MOHAMED SATHAK A.J. COLLEGE OF ENGINEERING

(Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai)

ISO 9001:2015 Certified by IRQS NABCB QM 006

PH3151 – ENGINEERING PHYSICS

(COMMON TO ALL B.E/B.TECH STUDENTS)

REGULATION – 2021

DEPARTMENT OF PHYSICS

MOHAMED SATHAK AJ COLLEGE OF ENGINEERING

CHENNAI – 603103

PH3151 - ENGINEERING PHYSICS (Syllabus) 2021-regulation Anna University

PH3151	ENGINEERING PHYSICS	LPTC
		3003

OBJECTIVES:

• To make the students effectively to achieve an understanding of mechanics.
• To enable the students to gain knowledge of electromagnetic waves and its applications.
• To introduce the basics of oscillations, optics and lasers.
• Equipping the students to be successfully understand the importance of quantum physics.
• To motivate the students towards the applications of quantum mechanics.

UNIT I — MECHANICS — 9

Multiparticle dynamics: Center of mass (CM) – CM of continuous bodies – motion of the CM – kinetic energy of system of particles. Rotation of rigid bodies: Rotational kinematics – rotational kinetic energy and moment of inertia - theorems of M .I – moment of inertia of continuous bodies – M.I of a diatomic molecule - torque – rotational dynamics of rigid bodies – conservation of angular momentum – rotational energy state of a rigid diatomic molecule - gyroscope - torsional pendulum – double pendulum –Introduction to nonlinear oscillations.

UNIT II — ELECTROMAGNETIC WAVES — 9

The Maxwell's equations - wave equation; Plane electromagnetic waves in vacuum, Conditions on the wave field - properties of electromagnetic waves: speed, amplitude, phase, orientation and waves in matter - polarization - Producing electromagnetic waves - Energy and momentum in EM waves: Intensity, waves from localized sources, momentum and radiation pressure - Cell-phone reception. Reflection and transmission of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence.

UNITIII — OSCILLATIONS, OPTICS AND LASERS — 9

Simple harmonic motion - resonance –analogy between electrical and mechanical oscillating systems - waves on a string - standing waves - traveling waves - Energy transfer of a wave - sound waves - Doppler effect. Reflection and refraction of light waves - total internal reflection - interference – Michelson interferometer –Theory of air wedge and experiment. Theory of laser - characteristics - Spontaneous and stimulated emission - Einstein's coefficients - population inversion - Nd-YAG laser, CO2 laser, semiconductor laser –Basic applications of lasers in industry.

UNIT IV — BASIC QUANTUM MECHANICS — 9

Photons and light waves - Electrons and matter waves –Compton effect - The Schrodinger equation (Time dependent and time independent forms) - meaning of wave function - Normalization –Free particle - particle in a infinite potential well: 1D,2D and 3D Boxes- Normalization, probabilities and the correspondence principle.

UNIT V — APPLIED QUANTUM MECHANICS — 9

The harmonic oscillator(qualitative)- Barrier penetration and quantum tunneling(qualitative)- Tunneling microscope - Resonant diode - Finite potential wells (qualitative)- Bloch's theorem for particles in a periodic potential –Basics of Kronig-Penney model and origin of energy bands.

TOTAL : 45 PERIODS

OUTCOMES: After completion of this course, the students should be able to

• Understand the importance of mechanics.
• Express their knowledge in electromagnetic waves.
• Demonstrate a strong foundational knowledge in oscillations, optics and lasers.
• Understand the importance of quantum physics.
• Comprehend and apply quantum mechanical principles towards the formation of energy bands.

TEXT BOOKS:

1. D.Kleppner and R.Kolenkow. An Introduction to Mechanics. McGraw Hill Education (Indian Edition), 2017.
2. E.M.Purcell and D.J.Morin, Electricity and Magnetism, Cambridge Univ.Press, 2013.
3. Arthur Beiser, Shobhit Mahajan, S. Rai Choudhury, Concepts of Modern Physics, McGraw-Hill (Indian Edition), 2017.

REFERENCES:

1. R.Wolfson. Essential University Physics. Volume 1 & 2. Pearson Education (Indian Edition), 2009.
2. Paul A. Tipler, Physic – Volume 1 & 2, CBS, (Indian Edition), 2004.
3. K.Thyagarajan and A.Ghatak. Lasers: Fundamentals and Applications, Laxmi Publications, (Indian Edition), 2019.
4. D.Halliday, R.Resnick and J.Walker. Principles of Physics, Wiley (Indian Edition), 2015.
5. N.Garcia, A.Damask and S.Schwarz. Physics for Computer Science Students. Springer- Verlag, 2012

UNIT 1

MECHANICS

DEPARTMENT OF SCIENCE AND HUMANITIES

Course Code/Name : PH3151 / ENGINEERING PHYSICS
Regulation : 2021 – R

Course Objective:

1. To make the students effectively to achieve an understanding of mechanics.
2. To enable the students to gain knowledge of electromagnetic waves and its applications.
3. To introduce the basics of oscillations, optics and lasers.
4. Equipping the students to successfully understand the importance of quantum physics.
5. To motivate the students towards the applications of quantum mechanics.

UNIT I MECHANICS

Multi particle dynamics: Center of mass (CM) – CM of continuous bodies – motion of the CM – kinetic energy of system of particles. Rotation of rigid bodies: Rotational kinematics – rotational kinetic energy and moment of inertia - theorems of M .I –moment of inertia of continuous bodies – M.I of a diatomic molecule - torque – rotational dynamics of rigid bodies – conservation of angular momentum –rotational energy state of a rigid diatomic molecule - gyroscope - torsional pendulum – double pendulum –Introduction to nonlinear oscillations.

After completion of this course, the students should be able to

COs	OUTCOMES	RBT
C103.1	Remember the concepts of Mechanics and understand the Fundamentals of static and dynamics of bodies.	K1
C103.2	Understand the properties of electro Magnetic waves and its practical applications.	K2 & K3
C103.3	Demonstrate a strong foundational knowledge, and understand the principles of sound, Light and optics with experimental examples.	K2
C103.4	Understand and deduce the basic quantum concepts and equations.	K2
C103.5	Understand the fundamentals of quantum applications	K2
Revised Bloom's Taxonomy **K1**- Remembering, **K2**- Understanding, **K3**- Applying, **K4**- Analyzing, **K5**- Evaluating, **K6**- Creating		

CO-PO Mapping

COs	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
C103.1	3	2	1									
C103.2	3	2	1									
C103.3	3	2	1									
C103.4	2											
C103.5	1											
Avg	2.4	2	1									

Centre of Mass (CM)

CM shall be understood with the help of following points

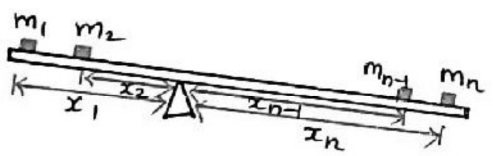
(i) A system consists of many particles with different masses and different position from the reference point.

(ii) The mass of the system is equal to the sum of the mass of each particle in the system.

If the mass of the entire particles of the system is connected at a particular point, then that point is called the centre of mass of the system.

CM in a One Dimensional System

Let us consider a fulcrum placed along the x-axis which is not at equilibrium position.



Let $m_1, m_2, m_3 \ldots m_n$ be mass of particles

$x_1, x_2, x_3 \ldots x_n$ position of particles from the supporting point

The tendency of a mass to rotate with respect to supporting point is called moment of mass.

The moment of mass for an element mass m_n with respect to the fulcrum can be written as $m_n x_n$

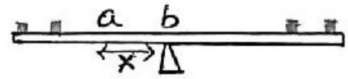
For the equilibrium system, the total moments is given by

$$m_1x_1 + m_2x_2 + \ldots + m_nx_n = \sum_{i=1}^{n} m_i x_i = 0 \quad \text{--- (1)}$$

If the total moment is equal to zero, the CM will lie at the supporting point.

But from the figure, the system is not equilibrium, therefore the supporting point is adjusted to a distance 'X'. to get balanced system.

Unbalanced state



Balanced state

Under equilibrium

$$\text{Eqn (1)} \Rightarrow \sum_{i=1}^{n} m_i x_i - \sum_{i=1}^{n} m_i X = 0$$

$$\sum_{i=1}^{n} m_i X = \sum_{i=1}^{n} m_i x_i$$

$$\boxed{X = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}}$$

$$\text{(ie)} \quad X = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

The system should be moved to a distance of X in order to attain the balanced position.

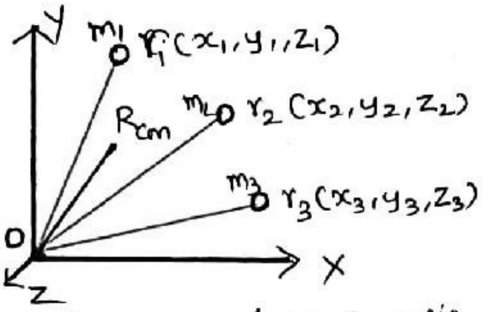
CM in a Three Dimensional System:

Consider a three dimensional system.

Here,

$m_1, m_2, m_3 \ldots \rightarrow$ Masses of particle

$r_1, r_2, r_3 \ldots \rightarrow$ Distance of particle from origin



Centre of mass along x-axis

$$X = \frac{\Sigma m_i x_1}{\Sigma m_i}$$

Centre of mass along Y-axis

$$Y = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

Centre of mass along Z-axis

$$Z = \frac{\Sigma m_i z_i}{\Sigma m_i}$$

Centre of mass of the system in a three dimensional system is

$$\vec{r}_{cm}(x, y, z) = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

$$\boxed{r_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}}$$

where $\vec{r}_i = x_i\vec{i} + y_i\vec{j} + z_i\vec{k}$

Motion of the CM:

The motion of the centre of mass is nothing but the force required to accelerate the system of particles with respect to the centre of mass.

Let $F \rightarrow$ External force acting on the system of particles along x-axis

CM along x-axis will be

$$x_{cm} = \sum_i \frac{m_i x_i}{m_i}$$

$$x_{cm} \Sigma m_i = \Sigma m_i x_i$$

Since $\Sigma m_i = M$

$$M x_{cm} = m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots \quad \text{--- (1)}$$

Differenting we get

$$M \frac{dx_{cm}}{dt} = m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt} + \ldots$$

Differentiating again we get

$$M \frac{d^2 x_{cm}}{dt^2} = m_1 \frac{d^2 x_1}{dt^2} + m_2 \frac{d^2 x_2}{dt^2} + \ldots \quad \text{--- (2)}$$

Since acceleration $a = \frac{d^2x}{dt^2}$

$$M a_{cm} = m_1 a_1 + m_2 a_2 + m_3 a_3 + \ldots \quad \text{--- (3)}$$

According to Newton's second law

$$F = ma$$

$\therefore$ (3) $\Rightarrow$ $F_{cm} = F_1 + F_2 + F_3 + \ldots$

$$\boxed{F_{cm} = \sum_i F_i}$$

The force acting on the centre of mass is equal to sum of forces acting on the system of particles.

Rotational kinetic Energy:

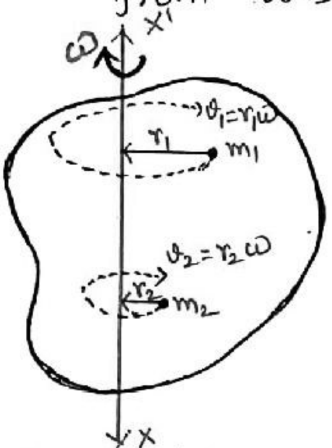
Consider a rigid body rotating about an axis XX'

$\omega \to$ Angular velocity (Constant)
$v \to$ Linear velocity.

Here, the velocity 'v' varies with radial distance from axis XX'

$v_1, v_2, v_3 \ldots$ be linear velocities of particles of masses $m_1, m_2 \ldots$

$r_1, r_2, r_3 \ldots$ Distance of particles from axis of rotation



K.E of particles of mass m_1 $= \frac{1}{2} m_1 v_1^2$

K.E of particles of mass m_2 $= \frac{1}{2} m_2 v_2^2$

$$\text{Total K.E of all particles} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \ldots \quad -(1)$$

The relation between linear velocity and angular velocity is given by

$$v_i = r_i \omega$$

$\therefore$ Eqn (1) becomes

$$\text{Total K.E} = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \ldots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$\text{Total K.E} = \frac{1}{2}\left[\sum m_i r_i^2\right]\omega^2 \quad -(2)$$

The moment of inertia of body about the xx' axis is given by

$$I = \sum m_i r_i^2$$

$\therefore$ Eqn (2) $\Rightarrow$ $\boxed{K.E = \frac{1}{2} I \omega^2}$

The above eqn represents the kinetic energy of the particles in a rigid body.

Theorem of Moment of Inertia

The moment of inertia of various bodies shall be calculated by using the following theorems.

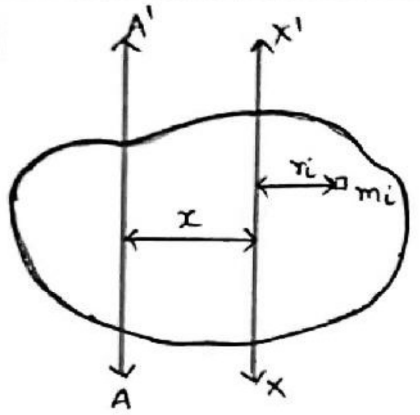
1. Parallel axis theorem
2. Perpendicular axis theorem

Parallel axis theorem:

Theorem:-

It states that the M.I with respect to any axis is equal to the sum of moment of inertia with respect to a parallel axis passing through the CM and the product of mass and square of the perpendicular distance between the parallel axes.

Proof :-



Let M → Mass of body.

G → Centre of mass

AA' → Rotational axis

XX' → Axis passing through cm

x → $\perp^r$ distance between AA' & xx'

Consider a particle of mass mi located at a distance r_i from the xx' axis

M.I of this particle about xx' is

$$dI_{xx'} = m_i r_i^2$$

M.I of the entire body with respect to xx' axis is

$$I_{xx'} = \sum dI_{xx'} = \sum m_i r_i^2 \quad \text{—①}$$

Illy

M.I of the body about AA'

$$I_{AA'} = \sum dI_{AA'} = \sum m_i (r_i + x)^2$$

$$I_{AA'} = \sum m_i (r_i^2 + 2r_i x + x^2)$$

$$I_{AA'} = \sum m_i r_i^2 + \sum 2 m_i r_i x + \sum m_i x^2 \quad \text{—②}$$

Substituting eqn ① in eqn ② we get

$$\therefore I_{AA'} = I_{xx'} + 2x \sum m_i r_i + Mx^2 \quad \text{—③}$$

where $M = \sum m_i$

According to CM of rigid body

$$\sum m_i r_i = 0$$

$$③ \Rightarrow \boxed{I_{AA'} = I_{xx'} + Mx^2}$$

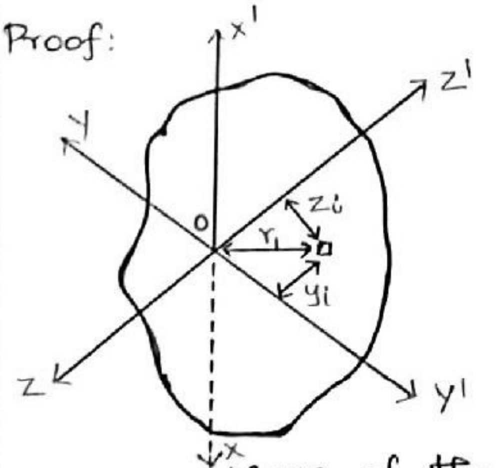
This eqn represents the eqn for parallel axis theorem.

Perpendicular Axis theorem:-

Theorem:-

It states that the MI of a thin plane body about an axis perpendicular to the thin plane surface is equal to the sum of the MI of a thin plane about ~~to~~ two $\perp^r$ axes lying in the surface of the plane.

Proof:



Let M → Mass of the body

xx', yy', zz' → Three mutually $\perp^r$ axes

O → Common point of 3 axes.

yy', zz' are parallel to surface

xx' is $\perp^r$ to surface.

Consider a particle of mass m_i located at a distance r_i from the point 'O'.

The moment of inertia of entire body about xx' is given by

$$I_{xx'} = \sum m_i r_i^2 \quad —(1)$$

From the figure, we can write

$$r_i^2 = y_i^2 + z_i^2 \quad —(2)$$

Substituting eqn (2) in eqn (1) we have

$$I_{xx'} = \sum m_i [y_i^2 + z_i^2]$$

$$I_{xx'} = \sum m_i y_i^2 + \sum m_i z_i^2 \quad —(3)$$

We know that

M.I of plane about YY' axis is

$$I_{YY'} = \sum m_i y_i^2$$

M.I of plane about ZZ' axis is

$$I_{ZZ'} = \sum m_i z_i^2$$

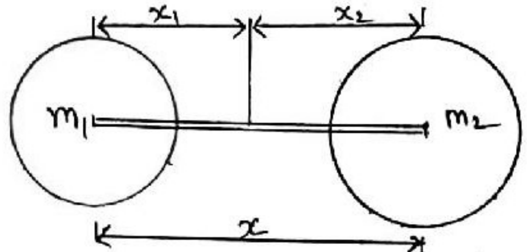
∴ Eqn (3) becomes

$$\boxed{I_{xx'} = I_{YY'} + I_{ZZ'}}$$

This equation represents the equation for perpendicular axis theorem.

M.I of a Diatomic Molecule:

Let us consider a rigid diatomic molecule containing two atoms of masses m_1 and m_2 separated by a distance 'x'.



Let $O \rightarrow$ CM of the system

$x_1, x_2 \rightarrow$ Distance of two atoms from the point 'O'.

From fig, we have

$$x = x_1 + x_2 \quad —(1)$$

Since the system is balanced with respect to the CM, we have

$$m_1 x_1 = m_2 x_2 \quad —(2)$$

From eqn (1), $x_2 = x - x_1$ —(3)

Substituting eqn (3) in eqn (2) we have

$$m_1 x_1 = m_2 (x - x_1)$$

$$m_1 x_1 = m_2 x - m_2 x_1$$

$$m_1 x_1 + m_2 x_1 = m_2 x$$

$$x_1 (m_1 + m_2) = m_2 x$$

$$x_1 = \frac{m_2 x}{m_1 + m_2} \quad —(4)$$

From eqn ① we have $x_1 = x - x_2$ —⑤

Substituting eqn ⑤ in eqn ② we get

$$m_1(x - x_2) = m_2 x_2$$

$$m_1 x - m_1 x_2 = m_2 x_2$$

$$m_1 x_2 + m_2 x_2 = m_1 x$$

$$x_2[m_1 + m_2] = m_1 x$$

$$x_2 = \frac{m_1 x}{m_1 + m_2} \quad \text{—⑥}$$

M.I of a diatomic molecule about an axis passing through the CM is given by

$$I = m_1 x_1^2 + m_2 x_2^2 \quad \text{—⑦}$$

Substituting eqn ④ & ⑥ in eqn ⑦ we get

$$I = m_1\left[\frac{m_2 x}{m_1 + m_2}\right]^2 + m_2\left[\frac{m_1 x}{m_1 + m_2}\right]^2$$

$$I = \frac{x^2}{(m_1 + m_2)^2}\left[m_1 m_2^2 + m_2 m_1^2\right]$$

$$I = \frac{x^2 (m_1 m_2) \cdot [m_1 + m_2]}{(m_1 + m_2)^2}$$

$$\boxed{I = \frac{m_1 m_2}{m_1 + m_2} x^2}$$

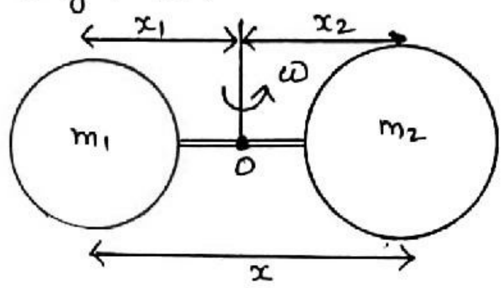
Since $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is called the reduced mass of the system, eqn ⑧ can be written as

$$\boxed{I = \mu x^2}$$

This eqn represents M.I of Diatmoic molecule

Rotational Energy state of a Rigid Diatomic molecule :-

Let us consider a rigid diatomic molecule having two atoms of masses m_1 and m_1 connected by a weightless rod of length 'x'.



Let $\omega \rightarrow$ angular velocity of rotating diatomic molecule.

$O \rightarrow$ centre of mass.

The K.E of diatomic molecule is

$$KE = \frac{1}{2} I \omega^2 \quad \text{—①}$$

The angular momentum of a rotating body is

$$L = I\omega$$

$$\omega = \frac{L}{I} \quad \text{—②}$$

Substituting eqn ② in eqn ①

$$KE = \frac{1}{2} I \frac{L^2}{I^2}$$

$$KE = \frac{1}{2}\frac{L^2}{I}$$

$$K.E = \frac{L^2}{2I} \quad \text{—③}$$

M.I of inertia of rotating diatomic molecule is

$$I = \mu x^2 \quad \text{—④}$$

Where μ is the reduced mass

Substituting equation ④ in eqn ③ we get

$$\boxed{KE = \frac{L^2}{2\mu x^2}} \quad —⑤$$

Eqn ⑤ represents the classical equation for K.E of a rigid diatomic molecule, in which ~~all~~ the energy levels are continuous for all possible values of 'L'.

But according to quantum mechanics, the energy levels are discrete.

∴ Based on quantum theory, the angular momentum 'L' is given by

$$L = \sqrt{J(J+1)}\ \hbar \quad —⑥$$

Where J ⇒ Total angular mom. quantum number

$J = 0, 1, 2, 3 \ldots$

Substituting eqn ⑥ in eqn ⑤ we have,

$$K.E = \frac{\left[\sqrt{J(J+1)}\ \hbar\right]^2}{2\mu x^2}$$

$$(ie)\ \boxed{E_J = \frac{J(J+1)\hbar^2}{2\mu x^2}} \quad —⑦$$

Eqn ⑦ represents the rotational K.E diatomic molecule quantum mechanically.

Special Cases:-

(i) When $J=0$, $E_0 = 0$

(ii) When $J=1$.

$$E_1 = \frac{\hbar^2}{\mu x^2}$$

(iii) When $J=2$

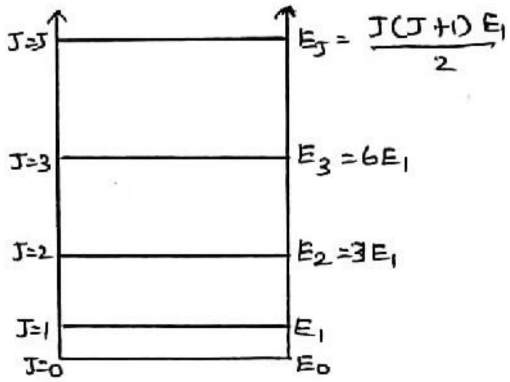
$$E_2 = \frac{2(3)\hbar^2}{2\mu x^2}$$

$$E_2 = \frac{3\hbar^2}{\mu x^2}$$

$$E_2 = 3E_1$$

The general equation for finding energy states of a diatomic molecule is given by

$$\boxed{E_J = \frac{J(J+1)}{2} E_1}$$

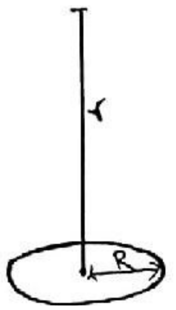


From the above results, we can confirm that rotational KE of rigid diatomic molecule is quantized and discrete.

Torsional Pendulum

A circular metallic disc is suspended using a thin wire that executes torsional oscillation is called torsional pendulum.

In torsional pendulum upper end is fixed and lower end is connected to the centre of a heavy circular disc



The restoring couple set up in the wire by applying a twist $= C\theta$ —(1)

where $\theta \rightarrow$ Angle of rotation

$C \rightarrow$ couple per unit twist

But applied couple $= I\dfrac{d^2\theta}{dt}$ —(2)

Where $\dfrac{d^2\theta}{dt^2} \rightarrow$ Angular Momentum

$I \rightarrow$ Moment of inertia

In equilibrium

Applied couple = Restoring couple

$$I\frac{d^2\theta}{dt^2} = C\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{C}{I}\theta \quad \text{—(3)}$$

Since, the acceleration is directly proportional to angular displacement, the motion of the disc is simple Harmonic Motion.

$\therefore$ Time period of the oscillation $\Big\}$ $T = 2\pi\sqrt{\dfrac{\text{Displacement}}{\text{Acceleration}}}$

$$T = 2\pi\sqrt{\frac{\theta}{C\theta/I}}$$

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{—(4)}$$

Determination of Torsional Rigidity:

* The disc is rotated through a small angle and set it free
* The time taken for 20 complete oscillation is noted. From this, the period of 1 oscillation is found.
* The diameter and mass of the disc are measured.

We know that

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{—(5)}$$

Squarring on bothsides

$$T^2 = 4\pi^2 \cdot \frac{I}{C} \quad \text{—(6)}$$

Substituting couple per unit twist $C = \dfrac{\pi n r^4}{2l}$ in eqn (6) we get

$$T^2 = 4\pi^2 I \times \frac{2l}{\pi n r^4}$$

Rearranging the above eqn

$$\boxed{n = \frac{8\pi I l}{T^2 r^4}}$$

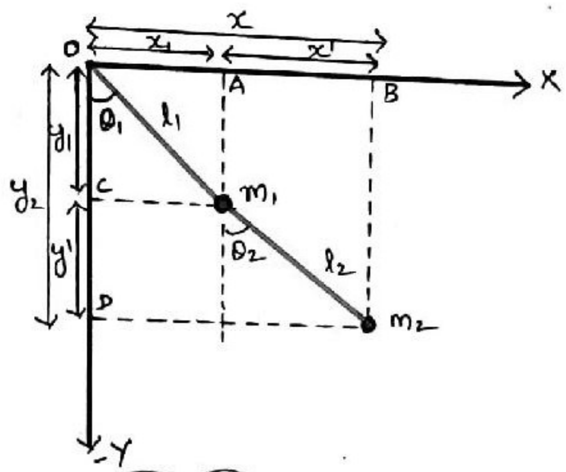
Where $I = \dfrac{MR^2}{2}$

$M \rightarrow$ Mass of the disc

$R \rightarrow$ Radius of the disc.

Double Pendulum:-

It consists of two pendulums in which one pendulum is attached to the other pendulum. If the motion is small then the pendulum behaves as a simple pendulum. If the motion is large then it behaves as a chaotic system.



Description:-

Consider a double pendulum suspended to a point 'o'.

$m_1 \rightarrow$ mass of pendulum-1

$m_2 \rightarrow$ mass of pendulum-2

$l_1 \rightarrow$ length of pendulum-1

$l_2 \rightarrow$ length of pendulum-2

$\theta_1 \rightarrow$ Angle of pendulum-1 during oscillation.

$\theta_2 \rightarrow$ Angle of pendulum-2 during oscillation.

Let us derive the expressions for the displacement, velocity, acceleration, kinetic energy.

Displacement:-

Let $x_1 \rightarrow$ Displacement of pendulum-1 along x axis

$x_2 \rightarrow$ Displacement of pendulum-2 along x axis.

$y_1 \rightarrow$ Displacement of pendulum-1 along y axis

$y_2 \rightarrow$ Displacement of pendulum-2 along y axis.

From figure (i)

$$\sin\theta_1 = \frac{x_1}{l_1}$$

$$x_1 = l_1 \sin\theta_1 \quad -(1)$$

From figure (ii)

$$\cos\theta_1 = \frac{-y_1}{l_1}$$

$$y_1 = -l_1 \cos\theta_1 \quad -(2)$$

From figure (iii)

$$\sin\theta_2 = \frac{x'}{l_2}$$

$$x' = l_2 \sin\theta_2 \quad -(3)$$

From figure (iv)

$$\cos\theta_2 = \frac{-y'}{l_2}$$

$$y' = l_2 \cos\theta_2 \quad -(4)$$

Displacement of pendulum-2 along X-axis is given by

$$x_2 = x_1 + x'$$

$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \quad -(5)$$

Displacement of pendulum-2 along Y-axis is given by

$$y_2 = y_1 + y'$$

$$y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2 \quad -(6)$$

Eqns ①, ②, ⑤, ⑥ represents the displacement at various positions of the double pendulum.

Velocity:

The velocity is the derivative with respect to time of the position.

$$\frac{dx_1}{dt} = l_1 \cos\theta_1 \dot{\theta}_1$$

(ie) $V_{x_1} = l_1 \cos\theta_1 \dot{\theta}_1$

$$\vartheta_{y_1} = l_1 \sin\theta_1 \dot{\theta}_1$$

$$\vartheta_{x_2} = l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2$$

$$\vartheta_{y_2} = l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \sin\theta_2 \dot{\theta}_2$$

The above eqns represents the velocity at various positions of the double pendulum.

Kinetic Energy:

K.E of the system is

$$T = \sum_{i=1}^{2} \frac{1}{2} m_i [\vartheta_{xi}^2 + \vartheta_{yi}^2]$$

$$\cancel{T = \frac{1}{2}\sum}$$

$$T = \frac{1}{2} m_1 [\vartheta_{x_1}^2 + \vartheta_{y_1}^2] + \frac{1}{2} m_2 [\vartheta_{x_2}^2 + \vartheta_{y_2}^2]$$

Substituting the expressions for respective velocities, we get

$$T = \frac{1}{2} m_1 [l_1^2 \cos^2\theta_1 \dot{\theta}_1^2 + l_1^2 \sin^2\theta_1 \dot{\theta}_1^2] + \frac{1}{2} m_2 [(l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2)^2 + (l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \sin\theta_2 \dot{\theta}_2)^2]$$

This eqn represents the kinetic energy of the double pendulum.

Potential Energy:

P.E of the system is

$$V = m_1 g y_1 + m_2 g y_2$$

Substituting expressions for y_1 and y_2 we get

$$V = -m_1 g l_1 \cos\theta_1 - m_2 g (l_1 \cos\theta_1 + l_2 \cos\theta_2)$$

This eqn represents the potential energy of the double pendulum.

Moment of inertia of a circular Disc

Let us find the moment of inertia of a circular disc with rotating axis at various positions

Position-1

Rotating axis is passing through the CM and $\perp^r$ to the disc plane.

Let $R \rightarrow$ Radius of circular Disc

$M \rightarrow$ Mass of disc

Assume the disc consists of large no. of small rings. Let dm and dr be mass and thickness of one of the rings.

MI of a small ring is given by

$$dI = (dm) r^2 \quad \text{—} ①$$

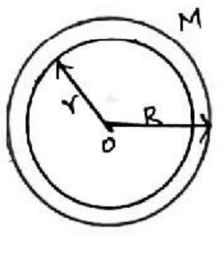
Mass of the small ring (dm) with radius 'r' is given by

Mass (dm) = Surface density × Cir. of the ring × Thickness of the ring

$$dm = \sigma \times 2\pi r \times dr$$

$$dm = \sigma 2\pi r\, dr \quad \text{—(2)}$$

We know that the surface mass density



$$\sigma = \frac{\text{Mass}}{\text{Area}} = \frac{M}{\pi R^2} \quad \text{—(3)}$$

Substituting eqn (3) in eqn (2) we get

$$dm = \frac{M}{\pi R^2} 2\pi r\, dr$$

$$dm = \frac{2M}{R^2} r\, dr \quad \text{—(4)}$$

Substituting eqn (4) in eqn (1)

$$dI = \frac{2M}{R^2} r\, dr \cdot r^2$$

$$dI = \frac{2M}{R^2} r^3 dr \quad \text{—(5)}$$

Integrating eqn (5) within the limits 0 to R, we will get total MI of disc.

(ie) $\int dI = \int_0^R \frac{2M}{R^2} r^3 dr$

$$I = \frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R$$

$$I = \frac{2M}{R^2} \cdot \frac{R^4}{4}$$

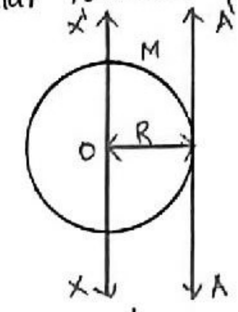
$$\boxed{I = \frac{MR^2}{2}} \quad \text{—(6)}$$

Eqn (6) gives MI of circular disc when the rotating axis is passing through the CM.

Position-2

Rotating axis at the edge of the disc and $\perp^r$ to the disc plane.

Let XX' & AA' are parallel and both are perpendicular to disc surface



Based on the parallel axis theorem

$$I_{AA'} = I_{XX'} + MR^2 \quad \text{—(7)}$$

Using eqn (6) we can write

$$I_{XX'} = \frac{1}{2} MR^2 \quad \text{—(8)}$$

Substituting eqn (8) in eqn (7) we get

$$I_{AA'} = \frac{1}{2} MR^2 + MR^2$$

$$\boxed{I_{AA'} = \frac{3}{2} MR^2} \quad \text{—(9)}$$

Eqn (9) represents the MI, when the rotational axis is at the edge of the disc.

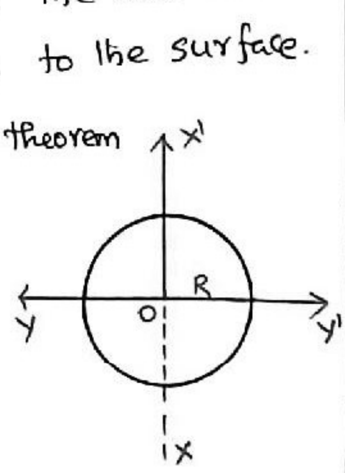
Position-3

Rotating axis is passing through the diameter of the disc

Let YY' → Rotating axis, passing through the diameter

XX' → $\perp^r$ axis to the surface.

Based on $\perp^r$ axis theorem



$$I_{XX'} = I_{YY'} + I_{ZZ'} \quad \text{—(10)}$$

For Circular Disc

$$I_{ZZ'} = I_{YY'}$$

$$(10) \Rightarrow I_{XX'} = 2I_{YY'}$$

$I_{yy'} = \frac{I_{xx'}}{2}$

We know $I_{xx'} = \frac{1}{2} MR^2$

$\therefore I_{yy'} = \frac{\frac{1}{2} MR^2}{2}$

$$\boxed{I_{yy'} = \frac{1}{4} MR^2} \quad — (11)$$

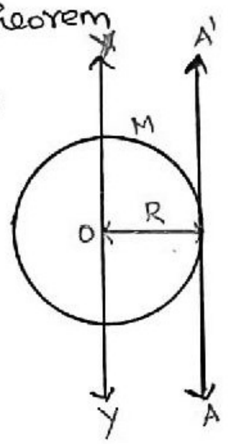
Eqn (11) represents MI, when the rotational axis is passing through the diameter of the disc.

Position-4

Rotating axis at the edge of disc and parallel to disc plane

Let YY' and AA' axes are ||^{l} to each other and also ||^{l} to disc surface

Based on the ||^{l} axis theorem



$I_{AA'} = I_{yy'} + MR^2$ — (12)

Substituting eqn (11) in eqn (12) we get

$I_{AA'} = \frac{1}{4} MR^2 + MR^2$

$$\boxed{I_{AA'} = \frac{5}{4} MR^2} — (13)$$

Eqn (13) represents the MI when the rotational axis at the edge of the disc and ||^{l} to the plane.

UNIT 2

ELECTROMAGNETIC

THEORY

Maxwell's Equations

The Maxwell's equations are used to explain the fundmental relations between electric and magnetic fields.

The formulated Maxwell's equations are

Eqn ① : $\nabla \cdot \vec{B} = \rho$ (Gauss law for elec)

Eqn ② : $\nabla \cdot \vec{B} = 0$ (Gauss law for mag)

Eqn ③ : $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday law)

Eqn ④ : $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ (Ampere law)

Derivations of Maxwell's Equations:

Maxwell's first equation from electric Gauss law

Let $S \rightarrow$ Surface of dielectric medium

$V \rightarrow$ Volume of dielectric medium.

$Q \rightarrow$ Total charge of dielectric

$\rho \rightarrow$ Charge density.

According to Gauss law, for electric field we can write

$$\oint_S \vec{E} \cdot ds = \frac{Q}{\varepsilon_0}$$

$$\oint \varepsilon_0 \vec{E} \cdot ds = Q \quad \text{—①}$$

We know Displacement vector

$$\vec{D} = \varepsilon \vec{E}$$

Since $\varepsilon = \varepsilon_0 \varepsilon_r$, $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

For Air $\varepsilon_r = 1$

$$\therefore \vec{D} = \varepsilon_0 \vec{E} \quad \text{—②}$$

Substituting eqn② in eqn① we get

$$\oint_S \vec{D} \cdot ds = Q \quad \text{—③}$$

Total charge Q interms of volume is given by

$$Q = \oint_V \rho \, dV \quad \text{—④}$$

Comparing eqn③ & eqn④ we have

$$\oint_S \vec{D} \cdot ds = \oint_V \rho \cdot dV \quad \text{—⑤}$$

Eqn⑤ Represents Maxwell's 1st equation in integral form.

Differential Form:-

Applying Gauss divergence theorem to LHS of eqn ⑤ we get

$$\oint_S \vec{D} \cdot ds = \oint_V \vec{\nabla} \cdot \vec{D} \, ds \quad \text{—⑥}$$

From eqns ⑤ & ⑥ we can write

$$\oint_V \vec{\nabla} \cdot \vec{D} ds = \oint \rho \cdot dV \quad \text{—⑦}$$

Two volume integrals are equal if these integrands are equal.

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho} \quad \text{—⑧}$$

Eqn ⑧ represents the Maxwell's 1st equation in differential form.

(ii) Maxwell's second equation from magnteic Gauss law

According to Gauss law for magnetic field, the net magnetic flux through any closed surface is equal to zero.

$$\text{(ie)}\ \phi = 0 \quad — (9)$$

We know that the magnetic flux (ϕ) in terms of magnetic induction (B) is

$$\phi = \oint \vec{B} \cdot ds \quad — (10)$$

Comparing eqns (9) & (10) we get

$$\oint \vec{B} \cdot ds = 0 \quad — (11)$$

Eqn (11) represents Maxwell's second equation in integral form.

Differential form:-

Using Gauss' Divergence theorem on eqn (11) we get

$$\oint_S \vec{B} \cdot ds = \oint_V \vec{\nabla} \cdot \vec{B}\, dV = 0 \quad — (12)$$

Here the surface bound volume is an arbitrary, therefore eqn (12) holds good only if the integral vanishes

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \quad — (13)$$

Eqn (13) represents the Maxwell's 2nd eqn in differential form.

Maxwell's third equation from Faraday's law

According to Faraday's law

$$\mathcal{E}mf = -\frac{d\phi}{dt} \quad — (14)$$

$\mathcal{E}mf \rightarrow$ Electro Motive force

$\phi \rightarrow$ Magnetic flux.

WKt

EMF in terms of electric field (E)

$$\mathcal{E} = \oint_l \vec{E} \cdot dl \quad — (15)$$

The magnetic flux ~~density~~ in terms of magnetic induction (B)

$$\phi = \oint_s \vec{B} \cdot ds \quad — (16)$$

Substituting the eqns (15), (16) in eqn (14) we get

$$\oint_l \vec{E} \cdot dl = -\frac{d\left(\oint_s \vec{B} \cdot ds\right)}{dt}$$

Since 'B' alone changes with time we can write,

$$\oint_l \vec{E} \cdot dl = -\oint_s \frac{d\vec{B}}{dt} \cdot ds \quad — (17)$$

Eqn (17) represents the Maxwell's 3rd eqn in integral form.

Differential form:

Apply Stoke's theorem to LHS of eqn (17) we can write

$$\oint_l \vec{E} \cdot dl = \oint_S \vec{\nabla} \times \vec{E}\, dl \quad — (18)$$

Comparing the eqns (17) & (18) we get

$$\oint_l \vec{E}\cdot dl = \oint_S \vec{\nabla}\times\vec{E}\, dl \quad -(19)$$

When surface is an arbitrary, the Integral must vanish.

∴ Eqn (19) becomes

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}} \quad -(20)$$

Eqn (20) represents the Maxwell's 3rd eqn in differential form.

(iv) Maxwell's fourth equation from Ampere's Law

From Ampere's circuit law,

$$\oint_l \vec{H}\cdot d\vec{l} = I \quad -(21)$$

We know, the relation between the current and current density is given by

$$I = \oint_S \vec{J}\cdot d\vec{s} \quad -(22)$$

Substituting eqn (22) in eqn (21) we get

$$\oint_l \vec{H}\cdot d\vec{l} = \oint_S \vec{J}\cdot d\vec{s} \quad -(23)$$

By Stoke's theorem

LHS of (23) ⇒ $\oint_l \vec{H}\cdot d\vec{l} = \oint \nabla\times\vec{H}\cdot ds \quad -(24)$

Comparing eqn (23), & (24) we have

$$\oint \vec{\nabla}\times\vec{H}\cdot ds = \oint \vec{J}\cdot d\vec{s} \quad -(25)$$

As the surface is arbitrary, integral must vanish.

(25) ⇒ $\vec{\nabla}\times\vec{H} = \vec{J} \quad -(26)$

Apply Gauss divergence theorem on both sides of eqn (26) we get

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J}$$

From vector identity $\vec{\nabla}\cdot(\vec{\nabla}\times H) = 0$

$$\therefore \quad \vec{\nabla}\cdot\vec{J} = 0 \quad -(27)$$

But according to eqn of continuity

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

(ie) $\vec{\nabla}\cdot\vec{J} = 0$ only if $\frac{\partial\rho}{\partial t} = 0 \quad -(28)$

From eqn (28) we know charge density is constant. Therefore Ampere's eqn is valid for steady state conditions and invalid for time varying fields.

∴ By adding current density J_D to equation (26), the equation will be valid for all conditions.

(ie) $\vec{\nabla}\times\vec{H} = \vec{J} + \vec{J}_d \quad -(29)$

Taking divergence on bothsides

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot(\vec{J}+\vec{J}_d)$$

Using vector identity

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0$$

$$\therefore \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{J}_d = 0$$

Here $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$

$$\therefore -\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\vec{J}_d = 0$$

$$\vec{\nabla}\cdot J_d = \frac{\partial \rho}{\partial t}$$

From 1st eqn $\nabla\cdot D = \rho$

$$\therefore \vec{\nabla}\cdot J_d = \frac{\partial(\vec{\nabla}\cdot\vec{D})}{\partial t}$$

$$\vec{\nabla}\cdot\vec{J_d} = \vec{\nabla}\cdot\frac{\partial D}{\partial t}$$

$$\vec{J_d} = \frac{\partial D}{\partial t} \quad \text{--- (30)}$$

Substitute eqn (30) in eqn (29)

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial D}{\partial t}} \quad \text{--- (31)}$$

Eqn (31) represents the Maxwell's fourth eqn in differential eqn.

Plane Electromagnetic waves in free space

Let us consider a plane electromagnetic wave which propagates in vacuum. Let the permeability (μ_0) and the permitivity (ε_0) in free space are constant and conductivity is zero.

Also charge density $\rho = 0$

The Maxwell's equation for free space shall be written as

$$\boxed{\begin{aligned} &\vec{\nabla}\cdot\vec{E} = 0 &&\text{--- (1)}\\ &\vec{\nabla}\cdot\vec{B} = 0 &&\text{--- (2)}\\ &\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} &&\text{--- (3)}\\ &\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} &&\text{--- (4)}\end{aligned}}$$

The wave equation in terms of electric field in free space

Taking curl on both sides of eqn (3), we get

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = \vec{\nabla}\times\left[-\frac{\partial\vec{B}}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\frac{\partial}{\partial t}[\vec{\nabla}\times\vec{B}] \quad \text{--- (5)}$$

Substituting eqn (4) in eqn (5) we get

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\frac{\partial}{\partial t}\left[\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} \quad \text{--- (6)}$$

Using vector identity

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = \vec{\nabla}[\vec{\nabla}\cdot\vec{E}] - \nabla^2\vec{E} \quad \text{--- (7)}$$

Comparing eqns (6), (7) we get

$$\cancel{\vec{\nabla}\times[\vec{\nabla}\times\vec{E}]}$$

$$\vec{\nabla}[\vec{\nabla}\cdot\vec{E}] - \nabla^2 E = -\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$$

From eqn (1) $\vec{\nabla}\cdot\vec{E} = 0$

$$\therefore \vec{\nabla}(0) - \nabla^2 E = -\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\nabla^2 E - \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} = 0} \quad \text{--- (8)}$$

Eqn (8) represents the wave eqn in terms of electric field in free space.

Wave equation in terms of magnetic field in free space

Taking curl on bothsides of eqn (4) we get

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \vec{\nabla}\times\left[\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \varepsilon_0\mu_0\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t} \quad \text{--- (9)}$$

Substituting eqn (3) in eqn (9) we get

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \varepsilon_0\mu_0\frac{\partial}{\partial t}\left[-\frac{\partial\vec{B}}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = -\varepsilon_0\mu_0\frac{\partial^2 B}{\partial t^2} \quad \text{—(10)}$$

Using vector identity, we can write

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} \quad \text{—(11)}$$

Comparing eqns (10) & (11) we get

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = -\varepsilon_0\mu_0\frac{\partial^2 B}{\partial t^2} \quad \text{—(12)}$$

Substituting eqn (2) in eqn (12) we get

$$\vec{\nabla}(0) - \nabla^2\vec{B} = -\varepsilon_0\mu_0\frac{\partial^2 B}{\partial t^2}$$

$$\boxed{\nabla^2 B - \varepsilon_0\mu_0\frac{\partial^2 B}{\partial t^2} = 0} \quad \text{—(13)}$$

Eqn (13) represents the wave eqn in terms of the magnetic field in free space.

Plane Electromagnetic waves in non-conducting medium.

The Maxwell's equations for a linear isotropic and homogeneous dielectric medium take the form as follows

$$\boxed{\begin{aligned} &\vec{\nabla}\cdot\vec{E} = 0 && \text{—(1)} \\ &\vec{\nabla}\cdot\vec{B} = 0 && \text{—(2)} \\ &\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} && \text{—(3)} \\ &\vec{\nabla}\times\vec{B} = \varepsilon\mu\frac{\partial\vec{E}}{\partial t} && \text{—(4)} \end{aligned}}$$

Wave equation interms of electric field in dielectric medium.

Taking curl on bothsides of equation (3) we get,

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = \vec{\nabla}\times\left[-\frac{\partial\vec{B}}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\frac{\partial}{\partial t}[\vec{\nabla}\times\vec{B}] \quad \text{—(5)}$$

Substituting eqn (4) in eqn (5) we get

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\frac{\partial}{\partial t}\left[\varepsilon_0\mu\frac{\partial E}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = -\varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2} \quad \text{—(6)}$$

Using vector identity we get

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{E}] = \vec{\nabla}[\vec{\nabla}\cdot\vec{E}] - \nabla^2\vec{E} \quad \text{—(7)}$$

Comparing (6) & (7) we get

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2 E = -\varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2}$$

From eqn (1) $\vec{\nabla}\cdot\vec{E} = 0$

$$\therefore \vec{\nabla}(0) - \nabla^2 E = -\varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 E - \varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{—(8)}$$

Eqn (8) represents the wave equation interms of electric field in a dielectric medium.

Wave equation interms of magnetic field in dielectric Medium.

Taking curl on both sides of equation (4)

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times\left[\varepsilon\mu\frac{\partial\vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\times(\vec{\nabla}\times B] = \varepsilon\mu\frac{\partial}{\partial t}[\vec{\nabla}\times\vec{E}] \quad \text{—(9)}$$

Substituting eqn (3) in eqn (9)

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \varepsilon\mu\frac{\partial}{\partial t}\left[-\frac{\partial\vec{B}}{\partial t}\right]$$

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = -\varepsilon\mu\frac{\partial^2 B}{\partial t^2} \quad \text{—(10)}$$

Using vector identity, we can write

$$\vec{\nabla}\times[\vec{\nabla}\times\vec{B}] = \vec{\nabla}[\vec{\nabla}\cdot\vec{B}] - \nabla^2 B \quad \text{--- (11)}$$

From (10) & (11)

$$\vec{\nabla}[\vec{\nabla}\cdot\vec{B}] - \nabla^2 B = -\varepsilon\mu\frac{\partial^2 B}{\partial t^2}$$

From eqn (2) $\vec{\nabla}\cdot\vec{B} = 0$

$$\therefore \vec{\nabla}(0) - \nabla^2 B = -\varepsilon\mu\frac{\partial^2 B}{\partial t^2}$$

$$-\nabla^2 B = -\varepsilon\mu\frac{\partial^2 B}{\partial t^2}$$

$$\boxed{\nabla^2 B - \varepsilon\mu\frac{\partial^2 B}{\partial t^2} = 0} \quad \text{--- (12)}$$

Eqn (12) represents the wave equation in terms of mag. field in a dielectric medium.

Energy in Electromagnetic waves:

Energy content:

Energy content in electromagnetic waves is the sum of time average of the energy density in electromagnetic waves due to electric field and magnetic field.

Tot. Energy Content in EM waves = Energy content due to elect field + Energy Content due to mag. field. --- (1)

Energy content due to electric field.

Wkt, Energy density due to electric field

$$u_E = \frac{1}{2}\varepsilon_0\vec{E}^2 \quad \text{--- (2)}$$

Substituting the solution of wave equation in sine form

(ie) $\vec{E} = \vec{E}_0\sin(kr-\omega t)$ in eqn (2) we get

$$U_E = \frac{1}{2}\varepsilon_0\vec{E}_0^2\sin^2(kr-\omega t)$$

Let us take the time avg. of energy density to find the energy content.

(ie) $$u_E = \frac{1}{T}\int_0^T \frac{1}{2}\varepsilon_0\vec{E}_0^2\sin^2(kr-\omega t)\,dt$$

$$u_E = \frac{1}{2}\varepsilon_0 E_0^2 \frac{1}{T}\int_0^T \sin^2(kr-\omega t)\,dt$$

Here $\frac{1}{T}\int_0^T \sin^2(kr-\omega t)\,dt = \frac{1}{2}$

$$\therefore u_E = \frac{1}{2}\varepsilon_0 E_0^2\cdot\frac{1}{2}$$

$$\boxed{\text{Energy } u_E = \frac{1}{4}\varepsilon_0 E_0^2} \quad \text{--- (3)}$$

Eqn (3) represents the energy content in electromagnetic waves due to electric field.

Energy content due to the magnetic field.

We know that energy density due to magnetic field.

$$U_B = \frac{1}{2\mu_0}\vec{B}^2 \quad \text{--- (4)}$$

The solution of eqn in sine form is

$$\vec{B} = \vec{B}_0\sin(kr-\omega t).$$

$$\therefore U_B = \frac{1}{2\mu_0}\vec{B}_0^2\sin^2(kr-\omega t)$$

Let us take the time average of energy density to find energy content

$$u_B = \frac{1}{T}\int_0^T \frac{1}{2\mu_0}\frac{\vec{E}_0^2}{c^2}\sin^2(kr-\omega t)\,dt \qquad \left[\because \vec{B}_0 = \frac{\vec{E}_0}{c}\right]$$

$$u_B = \frac{1}{2\mu_0 c^2}\vec{E}_0^2\,\frac{1}{T}\int_0^T \sin^2(kr-\omega t)\,dt$$

~~Since $\frac{1}{T}\int_0^T \frac{1}{2\mu_0}\frac{\vec{E_0}^2}{c}$~~

Since $\frac{1}{T}\int_0^T \sin^2(kr-\omega t)\,dt = \frac{1}{2}$

and $\varepsilon_0 = \frac{1}{\mu_0 c^2}$ we can write

the above equation as

$$\text{Energy } u_B = \frac{1}{2}\varepsilon_0 \vec{E_0}^2 \frac{1}{2}.$$

$$\boxed{u_B = \frac{1}{4}\varepsilon_0 \vec{E_0}^2} \quad -⑤$$

Eqn ⑤ represents the energy content in electromagnetic waves due to magnetic field.

Total energy content due to both fields:-

Substituting eqn ③ and ④ in eqn ① we get

Total Energy Content

$$u = \frac{1}{4}\varepsilon_0 \vec{E_0}^2 + \frac{1}{4}\varepsilon_0 \vec{E_0}^2$$

$$\boxed{u = \frac{1}{2}\varepsilon_0 \vec{E_0}^2} \quad -⑥$$

Eqn ⑥ represents the total energy content in electromagnetic wave due to electric and magnetic field.

Intensity of Electromagnetic waves:

The magnitude of time average of poynting vector is called intensity of electromagnetic wave.

Derivation:-

Wkt the poynting vector

$$\vec{S} = \vec{E}\times\vec{H}$$

(or) $\vec{S} = |\vec{E}||\vec{H}|\sin\theta\,\hat{n}$

Here $\theta = 90°$ ($\because \vec{E}$ & $\vec{H}$ are normal to each other)

$$\therefore \vec{S} = |\vec{E}||\vec{H}|\,\hat{n} \quad -①$$

Since $|B| = \mu_0|H|$ or $|\vec{H}| = \frac{|\vec{B}|}{\mu_0}$

$$① \Rightarrow \vec{S} = |\vec{E}|\left|\frac{\vec{B}}{\mu_0}\right|\hat{n} \quad -②$$

Solutions of wave eqn in sine form are given by

$$\vec{E} = \vec{E_0}\sin(\vec{K}\cdot\vec{r} - \omega t) \quad -③$$

$$\vec{B} = \vec{B_0}\sin(\vec{K}\cdot\vec{r} - \omega t) \quad -④$$

Substituting eqn ③ & ④ in eqn ② we get

$$\vec{S} = \vec{E_0}\sin(\vec{K}\cdot\vec{r} - \omega t)\cdot\frac{\vec{B_0}}{\mu_0}\sin(\vec{K}\cdot\vec{r} - \omega t)\hat{n}$$

$$\vec{S} = \frac{\vec{E_0}}{\mu_0}\cdot\frac{\vec{E_0}}{c}\,\hat{n}\sin^2(\vec{K}\cdot\vec{r} - \omega t) \quad \left[\because B_0 = \frac{E_0}{c}\right]$$

$$\text{(or) } \vec{S} = \frac{E_0^2}{\mu_0 c}\,\hat{n}\sin^2[\vec{K}\cdot\vec{r} - \omega t] \quad -⑤$$

$\therefore$ The time average of poynting vector shall be written as

$$\vec{S}_{\text{Time ave}} = \frac{1}{T}\int_0^T \frac{E_0^2}{\mu_0 c}\hat{n}\sin^2(\vec{K}\cdot\vec{r} - \omega t)\,dt$$

$$\vec{S}_{\text{Time Ave}} = \frac{E_0^2}{\mu_0 c}\hat{n}\cdot\frac{1}{T}\int_0^T \sin^2(\vec{K}\cdot\vec{r} - \omega t)\,dt$$

Since $\frac{1}{T}\int_0^T \sin^2(\vec{K}\cdot\vec{r} - \omega t)\,dt = \frac{1}{2}$

$$\vec{S}_{\text{Time ave}} = \frac{E_0^2}{\mu_0 c}\hat{n}\,\frac{1}{2}$$

$$\vec{S}_{\text{Time ave}} = \frac{1}{2}\frac{E_0^2}{\mu_0 c}\hat{n} \quad -⑥$$

We know that $\frac{1}{c} = \sqrt{\varepsilon_0 \mu_0}$

$$⑥ \Rightarrow \vec{S}_{Time\ Ave} = \frac{E_0^2 \sqrt{\varepsilon_0 \mu_0}}{2\mu_0} \hat{n}$$

$$\vec{S}_{Time\ Ave} = \frac{E_0^2}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} \hat{n} \quad -⑦$$

Intensity (or) magnitude of time average of poynting vector is given by

$$I = |\vec{S}| = \frac{E_0^2}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

Since $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ (or) $\frac{1}{\eta_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}}$

$$\boxed{I = \frac{E_0^2}{2\eta_0}} \quad -⑧$$

Eqn ⑧ represents the intensity of electromagnetic waves.

Momentum of Electromagnetic wave:

The effective mass will be taken into account for finding the momentum of the electromagnetic wave interms of

(i) Energy and

(ii) Poynting vector

(i) Momentum in terms of energy:

We know $E = mc^2$ —①

III^ly The energy 'u' with effective mass of electromagnetic radiation can be written as

$$u = mc^2$$

$$m = \frac{u}{c^2} \quad -②$$

Wkt the momentum of particle with mass 'm' and velocity 'ϑ' is

$$p = m\vartheta \quad -③$$

Substituting eqn ② in ③ we get

$$\text{Momentum } p = \frac{u}{c^2}\vartheta \quad -④$$

If the electromagnetic wave, which is travelling along Z axis with velocity c is represented by $c\hat{k}$, then eqn ④ becomes,

$$\vec{P} = \frac{u}{c^2} c\hat{k} \quad -⑤$$

$$\vec{P} = \frac{u\hat{k}}{c}$$

Magnitude of momentum

$$\boxed{p = \frac{u}{c}} \quad -⑦$$

Eqn ⑦ represents the momentum of electromagnetic waves in terms of energy 'u'.

(ii) Momentum in terms of Poynting Vector

We know the poynting vector

$$\vec{S} = (\vec{E} \times \vec{H}) = uc\hat{k}$$

$$\text{(or)} \quad u\hat{k} = \frac{\vec{S}}{c} \quad -⑧$$

Substituting eqn ⑧ in ⑥ we get

$$\vec{P} = \frac{1}{c}\frac{\vec{S}}{c}$$

$$\vec{P} = \frac{\vec{S}}{c^2} \quad -⑨$$

Since $c^2 = \frac{1}{\varepsilon_0 \mu_0}$

$$⑨ \Rightarrow \vec{P} = \varepsilon_0 \mu_0 \vec{S}$$

$$\text{(or)} \quad \boxed{\vec{P} = \varepsilon_0 \mu_0 [\vec{E} \times \vec{H}]} \quad -⑩$$

Eqn ⑩ represents the momentum per unit volume of the EM waves in terms of poynting vector

Radiation Pressure of EM wave:-

Definition:-

When the electromagnetic wave strike the surface, then a force will appear due to the change of momentum. The amount of pressure exerted per unit area on the surface due to the force is called radiation pressure.

Derivation:-

Wkt

the momentum vector

$$\vec{p} = \frac{u\vec{k}}{c}$$

∴ The magnitude of momentum

$$p = \frac{u}{c} \quad \text{—} ①$$

Similarly, we know

Poynting vector $\vec{S} = uc\vec{k}$

∴ The magnitude of energy flow of poynting vector

$$S = uc \quad \text{—} ②$$

According to poynting, theorem, the electromagnetic energy passing normal to the surface per unit area and unit time is given by

$$S = \frac{u}{At} \quad \text{—} ③$$

$A \rightarrow$ Area,

$t \rightarrow$ time

Comparing eqn ② & ③

$$uc = \frac{u}{At}$$

$$c = \frac{1}{At}$$

$$\frac{1}{c} = At \quad \text{—} ④$$

Substituting eqn ④ in eqn ① we get

$$p = uAt \quad \text{—} ⑤$$

According to Newton's law, the force acting on the surface is given by

$$F = \frac{p}{t} \quad \text{—} ⑥$$

Substituting the eqn ⑤ in eqn ⑥

$$F = \frac{uAt}{t}$$

$$F = uA \quad \text{—} ⑦$$

Wkt, the radiation pressure P_{rad} exerted on the surface is given by

$$P_{rad} = \frac{F}{A} \quad \text{—} ⑧$$

Substituting the eqn ⑦ in ⑧ we get

$$P_{rad} = \frac{uA}{A}.$$

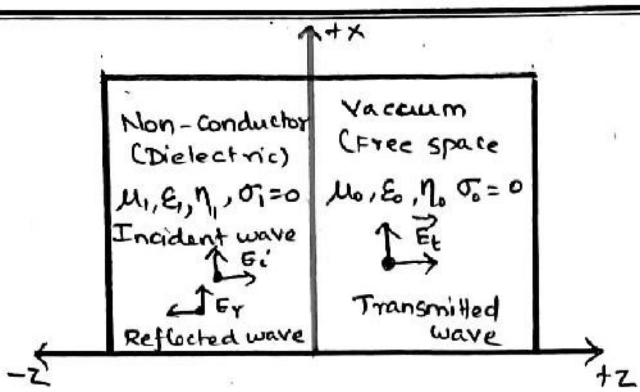
$$\boxed{P_{rad} = u} \quad \text{—} ⑨$$

From eqn ⑨ we can say that the radiation pressure of em wave is equal to the energy of the striking electromagnetic wave.

Reflection and Transmission of EM waves from a Non-conducting Medium to Vacuum:-

Consider an EM wave travel from a non-conducting to vacuum.

One part of the incident wave is reflected into same medium at interface and another part is transmitted into next medium.



$\epsilon_1, \epsilon_0 \rightarrow$ Permittivity of non-cond. medium & vacuum.

$\mu_1, \mu_0 \rightarrow$ Permeability of non-cond. medium & vacuum

$E_i, E_r \rightarrow$ Elec. field vectors of incident & reflected waves

$H_i, H_r \rightarrow$ Mag. field vector of incident & reflected waves

We can write

$$E_i + E_r = E_t \quad \text{--- (1)}$$

$$H_i + H_r = H_t \quad \text{--- (2)}$$

$i \rightarrow$ Represents incident wave
$r \rightarrow$ Represents reflected wave
$t \rightarrow$ Represents transmitted wave.

Transmission Co-efficient (T)

Let $\eta_1 \rightarrow$ intrinsic impedance of non-cond. medium.

$\eta_0 \rightarrow$ intrinsic impedance of vacuum.

WKT
$$\eta_1 = \frac{E}{H} \Rightarrow H = \frac{E}{\eta_1} \quad \text{--- (3)}$$

$$\eta_0 = \frac{E}{H} \Rightarrow H = \frac{E}{\eta_0} \quad \text{--- (4)}$$

Using eqn (3) & (4) eqn (2) can be written as $\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_0}$

-ve sign indicates that the reflected wave travels in the opposite direction to that of the incident wave.

$$\frac{1}{\eta_1}(E_i - E_r) = \frac{E_t}{\eta_0}$$

$$E_i - E_r = \frac{\eta_1}{\eta_0} E_t \quad \text{--- (5)}$$

Adding eqn (1) & eqn (5) we get

$$E_i + E_r + E_i - E_r = E_t + \frac{\eta_1}{\eta_0} E_t$$

$$2E_i = \left[1 + \frac{\eta_1}{\eta_0}\right] E_t$$

$$E_i = \frac{1}{2}\left[\frac{\eta_0 + \eta_1}{\eta_0}\right] E_t$$

$$E_t = \frac{2\eta_0}{\eta_0 + \eta_1} E_i \quad \text{--- (6)}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\eta_0}{\eta_0 + \eta_1}} \quad \text{--- (7)}$$

Transmission coefficient is the ratio of the intensity of the transmitted wave (I_t) to the intensity of the incident wave (I_i)

(ie) $$T = \frac{I_t}{I_i} \quad \text{--- (8)}$$

WKT $$I = \frac{E_0^2}{2\eta_0}$$

Intensity of transmitted wave $\Big\}$ $I_t = \frac{E_t^2}{2\eta_0}$ --- (9)

Intensity of incident wave $\Big\}$ $I_i = \frac{E_i^2}{2\eta_1}$ --- (10)

Substituting eqns (9) & (10) in (8)

$$T = \frac{E_t^2 / 2\eta_0}{E_i^2 / 2\eta_1}$$

$$T = \frac{\eta_1}{\eta_0}\left[\frac{E_t}{E_i}\right]^2 \quad \text{—(11)}$$

Substituting eqn (7) in (11) we get

$$T = \frac{\eta_1}{\eta_0}\left[\frac{2\eta_0}{\eta_0+\eta_1}\right]^2$$

$$\boxed{T = \frac{4\eta_0\eta_1}{(\eta_0+\eta_1)^2}} \quad \text{—(12)}$$

Eqn (12) represents transmission Coefficients.

Reflection co-efficient (R).

Substituting eqn (6) in eqn (1) we get

$$E_i + E_r = \frac{2\eta_0}{\eta_0+\eta_1} E_i$$

$$E_r = \frac{2\eta_0}{\eta_0+\eta_1} E_i - E_i$$

$$E_r = \left[\frac{2\eta_0}{\eta_0+\eta_1} - 1\right] E_i$$

$$E_r = \left[\frac{2\eta_0-\eta_0-\eta_1}{\eta_0+\eta_1}\right] E_i$$

$$E_r = \left[\frac{\eta_0-\eta_1}{\eta_0+\eta_1}\right] E_i$$

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_0-\eta_1}{\eta_0+\eta_1}} \quad \text{—(13)}$$

The reflection Coefficient is the ratio of intensity of the reflected wave (I_r) to the intensity of the incident wave (I_i)

i.e.) $R = \frac{I_r}{I_i}$ —(14)

Intensity of Reflected wave $\Big\}\ I_R = \frac{E_r^2}{2\eta_1}$ —(15)

Substituting eqn (10) & (15) in eqn (14)

$$R = \frac{E_r^2/\cancel{2\eta_1}}{E_i^2/\cancel{2\eta_1}}$$

$$R = \left[\frac{E_r}{E_i}\right]^2 \quad \text{—(16)}$$

Substituting eqn (13) in (16)

$$\boxed{R = \frac{(\eta_0-\eta_1)^2}{(\eta_0+\eta_1)^2}} \quad \text{—(17)}$$

Eqn (17) Represents Reflected Coefficient.

The sum of T+R:-

$$T+R = (12) + (17)$$

$$\therefore T+R = \frac{4\eta_0\eta_1}{(\eta_0+\eta_1)^2} + \frac{(\eta_0-\eta_1)^2}{(\eta_0+\eta_1)^2}$$

$$= \frac{4\eta_0\eta_1 + \eta_0^2 + \eta_1^2 - 2\eta_0\eta_1}{(\eta_0+\eta_1)^2}$$

$$= \frac{\eta_0^2+\eta_1^2+2\eta_0\eta_1}{(\eta_0+\eta_1)^2}$$

$$T+R = \frac{(\eta_0+\eta_1)^2}{(\eta_0+\eta_1)^2} \Rightarrow 1$$

$$\therefore \boxed{T+R=1}$$

(i.e.) The sum of the reflection and transmission coeffient is equal to one.

UNIT 3

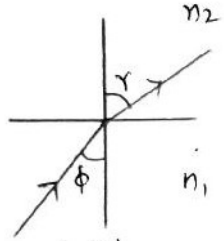
OSCILLATIONS,

OPTICS, LASER

OPTICS

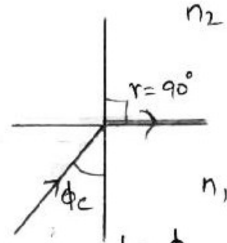
Total Internal Reflection

When a light wave is completely reflected while it travels from one medium to another medium, then this phenomenon is called, total internal reflection



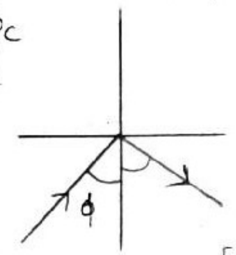
$\phi < \phi_c$

Fig 1



$\phi = \phi_c$

Fig 2



$\phi > \phi_c$ Fig 3

Let the light wave travels ~~passes~~ denser medium to rarer medium

Case (i)

When the angle of incidence (ϕ) is less than the critical angle (ϕ_c) (ie) $\phi < \phi_c$, the ray is refracted in to rarer medium. Fig(1)

Case (ii)

When $\phi = \phi_c$, the ray passes along the medium of separation as shown in fig(2), so that the angle of refraction is 90°. This angle ϕ_c is called as critical angle.

Case (iii)

When $\phi > \phi_c$, the ray is totally reflected into the denser medium itself as shown in fig(3).

From Snell's law

$$n_1 \sin \phi_c = n_2 \sin 90^\circ$$

$$\sin \phi_c = \frac{n_2}{n_1}$$

Critical angle $\phi_c = \sin^{-1}\left[\frac{n_2}{n_1}\right]$

Conditions for Total Internal Reflection:-

Condition 1:

Light should travel from denser medium to rarer medium.

(ie) $n_1 > n_2$.

Condition 2:

The angle of incidence at the interface should be greater than the critical angle.

(ie) $\phi > \phi_c$.

Interference:

When two light waves superimpose, then the resultant amplitude or intensity in the region of superposition is different than the amplitude of individual waves.

Condition for Constructive interference:-

(i) Waves are in phase and have phase differences of $0, 2\pi, 4\pi \ldots$

(ii) Path difference $= n\lambda$, when $n = 0, 1, 2 \ldots$

Thus if the path difference between the two waves is equal to the integral multiple of wave length (λ), then, Constructive interference occurs.

Condition for destructive interference:

(i) Waves are in out of phase and have phase differences of $\pi, 3\pi, 5\pi \ldots$

(ii) Path difference $= \frac{(2n+1)\lambda}{2}$

where $n = 0, 1, 2, 3 \ldots$

Thus, if the path difference between the two waves is equal to the odd integral multiple of $\frac{\lambda}{2}$, then destructive interference occurs.

Michelson Interferometer:-

Principle:-

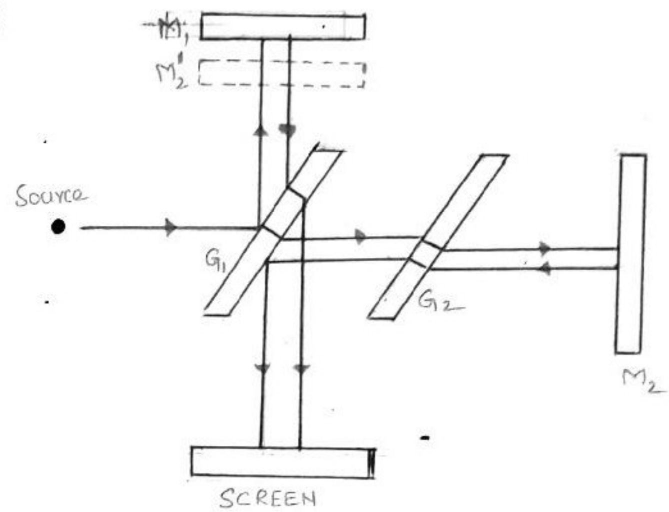
Producing interference pattern by splitting a light beam into two parts and then recombining them after they have travelled different optical paths.

Construction:

It consists of a movable mirror M_1 and a fixed mirror M_2 both are highly polished.

* Two glass plates G_1 (beam splitter) and G_2 (Compensatory glass plate) are placed parallel to each other between the mirrors at an angle of 45°.

* The rear side of glass plate G_1 is semi-silvered to make it as partially reflective glass plate such that splitting occurs and the light from a source is equally reflected and transmitted by it. In this way division of amplitude takes place.



Working:-

* Monochromatic light from source falls on the beam splitter glass plate G_1.

* Since G_1 is partially polished some part of the light gets reflected and some part of the light get transmitted producing two perpendicular beams of equal intensity.

* The Reflected light move towards M_1 and transmitted light move towards M_2 through the glass plate G_2.

* M_1 and M_2 reflect the beams back towards the beam splitter G.
* The ray reflected from M_1 is transmitted through the beam splitter to the screen and the ray reflected from M_2 is reflected again by the beam splitter to the screen.
* Because both beams originate from the same point on the source, they are coherent and therefore interface with each other
* An interference pattern of dark and bright ~~fr.~~ rings or fringes are observed on the viewing screen at S.

Theory:

Fringes are formed by the light reflected from mirror M_1 and M_2 which is equivalent to light reflected from upper and lower surface of the air film formed between mirror M_1 and M_2' the virtual image of mirror M_2.

Since the two interfering beams of light were split from the same initial beam, they were initially in phase.

For a given separation of 'd' between the mirrors M_1 and the virtual mirror M_2', the path difference (Δ) is given as

$$\Delta = 2d = n\lambda$$

where n is an integer, Path difference between the two rays can be varied by moving M_1.

The two waves will interface constructively or destructively as per the following conditions of path difference Δ.

When

(i) $\Delta = 0$ (no path difference, no interference pattern)

(ii) $\Delta = \frac{2n\lambda}{2} = n\lambda$ (Constructive interference - Bright Fringes)

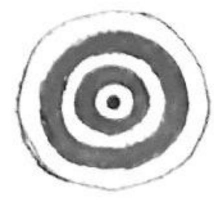
(iii) $\Delta = \frac{(2n+1)\lambda}{2}$ (Destructive interference - Dark Fringes)

Types of fringes:-

The fringes formed by the air film may be circular, curved or straight.

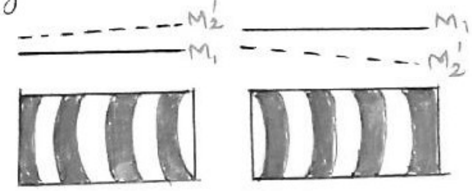
Circular Fringes:

When M_1 and M_2' are parallel to each other, then circular fringes can be observed.



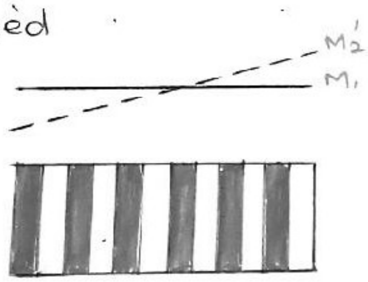
Curved Fringes:

When M_1 and M_2' are inclined to each other, the film enclosed is wedge shaped. Then curved fringes can be observed.



Straight Fringes:

With M_1 and M_2' intersect, Straight line fringes are obtained



Determination of wavelength from circular fringes:-

For a given separation of d between M_1 and M_2' the path difference is given by

$$2d = n\lambda \quad -(1)$$

When M_1 is moved a distance x, each fringe moves to the position previously occupied by an adjacent fringe. Let m be the number of fringes passing a given point as M_1 is moved

The path difference after moving the mirror is given as

$$2(d+x) = (m+n)\lambda \quad -(2)$$

$$2d + 2x = m\lambda + n\lambda \quad -(3)$$

Substituting eqn (1) in eqn (3)

$$n\lambda + 2x = m\lambda + n\lambda$$

$$2x = m\lambda$$

$$\lambda = \frac{2x}{m}$$

Applications:

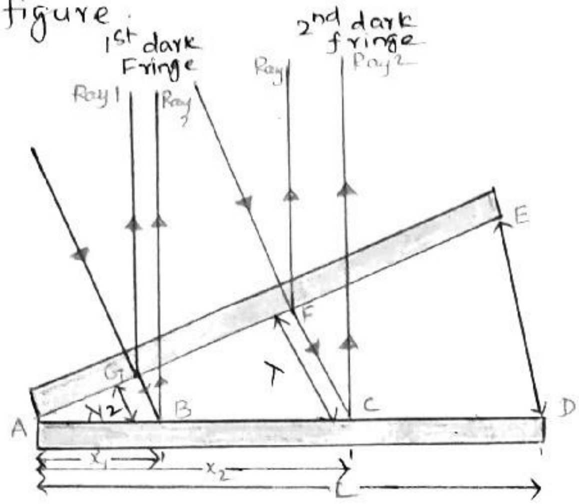
* It detected the existence of waves and confirmed the Space-time distortion
* It is used in optical coherence tomography.
* It is used in fiber optics.

Theory of Air Wedge & Experiment:

A thin film having zero thickness at one end and progressively increasing thickness at the other end is called a wedge.

This air wedge can be used to find the wavelength of the incident light. More importantly it can be used to measure the size of very small objects.

The arrangement for observing interference of light in a wedge shaped air film is shown in figure.



When light falls on wedge shaped thin film, it gets partly reflected from top of the air film, then transimitted through the air film. The trans-mitted beam again reflected at bottom of the air film.

The wedge angle is very small, and the two reflected rays interfere constructively or destructively producing alternate bright and dark fringes.

Consider points G and F, where the glass-to-glass distances across the air wedge are $\frac{\lambda}{2}$ and λ respectively.

At point G, there is both transmission and reflection. The reflected ray from top of the air film (ray 1) and the ray reflected from bottom of the air film have varying path difference. (ie) "Ray 2" travel more distance than "ray 1"

By the time ray 2 lines up with ray 1, it has travelled two widths of GB or one wavelength.

Thus the path difference between ray 1 and ray 2 is one wavelength and ray 1 and ray 2 interfere destructively, since they are 180° out of phase.

These similar phenomenon occurs at point 'F'.

Let us consider the triangles ΔABG and ΔADE.

By the property of similar triangles we have for the first dark fringe

$$\frac{x_1}{L} = \frac{\lambda/2}{t}$$

$$x_1 = \frac{\lambda L}{2t} \quad \text{—} (1)$$

Similarly, for the second dark fringe, let us consider the triangles ΔACF and ΔADE,

$$\frac{x_2}{L} = \frac{\lambda}{t}$$

$$x_2 = \frac{\lambda t}{t} \quad \text{—} (2)$$

The fringe width β is defined as the distance between any two consecutive bright or dark fringes,

Then we have

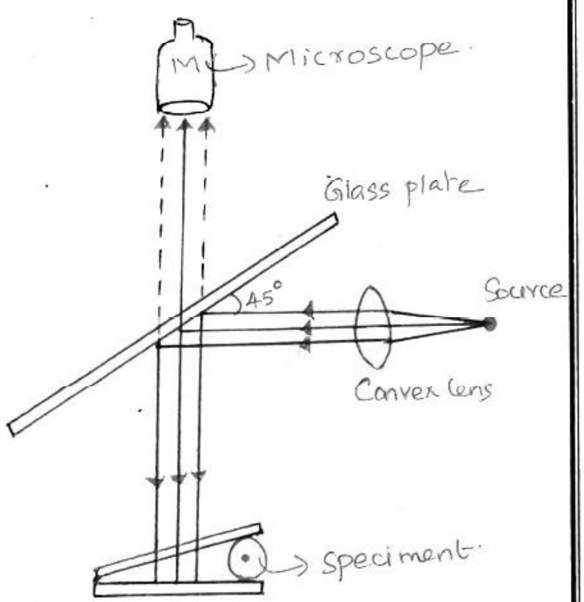
Fringe width $\beta = x_2 - x_1$

$$\beta = \frac{\lambda L}{t} - \frac{\lambda L}{2t}$$

$$\boxed{\beta = \frac{\lambda L}{2t}}$$

Experiment:

The experimental arrangement is shown in the figure.



* Two optically plane glass plates are placed one over other and tied together by means of a rubber band at one end. A thin wire is inserted between the plates at the other end.

* Now a wedge shaped air film is formed between the two glass plates.

* The light from a sodium vapour lamp is made to incident on a plano glass sheet held over the wedge at an angle of 45° with the vertical.

* When the light falls normally on air wedge arrangement, due to interference between the wave reflected from the top and bottom surface of the film, large number of interference fringes are formed.

* Now by focussing the microscope, the fringes are observed and readings are tabulated

* The horizontal position of the dark fringes in the order n, n+5, n+10, n+15 .... are measured.

* From this width of 5 fringes is calculated ($x = 5\beta$)

Then, the fringes width β is calculated as

$$\beta = \frac{x}{5}$$

WKT $\quad \beta = \frac{\lambda L}{2t}$

$$\boxed{t = \frac{\lambda L}{2\beta}}$$

Thus we can measure the thickness of the wire using airwedge.

EINSTEIN'S COEFFICIENTS

When a light radiation is incident on the assembly of atoms. we can observe three things

1. Stimulated absorption
2. Spontaneous Emission
3. Stimulated Emission

STIMULATED ABSORPTION:-

The Atoms in the Lower Energy State (E_1) absorbs radiation and is exited to the higher energy Level (E_2) This Process is known as "stimulated absorption".

The rate of stimulated absorption Nab is given by

$N_{ab} \propto N_1 Q$

$N_1 \rightarrow$ No of atoms in the lower Energy level

$Q \rightarrow$ Energy density (Incident radiation)

$$\therefore \boxed{N_{ab} = B_{12} N_1 Q} \longrightarrow (1)$$

$B_{12} \rightarrow$ Proportionality Constant

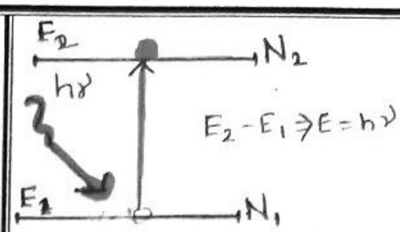
SPONTANEOUS EMISSION :-

The exited atoms in the Higher Energylevel E_2 Return to the lower Energy level E_1 by Emitting Photon of Energy 'hν' without any external factor This process is known as "spontaneous Emission".

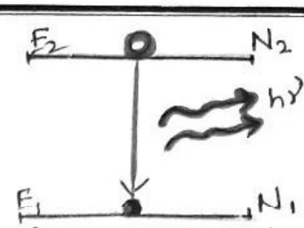
Rate of Emission $N_{sp} \propto N_2$

$$\boxed{N_{sp} = A_{21} N_2} \longrightarrow (2)$$

$N_2 \rightarrow$ atoms in the exites state A_{21} - constant



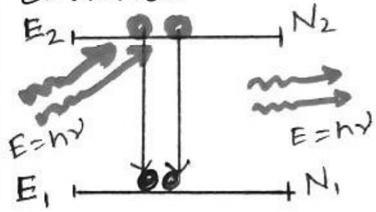
STIMULATED ABSORPTION



SPONTANEOUS EMISSION.

STIMULATED EMISSION:

exited Photons triggers the atom at higher energy level to make transition to lower Energy level (E_1) during this Transition photon of Energy 'hν' is Emitted. This kind of Emission is known as "Stimulated Emission



The rate of stimulated Emission N_{st} is given by $N_{st} \propto N_2 Q$

$$N_{st} = B_{21} N_2 Q \longrightarrow (3)$$

$B_{21} \rightarrow$ Proportionality Constant

These constants A_{12}, B_{12} & B_{21} are called Einstein's A & B Coefficients.

At equillibrium Condition

$$\left.\begin{matrix}\text{No. of downward} \\ \text{transitions}\end{matrix}\right\} = \left\{\begin{matrix}\text{No. of Upward} \\ \text{Transitions}\end{matrix}\right.$$

(ie)

$$N_{sp} + N_{st} = N_{ab} \longrightarrow (4)$$

Sub eqn (1) (2) & (3) in (4) we have

$$A_{21} N_2 + B_{21} N_2 Q = B_{12} N_1 Q$$

$$B_{12} N_1 Q - B_{21} N_2 Q = A_{21} N_2$$

$$(B_{12} N_1 - B_{21} N_2) Q = A_{21} N_2$$

$$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

÷ Numerator and denominator by $B_{21} N_2$

$$Q = \frac{A_{21} N_2 / B_{21} N_2}{\frac{B_{12} N_1}{B_{21} N_2} - \frac{B_{21} N_2}{B_{21} N_2}}$$

$$Q = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) \frac{N_1}{N_2} - 1}$$

From Boltzman's equation.

$$\frac{N_1}{N_2} = e^{h\nu/k_B T}$$

$$\boxed{\therefore Q \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/k_B T} - 1}} \rightarrow (5)$$

w.k.that according to Planck's Energy equation

$$Q = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/k_B T} - 1} \rightarrow (6)$$

Comparing equation (5) & (6)

$\frac{B_{12}}{B_{21}} \simeq 1$ since, $B_{12} \simeq B_{21}$

$$\boxed{\& \frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \text{ (or) } \frac{8\pi h}{\lambda^3}} \rightarrow (7)$$

Conclusion: equation (7) gives us the relation between spontaneous & stimulated Emission co-efficients and it is Proportional to 'ν^3'
Here, the Spontaneous Emission is more prodominant than the Stimulated Emission.

Nd-YAG LASER

SOLID STATE LASER

Nd-YAG Stands for Neodymium Yittrium Alumimium Garnet.

CHARECTERISTICS:

Type: Four level solid state laser

Active medium: Nd-YAG Rod

Pumping method: Optical pumping (xenon Lamp)

Resonator: fully Polished and Partially Polished Ends of Nd-YAG Rod.

Pw/output: 70 W

Nature: Pulzed (or) Continious wave form.

Wave length: 1.06 μm (IR-region)

PRINCIPLE

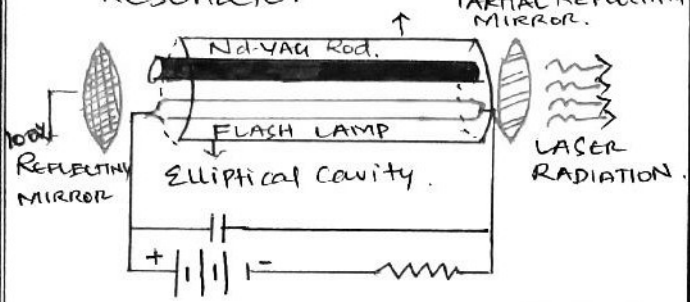
The neodimium (Nd^{3+}) atoms from Nd-YAG rod optically Pumped by xenon (or) crypton flash lamp. During the transition from meta stable State to groundstate Laser beam of wavelength 1.064 μm is Emitted.

CONSTRUCTION

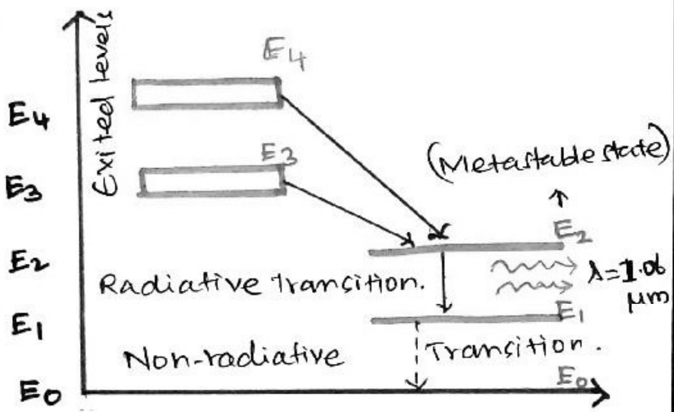
* The Nd-YAG crystal Cut in the form of cylender and the Ends are highly Polished so as to be optically flat & Polished.

* Cylinderical rod and a flash lamp (Pumping source) kept parallel and Placed inside a reflector Cavity

* Ends of the Cavity covered by 100% and Partially polished mirrors which acts as a Resonator



WORKING. - ENERGY LEVEL DIAGRAM.



(i) Due to absorption of Light radiation of wavelength 0.73 μm and 0.80 μm Nd^{3+} Neodimium atoms are exited from $E_0 \rightarrow E_3$ and E_4. levels

(ii) By spontaneous Emission atoms from E_3 & E_4 moves to E_2 the metastable state at which "Population Inversion" is achieved.

(iii) By stimulated Emission Transition from $E_2 - E_1$ intiated Leads to Laser action, wavelength of 1.06 μm (10600 Å) Laser Light is Emitted.

(iv) These Photons reflected. back and forth between the mirrors undergo Amlification hence high intence laser light Emitted through Partially Polished mirrors.

Advantage.

* High Energy O/P
* Repetition rate is Very high
* Population inversion achieved Easily

Applications

* Used as a range finder in millitary
* Used for cutting, drilling, welding and surface hardening in Industries.
* Used for Cataract Surgery, gall bladder surgery etc.... in Medical field.
* Used in Long haul communication.

* CARBON DIOXIDE (CO_2) LASER

It's a four level Molecular Gas laser operates in "far Infrared" region.

CHARECTERISTICS

Type : Four level molecular gas laser.

Active Medium : Mixer of CO_2, N_2, He gas.

Pumping Method : In elastic atom to atom collision.

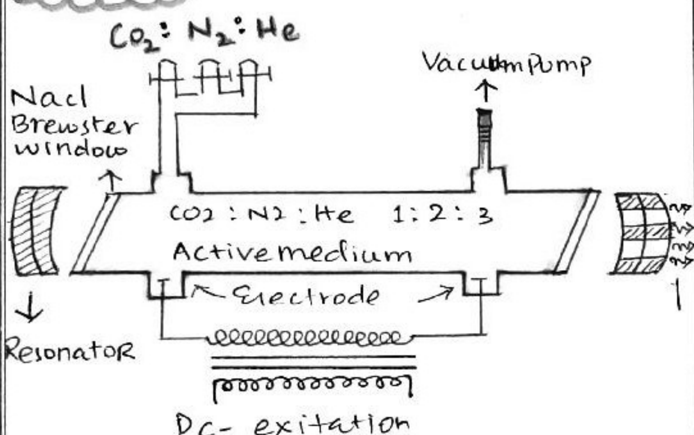
Resonator : fully Polished & Partially polished concave mirrors.

Pw/op : 40 w/m - 60kW/m

Nature : Pulzed (or) Continious

Wavelength : 9.6μm & 10 μm.

DIAGRAM.



CONSTRUCTION

(i) It consists a discharge tube of Diameter 2.5cm and length 5m.

(ii) CO_2, N_2, He filled inside the tube in the ratio 1:2:3 ratio. respectively.

(iii) Discharge is produced by Dc- exitation.

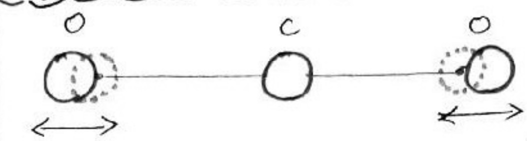
(iv) Nacl - Windows are Placed Inclined at Brewster angle to the resonator axis.

(v) Ends of the tube covered with Partially & fully polished concave mirrors. will act as a resonator.

VIBRATIONAL MODES

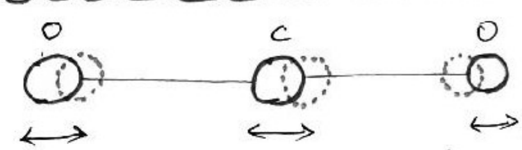
CO_2 Molecule is a Linear molecule Consisting of central atom with two oxygen atoms on either sides. It consist of 3 independent "Vibrational modes"

(i) Symmetric stretching mode



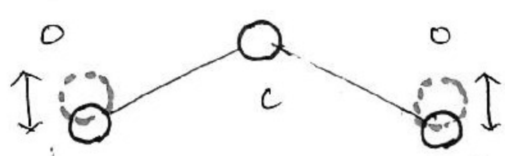
In this mode Carbon atom is Stationary where the two Oxygen atom Vibrate along the axis. Creates stretching.

(ii) Assymetric Stretching mode



In this mode all the three atoms Oscillate along the axis, but the Vibration direction of Carbon is opposite to that of the oxygen.

(iii) Bending mode



In this mode all the three atoms undergo Vibration Perpendicular to the bond axis and the movement of them (C-O) are opposite to each other.

ENERGY LEVEL DIAGRAM

* CO_2 Laser exited by a Dc exitation

(i) The exited electrons collided with Nitrogen molecules by which N_2 got exited to Metastable state

$$e^{-*} + N_2 \longrightarrow N_2^*$$

(ii) exited N_2 exites the ground state CO_2 molecule by inelastic collisions.

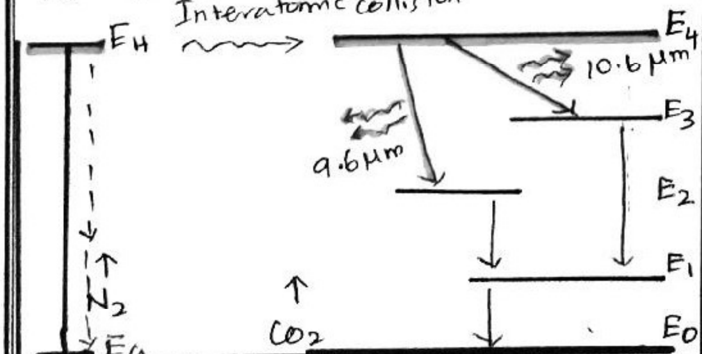
$$N_2^* + CO_2 \longrightarrow CO_2^*$$

(iii) due to this Population inversion is achieved in the 'E_4' level.

(iv) By Stimulated Emission CO_2 Molecules ~~Atom~~. make a Transition from E_4 to E_3 & E_2 through which a Coherent laser beam of wavelength 10μm and 9.6 μm is Emitted respectively in the IR region.

(v) Transition $E_2 \rightarrow E_1$ & $E_3 \rightarrow E_1$ happens due to Elastic collision.

(vi) finally helium helps to discharge the heat from CO_2 to come to ground state E_0



MERITS

* Construction is Simple
* O/P is Continious.
* High Efficiency.
* Very High O/P Power.
* O/P Pw Can be increased by Increasing the length of the gas tube.

DEMERITS:

* Carbon monoxide will contaminate the Oxygen.
* Depends on Operating Temp.
* Since it is in IR region it is invisible. Accidental Exposure cause some series damage.

APPLICATIONS:

* widely used in Material Processing welding, drilling, Cutting Soldering.
* Used for Open air Communication
* Used in remote Sensing.
* It is used in the Treatement of Liver & Lunch
* It is used to perform Micro surgery and blood loss less operation, ~~...~~

Solid - state semiconductor Diode Laser

* It is a most compact form of all laser
* It is also called injection laser.

It is broadly classified into two types,

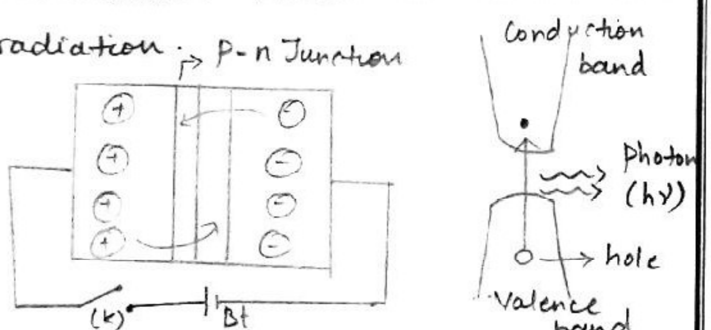
a) HomoJunction
b) Hetero Junction

Homo - Junction Laser:

It is specially fabricated PN Junction diode.

Principle:-

When the PN Junction diode is forwarded biased, due to recombination of electron and holes light radiation is emitted in direct band gap semiconductor known as recombination radiation.

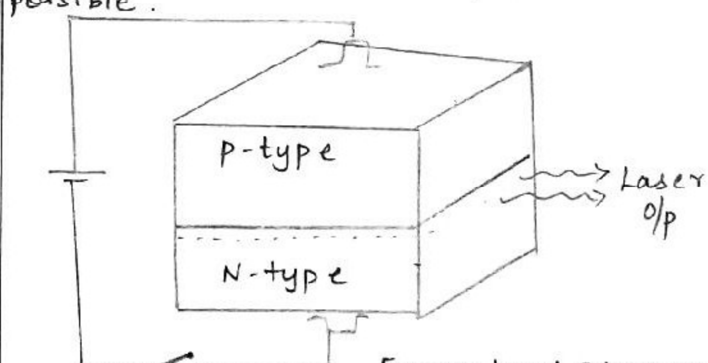


Construction:-

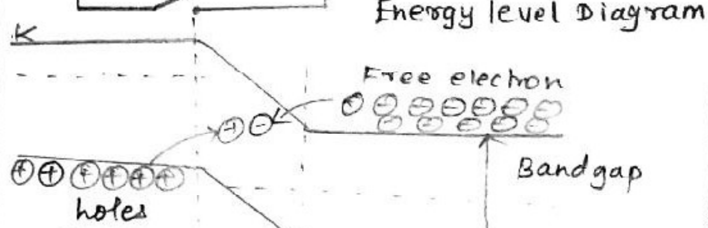
* p-type and n-type semiconductor combined to form a p-n junction
* Two electrodes connected on top and bottom side.
* Forward biasing is given through electrodes.
* The end faces of p-n junction are well polished which acts as an optical resonator.

Working:-

* The circuit in forward biased
* As shown in energy level diagram electron and holes are injected into junction region
* The large no. of electrons and holes recombine with each other.
* Due to this photons of hν emitted
* This get amplified by the polished side of diode.
* The more the biasing voltage more the emission of photons are possible.



Energy level Diagram



Advantages:

→ Small and compact
→ Highly efficient
→ operated with lesspower compared to other lasers
→ Output is continuous / pulsed

Disadvantage:
→ output has large divergence
→ poor coherence and stability

Application:
→ used in fiber optic communication
→ used in printer & CD players.

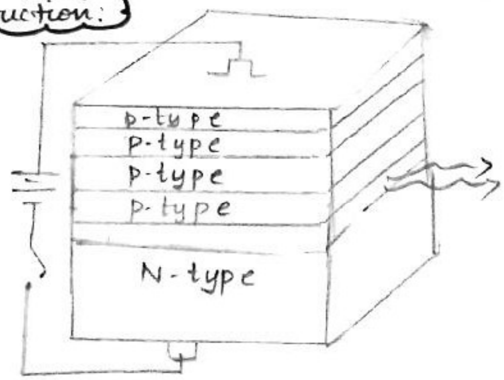
Hetero-junction Laser:

In this, p-n junction formed by more than on n-type and p-type material.

Principle:

when the pn junction diode is forwarded biased the electrons and holes recombine to produce photon.

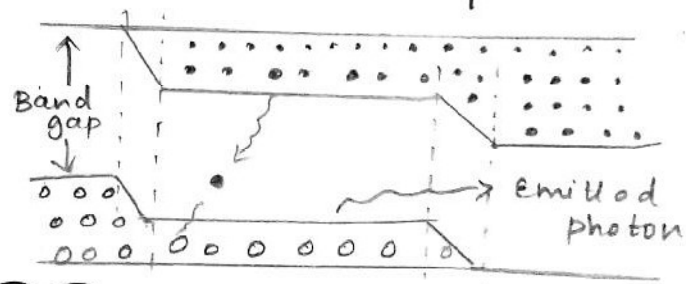
Construction:



* Three p-type layers and two n-type layers combine to form a p-n junction.
* Here the 'p' layer in the junction act as an active region
* this sandwiched placed in between two electrodes.
* Forward biased is given
* End faces are polished.

Working

* when the layers are forward biased large no. of electrons and holes are recombined.
* Recombination produce large output photons.
* The biasing voltage is directly proportional to the emission of electrons.
* the photons reflected by the polished ends and get amplified.
* A coherent beam of laser of wavelength 8000 Å emerges.



Advantages:

* It produces continuous waveform
* output power is very high

Disadvantage:-

* very difficult to form junction
* cost is very high

Applications:

* Very difficult to form junction
* cost is very high.

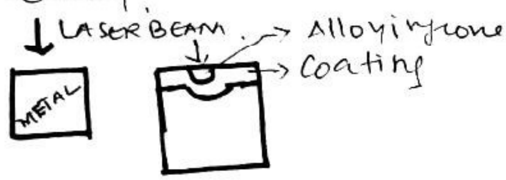
BASIC APPLICATIONS OF LASER

SURFACE ALLOYING:

It means that there is controlled melting of work of work piece surface to desired depth. with simultaneous addition of powdered alloying Element

LASER CLADDING

* In this the laser beam melts a thin Layer of work piece this will mixes with the liquid Cladding alloy.

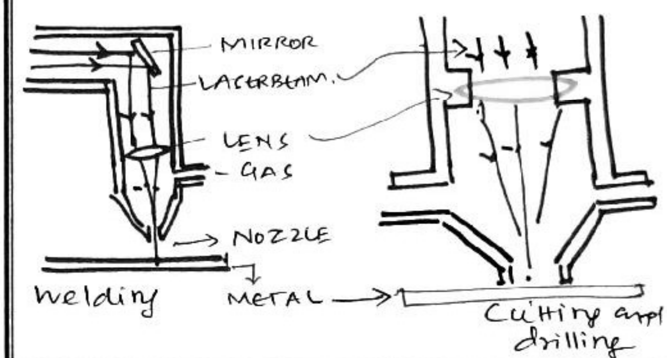


LASER WELDING

* welding is joining of two (or) or more metal pieces in to a single into single Unit
* In Laser welding the beam heats the Edges of the two Plates to their melting point and Cause them to fuse together.
* Hence NO possibilities of Impurities, welding rate is high, dissimilar metals Can be welded.

LASER CUTTING

* The Principle of Laser cutting is the vaporization of the material at a point of focus of the beam. The Vaporized material is removed with the help of a gas jet.
* It can be done at room Temp.
* Cutting speed is Very high.
* Heating and cooling are so rapid.



Oscillations

Differential Equation for a Simple Harmonic Motion:-

Definition of SHM:

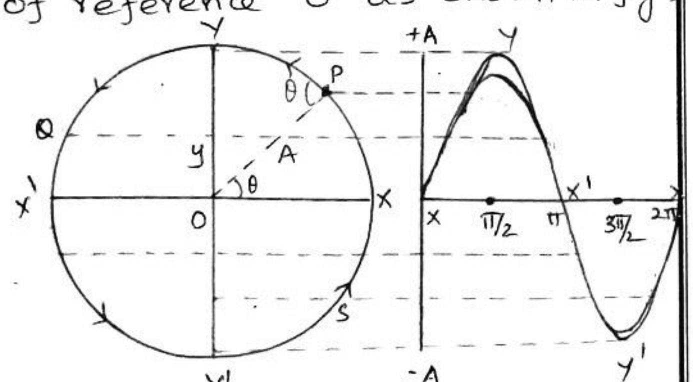
Simple Harmonic Motion is the motion in which the acceleration of a body is directly proportional to the displacement from a fixed point and is always directed towards the fixed point (or) equilibrium position.

Derivation:

Displacement:

The displacement of vibrating particle at any instant is defined as the distance moved by the particle from its mean position of rest.

Let us consider a particle 'P' moving in a circular path of radius 'A' with uniform velocity 'v' and angular velocity ω, with respect to the centre of the circle of reference O as shown in fig.



When the particle 'P' moves around the circle, then the foot of the perpendicular 'Q' vibrates along the diameter YY'. Further if the motion of the particle 'P' is uniform, then the motion of 'Q' is also periodic (ie) the particle will take the same time to vibrate between the points Y and Y'.

∴ If the particle 'P' completes one revolution, then the foot of the ⊥r 'Q' will complete one verticle oscillation.

Thus the distance OQ is termed as displacement of the particle and is denoted by the letter 'y'.

If the particle moves from X to P in t seconds, then the angle between POX is given by

~~From ΔQPO~~

~~sin θ or sin ωt~~

$$\angle POX = \angle QPO = \theta = \omega t$$

From ΔQPO

$$\sin\theta \text{ (or) } \sin\omega t = \frac{OQ}{OP} \quad \text{---(1)}$$

OQ = y and OP = A.

$$(1) \Rightarrow \sin\omega t = \frac{y}{A}$$

$$\boxed{y = A\sin\omega t} \quad \text{---(2)}$$

This eqn respresents displacement of vibrating particle.

(ii) Velocity:

Velocity of the vibrating particle is defined as the rate of change of displacement

$$\therefore \text{Velocity } (v) = \frac{dy}{dt} \quad -(3)$$

Substitute eqn (2) in eqn (3) we get

$$v = \frac{d}{dt}(A\sin\omega t)$$

$$\boxed{v = A\omega\cos\omega t} \quad -(4)$$

This eqn respresents the velocity of the vibrating particle.

(iii) Acceleration:-

Acceleration of the vibrating particle is defined as the rate of change of velocity.

$$\therefore \text{Acceleration} = \frac{dv}{dt} \quad -(5)$$

Substituting eqn (4) in eqn (5)

$$\text{Acceleration} = \frac{d}{dt}(A\omega\cos\omega t)$$

$$\frac{d^2y}{dt^2} = A\omega(-\omega\sin\omega t)$$

$$= -\omega^2 A\sin\omega t$$

$$\boxed{\frac{d^2y}{dt^2} = -\omega^2 y} \quad -(6)$$

This eqn respresents the acceleration of vibrating particle.

Unit - IV - Basic Quantum Mechanics.

Introduction:

The most outstanding development in modern science is the conception of quantum mechanics. The quantum mechanics is better than Newtonian mechanics in explaining the fundamental Physics.

The fundamental concept were not different from those of everyday experience, such as Particle, Position, Speed, mass, force, energy and even field. These concepts are referred as "classical"

The world of atoms cannot be described and understood with these concepts. Thus, it needed new concepts to understand the properties of atom.

A group of Scientists Neils Bohr. W. Heisenberg, E. Schrodinger, P. A. M. Dirac, W. Pauli and M. Born, conceived and formulated these new ideas in the beginning of 20^{th} century. This new formulation, a branch of Physics, was named as "Quantum mechanics"

Limitations of Classical Mechanics.

* The Phenomenas which Classical Physics failed to explain are black body Radiation, Photoelectric effect, emission of X-rays, etc..
* The other main difference is the quantized energy state. In classical Physics, an oscillating body can assume any Possible energy. on the contrary, quantum mechanics says that it can have only discrete non-zero energy.

Need of Quantum Mechanics:

* Classical mechanics successfully explained the motions of object which are observable directly or by instruments like microscope. But when classical mechanics is applied to the particles of atomic levels, it fails to explain actual behaviour. Therefore, the classical mechanics cannot used to explain in atomic level, e.g. motion of a electron in an atom.

* The Phenomena of black body radiation, Photoelectric effect, emission of X-rays, etc. were explained by Max Planck in 1990 by introducing of the formula

$$\boxed{E = nh\nu}$$

Where, $n = 0, 1, 2, \ldots$

$h =$ Planck's constant $= 6.63 \times 10^{-34}$ J/s

* This is known as "Quantum hypothesis" and marked the beginning of modern Physics. The whole microscopic world obeys the above formula.

—x—

Photons and Light Waves - (Duality of Radiation and matter)

The wave and Particle daudity of radiation is easily understood by knowing a difference between a wave and a Particle.

Wave:

* A Wave originate due to oscillations and it is spread out over a large region of space. A wave cannot be located at a Particular Place and mass cannot carried by a wave.

* Actually, a wave is a spread out disturbance specified by its amplitude 'A', frequency 'ν', wavelength 'λ' phase 'δ' and Intensity 'I'.

* The Phenomena of interference, diffraction and Polarisation require the Presence of two 'or' more waves at the same time and at the same Position.

It is very clear the two or more Particles cannot occupy the same position at the same time. So one has to conclude that radiation behaves like waves.

Particle:

* A Particle is located at some definite Point and it has mass. It can move from one Place to another. A Particle gains energy when it accelerated and it loses energy when it is slowed down.

* A Particle is Characterized by mass 'm', Velocity '$\vec{v}$' momentum 'P' and energy 'E'

* Spectra of black body radiation, Compton effect, Photo electric effect etc. could not be explained on wave nature of radiation.

These Phenomena established that radiant energy interacts with matter in the form of "Photons or quanta". Therefore, Planck's quantum theory came to conclude that radiation behaves like Particles.

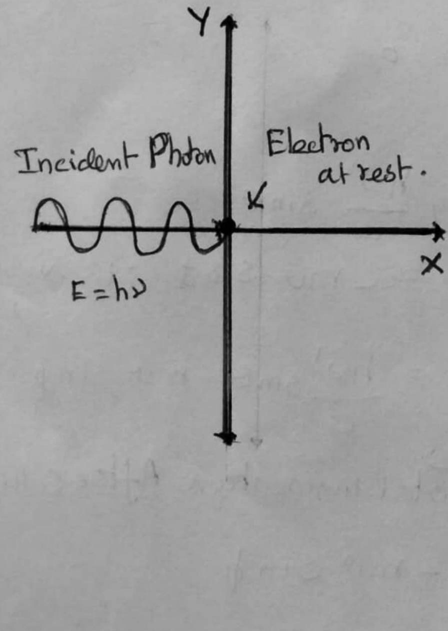
* Thus, radiation sometimes behaves as a wave and at some other times as a Particle. Now, wave - Particle duality of radiation is universally accepted.

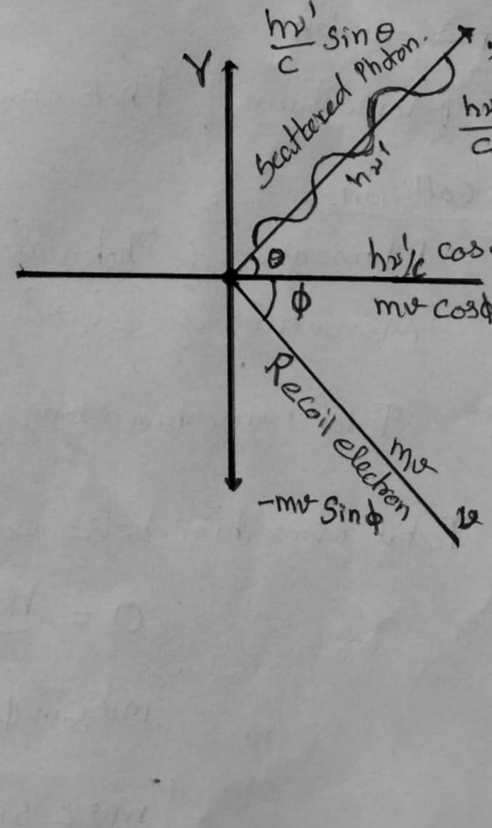
——— x ———

Compton Effect:

Statement:

"When a beam of 'X'-rays is scattered by a substance of low atomic number, the scattered X-ray radiation consists of two components. One component has the same wavelength 'λ' as the incident ray and the other component has a slightly longer wavelength 'λ''. The change in the wavelength of scattered X-rays is known as Compton shift. The Phenomenon is called Compton Effect."





Total energy before Collision = $h\nu + m_0c^2$

Total Energy after Collision = $h\nu' + mc^2$

Total Energy before collision = Total Energy after Collision

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = h\nu - h\nu' + m_0c^2$$

$$\boxed{mc^2 = h(\nu - \nu') + m_0c^2} \quad \text{——} (1)$$

$\left(\because p = \frac{h}{\lambda}\right.$

$\lambda = c/\nu$; $p = \frac{h}{c/\nu}$

$\left. p = \frac{h\nu}{c}\right)$

Total momentum along X-axis:

Before Collision : $= \frac{h\nu}{c}$

After collision
Momentum of Photon $\Big\}$ $= \frac{h\nu'}{c}\cos\theta$

moment of electron $= m\upsilon\cos\phi$

Total momentum $= \frac{h\nu'}{c}\cos\theta + m\upsilon\cos\phi$ — (2)

Total momentum before Collision = Total momentum after collision

$$\boxed{m\upsilon\cos\phi = h(\nu - \nu'\cos\theta)} \quad \text{—} (3)$$

Total momentum along Y-axis:

Before Collision:

Momentum of Photon and electron = 0.

After Collision:

Momentum of Photon along Y-axis = $\frac{h\nu'}{c}\sin\theta$

Momentum of electron along Y-axis = $-m\upsilon\sin\phi$ (in -ve Y direction)

Total momentum along Y axis $= \frac{h\nu'}{c}\sin\theta - m\upsilon\sin\phi$.

Total momentum before collision = Total momentum After Collision

$$0 = \frac{h\nu'}{c}\sin\theta - m\upsilon\sin\phi$$

$$m\upsilon\sin\phi = \frac{h\nu'}{c}\sin\theta \quad -(4)$$

$$\boxed{m\upsilon c\sin\phi = h\nu'\sin\theta} \quad \text{—} (5)$$

Squaring eqn(3) and eqn(5) and then adding, we get

$$(mvc\cos\phi)^2+(mvc\sin\phi)^2 = h^2(\nu-\nu'\cos\theta)^2+(h\nu'\sin\theta)^2 \quad -(6)$$

L.H.S of eqn(6)

$$= m^2v^2c^2\cos^2\phi + m^2v^2c^2\sin^2\phi$$

$$= m^2v^2c^2(\cos^2\phi+\sin^2\phi)$$

$$= m^2v^2c^2 \qquad [\because \sin^2\phi+\cos^2\phi = 1]$$

R.H.S of eqn(6)

$$= h^2(\nu^2-2\nu\nu'\cos\theta+\nu'^2\cos^2\theta)+h^2\nu'^2\sin^2\theta$$

$$= h^2[\nu^2-2\nu\nu'\cos\theta+\nu'^2\cos^2\theta+\nu'^2\sin^2\theta]$$

$$= h^2[\nu^2-2\nu\nu'\cos\theta+\nu'^2(\cos^2\theta+\sin^2\theta)]$$

$$(\because \cos^2\theta+\sin^2\theta=1)$$

$$= h^2[\nu^2-2\nu\nu'\cos\theta+\nu'^2]$$

L.H.S = R.H.S

$$\boxed{m^2v^2c^2 = h^2(\nu^2-2\nu\nu'\cos\theta+\nu'^2)} \quad -(7)$$

Squaring eqn (1) on both sides, we get

$$(mc^2)^2=[h(\nu-\nu')+m_oc^2]^2 \quad -(8)$$

$$m^2c^4 = h^2(\nu-\nu')^2+m_o^2c^4+2h(\nu-\nu')m_oc^2$$

$$\boxed{m^2c^4 = h^2(\nu-2\nu\nu'+\nu'^2)+2h(\nu-\nu')m_oc^2+m_o^2c^4} \quad -(9)$$

Subtracting eqn(7) from eqn (9), we get

$$m^2c^4-m^2v^2c^2 = h^2(\nu^2-2\nu\nu'+\nu'^2)+2h(\nu-\nu')m_oc^2+m_o^2c^4$$
$$-h^2(\nu^2-2\nu\nu'\cos\theta+\nu'^2)$$

$$m^2c^2(c^2-v^2) = \cancel{h^2\nu^2}-2h^2\nu\nu'+\cancel{h^2\nu'^2}+2h(\nu-\nu')m_oc^2+m_o^2c^4$$
$$-\cancel{h^2\nu^2}-2h^2\nu\nu'\cos\theta-\cancel{h^2\nu'^2}$$

$$\boxed{m^2c^2(c^2-v^2) = -2h^2\nu\nu'(1-\cos\theta)+2h(\nu-\nu')m_oc^2+m_o^2c^4} \quad (10)$$

From the theory of Relativity, the variation of mass with velocity is given by.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad — (11)$$

Squaring the eqn (11) on both sides, we have

$$m^2 = \frac{m_0^2}{1 - v^2/c^2} = \frac{m_0^2}{\frac{c^2 - v^2}{c^2}} = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$m^2 (c^2 - v^2) = m_0^2 c^2$$

multiplying "c^2" on both sides, we have

$$\boxed{m^2 c^2 (c^2 - v^2) = m_0^2 c^4} \quad — (12)$$

Substituting eqn (12) in eqn (10), we get

$$\cancel{m_0^2 c^4} = -2h^2 \nu\nu' (1 - \cos\theta) + 2h(\nu - \nu') m_0 c^2 + \cancel{m_0^2 c^4}$$

$$2h(\nu - \nu') m_0 c^2 = 2h^2 \nu\nu' (1 - \cos\theta)$$

$$(\nu - \nu') m_0 c^2 = \frac{\cancel{2} h^{\cancel{2}} \nu\nu'}{\cancel{2}\cancel{h}} (1 - \cos\theta)$$

$$\frac{(\nu - \nu')}{\nu\nu'} = \frac{h}{m_0 c^2} (1 - \cos\theta)$$

$$\frac{\cancel{\nu}}{\cancel{\nu}\nu'} - \frac{\cancel{\nu'}}{\nu\cancel{\nu'}} = \frac{h}{m_0 c^2} (1 - \cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2} (1 - \cos\theta) \quad — (13)$$

multiplying 'c' on both sides of eqn (13), we get

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h\cancel{c}}{m_0 c^{\cancel{2}}} (1 - \cos\theta) \qquad \left(\because \frac{c}{\nu} = \lambda, \; c/\nu' = \lambda'\right)$$

$$\lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos\theta)$$

$$\boxed{d\lambda = \frac{h}{m_0 c} (1 - \cos\theta)}$$

Therefore, the Change in wavelength is given by

$$\boxed{d\lambda = \frac{h}{m_o c}(1 - \cos\theta)}$$

It is found the change in wavelength ($d\lambda$) does not depend on the wavelength of the incident radiation and the scattering substance. But it depends only on the angle (θ)

Case(i): when $\theta = 0$, then

$$d\lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

$(\cos 0 = 1)$

$$= \frac{h}{m_o c}(1 - 1)$$

$$\boxed{d\lambda = 0}$$

(Along the Incident direction, There is no change in wavelength).

Cos (ii): When $\theta = 90°$, Then

$$d\lambda = \frac{h}{m_o c}(1 - \cos 90°)$$

$[\cos 90 = 0]$

$$d\lambda = \frac{h}{m_o c}(1 - 0)$$

$$\boxed{d\lambda = \frac{h}{m_o c}} \quad \boxed{d\lambda = 0.0243\ \text{Å}}$$

This difference in wavelength is known as Compton wavelength of electron

Case (iii): When $\theta = 180°$, Then.

$$d\lambda = \frac{h}{m_o c}(1 - \cos 180°)$$

$(\because \cos 180° = -1)$

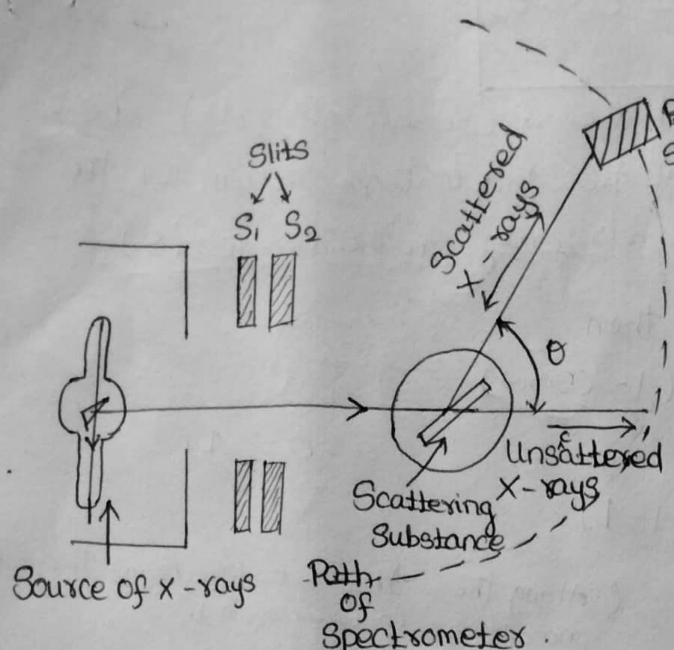
$$d\lambda = \frac{h}{m_o c}(1 - (-1))$$

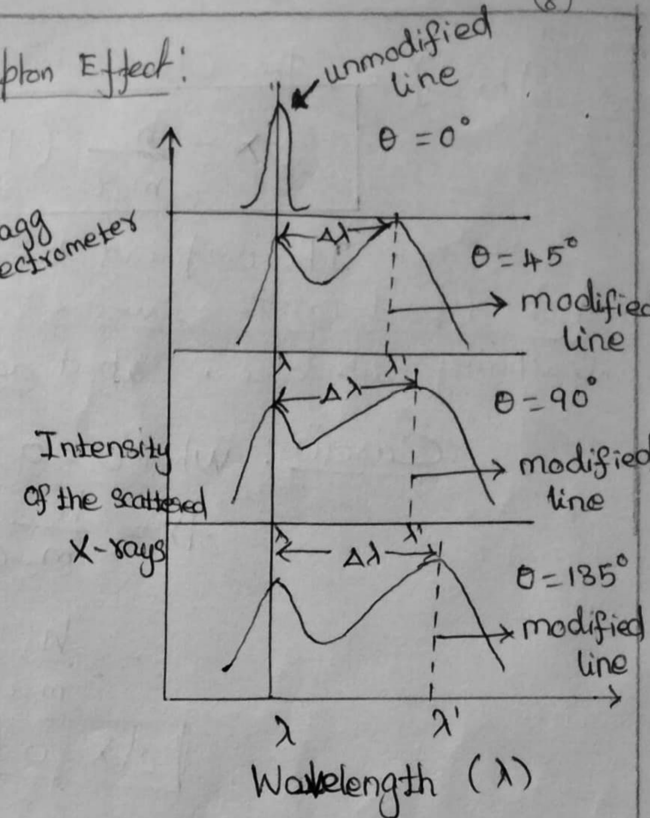
$$\boxed{d\lambda = \frac{2h}{m_o c}}$$

$$\boxed{d\lambda = 0.0486\ \text{Å}}$$

Thus, The Change in wavelength is maximum at $\theta = 180°$

Experimental Verification of Compton Effect:





A beam of monochromatic x-rays of wavelength λ is made to incident on a scattering substance. The scattered x-rays are received by Brag Spectrometer.

The intensity of scattered x-rays is measured for various scattering angles. The graph is plotted (Intensity and wavelength).

It is found that the curves have two peaks, one corresponding to unmodified radiation and other corresponding to modified radiation.

The Difference between two peaks on the wavelength axis gives "Compton Shift"

The curves shows that the greater the scattering angle, the greater is Compton Shift in accordance with expression.

$$\boxed{d\lambda = \frac{h}{m_0 c}(1 - \cos\theta)}$$

Electrons (Particles) And Matter Waves - (Concept of Matterwaves)

de-Broglie's Hypothesis:

Louis de-Broglie proposed a very bold and novel suggestion that "like light radiation, matter or material Particle also Posseses dual (two) characteristics i.e, Particle-like and wave like"

According to de-Broglie hypothesis, a moving Particle is always associated with waves.

* Waves and Particles are the only two modes through which energy can Propagate in nature.

* Our universe is fully Composed of light radiation and matter

* Since nature loves symmetry, matter and waves must be symmetric.

* If electromagnetic radiation like, light, x-rays can act like wave and a Particle, then material Particles (electron, Protons, etc) should also act like a Particle and a wave

* Every moving Particle is always associated with a wave.

de-Broglei waves and it's wavelength:

" The Waves associated with the matter Particles are called matter waves or de-Broglie waves.

From Planck's theory, the energy of a Photon (Particles) of frequency 'ν' is given by

$$E = h\nu \quad \text{——} (1)$$

According to Einstein's mass-energy relation

$$E = mc^2 \quad \text{——} (2)$$

where m - mass of the photon ; c - Velocity of the Photon.

Equating eqn (1) and (2), we get

$$h\nu = mc^2 \qquad (\because \nu = \frac{c}{\lambda})$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h\cancel{c}}{mc^{\cancel{2}}}$$

$$\boxed{\lambda = \frac{h}{mc}} \quad \text{———} \quad (3)$$

Since $mc = p$ (momentum of a Photon)

$$\boxed{\lambda = \frac{h}{p}}$$

Equation (3) is known as de-Broglie's wave equation.

de-Broglie wave length in terms of energy:

We know that the Kinetic energy $E = \frac{1}{2}mv^2$

multiplying by m on both sides we get

$$mE = \frac{1}{2}m^2v^2$$

$$2mE = m^2v^2$$

Taking square root

$$\sqrt{m^2v^2} = \sqrt{2mE}$$

$$mv = \sqrt{2mE}$$

we know that $\lambda = \frac{h}{mv}$.

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

De-Broglie's wavelength in terms of Energy is

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

de-Broglie's wavelength in terms of accelerating Potential associated with electrons:

When an electron of charge 'e' is accelerated by a Potential difference of 'V' volts, then the electron gains a velocity 'v' and hence.

$$\text{Workdone on the electron} = eV \quad -(1)$$

This workdone is converted into the kinetic energy of the electron as $\frac{1}{2}mv^2$

$$\text{Workdone} = \text{Kinetic energy}$$

$$eV = \frac{1}{2}mv^2 \quad -(2)$$

$$2eV = mv^2$$

$\times m$, on both sides

$$2meV = m^2v^2$$

Taking square root

$$mv = \sqrt{2meV} \quad -(3)$$

From the de-Broglie's concept

$$\lambda = \frac{h}{mv} \quad -(4)$$

Substituting eqn(3) in eqn(4), we have

$$\boxed{\lambda = \frac{h}{\sqrt{2meV}}} \quad -(5)$$

$h = 6.625 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C

$$\boxed{\lambda = \frac{12.25}{V} \times 10^{-10} \text{ metre}}$$

Properties of Matter Waves:

(1) If the mass of the Particle Smaller, then the wavelength associated with that Particle is longer.

(2) If the Velocity of the Particle is small, then the wavelength associated with that Particle is longer.

(3). If $V = 0$, then $\lambda = \infty$, i.e; the wave becomes indeterminate and if $V = \infty$, then $\lambda = 0$. This indicates that deBroglie' waves are generated by the motion of Particle's.

(4) These wave do not dependent on the charge of the Particles. This shows that these waves are not electromagentic waves

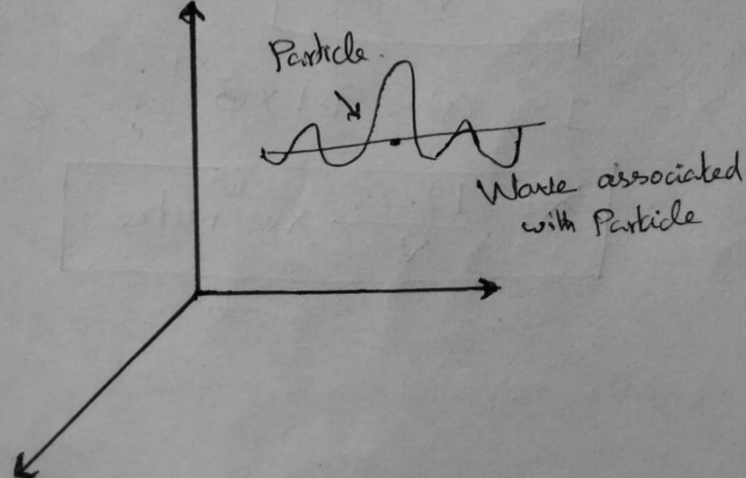
(5). The Velocity of de-Broglie's waves is not Constant Since it depends on the Velocity of the material Particle.

—— x ——

Schrodinger Time Independent Wave equation.

Consider a wave associated with a moving Particle.

Let x, y, z be the Coordinates of the particle and 'ψ' wavefunction for de-Broglie's waves at any given instant of time 't'.



Squaring eqn (6)

$$\frac{\omega^2}{v^2} = \frac{2^2\pi^2}{\lambda^2} = \frac{4\pi^2}{\lambda^2} \quad — (7)$$

Substituting eqn(7) in eqn (5)

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \quad — (8)$$

Subs, $\lambda = \frac{h}{mv}$ in eqn (8), we get

$$\nabla^2\psi + \frac{4\pi^2}{h^2/m^2v^2}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \quad — (9)$$

Total Energy = Potential Energy + Kinetic Energy.

$$E = V + \tfrac{1}{2}mv^2$$

$$(E - V) = \tfrac{1}{2}mv^2$$

$$2(E - V) = mv^2$$

$\times m$, on both sides

$$2m(E - V) = m^2v^2 \quad — (10)$$

Substituting eqn (10) in (9), we get

$$\nabla^2\psi + \frac{4\pi^2}{h^2} \times 2m(E - V)\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{8\pi^2}{h^2} 2m(E - V)\psi = 0} \quad — (11)$$

Let us Introduce $\boxed{\hbar = \frac{h}{2\pi}}$ in eqn(11)

$$\hbar^2 = \frac{h^2}{2^2\pi^2} = \frac{h^2}{4\pi^2} \quad — (12)$$

Where $\hbar$ is a reduced Planck's Constant.

eqn (11) is modified by Substituting $\hbar$

The classical differential equation for wave motion is given by:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad (1)$$

The eqn (1) is written as

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \quad (2)$$

The where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian's operator.

The solution of eqn(2) gives ψ as periodic variations in terms of time 't',

$$\psi(x,y,z,t) = \psi_0(x,y,z)\, e^{-i\omega t}$$

$$\psi = \psi_0 e^{-i\omega t} \quad (3)$$

Differentiating the eqn(3) with respect to 't', we have

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

Again Differentiating w.r.t time 't'

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)(-i\omega)\psi_0 e^{-i\omega t}$$

$(\because \psi = \psi_0 e^{-i\omega t})$

$$\frac{\partial^2 \psi}{\partial t^2} = -i^2\omega^2 \psi \quad (4)$$

$i^2 = -1$

Substituting eqn (4) in eqn(2), we have.

$$\nabla^2 \psi = -\frac{\omega^2}{v^2}\psi$$

$$\nabla^2 \psi + \frac{\omega^2}{v^2}\psi = 0 \quad 5$$

We know that Angular frequency $\omega = 2\pi\nu$

ν = frequency ; $\nu = \frac{c}{\lambda} = \frac{v}{\lambda}$

$$\omega = \frac{2\pi v}{\lambda} \; ; \; \frac{\omega}{v} = \frac{2\pi}{\lambda} \quad (6)$$

$$\nabla^2\psi + \frac{m}{h^2/8\pi^2}(E-V)\psi = 0 .$$

$$\nabla^2\psi + \frac{m}{h^2/4\pi^2\times 2}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{2m}{h^2/4\pi^2}(E-V)\psi = 0 \quad -(13)$$

on substituting eqn (12) in eqn (13), Schrodinger Time - independent wave eqn written as

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar}(E-V)\psi = 0} \quad -(14)$$

(or) $$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi} \quad -(15)$$

—x—

Schrodinger Time Dependent wave equation:

Schrodinger Time dependent wave equation is derived from Schrodinger's time independent wave equation

The Solution of classical differential equation of wave motion is given by,

$$\psi = \psi_0 e^{-i\omega t} \quad -(1)$$

Differentiating eqn (1) w.r.t to 'time' t'

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t} \quad -(2) \quad \because \psi = \psi_0 e^{(-i\omega t)}$$

$$\because \omega = 2\pi\nu .$$

$$\frac{\partial\psi}{\partial t} = -i2\pi\nu\,\psi$$

$$\frac{\partial\psi}{\partial t} = i2\pi\frac{E}{h}\psi \quad -(3) \quad \because E = h\nu, \; \nu = E/h$$

$$\frac{\partial\psi}{\partial t} = -i\frac{E}{h/2\pi}\psi \qquad (h/2\pi = \hbar)$$

$$\frac{\partial \psi}{\partial t} = -i\frac{E}{\hbar}\psi \quad -(4)$$

$\times i$ on both sides.

$$\frac{i\partial \psi}{\partial t} = -i \times i\frac{E}{\hbar}\psi \quad (\because i^2 = -1)$$

$$\boxed{i\hbar\frac{\partial \psi}{\partial t} = E\psi} \quad -(5)$$

Schrodinger time Independent eqn is

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi} \quad -(6)$$

From eqn (5) & (6)

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial \psi}{\partial t}$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial \psi}{\partial t} \quad -(7)$$

$$\boxed{H\psi = E\psi} \quad -(8)$$

Where $H = \left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]$ is Hamiltonian operator

$E = i\hbar\frac{\partial \psi}{\partial t}$ is Energy operator.

eqn (7) is known as Schrodinger's time Dependent wave equation

———— x ————

Physical Significance of wave function 'ψ':

1. The variable quantity which describes de-Broglie wave is called "wave function ψ".

2. It connects the particle nature and its associated wave nature statistically.

3. The wave function associated with the moving particle at a particular instant of time and at a particular point in space is related to the probability of finding the particle at that instant and at that point.

4. The probability 0 corresponds to the certainty of not finding the particle and '1' corresponds to certainty of finding the particle.

i.e $\iiint \psi^* \psi = 1$. if particle is present

$\psi^* \psi = 0$. if particle is not present.
($\psi^* \rightarrow$ complex conjugate of ψ')

5. The probability of finding a particle at a particular region must be real and positive, but the wave function 'ψ' is in general complex quantity.

Motion of a Free Particle:

Let us consider electrons propagating freely in space in the positive x-direction and not acted upon by any force. Their potential energy is zero.

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \text{ reduces to}$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m E}{h^2}\psi = 0 \; ; \text{ taking } k^2 = \frac{8\pi^2 m E}{h^2} \text{ we get}$$

$$\boxed{\frac{d^2\psi}{dx^2} + k^2\psi = 0}$$

The general solution of this eqn is

$$\psi(x) = A e^{ikx} + B e^{-ikx} \quad (A \& B \text{ are constant})$$

The wave only propagate in positive x-direction. we get,

$$\psi(x,t) = A e^{ikx} e^{-i\omega t}$$

The allowed energy values form a continuum and are given by

$$\boxed{E = \frac{h^2 k^2}{8\pi^2 m}} \qquad \boxed{E \propto k^2}$$

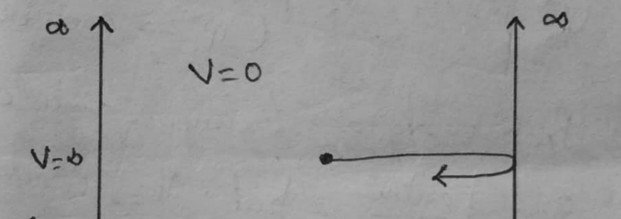
Particle in a infinite Potential:
(one-Dimensional Box)

Consider a Particle of mass 'm' moving between two rigid walls of a box at $x = 0$ and $x = a$ along x-axis. The Potential Energy (V) of the Particle inside the box is constant. It is taken as zero for Simplicity.

The walls are Infinitely high. The Potential Energy 'V' of the Particle is infinite outside the walls.

Thus, the Potential function is given by

$$\boxed{\begin{array}{ll} V(x) = 0 & \text{for } 0 < x < a \\ V(x) = \infty & \text{for } 0 \geq x \geq a \end{array}}$$



(Particle in a one Dimensional rigid box)

The Particle Cannot come out of the box. Also, it can not exist on the wall. So, wave function. So, wave function ψ is zero for $x \leq 0$ and $x \geq a$. Now, task is to find the Value ψ within the box ie between $x = 0$ and $x = a$.

Schrodinger's wave equation in one-dimensional is given by,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{——} (1)$$

Since $V=0$ between the walls, The eqn(1) reduces to

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad -(2)$$

Substituting $\frac{2mE}{\hbar^2} = k^2$ in eqn (2), we get

$$\boxed{\frac{d^2\psi}{dx^2} + k^2\psi = 0} \quad -(3)$$

The general solution of eqn (3) is given by

$$\boxed{\psi(x) = A \sin kx + B \cos kx} \quad -(4)$$

Here, A and B are two unknown constants.

The values of the constants A & B are determined by applying the boundary conditions.

<u>Boundary Condition (i)</u>

$\psi = 0$ at $x = 0$.

Applying this Condition to eqn (4), we have

$$0 = A \sin 0 + B \cos 0 \quad \left[\because \begin{array}{l} \sin 0 = 0 \\ \cos 0 = 1 \end{array}\right]$$

$$0 = 0 + B \times 1$$

Hence. $\boxed{B = 0}$

<u>Boundary Condition (ii)</u>

$\psi = 0$ at $x = a$

$$0 = A \sin ka + 0$$

$$A \sin ka = 0$$

It is found that either $A = 0$ (or) $\sin ka = 0$

'A' cannot be zero since already one of the constants B is '0'.
If A is zero, then the wave function is zero even in between walls of the box. Hence 'A' should not be zero.

$\therefore \sin ka = 0.$

Sin ka is '0' only when ka takes the value of $n\pi$

$$ka = n\pi$$

$$n = 1, 2, 3 \ldots$$

$$k = \frac{n\pi}{a} \quad \text{--- (5)}$$

$$k^2 = \frac{n^2\pi^2}{a^2} \quad \text{--- (6)}$$

$$(\because \hbar = h/2\pi)$$

we know that $k^2 = \frac{2mE}{\hbar^2} = \frac{2mE}{\left(h^2/2\pi^2\right)} = \frac{2mE \times 4\pi^2}{h^2}$

$$k^2 = \frac{8\pi^2 mE}{h^2} \quad \text{--- (7)}$$

Equating eqn (6) and (7)

$$\frac{n^2\cancel{\pi^2}}{a^2} = \frac{8\cancel{\pi^2} mE}{h^2}$$

$$\boxed{E_n = \frac{n^2h^2}{8ma^2}} \quad \text{--- (8)}$$

Substituting eqn (5) in eqn(4), we have.

$$\boxed{\psi_n(x) = A \sin \frac{n\pi x}{a}} \quad \text{--- (9)}$$

Here $n = 1, 2, 3$

For each value of n, there is an "energy level"

Each Value of $\underline{E_n}$ is known as <u>Energy Eigen Value</u> and the Corresponding $\underline{\psi_n}$ is Called as <u>eigen function</u>.

Normalisation of wave function:

The constant 'A' is determined by normalisation of wave function as follows.

Probability density is given by $\psi^* \psi$

w.k.t $\psi_n(x) = A \sin \frac{n\pi x}{a}$

$$\psi^* \psi = A \sin \frac{n\pi x}{a} \times A \sin \frac{n\pi x}{a}$$

[$\because$ $\psi = \psi^*$ The wave function is real (not complex)]

$$\psi^* \psi = A^2 \sin^2 \left[\frac{n\pi x}{a}\right] \quad —(10)$$

The Probability of finding the Particle inside the box. The Probability of finding the Particle inside the box of length 'a' is given by.

$$\int_0^a \psi^* \psi \, dx = 1 \quad —(11)$$

Substitute eqn (10) in eqn(11)

$$\int_0^a A^2 \sin^2\left(\frac{n\pi x}{a}\right) dx = 1 \qquad \left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$\frac{A^2}{2} \int_0^a [dx] - \int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx = 1 \qquad \left[\because \cos nx = \frac{\sin nx}{n}\right]$$

$$\frac{A^2}{2}\left[[x]_0^a - \left[\frac{\sin\left(\frac{2\pi n x}{a}\right)}{\frac{2n\pi}{a}}\right]_0^a\right] = 1$$

The second term of the Integral becomes zero at both limits

$$\frac{A^2}{2}[x]_0^a = 1$$

$$\frac{A^2 a}{2} = 1, \qquad A^2 = 2/a$$

$$\boxed{A = \sqrt{2/a}} \quad —(12)$$

on substituting eqn (12) in eqn (9), we have

$$\boxed{\psi_n = \sqrt{2/a}\ \sin \frac{n\pi x}{a}} \quad \text{—— (13)}$$

This expression (13) is known as "normalised eigen function"

From eqn (8) and (13), the following cases can be taken and they explain the motion of electron in one dimensional box.

Case (i): $\boxed{n = 1}$

$$E_1 = \frac{h^2}{8ma^2} \ ; \quad \psi_1(x) = \sqrt{2/a}\ \sin\left(\frac{\pi x}{a}\right)$$

$\psi_1(x)$ is maximum at exactly middle of the box.

Case (ii): $\boxed{n = 2}$

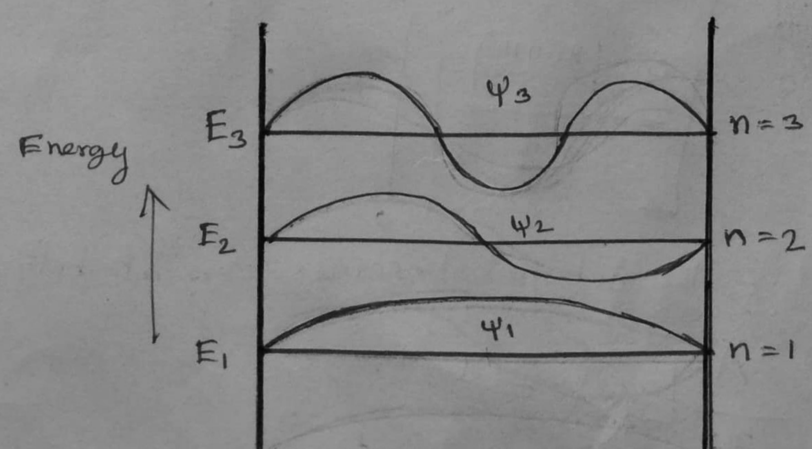
$$E_2 = \frac{4h^2}{8ma^2} = 4E_1 \ ; \quad \psi_2(x) = \sqrt{2/a}\ \sin\left[\frac{2\pi x}{a}\right]$$

$\psi_2(x)$ is maximum at quarter distance from either sides of the box

Case (iii) $\boxed{n = 3}$

$$E_3 = \frac{9h^2}{8ma^2} = 9E_1 ; \quad \psi_3(x) = \sqrt{2/a}\ \sin\left(\frac{3\pi x}{a}\right)$$

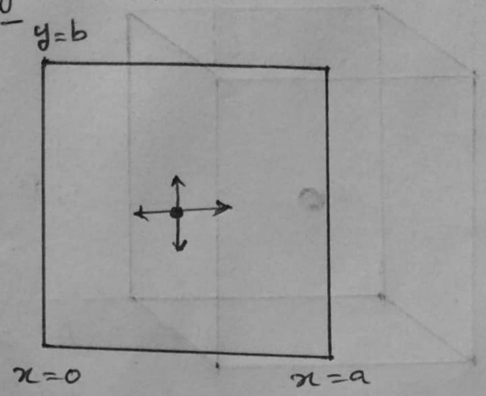
$\psi_3(x)$ is maximum at exactly middle and one - sixth distance from either of the sides of the box.



Extension to Two Dimension (2D Boxes)

In a Two dimensional Potential well, the Particle can freely move in two directions (x and y). n_x and n_y Corresponding to the two coordinate axes namely x and y respectively.

If 'a' and 'b' are the lengths of the well as shown fig along 'x' and 'y' axes, Then



Energy of the Particle $E = E_{nx} + E_{ny}$

$$E_{n_{xy}} = \frac{n^2h^2}{8ma^2} + \frac{n^2h^2}{8mb^2}$$

If $a=b$

$$E_{n_x n_y} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2\right]$$

The Corresponding normalised wave function of the Particle in the two dimensional well is written as

$$\psi_{n_x n_y} = \sqrt{2/a}\ \sin\left(\frac{n_x \pi x}{a}\right) \times \sqrt{2/a}\ \sin\left(\frac{n_y \pi y}{b}\right)$$

$$= \sqrt{2/a} \times \sqrt{2/a}\ \sin\left(\frac{n_x \pi x}{a}\right) \times \sin\left(\frac{\pi y n_y}{b}\right)$$

$$\therefore \psi_{n_x n_y} = \sqrt{\frac{4}{ab}}\ \sin\left(\frac{n_x \pi x}{a}\right) \times \sin\left(\frac{n_y \pi y}{b}\right)$$

example: $n_x = 1,\ n_y = 2$; similarly $n_x = 2,\ n_y = 1$

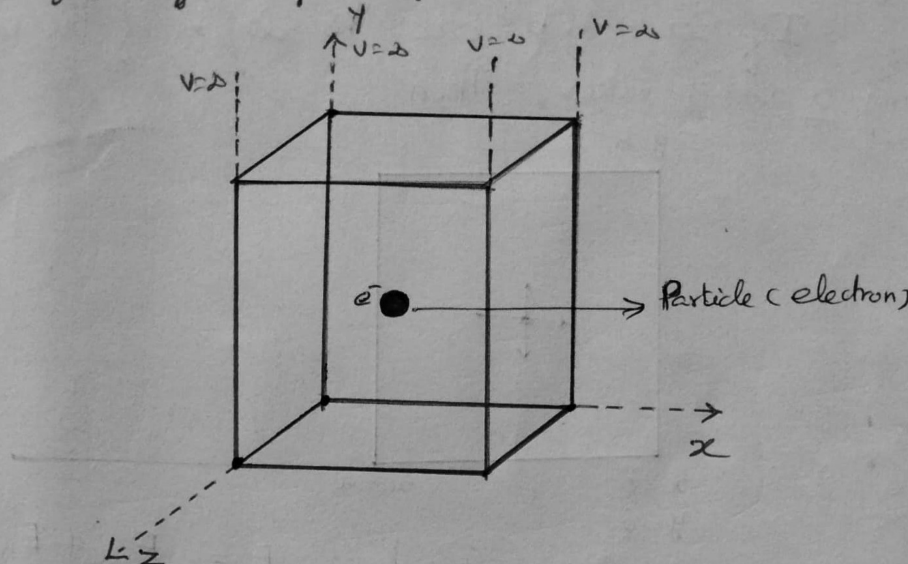
$$\boxed{E_{12} = E_{21} = \frac{5h^2}{8ma^2}}$$

$$\boxed{\begin{aligned}\psi_{12} &= \sqrt{4/ab}\ \sin\frac{\pi x}{a}\ \sin\frac{2\pi y}{b} \\ \psi_{21} &= \sqrt{4/ab}\ \sin\frac{2\pi x}{a}\ \sin\frac{\pi y}{a}\end{aligned}}$$

We understand that several combinations of the two quantum numbers (n_x & n_y) lead to different eigen value & eigen functions.

Particle in Three Dimensional Box:

In a Three dimensional box, The Particle can move in any direction in space. So, we have to use three quantum numbers, n_x, n_y, n_z Corresponding to the three Coordinate axes namely x, y and z respectively.



If a, b, c are the lengths of the box.

Energy of the Particle $= E_x + E_y + E_z$

$$E_{n_x n_y n_z} = \frac{n_x^2 h^2}{8ma^2} + \frac{n^2 h^2}{8mb^2} + \frac{n^2 h^2}{8mc^2}$$

If $a=b=c$ as for cubical box, Then

$$\boxed{E_{n_x n_y n_z} = \frac{h^2}{8ma^2}\left[n_x^2 + n_y^2 + n_z^2\right]} \quad — (1)$$

Corresponding normalised wave function

$$\Psi_{n_x n_y n_z} = \sqrt{2/a}\,\sin\left(\frac{n_x \pi x}{a}\right) \times \sqrt{2/c}\,\sin\left(\frac{n_y \pi y}{b}\right) \times \sqrt{2/c}\,\sin\left(\frac{n_z \pi z}{c}\right)$$

$$\therefore \boxed{\Psi_{n_x n_y n_z} = \sqrt{8/abc}\,\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{b}\right)\sin\left(\frac{n_z \pi z}{c}\right)} \quad — (2)$$

From eqn (1) & (2) Several Combination of the three quantum numbers (n_x, n_y and n_z) lead to different energy eigen value and eigen function

Example: $n_x = 1,\ n_y = 1,\ n_z = 2$

$n_x^2 + n_y^2 + n_z^2 = 1^2 + 1^2 + 2^2 = 6$ similarly, $n_x = 1, n_y = 2, n_z = 1$, and for $n_x = 2, n_y = 1, n_z = 1$ we have '6'

$$\boxed{\therefore\ E_{112} = E_{121} = E_{211} = \frac{6h^2}{8ma^2}} \quad — (3)$$

The corresponding Wave function is written as

$$\left.\begin{aligned}
\psi_{112} &= \sqrt{8/a^3}\ \sin\frac{\pi x}{a}\ \sin\frac{\pi y}{b}\ \sin\frac{2\pi z}{c} \\
\psi_{121} &= \sqrt{8/a^3}\ \sin\frac{\pi x}{a}\ \sin\frac{2\pi y}{b}\ \sin\frac{\pi z}{c} \\
\psi_{211} &= \sqrt{8/a^3}\ \sin\frac{2\pi x}{a}\ \sin\frac{\pi y}{b}\ \sin\frac{\pi z}{c}
\end{aligned}\right\} \quad — (4)$$

Degeneracy:

From eqn (3) and (4) several combinations of quantum numbers, we have the same Energy eigen Value but different eigen functions Such a State of energy level is called "degenerate state".

Non - degenerate State:

When only one wave function Corresponds to the energy eigen value, Such a State is Called " non- degenerate State.

Suppose: $n_x = 2,\ n_y = 2,\ n_z = 2$

Then, $E_{222} = \dfrac{12h^2}{8ma^2}$ and

$$\psi_{222} = \sqrt{\frac{8}{a^3}}\ \sin\frac{2\pi x}{a}\ \sin\frac{2\pi y}{b}\ \sin\frac{2\pi z}{c}.$$

— x —

Probability Density:

Probability of finding the Particle between Position x and $x + dx$.

$$P(x) = |\psi_n|^2 dx = \frac{2}{a} \sin^2\left(\frac{n\pi x}{a}\right) dx.$$

Probability density is maximum when

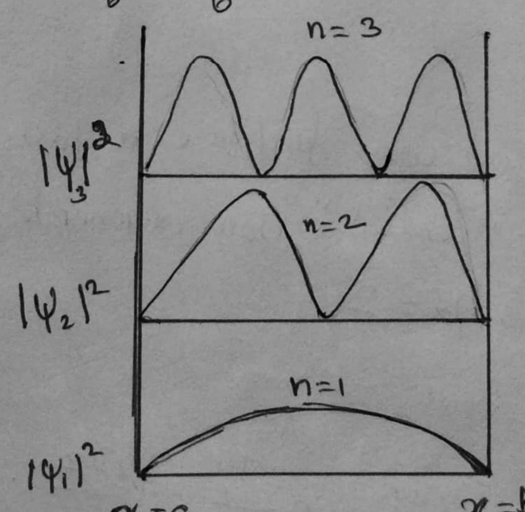
$$\frac{n\pi x}{a} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \cdots$$

$$x = \frac{a}{2n}, \frac{3a}{2n}, \frac{5a}{2n} \cdots$$

* $n=1$, $x = \frac{a}{2}$. The Particle most likely to be in the middle of the box (because $|\psi_1|^2$ is maximum there)

* For $n=2$, $x = \frac{a}{4}$ and $\frac{3a}{4}$, The Particle most likely to be at $\frac{a}{4}$ and $\frac{3a}{4}$ and never found in the middle of the box because $|\psi_2|^2$ is zero There.

* For $n=3$; The most likely Position of Particle are $x = \frac{a}{6}, \frac{3a}{6}, \frac{5a}{6}$



While classical mechanics Predicts the same Probability for the Particle being anywhere in the box. Quantum mechanics Predicts that the Probability is different at different Points and There are Points (nodes) where the Particle is never found.

Corcespondance Principle:

Quantum mechanics is highly successful in describing microscoping entities like atoms and elementary Particles. But, macroscopic system, like tennis ball, automobile etc, are accurately described by classical mechanics.

Bohr's Corcespondance Principle bridges the gap between the Classical mechanics and Quantum mechanics. it removes The apparent discontinuity between these two.

Statement:

"The Principle States that for large quantum numbers, quantum Physics gives the same results as those of classical Physics."

In fact, " the greater The quantum number, The Closer quantum Physics approaches Classical Physics"

Example:

Einstein's Special relativity Satisfies the Corcespondance Principle, because it reduces to classical mechanics in the limit of Velocities small compared to the speed of light.

Significance:

The corcespondance Principle has Proved to be of great use in the computation of the Intensity, Polarisation and Coherance of Spectral radiation. It has also been ~~helpfull~~ helpful in the formulation of "Selection rules"

— ∝ —

Unit - V

APPLIED QUANTUM MECHANICS

5.1. THE HARMONIC OSCILLATOR:

Any Oscillation system for which the net restoring force is directly proportional to the negative of the displacement is called an harmonic oscillator.

A pendulum, a particle attached to a spring, or many vibrations in atoms and molecules can be described as a harmonic oscillator.

A simple realization of the harmonic oscillator is a mass, attached to the end of a simple spring as shown in fig.5.1

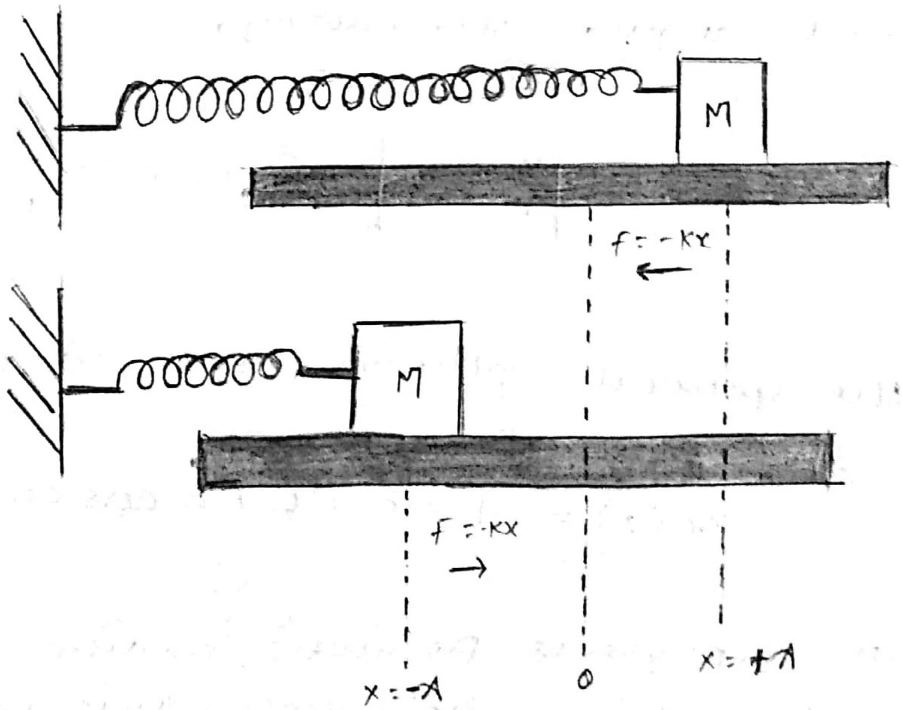


Fig. 5.1 Harmonic oscillator

The equation of motion for the simple harmonic oscillator is given by

$$\frac{d^2 x}{dt^2} + \frac{k}{m} x = 0 \quad \rightarrow (2)$$

Where x is the position of the mass as a function of time, t. The constant k is known as the force constant; the larger the force constant, the larger the restoring force for a given displacement from the equilibrium position.

Equ (2) can be rewritten as

$$\frac{d^2 x}{dt^2} + \omega^2 x = 0 \quad \rightarrow (3)$$

Where the oscillation occurs with a constant angular frequency,

$$\omega = \sqrt{\frac{k}{m}} \quad \left[\text{since, } \omega^2 = \frac{k}{m} \right]$$

The general solution to eq (3) is

$$x(t) = A \sin \omega t + B \cos \omega t \quad \rightarrow (4)$$

With represents periodic motion with a sinusoidal time dependence. This is known as simple harmonic motion and the corresponding system is known as harmonic oscillator.

Energy in the Harmonic Oscillator:

The total energy E of the oscillator is the sum of its kinetic energy and the elastic potential energy of the force given by

$$E = \frac{1}{2} mv^2 + \frac{1}{2} mx^2$$

Where v is the velocity of the mass m when it is at a distance x from the equilibrium position.

The study of quantum mechanical harmonic motion begins with the specification of the schrodinger equation.

The classical potential energy is given by

$$V = \frac{1}{2} Kx^2$$

and so we can write down the schrodinger equation as

$$\left[\underbrace{-\frac{\hbar^2}{2m} \frac{d^2}{dx^2}}_{\text{Kinetic Energy}} + \underbrace{\frac{1}{2} Kx^2}_{\text{Potential Energy}} \right] \psi(x) = E\psi(x)$$

Since harmonic motion has a characteristic angular frequency, it makes sense to measure energy in terms of ω.

Hence, the allowed energies are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \quad \text{for } n = 0, 1, 2, 3, \ldots$$

where the ground state is usually designated with the quantum number, $n=0$

Therefore, we have

$$E_0 = \left(\frac{1}{2}\right)\hbar\omega$$

$$E_1 = \left(\frac{3}{2}\right)\hbar\omega$$

$$E_2 = \left(\frac{5}{2}\right)\hbar\omega$$

$$E_3 = \left(\frac{7}{2}\right)\hbar\omega \quad \text{and so on.}$$

It is clear that the difference between successive energy eigen values has a constant value given by,

$$\therefore \Delta E = E_{n+1} - E_n = \hbar\omega$$

The potential, energy function and first few energy levels for the harmonic oscillator as shown in Fig. 5.2

As the quantum member number n increases, the energy of the oscillator and therefore the amplitude of oscillation increases.

A packet of energy $\hbar\omega$ is needed to make the quantum harmonic oscillator to move from a lower energy state to higher energy state.

Here, the ground-state energy, $E_0 = (\frac{1}{2})\hbar\omega$ is greater than the classical value of zero, which is a consequence of the uncertainty principle. This means that the oscillator is always oscillating.

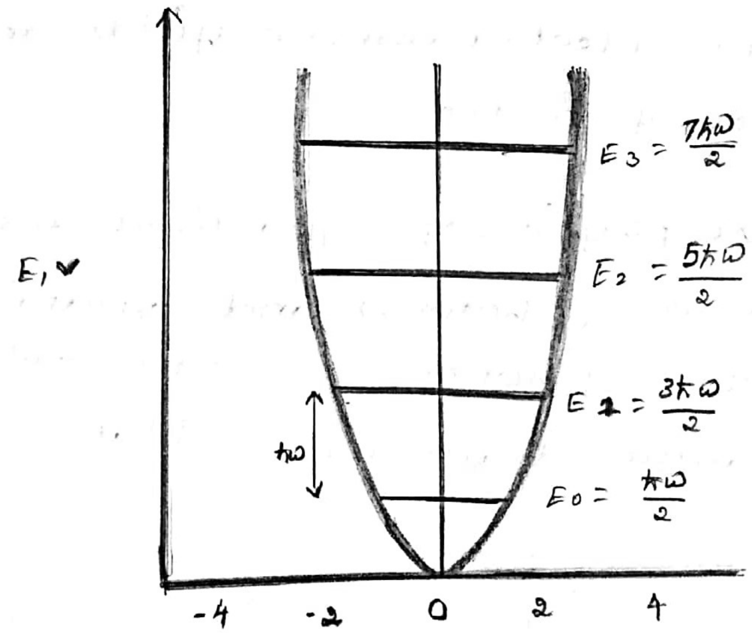


Fig. 5.2. Energy States and potential well of a quantum harmonic oscillator

The salient features of harmonic oscillator are,

The energies are quantized and the energy levels are evenly spaced.

There is a non-zero ground state energy.

E = 0 is not allowed by Heisenberg uncertainly principle.

Significance :

(i) It serves as a prototype in the mathematical treatment of phenomena like elasticity, acoustics, AC circuits, molecular and crystal vibrations, electromagnetic fields and optical properties of matter.

(ii) The physics of quantized electromagnetic oscillations (photons) and quantized mechanical oscillations (phonons) is intimately related to the quantum harmonic oscillator.

5.2. BARRIER PENETRATION AND QUANTUM TUNNELLING:

According to quantum mechanics, a particle such as electron can penetrate a barrier into a region forbidden by classical mechanics.

This phenomenon is known as barrier penetration and can happen only when the particle exhibits wave nature.

Tunnelling is a quantum phenomenon where particles with less energy than that of a potential barrier can still cross the energy barrier, by penetrating through it.

Explanation:

Let us consider a particle of mass m travelling to the right along the x-axis. The particle encounters a narrow potential barrier whose height V_0 is greater than E and whose thickness is L.

- classically all the particles.

(i) Will be reflected back (at $x=0$) if $E < V_0$ and

(ii) Will be transmitted to $x > L$ if $E > V_0$

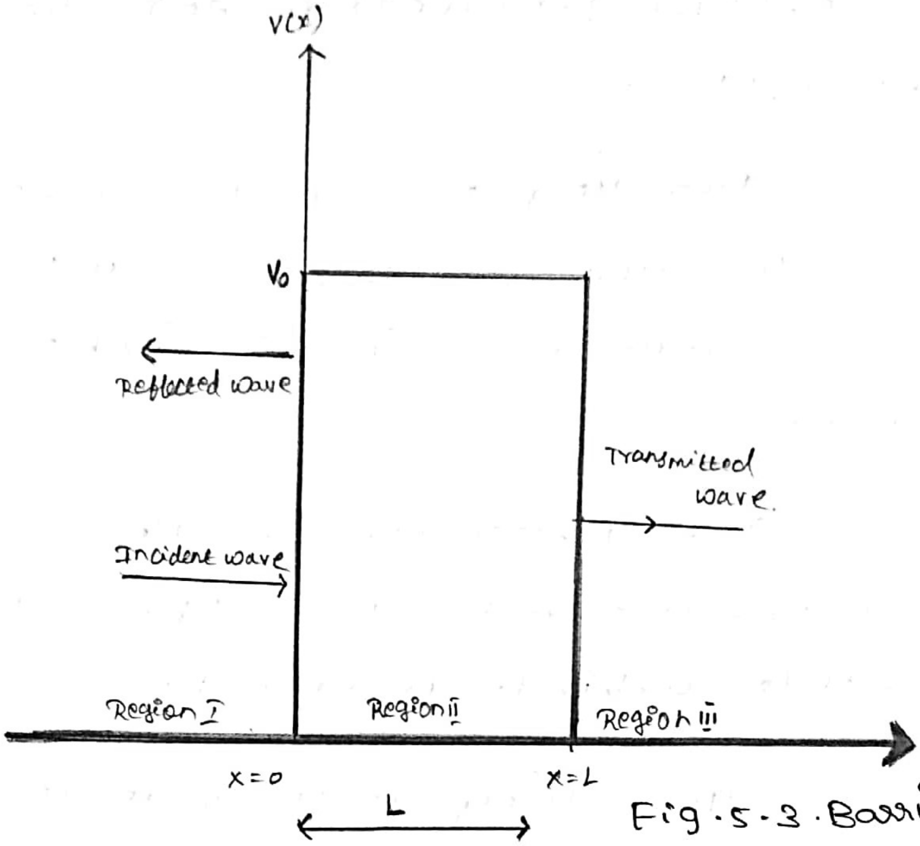


Fig. 5.3. Barrier penetration

But, quantum mechanics predicts a non-zero probability for finding the particle on the other side of the barrier even when $E<V_0$.

This can happen as the approaching particle has a sinusoidal wave function as shown in fig. 5.4.

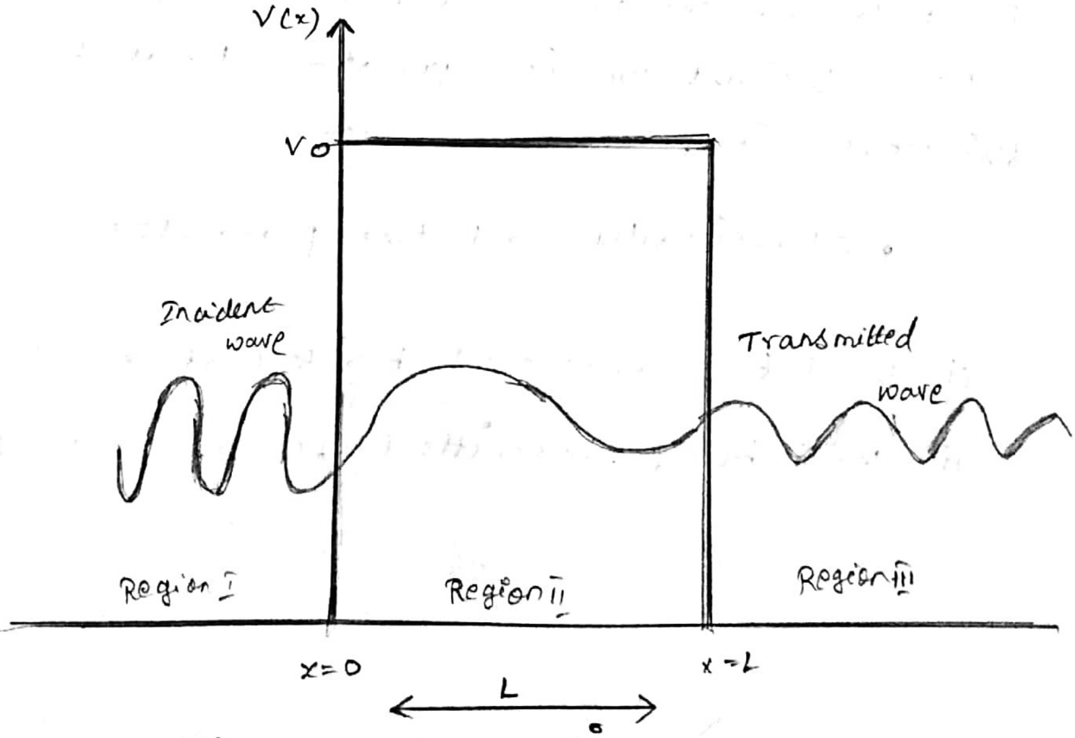


Fig. 5.4. Tunnelling

Within the barrier the wave is decaying and before it dies away to zero, there is again a sinusoidal wave function, at $x=L$. But it is a sine wave of greatly reduced amplitude.

Since, $[\psi]^2$ is non-zero beyond the barrier it is evident that there is a non-zero probability that the particle penetrates the barrier. This process is called tunnelling through the barrier or barrier penetration. Thus, tunnelling is a result of the wave properties of material particles.

Tunnelling probability:

The tunnelling probability can be described with a transmission co-efficient, T and a reflection co-efficient R.

Since an incident particle must either reflect or tunnel through, we have $T+R=1$

Transmission coefficient:

The probability that the particle gets through the barrier is called transmission coefficient (T).

$$T = \frac{\text{Probability density of transmitted wave}}{\text{probability density of incident wave}}$$

The transmission coefficient is given by

$$T = e^{-2G_1L}$$

Where,

$$G_1 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

The transmission probability increases with decrease in height and width of the barrier.

5.3. TUNNELLING MICROSCOPE :

An electron microscope that works by quantum tunnelling phenomenon and creates atomic scale imaging of surfaces is known as tunnelling microscope.

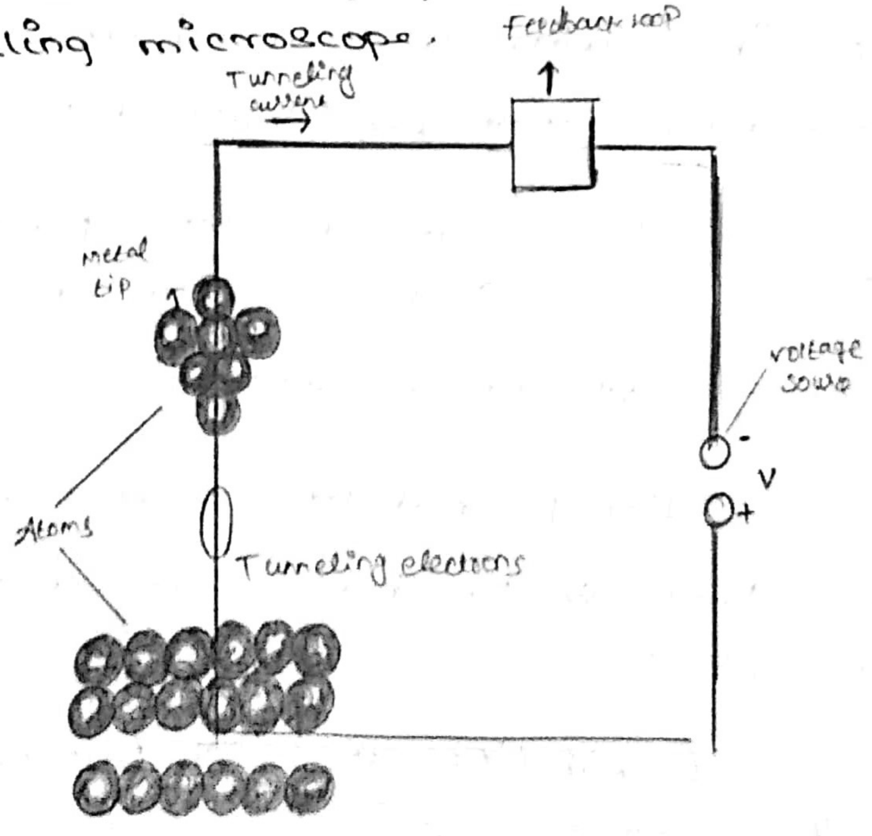


Fig. 5.5. Tunnelling process

For the electrons in the sample and in the metal tip, it is forbidden to stop in the gap between sample and tip. However, this gap is so small that the electrons are able to tunnel and flow through the gap.

Principle:

When a voltage is applied between a conducting tip and a surface close to it, electron can tunnel through the vaccum between the atoms of the tip and the surface. The tunnelling currents that results depends upon the distance between probe tip and sample surface.

Construction

The basic components are,

piezoelectric tube
Tunnelling current amplifier
Distance control unit & scanning unit
Data processing and display.

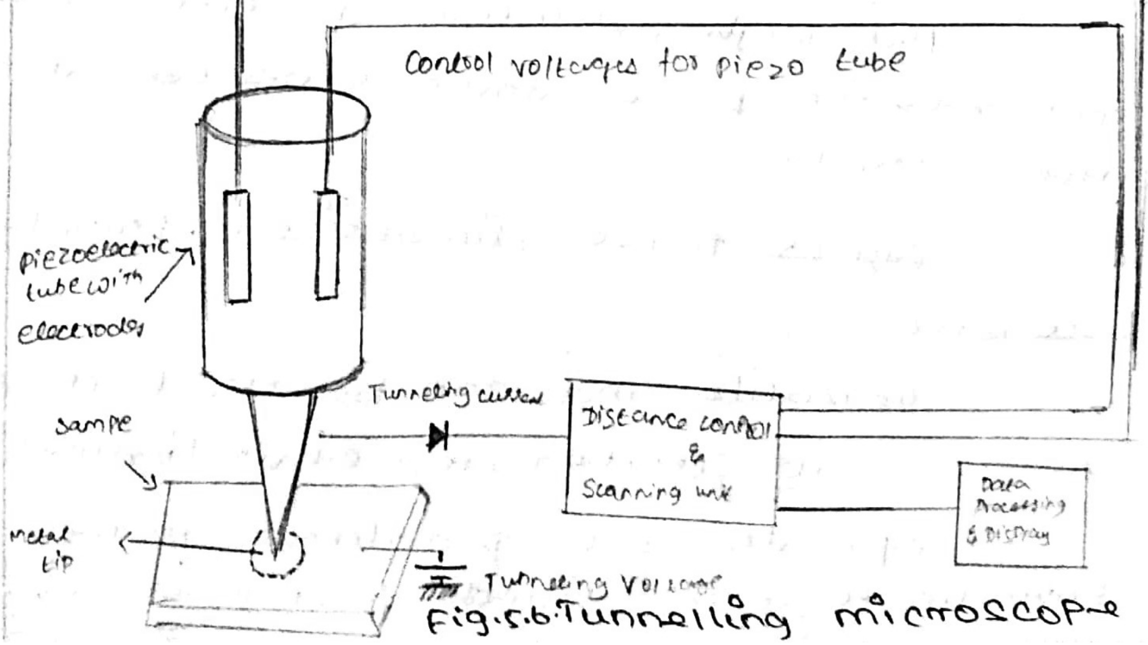


Fig.5.6 Tunnelling microscope

Working

The sharp conducting probe tip attached to a piezoelectric tube is positioned at a distance few angstroms from the sample surface.

A small voltage applied between the probe tip and the surface causes electrons to tunnel from the tip to the sample surface.

As the probe is scanned over the surface, the voltage applied to the piezotube is altered to maintain a constant tip-surface distance.

changes in this voltage registers variations in the tunnelling current.

The changes in the tunnelling current are recorded and then used to generate a map of the sample surface on the display unit

Merits:

The high resolution of STMs enables researchers to examine surfaces at an atomic level.

Gives three dimensional profile of a surface.

Versatile and can be used in ultrahigh vaccum, air, water and other liquids and gases

operates in temperatures as low as zero Kelvin up to a few hundred degrees celsius.

Demerits:

Require very stable, clean surfaces and conducting surfaces.

Difficult to use effectively.

The electronics required are extremely sophisticated as well as very expensive.

Applications:

The tunnelling microscope is widely used in both industrial and fundamental research to obtain atomic-scale images of metal surfaces

Used as diagnostic tool in the fields like solid state physics, electrochemistry, biology, organic, chemistry, nano machining etc..,

Defects and physical structure of synthetic chemical compounds can be studied.

To study charge transport mechanism in molecules.

Used in research surrounding semiconductors and microelectronics.

5.4. RESONANT DIODE

A diode with a resonant tunnelling structure that allows electrons to tunnel through various resonant states at certain energy level is known as resonant diode.

Principle:

Tunnelling of electrons through a finite - height potential well that occurs only when electron energies match an energy level in the well.

Construction:

The structure of resonant diode is shown below.

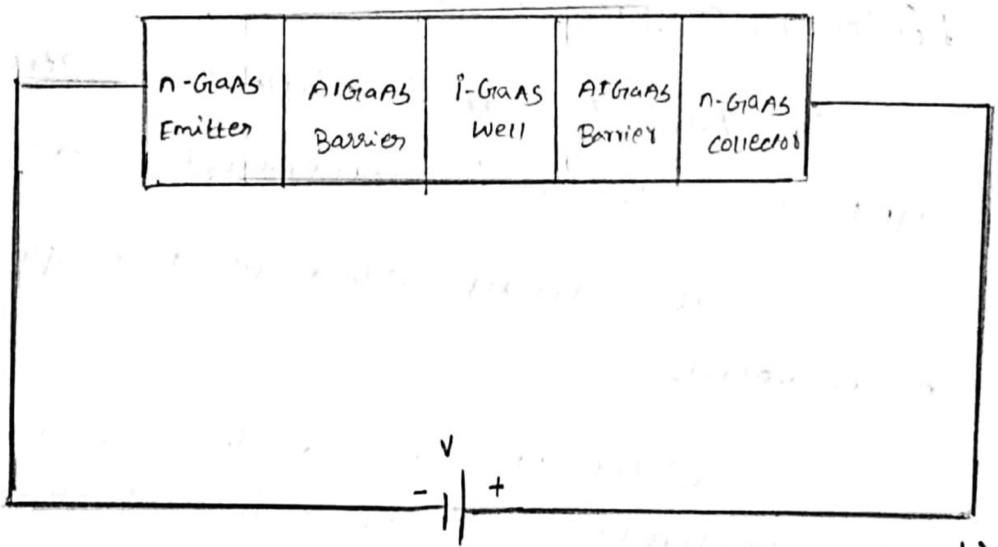


Fig. 5.7. Structure of resonant diode

It consists of an intrinsic GaAs quantum well region sandwiched between two thin barrier regions made of AlGaAs.

The regions at the extreme ends on both sides are made of heavily doped n-GaAs and they serve the purpose of emitter and collector.

By applying a bias voltage, the tunnelling current across the diode can be controlled.

Working :

According to quantum mechanics, electrons can tunnel from outside into the well through barrier under suitable conditions

The Energy band diagram of the resonant diode is shown in fig. 5.8

Without any voltage bias, the electron energy level in the well is higher than the incident electron energy (E).

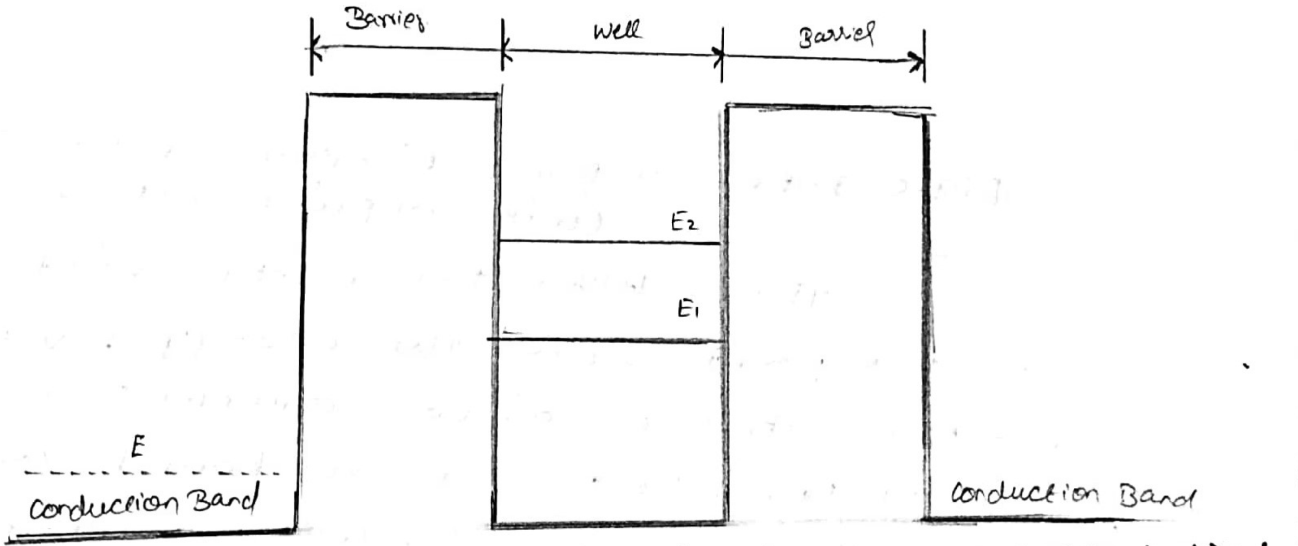


Fig. 5.8. Energy Band Diagram of resonant diode (with no voltage bias)

So, no electron in the conduction band can tunnel to the well, and there is no current.

On increasing the bias voltage, the incident electron energy level, E on the left becomes higher and matches an energy level in the potential well. Now some electrons can tunnel into the well and the current from left to right increases due to tunnelling.

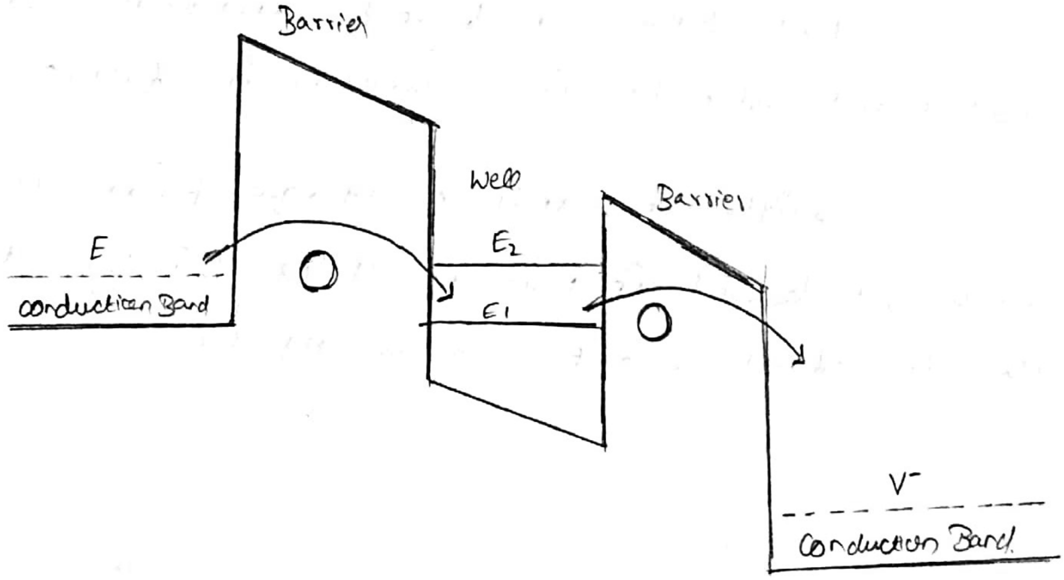


Fig.5.9. Energy Band Diagram of resonant diode (with applied voltage bias)

Thus, when the electric field increases to the point where the energy level of the electrons in the emitter coincides with the quasi-bound state of energy level to the well, the current reaches a maximum. This type of tunnelling is known as resonant tunnelling.

I-V characteristics of resonant diode:

The current - voltage (V) characteristic for resonant diode is shown in Fig.5.10.

As voltage increase, E also increases and hence the tunnelling current increases and reaches a peak point.

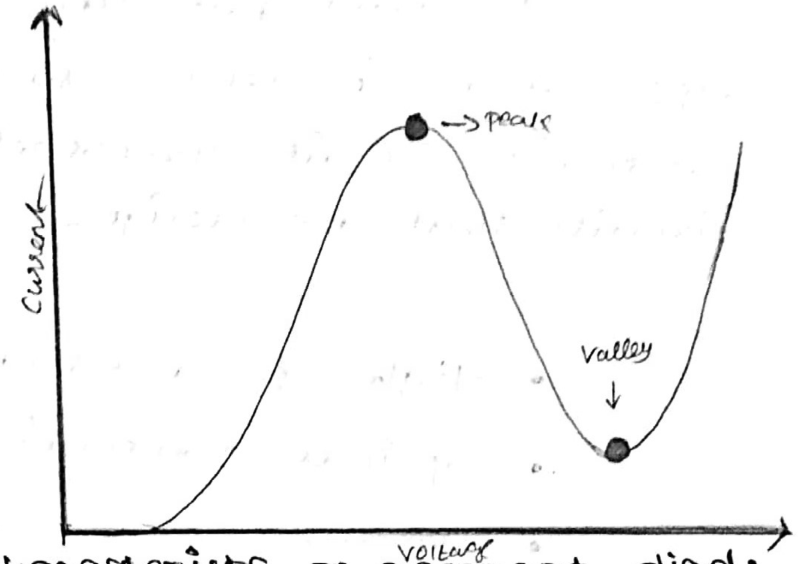


Fig.5.10. I-V-characteristic of resonant diode

Further increases in voltage alters the energy value E and hence the transmission is low. This results in decrease in current which reaches a minimum value point called valley. The decrease of current with an increase in voltage is known as negative resistance.

As the applied voltage continues to increase, current begins to rise again because of substantial thermionic emission.

Let us consider a particle with total energy $E < V_0$ confined within a well bouncing back and forth between the turning points at $x = -L$ and $x = L$.

The potential well is divided into three regions (1, 2, 3) with associated wave function as follows.

$$\psi = \begin{cases} \psi_1, \text{ if } x < -L & \text{(the region outside the box)} \\ \psi_2, \text{ if } -L < x < L & \text{(the region inside the box)} \\ \psi_3, \text{ if } x > L & \text{(the region outside the box)} \end{cases}$$

The time independent Schroedinger's equation can be written as

$$\frac{-\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + V\psi = E\psi \quad \rightarrow (1)$$

Inside the box

For this region, inside the box $V(x) = 0$ and eqn (1) reduces to

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_2}{dx^2} = E\psi_2 \quad \rightarrow (2)$$

Letting

$$K = \frac{\sqrt{2mE}}{\hbar}$$

eqn (2) becomes

$$\frac{d^2\psi_2}{dx^2} = -K^2\psi_2$$

The general solution for the above differential equation is

$$\psi_2 = A\sin(Kx) + B\cos(Kx) \rightarrow (3)$$

Here, A & B are constants and K can be any real number

Hence, $\boxed{E = \frac{K^2\hbar^2}{2m}}$

Outside the box

In the regions, $x < -L$ and $x > L$, we have, $V = V_0$ and now the Schrodinger equation is

$$\frac{-\hbar^2}{2m}\cdot\frac{d^2\psi}{dx^2} + V_0\psi = E\psi$$

We rewrite this as

$$\frac{d^2\psi}{dx^2} - \left[\frac{2m(V_0 - E)}{\hbar^2}\right]\psi = 0 \rightarrow (4)$$

Let us assume that E is less than V_0, so the particle is "trapped" in the well. There might be only one such bound state or more.

We define a constant G by

$$G^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

and rewrite the Schrodinger equation as

$$\frac{d^2\psi}{dx^2} - G^2\psi = 0$$

This equation has the general solution

$$\psi_{1,3} = Ce^{Gx} + De^{-Gx}$$

In the region, $x < -L$, x is always negative, so, D must be zero. Similarly in region, $x > L$, where x is always positive, C must be zero.

Hence,
$$\psi_1 = Ce^{Gx}$$
$$\psi_3 = De^{-Gx}$$

Conclusion:

In a finite potential well, the wave function extends into these classically forbidden region where the total energy is less than the potential energy. The situation is possible and is consistent with the uncertainty principle.

However, in both these regions the wave function decreases exponentially with distance from the well.

If the particle manages to acquire energy, $E > V_0$, then it will escape from the well.

An electron confined within a semiconductor by an electric force has a potential energy that can be modelled as a finite potential well. similarly a proton confined within the nucleus by the nuclear force has a potential energy that can be modelled as a finite potential well. Hence any situation in which a particle is confined can be modelled as a finite potential well.

Characteristics of finite potential well.

The number of bound state energies is finite.

The number of bound states increases with the width and depth of the well.

Tunnelling into the barrier (wall) is possible.

Higher energy states are less tightly bound than lower ones.

A particle provided with enough energy can escape the well (unbound state).

5.6. BLOCH THEOREM

One of the characteristic features of many solids is the regular arrangement of their atom forming a crystal. The potential energy of electrons in such a crystal is the result of the positively charged ion producing a columbic attraction.

In a crystal, electrons move in a potential V(x) which is produced by regularly-spaced ion cores as shown in fig.5.12 (a)

The potential of the electron at the site of positive ions is zero and is maximum in between the sites of two positive ions as shown in fig.5.12(b)

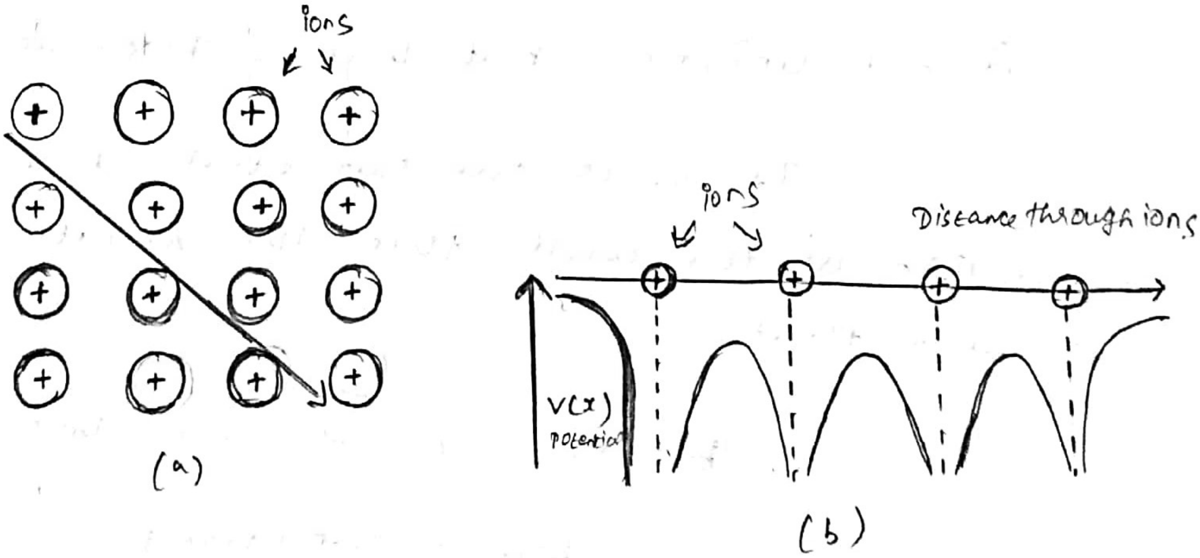


Fig.5.12. Electrons in a periodic potential

Let a one-dimensional lattice (i.e.) only an array of ionic cores along x-axis is considered.

Since the potential energy of any particle bound in a field of attraction is negative, and since the conduction electron is bound to the solid, its potential energy V is negative.

Further, as it approaches the site of an ionic core, $V \to -\infty$

Since this occurs symmetrically on either side of the core, it is referred to as potential well.

The width of the potential well 'b' is not uniform, but has a tapering shape.

If V_0 is the potential at a given depth of the well, then the variation is such that,

$b \to 0$, as $V \to \infty$ and hence,

bV_0 = constant

Now, since the lattice is a repetitive structure of the ion arrangement in a crystal, the type of variation at also repeats itself.

If "a" is the inter-ionic distance, then, as we move in x-direction, the value of V will be same at all points which are seperated by a distance equal to "a".

$$\text{ie, } V(x) = V(x+a)$$

where, x is distance of the electron from the core.

Such a potential is said to be a periodic potential.

The Bloch theorem states that, for a particle moving in a periodic potential, the eigen functions for a conduction electron are of the form,

$$\psi(x) = U(x) \cos Kx$$

$$\text{where, } U(x) = U(x+a)$$

The function U(x) has the same periodicity as the potential energy of the electron, and is called the modulating function.

Significance:

A large number of materials are well described by regular atomic spacing and a periodic potential for a crystal lattice which is like a string of finite wells.

The presence of periodic potential in a crystal leads to energy bands, which are essentially energy intervals between which energy levels are nearly continuous.

5.7. THE KRONIG PENNEY MODEL:

The study of essential behaviour of electrons by approximating the potential inside a crystal to the shape of rectangular steps is called Kronig-Penney model of potentials.

The Kronig-penney model describes the one dimensional representation of electron potential in a periodic lattice (fig.5.18)

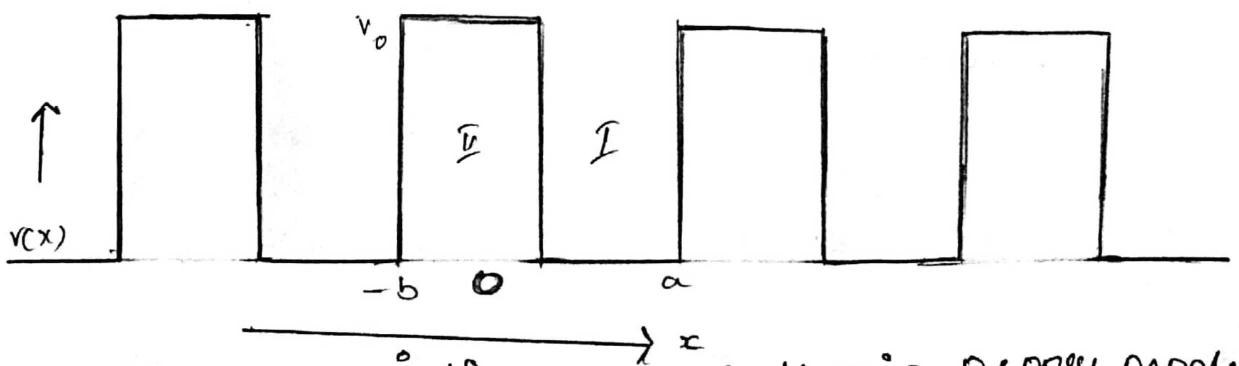


Fig: 5.18 Periodic potential - Kronig-penney model

This model consist of an infinite row of rectangular potential wells separated by barrier of width "b". Each well has a width "a" and a depth V_0.

It is assumed that when an electron is near the positive ion site, potential energy is taken as zero. Whereas, outside the well, that is, in between two positive ions, potential energy is assumed to be V_0.

Hence, we have

$$V(x) = V_0 \quad \text{for} \quad -b < x < 0$$

and

$$V(x) = 0 \quad \text{for} \quad 0 < x < a$$

The period of the potential is, $(a+b)$

The possible states that the electron can occupy are determined by the schrodinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

The schrodinger equation for the two regions can be written as

$\boxed{\text{Region I}}$ $$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E)\psi = 0 \quad , 0 < x < a \rightarrow (1)$$

[since, $V=0$]

Region (ii) $\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$, $-b < x < 0$ → (5)

[since, V = 0]

We rewrite the above equation as

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \text{ for } 0 < x < a \quad \rightarrow (6)$$

and

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \text{ for } -b < x < 0 \quad \rightarrow (7)$$

where,

$$\alpha^2 = \frac{2mE}{\hbar^2}$$

$$\beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$$

Bloch has given the solution for Schrodinger equation as

$$\psi(x) = U(x)\cos kx$$

where, $U(x) = U(x+a)$

Solving the above equation (6) & (7) by applying boundary conditions, we get

$$\frac{8\pi^2 m V_0}{2h^2 a} b \cdot \sin(\alpha a) + \cos(\alpha a) = \cos(ka) \quad \rightarrow (7)$$

$$\frac{P}{\alpha a}\sin(\alpha a) + \cos(\alpha a) = \cos(ka) \quad \rightarrow (8)$$

where, $P = \frac{8\pi^2 m V_0 \alpha b}{2h^2}$ = potential barrier strength

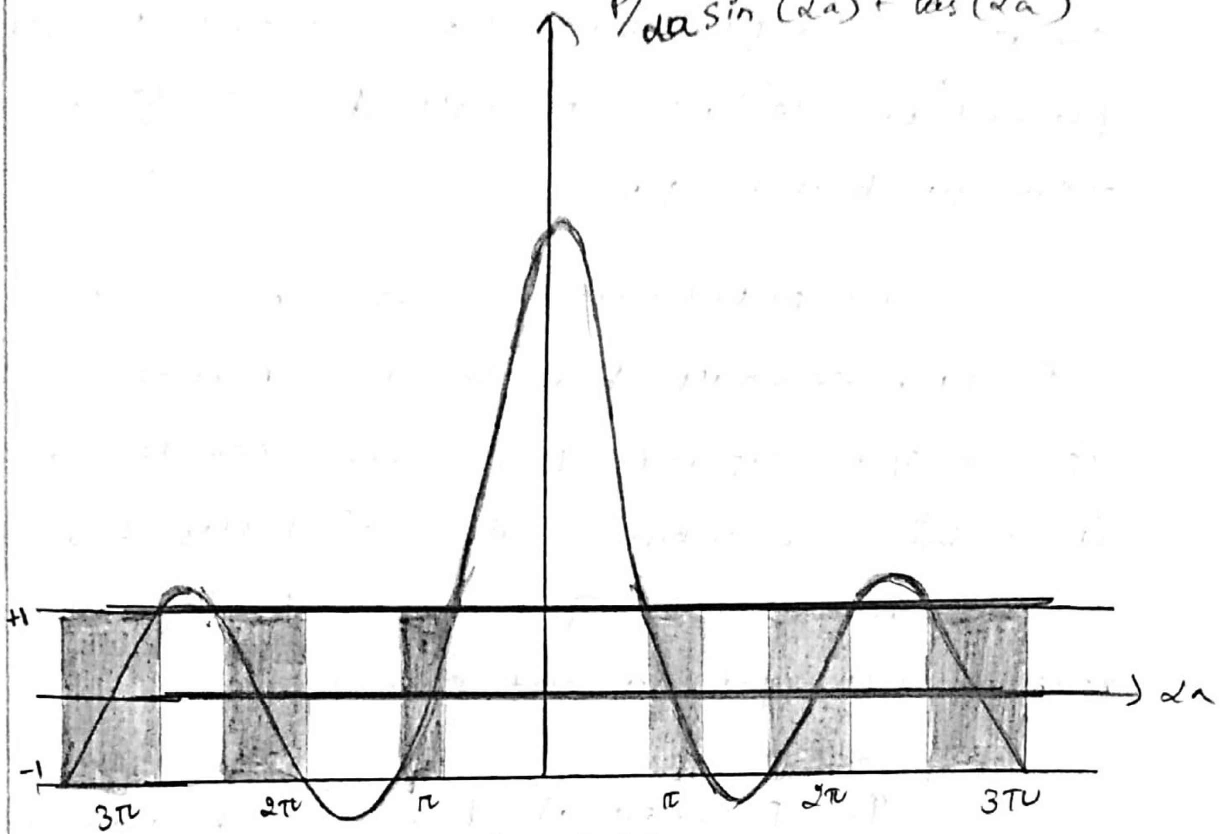


Fig. 5.14

By varrying α, a, a wave function mechanical nature could be plotted as shown in fig. 5.14.

The shaded portion of the wave shows the bands of allowed energy with the forbidden region as unshaded portion.

Thus, the energies of an electron moving under a periodic lattice potential lie only in certain allowed zones; other energies are forbidden.

Results from Kronig - Penney Model:

The Kronig - penney model demonstrates that a simple one dimensional periodic potential yeilds energy bands as well as energy band gaps.

If potential barrier between wells is strong, energy bands are narrowed and speed far apart. This corresponds to crystals in which electrons are tightly band to ion cores, and wave functions donot overlap much with adjacent cores.

If potential barrier between wells is weak, energy bands are wide and spaced close together.

The energy spectrum of electrons consist of an infinite number of allowed energy bands seperated by intervals in which, there are no allowed energy levels. These are known as forbidden regions.

When "α" increases, the width of the allowed energy band also increases and forbidden energy regions become narrow

5.8. ORIGIN OF ENERGY BAND (OR) BAND THEORY OF SOLIDS:

An easy way to consider how energy bands arise in a solid is to look at what happens to the energy levels of isolated atoms as they are brought closer and closer together to form the solid.

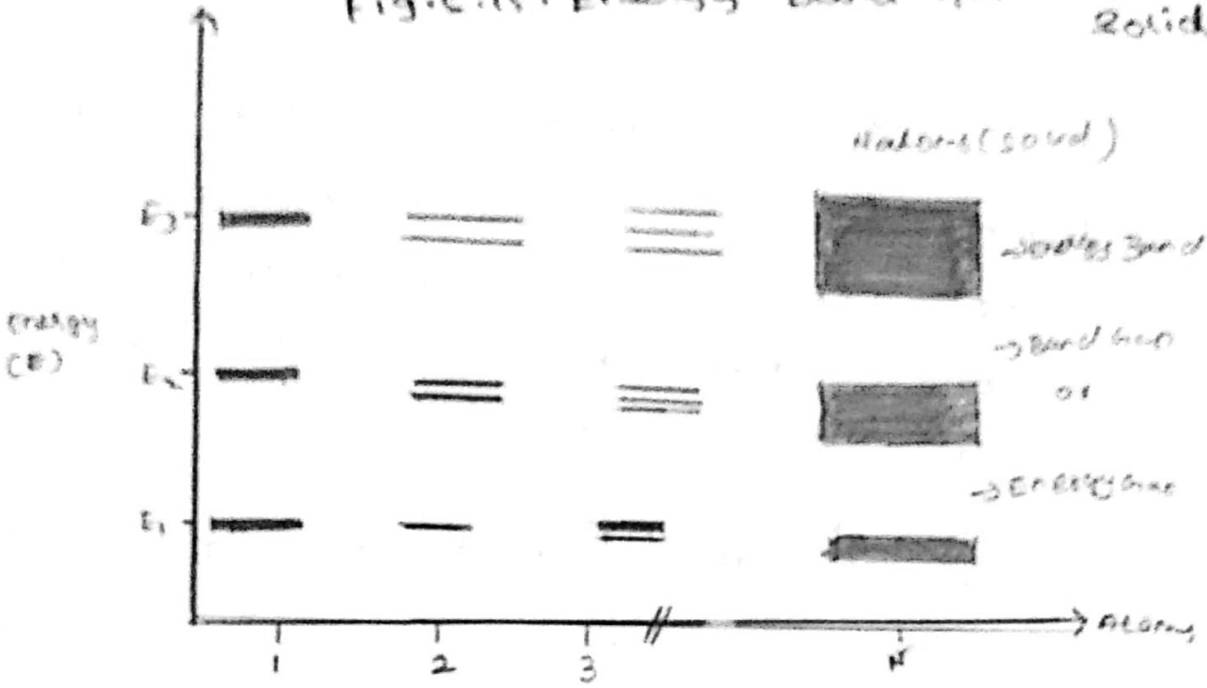
In an isolated atom, the electrons are tightly bound and have discrete, sharp energy levels.

As the atoms are brought closer together, each of the energy level for each atom changes because of the influence of the other atom.

Hence, when two identical atoms are brought closer, the outermost orbit of these atoms overlap and interacts with the wave functions of the electrons of the different atoms, then the energy levels corresponding to those wave functions split into two.

Thus, an energy level split into two levels of slightly different energies for the two-atom system and three levels of slightly different energies for the three-atom system and so on as shown in fig.5.15

Fig.5.15. Energy band formation in a solid



When a large number of atoms (of order 10^{23} or more) are brought together to form a solid, the number of orbitals becomes exceedingly large, and the difference in energy between them becomes very small. So the levels may be considered to form continuous bands of energy rather than the discrete energy levels of the atoms in isolation.

Within an energy band, energy level are so numerous and are so close to each other that they form almost continuous band.

However, some intervals of energy contain no orbitals i.e., the forbidden energy levels, no matter how many atoms are aggregated, forming band gaps.

KEY POINTS:

Energy Band

A set of closely packed energy level is called as energy band.

Width of a band

The overall range of energies from the lowest to the highest level for a band is called the width of a band

Valence band

A band which is occupied by the valence electrons is called as valence band. The valence band may be partially or completely filled up depending on the nature of the material.

Conduction band

The lowest unfilled energy band is called as conduction band. This band may be empty or partially filled. In conduction band the electrons can move freely.

Forbidden gap or band gap:

The energy gap between valence band and conduction band is called forbidden energy gap or forbidden gap or band gap.