(\theta_2) - \ln(\theta_1) = -\,U\,A\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

. .

.

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

$$Q = C_h(T_{h1}-T_{h2})$$

$$\frac{1}{C_h} = \frac{T_{h1} - T_{h2}}{Q}$$

$$Q = C_c(T_{c2}-T_{c1})$$

$$\frac{1}{C_c} = \frac{T_{c2} - T_{c1}}{Q}$$

Substitute $1/C_h$, $1/C_c$ Value in equation -1

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left(\frac{T_{h1}-T_{h2}}{Q}+\frac{T_{c2}-T_{c1}}{Q}\right)$$

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left(\frac{(T_{h1}-T_{c1})-(T_{h2}-T_{c2})}{Q}\right)$$

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -UA\left(\frac{\theta_1-\theta_2}{Q}\right)$$

$$Q = -UA\left(\frac{\theta_1-\theta_2}{\ln\left(\frac{\theta_2}{\theta_1}\right)}\right)$$

$$Q = UA\frac{\theta_1-\theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)}$$

$$Q = UA\,\theta_m$$

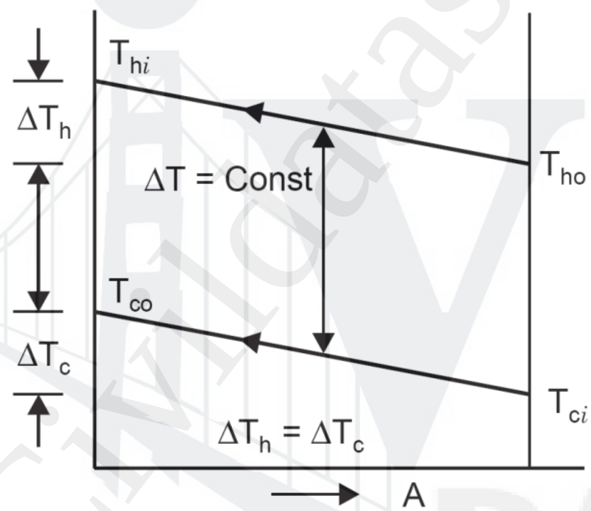
. .

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \Delta T_m$$ = Logarithmic Mean Temperature Difference (LMDT)

$$\theta_m = \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{\ln\left(\frac{T_{h1} - T_{c1}}{T_{h2} - T_{c2}}\right)}$$

Logarithmic Mean Temperature Difference (LMDT) for "Counter flow"

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary are,



$$dQ = U\,dA\,(T_h - T_c)$$

$$dQ = U\,dA\,\Delta T_m$$

in a counter flow system , the temperature of both the fluids decrease in the direction of heat exchanger length, hence the – Ve sign

Heat lost by the hot fluid

$$dQ = m_h C_{ph}(T_{h1} - T_{h2})$$

$$dQ = -\,m_h C_{ph}(T_{h2} - T_{h1})$$

$$dQ = - m_h C_{ph}\, dT_h$$

$$dQ = - m_h C_{ph}\, dT_h$$

$$dT_h = - \frac{dQ}{m_h C_{ph}}$$

$$dT_h = - \frac{dQ}{C_h}$$

C_h = Heat capacity of hot fluid

Heat gain by the cold fluid

$$dQ = - m_c C_{pc}(T_{c2}-T_{c1})$$

$$dQ = - m_c C_{pc}(T_{c2}-T_{c1})$$

$$dQ = - m_c C_{pc}\, dT_c$$

$$dQ = - m_c C_{pc}\, dT_c$$

$$dT_c = - \frac{dQ}{m_c C_{pc}}$$

$$dT_c = - \frac{dQ}{C_c}$$

C_c = Heat capacity of cold fluid

$$dT_h - dT_c = - \frac{dQ}{C_h} + \frac{dQ}{C_c}$$

$$= - dQ\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$= -\,UdA(T_h - T_c)\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$dQ = -\,U\,dA\,\theta\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$\frac{d\theta}{\theta} = -\,U\,dA\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

Integrating between inlet and outlet conditions

Area A= 0 to A = A

$$\int_1^2 \frac{d\theta}{\theta} = \int_0^A -\,U\,dA\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$[\ln(\theta)]_1^2 = -\,U\left(\frac{1}{C_h} - \frac{1}{C_c}\right)[A]_0^A$$

$$\ln(\theta_2) - \ln(\theta_1) = -\,U\,A\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = -\,U\,A\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$Q = C_h(T_{h1} - T_{h2})$$

$$\frac{1}{C_h} = \frac{T_{h1} - T_{h2}}{Q}$$

$$Q = C_c(T_{c2} - T_{c1})$$

$$\frac{1}{C_c} = \frac{T_{c2} - T_{c1}}{Q}$$

Substitute $1/C_h$, $1/C_c$ Value in equation -1

.

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = - U A\left(\frac{(T_{h1}-T_{h2})}{Q} - \frac{(T_{c2}-T_{c1})}{Q}\right)$$

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = - U A\left(\frac{(T_{h1}-T_{c2})-(T_{h2}-T_{c1})}{Q}\right)$$

$$\ln\left(\frac{\theta_2}{\theta_1}\right) = - U A\left(\frac{\theta_1-\theta_2}{Q}\right)$$

$$Q = - U A\left(\frac{\theta_1-\theta_2}{\ln\left(\frac{\theta_2}{\theta_1}\right)}\right)$$

$$Q = U A\frac{\theta_1-\theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)}$$

$$\boxed{Q = U A \theta_m}$$

$$\theta_m = \frac{\theta_1-\theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \Delta T_m$$ = Logarithmic Mean Temperature Difference (LMDT)

$$\theta_m = \frac{(T_{h1}-T_{c2})-(T_{h2}-T_{c1})}{\ln\left(\frac{T_{h1}-T_{c2}}{T_{h2}-T_{c1}}\right)}$$

<u>Overall heat transfer co-efficient:-</u>

If the fluids are separated by a tube wall as shown in fig. the overall heat transfer coefficient is given by ,

Considering inner surface :

. .

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{K}\ln\left(\frac{r_o}{r_i}\right) + \frac{r_i}{r_o}\left(\frac{1}{h_o}\right)}$$

Considering Outer surface :

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_0}{r_i}\right) + \frac{r_0}{K}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}}$$

The heat exchanger ,considering the thermal resistance due to scale formation is given by,

Considering inner surface

$$U_i = \frac{1}{\frac{1}{h_i} + R_{fi} + \frac{r_i}{K}\ln\left(\frac{r_o}{r_i}\right) + \frac{r_i}{r_o}R_{f0} + \frac{r_i}{r_o}\left(\frac{1}{h_o}\right)}$$

Considering Outer surface :

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_0}{r_i}\right) + \frac{r_0}{r_i}R_{fi} + \frac{r_0}{K}\ln\left(\frac{r_o}{r_i}\right) + R_{f0} + \frac{1}{h_o}}$$

Incase of thin walled surface

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

When only, fouling factors are neglected

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_0}{r_i}\right) + \frac{r_0}{K}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}}$$

<u>Problems on Parallel flow</u>

1. the flow rate of hot and cold water streams running through a parallel flow heat exchanger are 0.2kg/s and 0.5kg/s respectively. The inlet temperatures on the hot and cold sides are 75 ºC and 20 ºC respectively. The exit temperature of hot water is 45 ºC. if the overall heat transfer co-efficient is 350 W/m^2K. Calculate the area of the heat exchanger.

Given :

m_h = 0.2 kg/s

m_c = 0.5 kg/s

T_{h1} = 75 ºC

T_{c1} = 20 ºC

T_{h2} = 45 ºC

U = 350 W/m^2K

C_{ph} = 4187 J/kgK = C_{pc}

To find :

Area of the heat exchanger,

Solution:

Heat lost by the hot fluid

$Q = m_h C_{ph}(T_{h1}-T_{h2})$

= 0.2x4187 x(75-45)

= 25,122 W

Heat lost by the hot fluid = Heat gain by the cold fluid

$Q = m_c C_{pc}(T_{c2}-T_{c1})$

$$25{,}122 = 0.5 \times 4187 \times (T_{c2} - 20)$$

$$T_{c2} = \frac{25{,}122}{0.5 \times 4187} + 20$$

$$T_{c2} = 32 \text{ ºC}$$

$$\theta_m = \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{\ln\left(\frac{T_{h1} - T_{c1}}{T_{h2} - T_{c2}}\right)}$$

$$\theta_m = \frac{(75 - 20) - (45 - 32)}{\ln\left(\frac{75 - 20}{45 - 32}\right)}$$

$$\theta_m = 29.12 \text{ ºC}$$

Heat transfer rate

$$Q = UA\,\theta_m$$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{25{,}122}{350 \times 29.12}$$

$$A = 2.46 \text{ m}^2$$

Result :

Heat transfer area A = 2.46 m^2

2. in a double pipe counter flow heat exchanger 10,000kg/hr of an oil having a specific heat of 2095 J/kgK is cooled from 80 ºC to 50 ºC by 8000 kg/hr of water entering at 25 ºC. Determine the heat exchanger area for an overall heat transfer coefficient of 300 W/m^2K. take Cp for water as 4180 J/kgK.

.

Given :

Hot fluid = Oil

Cold fluid = Water

m_h = 10,000 kg/hr

=10,000 / 3600 = 2.78 kg/s

m_c = 8000 kg/hr

= 8000 / 3600 = 2.22 kg/s

T_{h1} = 80 ºC

T_{h2} = 50 ºC

T_{c1} = 25 ºC

U = 300 W/m^2K

C_{ph} = 2095 J/kgK

C_{pc} = 4180 J/kgK

To find :

Area of the heat exchanger,

Solution:

Heat lost by the hot fluid

$Q = m_h C_{ph}(T_{h1}-T_{h2})$

= 2.78 x 2095 x(80 – 50)

= 174583 W

. .

Heat lost by the hot fluid = Heat gain by the cold fluid

$$Q = m_c C_{pc}(T_{c2} - T_{c1})$$

$$174583 = 2.22 \times 4180 \times (T_{c2} - 25)$$

$$T_{c2} = \frac{174583}{2.22 \times 4180} + 25$$

$$T_{c2} = 43.8\ ºC$$

For counter Flow, $$\theta_m = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

$$\theta_m = \frac{(80 - 43.8) - (50 - 25)}{\ln\left(\frac{80 - 43.5}{50 - 25}\right)}$$

$$\theta_m = 29.6\ ºC$$

Heat transfer rate

$$Q = UA\,\theta_m$$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{174583}{300 \times 29.6}$$

$$A = 19.66\ m^2$$

Result :

Heat transfer area $A = 19.66\ m^2$

3. Hot oil with a capacity rate of 2500 W/K flows through a double pipe heat exchanger. It enters at 360 ºC and leaves at 300 ºC cold fluid enters at 30 ºC and leaves at 200 ºC. If the overall heat transfer coefficient is 800 W/m^2K, Determine the heat exchanger area required for a) Parallel Flow b) Counter Flow.

Given :

Hot fluid = Oil

Cold fluid = Water

T_{h1} = 360 ºC

T_{h2} = 300 ºC

T_{c1} = 30 ºC

T_{c2} = 200 ºC

U = 800 W/m^2K

Heat Capacity C_h = 2095 J/kgK = $m_h C_{ph}$

To find :

Area of the heat exchanger, for a) Parallel flow

b) Counter flow

Solution:

Heat lost by the hot fluid

$$Q = m_h C_{ph}(T_{h1} - T_{h2})$$

$$= 2500 \times (360 - 300)$$

$$= 150000 \text{ W}$$

a) For parallel flow :

.

$$\theta_m = \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{\ln\left(\frac{T_{h1} - T_{c1}}{T_{h2} - T_{c2}}\right)}$$

$$\theta_m = \frac{(360 - 30) - (300 - 200)}{\ln\left(\frac{360 - 30}{300 - 200}\right)}$$

θ_m = 192.64 ºC

Heat transfer rate

$Q = UA\ \theta_m$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{15000}{800 \times 192.64}$$

$A = 0.973\ m^2$

b) For counter Flow, $$\theta_m = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

$$\theta_m = \frac{(360 - 200) - (300 - 30)}{\ln\left(\frac{360 - 200}{300 - 30}\right)}$$

θ_m = 210.22 ºC

Heat transfer rate

$Q = UA\ \theta_m$

. .

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{150000}{800 \times 210.22}$$

$$A = 0.892 \text{ m}^2$$

Result :

The surface area required for a counter flow arrangement is less than that in a parallel flow arrangement

4. A counter flow concentric tube heat exchanger is used to cool engine oil (C_p =2130 J/kgK) from 160 ºC to 60 ºC with water available at 25 ºC as the cooling medium. The flow rate of cooling water through the inner tube of 0.5m diameter is 2 kg/s while the flow rate of oil through the outer tube is 2 kg/s. If the value of the overall heat transfer coefficient is 250 W/m^{2}K,What length must the heat exchanger be to meet its cooling requirement?

Given:

Hot fluid = Engine Oil

Cold fluid = Water

m_h = 2 kg/s

m_c = 2 kg/s

T_{h1} = 160 ºC

T_{h2} = 60 ºC

T_{c1} = 25 ºC

U = 250 W/m^{2}K

C_{ph} = 2130 J/kgK

C_{pc} = 4187 J/kgK

d = 0.5 m

To find :

Length of heat exchanger,

Solution:

Heat lost by the hot fluid

$$Q = m_h C_{ph}(T_{h1}-T_{h2})$$

$$= 2 \times 2130 \times (160 - 60)$$

$$= 426000 \text{ W}$$

Heat lost by the hot fluid = Heat gain by the cold fluid

$$Q = m_c C_{pc}(T_{c2}-T_{c1})$$

$$426000 = 2 \times 4187 \times (T_{c2}-25)$$

$$T_{c2} = \frac{426000}{2 \times 4187} + 25$$

$$T_{c2} = 75.87 \text{ ºC}$$

For counter Flow, $$\theta_m = \frac{(T_{h1}-T_{c2})-(T_{h2}-T_{c1})}{\ln\left(\frac{T_{h1}-T_{c2}}{T_{h2}-T_{c1}}\right)}$$

$$\theta_m = \frac{(160-75.87)-(60-25)}{\ln\left(\frac{160-75.87}{60-25}\right)}$$

$$\theta_m = 56.02 \text{ ºC}$$

Heat transfer rate

$$Q = UA\,\theta_m$$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{426000}{250 \times 56.02}$$

$$A = 30.417\ m^2$$

$$A = \pi \times d \times L$$

$$L = \frac{A}{\pi \times d}$$

$$L = \frac{30.417}{\pi \times 0.5}$$

$$L = 19.36\ m$$

Result :

Length of heat exchanger L = 19.36 m

5. Saturated steam at 120 ºC is condensing on the outer tube surface of a single pass heat exchanger. The heat transfer co-efficient is U_0 =1800 W/m²K. Determine the surface area of a heat exchanger capable of heating 1000 kg /hr of water 20 ºC to 90 ºC. Also compute the rate of condensation of steam. Take h_{fg} = 2200 kJ/kg

Given :

Hot fluid = Steam

Cold fluid = Water

.

T_{h1} = 120 ºC = T_{h2} (For condenser)

T_{c1} = 20 ºC

T_{c2} = 90 ºC

m_c = 1000 kg/hr

= 1000 / 3600 = 0.278 kg/s

U_o = 1800 W/m^{2}K

C_{pc} = 4187 J/kgK

h_{fg} = 2200 kJ/kg

To find :

i) Heat transfer Area

ii) The rate of condensation of steam

Solution:

Heat lost by the hot fluid

Q = $m_c C_{pc}(T_{c2}-T_{c1})$

= 0.278 x 4187 x (90 – 20)

= 81413.89 W

For parallel flow :

$$\theta_m = \frac{(T_{h1}-T_{c1})-(T_{h2}-T_{c2})}{\ln\left(\frac{T_{h1}-T_{c1}}{T_{h2}-T_{c2}}\right)}$$

$$\theta_m = \frac{(120-20)-(120-90)}{\ln\left(\frac{120-20}{120-90}\right)}$$

. .

θ_m = 58.14 ºC

Heat transfer rate

$$Q = UA\,\theta_m$$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{81413.89}{1800 \times 58.14}$$

$$A = 0.78\ m^2$$

$$Q = m \times h_{fg}$$

$$m = \frac{Q}{h_{fg}}$$

$$m = \frac{81413.89}{2200 \times 10^3}$$

m = 0.037 kg/s (or) 133.22 kg/hr

Result :

i) Heat transfer area A = 0.78 m^2

ii) The rate of condensation of steam m = 133.22 kg/hr

6. in a counter flow double pipe heat exchanger , water is heated from 25 ºC to 65 ºC by an oil with a specific heat of 1.45 kJ/kgK and mass flow rate of 0.9 kg/s. the oil is cooled from 230 ºC to 160 ºC. if the overall heat transfer coefficient is 420 W/m2K, Calculate the following

   i. The rate of heat transfer

ii. The mass flow rate of water

iii. The surface area of the Heat exchanger

Given:

Hot fluid = Engine Oil

Cold fluid = Water

T_{h1} = 230 ºC

T_{h2} = 160 ºC

T_{c1} = 25 ºC

T_{c2} = 65 ºC

m_h = 0.9 kg/s

U = 420 W/m^2K

C_{ph} = 1.450 kJ/kgK = 1450 J/kgK

C_{pc} = 4187 J/kgK

To find :

i. The rate of heat transfer
ii. The mass flow rate of water
iii. The surface area of the Heat exchanger

Solution:

i) The rate of heat transfer:

Heat lost by the hot fluid

$$Q = m_h C_{ph}(T_{h1}-T_{h2})$$

$= 0.9 \times 1450 \times (230 - 160)$

$= 91350$ W

ii) The mass flow rate of water:

Heat lost by the hot fluid = Heat gain by the cold fluid

$Q = m_c C_{pc}(T_{c2} - T_{c1})$

$91350 = m_c \times 4187 \times (65 - 25)$

$$m_c = \frac{91350}{4187 \times (65 - 25)}$$

$m_c = 0.545$ kg/s

iii) surface area of heat exchanger:

For counter Flow,
$$\theta_m = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

$$\theta_m = \frac{(230 - 65) - (160 - 25)}{\ln\left(\frac{230 - 65}{160 - 25}\right)}$$

$\theta_m = 149.5$ ºC

Heat transfer rate

$Q = UA\,\theta_m$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{91350}{420 \times 149.5}$$

$A = 1.45\ m^2$

Result :

i. The rate of heat transfer Q = 91350 W

ii. The mass flow rate of water m_c = 0.545 kg/s

iii. The surface area of heat exchanger $A = 1.45\ m^2$

7. An oil cooler for a lubrication system has to cool 1000 kg/hr of oil (Cp = 2.09 kJ/kgK) from 80 ºC to 40 ºC by using a cooling water flow of 1000 kg/hr at 30 ºC. Give your choice for a parallel flow or counter flow heat exchanger with reasons. Calculate the surface area of the heat exchanger, if the overall heat transfer coefficient is 24 W/m²K.

Given:

Hot fluid = Engine Oil

Cold fluid = Water

C_{ph} = 2.09 kJ/kgK = 2090 J/kgK

m_h = 1000 kg/hr

= 1000 / 3600 = 0.278 kg/s

T_{h1} = 80 ºC

T_{h2} = 40 ºC

T_{c1} = 30 ºC

m_c = 1000 kg/hr

= 1000 / 3600 = 0.278 kg/s

$U = 24\ W/m^2K$

.

C_{pc} = 4187 J/kgK

To find :

The surface area of the Heat exchanger

Solution:

Heat lost by the hot fluid

$Q = m_h C_{ph}(T_{h1}-T_{h2})$

$= 0.278 \times 2090 \times (80 - 40)$

$= 23222$ W

ii) The mass flow rate of water:

Heat lost by the hot fluid = Heat gain by the cold fluid

$Q = m_c C_{pc}(T_{c2} - T_{c1})$

$23222 = 0.278 \times 4187 \times (T_{c2} - 40)$

$$T_{c2} = \frac{23222}{0.278 \times 4187} + 30$$

T_{c2} = 50 ºC

$T_{c2} > T_{h2}$

So, counter flow arrangement must be used

Surface area of heat exchanger:

For counter Flow,
$$\theta_m = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

. .

$$\theta_m = \frac{(80-50)-(40-30)}{\ln\left(\frac{80-50}{40-30}\right)}$$

θ_m = 18.2 ºC

Heat transfer rate

$Q = UA\,\theta_m$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{23222}{24 \times 18.2}$$

A = 53.15 m^2

Result :

Surface area of the heat exchanger A = 53.15 m^2

8. A counter flow double pipe heat exchanger using superheated steam is used to hot water at the rate of 10500 kg/hr. the steam enter the heat exchanger at 200 ºC and leaves at 130 ºC. The inlet and exit temperature of water are 30 ºC and 80 ºC respectively. If overall heat transfer coefficient from steam t water is 814 W/m2K, calculate the heat transfer area. What would be the increase in area in the fluid flows were in parallel?

Given:

Hot fluid = steam

Cold fluid = Water

m_c = 10500 kg/hr

= 10500 / 3600 = 2.917 kg/s

.

T_{h1} = 200 ºC

T_{h2} = 130 ºC

T_{c1} = 30 ºC

T_{c2} = 80 ºC

U = 814 W/m^{2}K

C_{pc} = 4187 J/kgK

To find :

% of increase in area if the fluid flows were in parallel

Solution:

Heat gain by the hot fluid

$$Q = m_c C_{pc}(T_{c2} - T_{c1})$$

$$= 2.917 \times 4187 \times (80 - 30)$$

$$= 610670 \text{ W}$$

For counter Flow,

$$\theta_m = \frac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\frac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$$

$$\theta_m = \frac{(200 - 80) - (130 - 30)}{\ln\left(\frac{200 - 80}{130 - 30}\right)}$$

$$\theta_m = 109.7 \text{ ºC}$$

Heat transfer rate

$$Q = UA_1 \theta_m$$

. .

$$A_1 = \frac{Q}{U\theta_m}$$

$$A_1 = \frac{610670}{814 \times 109.7}$$

A_1 = 6.8 m^2

For parallel flow :

$$\theta_m = \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{\ln\left(\frac{T_{h1} - T_{c1}}{T_{h2} - T_{c2}}\right)}$$

$$\theta_m = \frac{(200 - 30) - (130 - 80)}{\ln\left(\frac{200 - 30}{130 - 80}\right)}$$

θ_m = 98.05 ºC

Heat transfer rate

$Q = UA_1 \theta_m$

$$A_2 = \frac{Q}{U\theta_m}$$

$$A_2 = \frac{610670}{814 \times 98.05}$$

A_2 = 7.65 m^2

% of increase in area $= \frac{A_2 - A_1}{A_2} \times 100$

$= \frac{7.65 - 6.8}{7.65} \times 100$

= 11.11 %

Result :

% of increase in area = 11.11%

9. Determine the overall heat transfer coefficient U_0 based on the outer surface of a 2.54 cm O.D 2.286 cm I.D. heat exchanger tube (K= 102 W/mK).If the heat transfer co-efficients at the inside and out side of the tube are h_i = 5500 W/m^2K and h_o = 3800 W/ m^2K respectively and the fouling factors are $R_{fo} = R_{fi}$ = 0.0002 m^2WK.

Given :

$$r_1 = \frac{2.286}{2} = 1.143 \text{ cm} = 1.143 \times 10^{-2} \text{ m}$$

$$r_2 = \frac{2.54}{2} = 1.27 \text{ cm} = 1.27 \times 10^{-2} \text{ m}$$

h_o = 3800 W/ m^2K

h_i = 5500 W/m^2K

$R_{fo} = R_{fi}$ = 0.0002 m^2WK

K = 102 W/mK

To find

Overall heat transfer co-efficient

Solution:

Over all heat transfer co-efficient based on outer surface

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_0}{r_i}\right)+\frac{r_0}{r_i}R_{fi}+\frac{r_0}{K}\ln\left(\frac{r_o}{r_i}\right)+R_{f0}+\frac{1}{h_o}}$$

$$= \frac{1}{\frac{1}{5500}\left(\frac{1.27\times10^{-2}}{1.143\times10^{-2}}\right)+\frac{1.27\times10^{-2}}{1.143\times10^{-2}}(0.0002)+\frac{1.27\times10^{-2}}{102}\ln\left(\frac{1.27\times10^{-2}}{1.143\times10^{-2}}\right)+0.0002+\frac{1}{3800}}$$

= 1110.47 W/m^2K

Result :

Overall heat transfer coefficient U_o = 1110.47 W/m^2K

10. Steam enters a counter flow heat exchanger dry saturated at 10 bar and leaves at 350 ºC. The mass flow of steam is 800 kg/min. the gas enters the heat exchanger at 650 ºC and mass flow rate is 1350 kg/min. if the tubes are 30mm diameter and 3m long. Determine the number of tubes required. Neglect the resistance offered by metallic tubes use following data

For steam: Tsat = 180 ºC (at 10 bar)

C_{ps} = 2.71 kJ/kgK,

h_s = 600 W/m^2K

For Gas : C_{pg} =1 kJ/kgK

h_g = 250 W/m^2K

Given :

Hot fluid = Gas

Cold fluid = Steam

m_c = 800 kg/min

= 800 / 60 = 13.33 kg/s

m_h = 1350 kg/min

= 1350 / 60 = 22.5 kg/s

T_{h1} = 650 ºC

T_{c1} = 180 ºC (at 10 bar, take saturation temperature from steam table)

T_{c2} = 350 ºC

C_{pc} = 2.71 kJ/kgK = 2710 J/kgK

C_{ph} = 1 kJ/kgK = 1000 J/kgK

h_o = 600 W/m²K

h_i = 250 W/m²K

To find :

No of tubes required

Solution:

Heat gain by the hot fluid

$$Q = m_c C_{pc}(T_{c2} - T_{c1})$$

$$= 13.33 \times 2710 \times (350 - 180)$$

$$= 6141131 \text{ W}$$

Heat lost by the hot fluid = Heat gain by the cold fluid

$$Q = m_h C_{ph}(T_{h1} - T_{h2})$$

$$6141131 = 22.5 \times 1000 \times (650 - T_{h2})$$

$$T_{h2} = 650 - \frac{6141131}{22.5 \times 1000}$$

T_{c2} = 377 ºC

For counter Flow, $\theta_m = \dfrac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\dfrac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$

$$\theta_m = \frac{(650 - 350) - (377 - 180)}{\ln\left(\dfrac{650 - 350}{377 - 180}\right)}$$

θ_m = 245 ºC

Overall heat transfer coefficient

$$U_o = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}}$$

$$U_o = \frac{1}{\dfrac{1}{600} + \dfrac{1}{250}}$$

U_o = 176.47 W/m^{2}K

Heat transfer rate

$Q = UA\,\theta_m$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{6141131}{176.47 \times 245}$$

A = 142.04 m^2

A = N π d L

$$N = \frac{A}{\pi \times d \times L}$$

$$N = \frac{142.04}{\pi \times 0.03 \times 3}$$

N = 502 tubes

Result :

No of tubes required N = 502

11. In a shell and tube counter flow heat exchanger water flows through a copper tube 20mm I.D and 23mm O.D, while oil flows through the shell. Water enters at 20 ºC and comes out at 30 ºC. While oil enters at 75 ºC and comes out at 60 ºC. The water and oil side film coefficients are 4500 and 1250 W/m2K. Respectively. The thermal conductivity of the tube wall is 355 W/mK. The fouling factors on the water and oil sides may be taken to be 0.0004 and .001 respectively if the length of the tube is 2.4m. Calculate the following
    (i) overall heat transfer coefficient
    (ii) heat transfer rate.

Given :

Hot fluid = oil

Cold fluid = water

r_i = 20/2 = 10mm = 0.01

r_0 = 23/2 = 11.5mm = 0.0115

T_{h1} = 75 ºC

T_{h2} = 60 ºC

T_{c1} = 20 ºC

T_{c2} = 30 ºC

h_o = 1250 W/m^{2}K

h_i = 4500 W/m^{2}K

K = 355 W/mK

R_{fi} = 0.0004

R_{fo} = 0.001

L = 2.4 m

To find :

(i) overall heat transfer coefficient

(ii) Heat transfer rate.

Solution:

$$U_o = \frac{1}{\frac{1}{h_i}\left(\frac{r_0}{r_i}\right) + \frac{r_0}{r_i}R_{fi} + \frac{r_0}{K}\ln\left(\frac{r_o}{r_i}\right) + R_{f0} + \frac{1}{h_o}}$$

$$= \frac{1}{\frac{1}{5500}\left(\frac{1.27\times10^{-2}}{1.143\times10^{-2}}\right) + \frac{1.27\times10^{-2}}{1.143\times10^{-2}}(0.0002) + \frac{1.27\times10^{-2}}{102}\ln\left(\frac{1.27\times10^{-2}}{1.143\times10^{-2}}\right) + 0.0002 + \frac{1}{3800}}$$

= 1110.47 W/m^{2}K

For counter Flow, θ_m $= \dfrac{(T_{h1} - T_{c2}) - (T_{h2} - T_{c1})}{\ln\left(\dfrac{T_{h1} - T_{c2}}{T_{h2} - T_{c1}}\right)}$

$$\theta_m = \frac{(650 - 350) - (377 - 180)}{\ln\left(\frac{650 - 350}{377 - 180}\right)}$$

$\theta_m = 245$ ºC

Overall heat transfer coefficient

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}}$$

$$U_o = \frac{1}{\frac{1}{600} + \frac{1}{250}}$$

$U_o = 176.47$ W/m^{2}K

Heat transfer rate

$Q = UA\,\theta_m$

$$A = \frac{Q}{U\theta_m}$$

$$A = \frac{6141131}{176.47 \times 245}$$

$A = 142.04$ m^2

$A = N\,\pi\,d\,L$

$$N = \frac{A}{\pi \times d \times L}$$

$$N = \frac{142.04}{\pi \times 0.03 \times 3}$$

N = 502 tubes

Result :

No of tubes required N = 502

VI: RADIATION

Introduction

Radiation heat transfer is defined as the transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused solely by a temperature difference. Radiation heat transfer does not require a medium.

Application:-

(i). In furnaces, combustion chambers, nuclear explosion and in space applications.
(ii). Solar energy incident upon the earth.

Surface Emission Properties

The rate of emission of radiation by a body depends upon the following factors:

(i). The temperature of the surface
(ii). The nature of the surface and
(iii). The wavelength or frequency of radiation.

Total emissive power (E_b):

The emissive power is defined as the total amount of radiation emitted by a body per unit area and unit time.

Monochromatic emissive power (E_λ)

At any Given Data temperature the amount of radiation emitted per unit wave length varies at different wavelength. It is defined as the rate of energy radiated per unit area of the surface per unit wavelength.

Emissivity:-

It is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of the emissive power of any body to the emissive power of a block body of equal temperature.

$$\epsilon = \frac{E}{E_b}$$

Its value ranging from 0 to 1

For a black body $\epsilon = 1$

For a white body $\epsilon = 0$

For a gray body $\epsilon = 0$ to 1

Absorptivity (α) :-

It is defined as the ratio of the radiation absorbed to the incident radiation

$$\alpha = \frac{\text{radiation absorbed}}{\text{incident radiation}}$$

Reflectivity (ρ):-

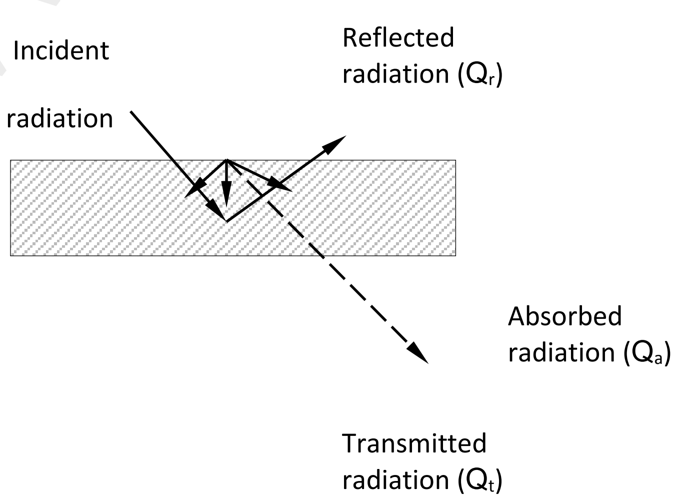
It is defined as the ratio of the radiation reflected to the incident radiation.

$$\rho = \frac{\text{radiation reflected } (Q_r)}{\text{Incident radiation } (Q)}$$

Transmissivity (τ) :-

It is defined as the ratio of the radiation transmitted to the incident.

$$\tau = \frac{\text{radiation transmitted } (Q_t)}{\text{Incident radiation } (Q)}$$



By the conservation of energy principle

$$Q_a + Q_r + Q_t = Q$$

Dividing both sides by Q , we get

$$\frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q} = \frac{Q}{Q}$$

$$\alpha + \rho + \tau = 1$$

For black body: - $\alpha = 1$, $\rho = 0$, $\tau = 0$

(i.e.) a black body is one which neither reflects nor transmits any part of the incident radiation but absorbs all of it. In practice, a perfect black body ($\alpha = 1$) does not exists

For opaque body:-

When no incident radiation is transmitted through the body it is called an opaque body.

$\tau = 0$

$\therefore \quad \alpha + \rho = 1$

Example: gasses and liquids.

White body:-

If all the incident radiation falling on the body are reflected it is called a “white body”.

For a white body, $\rho = 1$, $\alpha = 0$, $\tau = 0$

Example: gasses such as hydrogen, oxygen nitrogen.

.

Gray body:-

A gray body is defined as one whose absorptive of surface does not vary with temperature and wavelength of the incident radiation ($\alpha = \alpha_\lambda$ = constant)

Concept of a black body:-

A black body has the following properties:-

(i). It absorbs all the incident radiation falling on it and does not transmit (or) reflect regardless of wave length and direction

(ii). It emits maximum amount of thermal radiation at all wavelength at any specified temperature.

(iii). It is a perfect emitter (i.e. the radiation emitted by a black body is independent of direction).

The STEFAN BOLTZMANN LAW:-

The law states that the emissive power of a black body is directly proportional to the fourth power of its absolute temperature.

(i.e.) $E_b = \sigma T^4$

E_b = Emissive power of a black body

σ = Stefan Boltzmann constant = $5.67 \times 10^{-8}\ {}^{W}/_{m^2k^4}$

T = absolute temperature.

Plank's law:-

The monochromatic distribution of the radiation intensity of a black body is Given Data by

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{e^{(C_2/\lambda T)} - 1}$$

$(E_\lambda)_b$ = Monochromatic emissive power ${}^{W}/_{m^2}$

$C_1 = 3.742 \times 10^8\ \mu wm^2$

. .

$C_2 \quad = 1.4388 \times 10^4$ μmk

Wien's Displacement law:-

A relationship between the temperature of a black body and the wave length at which the maximum valve of monochromatic emissive power, occurs.

Wien's displacement law states that the product of λ_{max} and T is constant,

(i.e.)

$$\lambda_{max} T = \text{constant}$$

$$\lambda_{max} T = 2898 \ \mu mk$$

Another form of wien's law,

$$\frac{E_b\lambda_{max}}{T^5} = \text{constant (or)} \quad E_b\lambda_{max} = C_4T^5$$

$$C_4 = 1.307 \times 10^{-5} \ {}^{W}/{m^2 K^5}$$

Krichoff's law:-

The law states that at any temperature the ratio of total emissive power E to the total absorptivity α is a constant for all substance which are in thermal equilibrium with their environment.

Kirchoff's law also states that the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings

$$\epsilon = \alpha$$

Intensity of radiation:- (I)

The intensity of radiation (I) is defined as the rate of energy leaving a surface in a Given Data direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

The total emissive power of a diffuser surface is equal to π times its intensity of radiation.

$$E = \pi I$$

.

Lambert's cosine law:-

The law states that the total emissive power (E_b) form a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission

$$E_b \propto \cos\theta$$

$$E_b = E_n \cos\theta$$

Problems

1) The effective temperature of a body having an area of $0.12m^2$ is 527ºC. calculate the following.
   (i). The total rate of energy emission.
   (ii). The intensity of normal radiation, and
   (iii). The wavelength of maximum monochromatic emissive power.

Given Data:-

$A = 0.12m^2$

$T = 527ºC + 273 = 800K$

To find :-
   (i). The total rate of energy emission.(E_b)
   (ii). The intensity of normal radiation, and (I_{bn})
   (iii). The wavelength of maximum monochromatic emissive power.(λ_{max})

Soln.:-

(i). $E_b = \sigma A T^4$

$= 5.67 \times 10^{-8} X\ 0.12\ X\ 800^4$

$= 2786.9$ W

(ii). $I_{bn} = \frac{E_b}{\pi}$

.   .

$= \frac{\sigma T^4}{\pi} = 7392.5\ ^{W}/_{m^2 sr}$

(iii). λ_{max} , from wien's displacement Law.

$\lambda_{max} = 2898\ \mu mk$

$\lambda_{max} = \frac{2898}{800}$

$= 3.622\ \mu m$

2) Assuming the sun to be a black body emitting radiation with maximum intensity at $\lambda = 0.49\mu m$, calculate the following.
   (i). The surface temperature of the sun and.
   (ii). The heat flux at surface of the sun.

Given Data:-

$\lambda_{max} = 0.49\mu m.$

To find:-
(i). The surface temperature of the sun and.
(ii). The heat flux at surface of the sun.

Soln.:-

(i). The surface temperature of the sun (T):

According to Wien's displacement Law.

$\lambda_{max} T = 2898 \mu mk.$

$$T = \frac{2898}{\lambda_{max}}$$

$$= \frac{2898}{0.49}$$

$$= 5914k$$

(ii). The heat flux at surface of the sun $(E)_{sun}$:

$$(E)_{sun} = \sigma T^4$$

$$= 5.67 \times 10^{-8} \times (5914)^4$$

$$(E)_{sun} = 6.936 \times 10^7 \ {}^{W}/{m^2}$$

3) Calculate the following for an industrial furnace in the form of a black body and emitting radiation at 2500ºC.

(i). Monochromatic emissive power at 1.2 µm length.
(ii). Wavelength at which the emission is maximum
(iii). Maximum emissive power,
(iv). Total emissive power ,
(v). Total emissive power of the furnace if it is assumed as a real surface with emissivity equal to 0.9.

Give Data:-

T = 2500 + 273 = 2773k

λ = 1.2µm

Є = 0.9

To find:-

(i). Monochromatic emissive power at 1.2 µm length.
(ii). Wavelength at which the emission is maximum
(iii). Maximum emissive power,
(iv). Total emissive power ,
(v). Total emissive power of the furnace if it is assumed as a real surface with emissivity equal to 0.9.

Soln.:

(i). Monochromatic emissive power at 1.2 µm length $(E_\lambda)_b$:

According to plank's law.

$C_1 = 3.742 \times 10^2\ {}^{W}/_{m^2}$

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{e^{(C_2/\lambda T)} - 1}$$

$$= \frac{(3.742 \times 10^2)(1.2 \times 10^{-6})^{-5}}{e^{(1.4388 \times 10^{-2}/2773 \times 1.2 \times 10^{-6})} - 1}$$

$$= \frac{1.5038 \times 10^{38}}{74.4776}$$

$$(E_\lambda)_b = 2.014 \times 10^{12}\ {}^{W}/_{m^2}$$

(ii). Wavelength at which the emission is maximum (λ_{max})

According to Wien's displacement Law.

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{2773} = 1.045\ \mu m$$

(iii). Maximum emissive power $(E_\lambda)_{b\ max}$

$$(E_\lambda)_{b\ max} = 1.285 \times 10^{-5}\ T^5$$

$$= 1.285 \times 10^{-5} \times (2773)^5$$

$$= 2.1 \times 10^{12}\ {}^{W}/{m^2} \text{ per m length}$$

(iv). Total emissive power (E_b)

$$E_b = \sigma T^4$$

$$= 5.67 \times 10^{-8} (2773)^4$$

$$= 3.352 \times 10^6\ {}^{W}/{m^2}$$

(v). Total emissive power (E with emissivity (ϵ) = 0.9

$$E = \epsilon \sigma T^4$$

$$= 0.9 \times 5.67 \times 10^{-8} \times 2773^4$$

.

$= 3.017 \times 10^6\ ^{W}/_{m^2}$

4) Assuming the sun (diameter = 1.4×10^9m) as a black body having a surface temperature of 5750k and at a mean distance of 15×10^{10}m from the earth (diameter = 12.8×10^6m) estimate the following.
   (i). The total energy emitted by the sun.
   (ii). The emission received per m^2 just outside the atmosphere of the earth.
   (iii). The total energy received by the earth if no radiation is blocked by the atmosphere of the earth and,
   (iv). The energy received by a 1.6m X 1.6m solar collector whose normal is inclined at 50ºC to the sun. the energy loss through the atmosphere is 42% and diffuse radiation is 22% of direct radiation :-

Given Data:-

Diameter of sun= 1.4×10^9m

Radius of sun = 0.7×10^9m

Mean distance of the sun from the earth $R = 15 \times 10^{10}$m

Radius of the earth $r_e = 12.8 \times 10^6/2 = 6.4 \times 10^6$m

Surface temp. of the sun $T = 5750$k

To find:-

(i). The total energy emitted by the sun
(ii). The emission received per m^2
(iii). The total energy received by the earth
(iv). The energy received by the solar collector.

. .

Soln.:-

(i). The total energy emitted by the sun:

$$E_b = \sigma A T^4$$

$$= 5.67 \times 10\text{-}8 \times 4\pi r_s^2 \times (5750)^4$$

$$= 5.67 \times 10^{-8} \times 4\pi \times (0.7 \times 10^9)^2 \times (5750)^4$$

$$= 3.816 \times 10^{26} \text{ w.}$$

(ii). The emission received per m^2

$$\frac{E_b}{A} = \frac{3.816 \times 10^{26}}{4\pi R^2}$$

$$= \frac{3.816 \times 10^{26}}{4\pi(15 \times 10^{10})^2}$$

$$= 1349.6 \; {}^{W}/{m^2}$$

(iii). The total energy received by the earth :

Assume the earth a spherical body, the energy received by it will be proportional to the perpendicular projected area,

Energy recived by the earth $= 1349.6 \text{ X } \pi r_e^2$

$= 1349.6 \text{ X } \pi \text{ X } (6.4 \text{ X } 10^6)^2$

$= 1.736 \text{ X } 10^{17}$ w

(iv). The energy received by the solar collector:

% of the direct energy reading

the earth = (1- 0.42) X 100

= 58%

The direct energy reading

the earth = 0.58 X 1349.6

$= 782.77\ ^{w}/m^2$

% of diffuse radiation = 0.22 X 782.77

$= 172.21\ ^{w}/m^2$

Total radiation reaching the

Collector = 782.77 + 172.21

$= 955\ ^{w}/_{m^2}$

Projected area $= A \cos\phi$

$= 1.6 \times 1.6 \cos 50$

$= 1.961\ m^2$

Energy received by the

Solar Collector $= 955 \times 1.961$

$= 1872.7\ w$

5) A black body is kept at a temperature of 727˚ C. Estimate the fraction of the thermal radiation emitted by the surface in the wavelength band 1 and 5 μ

Given Data:-

727˚ C + 273 = 1000 K

$\lambda_1 = 1\ \mu$

$\lambda_2 = 5\ \mu$

To find:-

The fraction of thermal radiation.

Solution:-

$\lambda_1 T = 1 \times 1000 = 1000\ \mu k$

$\lambda_2 T = 5 \times 1000 = 5000\ \mu k$

[From table HMT data book pg]

corresponding 1000 μk

$F_o - \lambda_1 T = \dfrac{E_{bo} - \lambda_1 T}{\sigma T^4}$

$= 0.3 \times 10^{-3}$

$= 0.0003.$

Corresponding to 5500μk,

$F_o - \lambda_2 T = \dfrac{E_{bo} - \lambda_2 T}{\sigma T^4}$

$= 0.6337$

$\sigma T^4 = (5067 \times 10^{-8})(10000)^4$

$= 56.7 kw/m^2$

$F\lambda_1 T - \lambda_2 T = (F_o - \lambda_2 T) - (F_o - \lambda_1 T)$

$= 0.6337 - 0.0003$

.

$= 0.6334$

$E_b(\lambda_1 - \lambda_2) = 0.6334 \times 56.7$

$= 35.9\ kw/m^2$

6) It is observed that the intensity of the radiation emitted by the sun is maximum at a wavelength of 0.5μ. Assuming the sun to be a black body, estimate the surface temperature and emissive power.

Given Data:-

$\lambda = 0.5\mu$

To find

(i). Temperature of surface
(ii). Emissive power

Solution

According to Wien's Displacement law

$$\lambda_{max} T = 0.289 \times 10^{-2} mk$$

$$T = \frac{0.289 \times 10^{-2}}{\lambda_{max}}$$

$$T = \frac{0.289 \times 10^{-2}}{0.5 \times 10^{-6}}$$

$$T = 5780\ k$$

. .

.

Using Stefan Boltzman law

$$(E_b)_{sun} = \sigma T^4$$

$$= 5.67 \times 10^{-8} \times (5780)^4$$

$$= 63.3 Mw/m^2$$

7) A Gray surface is maintained at a temperature of 827˚C. If the maximum spectral emissive power at a temperature is 1.37 X 10^{10} W/m3 determine the emissivity of the body and the wavelength corresponding to the maximum spectral intensity of radiation.

Given Data:-

$$`T = 827^\circ C + 273 = 1100K$$

$$E_{\lambda max} = 1.37 \times 10^{10}\ W/m^3$$

To find

(i) Emissivity

(ii) λ_{max}

Soln.:-

According to Wein's law,

$$\frac{Eb\ \lambda_{max}}{T^5} = 1.307 \times 10^{-5}$$

$$Eb\ \lambda max = 1.307 \times 10^{-5} \times (1100)^5$$

$$= 2.1 \times 10^{10} W/m^3$$

$$\text{Emissivity} = \frac{E\ \lambda_{max}}{E_b\ \lambda_{max}}$$

. .

$$= \frac{1.37 \times 10^{10}}{2.1 \times 10^{10}}$$

$= 0.65$

According to Wein law

$\lambda_{max} T = 0.289 \times 10^{-2} mk$

$$\lambda_{max} = \frac{0.289 \times 10^{-2}}{T}$$

$$= \frac{0.289 \times 10^{-2}}{1100}$$

$= 2.627\mu$

Result:-

(i) $E = 0.65$

(ii) $\lambda_{max} = 2.67\mu$

8) A Pyrometer records the temperature of the body as 1400˚C with a red light filter ($\lambda = 0.65\ \mu$). Find the true temperature of the body .If it emissivity at 0.65μ is 0.6.

Given Data:-

$T = 1400^{\circ}C$

$\lambda = 0.65\ \mu$

E = 0.6

To find:-

True temperature of the body

Soln.:-

According to the plank's law

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{e^{(C_2/\lambda T_b)} - 1}$$

For gray body

$$(E_\lambda)_b = \frac{C_1 \epsilon_\lambda \lambda^{-5}}{e^{(c_2/\lambda T)} - 1}$$

$$(E_\lambda)_b = E_\lambda$$

$$\frac{C_1 \lambda^{-5}}{e^{(C_2/\lambda T_b)} - 1} = \frac{C_1 \epsilon \lambda^{-5}}{e^{(C_2/\lambda T)} - 1}$$

$$\frac{C_1 \lambda^{-5}}{e^{(C_2/\lambda T_b)} - 1} = \frac{C_1 \lambda^{-5}}{\frac{1}{\epsilon_\lambda}\left[e^{(C_2/\lambda T)} - 1\right]}$$

$$e^{(C_2/\lambda T_b)} - 1 = \frac{1}{\epsilon_\lambda}\left[e^{(C_2/\lambda T)} - 1\right]$$

$$\epsilon_\lambda \left[e^{(C_2/\lambda T_b)} - 1\right] = e^{(C_2/\lambda T)} - 1$$

$$\epsilon_\lambda \left[e^{(C_2/\lambda T_b)} - 1\right] + 1 = e^{(C_2/\lambda T)}$$

$$\ln\left[\epsilon_\lambda \left(e^{(C_2/\lambda T_b)} - 1\right) + 1\right] = C_2/\lambda T$$

$$\ln\epsilon_\lambda + \ln e^{(C_2/\lambda T_b)} - \ln(1) + \ln(1) = C_2/\lambda T$$

$$\ln\epsilon_\lambda + C_2/\lambda T_b = C_2/\lambda T$$

$$C_2/\lambda T = C_2/\lambda T_b + \ln\epsilon_\lambda$$

$$1/T = 1/T_b + \lambda/C_2 \ln\epsilon_\lambda$$

$$T = \frac{1}{1/T_b + \lambda/C_2 \ln\epsilon_\lambda}$$

$$T = \frac{1}{{}^{1}/_{1673} + \left[{0.65 \times 10^{-6}}/{1.439 \times 10^{-2}}\ \ln(0.6)\right]}$$

= 1740 K

= 1467ºC

Result:

The temperature of the body = 1467ºC

9) A metallic bar at 37ºCis placed inside an oven whose interior is maintained at a temperature of 1100K the absorptivity of the bar (at 37ºC) a function of the temperature of incident radiation and a few representative values are Given Data below.

Temp (k)	310	700	1100
α	0.8	0.68	0.52

Estimate the rate of absorption and emission by the metallic bar.

Given Data:-

Temp of metallic bar = 37ºC

Temp at oven = 1100K

α (at 1100K) = 0.52

α (at 310K) = 0.8

To find:-

.

(i) rate of absorption
(ii) rate of emission

Soln.:-

(i). Rate of absorption
(ii).

Q_a $= \alpha.\sigma\, T^4$

$= 0.52 \times (5.67 \times 10^{-8})1100^4$

$= 43.15$ k W/m^2

(iii). Rate of emission
(iv).

Q_e $= E\, \sigma\, T^4$

According to Kirchoff's law

$\epsilon = \alpha$

$\epsilon = 0.8$ (at 37˚C) or 310 K

Qe $= E\, \sigma T^4$

$= 0.8 \times 5.67 \times 10^{-8} \times (310)^4$

$= 418.9\ ^{W}/_{m^2}$

. .

Radiation exchange between Surfaces:

The radiation heat transfer between different types of surfaces both in non participating and participating media and the following assumptions made.

(i) All surfaces have uniform properties over their whole extent
(ii) Each surface is considered to be either gray or black.
(iii) The absorptivity of surface is independent of the temperature of the source of the incident radiation and equals is emissivity and
(iv) Radiation and reflection process is diffuse.

Radiation exchange between two black bodies separated by a non absorbing medium:

Lets us consider heat exchange between elementary between dA_1 and dA_2 of two black radiating bodies, separated by a non absorbing medium, and having area A_1 and A_2 and temperature T_1 and T_2 respectively.

The elementary areas at a distances r apart and the normal to these areas make angles θ_1 and θ_2 with the line joining them.

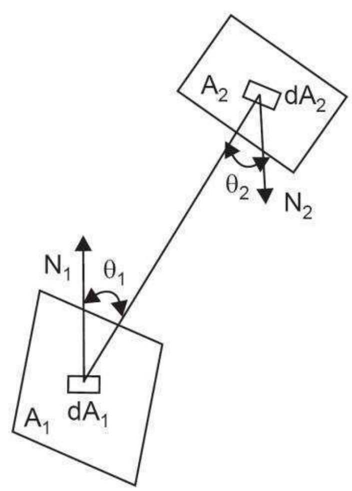
Projected area $dA_1 \cos\theta$.

Energy leaving dA_1 in direction θ_1

$= I_{n1} dA_1 \cos\theta_1$

In_1 = Intensity of radiation at surface A_1

$$In_1 = \frac{E_{b1}}{\pi}$$



Let d_{W1} be subtended at dA_1 by dA_2, and d_{W2} subtended at dA_2 by dA_1,

So,

$$d_{W1} = \frac{dA_2 \cos\theta_2}{r^2}$$

.

$$d_{W2} = \frac{dA_1 \cos\theta_1}{r^2}$$

The rate of radiant energy leaving dA_1 and striking on dA_2 is Given Data by,

$$dQ_{1\text{-}2} = In_1\, d_{A1} \cos\theta_1 \times d_{W1}$$

$$= In_1\, d_{A1} \cos\theta_1 \times \frac{dA_2 \cos\theta_2}{r^2}$$

$$dQ_{1\text{-}2} = \frac{In_1 \cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2}$$

The rate of energy radiated by dA_2 and absorbed by dA_1 is Given Data by,

$$dQ_{2\text{-}1} = In_2\, d_{A2} \cos\theta_2 \times d_{W2}$$

$$= In_2\, d_{A2} \cos\theta_2 \times \frac{dA_1 \cos\theta_1}{r^2}$$

$$dQ_{2\text{-}1} = \frac{In_1 \cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2}$$

The net rate of heat transfer between dA_1 and dA_2 is,

. .

$dQ_{1-2} = dQ_{1-2} – dQ_{2-1}$

$$= \frac{In_1 \cos\theta_1 \cos\theta_2 dA_1\, dA_2}{r^2} - \frac{In_2 \cos\theta_1 \cos\theta_2 dA_1\, dA_2}{r^2}$$

$$dQ_{1-2} = \frac{(In_1 - In_2) \cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2}$$

$$dQ_{1-2} = \frac{(Eb_1 - Eb_2)}{\pi} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{r^2}$$

the net flow is the difference between the quantities

$$Q_{1-2} = (E_{b1} - E_{b2}) \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

This is also equal to $(Eb_1\text{-}Eb_2)A_1F_{12}$

Shape factor relationships

As the shape factor values are available for limited geometric situations only, it becomes necessary to use some basic relationships between shape factors to evaluate the shape factor for other connected geometries. For example shape factor value are available for perpendicular surfaces with a common edge. But shape factor values for perpendicular surfaces will meet only if extended, is needed. The shape factor relationship together with the reciprocity theorem are used to evaluate shape of factor value in such situations.

Consider surfaces A_1, A_2 and A_3 shown in Fig. The first of such rules is

$$F_{1-2} = \frac{1}{A_1} \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2\, dA_1\, dA_2}{\pi r^2}$$

$$F_{3-1,\,2} = F_{3-1} + F_{3-2}$$

This is an obvious relation as the energy reaching an area is the sum of energies reaching individual parts of the area. Generally

$$F_{i-j,\,k,\,l,\,m,\,n.....} = F_{i-j} + F_{i-k} + F_{i-j} + F_{i-m} + ...$$

Multiplying the RHS and LHS of equation, by the area

$$A_3F_{3-1,\,2} = A_3F_{3-1} + A_3F_{2-3}$$

Then using the reciprocity theorem,

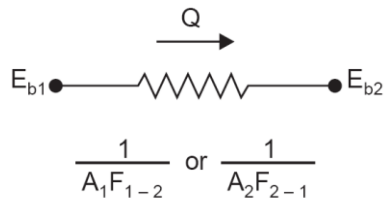
$$(A_1 + A_2)\, F_{1,2-3} = A_1\, F_{1-3} + A_2\, F_{2-3}$$

RADIANT HEAT EXCHANGE BETWEEN BLACK SURFACES

To determine radiant heat exchange between black surfaces

$$Q_{1-2} = A_1 F_{1-2} (E_{b1} - E_{b2})$$

This can be represented by **electrical analogue** shown in



The temperatures and geometric parameters should be specified for solution

HEAT EXCHANGE BY RADIATION BETWEEN GRAY SURFACES

The calculation of heat exchange involves the summation of the energy absorbed on each incidence on the surface. Additional resistance to heat absoption is introduced by the emissivity/absorptivity of the surface. In order to simplify the process of calculation two new terms called **“radiosity” and “irradiation”** are introduced. **Irradiation (G) is the total radiation incident upon a surface per unit time and unit area (W/m2).**

This quantity consists of the radiation from other surfaces and the reflected radiation from other surfaces. **Radiosity (J) is defined as the total radiation that leaves a surface per unit time and unit area (W/m2).** This quantity consists of the emissive power of the surface and the reflections by the surface. From these definitions we get

$$J = \varepsilon E_b + \rho G$$
$$\rho = 1 - \alpha = 1 - \varepsilon$$
$$J = \varepsilon E_b + (1 - \varepsilon) G$$

In the calculation of heat transfer between gray surfaces an important assumption is that radiosity and irradiation are uniform over the surface. Considering a heat balance over the surface, the net energy leaving the surface is the difference between radiosity and irradiation.

$$Q/A_1 = J_1 - G_1$$

After simplifying

$$Q = \frac{\varepsilon_1 A_1}{1 - \varepsilon_1} (E_{b1} - J_1) = \frac{E_{b1} - J_1}{(1 - \varepsilon_1)/A_1 \varepsilon_1} \text{ W}$$

Radiation Shields:

Any surface placed in between two surfaces introduces additional surface resistance reducing heat transfer. This is known as radiation shield and is extensively used in practice.

UNIT - III

1. **Two large plates are maintained at a temperature of 900 K and 500 K respectively. Each plate has area of 6^2. Compare the net heat exchange between the plates for the following cases.**
   (i) Both plates are black
   (ii) Plates have an emissivity of 0.5

Given:

$T_1 = 900$ K

$T_2 = 500$ K

$A = 6\ m^2$

To find:

(i) $(Q_{12})_{net}$ Both plates are black $\epsilon = 1$

(ii) $(Q_{12})_{net}$ Plates have an emissivity of $\epsilon = 0.5$

Solution

Case (i) $\epsilon_1 = \epsilon_2 = 1$

$$(Q_{12})_{net} = \frac{A\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$(Q_{12})_{net} = \frac{A \times 5.67\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$(Q_{12})_{net} = \frac{6 \times 5.67\left[\left(\frac{900}{100}\right)^4 - \left(\frac{500}{100}\right)^4\right]}{\frac{1}{1} + \frac{1}{1} - 1}$$

$$(Q_{12})_{net} = 201.9 \times 10^3 W$$

Case (ii) $\epsilon_1 = \epsilon_2 = 0.5$

$$(Q_{12})_{net} = \frac{A\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$(Q_{12})_{net} = \frac{6 \times 5.67\left[\left(\frac{900}{100}\right)^4 - \left(\frac{500}{100}\right)^4\right]}{\frac{1}{0.5} + \frac{1}{0.5} - 1}$$

$$(Q_{12})_{net} = 67300\ W$$

.

2. The sun emits maximum radiation at λ = 0.52 μ. Assuming the sun to be a black body, calculate the surface temperature of the sun. Also calculate the monochromatic emissive power of the sun's surface.

Given:

$\lambda_{max} = 0.52\ \mu = 0.52 \times 10^{-6}$ m

To find:

(i) Surface temperature, T.

(ii) Monochromatic emissive power, $E_{b\lambda}$

(iii) Total emissive power, E

(iv) Maximum emissive power, E_{max}

Solution:

1. **From Wien's law,**

$\lambda_{max}\ T = 2.9 \times 10^{-3}$ mK

[From HMT Data book, page no 82, sixth editions]

$$T = \frac{2.9 \times 10^{-3}}{0.52 \times 10^{-6}}$$

$$T = 5576\ K$$

2. **Monochromatic emissive power, ($E_{b\lambda}$)**

From Planck's law,

$$E_{b\lambda} = \frac{c_1 \lambda^{-5}}{\left[e^{\left(\frac{c_2}{\lambda T}\right)} - 1\right]}$$

[From HMT Data book, page no 82, sixth editions]

Where

$$c_1 = 0.374 \times 10^{-15}\ Wm^2$$

$$c_2 = 14.4 \times 10^{-3}\ mK$$

$$\lambda = 0.52 \times 10^{-6}\ \text{m}$$

T = 5576 K

$$E_{b\lambda} = \frac{0.374 \times 10^{-15}[0.52 \times 10^{-6}]^{-5}}{\left[e^{\left(\frac{14.4 \times 10^{-3}}{0.52 \times 10^{-6} \times 5576}\right)} - 1\right]}$$

$$E_{b\lambda} = 6.9 \times 10^{13}\ \text{W/m}^2$$

3. Total emissive power

$$E = \sigma T^4 = 5.67 \times 10^{-6} \times (5576)^4 \quad \text{W/m}^2$$

4. Maximum emissive power

$E_{max} = 1.285 \times 10^{-5}\ T^5 = 1.285 \times 10^{-5} (5576)^5$ W/m^2

3. A 70 mm thick metal plate with a circular hole of 35 mm diameter along the thickness is maintained at a uniform temperature 250 ° C. Find the loss of energy to the surroundings at 27 °, assuming the two ends of the hole to be as parallel discs and the metallic surfaces and surroundings have black body characteristics.

Given:

$$r_2 = (r_3) = \frac{35}{2} = 17.5\, mm = 0.0175\, m$$

L = 70 mm =0.07 m

$T_1 = 250 + 273 = 523$ K

$T_{surr} = 27 + 273 = 300$ K

Let suffix 1 designate the cavity and the suffices 2 and 3 denote the two ends of 35 mm dia. Hole which are behaving as discs. Thus,

$$\frac{L}{r_2} = \frac{0.07}{0.0175} = 4$$

$$\frac{r_3}{L} = \frac{0.0175}{0.07} = 0.25$$

The configuration factor, $F_{2\text{-}3}$ is 0.065

Now, $F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} = 1$ …….By summation rule

But, $F_{2\text{-}2} = 0$

$$F_{2\text{-}1} = 1 - F_{2\text{-}3} = 1 - 0.065 = 0.935$$

Also,

$A_1 F_{1\text{-}2} = A_2 F_{2-1}$ .....By reciprocating theorem

$$F_{1-2} = \frac{A_2 F_{2-1}}{A_1} = \frac{\pi \times (0.0175)^2 \times 0.935}{\pi \times 0.035 \times 0.07} = 0.1168$$

$F_{1-3} = F_{1-2} = 0.1168$ ………. By symmetry

The total loss of energy = loss of heat by both ends

$$= A_1 F_{1-2}\, \sigma \left(T_1^{\,4} - T_{surr}^{\,4}\right) + A_1 F_{1-3}\, \sigma \left(T_1^{\,4} - T_{surr}^{\,4}\right)$$

therefore ($F_{1-2} = F_{1-3}$)

$$= 2 A_1 F_{1-2}\, \sigma \left(T_1^{\,4} - T_{surr}^{\,4}\right)$$

$$= 2\,(\pi \times 0.035 \times 0.07) \times 0.1168 \times 5.6 \left[\left(\frac{523}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right] = 6.8\ \text{W}$$

November 2011

4. The filament of a 75 W light bulb may be considered as a black body radiating into a black enclosure at 70^0 C. the filament diameter is 0.10 mm and length is 5 cm. considering the radiation, determine the filament temperature .

Given:

Q = 75W = 75 J/s

$T_2 = 70 + 273 = 343$ K

d = 0.1 mm

l = 5 cm

Area = π dl

Solution:

$\epsilon = 1$ for black body

$$Q = \sigma\epsilon A\left(T_1^{\ 4} - T_2^{\ 4}\right)$$

$$75 = 5.67 \times 10^{-8} \times 1 \times \pi \times 0.1 \times 10^{-3} \times 5 \times 10^{-2}\left(T_1^{\ 4} - (343)^4\right)$$

$$T_1^{\ 4} = \frac{75}{8.906 \times 10^{-13}} + (343)^4$$

$$T_1 = 3029\,K$$

$$T_1 = 3029 - 273 = 2756^0C$$

November 2011 (old regulation)

5. **Two parallel plates of size 1.0 m by 1.0 m spaced 0.5 m apart are located in a very large room, the walls of which are maintained at a temperature of 27^0 C. one p[late is maintained at a temperature of 900^0 C and other at 400^0C. their emissivities are 0.2 and 0.5 respectively. If the plates exchange heat between themselves and the surroundings, find the net heat transfer to each plate and to the room. Consider only the plate surface facing each other.**

Given:

Three surfaces (2 plates and wall)

$$T_1 = 900^0C = 1173\,K$$

$$T_2 = 400^0C = 673\,K$$

$$T_3 = 27^0C = 300\,K$$

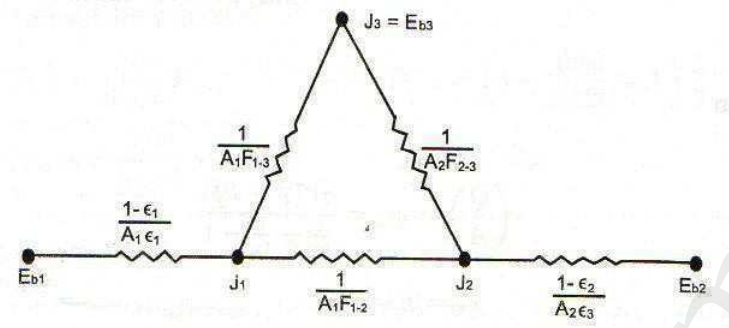
$$A_1 = A_2 = 1.0\,m^2$$

$$\epsilon_1 = 0.2$$

$$\epsilon_2 = 0.2$$

Room size is much larger than the plate size

$$Surface\ resistance\ \frac{1-\epsilon_3}{\epsilon_3\ A_3} = 0\ and\ then\ E_{b3} = J_3$$



1. To find the shape factor F_{1-2}.

Ratio of smaller side to distance between plane.

$$= \frac{1}{0.5} = 2$$

Corresponding to 2 and curve 2 in HMT Data book

$$F_{1-2} = 0.4$$

By summation rule

$F_{1-2} + F_{1-3} = 1$

$F_{1-3} = 1 - F_{1-2}$

$F_{1-3} = 1 - 0.4 = 0.6$

$F_{1-3} = 0.6$

$F_{2-1} + F_{2-3} = 1$

$F_{2-3} = 1 - F_{2-1}$

$F_{2-3} = 1 - 0.4$

$F_{2-3} = 0.6$

The resistances are

$$R_1 = \frac{1-\epsilon_1}{\epsilon_1\ A_1} = \frac{1-0.2}{0.2 \times 1} = 4.0$$

$$R_2 = \frac{1-\epsilon_2}{\epsilon_2\ A_2} = \frac{1-0.5}{0.5 \times 1} = 1.0$$

$$R_{1-2} = \frac{1}{A_1 F_{1-2}} = \frac{1}{1 \times 0.4} = 1.0$$

$$R_{1-3} = \frac{1}{A_1 F_{1-3}} = \frac{1}{1 \times 0.6} = 1.67$$

$$R_{2-3} = \frac{1}{A_2 F_{2-3}} = \frac{1}{1 \times 0.6} = 1.67$$

To find radiosities $J_1 J_2$ and J_3, find total emissive power (E_b)

$$E_{b1} = \sigma T_1{}^4 = 5.67\left(\frac{1173}{100}\right)^4 = 107.4\ kW/m^2$$

$$E_{b2} = \sigma T_2{}^4 = 5.67\left(\frac{673}{100}\right)^4 = 11.7\ kW/m^2$$

$$E_{b3} = \sigma T_3{}^4 = 5.67\left(\frac{300}{100}\right)^4 = 0.46\ kW/m^2$$

Node J_1 :

$$\frac{E_{b1}-J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}} + \frac{J_2-J_1}{\frac{1}{A_1F_{1-2}}} + \frac{E_{b3}-J_1}{\frac{1-\epsilon_1}{A_1F_{1-3}}} = \frac{107.4-J_1}{4.0} + \frac{J_2-J_1}{2.5} + \frac{0.46-J_1}{1.67}$$

J_1 in terms of J_2

Node J_2

$$\frac{J_1-J_2}{R_{1-2}} + \frac{E_{b3}-J_2}{R_{2-3}} + \frac{E_{b2}-J_2}{R_2}$$

Here J_1 in terms of J_2

$$J_2 = 11.6\text{kW/m}^2$$

And $J_1 = 25.0\text{kW/m}^2$

The total heat loss by plate (1) is

$$Q_1 = \frac{E_{b1}-J_1}{\frac{1-\epsilon_1}{\epsilon_1 A_1}} = \frac{107.4-25}{4.00} = 20.6\ kW$$

The total heat loss by plate (2) is

$$Q_1 = \frac{E_{b2}-J_2}{\frac{1-\epsilon_2}{\epsilon_2 A_2}} = \frac{11.7-11.6}{1.00} = 0.1\ kW$$

The total heat received by the room is

$$Q_3 = Q_1 + Q_2$$

$$Q_3 = 20.6 + 0.1$$

$$Q_3 = 20.7\ kW$$

Net energy lost by the plates = Absorbed by the room.

6. Two large parallel planes with emissivities of 0.3 and 0.5 are maintained at temperatures of 527^0 C and 127^0C respectively. A radiation shield having emissivities of 0.05 on both sides is placed between them. Calculate

(i) Heat transfer rate between them without shield.

(ii) Heat transfer rate between them with shield.

Given:

$\epsilon_1 = 0.3$

$\epsilon_2 = 0.5$

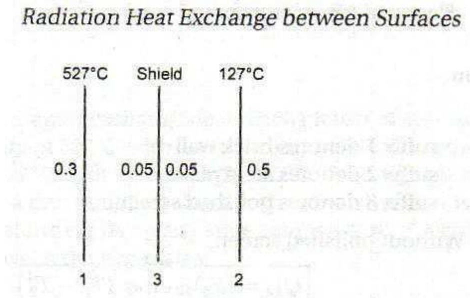
$\epsilon = 0.05$

$T_1 = 527 + 273 = 800$ K

$T_2 = 127 + 273 = 400$ K

Find:

$Q_{w/o\ shield}$ and $Q_{with\ shield}$



Solution:

$$(Q_{12})_{net\ without\ shield} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{5.67\left(\left(\frac{800}{100}\right)^4 - \left(\frac{400}{100}\right)^4\right)}{\frac{1}{0.3} + \frac{1}{0.5} - 1}$$

$$(Q_{12})_{net\ without\ shield} = 5024.5\ W/m^2$$

$$(Q_{12})_{with\ shield} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right) + \left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right)}$$

$$= \frac{5.67(8^4 - 4^4)}{\left(\frac{1}{0.3} + \frac{1}{0.05} - 1\right) + \left(\frac{1}{0.05} + \frac{1}{0.5} - 1\right)}$$

$$(Q_{12})_{with\ shield} = 859.45\ W/m^2$$

November 2012

7. Emissivities of two large parallel plates maintained at 800° C and 300° C are 0.3 and 0.5 respectively. Find the net radiant heat exchange per square meter of the plates. If a polished aluminium shield (Є = 0.05) is placed between them. Find the percentage of reduction in heat transfer.

Given:

$T_1 = 800^o$ C +273 = 1073 K

$T_2 = 300^o$ C +273 = 573 K

$\varepsilon_1 = 0.3$

$\varepsilon_2 = 0.3$

Radiation shield emissivity $\varepsilon_3 = 0.05$

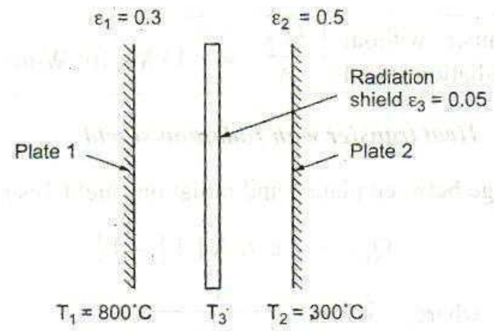


Fig. 4.27.

To find:

(i) Net radiant heat exchange per square meter $\left[\frac{Q_{12}}{A}\right]$

(ii) Percentage of reduction in heat transfer due to radiation shield.

Solution:

Case I: **Heat transfer without radiation shield**:

Heat exchange between two large parallel plates without radiation shield is given by

$$Q_{12} = \vec{\varepsilon}\sigma A\left[T_1^{\ 4} - T_2^{\ 4}\right]$$

Where

$$\vec{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{1}{\frac{1}{0.3} + \frac{1}{0.5} - 1}$$

$$\vec{\varepsilon} = 0.230$$

$$Q_{12} = 0.230 \times 5.67 \times 10^{-8} \times A \times [(1073)^4 - (573)^4]$$

Heat transfer without radiation shield $\left[\frac{Q_{12}}{A}\right]$ **= 15.8 X10^{3}W/m^2**

Case II: **Heat transfer with radiation shield**:

Heat exchange between plate I and radiation shield 3 is given by

$$Q_{13} = \vec{\varepsilon}\sigma A\left[T_1^{\ 4} - T_3^{\ 4}\right]$$

Where

$$\vec{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$Q_{13} = \frac{\sigma A\left[T_1^{\ 4} - T_3^{\ 4}\right]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \qquad \ldots\ldots\ldots\ldots..(1)$$

Heat exchange between radiation shield 3 and plate 2 is given by

$$Q_{32} = \vec{\varepsilon}\sigma A[T_3{}^4 - T_2{}^4]$$

Where

$$\vec{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$Q_{32} = \frac{\sigma A[T_3{}^4 - T_2{}^4]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \qquad \ldots\ldots\ldots\ldots..(2)$$

We know that,

$$Q_{13} = Q_{32}$$

$$\frac{\sigma A[T_1{}^4 - T_3{}^4]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\sigma A[T_3{}^4 - T_2{}^4]}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{(1073)^4 - T_3{}^4}{\frac{1}{0.3} + \frac{1}{0.05} - 1} = \frac{T_3{}^4 - (573)^4}{\frac{1}{0.05} + \frac{1}{0.5} - 1}$$

$$= \frac{(1073)^4 - T_3{}^4}{22.3} = \frac{T_3{}^4 - (573)^4}{21}$$

$$= 2.78 \times 10^{13} - 21\, T_3{}^4 = 22.3\, T_3{}^4 - 2.4 \times 10^{12}$$

$$= 3.02 \times 10^{13} = 43.3\, T_3{}^4$$

Shield temperature $T_3 = 913.8\, K$

Heat transfer with radiation shield Q_{13} =

$$Q_{13} = \frac{\sigma A[T_1{}^4 - T_3{}^4]}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$Q_{13} = \frac{5.67 \times 10^{-8} \times A \times [(1073)^4 - (913.8)^4]}{\frac{1}{0.3} + \frac{1}{0.05} - 1}$$

$$\frac{Q_{13}}{A} = 1594.6\ W/m^2 \ \ldots\ldots\ldots\ldots\ldots(3)$$

$$\%\ of\ reduction\ in\ heat\ transfer = \frac{Q_{without\ shield} - Q\ with\ shield}{Q_{without\ shield}}$$

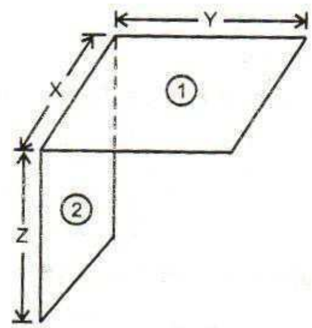
$due\ to\ radiation\ shield$

$$= \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= \frac{15.8 \times 10^3 - 1594.6}{15.8 \times 10^3}$$

$$= 0.899 = 89.9\ \%$$

8. **Two rectangular surfaces are perpendicular to each other with a common edge of 2 m. the horizontal plane is 2 m long and vertical plane is 3 m long. Vertical plane is at 1200 K and has an emissivity of 0.4. the horizontal plane is 18^0 C and has a emissivity of 0.3. Determine the net heat exchange between the planes.**



Solution:

$Q_{12} = ?$

$$Q_{12} = (Fg)_{1-2} A_1 \sigma (T_1{}^4 - T_2{}^4)$$

Here

$$(Fg)_{1-2} = \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{1-2}} + \left(\frac{1-\epsilon_2}{\epsilon_2}\right)\frac{A_1}{A_2}}$$

A_1 = Area of horizontal plane = XY = 2x2 = 4 m^2

A_2 = Area of vertical plane = ZX = 3x2 = 6 m^2

Both surfaces have common edge for which

$$\frac{Z}{X} = \frac{3}{2} = 1.5 \;\; and \;\; \frac{Y}{X} = \frac{2}{2} = 1$$

From HMT data book the shape factor $F_{1-2} = 0.22$

$$Q_{12=} \frac{4 \times 5.67\left(\left(\frac{1200}{100}\right)^4 - \left(\frac{18+273}{100}\right)^4\right)}{\frac{1-0.4}{0.4} + \frac{1}{0.22} + \left(\frac{1-0.3}{0.3}\right)\frac{4}{6}}$$

$$Q_{12=} \; 61657.7 \; W$$

9. **Determine the view factor (F_{14}) for the figure shown below.**

From Fig. We know that

$A_5 \quad = \quad A_1 + A_2$

$A_6 \quad = \quad A_3 + A_4$

Further,

$A_5 F_5 \quad = \quad A_1 F_{1-6} + A_2 F_{2-6}$

$[\because A_5 = A_1 + A_2;\; F_{5-6} = F_{1-6} + F_{2-6}]$

$= \quad A_1 F_{1-3} + A_1 F_{1-4} + A_2 F_{2-6}$

$[\because A_5 = A_1 + A_2;\ F_{5-6} = F_{1-6} + F_{2-6}]$

$A_5 F_{5-6} = A_5 F_{5-3} - A_2 F_{2-3} + A_1 F_{1-4} + A_2 F_{2-6}$

$[\because A_1 = A_5 + A_2;\ F_{1-3} = F_{5-3} - F_{2-3}]$

$\Rightarrow A_1 F_{1-4} = A_5 F_{5-6} - A_5 F_{5-3} + A_2 F_{2-3} - A_2 F_{2-6}$

$$\Rightarrow F_{1-4} = \frac{A_5}{A_1}[F_{5-6} - F_{5-3}] + \frac{A_2}{A_1}[F_{2-3} - F_{2-6}] \qquad ......(1)$$

[Refer HMT Data book, page No.94 (sixth Edition)

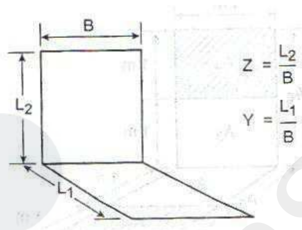


Fig. 4.52.

Shape factor for the area A_5 and A_6

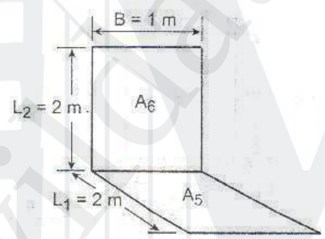


Fig. 4.53.

$$Z = \frac{L_2}{B} = \frac{2}{1} = 2$$

$$Y = \frac{L_1}{B} = \frac{2}{1} = 2$$

Z value is 2, Y value is 2. From that, we can find corresponding shape factor value is 0.14930. (From tables)

$F_{5-6} = 0.14930$

Shape factor for the area A_5 and A_3

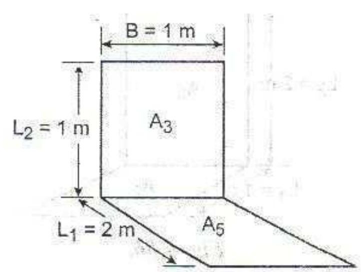


Fig. 4.54.

$$Z = \frac{L_2}{B} = \frac{1}{1} = 1$$

$$Y = \frac{L_1}{B} = \frac{2}{1} = 2$$

F_{5-3} = 0.11643

Shape factor for the area A_2 and A_3

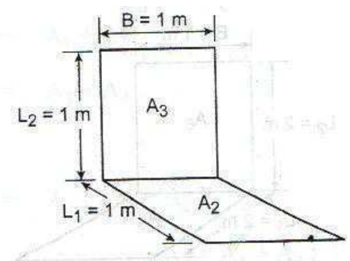


Fig. 4.55.

$$Z = \frac{L_2}{B} = \frac{1}{1} = 1$$

$$Y = \frac{L_1}{B} = \frac{1}{1} = 1$$

F_{2-3} = 0.20004

Shape factor for the area A_2 and A_6

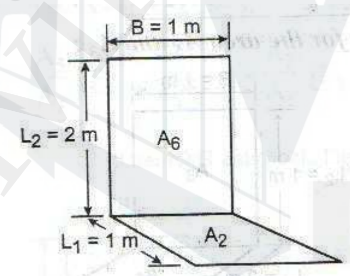


Fig. 4.56.

$$Z = \frac{L_2}{B} = \frac{2}{1} = 1$$

$$Y = \frac{L_1}{B} = \frac{1}{1} = 1$$

F_{2-6} = 0.23285

Substitute F_{5-6}, F_{5-3}, F_{2-3}, and F_{2-6} values in equation (1),

$$\Rightarrow F_{1-4} = \frac{A_5}{A_1}[0.14930 - 0.11643] + \frac{A_2}{A_1}[0.20004 - 0.23285]$$

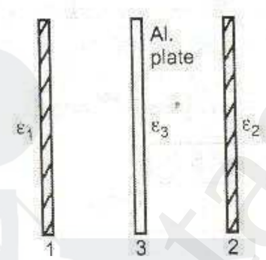
$$= \frac{A_5}{A_1}[0.03287] - \frac{A_2}{A_1}[0.03281]$$

$$F_{1-4} = 0.03293$$

Result :

View factor, $F_{1-4} = 0.03293$

10. Calculate the net radiant heat exchange per m^2 area for two large parallel plates at temperatures of 427^0 C and 27^{0}C. $\epsilon_{(hot\ plate)} = 0.9$ and $\epsilon_{(cold\ plate)} = 0.6$.If a polished aluminium shield is placed between them, find the % reduction in the heat transfer $\epsilon_{(shield)} = 0.4$



Net radiation heat transfer $(Q_{12})_{net}$ = ?

Given:

$T_1 = 427 + 273 = 700$ K

$T_2 = 27 + 273 = 300$ K

$\epsilon_1 = 0.9$

$\epsilon_2 = 0.6$

$\epsilon = 0.4$

Solution:

$$(Q_{12})_{net\ without\ shield} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{5.67\left(\left(\frac{700}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right)}{\frac{1}{0.9} + \frac{1}{0.6} - 1}$$

$$(Q_{12})_{net} = 7399.35\ W/m^2$$

Percentage reduction in the heat transfer flow

$$= \frac{Reduction\ in\ heat\ flow\ due\ to\ shield}{Net\ heat\ flow} \times 100$$

$$Reduction\ in\ heat\ flow\ due\ to\ shield = (Q_{12})_{net} - (Q_{13})_{net}$$

$$(Q_{13})_{net\ with\ shield} = \frac{A\sigma\left(T_1{}^4 - T_3{}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1}$$

To find T₃ shield temperature $(Q_{13})_{net} = (Q_{32})_{net}$

$$\frac{A\sigma\left(T_1{}^4 - T_3{}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{A\sigma\left(T_3{}^4 - T_2{}^4\right)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1}$$

Let $\frac{T_3}{100} = x$

$$\frac{\left(\left(\frac{700}{100}\right)^4 - \left(\frac{T_3}{100}\right)^4\right)}{\frac{1}{0.9} + \frac{1}{0.4} - 1} = \frac{\left(\left(\frac{T_3}{100}\right)^4 - \left(\frac{300}{100}\right)^4\right)}{\frac{1}{0.4} + \frac{1}{0.6} - 1}$$

$$\frac{2401 - x^4}{1.11 + 25 - 1} = \frac{x^4 - 81}{25 + 1.67 - 1}$$

$$x^4 = 1253.8$$

$$\frac{T_3}{100} = (1253.8)^{1/4} = 5.95 \qquad (or)$$

$$T_3 = 595\ K$$

$$(Q_{13})_{net} = \frac{\sigma\left(T_1{}^4 - T_3{}^4\right)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1}$$

$$= \frac{5.67\left(\left(\frac{700}{100}\right)^4 - \left(\frac{595}{100}\right)^4\right)}{\frac{1}{0.9} + \frac{1}{0.4} - 1}$$

$$(Q_{13})_{net} = 2492.14\ W/m^2$$

$$Reduction\ in\ heat\ flow\ due\ to\ shield = (Q_{12})_{net} - (Q_{13})_{net}$$

= 7399.35 -2492.14

= 4907.21 W/m^2

Percentage reduction = $\frac{4907.21}{7399.35}\ x\ 100 = 66.32\%$

11. There are two large parallel plane with emissivities 0.3 and 0.8 exchange heat. Find the percentage reduction when an aluminium shield of emissivity 0.04 is p[laced between them. Use the method of electrical analogy.

Solution:

Given:

$\epsilon_1 = 0.3$

$\epsilon_2 = 0.8$

$\epsilon = 0.04$

Percentage reduction in heat transfer

$$= \frac{Reduction\ in\ heat\ transfer\ due\ to\ shield}{Net\ heat\ transfer\ rate} \times 100$$

$$Reduction\ in\ heat\ flow\ due\ to\ shield = \frac{(Q_{12})_{net} - (Q_{13})_{net}}{(Q_{12})_{net}}$$

$$(Q_{12})_{net\ w/o\ shield} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{\frac{1}{0.3} + \frac{1}{0.8} - 1} = \frac{\sigma(T_1{}^4 - T_2{}^4)}{3.58}$$

$$(Q_{13})_{net\ with\ shield} = \frac{\sigma(T_1{}^4 - T_3{}^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{\sigma(T_1{}^4 - T_3{}^4)}{\frac{1}{0.3} + \frac{1}{0.04} - 1} = \frac{\sigma(T_1{}^4 - T_3{}^4)}{27.33}$$

Percentage reduction in heat transfer

$$= 1 - \frac{(Q_{13})}{(Q_{12})}$$

Here T_3 = in terms of T_1 and T_2

To find the values of T_3

$$(Q_{13})_{net} = (Q_{32})_{net}$$

$$\frac{T_1{}^4 - T_3{}^4}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{T_3{}^4 - T_2{}^4}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1}$$

$$\frac{T_1{}^4 - T_3{}^4}{27.33} = \frac{T_3{}^4 - T_2{}^4}{25.25}$$

$$T_1{}^4 - T_3{}^4 = \frac{27.33}{25.25}(T_3{}^4 - T_2{}^4)$$

$$T_3{}^4 = 0.48\,(T_1{}^4 + 1.08T_2{}^4)$$

Percentage reduction in heat transfer

$$= 1 - \frac{(Q_{13})}{(Q_{12})}$$

$$= 1 - \frac{\sigma(T_1{}^4 - T_3{}^4)/27.33}{\sigma\,(T_1{}^4 - T_2{}^4)/27.33}$$

$$= 1 - \frac{3.58}{27.33}\left[\frac{\left({T_1}^4 - {T_3}^4\right)}{\left({T_1}^4 - {T_2}^4\right)}\right]$$

$$= 1 - 0.131\left[\frac{{T_1}^4 - 0.48\left({T_1}^4 + 1.08\,{T_2}^4\right)}{\left({T_1}^4 - {T_2}^4\right)}\right]$$

$$= 1 - 0.131\left[\frac{0.52\left({T_1}^4 - \ {T_2}^4\right)}{\left({T_1}^4 - {T_2}^4\right)}\right]$$

$$= 1 - 0.131(0.52)$$

$$= 0.932$$

$$= 93.2\%$$

V: MASS TRANSFER

Mass transfer is different from the flow of fluid which was discussed in previous chapters. Mass transfer is the flow of molecules from one body to another when these bodies are in contact or within a system consisting of two components when the distribution of materials is not uniform. When a copper plate is placed on a steel plate, some molecules from either side will diffuse into the other side **Usually mass transfer takes place from a location where the particular component is proportionately high to a location where the component is proportionately low.** Mass transfer may also take place due to potentials other than concentration difference. But in this chapter only transfer due to concentration gradient is discussed

PROPERTIES OF MIXTURE

In a mixture consisting of two or more materials the mass per unit volume of any component is called mass concentration of that component. If there are two components *A* and *B*, then the mass concentration of *A* is

$$M_a = \frac{\text{mass of in the mixture}}{\text{volume of the mixture}}$$

and concentration of *B*,

$$m_b = \frac{\text{mass of in the mixture}}{\text{volume of the mixture}}$$

The total mass concentration is $m_a + m_b$, which is also the density of the mixture.
Mass concentration can also be expressed in terms of individual and total densities of the mixture *i.e.*,

$$m_a = \frac{\rho_a}{\rho}$$

where ρ_a is the density of *A* in the mixture and ⟩ is the density of the mixture.
It is more convenient to express the concentration in terms of the molecular weight of the component.
Mole fraction N_a can be expressed as

$$N_a = \frac{\text{Number of moles of component A}}{\text{Total number of moles in the mixture}}$$

$$\text{Number of Mole} = \text{mass / molecular weight}$$

For gases as $\rho_i = \dfrac{P}{R_i T}$

Or $N_i = \dfrac{P}{RT_i}$

where R is universal gas constant.
At the temperature *T* of the mixture then

$$N_i \propto P_i$$

where

$$C_a = \frac{N_a}{Nt} = \frac{P_a}{P_T}$$

where P_a is the partial pressure of *A* in the mixture and P_T is the total pressure of the mixture. C_a is the mole concentration of *A* in the mixture.
Also $C_a + C_b = 1$ for a two component mixture.

DIFFUSION MASS TRANSFER

Consider a chamber in which two different gases at the same pressure and temperature are kept separated by a thin barrier. When the barrier is removed, the gases will begin to diffuse into each other's, volume. After some time, a steady condition of uniform mixture would be reached. This type of diffusion can occur in solids also. The rate in solids will be extremely slow. Diffusion in these situations occurs at the molecular level and the governing equations are similar to those in heat conduction where energy transfer occurs at the molecular level

FICK'S LAW OF DIFFUSION

The Fick's law can be stated as

$$N_a = -D_{ab}\frac{dC_a}{dx}$$

Where N_a—> number of moles of 'a' diffusing perpendicular to area A, moles/m² sec

D_{ab}—> Diffusion coefficient or mass diffusivity, m²/s, a into b

C_a —> mole concentration of '*a*' moles/m³

x —> diffusion direction

The diffusion coefficient is similar to thermal diffusivity, 〈 and momentum diffusivity *v*. Number of moles multiplied by the molecular mass (or more popularly known as molecular weight) will provide the value of mass transfer in kg/s.

Equation above can also be written as

$$\frac{m_a}{A} = -D_{ab}\cdot\frac{d\rho_a}{dx}$$

The value of D_{ab} for certain combinations of components are available in literature. It can be proved that $D_{ab} = D_{ba}$. When one molecule of '*A*' moves in the *x* direction, one molecule of '*B*' has to move in the opposite direction. Otherwise a macroscopic density gradient will develop, which is not sustainable

$$\frac{N_a}{A} = -D_{ab}\frac{dC_a}{dx}$$

$$\frac{N_b}{A} = -D_{ba}\frac{dC_b}{dx} = -D_{ba}\frac{d(1-C_a)}{dx} = D_{ba}\frac{dC_a}{dx}$$

$$\frac{N_a}{A} = -\frac{N_b}{A} \text{ and so } D_{ab} = D_{ba}$$

EQUIMOLAL COUNTER DIFFUSION

The total pressure is constant all through the mixture. Hence the difference in partial pressures will be equal. The Fick's equation when integrated for a larger plane volume of thickness *L* will give

$$\frac{N_a}{A} = D_{ab}\frac{(C_{a1}-C_{a2})}{L}$$

$$\frac{N_b}{A} = D_{ba}\frac{(C_{b2}-C_{b1})}{L}$$

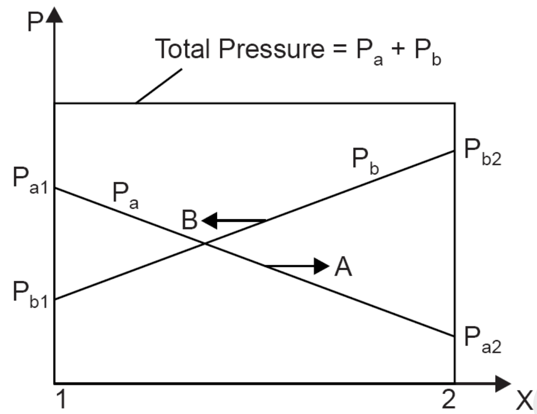
$$\frac{N_b}{A} = -\frac{N_a}{A}, \text{ and } (C_{a1}-C_{a2}) = (C_{b2}-C_{b1}),$$

D_{ab} equals D_{ba} Where C_{a1} and C_{b1} are the mole concentrations at face 1 and C_{a2} and C_{b2} are mole concentrations at face 2 which is at a distance *L* from the first face. When applied to gases,

$$\frac{N_a}{A} = \frac{D}{\Re T}\cdot\frac{P_{a1} - P_{a2}}{(x_2 - x_1)}$$

Where P_{a1} and P_{a2} are partial pressures of component '*A*' at x_1 and x_2 and R is the universal gas constant in J/kg mol *K*. *T* is the temperature in absolute units. The distance should be expressed in metre.

The partial pressure variation and diffusion directions are shown in Fig



DIFFUSION OF ONE COMPONENT INTO A STATIONARYCOMPONENT OR UNIDIRECTIONAL DIFFUSION

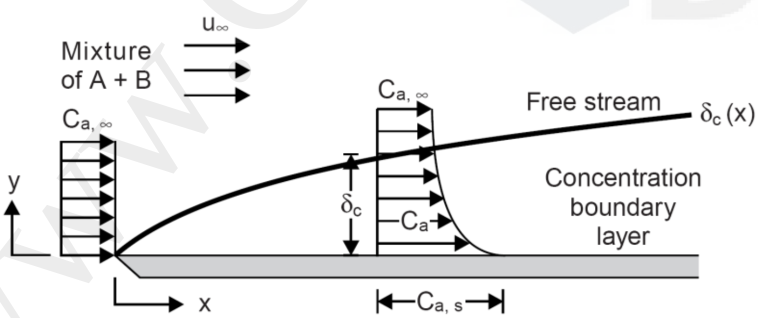
In this case one of the components diffuses while the other is stationary. For steady conditions the mass diffused should be absorbed continuously at the boundary. In certain cases this is not possible. The popular example is water evaporating into air. In this case, as mentioned earlier, a bulk motion replaces the air tending to accumulate at the interface without being absorbed, causing an increase in the diffusion rate. The diffusion equation for gases can be derived as (with '*a*' as the diffusing medium and P = total pressure)

$$\frac{N_a}{A} = \frac{P}{\Re T} \cdot \frac{D}{(x_2 - x_1)} \cdot \ln\left(\frac{P - P_{a2}}{P - P_{a1}}\right)$$

CONVECTIVE MASS TRANSFER

When a medium deficient in a component flows over a medium having an abundance of the component, then the component will diffuse into the flowing medium. Diffusion in the opposite direction will occur if the mass concentration levels of the component are interchanged. In this case a boundary layer develops and at the interface mass transfer occurs by molecular diffusion (In heat flow at the interface, heat transfer is by conduction).

Velocity boundary layer is used to determine wall friction. Thermal boundary layer is used to determine convective heat transfer. Similarly concentration boundary layer is used to determine convective mass transfer. The Fig. shows the flow of a mixture of components *A* and *B* with a specified constant concentration over a surface rich in component *A*. A concentration boundary layer develops. The concentration gradient varies from the surface to the free stream. At the surface the mass transfer is by diffusion. Convective mass transfer coefficient h_m is defined by the equation, where h_m has a unit of m/s.



$$\text{Mole flow} = h_m(C_{as} - C_{a\infty})$$

By similarity the solutions for boundary layer thickness for connective mass transfer can be obtained. This is similar to the heat transfer by analogy. In this case, in the place of Prandtl number Schmidt number defined by

$$Sc = \nu/D_{ab} \ldots$$

Nondimensionalising the equation leads to the condition as below:

$$\delta_m = f(\text{Re, Sc})$$

$$\text{Sh} = f(\text{Re, Sc})$$

where Sherwood number Sh is defined as

$$Sh = h_m x / D_{ab}$$

In the laminar region flow over plate :

$$\delta_{mx} = \frac{5x}{Re_x^{1/2}} \quad Sc^{-1/3}$$

$$Sh_x = \frac{h_m x}{D_{ab}} = 0..332\ Re^{1/2} Sc^{1/3}$$

In turbulent region Re > $5x10^5$

$$\delta_m = \delta_v$$

$$Sh_x = 0.0296\ Rex^{0.8} Sc^{1/3}$$

$$\overline{ShL} = 0.037 Re_L^{0.8} Sc^{1/3}$$

SIMILARITY BETWEEN HEAT AND MASS TRANSFER

It is possible from similarity between the heat convection equation and mass convection equation to obtain value of h_m. (*i.e.,* called as Lewis number)

$$\frac{h}{h_m} = \rho C_p / Le^{2/3}$$

Where

$$Le = \frac{\alpha}{D}$$

Many of the correlation in heat transfer can be applied to mass transfer under similar condition, by replacing Nusselt number by Sherwood number and Prandtl number by Schmidt number

UNIT-V

1. Water flows at the rate of 65 kg/min through a double pipe counter flow heat exchanger. Water is heated from 50^o C to75^{o}C by an oil flowing through the tube. The specific heat of the oil is 1.780 kj/kg.K. The oil enters at 115^{o}C and leaves at 70^{o}C.the overall heat transfer co-efficient is 340 W/m2K.calcualte the following

1. Heat exchanger area
2. Rate of heat transfer

Given:

Hot fluid – oil, (T_1 , T_2)

Cold fluid – water (t_1 , t_2)

Mass flow rate of water (cold fluid), m_c = 65 kg/min

= 65/60 kg/s

m_c = 1.08 kg/s

Entry temperature of water, t_1 =50^o C

Exit temperature of water, t_2 =75^o C

Specific heat of oil (Hot fluid), C_{ph} = 1.780 KJ/kg K

= 1.780 x 10^3 J/kg K

Entry temperature of oil, T_1 =115^o C

Exit temperature of water, T_2 =70^o C

Overall heat transfer co-efficient, U = 340 w/m^2 K

To find:

1. Heat exchanger area, (A)
2. Rate of heat transfer, (Q)

Solution:

We know that,

Heat transfer, $Q = m_c\, c_{pc}\, (t_2 – t_1)$ (or) $m_h\, c_{ph}\, (T_1 - T_2)$

$Q = m_c\, C_{pc}\, (t_2 – t_1)$

Q = 1.08 x 4186 x (75 – 50)

[Specific heat of water, c_{pc} = 4186 J/kg K]

Q = 113 x 10^3 W

We know that,

Heat transfer, $Q = U \times A\, (\Delta T)_m$ …….. (1)

[From HMT data book page no:152(sixth edition)]

Where

ΔT_m – Logarithmic Mean Temperature Difference. (LMTD)

For counter flow,

$$\Delta T_{lm} = \frac{[(T_1 - t_2) - (T_2 - t_1)]}{ln\left[\frac{T_1 - t_2}{T_2 - t_1}\right]}$$

$$\Delta T_{lm} = \mathbf{28.8^oC}$$

Substitute $(\Delta T)_{lm}$, Q and U values in Equn (1)

(1) $Q = UA\,(\Delta T)_{lm}$

$113 \times 10^3 = 340 \times A \times 28.8$

$A = 11.54\ m^2$

2. **A parallel flow heat exchanger is used to cool 4.2 kg/min of hot liquid of specific heat 3.5 kJ/kg K at 130^o C. A cooling water of specific heat 4.18 kJ/kg K is used for cooling purpose of a temperature of 15^o C. The mass flow rate of cooling water is 17 kg/min. calculate the following.**
    1. Outlet temperature of liquid
    2. Outlet temperature of water
    3. Effectiveness of heat exchanger

Take

Overall heat transfer co-efficient is 1100 W/m^2 K.

Heat exchanger area is $0.30m^2$

Given:

Mass flow rate of hot liquid, $m_h = 4.2$ kg/min

$m_h = 0.07$ kg/s

Specific heat of hot liquid, $c_{ph} = 3.5$ kJ/kg K

$c_{ph} = 3.5 \times 10^3$ J/kg K

Inlet temperature of hot liquid, $T_1 = 130^0$ C

Specific heat of hot water, $C_{pc} = 4.18$ kJ/kg K

$C_{pc} = 4.18 \times 10^3$ J/kg K

Inlet temperature of hot water, $\mathbf{t_1} = 15^0$ C

Mass flow rate of cooling water, $m_c = 17$ kg/min

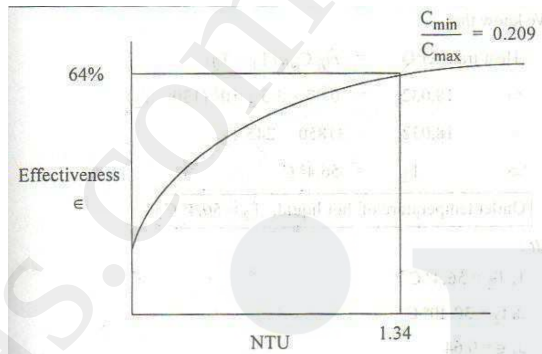
$$\mathbf{m_c = 0.28 \text{ kg/s}}$$

Overall heat transfer co – efficient, U = 1100 w/m^2 K

Area, A = 0.03 m^2

To find :

1. Outlet temperature of liquid, (T_2)
2. Outlet temperature of water, (t_2)
3. Effectiveness of heat exchanger, (ε)



Solution :

Capacity rate of hot liquid, $C_h = m_h \times C_{ph}$

$= 0.07 \times 3.5 \times 10^3$

$\mathbf{C_h = 245 \text{ W/K}}$ **……… (1)**

Capacity rate of water, $C_c = m_c \times C_{pc}$

$= 0.28 \times 4.18 \times 10^3$

$\mathbf{C_c = 1170.4 \text{ W/K}}$ **……… (2)**

From (1) and (2),

$C_{min} = 245$ W/K

$C_{max} = 1170.4$ W/K

=> $\frac{C_{min}}{C_{max}} = \frac{245}{1170.4} = 0.209$

$\mathbf{\frac{C_{min}}{C_{max}} = 0.209}$ **…………. (3)**

Number of transfer units, NTU = $\frac{UA}{C_{min}}$

[From *HMT data book page no. 152]*

=> NTU = $\frac{1100 \times 0.30}{245}$

NTU = 1.34 ……………(4)

To find effectiveness ε, refer HMT data book page no 163

(Parallel flow heat exchanger)

From graph,

$X_{axis} \rightarrow$ NTU = 1.34

Curve $\rightarrow \frac{C_{min}}{C_{max}} = 0.209$

Corresponding Y_{axis} value is 64 %

i.e., $\boldsymbol{\varepsilon} = \mathbf{0.64}$

from HMT data Book

$$\in = \frac{m_h cp_h (T_1 - T_2)}{C_{min} (T_1 - t_1)}$$

$$0.64 = \frac{130 - T_2}{130 - 15}$$

$T_2 = 56.4\ ^oC$

To find t_2

$m_h\ cp_h(T_1\text{-}T_2) = m_c Cp_c\ (t_2\text{-}t_1)$

$0.07 \times 3.5 \times 10^3\ (130\text{-}56.4) = 0.28 \times 4186\ (t_2\text{-}15)$

$t_2 = 30.4^oC$

Maximum possible heat transfer

$Q_{max} = C_{min} (T_1 - t_1)$

$= 245\ (130 - 15)$

Q_{max} = 28.175 W

Actual heat transfer rate

$Q = \varepsilon \times Q_{max}$

$= 0.64 \times 28.175$

Q = 18.032 W

We know that,

Heat transfer, $Q = m_c\ C_{pc}(t_2 - t_1)$

=> $18.032 = 0.28 \times 4.18 \times 10^3\ (t_2 - 15)$

=> $18.032 = 1170.4\ t_2 - 17556$

=> $t_2 = 30.40^oC$

Outlet temperature of cold water, $t_2 = 30.40^oC$

We know that,

Heat transfer, $Q = m_h\ C_{ph}(T_1 - T_2)$

=> $18.032 = 0.07 \times 3.5 \times 10^3\ (130 - T_2)$

=> $18.032 = 31850 - 245\ T_2$

=> $T_2 = 56.4^oC$

Outlet temperature of hot liguid, $T_2 = 56.4^oC$

3.Hot chemical products (C_{ph} = 2.5 kJ/kg K) at 600° C and at a flow rate of 30 kg/s are used to heat cold chemical products (C_p = 4.2 kJ/kg K) at 200° C and at a flow rate 20 kg/s in a parallel flow heat exchanger. The total heat transfer is 50 m^2 and the overall heat transfer coefficient may be taken as 1500 W/m^2 K. calculate the outlet temperatures of the hot and cold chemical products.

Given: Parallel flow heat exchanger

$T_{h1} = 600^o$ C ; m_h = 30 kg/s

C_{ph} = 2.5 kJ/kg K

$T_{c1} = 100^o$C ; m_c 28 kg/s

C_{pc} = 4.2kJ/kg K

A = 50m^2

U = 1500 W/m^2K

Find:

(i) T_{h2} (ii) T_{c2} ?

Solution

The heat capacities of the two fluids

$C_h = m_h c_{ph}$ = 30 x 2.5 =75 kW/K

$C_c = m_c c_{pc}$ = 28 x 4.2 = 117.6 kW/K

The ratio $\frac{C_{min}}{C_{max}} = \frac{75}{117.6} = 0.64$

NTU = $\frac{UA}{Cmin} = \frac{1500 \ x \ 50}{75 \ x \ 10^3}$ = 1.0

For a parallel flow heat exchanger, the effectiveness from Fig. 13.15 corresponding to $\frac{C_{min}}{C_{max}}$ and NTU

∈ = 0.48

We know that

$$\in = \frac{Actual\ heat\ transfer}{Max.possible\ heat\ transfer}$$

$$= \frac{m_h C_{ph}\,(T_{h1} - T_{h2})}{C_{min}\,(T_{h1} - T_{c1})}$$

$$\in = \frac{(T_{h1} - T_{h2})}{(T_{h1} - T_{c1})}$$

$$0.48 = \frac{600 - T_{h2}}{600 - 100}$$

$T_{h2} = 360^oC$

We know that

Heat lost by the hot product = Heat gained by the cold product

$$m_h c_{ph} (T_{h1} - T_{h2}) = m_c c_{ph} (T_{c2} - T_{c1})$$

$$75(600 - 360) = 117.6\ (T_{c2} - 100)$$

$$\boldsymbol{T_{c2} = 253.06^o\ C}$$

4. Estimate the diffusion rate of water from the bottom of a tube of 10mm diameter and 15cm long into dry air 25°C. Take the diffusion coefficient of water through air as 0.235 x $10^{-4}m^2/s$

Given:

$D = 0.255 \times 10^{-4} m^2/s$

Area (A) $= \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.01)^2 = 7.85 \times 10^{-5}\ m^2$

$R_o = 8314$ J/kg – mole K

T = 25 + 273 = 298 K

M_w = molecular weight of water = 18

P = Total pressure = $1.01325 \times 10^5\ N/m^2$

$X_2 - X_1 = 0.15m$

P_{w1} = partial pressure at 25° C = $0.03166 \times 10^5\ N/m^2$

$P_{w2} = 0$

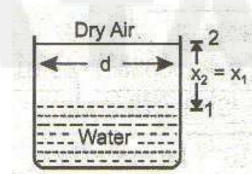
Find:

Diffusion rate of water (or) Mass transfer rate of water.

Solution

We know that

Molar rate of water (M_a)



$$M_a = \frac{DA}{R_o T} \cdot \frac{P}{x_2 - x_1} \ln\left(\frac{P_{a2}}{P_{a1}}\right)$$

$$= \frac{0.255 \times 10{-}4 \times 7.85 \times 10{-}5 \times 1.01325 \times 105}{8314 \times 298 \times 0.15} \times \left(\frac{1.01325 - 0}{1.01325 - 0.03166}\right)$$

Here $P_{a2} = P - P_{w2}$, $P_{a1} = P - P_{w1}$

$$\mathbf{M_a = 1.72 \times 10^{-11}\ kg\text{-}mole/s}$$

Mass transfer rate of water (or) Diffusion rate of water } = Molar rate of water X molecular weight of steam

$M_w = 1.72 \times 10^{-11} \times 18$

Diffusion rate of water (M_w) = 3.1×10^{-10} kg/s

5. A vessel contains a binary mixture of O_2 and N_2 with partial pressure in the ratio of 0.21 and 0.79 at 15°C. The total pressure of the mixture is 1.1 bar. Calculate the following

1. Molar concentration
2. Mass densities
3. Mass fractions
4. Molar fractions.

Given:

T = 15 + 273 = 288 K

P = 1.1bar = 1.1 x 10^5 N/m^2

P_{o_2} = 0.21 bar

P_{N_2} = 0.21 bar

Solution

1. To find Molar concentration (C_{o_2} and C_{o_2})

$$C_{O_2} = \frac{P_{o_2}}{R_o T} = \frac{0.21 \text{ x } 1.1 \text{ x } 10^{\ 5}}{8314 \text{ x } 288}$$

$$\mathbf{C_{O_2} = 0.00965\ kg\ mole/m^3}$$

$$C_{N_2} = \frac{P_{N_2}}{R_o T} = \frac{0.79 \text{ x } 1.1\text{x } 10^{\ 4}}{8314 \text{ x } 288}$$

$$\mathbf{C_{N_2} = 0.0363\ kg\ mole/m^3}$$

2. To find mass densities (p_{o_2} and p_{N_2})

P = MC

Where, M: Molecular weight

$$P_{o_2} = M_{o_2} \text{ x } C_{o_2} = 32 \text{ x } 0.00965$$

$$\mathbf{P_{o_2} = 0.309\ kg/m^3}$$

$$P_{N_2} = M_{N_2} \text{ x } C_{N_2} = 28 \text{ x } 0.0363$$

$$\mathbf{P_{N_2} = 1.016\ kg/m^3}$$

3. To find mass fractions (M_{o_2} and M_{N_2})

We know that

$$\rho = \rho_{o_2} + \rho_{N_2} = 0.309 + 1.016$$

$$\boldsymbol{\rho = 1.375\ kg/m^3}$$

$$M_{o_2} = \frac{\rho_{o_2}}{\rho} = \frac{0.309}{1.325}$$

$$\boldsymbol{M_{o_2} = 0.233}$$

$$M_{N_2} = \frac{\rho_{N_2}}{\rho} = \frac{1.016}{1.325}$$

$$\boldsymbol{M_{N_2} = 0.767}$$

4. To find molar fraction (n_{o_2} and n_{N_2})

   We know that

$$C = C_{o_2} + C_{N_2} = 0.00965 + 0.0363$$

$$C = \mathbf{1.375\ kg\ mole/m^3}$$

$$n_{o_2} = \frac{C_{o_2}}{C} = \frac{0.00965}{0.046}$$

$$\boldsymbol{n_{o_2} = 0.21}$$

$$n_{N_2} = \frac{C_{N_2}}{C} = \frac{0.0363}{0.046}$$

$$\boldsymbol{n_{N_2} = 0.79}$$

6. A counter flow heat exchanger is employed to cool 0.55 kg/s (C_p = 2.45 kj/kg^{o}C) of oil from 115^{o}C to 40^{o}C by the use of water. The inlet and outlet temperature of cooling water are 15^{o}C and 75^{o}C respectively. The overall heat transfer coefficient is expected to be 1450 W/m^{2o}C.

Using NTU method, calculate the following:

(i) The mass flow rate of water.

(ii) The effectiveness of heat exchanger.

(iii) The surface area required.

Given:

Counter flow HE

M_h = 0.55 kg/s

C_{p_h} = 2.45kj/kg^{o}C

T_1 = 115^{o}C

T_2 = 40^{o}C

t_1 = 15^{o}C

t_2 = 75^{o}C

U = 1450 W/m^{2o}C

To find:

1.The mass flow rate of water. (m_c)

2.The effectiveness of heat exchanger. (∈)

3.The surface area required.(A)

Solution:

For $\in -NTU$ method from HMT date book

$$\mathbf{Q} = \in \mathbf{C_{min}} (\mathbf{T_1} - \mathbf{t_1})$$

To find m_c

Use energy balance equation.

Heat lost by hot fluid = Heat gained by cold fluid

$$m_h C_{p_h}(T_1 - T_2) = m_c C_{p_c}(t_2 - t_1)$$

0.55 x 2450 (115 - 40) = m_c x 4186 (75 - 15)

m_c = 0.40kg/s

Heat capacity rate of hot fluid = $C_h = m_h - C_{p_h}$

= 0.55 x 2.45

C_h = 1.35 kw/K

Heat capacity rate of cold fluid = $C_c = m_c - C_{p_c}$

= 0.40 x 4.186

C_c = 1.67kw/K

$C_h < C_c$

$C_h = C_{min}$

$$\in = \frac{m_h C_{p_h}(T_1 - T_2)}{C_{min}\ (T_1 - T_2)}$$

$$= \frac{115-40}{115-15}$$

$$\in = 0.75 = 75\%$$

Q = 0.75 x 1350 (115 – 15)

Q = 101.250W

Q = UA $(\Delta T)_{lm}$

A = Q/U$(\Delta T)_{lm}$

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{ln\left[\frac{(T_1 - t_2)}{(T_2 - t_1)}\right]}$$

$$= \frac{(115-75)-(40-15)}{ln\left[\frac{115-75}{40-15}\right]}$$

$(\Delta T)_{lm}$ = 31.9°C

$$A = \frac{101.250}{1450\ x\ 31.9}$$

A = 2.19 m²

7. A pan of 40 mm deep, is filled with water to a level of 20 mm and is exposed to dry air at 30^0 C. Calculate the time required for all the water to evaporate. Take, mass diffusivity is $0.25X10^{-4}$ m^2/s.

Given:

Deep, $(x_2 - x_1)$ = 40 – 20 = 20 mm = 0.020 m

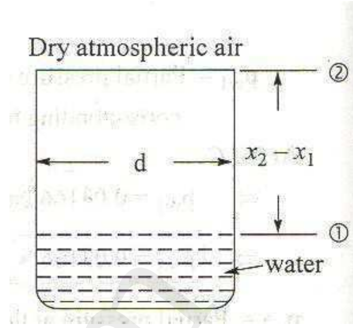
Temperature, T = 30^0 C + 273 = 303 K

Diffusion co- efficient , D_{ab} =$0.25X10^{-4}$ m^2/s.

To find:

Time required for all the water to evaporate, t.

Solution:



We know that, for isothermal evaporation

Molar flux, $\frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{p}{(x_2 - x_1)} \times \ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$ …………(1)

Where,

G – Universal gas constant = 8314 j/kg – mole-K

P – Total pressure = 1 atm = 1.013 bar = $1.013 \text{ X } 10^5$ N/m^2

p_{w1} - Partial pressure at the bottom of the pan Corresponding to saturation temperature 30^oC

At 30^oC

$\Rightarrow p_{w1} = 0.04242$ bar (From steal table page no.2)

$\Rightarrow p_{w1} = 0.4242 \times 10^5 \text{ N/m}^2$

P_{w2} – partial pressure at the top of the pan, which is zero.

$\Rightarrow P_{w2} = 0$

(1) $\Rightarrow \frac{m_a}{A} = \frac{0.25 \times 10^{-4}}{8314 \times 303} \times \frac{1.013 \times 10^5}{0.020} \times \left[\frac{1.013 \times 10^5 - 0}{1.013 \times 10^5 - 0.04242 \times 10^5}\right]$

$\frac{m_a}{A} = 2.15 \times 10^{-6} \frac{kg - mole}{s}$

For unit Area, A = $1m^2$

Molar rate of water, $m_a = 2.15 \times 10^{-6} \frac{kg - mole}{sm^2}$

We know that,

Mass Rate of water vapour = Molar Rate of water vapour × Molecular weight of steam

$= 2.15 \times 10^{-6} \times 18.016$

Molar rate of water vapour $= 3.87 \times 10^{-5}$ kg/s-m^2

The total amount of water to be evaporated per m^2 area

$= (0.20 \times 1) \times 1000$

$= 20 \text{ kg/m}^2$ Area

Time required, $t = \frac{20}{\text{Mass rate of water vapour}}$

$$= \frac{20}{3.87\times10^3 s}$$

Result :

Time required for all the water to evaporate, $t=516.79\times10^3 S$

8. A heat exchanger is to be designed to condense an organic vapour at a rate of 500 kg/min. Which is available at its saturation temperature of 355 K. Cooling water at 286 K is available at a flow rate of 60 kg/s. The overall heat transfer coefficient is 475 W/m²C Latent heat of condensation of the organic vapour is 600 kJ/kg. Calculate

1. The number of tubes required, if tubes of 25 mm otuer diameter, 2mm thick and 4.87m long are available, and
2. The number of tube passes, if cooling water velocity (tube side) should not exceed 2m/s.

Given:

d_o = 25 mm = 0.025

d_i = 25-(2×2)= 21 mm = 0.21 m

L = 4.87 m

V = 2 m/s

T_{c1} = 286-273 = 13°C

T_{sat} = T_{h1} = T_{h2} = 355-273 = 82°C

U = 475 /m² K

h_{fg} = 600 kj/kg

$m_h = \frac{500}{60} = 8.33$ kg/s

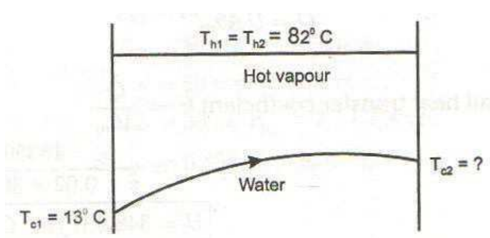
m_c = 60 kg/s

Find

(i) Number of tubes (N)

(ii) Number of tube passes (P)

Solution



$$Q = UA\theta_m = U(\pi d_o LN)\theta m$$

$$Q = m_h h_{fg} = m_c C_{pc} (T_{c2} - T_{c1})$$

i.e. Heat lost by vapour = heat gained by ater

$Q = 8.33 \times 600 \times 10^3$

$Q = m_c c_{pv} (T_{c2} - T_{c1})$

$8.33 \times 600 \times 10^3 = 60 \times 4.18\ (T_{c2} - 13)$

$T_{c2} = 32.9^oC$

$$\therefore\ \theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)}$$

$$\theta_m = \frac{(T_{h1} - T_{c1}) - (T_{h2} - T_{c2})}{\ln\left(\frac{(T_{h1} - T_{c1})}{(T_{h2} - T_{c2})}\right)}$$

$$= \frac{(82 - 13) - (82 - 32.9)}{\ln\left(\frac{(82 - 13)}{(82 - 32.9)}\right)}$$

$$\theta_m = 58.5^o C$$

Heat transfer rate is given by

$$Q = m_h h_{fg} = UA\theta_m$$

$8.33 \times 600 \times 10^3 = 475 \times (\pi \times 0.025 \times 4.87 \times N \times 58.5)$

$N = 470$ Tubes

To find N. of tube passes (P)

$$N = P \times N_p$$

Where

N	:	No. of tubes
P	:	No. of tube passes
N_p	:	No. of tubes in each pass

i.e. The cold water flow passing through each pass.

$$m_c = AV_p N_p$$

$$60 = \frac{\pi}{4} di^2 V_\rho \times N_p$$

$$60 = \frac{\pi}{4} (0.021)^2 \times 2 \times 1000 \times N_p$$

$N_p = 95.5$

We know that

$$N = P \times N_p$$

$\therefore$ No. of passes (P) = $\dfrac{N}{N_p}$

$$= \frac{470}{95.5} = 4.91$$

P = 5

$\therefore$ **Number of passes (P) = 5**

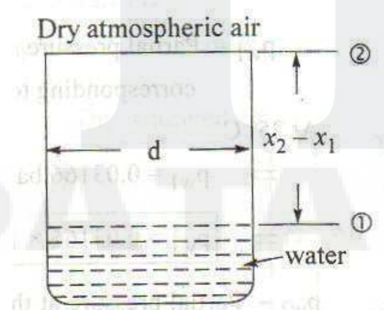
9. An Open pan 20 cm in diameter and 8 cm deep contains water at 25°C and is exposed to dry atmospheric air. If the rate of diffusion of water vapour is 8.54×10^{-4} kg/h, estimate the diffusion co-efficient of water in air.

Given

Diameter d	=	20 cm	=	0.20 m
Length (x_2-x_1) =		8cm	=	0.08 m
Temperature ,T=		25°C+273	=	298 K
Diffusion rate (or)				
Mass rate of water vapour			=	8.54×10^{-4} kg/h
			=	$\dfrac{8.54\times10^{-4}\,kg}{3600\,s}$
			=	2.37×10^{-7} kg/s

To find

Diffusion co-efficient, D_{ab}



Solution

We know that

Molar rate of water vapour

$$\frac{m_a}{A} = \frac{D_{ab}}{GT}\frac{p}{(x_2-x_1)}\times\ln\left[\frac{p-p_{w2}}{p-p_{w1}}\right]$$

$$\Rightarrow m_a = \frac{D_{ab}}{GT}\frac{p}{(x_2-x_1)}\times\ln\left[\frac{p-p_{w2}}{p-p_{w1}}\right]$$

We know that,

Mass rate of water vapour = Molar rate of water vapour + Molecular weight of steam

$$2.37\times10^{-7} = \frac{D_{ab}}{GT}\frac{p}{(x_2-x_1)}\times\ln\left[\frac{p-p_{w2}}{p-p_{w1}}\right]\times 18.016 \quad \text{......... (1)}$$

where,

Area, A = $\dfrac{\pi}{4}d^2$

$$= \frac{\pi}{4}(0.20)^2$$

$$A = 0.0314\ m^2$$

G - Universal gas constant = $8314 \dfrac{J}{kg - mole - K}$

p - Total pressure = 1 atm = 1.013 bar

= $1.013 \times 10^5\ N/m^2$

p_{wl} = Partial pressure at the bottom of the test tube corresponding to saturation temperature 25°C

At 25°C

$\Rightarrow p_{wl}$ = 0.03166 bar [From (R.S. Khurami) Steam table, Page no.2]

$\Rightarrow p_{wl}$ = $0.03166 \times 10^5\ N/m^2$

p_{w2} - Partial pressure at the top of the pan. Here, air is dry and there is no water vapour. So, pw2 – 0.

$\Rightarrow p_{w2} = 0$

(1) 2.37×10^{-7} =

$$\frac{D_{ab} \times 0.0314}{8314 \times 298} \times \frac{1.013 \times 10^5}{0.08} \times in\left[\frac{1.013 \times 10^5 - 0}{1.013 \times 10^5 - 0.03166 \times 10^5}\right] \times 18.016$$

D_{ab} = $2.58 \times 10^{-5}\ m^2/s$

Result:

Diffusion co-efficient, $D_{ab} = 2.58 \times 10^{-5}\ m^2/s$.

10. A counter flow double pipe heat exchanger using super heated steam is used to heat water at the rate of 10500 kg/hr. The steam enters the heat exchanger at 180°C and leaves at 130°C. The inlet and exit temperature of water are 30°C and 80°C respectively. If the overall heat transfer coefficient from steam to water is 814 W/m² K, calculate the heat transfer area. What would be the increase in area if the fluid flow were parallel?

Given

Counter flow heat exchanger

$$\dot{m}_w = \dot{m}_c = \frac{10500}{3600} = 2.917\, kg/s$$

T_1 = 180°C t_1=30°C

T_2 = 130°C t_2 = 80°C

$U = 814\ W/m^2\ K$

Find

(i) Area of heat transfer (A)

(ii) Increase in area

Solution

(i) When the flow is counter:

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)}$$

$$\theta_1 = T_1 - t_2 = 180 - 80 = 100^o C$$

$$\theta_2 = T_2 - t_1 = 130 - 30 = 100^o C$$

LMTD = 0 °C

If LMTD = 0 °C use AMTD

So, AMTD $= \frac{\theta_1 + \theta_2}{2}$ [AMTD: Arithmetic mean temperature difference]

AMTD $= \frac{100+100}{2}$

AMTD = 100°C

θ_m = 100°C

Here ΔT_{lm} = AMTD

∴ To find heat transfer rate

$$Q = U\,A\,\Delta T_{lm}$$

$$Q = \dot{m}_c c_{pc}(t_2 - t_1)$$

$$Q = 2.917\times4.187\times10^3\,(80\text{-}90)$$

$$2.917\times4.187\times10^3\times50 = 814\times A\times100$$

$$A = 7.5\ m^2$$

ii) When the flow is parallel

$$\Delta T_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln[((T_1 - t_1)/(T_2 - t_2))]}$$

$$= \frac{(180-30)-(130-80)}{\ln[(180-30)/(130-80)]}$$

$$= \frac{150-50}{\ln[150/50]} = 91\,^oC$$

Q = U A ΔT_{lm}

or $2.917 \times (4.187\times10^3) \times (80-30) = 814 \times A \times 91$

A = $8.24\, m^2$

$\therefore$ Increase in Area = $\frac{8.24-7.5}{7.5} = 0.0987$ *or* 9.87%