MA3151-MATRICES AND CALCULUS

QUESTION BANK

# UNIT – I – MATRICES

## PART – A

1. The product of two eigen values of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16, find the third eigen value..
2. Write the matrix of the Q.F $2x_1^2-2x_2^2+4x_3^2+2x_1x_2-6x_1x_3+6x_2x_3$
3. If $\lambda$ is an eigen value of a square matrix A, P.T $1/\lambda$ is an eigen values of $A^{-1}$.
4. Find the eigen values of $A^2$, given $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$
5. Find the sum and product of the eigen values of $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 3 & 6 & 7 \end{pmatrix}$
6. Discuss the nature of the following Q.F $3x_1^2+3x_2^2-5x_3^2-2x_1x_2-6x_2x_3-6x_3x_1$.
7. If the sum of two eigen values and trace of 3 X 3 matrix A are equal, Find the value of $|A|$.
8. Using Cayley-Hamilton theorem, Show that $A=A^{-1}$, given $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
9. Verify Cayley – Hamilton theorem for $A = \begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$
10. Find the eigen values of $A^{-1}$ given that $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{pmatrix}$
11. Two eigen values of $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ are equal to 1 each. Find the eigen values of$A^{-1}$.
12. State Cayley – Hamilton theorem.
13. Prove that eigen values of $-3A^{-1}$ are the same as those of $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$
14. Find the constants a & b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3 and -2 as its eigen values.
15. Determine the nature of the Q.F without reducing them to canonical form $2x_1^2+x_2^2-3x_3^2+12x_1x_2-8x_2x_3-4x_3x_1$.
16. If $x = (-1 , 0, 1)^T$ is the eigen vector of the Matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$, Find the corresponding eigen value.

17. If 2 & 3 are the eigen values of a 2 X 2 matrix A. express $A^2$ in terms of A and I.

## PART – B

1. Find the eigen values and the eigen vectors of the matrix $\begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$

2. Find the eigen values and the eigen vectors of $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

3. Find the eigen values and the eigen vectors of the matrix A = $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$

4. Find the eigen values and eigen vectors of the Matrix A = $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$ and hence find the eigen values of $A^2$ , 5A and $A^{-1}$ using properties.

5. Verify Cayley – Hamilton theorem and hence find $A^{-1}$ if A = $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

6. Find the characteristic equation of A = $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and hence express the matrix $A^5$ interms of $A^2$, A and I.

7. If A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ find $A^{-1}$ and $A^3$ using Cayley – Hamilton theorem.

8. If A = $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ find $A^{-1}$ and $A^4$ using Cayley – Hamilton theorem.

9. Using Cayley-Hamilton theorem, evaluate the matrix

$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2+2A-I$ if A = $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$

10. If A = $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ , then S.T $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$.
Using Cayley-Hamilton theorem.

11. If -1 , 1 , 4 are the eigen values of a matrix A of order 3 X 3 and $(0, 1, 1)^t$,

$(2, -1, 1)^t$, $(1, 1, -1)^t$ are corresponding eigen vectors determine the matrix A.

12. Find the matrix A, whose eigen values are 2, 3 and 6 and the eigen vectors are $\{1, 0, -1\}^T$, $\{1, 1, 1\}^T$ & $\{1, -2, 1\}^T$.
13. Diagonalise the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ through an orthogonal transformation.
14. Reduce the quadratic form below to its normal form by an orthogonal reduction

    $q = 3x_1^2+2x_2^2+3x_3^2-2x_1x_2-2x_2x_3$ .
15. Diagonalise $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ by an orthogonal transformation.
16. Reduce $6x^2+3y^2+3z^2-4xy-2yz+4xz$ into a canonical form by an orthogonal reduction and find the rank, index, signature and the nature of the quadratic form.
17. Reduce $2x_1x_2+2x_2x_3+2x_3x_1$ to canonical form by an orthogonal transformation.
18. Reduce the Q.F $x^2+5y^2+z^2+2xy+6xz+2yz$ to a canonical form through an orthogonal transformation.
19. Reduce the Q.F given below to its canonical form by an orthogonal reduction $x_1^2+2x_2^2+x_3^2-2x_1x_2+2x_2x_3$.
20. Reduce the Q.F $2x^2+5y^2+3z^2+4xy$ to canonical form by an orthogonal reduction and find the rank, index, signature and nature of the Q.F.

## Unit II Differential Calculus

1. **For what value of the constant C is the function $\boldsymbol{f}$ continuous on $(-\infty, \infty)$,** $\boldsymbol{f(x) = \begin{cases} cx^2 + 2x; & x < 2 \\ x^3 - cx; & x \geq 2 \end{cases}}$.
2. **Find the values of a and b that make f continuous on $(-\infty, \infty)$. $\boldsymbol{f(x) = \begin{cases} \frac{x^3-8}{x-2} & if\ x < 2 \\ ax^2 - bx + 3 & if\ 2 \leq x < 3 \\ 2x - a + b & if\ x \geq 3 \end{cases}}$**
3. **Find $\boldsymbol{\frac{dy}{dx}}$ if $\boldsymbol{y = x^2 e^{2x}(x^2 + 1)^4}$**
4. **Find $\boldsymbol{y''}$ if $\boldsymbol{x^4 + y^4 = 16}$.**
5. **Find the derivative of $\boldsymbol{f(x) = cos^{-1}\left(\frac{b+acosx}{a+bcosx}\right)}$.**
6. **Find $\boldsymbol{y'}$ for $\boldsymbol{\cos(xy) = 1 + siny}$.**
7. Find the tangent line to the equation $x^3 + y^3 = 6xy$ at the point (3, 3) and at what point the tangent line horizontal in the first quadrant.
8. Guess the value of the limit (if it exists) for the function $\lim_{n\to\infty} \frac{e^{5x}-1}{x}$ by evaluating the function at the points $x = \pm 0.5, \pm 0.1, \pm 0.01 \pm 0.001, \pm 0.0001$ (correct to 6 places).
9. **For the function $\boldsymbol{f(x) = 2 + 2x^2 - x^4}$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity.**
10. **For the function $\boldsymbol{f(x) = 2x^3 + 3x^2 - 36x}$, find the intervals of increase or decrease, local maximum and minimum values, the intervals of concavity.**
11. **Find the local maximum and minimum values of $\boldsymbol{f(x) = \sqrt{x} - \sqrt[4]{x}}$ using both first and second derivatives tests.**
12. For the function $f(x) = sin\, x + cos\, x,\ 0 \leq x \leq 2\pi$ (i) find the intervals on which it is increasing or decreasing (ii) find the local maximum and minimum values of *f* (iii) find the intervals of concavity and the inflection points.
13. Find the local maximum and local minimum of $f(x) = x^4 - 2x^2 + 3$.
14. Calculate the absolute maximum and minimum of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ in $[-2,3]$.

## Unit – III FUNCTIONS OF SEVERAL VARIABLES

1. If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, then evaluate the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$.
2. If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ find $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z}$.

3. If $z = f(x, y)$ where $x = rcos\theta, y = rsin\theta$, then prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.

4. **Find the Jacobian of $y_1, y_2, y_3$ with respect to $x_1, x_2, x_3$ if $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$.**

5. **Find the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$ of the transformation $x = rsin\theta cos\emptyset, y = rsin\theta sin\emptyset$ and $z = rcos\theta$.**

6. **A rectangular box open at the top, is to have a volume of 32cc. Find dimensions of box which requires least amount of material for its construction.**

7. **Classify the shortest and the longest, distances from the point (1,2,-1) to the sphere $x^2 + y^2 + z^2 = 24$.**

8. Find the maximum volume of the largest rectangular parallelepiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

9. Find the dimension of the rectangular box without a top of maximum capacity, whose area is 108squ.cm.

10. Find the minimum distance from the point $(1, 2, 0)$ to the cone $z^2 = x^2 + y^2$.

11. Expand $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x + 2)$ and $(y - 1)$ using Taylor's series upto third degree terms.

12. **Expand $e^x siny$ in powers of $x$ and $y$ using Taylor's series upto third degree terms.**

13. **Expand $e^x cosy$ about $\left(0, \frac{\pi}{2}\right)$ using Taylor's series upto third degree terms**

14. Find Taylor's series expansion of function of $f(x, y) = \sqrt{1 + x^2 + y^2}$ in powers of $(x - 1)$ and $y$ upto second degree terms.

15. **Obtain Taylor's series expansion of $x^3 + y^3 + xy^2$ in terms powers of $(x - 1)$ and $(y - 2)$ upto third degree terms**

16. **Find the maxima and minima of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.**

17. **Find the maxima and minima of $f(x, y) = 3x^2 - y^2 + x^3$.**

18. **Examine $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values.**

19. Find the maxima and minima of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$.

20. Find the maxima and minima of $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$.

## Unit – IV Integral Calculus.

1. **Evaluate $\int \frac{2x+5}{\sqrt{x^2-2x+10}} dx$.**

2. Evaluate $\int \frac{x}{\sqrt{x^2+x+1}} dx$

3. **Find $\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5\sqrt{9x^2-1}}$**

4. **Evaluate $\int_0^{\frac{\pi}{2}} sin^8 x\, dx$**

5. Find $\int_0^{\frac{\pi}{4}} x\, tan^2 x\, dx$.

6. **Establish a reduction formu** $\int \sin x\, dx$ **Hence, find** $\int_0^{\frac{\pi}{2}} \sin^n x\, dx.$
7. **Establish a reduction formula for** $I_n = \int \cos^n x\, dx$. **Hence, find** $\int_0^{\frac{\pi}{2}} \cos^n x\, dx.$
8. **Establish a reduction formula for** $I_n = \int \sec^n x\, dx$. **and** $I_n = \int \tan^n x\, dx.$
9. Evaluate $\int_0^{\frac{\pi}{2}} \sin^8 x\, dx$
10. **Evaluate** $\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$
11. **Evaluate** $\int \frac{(\ln x)^2}{x^2} dx$
12. Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x\, dx.$
13. **Evaluate** $\int \frac{\tan x}{\sec x + \tan x} dx$
14. **Evaluate** $\int \frac{\tan x}{\sec x - \tan x} dx.$
15.
16. **Evaluate** $\int_0^{\infty} e^{-ax} \sin bx\, dx\ (a > 0)$ **using integration by parts**
17. **Evaluate** $\int_0^{\infty} e^{-ax} \cos bx\, dx\ (a > 0)$ **using integration by parts**
18. Evaluate $\int \frac{x^2+x+1}{(x-1)^2(x-2)} dx$
19. **Evaluate** $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} dx$
20. *Evaluate* $\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)}$dx by partical fraction method
21. For what value of $p$ is $\int_0^{\infty} \frac{1}{x^p} dx$ convergent?

## UNIT – V MULTIPLE INTEGRALS

1. **Change of order of integration for the given integral** $\int_0^a \int_0^{2\sqrt{ax}} x^2 dy\, dx$ **and evaluate it.**
2. **Change of order of integration for the given integral** $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy dy\, dx$ **and evaluate it.**
3. **Change of order of integration for the given integral** $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dy\, dx$ **and evaluate it.**
4. Evaluate by change of order of integration for the given integral $\int_0^{\infty} \int_0^{y} y e^{-\frac{y^2}{x}} dy\, dx$ and evaluate it.
5. Evaluate by change of order of integration for the given integral $\int_1^3 \int_0^{\frac{6}{x}} x^2 dy\, dx$ and evaluate it
6. **Using double integral find the area bounded by** $y = x$ **and** $y = x^2$.
7. **Find the area of the ellipse** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

**8. Find by double integrat d by the curves $y^2 = 4ax$ and $x^2 = 4ay$.**

9. Evaluate $\iint xydxdy$ over the positive quadrant of the circle. $x^2 + y^2 = a^2$.

**10. Evaluate by changing to polar coordinates $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$.**

11. Express $\int_0^a \int_y^a \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}} dx\, dy$ in polar coordinates and then evaluate it

**12. Find, using a double integral , the area of the cardioid $r = a(1 + cos\theta)$.**

13. Calculate the area which is inside the cardioids $r = 2(1 + cos\theta)$ and outside the circle $r = 2$.

**14. Evaluate $\iiint xyz\, dxdydz$ over the first octant of $x^2 + y^2 + z^2 = a^2$.**

**15. Find the volume of that portion of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant.**

**16. Compute the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0, y + z = 4$.**

17. Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$.

18. Evaluate $\iiint_v dx\, dy\, dz$, where V is the finite region of space (tetrahedron) bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 3y + 4z = 12$.

**19. By changing to polar coordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$.**

**20. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) dy\, dx$ by changing into polar coordinates.**

**All the Best**